



**The Abdus Salam
International Centre for Theoretical Physics**



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Generation, Propagation and their Inversion**

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Relative location, relative moment tensor inversion.

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Relative location, relative moment tensor inversion

ICTP Course 2008 Trieste

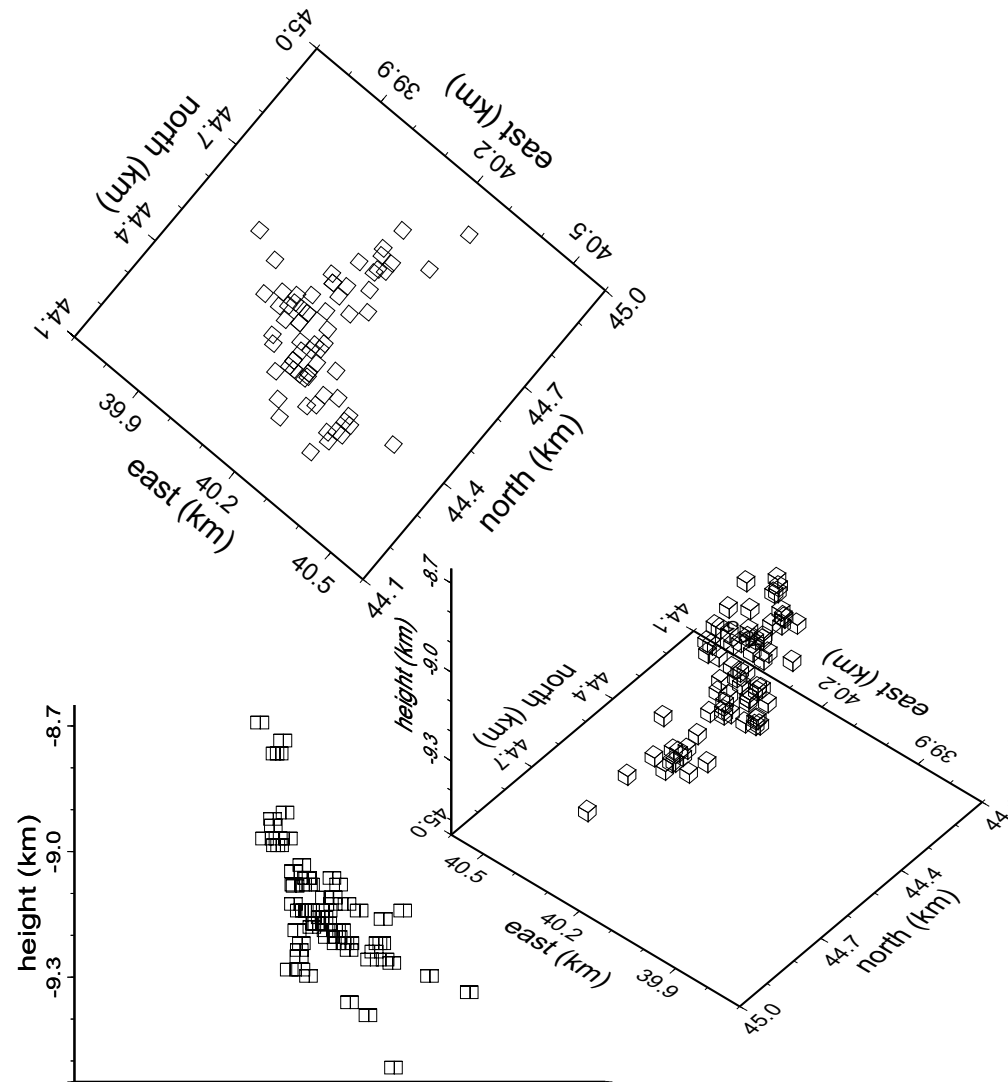
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contributions from: Th. Fischer, J. Reinhardt

Earthquake swarms & cluster



700 clustered events from the 1997 Bohemian massive earthquake swarm (see Dahm, Sileny and Horalek, 2000).

A. Precise earthquake location

The simultaneous location of clustered earthquakes and multiplets increases the location accuracy and reduces model-dependent bias.

Single source location (Geigers method)

Equation for one observation at $\vec{X} = (X, Y, Z)$:

$$t = t^{\text{obs}} = h + T(\vec{x}, \vec{X}, \text{velocity})$$

linearizing around starting model \vec{m}_0 (summ. conv.)

$$t^{\text{obs}} - t^{\text{theo}}(\vec{m}_0) \approx \frac{\partial t}{\partial m_k} (m_{0k} - m_k) = \frac{\partial t}{\partial m_k} \Delta m_k$$

with t : arrival time

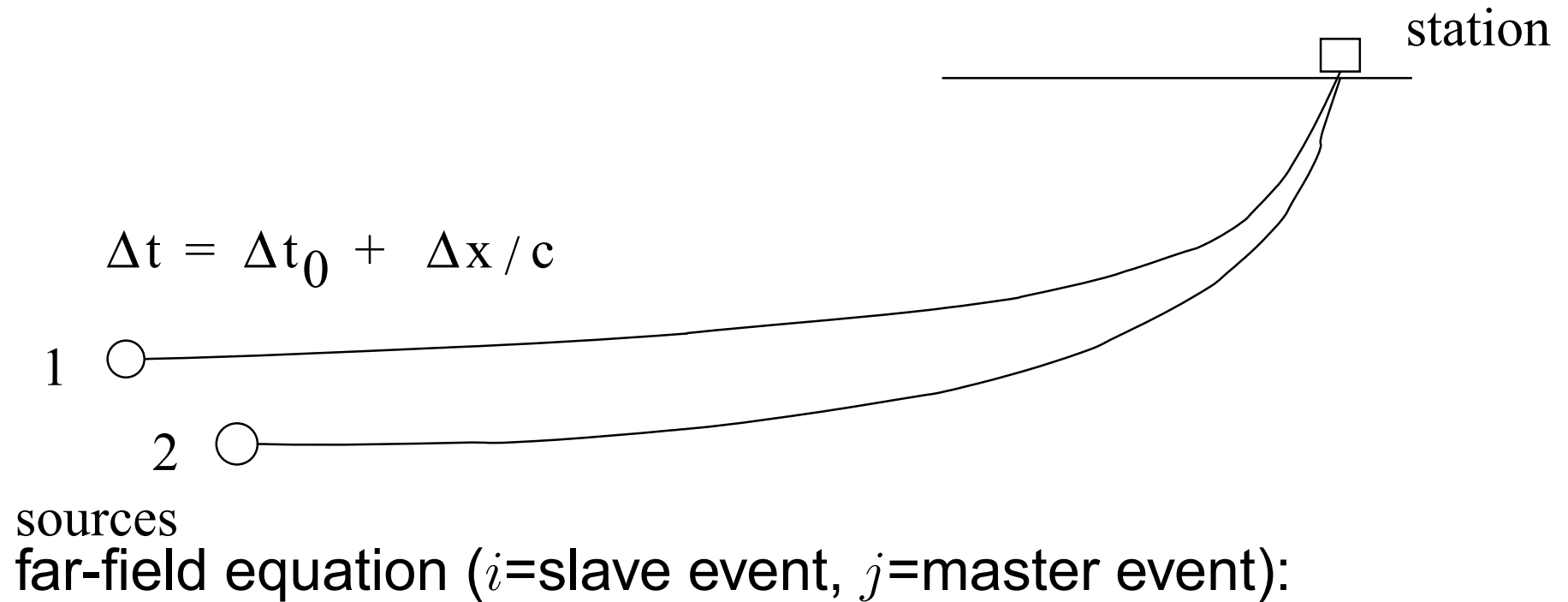
h : origin time

T : travel time

$\vec{x} = (x, y, z)$: location vector

$\vec{m} = (h, X - x, Y - y, Z - z)$: model vector

master event location



$$t_i^{\text{obs}} - t_j^{\text{obs}} = (h_i - h_j) + \frac{\vec{n}}{c}(\vec{x}_i - \vec{x}_j)$$

