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1966-10

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Weather Predictability and Modelling**

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**Physic of stable ABL and PBL? Possible improvements of their parameterizations
in atmospheric models**

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Atmospheric Planetary Boundary Layer (ABL / PBL)

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Trieste, October 2008



Lecture 1

THEORY AND PARAMETERIZATION OF THE STABLY STRATIFIED ATMOSPHERIC BOUNDARY LAYER (SBL)



References

- Zilitinkevich, S., and Calanca, P., 2000: An extended similarity-theory for the stably stratified atmospheric surface layer. *Quart. J. Roy. Meteorol. Soc.*, 126, 1913-1923.
- Zilitinkevich, S., 2002: Third-order transport due to internal waves and non-local turbulence in the stably stratified surface layer. *Quart. J. Roy. Met. Soc.* 128, 913-925.
- Zilitinkevich, S.S., Perov, V.L., and King, J.C., 2002: Near-surface turbulent fluxes in stable stratification: calculation techniques for use in general circulation models. *Quart. J. Roy. Met. Soc.* 128, 1571-1587.
- Zilitinkevich S. S., and Esau I. N., 2005: Resistance and heat/mass transfer laws for neutral and stable planetary boundary layers: old theory advanced and re-evaluated. *Quart. J. Roy. Met. Soc.* 131, 1863-1892.
- Zilitinkevich, S., Esau, I. and Baklanov, A., 2007: Further comments on the equilibrium height of neutral and stable planetary boundary layers. *Quart. J. Roy. Met. Soc.* 133, 265-271.
- Zilitinkevich, S. S., and Esau, I. N., 2007: Similarity theory and calculation of turbulent fluxes at the surface for stably stratified atmospheric boundary layers. *Boundary-Layer Meteorol.* 125, 193-296.
- Zilitinkevich, S.S., Elperin, T., Kleeorin, N., and Rogachevskii, I., 2007: Energy- and flux-budget (EFB) turbulence closure model for the stably stratified flows. Part I: Steady-state, homogeneous regimes. *Boundary-Layer Meteorol.* 125, 167-192.
- Zilitinkevich, S. S., Mammarella, I., Baklanov, A. A., and Joffre, S. M., 2008: The effect of stratification on the roughness length and displacement height. *Boundary-Layer Meteorol.*
- DOI: 10.1007/s10546-008-9307-9.



Motivation

NWP, climate and air pollution modeling require

- Surface fluxes (lower boundary conditions in all models)
 - surface layer
 - roughness layer
- SBL height
 - in advanced surface-flux scheme (especially for shallow SBLs)
 - in air-pollution modeling
- Turbulent fluxes in any stratification (to close Reynolds equations in all models)
 - critical Richardson number?
 - turbulent Prandtl number
 - where to go?
- Depth/strength of and fluxes within capping inversions (especially in Polar regions)



State of the art

Surface fluxes

Surface layer concept:

$$\tau, F_\theta, F_q = \text{constant}$$

Local M-O (1954) scaling:

$$L = -u_*^3 / F_{bs}$$

Roughness length $z_{0u} \sim h_0$:

no stability effect

SBL height

Local (RM,1935) \Leftrightarrow Z(1974):

$$N|_{\text{free-flow}} \text{ neglected}$$

Closure

Down-gradient, Kolmogorov (1941):

$$K_M, K_H, K_D \sim E_K^{1/2} l_T$$

TKE and ,e.g., \mathcal{E} -budgets:

TPE disregarded

Improvements:

to avoid Ri_{cr} and correct Pr_{turb}
low interest / no parameterization

Capping inversions

Data

Mid latitudes \rightarrow residual layers ($N=0$) \rightarrow SBL = nocturnal BL



Basic types of the SBL

- Until recently ABLs were distinguished accounting only for $F_{bs} = F_*$:
 - neutral at $F_* = 0$
 - stable at $F_* < 0$
- Now more detailed classification:
 - truly neutral (TN) ABL: $F_* = 0, N = 0$
 - conventionally neutral (CN) ABL: $F_* = 0, N > 0$
 - nocturnal stable (NS) ABL: $F_* < 0, N = 0$
 - long-lived stable (LS) ABL: $F_* < 0, N > 0$
- Realistic surface flux calculation scheme should be based on a model applicable to all these types of the ABL



Content

- Revision of the similarity theory for the stably stratified ABL
- Analytical approximations for the wind velocity and potential temperature profiles across the ABL
- Validation of new theory against LES and observational data
- Improved surface flux scheme for use in operational models

Zilitinkevich, S. S., and Esau, I. N., 2007: Similarity theory and calculation of turbulent fluxes at the surface for stably stratified atmospheric boundary layers. *Boundary-Layer Meteorol.* **125**, 193-296.



Turbulence in atmospheric models

- turbulence closure – to calculate vertical fluxes: $\vec{\tau}$ and F_θ through mean gradients: $d\vec{U} / dz$ and $d\Theta / dz$
- flux-profile relationships – to calculate the surface fluxes: $u_*^2 = \tau_* = \tau|_{z=0}$, $F_* = F_\theta|_{z=0}$ through wind speed $U_1 = U|_{z=z_1}$ and potential temperature $\Theta_1 = \Theta|_{z=z_1}$ at a given level z_1
- in NWP and climate models, the lowest computational level is $z_1 \sim 30$ m



Neutral stratification (no problem)

From logarithmic wall law:

$$\frac{dU}{dz} = \frac{\tau^{1/2}}{kz}, \quad \frac{d\Theta}{dz} = \frac{-F_\theta}{k_T \tau^{1/2} z}, \quad U = \frac{\tau^{1/2}}{k} \ln \frac{z}{z_{0u}}, \quad \Theta - \Theta_0 = \frac{-F_\theta}{k_T \tau^{1/2}} \ln \frac{z}{z_{0u}}$$

k , k_T von Karman constants; z_{0u} aerodynamic roughness length for momentum;
 Θ_0 aerodynamic surface potential temperature (at z_{0u}) [$\Theta_0 - \Theta_s$ through z_{0T}]

It follows: $\tau_1^{1/2} = kU_1(\ln z/z_{0u})^{-1}$, $F_{\theta 1} = -kk_T U_1 (\Theta_1 - \Theta_0)(\ln z/z_{0u})^{-2}$

$\tau_1 = \tau_*$, $F_{\theta 1} = F_*$ when $z_1 \approx 30$ m $\ll h$ \rightarrow OK in neutral stratification



Stable stratification: current theory

(i) local scaling, (ii) log-linear Θ -profile \rightarrow both questionable

- When z_1 is much above the surface layer $\rightarrow \tau_1 \neq \tau_*, F_{\theta 1} \neq F_*$
- Monin-Obukhov (MO) theory $\rightarrow L = \frac{\tau^{3/2}}{-\beta F_\theta}$ (**neglects other scales**) \rightarrow
$$\frac{kz}{\tau^{1/2}} \frac{dU}{dz} = \Phi_M(\xi), \quad \frac{k_T \tau^{1/2} z}{F_\theta} \frac{d\Theta}{dz} = \Phi_H(\xi), \quad \text{where} \quad \xi = \frac{z}{L}$$
- $\Phi_M = 1 + C_{U1}\xi, \Phi_H = 1 + C_{\Theta 1}\xi$ from z -less stratification concept
$$U = \frac{u_*}{k} \left(\ln \frac{z}{z_{u0}} + C_{U1} \frac{z}{L_s} \right), \quad \Theta - \Theta_0 = \frac{-F_*}{k_T u_*} \left(\ln \frac{z}{z_{u0}} + C_{\Theta 1} \frac{z}{L_s} \right)$$
- $Ri \equiv \beta(d\Theta/dz)(dU/dz)^{-2} \rightarrow Ri_c = k^2 C_{\Theta 1} k_T^{-1} C_{U1}^{-2}$ (**unacceptable**)
- $C_{U1} \sim 2, C_{\Theta 1}$ also ~ 2 (**factually increases with z/L**)



Stable stratification: current parameterization

To avoid critical Ri modellers use empirical, heuristic correction functions to the neutral drag and heat/mass transfer coefficients

- Drag and heat transfer coefficients: $C_D = \tau / (U_1)^2$, $C_H = -F_{\theta s} / (U_1 \Delta \Theta)$
- Neutral: C_{Dn} , C_{Hn} – from the logarithmic wall law
- To account for stratification, correction functions (dependent only of Ri):

$$f_D(\text{Ri}_1) = C_D / C_{Dn} \text{ and } f_H(\text{Ri}_1) = C_H / C_{Hn}$$

$\text{Ri}_1 = \beta(\Delta \Theta) z_1 / (U_1)^2$ (surface-layer “Richardson number”) - given parameter



SS Zilitinkevich et al., 2002: Near-surface turbulent fluxes in stable stratification: Calculation techniques for use in general-circulation models. *Boundary-layer Meteorol.* **128**, 1571-1587

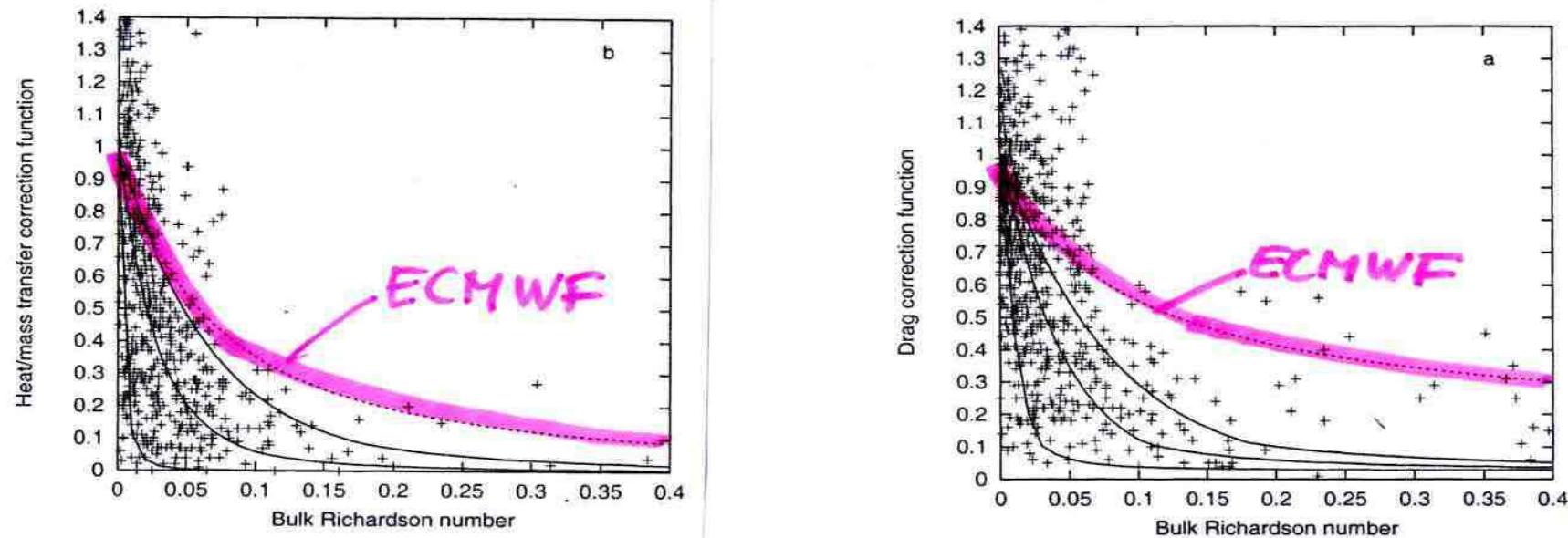


Figure 1. The correction functions (a) to the drag coefficient, f_D , and (b) to the heat and mass-transfer coefficients, $f_H = f_M$, versus the surface-layer bulk Richardson number Ri , see Eq. (7). Crosses are data from measurements at Halley, Antarctica. The correction functions from Louis *et al.* (1982) are shown by dashed lines.

