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Fall Colloquium on the Physics of Weather and Climate: Regional Weather Predictability and Modelling

29 September - 10 October, 2008

Dynamics of the Eta model
Part I

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The Eta Model Dynamics, Part I

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Workshop on
"Design and Use of Regional Weather Prediction Models"

The Abdus Salam International Centre for Theoretical Physics (ICTP),
Trieste, Italy

29 September - 7 October 2008

Part I:

- Approach;
- Lateral boundary conditions;
- Gravity-wave coupling/ time differencing;
- Nonhydrostatic effects;
- Advection:
- Energy transformations kinetic to potential

"Philosophy" of the Eta numerical design:

"Arakawa approach"

Attention focused
on the physical properties
of the finite difference analog
of the continuous equations

- Formal, Taylor series type accuracy: not emphasized;
- Help not expected from merely increase in resolution

"Physical properties . . . " ?

Properties (e.g., kinetic energy, enstrophy) defined using grid point values as model grid box averages /

as opposed to their being values of continuous and differentiable functions at grid points

(Note "physics": done on grid boxes ! !)

Arakawa, at early times:

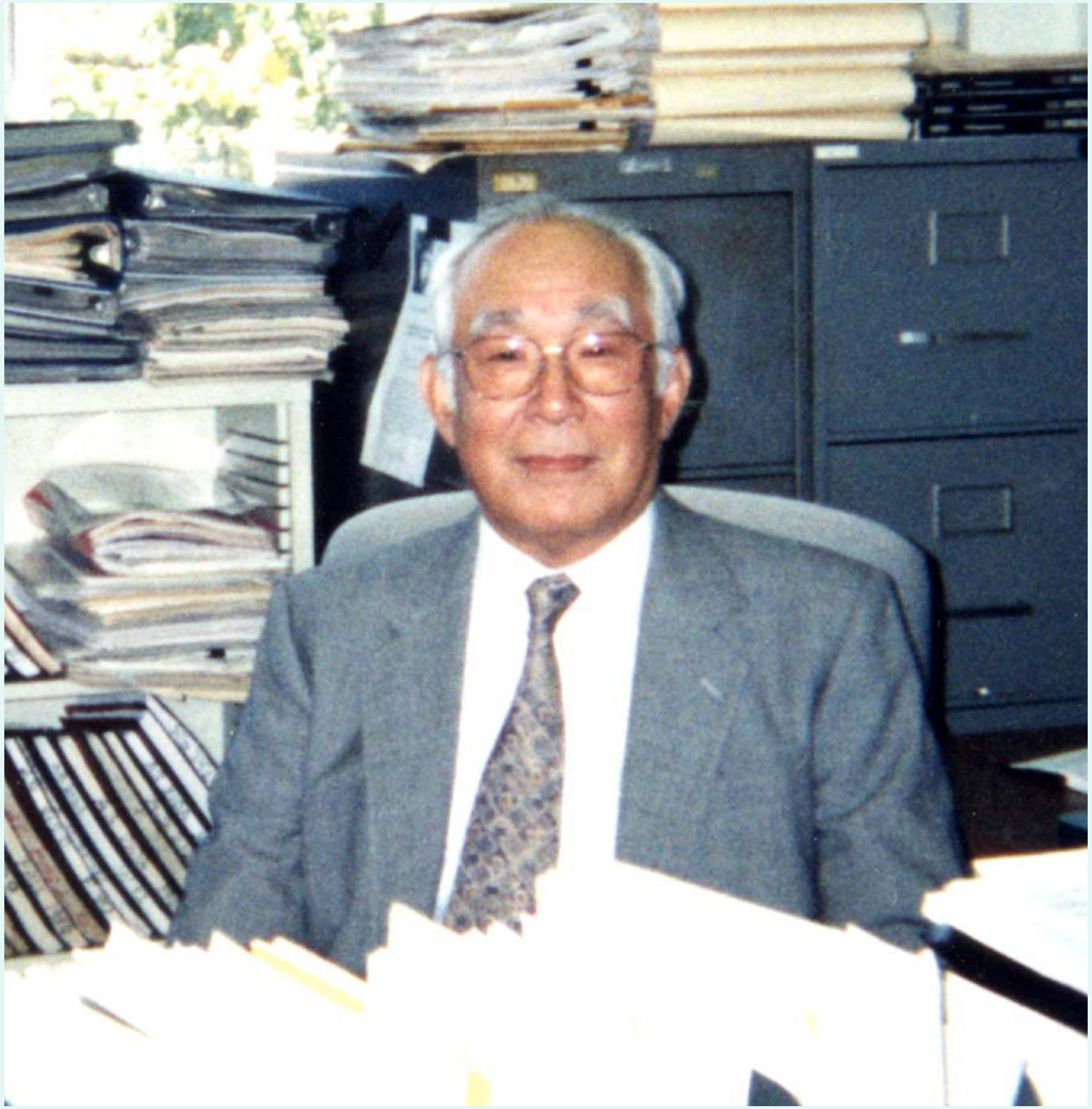
- Conservation of energy and enstrophy;
- Avoidance of computational modes;
- Dispersion and phase speed;
- . . .

Akio Arakawa:

Design schemes so as to emulate as much as possible physically important features of the continuous system !

Understand/ solve issues by looking at schemes for the minimal set of terms that describe the problem