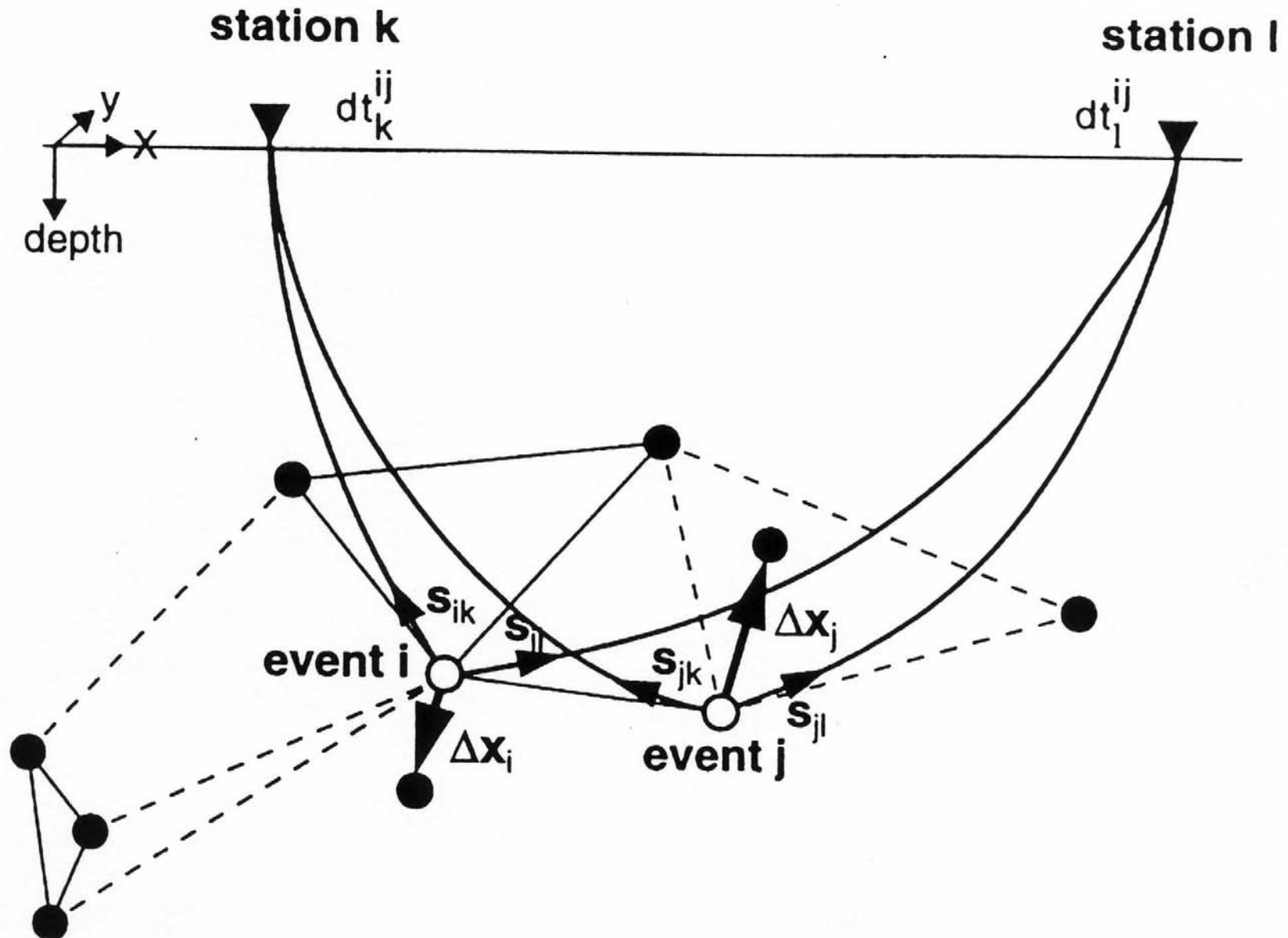
with

\vec{n} : unit vector of ray to \vec{X} c : velocity at source

master event location

- relative locations with high accuracy, but absolute location is not improved
- location of master event is not changed
- high precision time-difference measurements when waveforms are similar
- a minor influence of the unmodelled unknown structure far away from the cluster
- systematic time errors at a station have no effect on the results

Double difference method



Double difference method (linearized)

The parallel ray assumption is relaxed. For each observation at \vec{X} :

$$\begin{aligned} (t_{(i)}^{\text{obs}} - t_{(j)}^{\text{obs}}) - (t_{(i)}^{\text{theo}} - t_{(j)}^{\text{theo}}) &= \\ &= \frac{\partial t^{(i)}}{\partial m_k} \Delta m_k^{(i)} - \frac{\partial t^{(j)}}{\partial m_k} \Delta m_k^{(j)} \end{aligned}$$

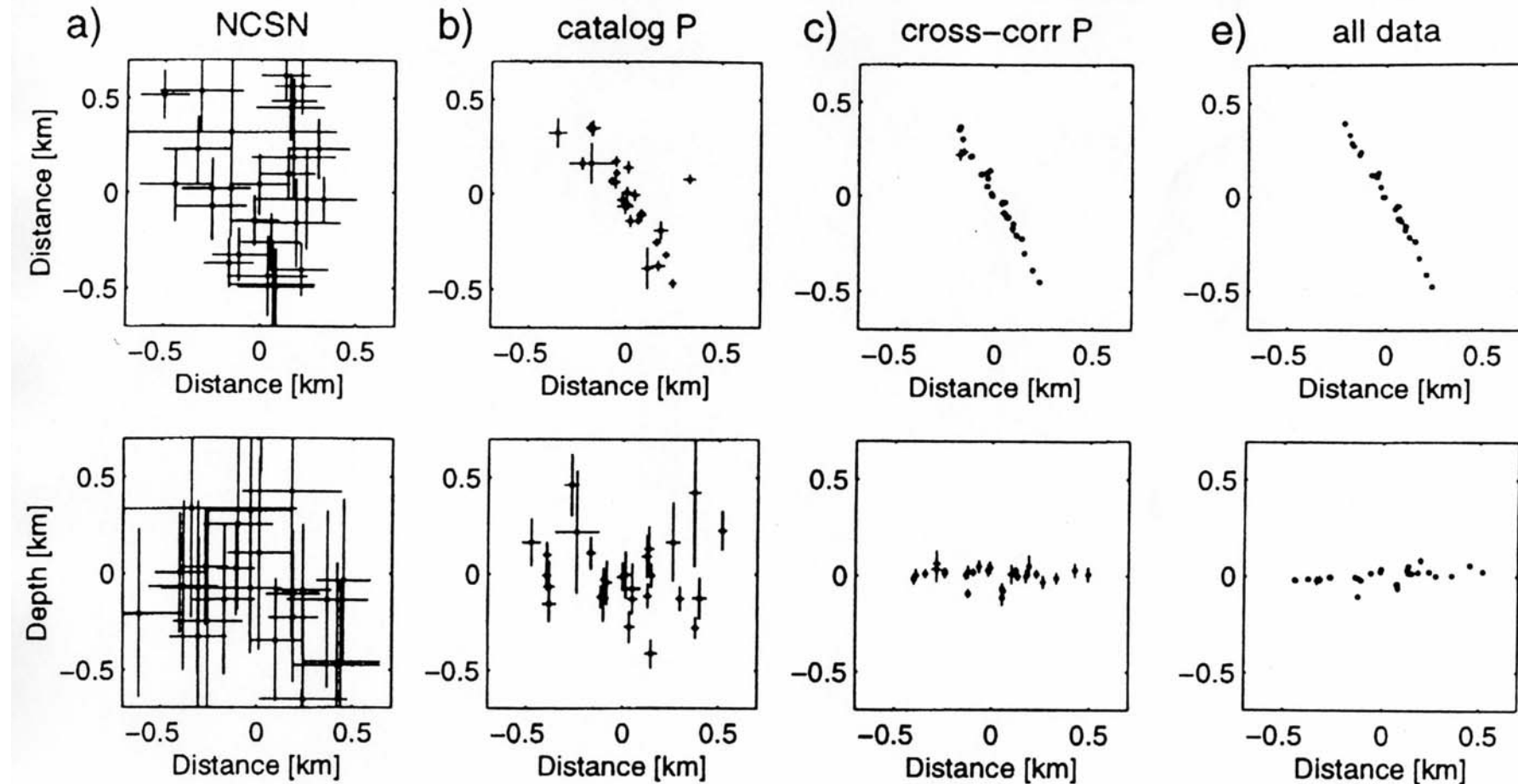
advantage : bended rays, mixed input data