$$C_D \equiv \frac{\tau_s}{u^2}, \quad C_H \equiv -\frac{F_{\theta s}}{u \Delta \theta}, \quad C_M \equiv -\frac{F_{qs}}{u \Delta q}.$$

$$Ri \equiv \frac{(\beta \Delta \theta + 0.61 g \Delta q) z_1}{u^2}$$

$$f_D = C_D / C_{Dn}, \quad f_H = C_H / C_{Hn}, \quad f_M = C_M / C_{Mn}$$

SS Zilitinkevich et al., 2002: Near-surface turbulent fluxes in stable stratification: Calculation techniques for use in general-circulation models. *Boundary-layer Meteorol.* **128**, 1571–1587

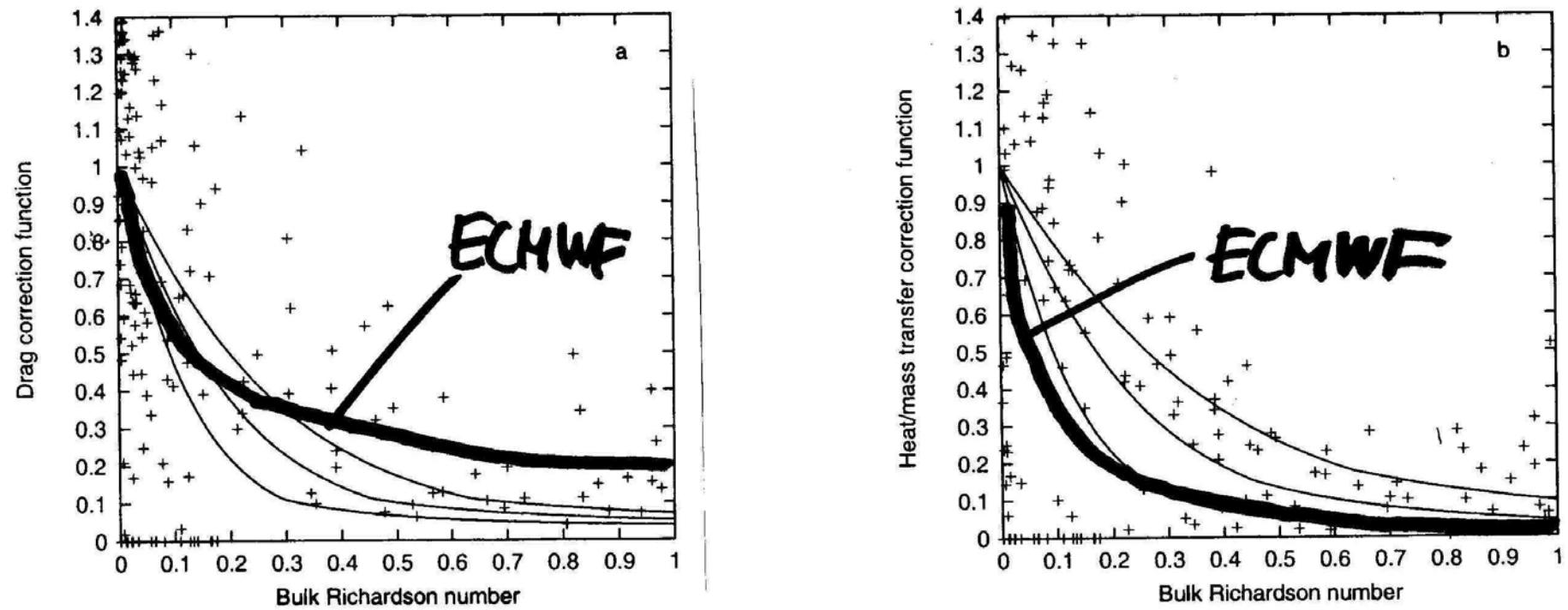


Figure 2. The same as in Fig. 1, but for Sodankyla, Arctic Finland: (a) f_D and (b) $f_H = f_M$. Crosses are measurements at this site.

Stable stratification: revised theory

Zilitinkevich and Esau (2005) → two additional length scales besides L :

$$L_N = \frac{\tau^{1/2}}{N} \quad \text{non-local effect of the free flow static stability}$$

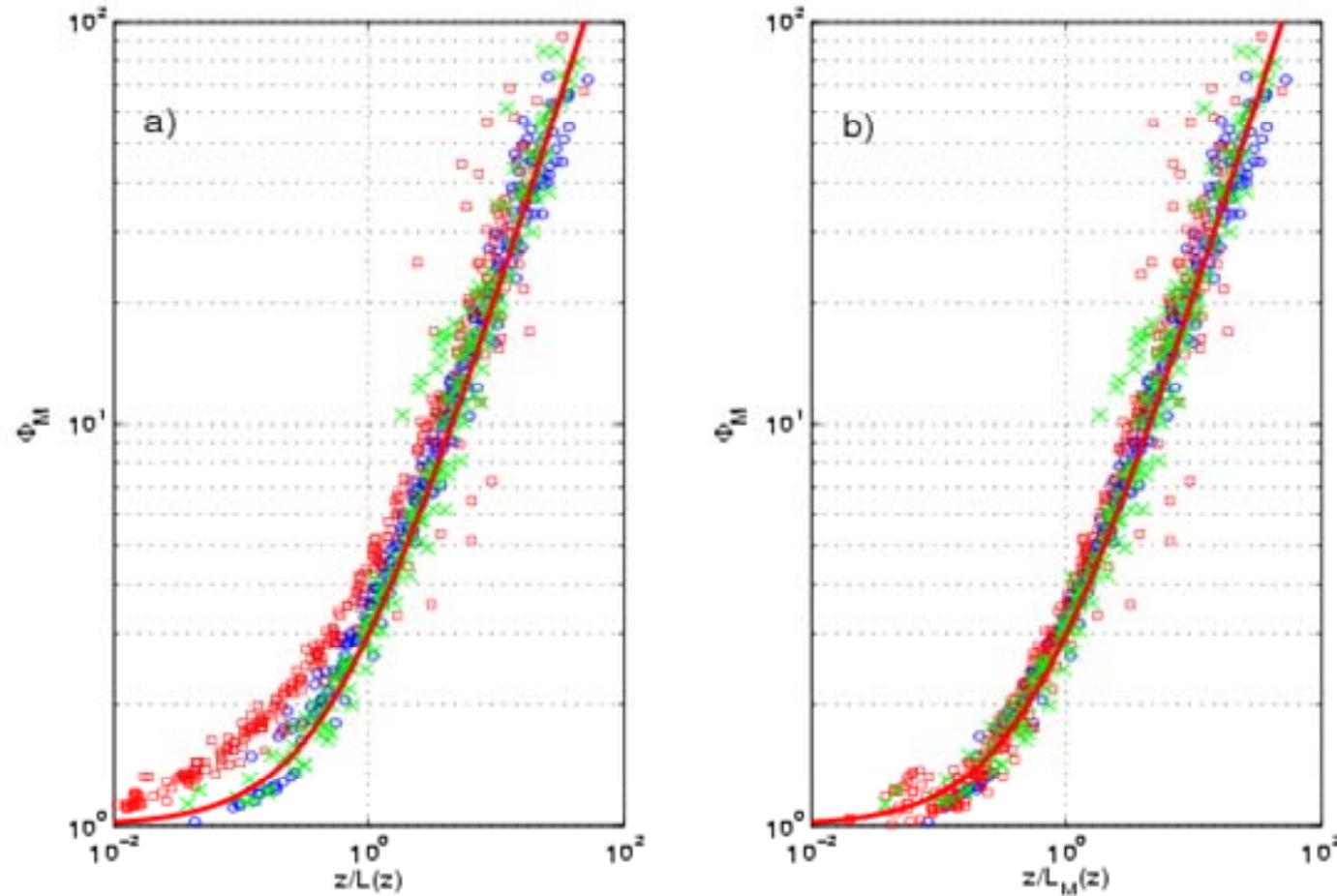
$$L_f = \frac{\tau^{1/2}}{|f|} \quad \text{the effect of the Earth's rotation}$$

N is the Brunt-Väisälä frequency at $z>h$ ($N \sim 10^{-2} \text{ s}^{-1}$), f is the Coriolis parameter

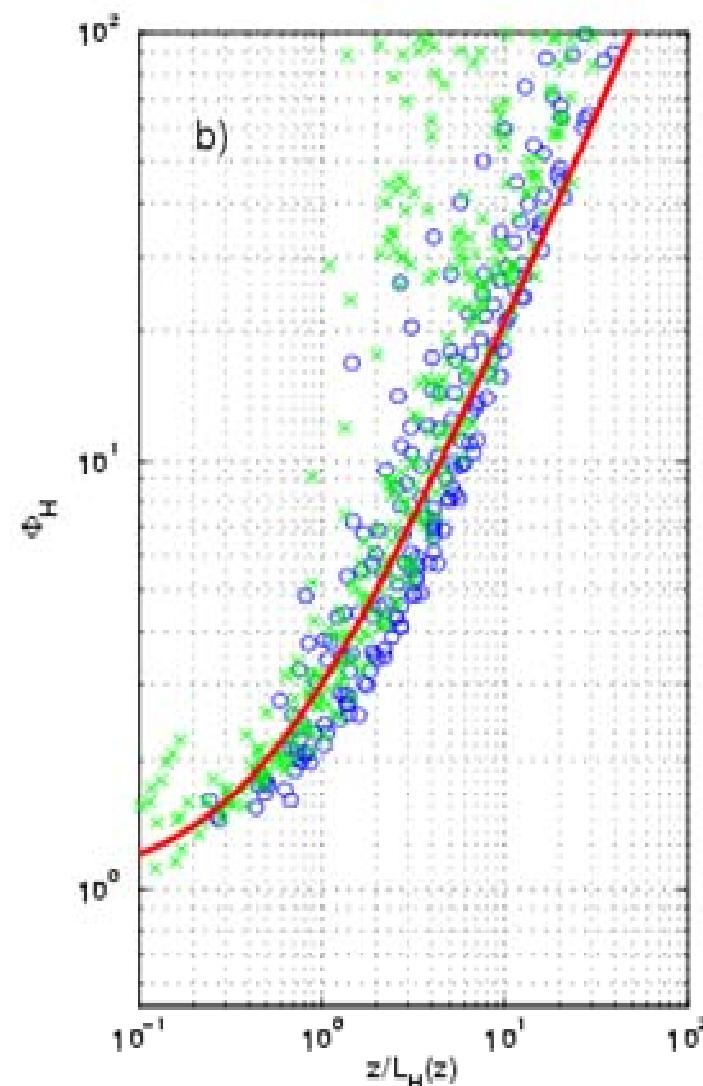
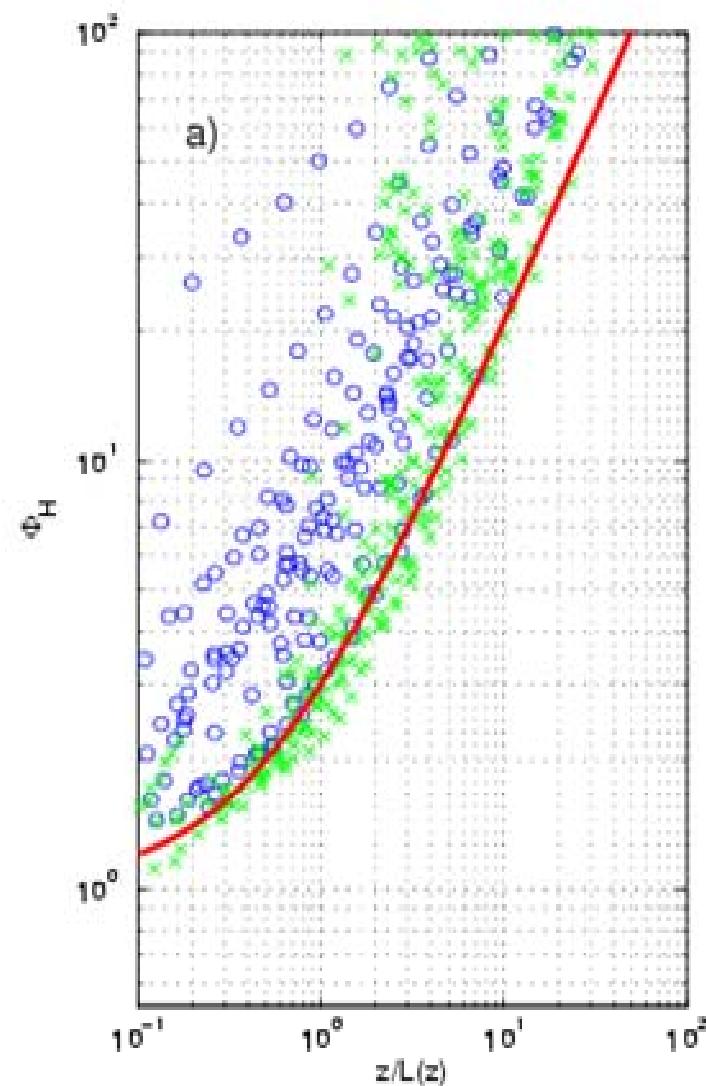
Interpolation: $\frac{1}{L_*} = \left[\left(\frac{1}{L} \right)^2 + \left(\frac{C_N}{L_N} \right)^2 + \left(\frac{C_f}{L_f} \right)^2 \right]^{1/2}$ where $C_N=0.1$ and $C_f=1$