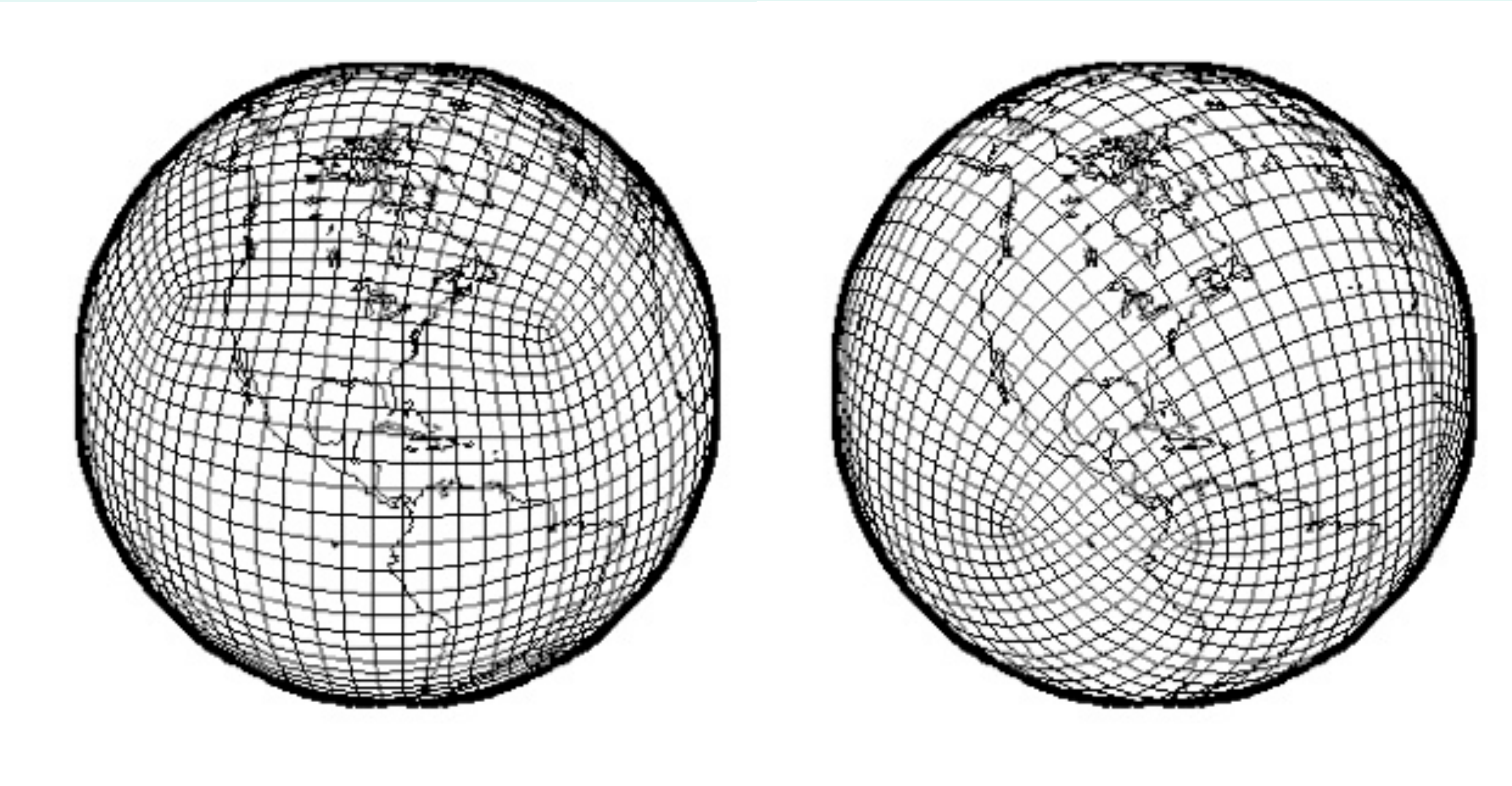
Akio Arakawa:



The Eta (as mostly used up to now) is a regional model:
Lateral boundary conditions (LBCs) are needed

There is now also a global Eta Model:

Zhang, H., and M. Rancic: 2007: A global Eta model on quasi-uniform grids. *Quart. J. Roy. Meteor. Soc.*, **133**, 517-528.



Eta dynamics: What is being done ?

- Lateral boundary conditions, defined only along the outer boundary row;
- Gravity-wave terms, on the B/E grid: forward-backward scheme that

(1) avoids the time computational mode of the leapfrog scheme, and is neutral with time steps twice leapfrog;

(2) modified to enable propagation of a height point perturbation to its nearest-neighbor height points/suppress space computational mode;

- Nonhydrostatic option;
- Horizontal advection scheme that conserves energy and C-grid enstrophy, on the B/E grid, in space differencing (Janjić 1984);
- Conservation of energy in transformations between the kinetic and potential energy, in space differencing;
- The eta vertical coordinate, ensuring hydrostatically consistent calculation of the pressure gradient ("second") term of the pressure-gradient force (PGF);
- . . . . .

• Lateral boundary condition scheme(s)

The problem:
Considered already in Charney (1962):

Linearized shallow-water eqs., one space dimension, characteristics;

"at least two conditions have to be specified at inflow points and one condition at outflow".

Sundström (1973);

Davies (1976): "boundary relaxation scheme"

Almost all LA models:

Davies ("relaxation LBCs"):

Outside row: specify all variables

Row 1 grid line inside: specify, e.g.,

$0.875 * Y_{DM} + 0.125 * Y_{LAM}$

Row 2 grid lines inside:

$0.750 * Y_{DM} + 0.250 * Y_{LAM}$

. . .

• Lateral boundary condition scheme(s)

The problem:
Considered already in Charney (1962):

Linearized shallow-water eqs., one space dimension, characteristics;

"at least two conditions have to be specified at inflow points and one condition at outflow".

Sundström (1973);

Davies (1976): "boundary relaxation scheme"

Res. Activities ..., 1999:

A TEST OF THE ETA LATERAL

Thomas L. Black, Geoffrey
U.S. National Centers for Environme

Over the years considerable degree of concern has been expressed by various investigators regarding the non well-posedness of the one-way boundary conditions of hydrostatic limited-area models. To aggravate the feelings, it is perhaps universally considered that "A common and essential ingredient of limited-area strategies is the introduction of an adjustment region immediately adjacent to the lateral boundaries, where one or both of the techniques of blending and diffusion, either explicit or implicit, are applied" (Côté et al. 1998). As a summary, Côté et al. cite as many as ten papers stating that they "all indicate that lateral boundary condition error can, depending upon the meteorological situation, importantly contribute to the total error." This assessment seems to have played a crucial role in their favoring a global variable resolution as opposed to a limited-area strategy.

Warner, T. T., R. A. Peterson, and R. E. Treadon, 1997: A tutorial on lateral boundary conditions as a basic and potentially serious limitation to regional numerical weather prediction. Bull. Amer. Meteor. Soc., **78**, 2599-2617.

(Emphasis FM)

The Eta LBC scheme:

LBCs needed along a single outer bndry line of grid points

(as required by the mathematical nature of the initial-boundary value problem we are solving)

The scheme (Mesinger 1977)