disadvantage : nonlinear, iterative scheme

accurate theoretical travelttime differences

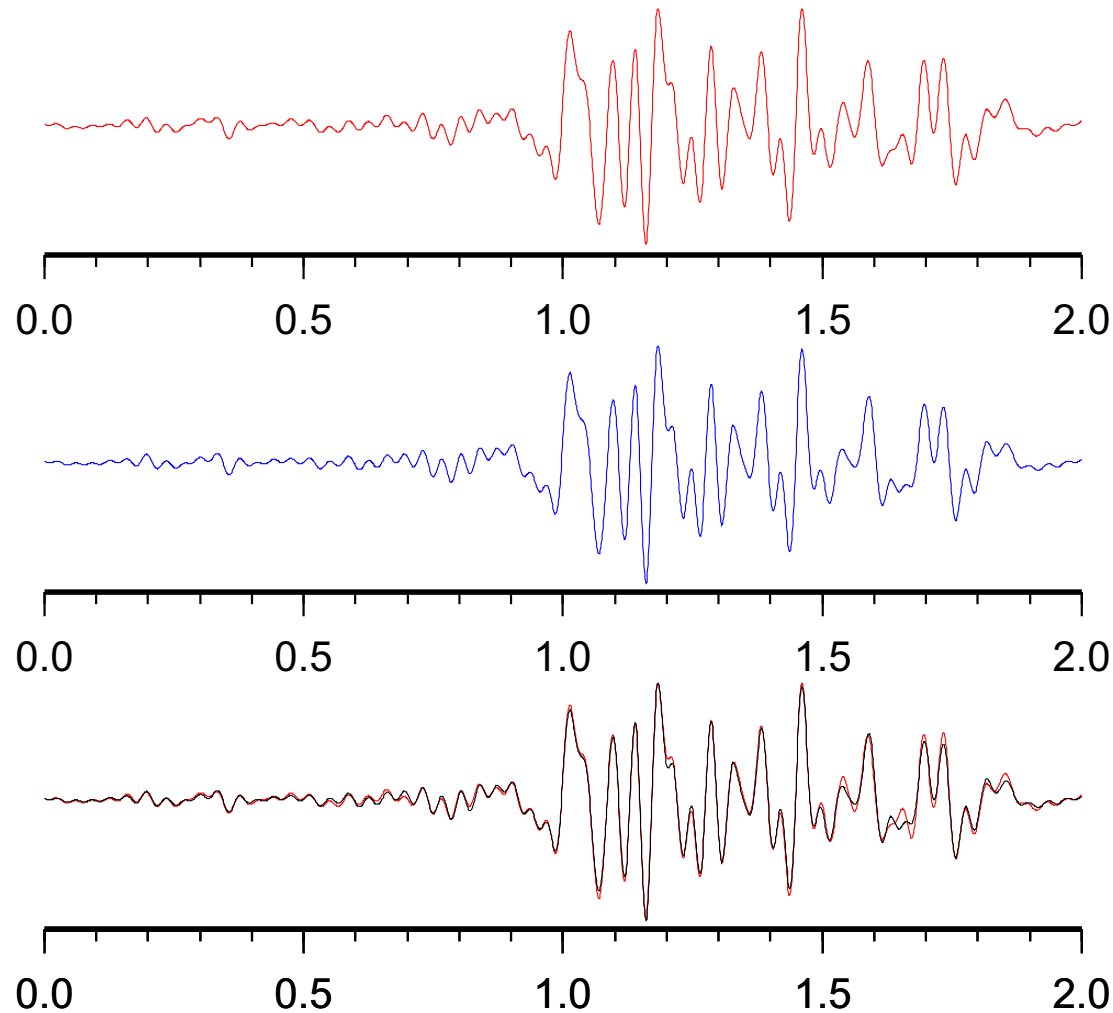
(see Waldhauser and Ellsworth, BSSA, 2000)

Double difference method



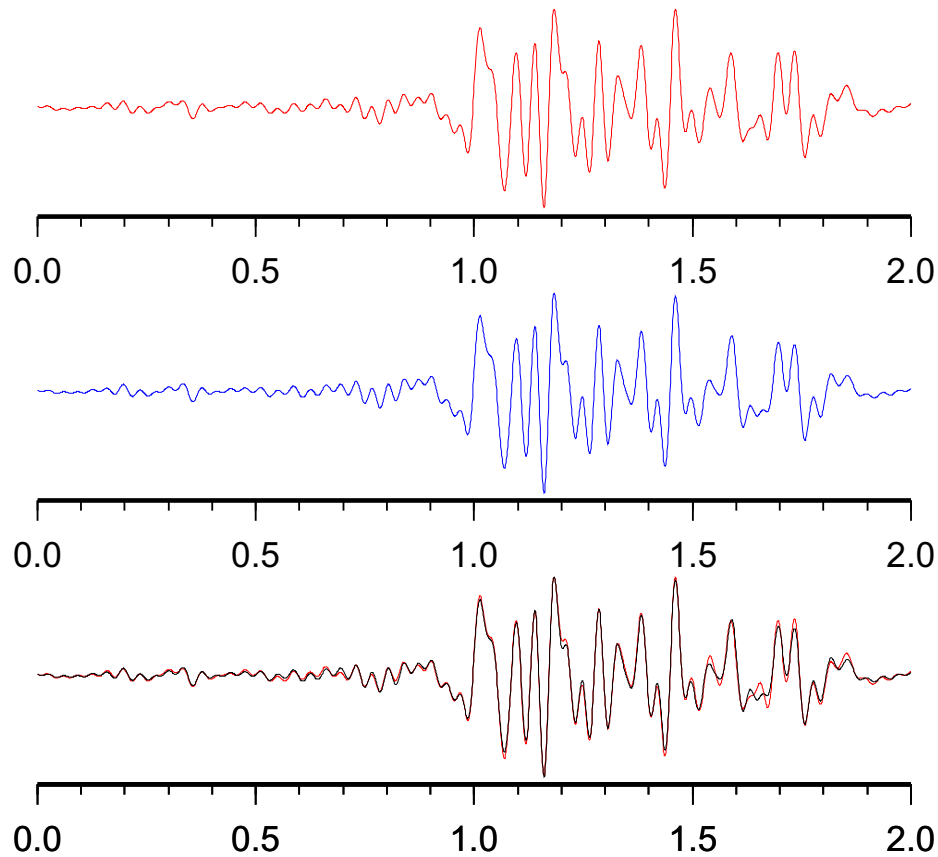
Relocation of 28 events from the Berkeley cluster
(Waldhauser and Ellsworth, BSSA, 2000)

Application to 1997 Vogtland swarm



- WEBNET waveform data available (Horálek, pers. comm.)
- Precise estimate of time shift for best coherence
 - Arrival time differences with high accuracy
 - Easy amplitude picking

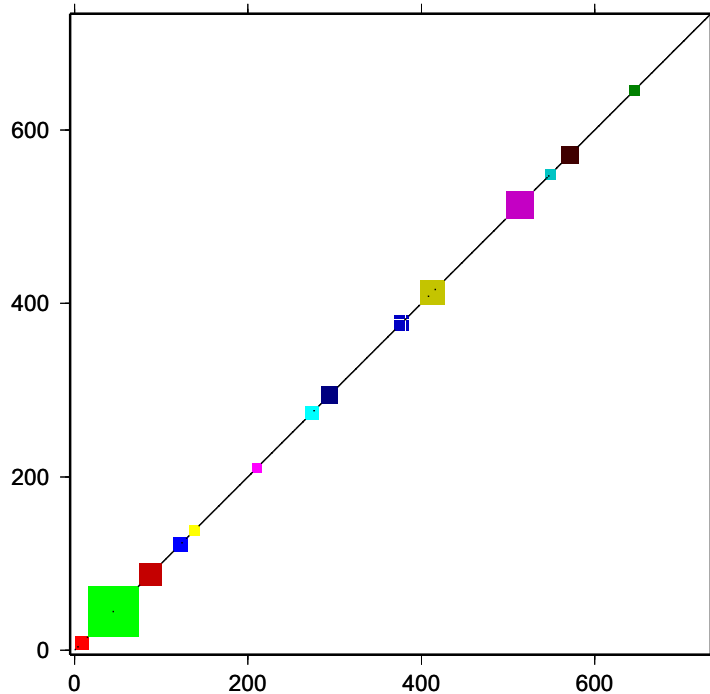
A. accurate time difference measurement



very accurate arrival time dif-

ference measurements when waveforms are similar (correlation approach)