$kz\tau^{1/2}dU/dz$ vs. z/L (a), z/L_* (b) nocturnal; long-lived; conventionally neutral



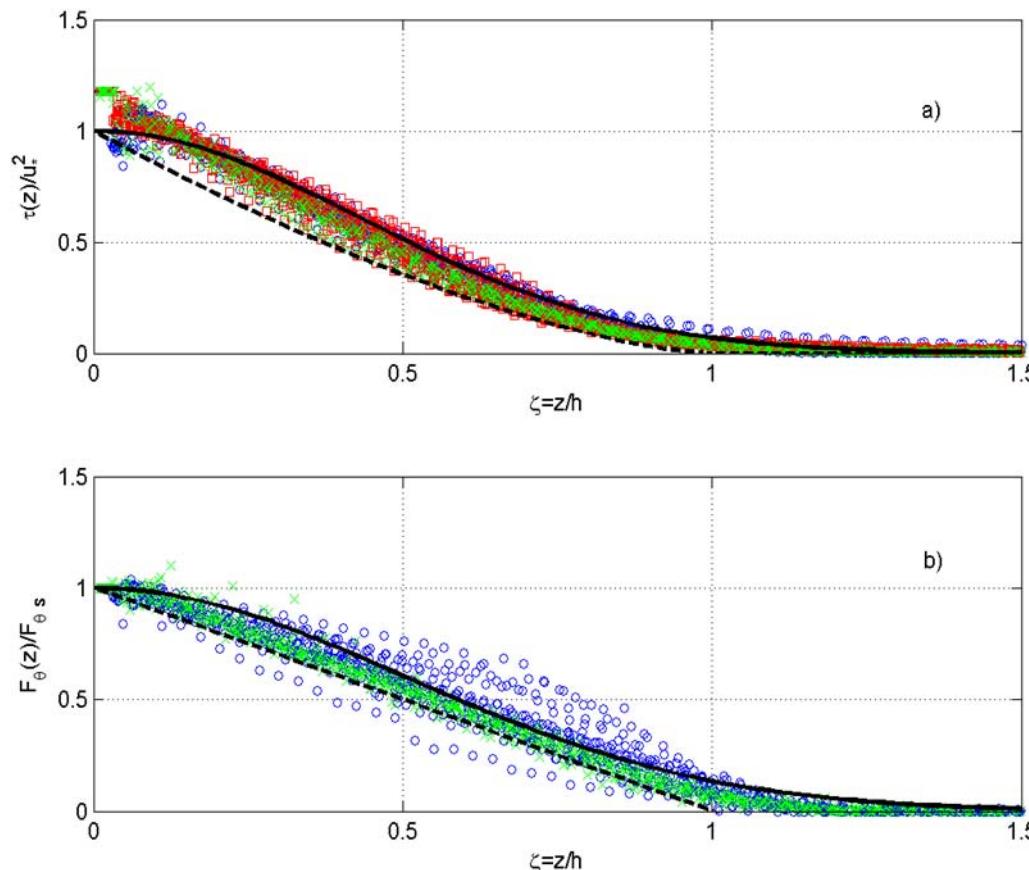
$$\Phi_H = (k_T \tau^{1/2} z / F_\theta) d\Theta / dz \text{ vs. } z/L \text{ (a), } z/L_* \text{ (b)} \quad x \text{ } \underline{\text{nocturnal}}; \text{ } o \text{ } \underline{\text{long-lived}}$$



Vertical profiles of turbulent fluxes

LES turbulent fluxes: solid lines $\tau/u_*^2 = \exp(-\frac{8}{3}\zeta^2)$, $F_\theta/F_{\theta s} = \exp(-2\zeta^2)$

Approximation based on atmospheric data (e.g. Lenshow, 1988): dashed lines



New mean-gradient formulation (no critical Ri)

Flux Richardson number is limited:

$$Ri_f = \frac{-\beta F_\theta}{\tau dU/dz} > Ri_f^\infty \approx 0.2$$

Hence asymptotically $\frac{dU}{dz} \rightarrow \frac{\tau^{1/2}}{Ri_f^\infty L}$, and interpolating $\Phi_M = 1 + C_{U1}\xi$

Gradient Richardson number becomes

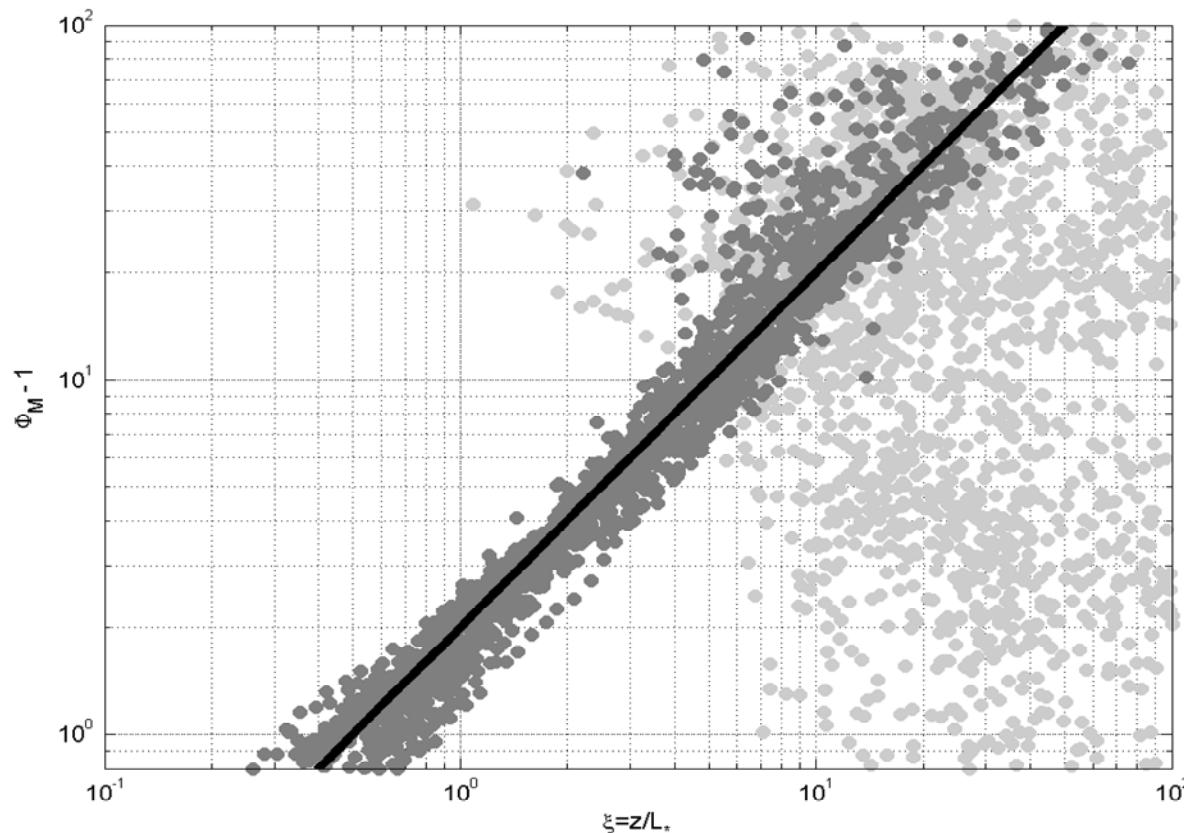
$$Ri \equiv \frac{\beta d\Theta / dz}{(dU / dz)^2} = \frac{k^2}{k_T} \frac{\xi \Phi_H(\xi)}{(1 + C_{U1}\xi)^2}$$

To assure no Ri-critical, ξ -dependence of Φ_H should be **stronger than linear**.

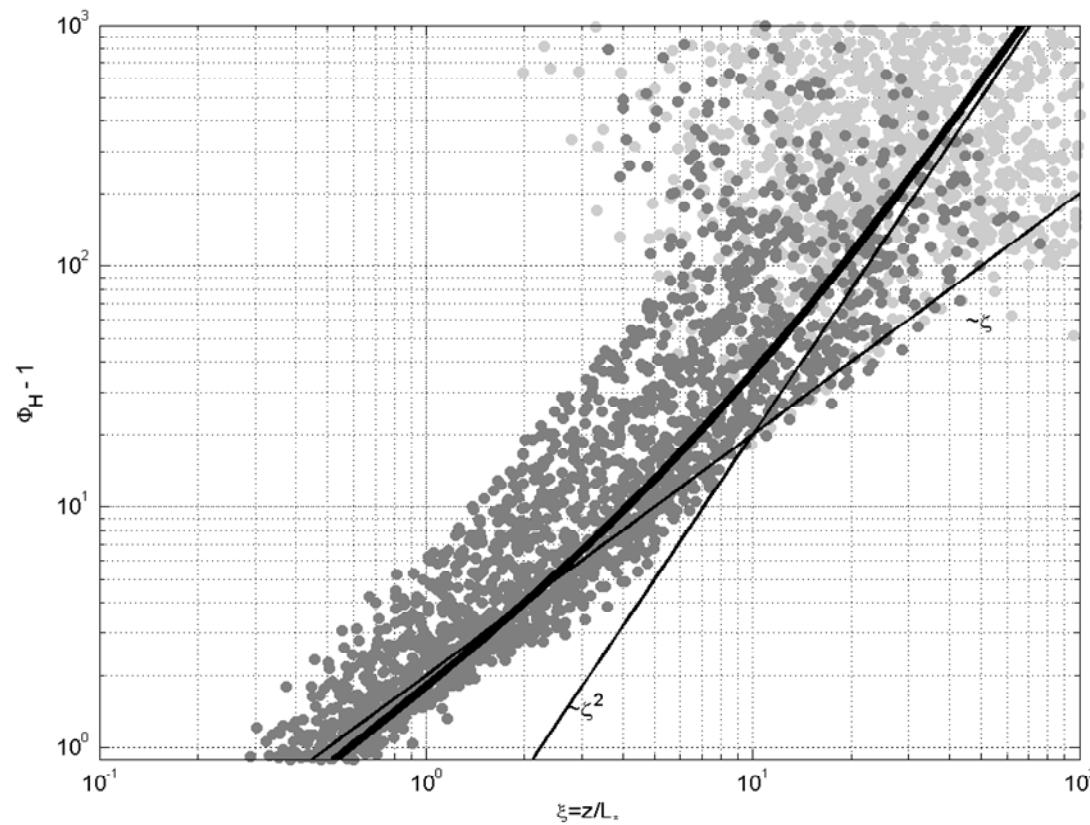
Including CN and LS ABLs: $\Phi_M = 1 + C_{U1} \frac{z}{L_*}$,

$$\Phi_H = 1 + C_{\Theta 1} \frac{z}{L_*} + C_{\Theta 2} \left(\frac{z}{L_*} \right)^2$$