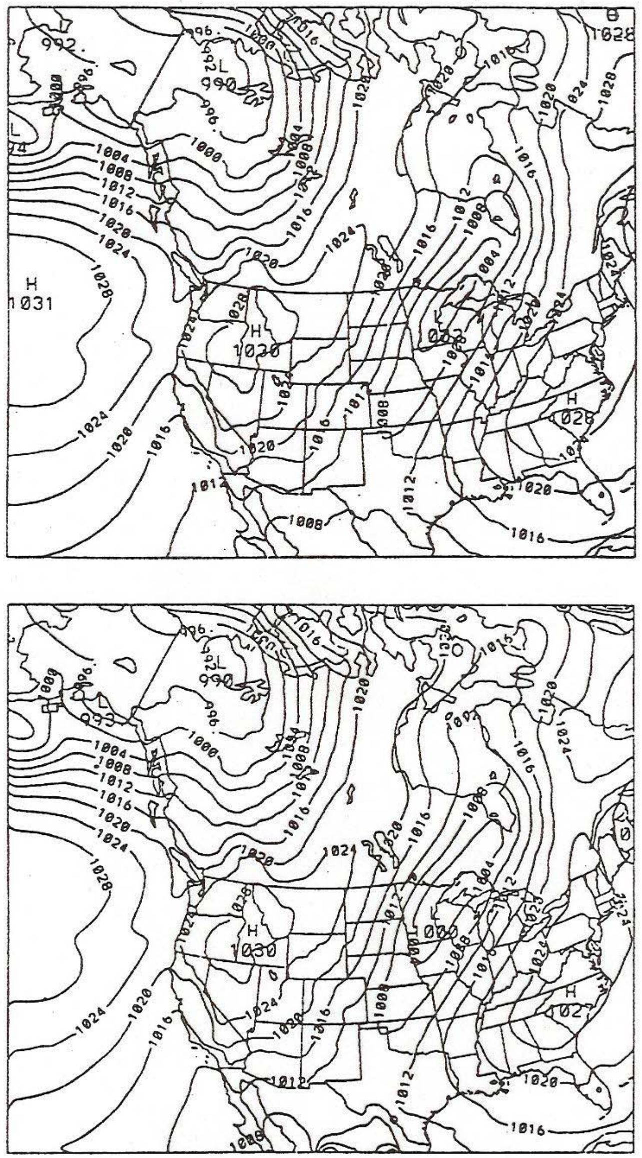
- At the inflow boundary points, all variables prescribed;
- At the outflow boundary points, tangential velocity extrapolated from the inside (characteristics!);
- The row of grid points next to the boundary row, "buffer row"; variables four-point averaged (this couples the gravity waves on two C-subgrids of the E-grid, will be explained as the next item)

Thus: No "boundary relaxation" !

"Control"

No space interpolation errors at the lateral boundary

Black et al., RAAOM, '99; 50th Anniv. of NWP, 2001



"Driven using the control lateral bnd. condition"

Figure 4: A section of the then operational 32-km Eta 48-h sea level pressure forecast, valid at 1200 UTC 17 October 1998, top panel; same except for a run over a smaller domain, done using the operational forecast to supply its boundary conditions, bottom panel. Boundaries of the plots shown are the outermost boundaries of the smaller domain, thus, in the bottom panel, all of the forecast domain of the nested run is shown.

“limitation”:

Near inflow boundaries, LA model cannot do better - it can only do worse - that its driver model

- Gravity-wave coupling scheme

Arakawa 1997:

Reviews of various discretization methods applied to atmospheric models include Mesinger and Arakawa (1976), GARP (1979), ECMWF (1984), WMO (1984), Arakawa (1988) and Bourke (1988) for finite-difference, finite-element and spectral methods and Staniforth and Côté (1991) for the semi-Lagrangian method.

7.2 Horizontal computational mode and distortion of dispersion relations

Among problems in discretizing the basic governing equations, computational modes and computational distortion of the dispersion relations in a discrete system require special attention in data assimilation. Here a computational mode refers to a mode in the solution of discrete equations that has no counterpart in the solution of the original continuous equations. The concept of the order of accuracy, therefore, which is based on the Taylor expansion of the residual when the solution of the continuous system is substituted into the discrete system, is not relevant for the existence or non-existence of a computational mode.

geostrophic ajustm.:

"the green book"

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv,$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - fu,$$

$$\frac{\partial h}{\partial t} = -H\,\nabla\cdot\mathbf{v}$$

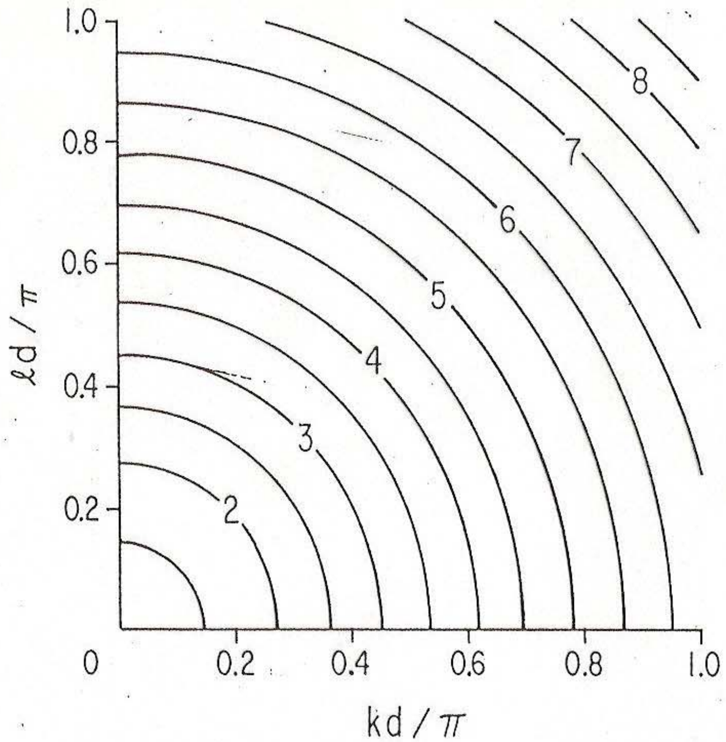


FIG. 9. Contours of the (nondimensional) frequency as a function of the (nondimensional) horizontal wave numbers for the differential shallow water equation for $\lambda/d = 2$, presented for comparison with Fig. 8.

$$\lambda \equiv \sqrt{gH}/f$$

Arakawa, dynamics:
- geostrophic adjustment
- simulation of slow, quasi-geostrophic motions