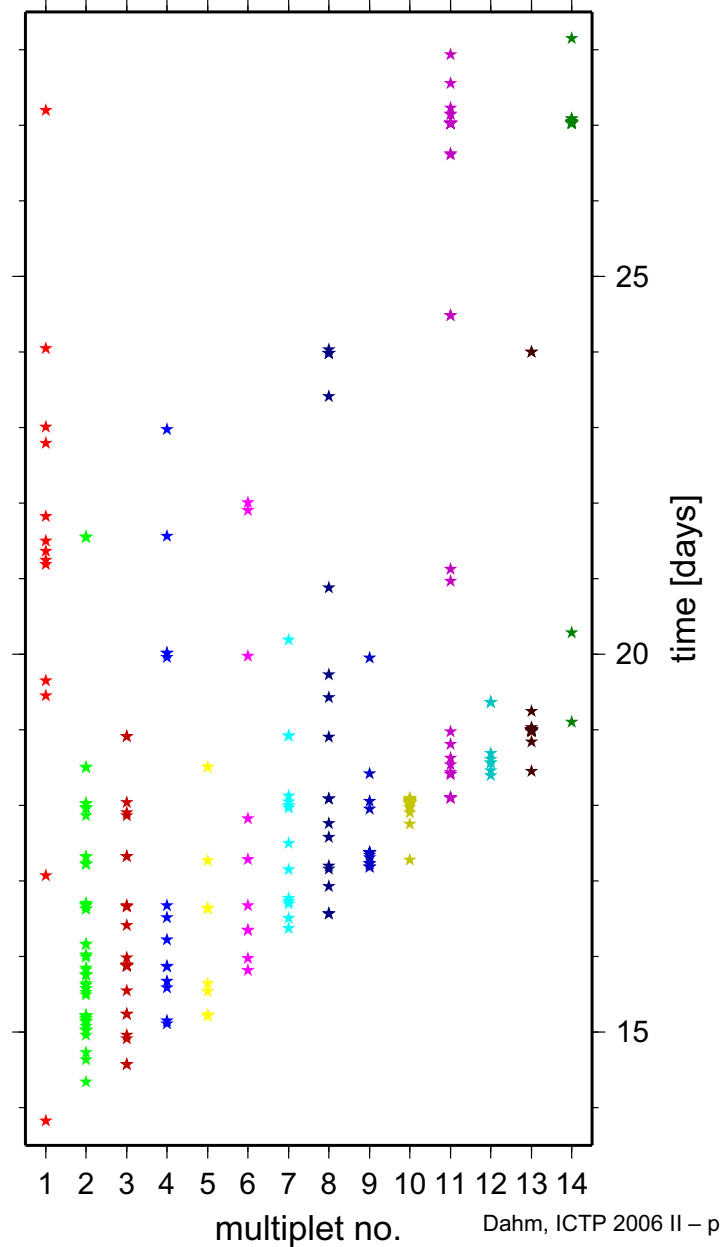
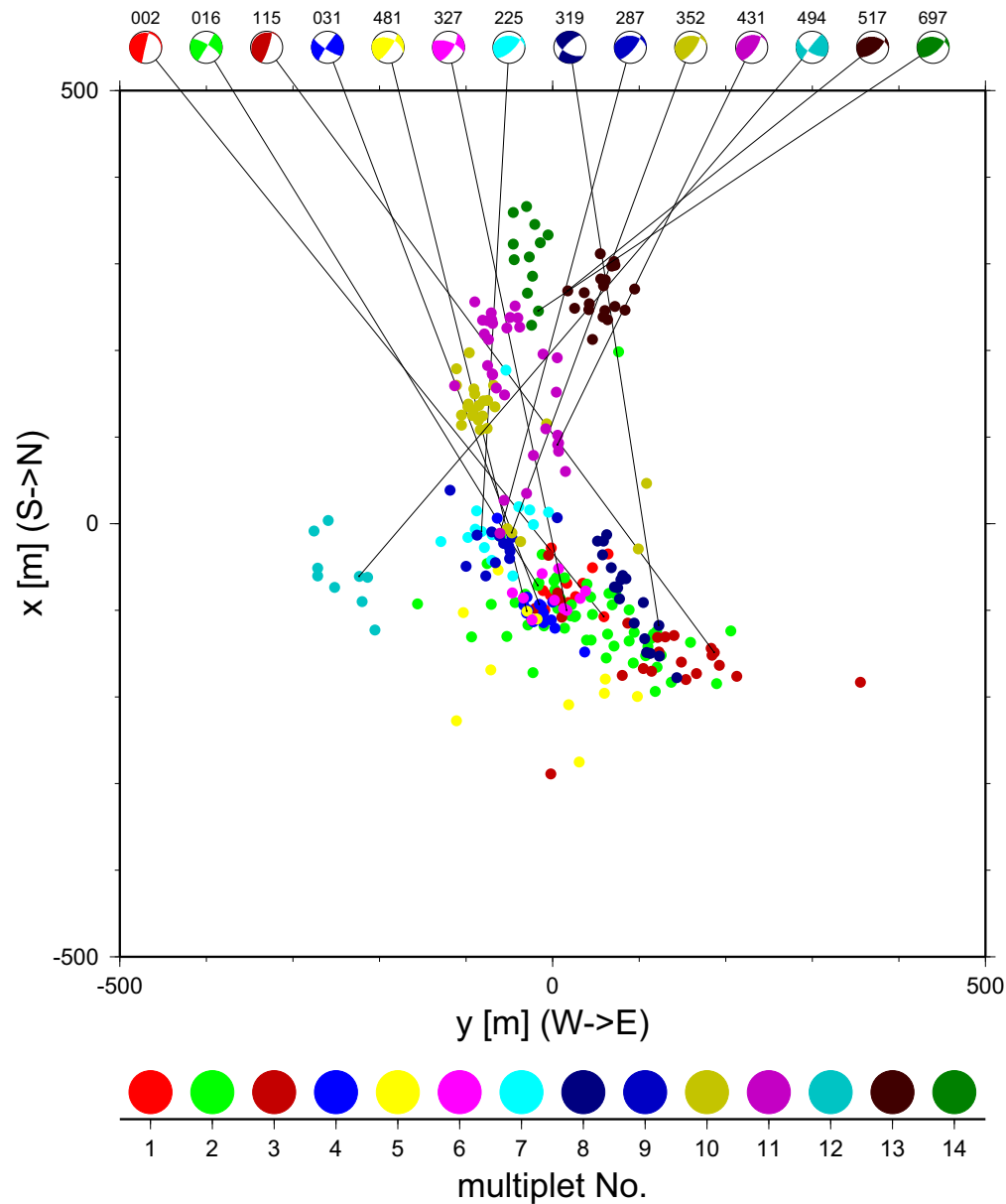
B. multiplet analysis



Coherence analysis (Maurer and Deichmann, 1995) with events of the 1997 earthquake swarm beneath Novy Kostel

- 37 % of the events (274) grouped in 14 multiplets
- relative time differences extracted
- correlation coefficients calculated