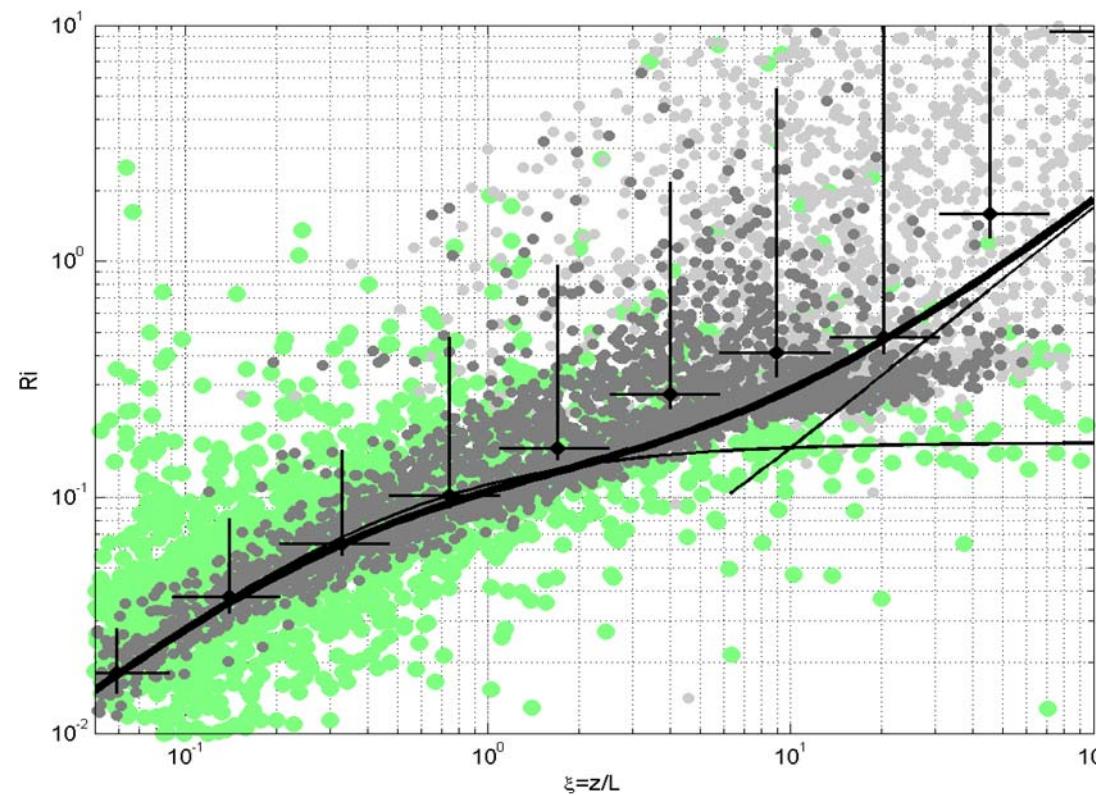




Φ_M vs. $\xi = z / L_*$, after LES DATABASE64 (all types of SBL). Dark grey points for $z < h$; light grey points for $z > h$; the line corresponds to $C_{U1} = 2$.



Φ_H vs. $\xi = z / L_*$ (all SBLs). Bold curve is our approximation: $C_{\Theta 1} = 1.8$, $C_{\Theta 2} = 0.2$; thin lines are $\Phi_H = 0.2 \xi^2$ and traditional $\Phi_H = 1 + 2\xi$.



Ri vs. $\xi = z / L$, after LES and field data (SHEBA - green points). Bold curve is our model with $C_{U1}=2$, $C_{\Theta_1}=1.6$, $C_{\Theta_2}=0.2$. Thin curve is $\Phi_H=1+2\xi$.

Mean profiles and flux-profile relationships

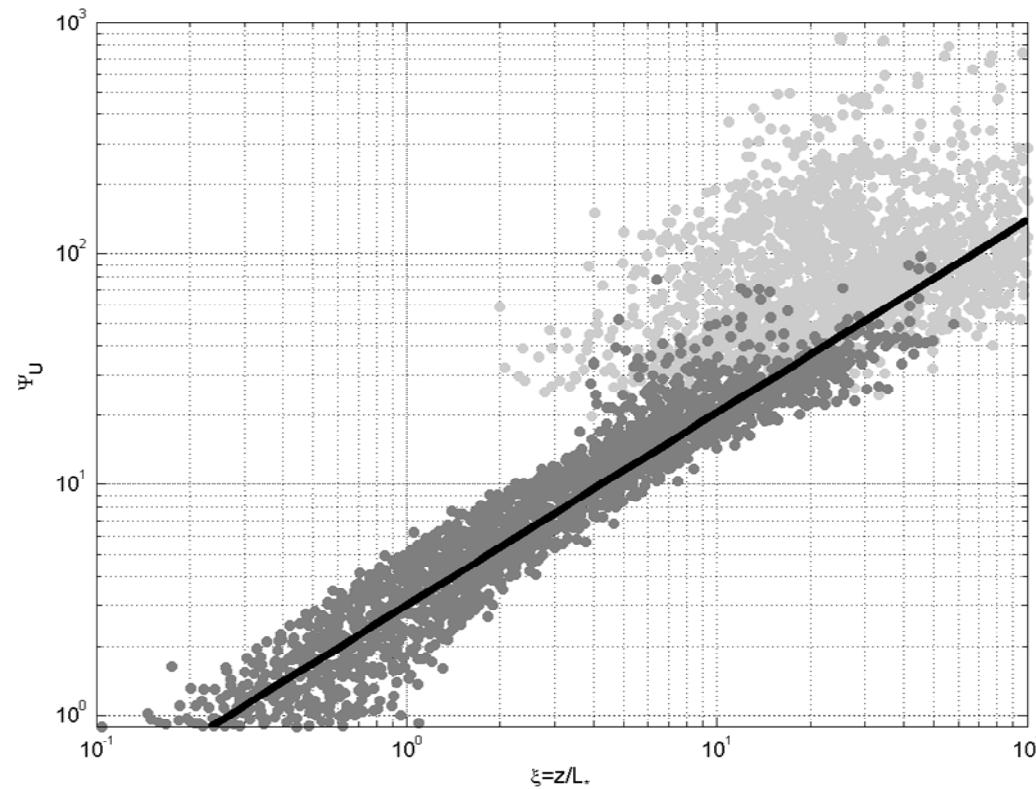
We consider wind/velocity and potential/temperature functions

$$\Psi_U = \frac{kU(z)}{\tau^{1/2}} - \ln \frac{z}{z_{0u}} \quad \text{and} \quad \Psi_\Theta = \frac{k_T \tau^{1/2} [\Theta(z) - \Theta_0]}{-F_\theta} - \ln \frac{z}{z_{0u}}$$

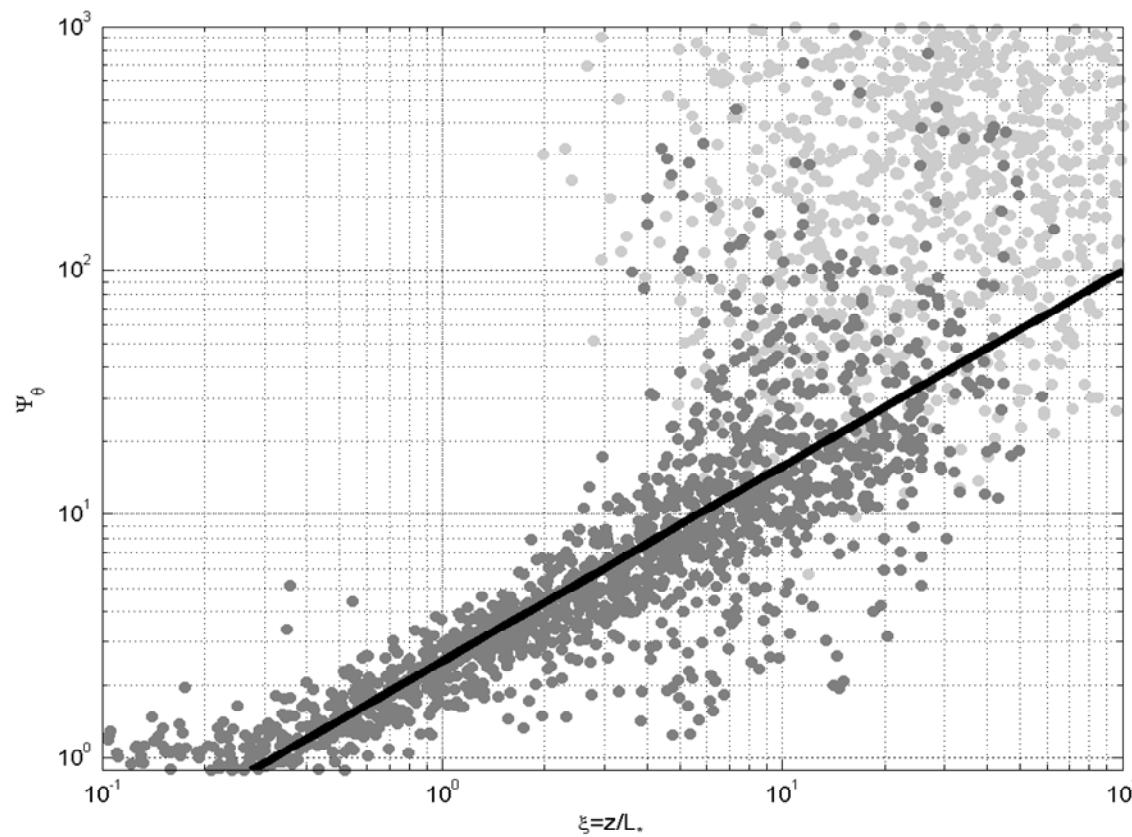
Our analyses show that Ψ_U and Ψ_Θ are universal functions of $\xi = z / L_*$

$$\Psi_U = C_U \xi^{5/6}, \quad \Psi_\Theta = C_\Theta \xi^{4/5}, \quad \text{with } C_U = 3.0 \text{ and } C_\Theta = 2.5$$





Wind-velocity function $\Psi_U = k\tau^{-1/2}U - \ln(z/z_{0u})$ vs. $\xi = z/L_*$, after LES DATABASE64 ([all types of SBL](#)). The line: $\Psi_U = C_U \xi^{5/6}$, $C_U = 3.0$.



Pot.-temperature function $\Psi_{\Theta} = k\tau^{-1/2}(\Theta - \Theta_0)(-F_{\theta})^{-1} - \ln(z/z_{0u})$
(all types of SBL). The line: $\Psi_{\Theta} = C_{\Theta}\xi^{4/5}$ with $C_U = 3.0$ and $C_{\Theta} = 2.5$.



Analytical wind and temperature profiles (SBL)

$$\frac{kU}{\tau^{1/2}} = \ln \frac{z}{z_{0u}} + C_U \left(\frac{z}{L} \right)^{5/6} \left[1 + \frac{(C_N N)^2 + (C_f f)^2}{\tau} L^2 \right]^{5/12}$$

$$\frac{k_T \tau^{1/2} (\Theta - \Theta_0)}{-F_\theta} = \ln \frac{z}{z_{0u}} + C_\Theta \left(\frac{z}{L} \right)^{4/5} \left[1 + \frac{(C_N N)^2 + (C_f f)^2}{\tau} L^2 \right]^{2/5}$$

where $C_N = 0.1$ and $C_f = 1$. Given $U(z)$, $\Theta(z)$ and N , these equations allow determining τ , F_θ , and $L = \tau^{3/2} (-\beta F_\theta)^{-1}$, at the computational level z .