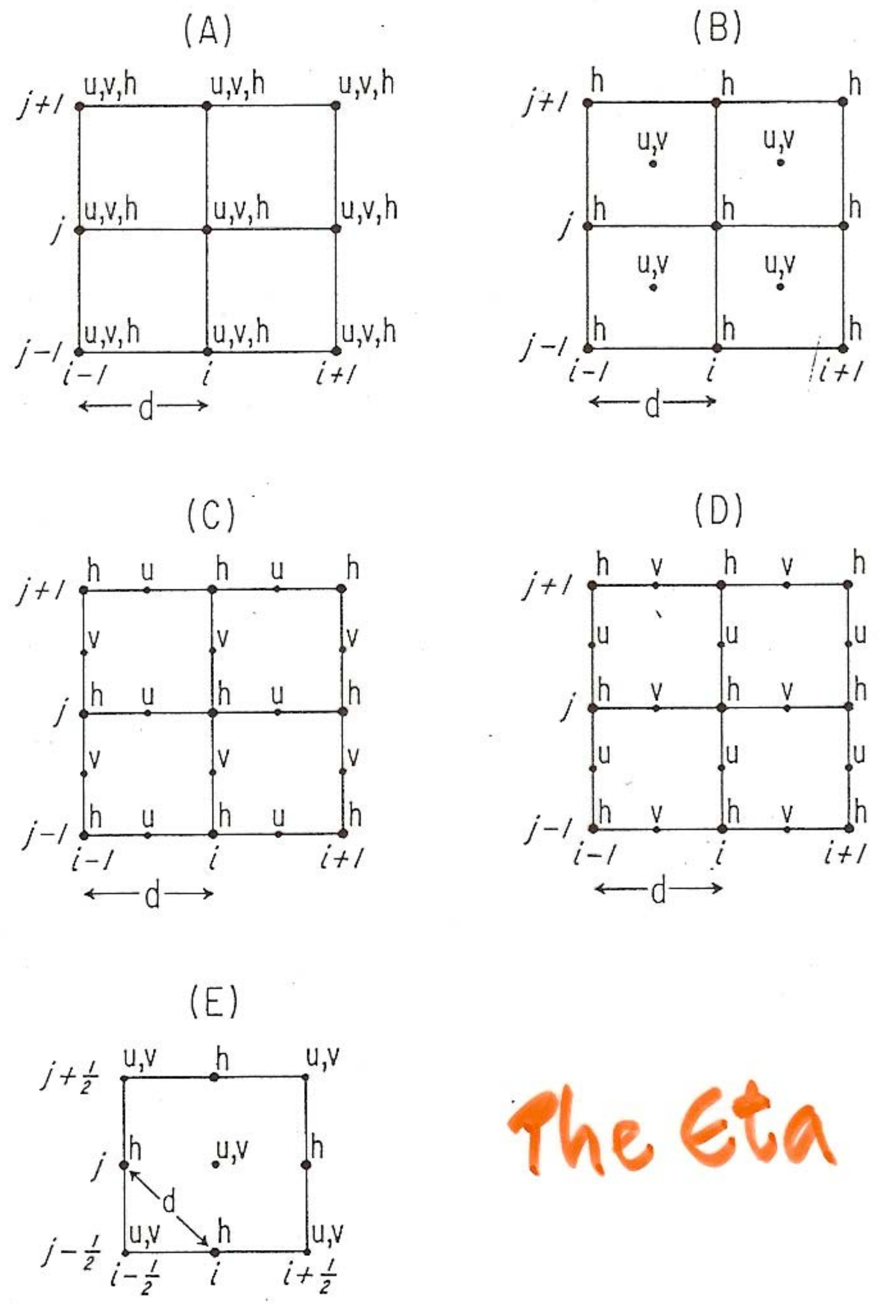
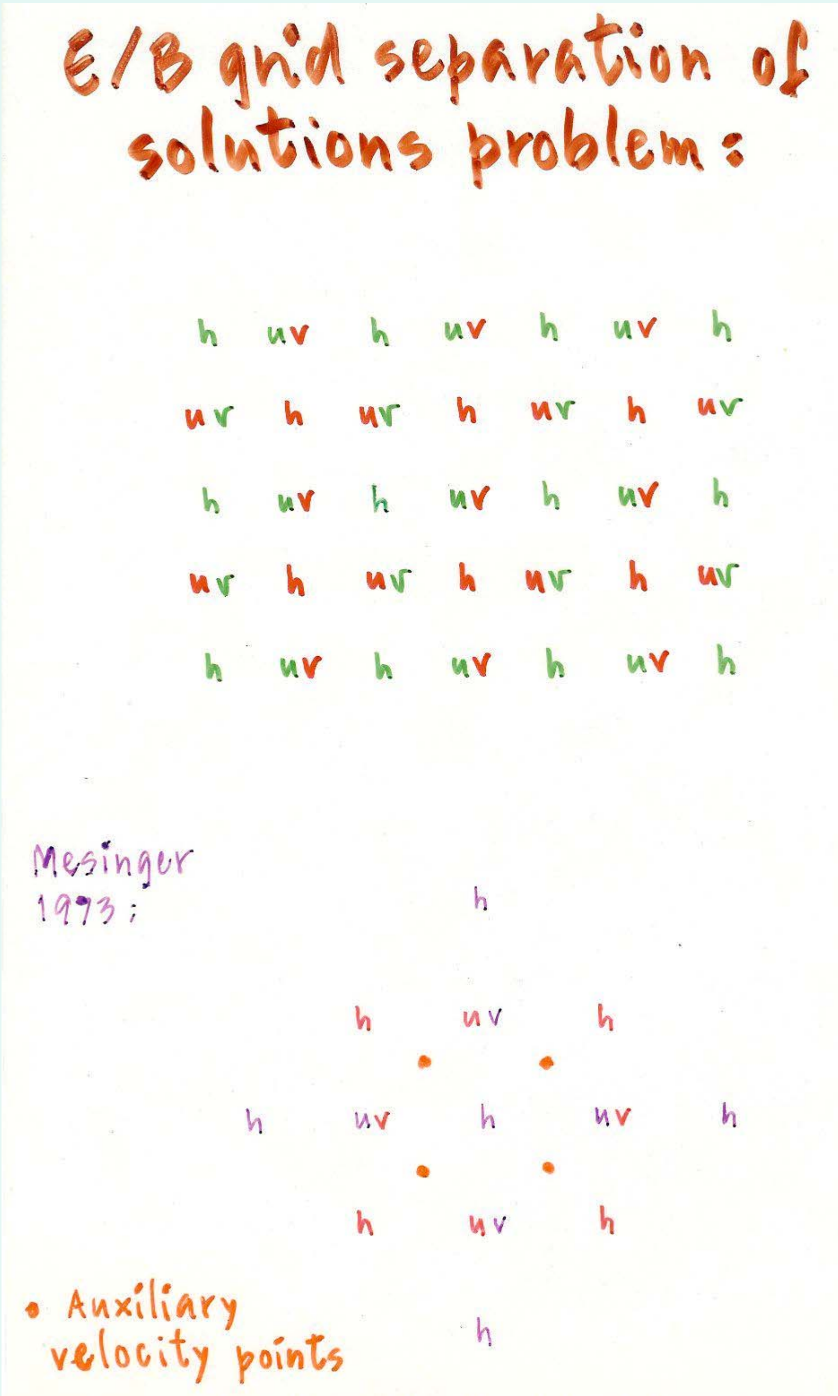


FIG. 3. Spatial distributions of the dependent variables on a square grid.