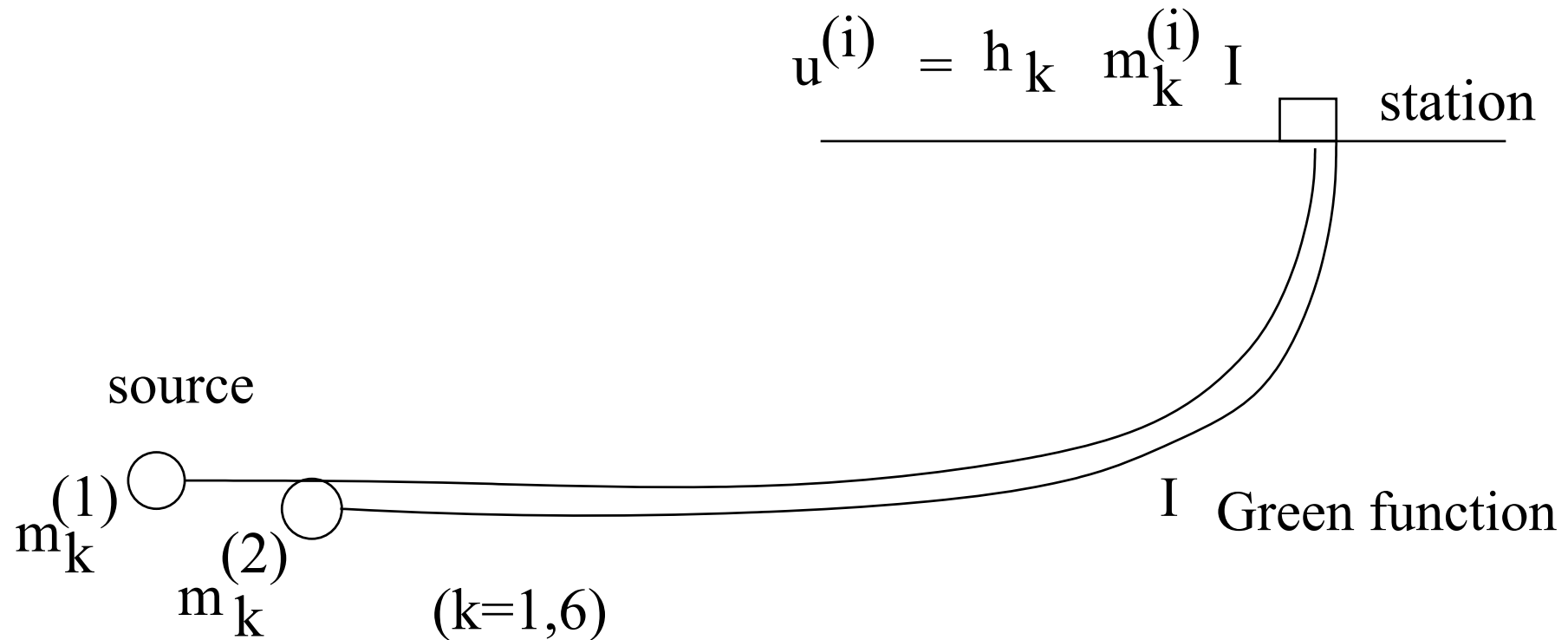
C. high precision relocation



D. Precise source mechanisms

For clustered earthquakes and for multiplets a relative amplitude inversion (relative moment tensor) is possible and allows to analyse weak events. Having accurate locations and source mechanism is useful for the identification of micro-faults and stress inversion

relative moment tensor inversion



Body waves amplitudes depend on a single scalar Green function, that can be eliminated when one master event mechanism is known

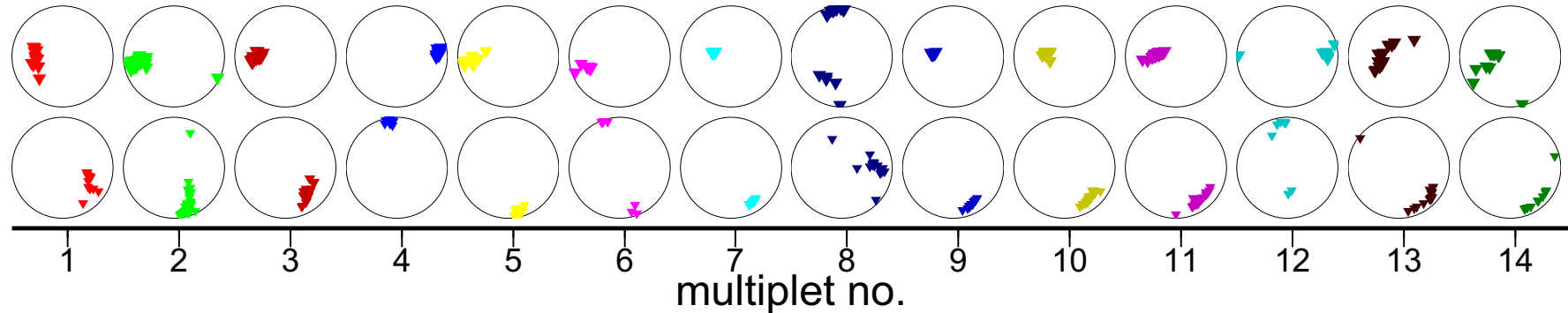
relative moment tensor inversion

- moment tensors relative to the tensor of a master event, but absolute tensors (moments) are not improved
- moment tensor of the master event is not improved
- relative amplitudes are measured with high precision when waveforms are similar
- unmodelled unknown structure far away from the cluster has a minor influence on the results
- systematic station effects have no impact on the results

assumptions

- the mechanism of the reference event is well known a priori
- events are narrow clustered and seismograms are lowpass filtered (temporal and spatial point source, all events occur within approx. one wavelength from the master event)
- used frequency range below the corner frequency of the largest studied event of the cluster
- isolated, non-interfering body-waves are used

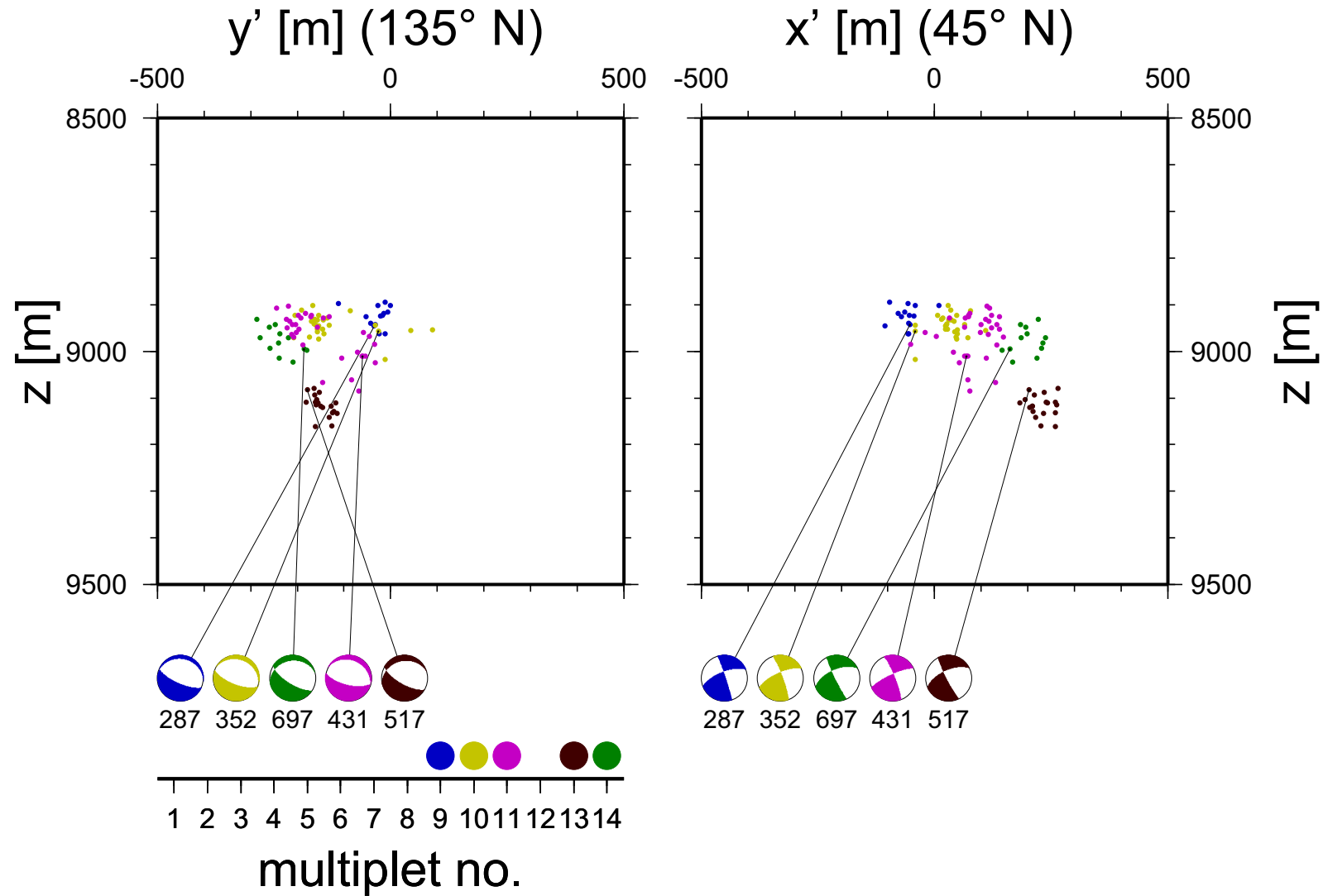
Estimation of Moment Tensors



Relative moment tensor inversion (Dahm, 1996)

- automated picking of P- and S-phase amplitudes
- 522 moment tensors inverted
- Classification with multiplets possible

Plane with en echelon faults



body-wave approach

for each ray (P, SV, SH): $\tilde{u} = h_k m_k I$,

with

$$\begin{aligned} h_1^P &= -\cos(2\varphi)\sin^2\alpha, & h_1^{SV} &= -0.5\cos(2\varphi)\sin 2\alpha, \\ h_2^P &= \sin(2\varphi)\sin^2\alpha, & h_2^{SV} &= 0.5\sin(2\varphi)\sin 2\alpha, \\ h_3^P &= \cos\varphi\sin 2\alpha, & h_3^{SV} &= \cos\varphi\cos 2\alpha, \\ h_4^P &= \sin\varphi\sin 2\alpha, & h_4^{SV} &= \sin\varphi\cos 2\alpha, \\ h_5^P &= 2 - 3\sin^2\alpha, & h_5^{SV} &= -1.5\sin 2\alpha, \\ h_6^P &= 1, & h_6^{SV} &= 0, \end{aligned}$$

and similar for SH-waves.