Algorithm

Given τ , F_θ , surface fluxes are calculated using empirical dependencies

$$\frac{\tau}{\tau_*} = \exp\left[-\frac{8}{3}\left(\frac{z}{h}\right)^2\right], \quad \frac{F_\theta}{F_*} = \exp\left[-2\left(\frac{z}{h}\right)^2\right] \quad (\text{Figures above})$$

The equilibrium ABL height, h_E , is determined diagnostically (Z. et al., 2006a):

$$\frac{1}{h_E^2} = \frac{f^2}{C_R^2 \tau_*} + \frac{N |f|}{C_{CN}^2 \tau_*} + \frac{|f\beta F_*|}{C_{NS}^2 \tau_*^2} \quad (C_R = 0.6, C_{CN} = 1.36, C_{NS} = 0.51)$$

The actual ABL height, after prognostic equation (Z. and Baklanov, 2002):

$$\frac{\partial h}{\partial t} + \vec{U} \cdot \nabla h - w_h = K_h \nabla^2 h - C_t \frac{u_*}{h_E} (h - h_E) \quad (C_t = 1)$$

Given h , the free-flow Brunt-Väisälä frequency is

$$N^4 = \frac{1}{h} \int_h^{2h} \left(\beta \frac{\partial \Theta}{\partial z} \right)^2 dz$$



Conclusions (SBL)

Background: Generalised scaling accounting for the free-flow stability,
No critical Ri (TTE closure)
Stable ABL height model

Verified against

LES DATABASE64 (4 ABL types: TN, CN, NS and LS)
Data from the field campaign SHEBA

Deliverable 1: **analytical wind & temperature profiles in SBLs**

Deliverable 2: **surface flux scheme for use in operational models**

Remaining tasks: **(ii) ABL height and (i) roughness lengths**



Lecture 2

THE EFFECT OF STRATIFICATION ON THE ROUGHNESS LENGTH AND DISPLACEMENT HEIGHT

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Reference

S. S. Zilitinkevich, I. Mammarella, A. A. Baklanov, and S. M. Joffre,
2007: The roughness length in environmental fluid mechanics: the
classical concept and the effect of stratification. *Boundary-Layer
Meteorology* DOI: 10.1007/s10546-008-9307-9



Content

- Roughness length and displacement height:

$$u(z) = \frac{u_*}{k} \left[\ln \frac{z - d_{0u}}{z_{0u}} + \Psi_u \left(\frac{z}{L} \right) \right]$$

- No stability dependence of z_{0u} (and d_{0u}) in engineering fluid mechanics:
neutral-stability z_0 = level, at which $u(z)$ plotted vs. $\ln z$ approaches zero;
 $z_0 \sim 1/25$ of typical height of roughness elements, h_0

- Meteorology / oceanography: h_0 comparable with MO length $L = \frac{u_*^3}{-\beta F_{\theta_s}}$

- Stability dependence of the actual roughness length, z_{0u} :
 $z_{0u} < z_0$ in stable stratification; $z_{0u} > z_0$ in unstable stratification



Surface layer and roughness length

- Self similarity in the surface layer (SL) $5h_0 < z < 10^{-1}h$
- Height-constant fluxes: $\tau \approx \tau|_{z=5h_0} \equiv u_*^2$
- u_* and z serve as turbulent scales: $u_T \sim u_*, l_T \sim z$
- Eddy viscosity ($k \approx 0.4$) $K_M (\sim u_T l_T) = k u_* z$
- Velocity gradient $\partial U / \partial z = \tau / K_M = u_* / kz$
- Integration constant: $U = k^{-1} u_* \ln z + \text{constant} = k^{-1} u_* \ln(z / z_{0u})$
- z_{0u} (redefined constant of integration) is “roughness length”
- “Displacement height” d_{0u} $U = k^{-1} u_* \ln[(z - d_{0u}) / z_{0u}]$
- Not applied to the roughness layer (RL) $0 < z < 5h_0$



Parameters controlling z_{0u}

Smooth surfaces: viscous layer $\rightarrow z_{0u} \sim \nu / u_*$

Very rough surfaces: pressure forces depend on:

obstacle height h_0

velocity in the roughness layer $U_R \sim u_*$

$z_{0u} = z_{0u}(h_0, u_*) \sim h_0$ (in sand roughness experiments $z_{0u} \approx \frac{1}{30} h_0$)

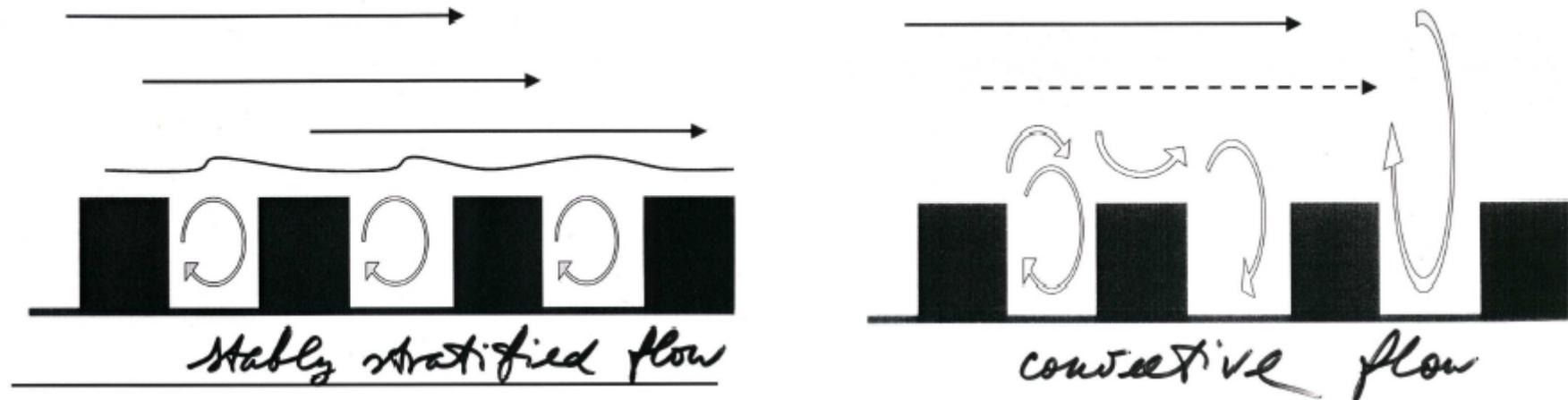
No dependence on u_* ; surfaces characterised by $z_{0u} = \text{constant}$

Generally $z_{0u} = h_0 f_0(\text{Re}_0)$ where $\text{Re}_0 = u_* h_0 / \nu$

Stratification at M-O length $L = -u_*^3 F_b^{-1}$ comparable with h_0



Stability Dependence of Roughness Length



For urban and vegetation canopies with roughness-element heights (20-50 m) comparable with the Monin-Obukhov turbulent length scale, L , the surface resistance and roughness length depend on stratification

Background physics and effect of stratification

Physically z_{0u} = depth of a sub-layer within RL ($0 < z < 5h_0$)
with 90% of the velocity drop from $U_R \sim u_*$ (approached at $z \sim h_0$)

From $\tau = K_{M(RL)} \partial U / \partial z$, $\tau \sim u_*^2$ and $\partial U / \partial z \sim U_R / z_{0u} \sim u_* / z_{0u}$

$$z_{0u} \sim K_{M(RL)} / u_*$$

$K_M(\text{RL}) = K_M(h_0 + 0)$ from matching the RL and the surface-layer

Neutral: $K_M \sim u_* h_0 \Rightarrow$ **classical formula** $z_{0u} \sim h_0$

Stable: $K_M = k u_* z (1 + C_u z / L)^{-1} \sim u_* L \Rightarrow z_{0u} \sim L$

Unstable: $K_M = k u_* z + C_U^{-1} F_b^{1/3} z^{4/3} \sim F_b^{1/3} z^{4/3} \Rightarrow z_{0u} \sim h_0 (-h_0 / L)^{1/3}$



Recommended formulation

Neutral \Leftrightarrow stable

$$\frac{z_{0u}}{z_0} = \frac{1}{1 + C_{SS} h_0 / L}$$

Neutral \Leftrightarrow unstable

$$\frac{z_{0u}}{z_0} = 1 + C_{US} \left(\frac{h_0}{-L} \right)^{1/3}$$

Constants: $C_{SS} = 8.13 \pm 0.21$, $C_{US} = 1.24 \pm 0.05$



Experimental datasets



Sodankyla Meteorological Observatory, Boreal forest (FMI)

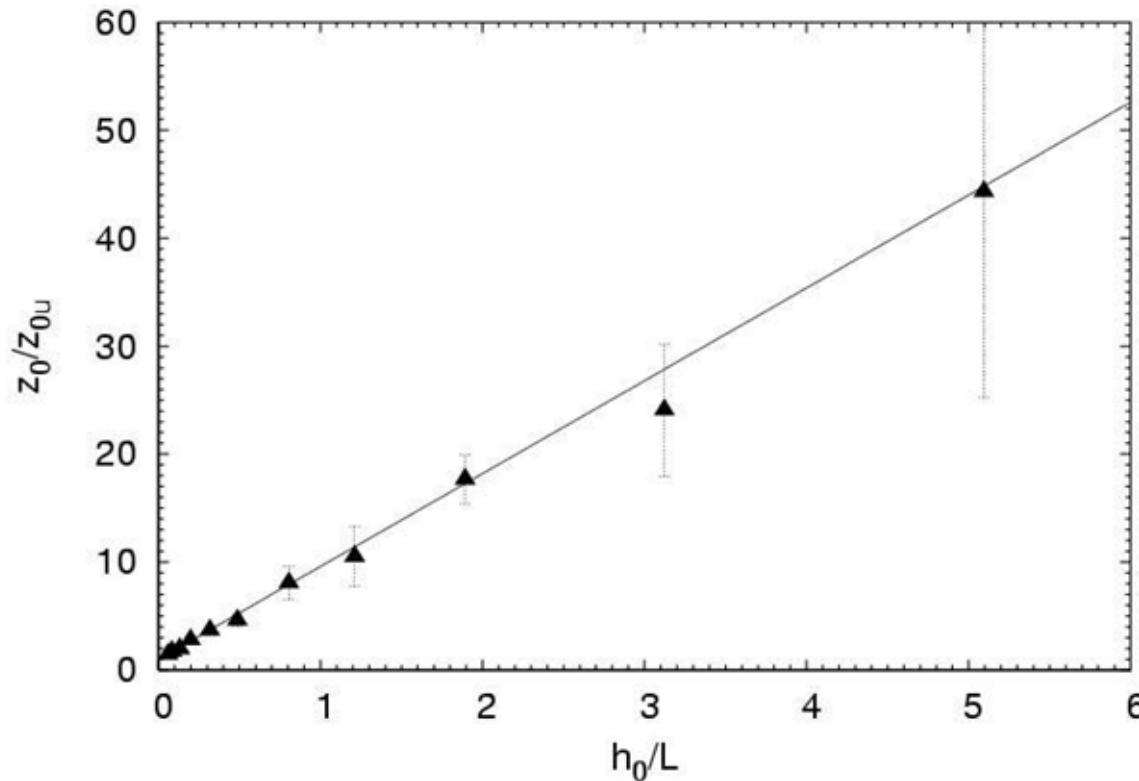
$h \approx 13$ m, measurement levels 23, 25, 47 m



BUBBLE urban BL experiment, Basel, Sperrstrasse (Rotach et al., 2004)