Note:

E grid is same as B, but rotated 45°. Thus, often: E/B, or B/E



(Two C-subgrids)

"The modification"

Pointed out (1973) that divergence equation can be used just as well; result is the same as when using the auxiliary velocity points

The method, 1973, applied to a number of time differencing schemes;

In Mesinger 1974:

applied to the "forward-backward" scheme

Linearized shallow-water equations:

The forward-backward scheme:

(Richtmyer ?)

It seems not/
the original author?

$$u^{n+1} = u^n - g\Delta t\, \delta_x h^{n+1},$$

$$v^{n+1} = v^n - g\Delta t\, \delta_x h^{n+1},$$

$$h^{n+1} = h^n - H\Delta t\,(\delta_x u + \delta_y v)^n.$$

Stable, and neutral, for time steps twice those of the leapfrog scheme;

No computational mode

Coriolis terms: trapezoidal scheme

$$u^{n+1} = \dots \quad + \tfrac{1}{2} f \Delta t\,(v^n + v^{n+1})$$

$$v^{n+1} = \dots \quad - \tfrac{1}{2} f \Delta t\,(u^n + u^{n+1})$$

Unconditionally neutral

(Fischer, MWR, 1965)

Elimination of u,v from pure gravity-wave system leads to the wave equation, (5.6):

(From Mesinger, Arakawa, 1976)

$$\frac{\partial^2 h}{\partial t^2} - gH\frac{\partial^2 h}{\partial x^2} = 0. \qquad (5.6)$$

We can perform the same elimination for each of the finite difference schemes; forward-backward and leapfrog.

The forward-backward and space-centered approximation to (5.5) is

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + g\frac{h_{j+1}^n - h_{j-1}^n}{2\Delta x} = 0,$$

$$\frac{h_j^{n+1} - h_j^n}{\Delta t} + H\frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2\Delta x} = 0, \qquad (5.7)$$

We now substract from the second of these equations an analogous equation for time level $n-1$ instead of n, divide the resulting equation by Δt, and, finally, eliminate all u values from it using the first of Eqs. (5.7), written for space points $j+1$ and $j-1$ instead of j. We obtain

$$\frac{h_j^{n+1} - 2h_j^n + h_j^{n-1}}{(\Delta t)^2} - gH\frac{h_{j+2}^n - 2h_j^n + h_{j-2}^n}{(2\Delta x)^2} = 0. \qquad (5.8)$$

This is a finite difference analogue of the wave equation (5.6). Note that although each of the two equations (5.7) is only of the first order of accuracy in time, the wave equation analogue equivalent to (5.7) is seen to be of the second order of accuracy.

If we use a leapfrog and space-centered approximation to (5.5), and follow an elimination procedure like that used in deriving (5.8), we obtain

$$\frac{h_j^{n+1} - 2h_j^{n-1} + h_j^{n-3}}{(2\Delta t)^2} - gH\frac{h_{j+2}^{n-1} - 2h_j^{n-1} + h_{j-2}^{n-1}}{(2\Delta x)^2} = 0. \tag{5.9}$$

This also is an analogue to the wave equation (5.6) of second-order accuracy. However, in (5.8) the second time derivative was approximated using values at three consecutive time levels; in (5.9) it is approximated by values at every second time level only, that is, at time intervals $2\Delta t$. Thus, while the time step required for linear stability with the leapfrog scheme was half that with the forward-backward scheme, (5.9) shows that we can omit the variables at every second time step, and thus achieve the same computation time as using the forward-backward scheme with double the time step.

Back to "modification", gravity wave terms only:

on the lattice separation problem. If, for example, the forward–backward time scheme is used, with the momentum equation integrated forward,

$$u^{n+1} = u^n - g\Delta t \delta_x h^n, \qquad v^{n+1} = v^n - g\Delta t \delta_y h^n, \tag{2}$$

instead of

$$h^{n+1} = h^n - H\Delta t\left[(\delta_x u + \delta_y v) - g\Delta t \nabla_+^2 h\right]^n, \tag{3}$$

the method results in the continuity equation (Mesinger, 1974):

$$h^{n+1} = h^n - H\Delta t\left[(\delta_x u + \delta_y v) - g\Delta t\left(\frac{3}{4}\nabla_+^2 h + \frac{1}{4}\nabla_\times^2 h\right)\right]^n. \tag{4}$$

Single-point perturbation spreads to both *h* and *h* points !

Extension to 3D: Janjić, Contrib. Atmos. Phys., 1979

Eq. (4) (momentum eq. forward):

Following a pulse perturbation (height increase) at the initial time, at time level 1 increase in height occurs at four nearest points equal to 2/3 of the increase which occurs in four second nearest points.

This is not ideal, but is a considerable improvement over the situation with no change at the four nearest height points !

In the code: continuity eq. is integrated forward.

"Historic reasons". With this order, at time level 1 at the four second nearest points a decrease occurs, in the amount of 1/2 of the increase at the four nearest points !

Might well be worse? However:

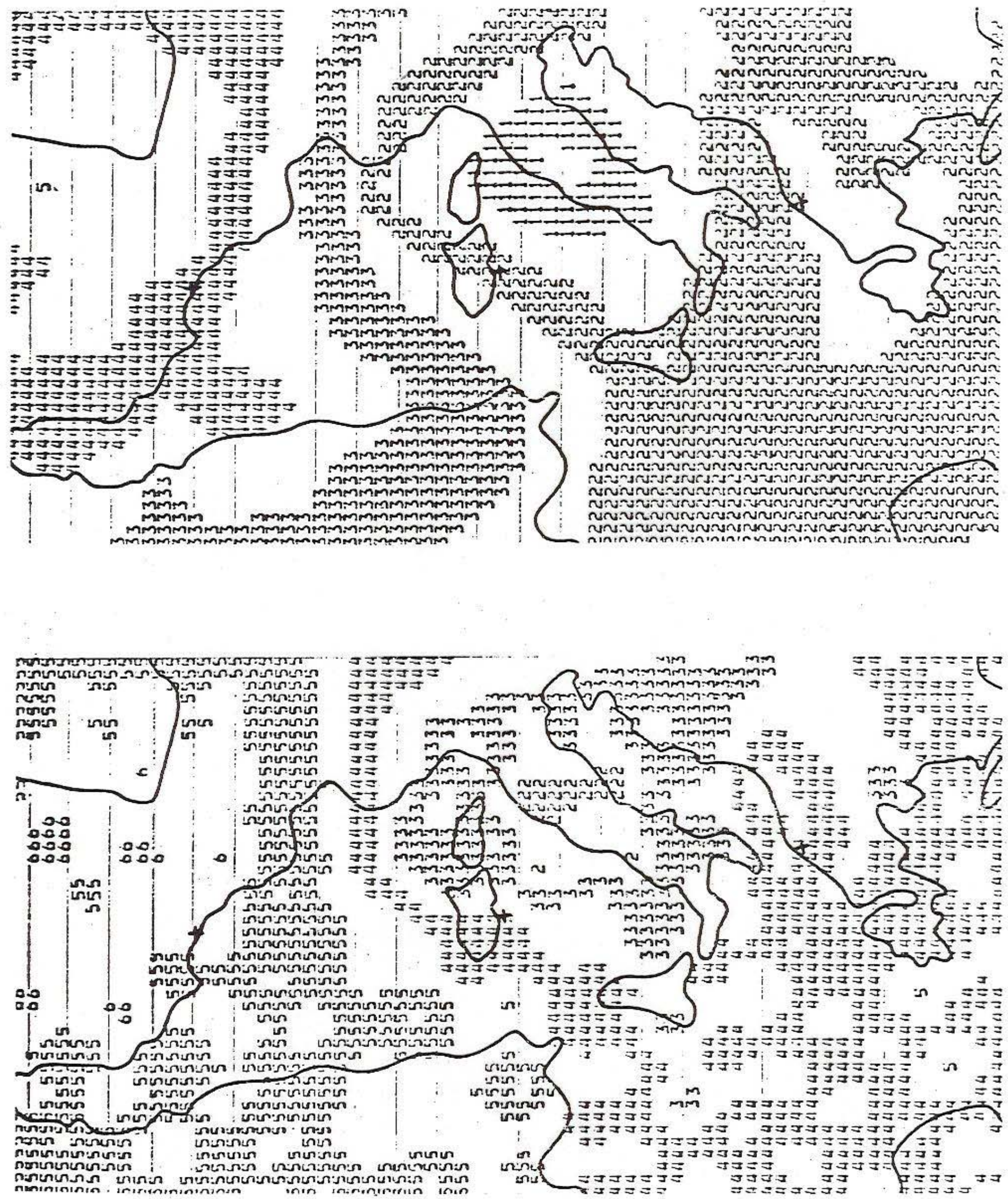
Experiments recently (2006) made, doing 48 h forecasts, with full physics, at two places, comparing

continuity eq. forward, vs momentum eq. forward

No visible difference !

Impact of "modification":

upper panel, used
lower panel, not used



• **Figure 8** Sea level pressure, 00 GMT 24 August 1975, 24 hr forecast with variable boundary conditions. Above: with w = .25; below: with w = 0.

Time differencing sequence ("splitting" is used):

Adjustment stage: cont. eq. forward, momentum backward

(the other way around might still be a little better?)

Vertical advection over 2 adj. time steps

Repeat (except no vertical advection now, if done for two time steps)

Horizontal diffusion;

Horizontal advection over 2 adjustment time steps

(first forward then off-centered scheme, approx. neutral);

Some physics calls;

Repeat all of the above;

More physics calls;

. . . . .

Splitting used:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -f\mathbf{k} \times \mathbf{v} - g\nabla h,$$
$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = 0. \qquad (1)$$

is replaced by

$$\frac{\partial \mathbf{v}}{\partial t} = -f\mathbf{k} \times \mathbf{v} - g\nabla h,$$
$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = 0. \qquad (2)$$

as the "adjustment step",

and

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = 0, \qquad (3)$$

as the "advection step"

Note that height advection $\mathbf{v} \cdot \nabla h$ (corresponding to pressure in 3D case) is carried in the adjustment step (or, stage), even though it represents advection!

This is a necessary, but not sufficient, condition for energy conservation in time differencing in the energy transformation ("ωα") term (transformation between potential and kinetic energy). Splitting however, as above, makes exact conservation of energy in time differencing not possible (correction of Mesinger (BAMS 1996), and amendment to Janjic et al. 1995, slides that follow). Energy conservation in the Eta, in transformation between potential and kinetic energy is achieved in space differencing.

Time differencing in the Eta: two steps of (2) are followed by one, over 2Δt, step of (3).

How is this figured out?

To achieve energy conservation in time differencing one needs to replicate what happens in the continuous case. Energy conservation in the continuous case, still shallow water eqs. for simplicity:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -f\mathbf{k}\times\mathbf{v} - g\nabla h, \qquad (1.1)$$

$$\frac{\partial h}{\partial t} + \nabla\cdot(h\mathbf{v}) = 0. \qquad (1.2)$$

To get the kinetic energy eq., multiply (1.1) by *h* **v**, multiply (1.2) by $\frac{1}{2}\mathbf{v}\cdot\mathbf{v}$, and add,

$$\frac{\partial}{\partial t}\frac{1}{2}h\mathbf{v}\cdot\mathbf{v} + h(\mathbf{v}\cdot\nabla)\frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \frac{1}{2}\mathbf{v}\cdot\mathbf{v}\nabla\cdot(h\mathbf{v}) = -gh\mathbf{v}\cdot\nabla h \qquad (4)$$

For the potential energy eq., multiply (1.2) by *gh,*

$$\frac{\partial}{\partial t}\frac{1}{2}gh^2 + gh\nabla\cdot(h\mathbf{v}) = 0 \qquad (5)$$

Adding (4) and (5) we obtain

$$\frac{\partial}{\partial t}(\frac{1}{2}h\mathbf{v}\cdot\mathbf{v} + \frac{1}{2}gh^2) + \nabla\cdot(\frac{1}{2}\mathbf{v}\cdot\mathbf{v}\,h\mathbf{v}) + \nabla\cdot(gh^2\mathbf{v}) = 0. \qquad (6)$$

Thus, the total energy in a closed domain is conserved

For conservation *in time differencing* terms that went into one and the other divergence term have to be available at the same time;

- Kinetic energy in horizontal advection (the 1st divergence term):

Formed of contributions of horizontal advection of **v** in (1.1), and mass divergence in (1.2)
Not available at the same time with the split-explicit approach;

cannot be done;

- Energy in transformations potential to kinetic (the 2nd divergence term):

Formed of the advection of *h* term on the right side of (2), coming from the pressure-gradient force, and the mass divergence term of (3), coming from the continuity eq.;

Both are done in the adjustment stage with the splitting as in (2) and (3);
cancellation is thus possible if the two are done at the same time

However: they are done separately with the forward-backward scheme;

Thus, with the forward-backward scheme, cannot be done;

Time steps used for the adjustment stage very small;
not considered a serious weakness

(Eta at 10 km resolution is typically using adjustment time step of 20 s)

Nonhydrostatic option (a switch available),

Janjic et al. 2001:

$$\left(\frac{\partial w}{\partial t}\right)^{\tau+1/2} \rightarrow \frac{w^{\tau+1}-w^{\tau}}{\Delta t}$$

- Advection

Horizontal advection: conserve enstrophy ($\sum \frac{1}{2}\zeta^2$) and kinetic energy for nondivergent barotropic part of the flow!