(e.g. Aki & Richards, 1982)

used convention

take-off angle: α

azimuth angle: φ

moment tensor: $m_1 = \frac{1}{2}(M_{22} - M_{11})$

$$m_2 = M_{12}$$

$$m_3 = M_{13}$$

$$m_4 = M_{23}$$

$$m_5 = \frac{1}{3}(0.5(M_{22} - M_{11}) - M_{33})$$

$$m_6 = \frac{1}{3}(M_{11} + M_{22} + M_{33})$$

what is different?

- the "Green function I " is **a scalar** for each body-wave mode

⇒ it can be eliminated when using two earthquakes from the same source region

$$\tilde{u}^{(1)} = h_k^{(1)} m_k^{(1)} I$$

$$I = \frac{\tilde{u}^{(2)}}{h_k^{(2)} \mathbf{m}_k^{(2)}}$$

leading to

$$\tilde{u}^{(2)} h_l^{(1)} m_l^{(1)} = \tilde{u}^{(1)} h_k^{(2)} m_k^{(2)}$$

$$[\text{data}] = [\text{vector}]^T [\text{unknown model}]$$

additional polarity constraints

$$\begin{aligned}\tilde{u}^{(2)} h_l^{(1)} m_l^{(1)} &= \tilde{u}^{(1)} h_k^{(2)} m_k^{(2)} \\ 0 &\leq \frac{\tilde{u}^{(2)}}{|\tilde{u}^{(2)}|} h_k^{(2)} m_k^{(2)}\end{aligned}$$

$$[\text{data}] = [\text{vector}]^T [\text{unknown model}]$$

$$0 \leq \text{polarity} \cdot [\text{geometry}]^T [\text{unknown model}]$$

relative method without master

From

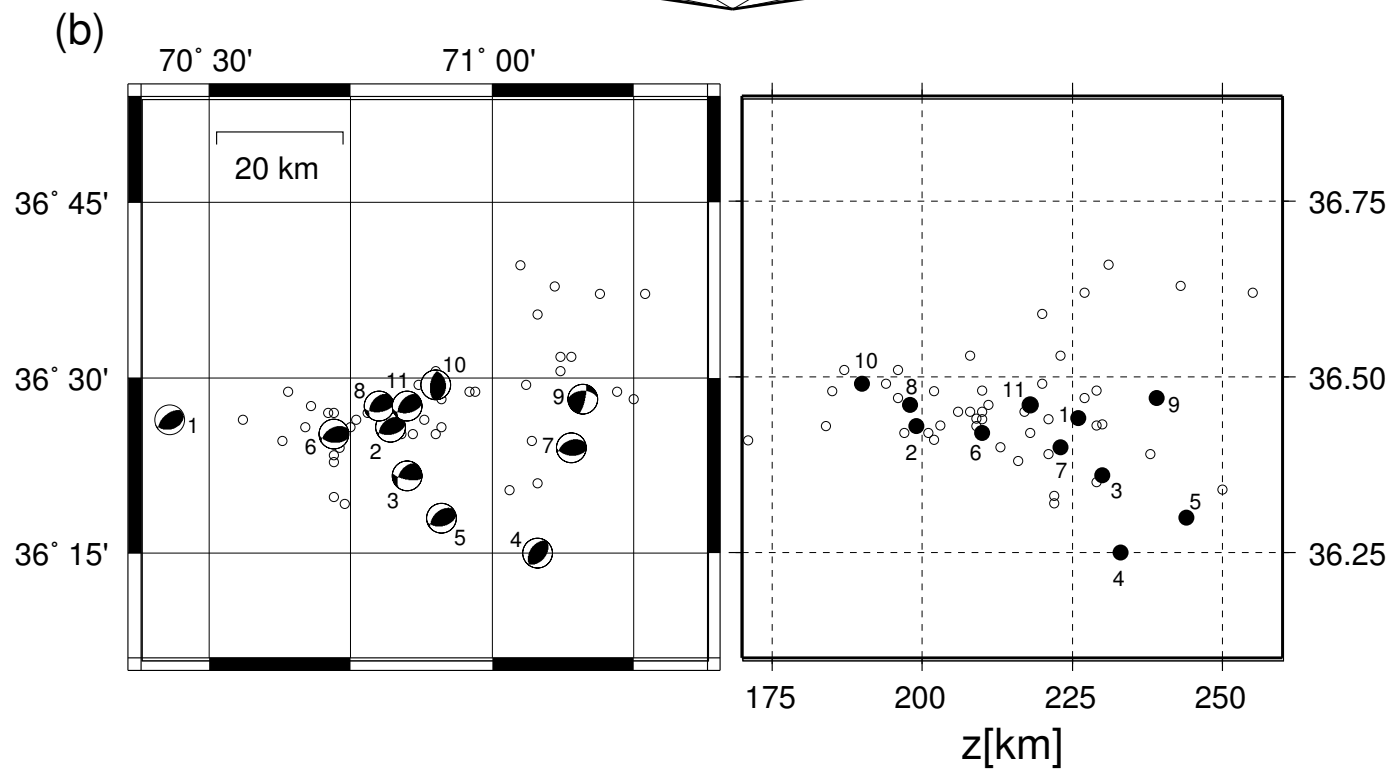
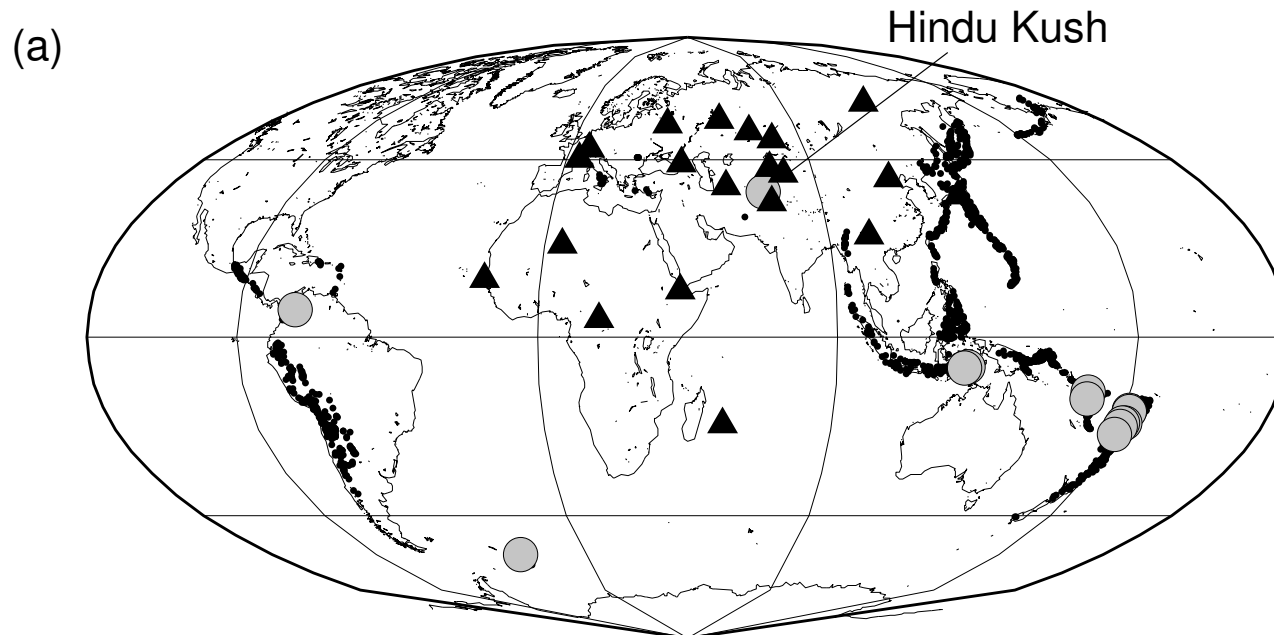
$$\tilde{u}^{(2)} h_l^{(1)} m_l^{(1)} = \tilde{u}^{(1)} h_k^{(2)} m_k^{(2)}$$