$h \approx 14.6$ m, measurement levels 3.6, 11.3, 14.7, 17.9, 22.4, 31.7 m

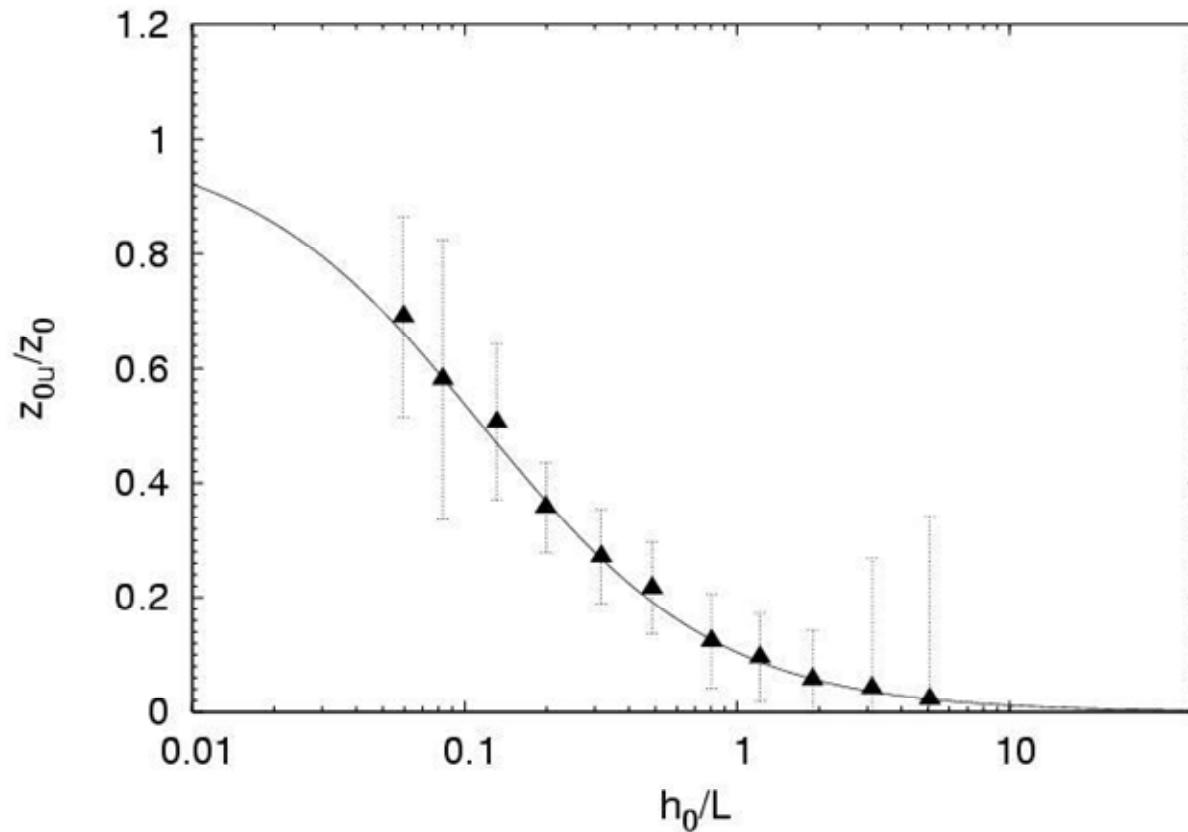
Stable stratification



Bin-average values of z_0 / z_{0u} (neutral- over actual-roughness lengths) versus h_0 / L in stable stratification for Boreal forest ($h_0=13.5$ m; $z_0=1.1\pm 0.3$ m). Bars are standard errors; the curve is $z_0 / z_{0u} = 1 + 8.13h_0 / L$.



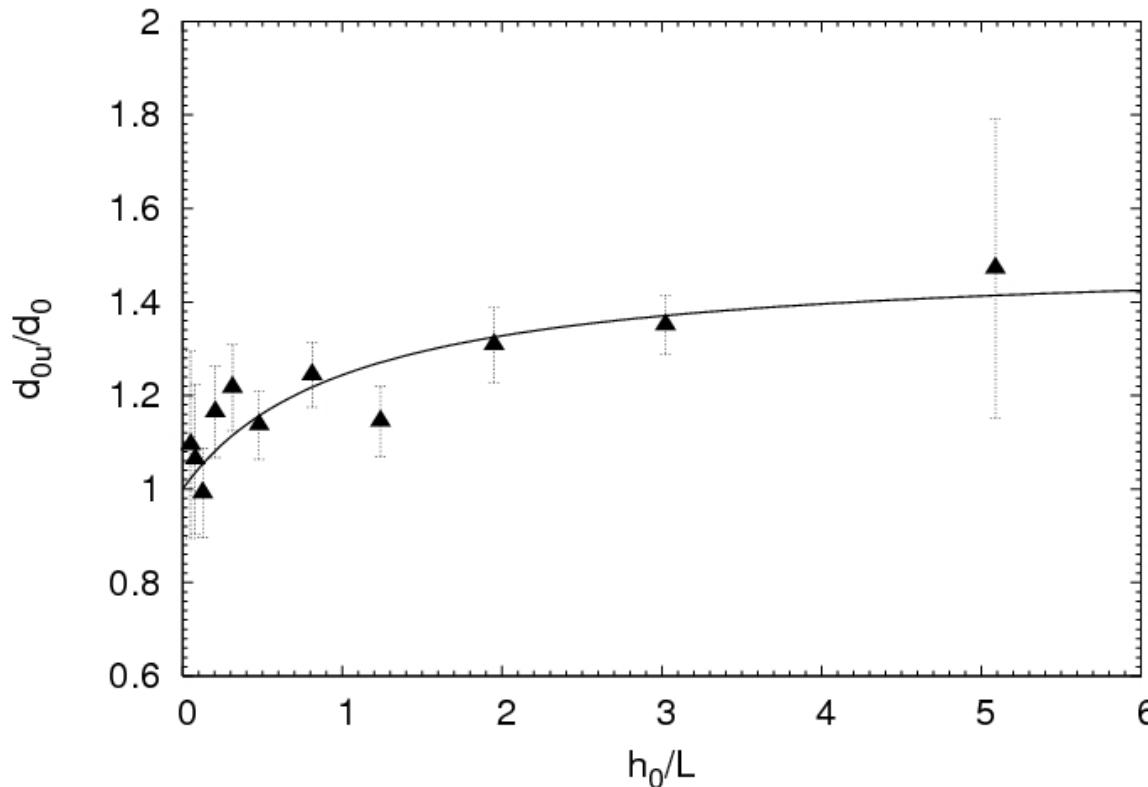
Stable stratification



Bin-average values of z_{0u}/z_0 (actual- over neutral-roughness lengths) versus h_0/L in stable stratification for boreal forest ($h_0=13.5$ m; $z_0=1.1\pm0.3$ m). Bars are standard errors; the curve is $z_{0u}/z_0=(1+8.13h_0/L)^{-1}$.



Stable stratification



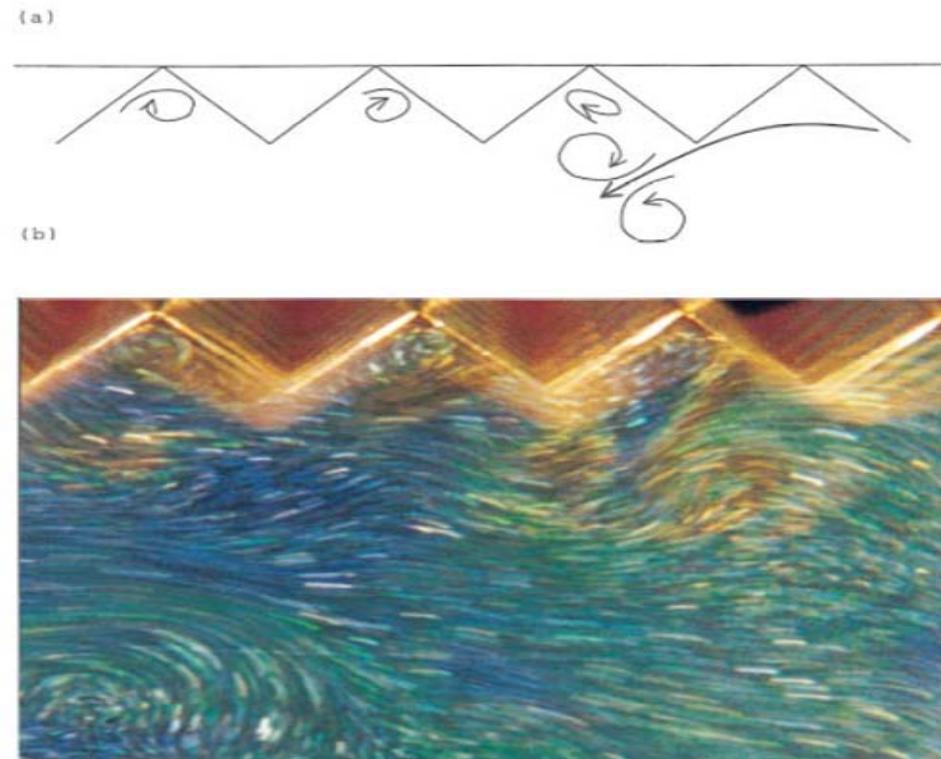
Displacement height over its neutral-stability value in stable stratification.
Boreal forest ($h_0 = 15$ m, $d_0 = 9.8$ m).

The curve is $d_{0u} / d_0 = 1 + 0.5(h_0 / L)(1.05 + h_0 / L)^{-1}$

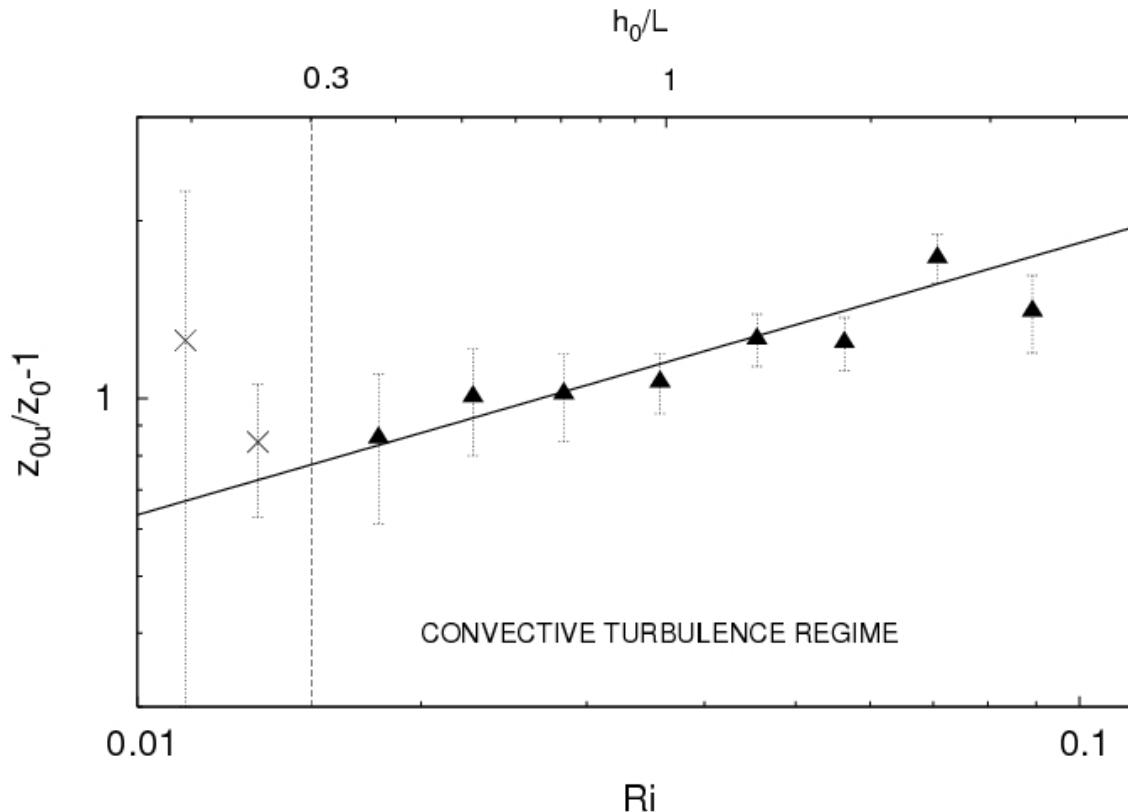
Unstable stratification

Convective eddies extend in the vertical causing $z_0 > z_{0u}$

VOLUME 81, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1998
Y.-B. Du and P. Tong, Enhanced Heat Transport in Turbulent Convection over a Rough Surface