MWR, 1984

ZAVIŠA I. JANJIĆ

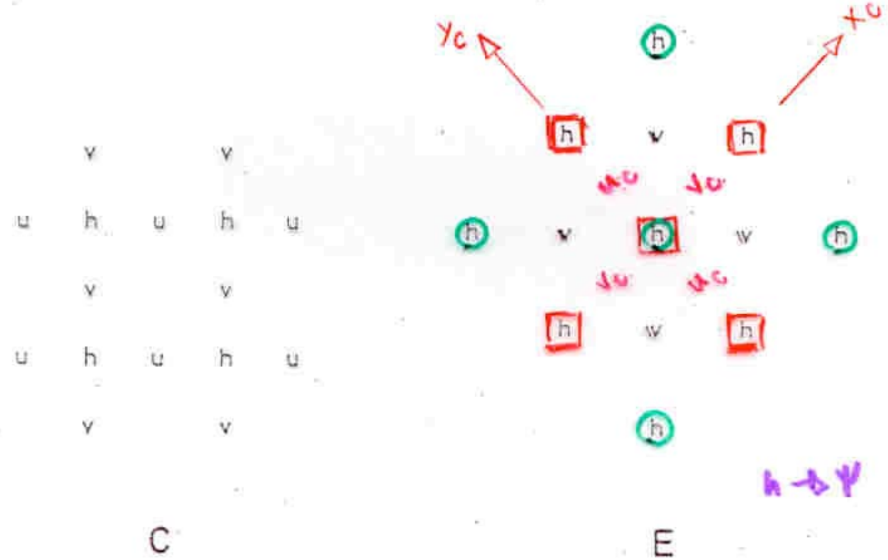


FIG. 1. Distributions of variables over grid points C and E.

Problem: ζ_E defined by simple differencing of the E grid u, v components eq. $\nabla_+^2\psi$, using ψ values at o points.

ζ_C, defined by differencing u_C, v_C, is equal to $\nabla_\times^2\psi$, using ψ values at □ points!

Janjic 1984:

- Arakawa-Lamb *C* grid scheme written in terms of u_C, v_C;
- write in terms of stream function values (at h points of the right hand plot);
- these same stream function values (square boxed in the plot) can now be transformed to u_E, v_E

Horizontal velocity components:

The horizontal advection scheme:

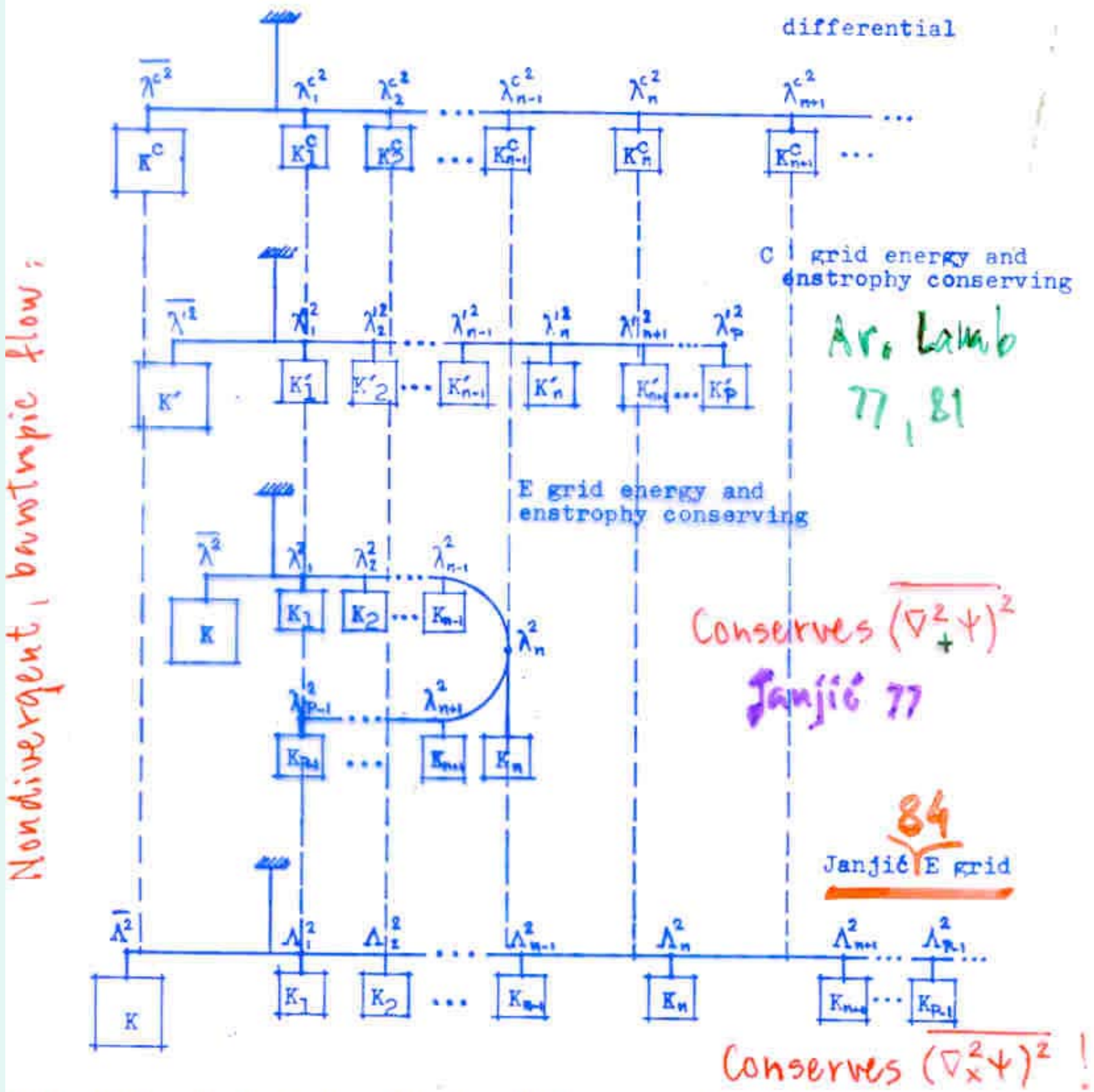


Fig. 3.12. Mechanical analogies of the constraints imposed on the non-linear energy cascade in the continuous case, in the case of the C-grid energy and enstrophy conserving scheme, in the case of the E-grid energy and enstrophy conserving scheme, and in the case of the scheme due to Janjić (1984).