we formally write

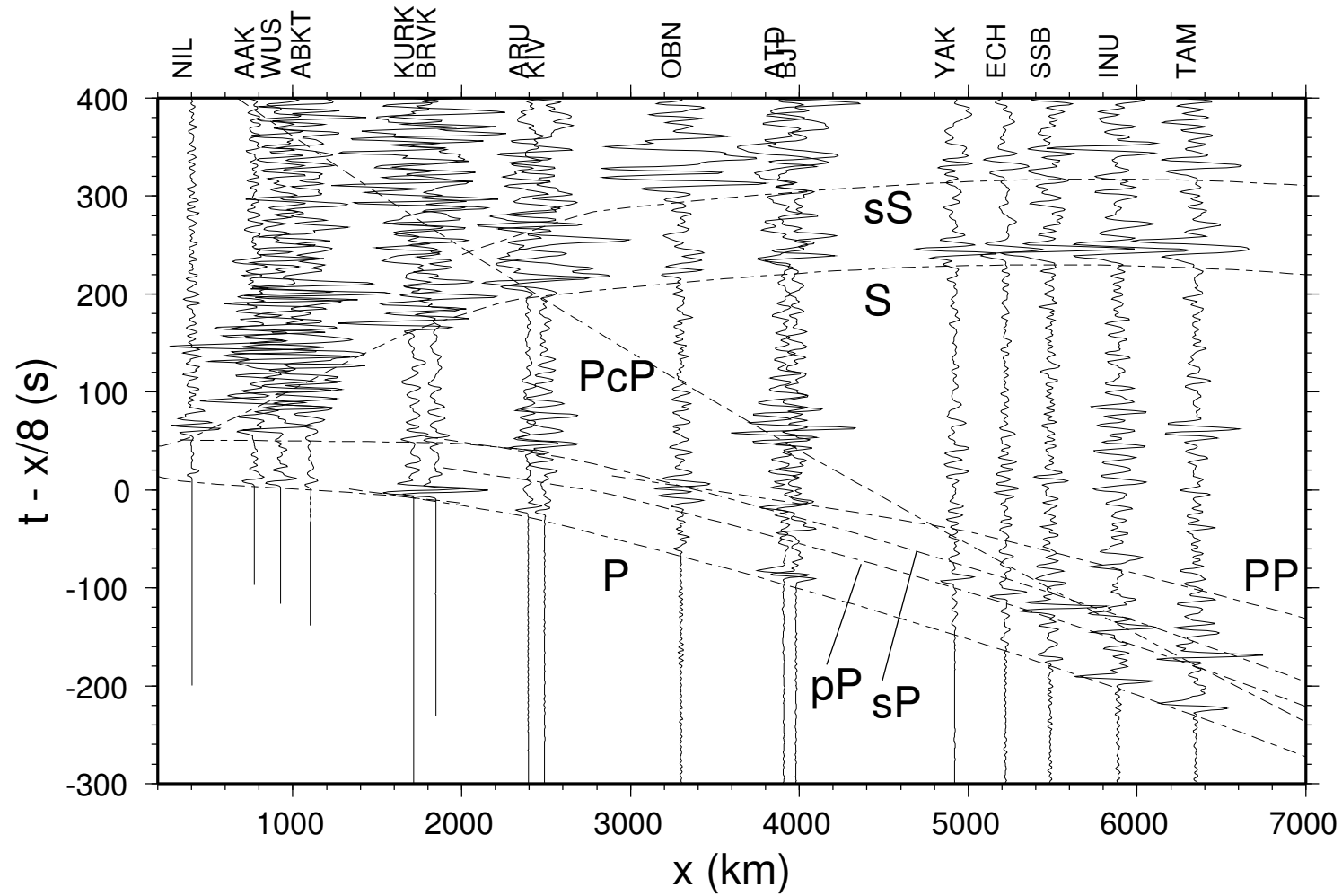
$$0 = -\tilde{u}^{(2)} h_l^{(1)} m_l^{(1)} + \tilde{u}^{(1)} h_k^{(2)} m_k^{(2)}$$

$$\begin{bmatrix} 0 \\ \text{const} \end{bmatrix} = \begin{bmatrix} -\tilde{u}^{(2)} h_1^{(1)} & \dots & -\tilde{u}^{(2)} h_6^{(1)} & \tilde{u}^{(1)} h_1^{(2)} & \dots & \tilde{u}^{(1)} h_6^{(2)} \\ 1 & \dots & 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} m_1^{(1)} \\ m_2^{(1)} \\ \vdots \\ m_6^{(1)} \\ m_1^{(2)} \\ m_2^{(2)} \\ \vdots \\ m_6^{(2)} \end{bmatrix}$$

Hindu Kush deep cluster



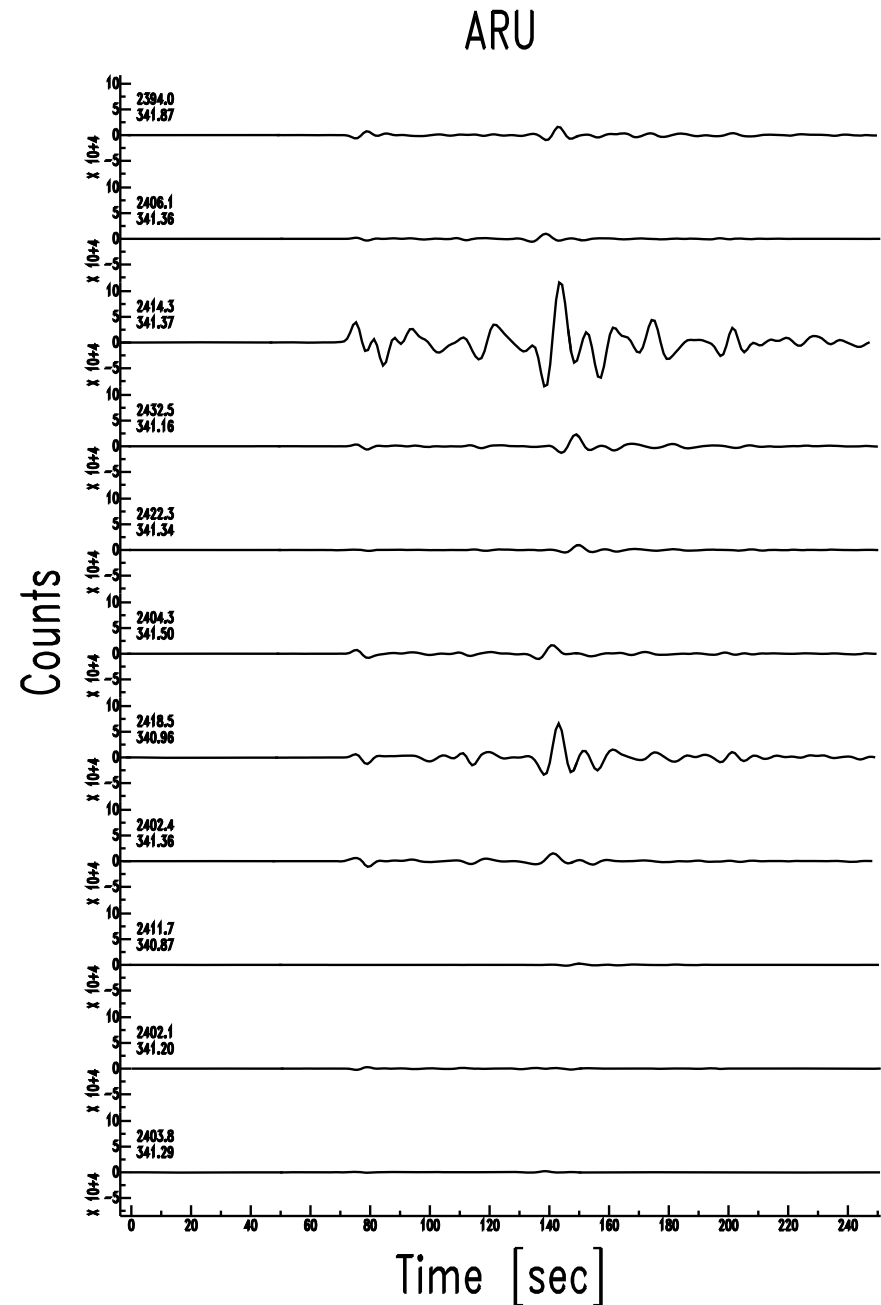
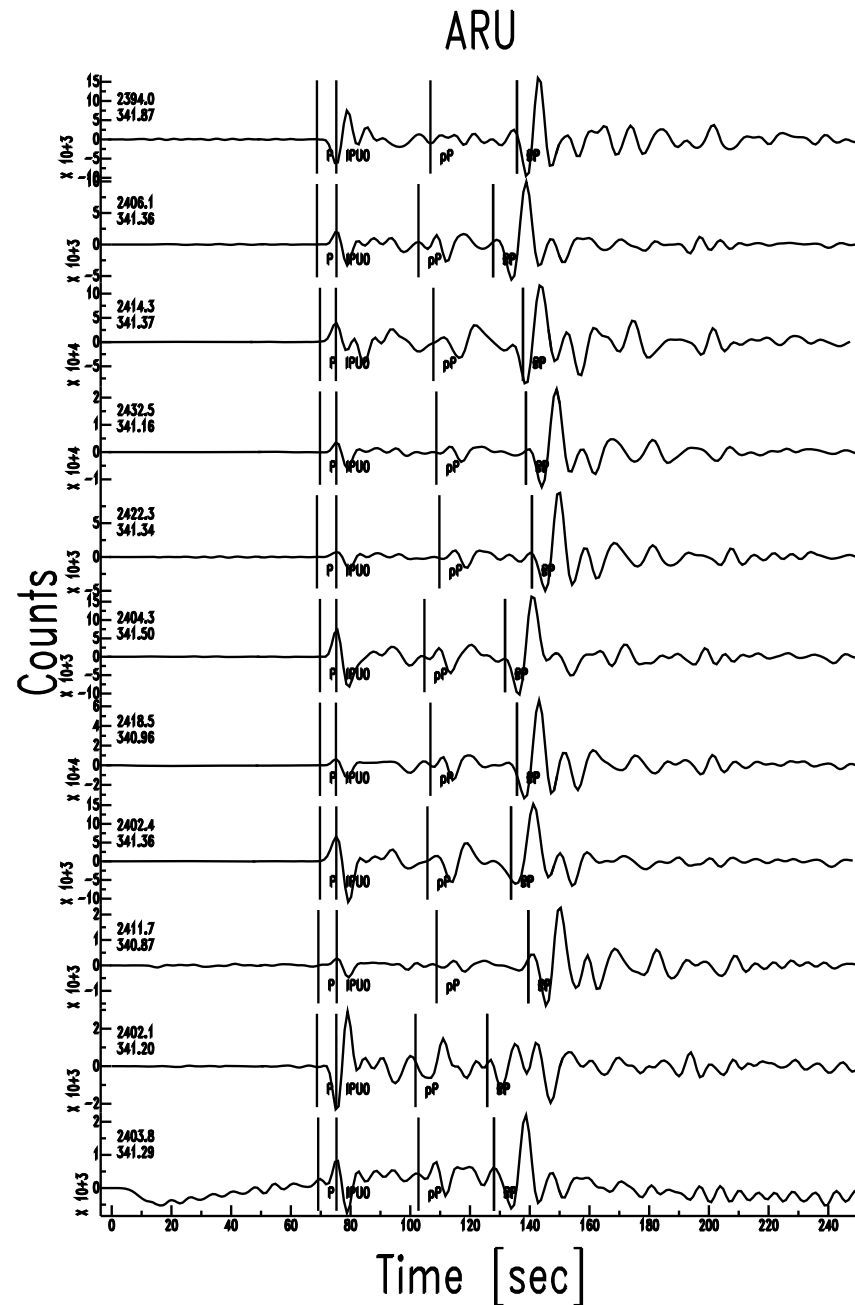
record section for event No 1