Unstable stratification



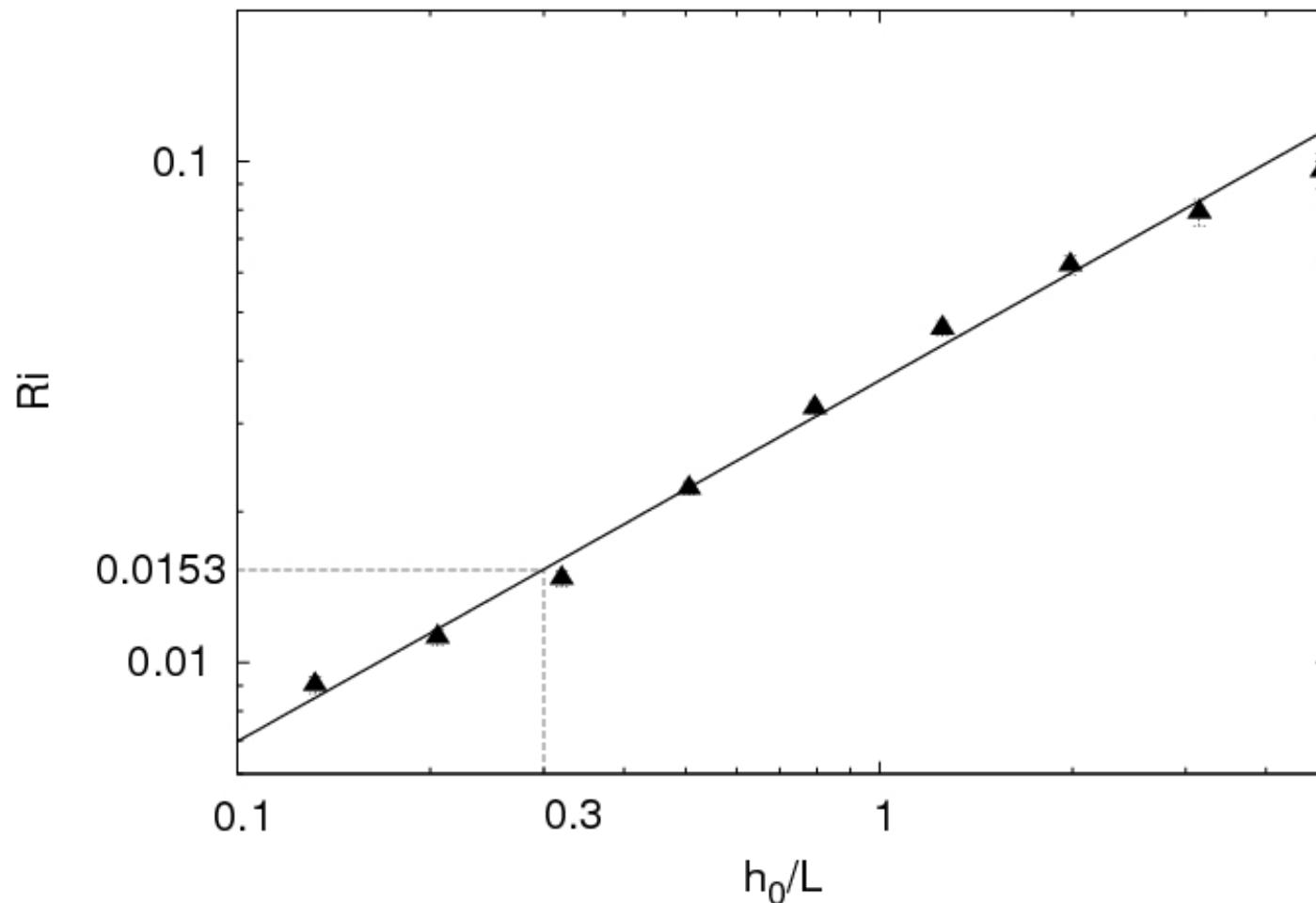
Unstable stratification, Basel, z_0/z_{0u} vs. $Ri = (gh_0/\Theta_{32})(\Theta_{18}-\Theta_{32})/(U_{32})^2$

Building height = 14.6 m, neutral roughness $z_0 = 1.2$ m; BUBBLE, Rotach et al., 2005).

h_0/L through empirical dependence on Ri on (next figure)

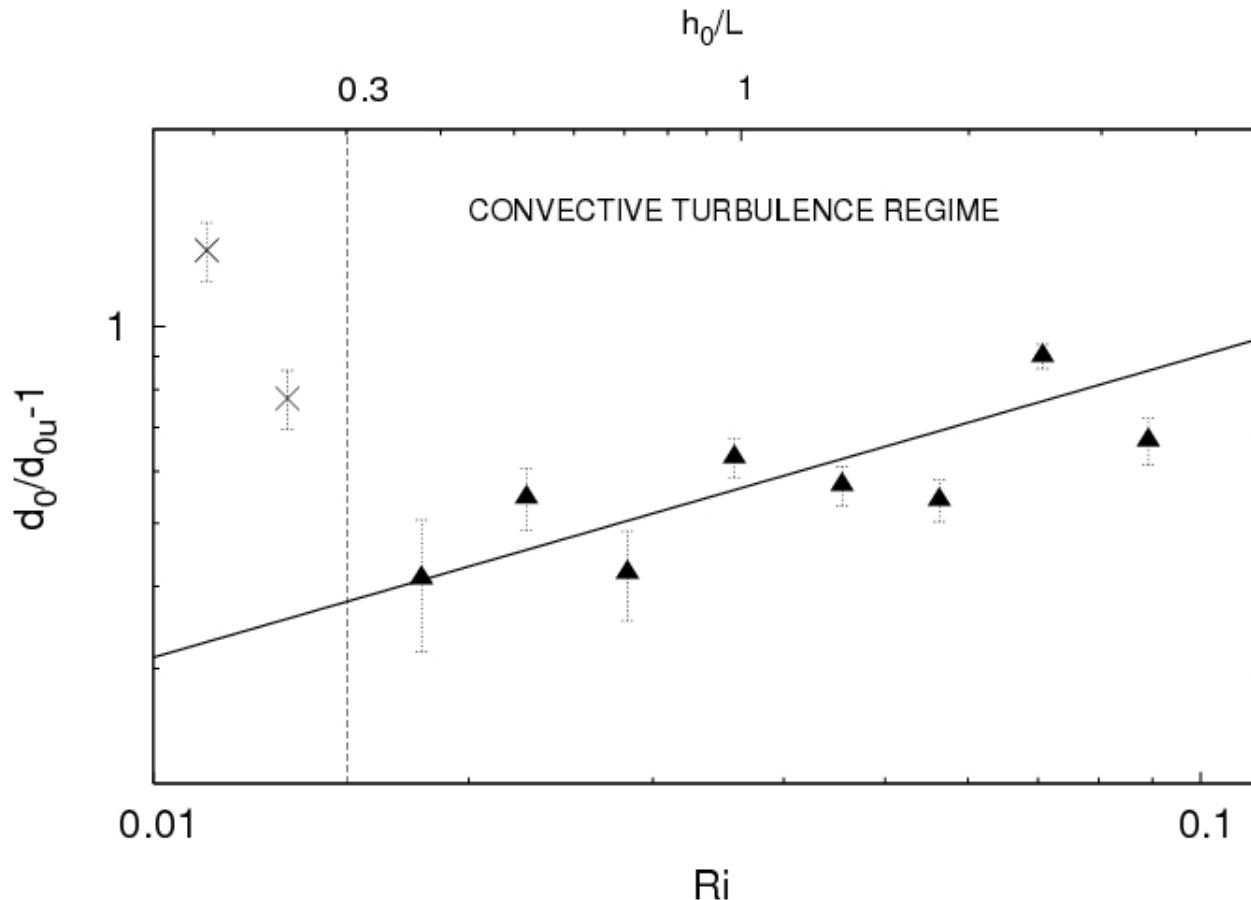
The curve ($z_0/z_{0u} = 1 + 5.31Ri^{6/13}$) confirms theoretical $z_{0u}/z_0 = 1 + 1.15(h_0/-L)^{1/3}$

Unstable stratification



$$\text{Empirical } Ri = 0.0365 (h_0/-L)^{13/18}$$

Unstable stratification



Displacement height in unstable stratification (Basel): $d_0 / d_{ou} - 1$ versus Ri

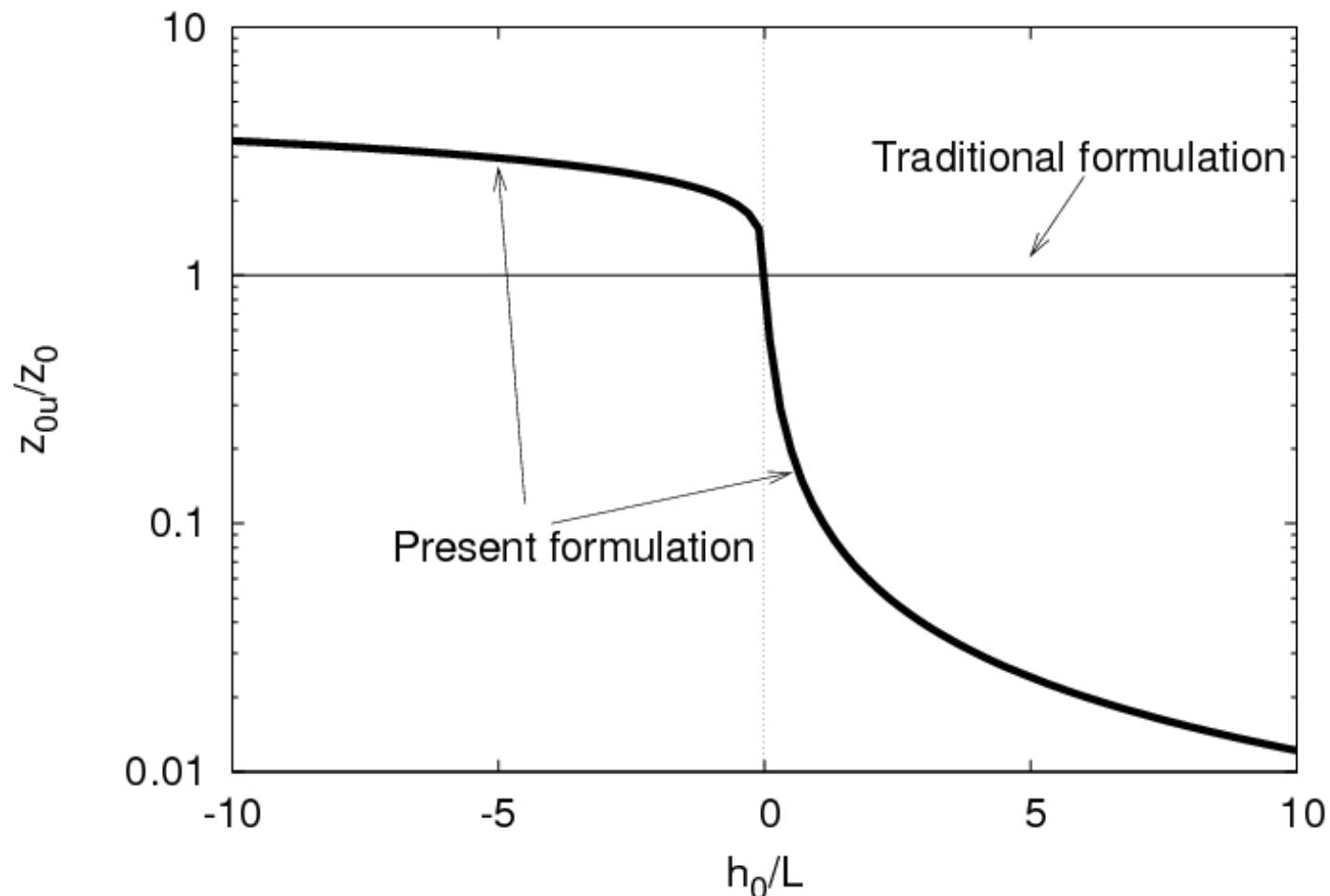
The line confirms theoretical dependence: $d_{0u} = \frac{d_0}{1 + C_{DC}(h_0 / -L)^{1/3}}$