(ECMWF Seminar '83)

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Janjić adv. scheme

	Conserves: No inter. bnd.	Conserves: With int. bnd.
Nondivergent part of the flow		
C-grid enstrophy	✓	✓
vorticity	✓	✓
rotat. energy	✓	✓
Divergent part incl.		
mass	✓	✓
E-grid kin. energy	✓	✓
momentum	✓	○
Passive quantity (T, q, q2, hor. adv.)		
1st moment	✓	✓
2nd moment	✓	✓

Vertical advection: Centered Lorenz-Arakawa, e.g.:

$$\frac{\partial T}{\partial t} = \ldots - \overline{\dot{\eta}\frac{\partial T}{\partial \eta}}^{\eta}$$

E.g., Arakawa and Lamb (1977, "the green book", p. 222). Conserves first and second moments (e.g., for u,v: momentum, kin. energy).

There is a problem however: false advection occurs from below ground. Experiments in progress to replace the scheme with a piecewise linear scheme of Mesinger and Jovic (2002)

(Code available: possible lab problem)

Advection of passive scalars (moisture, cloud water/ice):

In "standard" Eta:

Horizontal: Janjic (1997) "antidiffusion scheme"
Vertical: Piecewise-linear (Mesinger and Jovic 2002)

From Mesinger and Jovic :

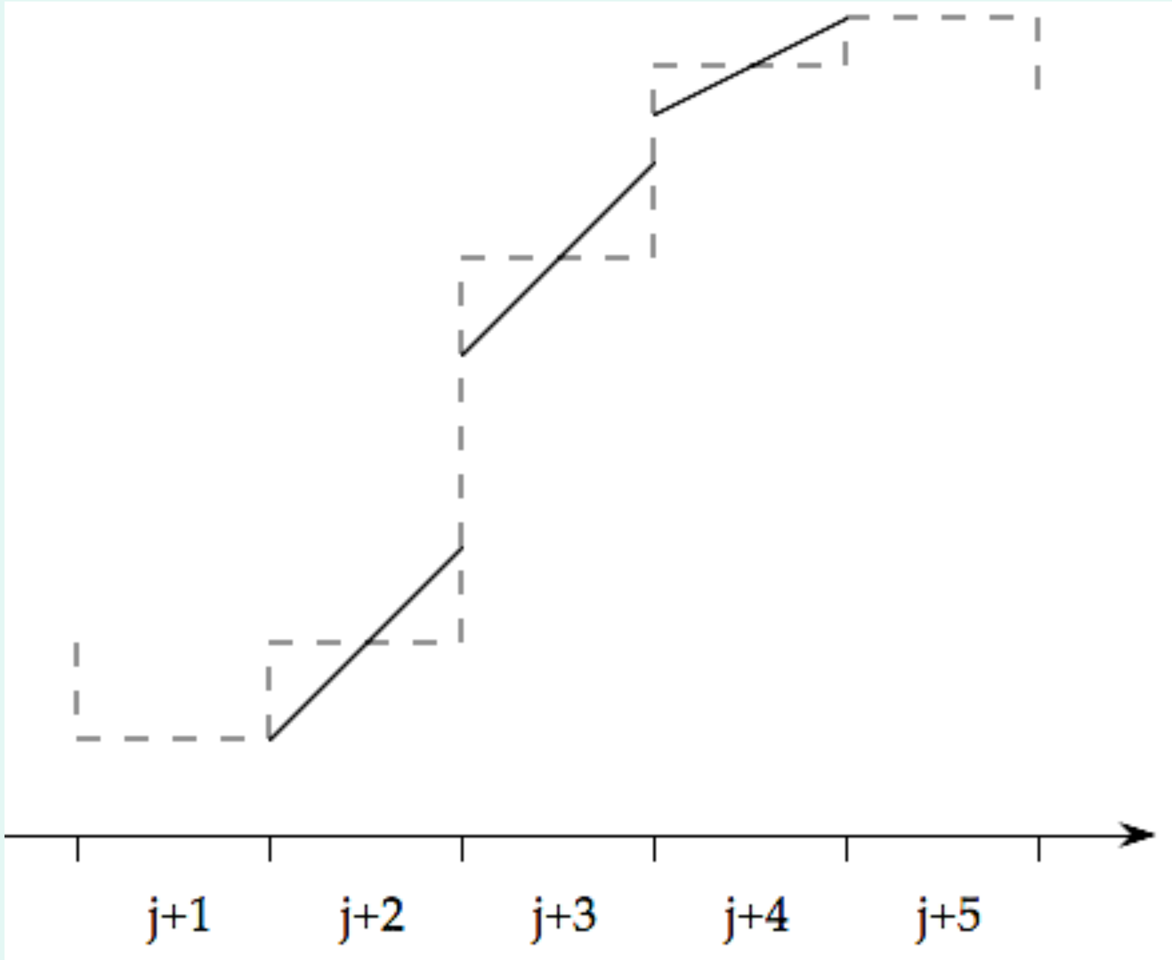

Figure 1. An example of the Eta iterative slope adjustment algorithm. The initial distribution is illustrated by the dashed line, with slopes in all five zones shown equal to zero. Slopes resulting from the first iteration are shown by the solid lines. See text for additional detail.

Mesinger, F., and D. Jovic, 2002: The Eta slope adjustment: Contender for an optimal steepening in a piecewise-linear advection scheme? Comparison tests. NCEP Office Note 439, 29 pp (available online at http://www.emc.ncep.noaa.gov/officenotes).

A comprehensive study of the Eta piecewise linear scheme including comparison against five other schemes (three Van Leer's, Janjic 1997, and Takacs 1985):

Most accurate; only one of van Leer's schemes comes close!

- Conservation of energy in transformation
  kinetic to potential

  - Evaluate generation of kinetic energy over the model's **v** points;
  - Convert from the sum over **v** to a sum over T points;
  - Identify the generation of potential energy terms in the thermodynamic equation, use appropriate terms from above

(2D: Mesinger 1984, 3D: Dushka Zupanski (then Gavrilov) in Mesinger et al. 1988)

References (if missing, check the "A Guide to the Eta Model" distributed):

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Mesinger, F., and D. Jovic, 2002: The Eta slope adjustment: Contender for an optimal steepening in a piecewise-linear advection scheme? Comparison tests. NCEP Office Note 439, 29 pp (Available online at http://wwwt.emc.ncep.noaa.gov/officenotes).

Zhang, H., and M. Rancic: 2007: A global Eta model on quasi-uniform grids. *Quart. J. Roy. Meteor. Soc.*, **133**, 517-528.