data processing

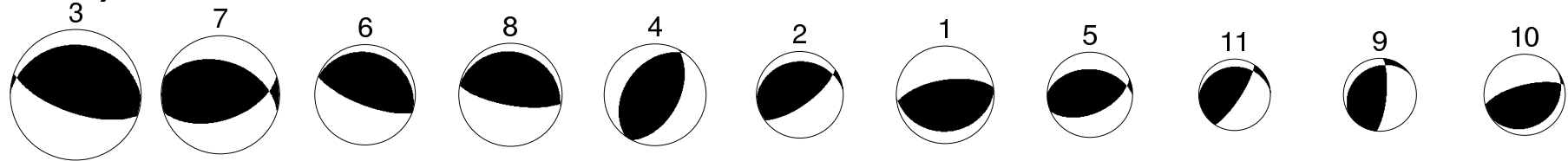
- 11 events with max. distance of 35 km.
- band-pass filter between 0.05 Hz and 0.1 Hz;
wavelength between $\approx 80 \text{ km}$ (P) and $\approx 46 \text{ km}$ (S)
- P-, S-, pP-, sP-, PP-phases selected where no interference
- peak amplitudes measured

measuring amplitudes

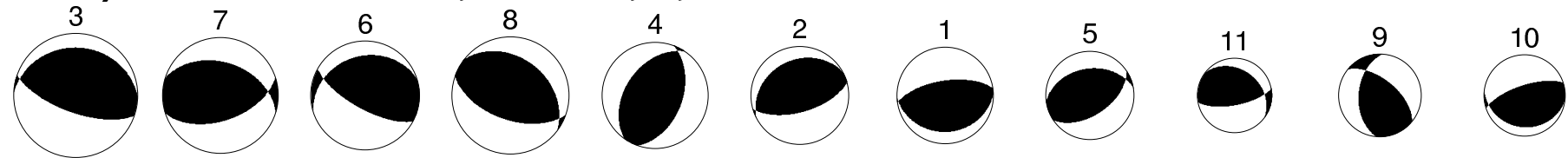


inversion results

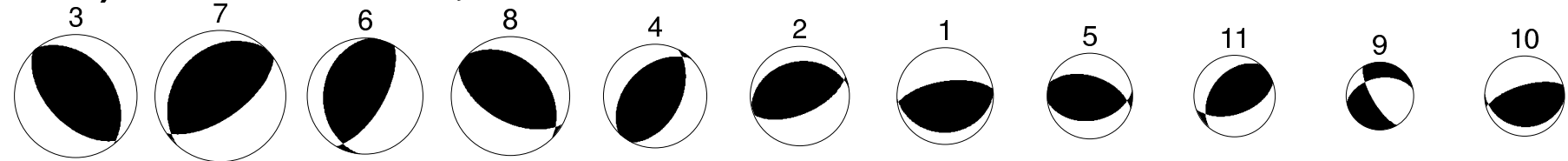
a) Harvard CMT



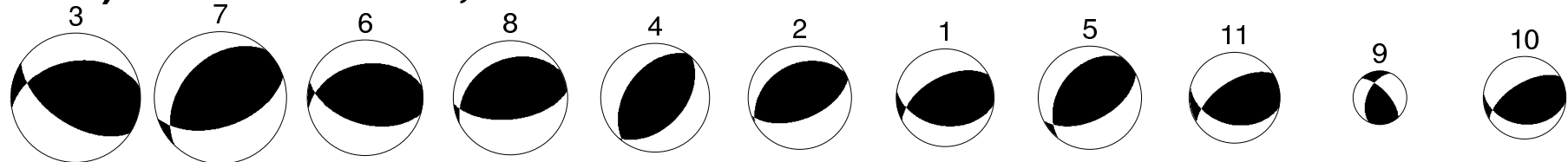
b) Relative method, event 1, 3, and 7 fixed



c) Relative method, event 1 fixed



d) Relative method, no mechanism fixed



programs and directories

directory *example*

1. *relref.exe*: relative inversion with reference mechanism
2. *relrpos.exe*: relative inversion without reference mechanism
3. *syndat4relef.f*: generate synthetic input to test geometry
4. *pltbeachball.cmd*: plot solutions in comparison to Harvard CMT

relref.f

Input: *relref.inp*

Output:

1. *relref.out*: full listing of result
2. *result.par*: short version of result for plotting
3. *relray.pola*: polarities used
4.

description of input file

HK, 11ev, 28.07.99, Pol

iev ibit inorm idyn iclust imat idev ipol

11 56 +1 0 0 0 6 0

fact(i),i=1,iev (16f5.1)

1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0

.00E+00 .0000

wt cmp azi toff / u(i,j), i=1,iev

1.00 1 23.062 102.630 AAK P 0.000 0.000

0 -7.952200000000000e+03 25.644 98.997 0.0000 1.000e+00 -1

0 -1.928310000000000e+05 23.072 99.817 0.0000 1.000e+00 -1

1 0.000000000000000e+00 21.440 100.23 4 0.0000 1.000e+00 0

0 -7.277300000000000e+04 23.992 106.039 0.0000 1.000e+00 -1

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Reference Mechanisms

1 0 -0.76 0.07 0.00 -0.63 0.12 0.76 0.4

practical

1. reproduce results using relref.exe and relrpos.exe
2. introduce deviatoric source constraint
3. introduce polarity constraint
4. use only event 1 as reference event (relref.exe)
5. change moment tensor of reference event