STABILITY DEPENDENCE OF THE ROUGHNESS LENGTH

in the “meteorological interval” $-10 < h_0/L < 10$ after new theory and experimental data

Solid line: z_{0u}/z_0 versus h_0/L

Thin line: traditional formulation $z_{0u} = z_0$

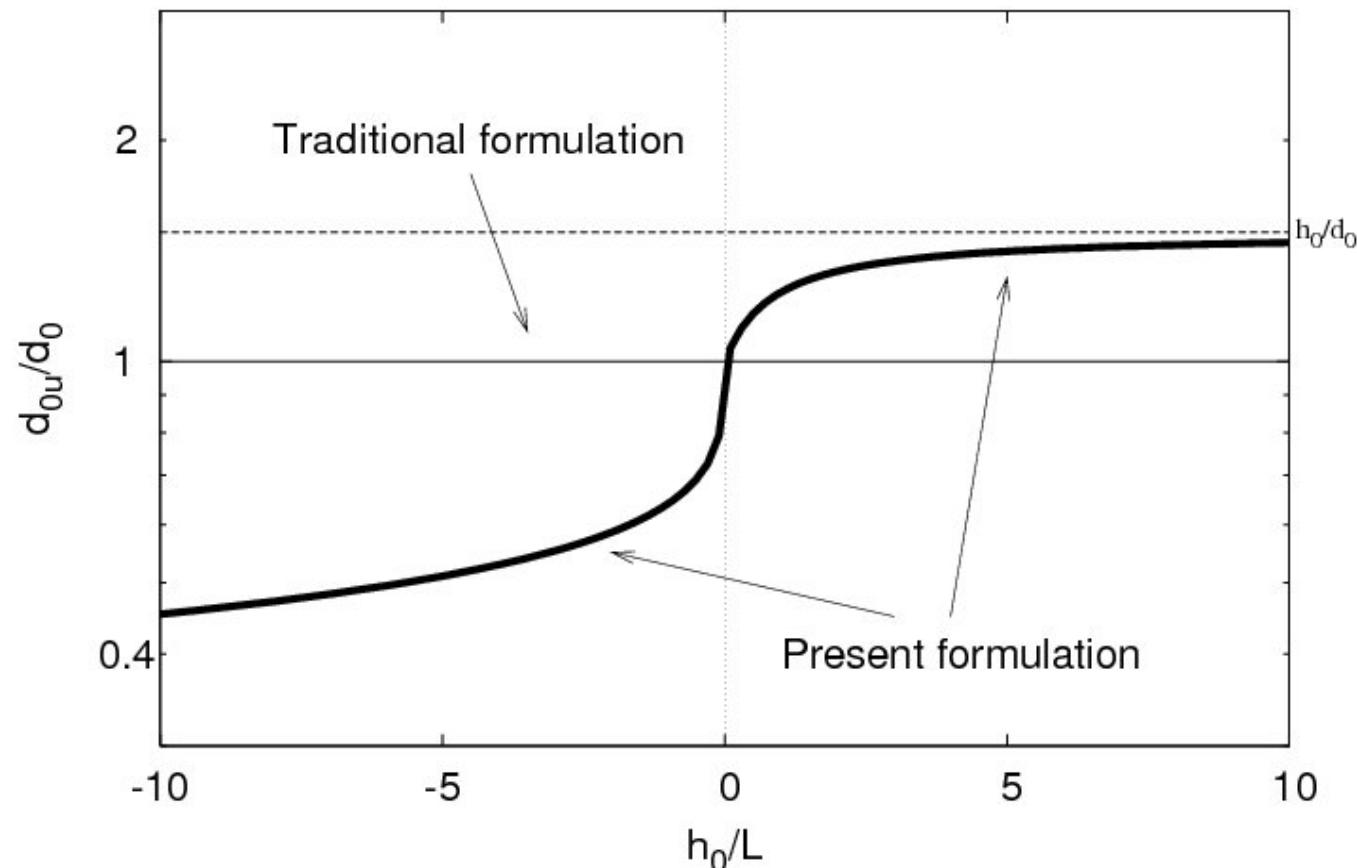


STABILITY DEPENDENCE OF THE DISPLACEMENT HEIGHT

in the “meteorological interval” $-10 < h_0/L < 10$ after new theory and experimental data

Solid line: d_{0u}/d_0 versus h_0/L

Dashed line: the upper limit: $d_0 = h_0$



Conclusions (Roughness Length)

- **Traditional concept:** roughness length and displacement height fully characterised by geometric features of the surface
- **New theory and data:** essential dependence on hydrostatic stability especially strong in stable stratification
- **Applications:** to urban and terrestrial-ecosystem meteorology
- **Especially:** urban air pollution episodes in very stable stratification



Lecture 3

NEUTRAL and STABLE ABL HEIGHT

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References

- Zilitinkevich, S., Baklanov, A., Rost, J., Smedman, A.-S., Lykosov, V., and Calanca, P., 2002: Diagnostic and prognostic equations for the depth of the stably stratified Ekman boundary layer. *Quart. J. Roy. Met. Soc.*, 128, 25-46.
- Zilitinkevich, S.S., and Baklanov, A., 2002: Calculation of the height of stable boundary layers in practical applications. *Boundary-Layer Meteorol.* 105, 389-409.
- Zilitinkevich S. S., and Esau, I. N., 2002: On integral measures of the neutral, barotropic planetary boundary layers. *Boundary-Layer Meteorol.* 104, 371-379.
- Zilitinkevich S. S. and Esau I. N., 2003: The effect of baroclinicity on the depth of neutral and stable planetary boundary layers. *Quart. J. Roy. Met. Soc.* 129, 3339-3356.
- Zilitinkevich, S., Esau, I. and Baklanov, A., 2007: Further comments on the equilibrium height of neutral and stable planetary boundary layers. *Quart. J. Roy. Met. Soc.*, 133, 265-271.



Factors controlling PBL height

Basic factors:

- Deepening due shear-generated turbulence
- Swallowing by earth's rotation and negative buoyancy forces:
 - (i) flow-surface interaction, (ii) free-flow stability atmosphere

Additional factors:

- baroclinic shears (enhances deepening)
- large-scale vertical motions (both ways))
- temporal and horizontal variability

Strategy:

Basic regimes → theoretical models → general formulation



Scaling analysis

Ekman (1905): $h_E \sim \sqrt{K_M / |f|}$; K_M in three basic regimes:

$$h_E^2 \sim \frac{K_M}{|f|}, \quad K_M \sim u_T l_T \sim \begin{cases} u_* h_E & \text{for TN} \\ u_* L_N & \text{for CN} \\ u_* L & \text{for NS} \end{cases}$$

$$l_T \sim h_E \text{ in TN} \quad L_N = u_* N^{-1} \text{ in CN} \quad L = -u_*^3 F_{bs}^{-1} \text{ in NS}$$

Basic formulations

$$h_E \sim \begin{cases} u_* |f|^{-1} \text{ Rossby, Montgomery (1935) TN} \\ u_* |fN|^{-1/2} \text{ Pollard et al. (1973) CN} \\ u_*^2 |fB_s|^{-1/2} \text{ Zilitinkevich (1972, 74) NS} \end{cases}$$



Dominant role of the smallest scale

$$\frac{1}{h_E^2} = \frac{f^2}{(C_R u_*)^2} + \frac{N |f|}{(C_{CN} u_*)^2} + \frac{|fB_s|}{(C_{NS} u_*^2)^2}, \quad C_R, C_{CN}, C_{NS} = \text{constant}$$

Four parameters u_*, f, N, B_s ; hence two dimensionless numbers:

$$\mu = u_* |fL|^{-1} \text{ and } \mu_N = N / |f|$$

More generally, h_E depends also on

- geostrophic shear $\Gamma = |\partial \mathbf{u}_g / \partial z|$ (increases h_E : Z & Esau, 2003)
- vertical velocity w_h ($\pm w_h t_{PBL}$, $t_{PBL} \sim h_E / u_*$: Z & Baklanov, 2002).

Hence, additional (usually unavailable) parameters:

$$\mu_\Gamma = \Gamma / N \text{ and } \mu_w = w_h / u_*$$



How to verify h -equations?

Stage I: TN $h_E = C_R u_* / f$ transitions TN \rightarrow CN and TN \rightarrow NS

$$\left(\frac{u_*}{fh_E}\right)^2 = \begin{cases} C_R^{-2} + C_{CN}^{-2} \mu_N & \text{TN - CN} \\ C_R^{-2} + C_{NS}^{-2} \mu & \text{TN - NS} \end{cases}$$

to determine constants C_R , C_{CN} , C_{NS} from selected high-quality data

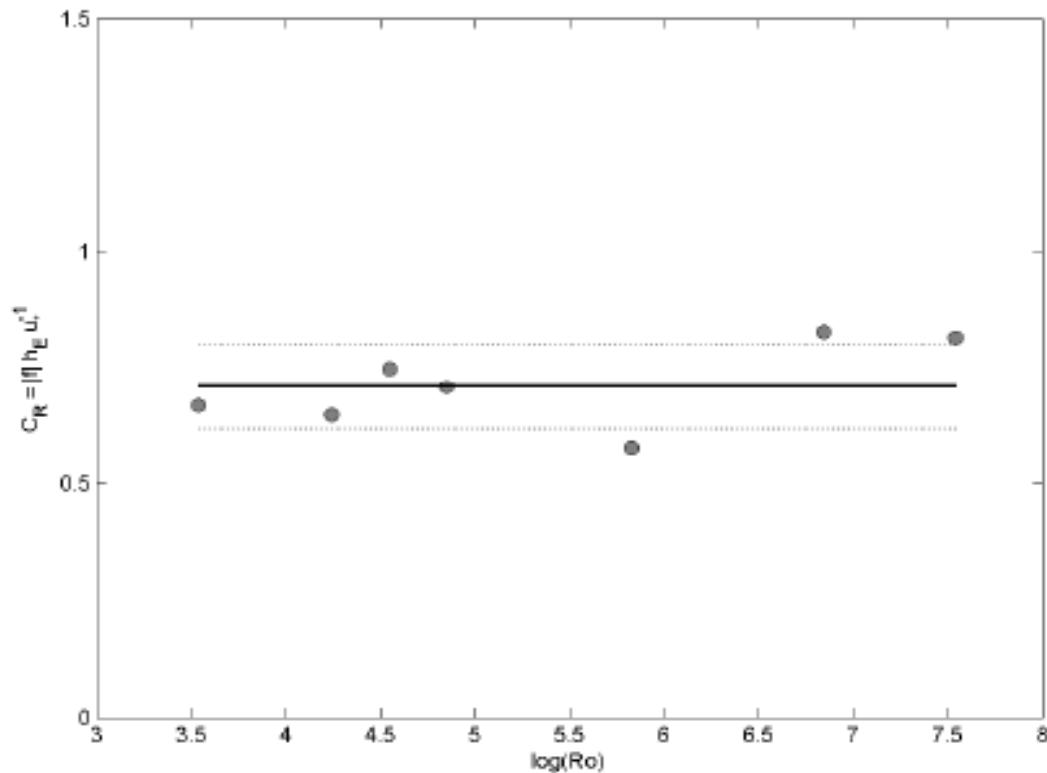
Stage II: Substitute constants in

$$\frac{1}{h_E^2} = \frac{f^2}{(C_R u_*)^2} + \frac{N |f|}{(C_{CN} u_*)^2} + \frac{|fB_s|}{(C_{NS} u_*^2)^2}$$

and verify against all available data



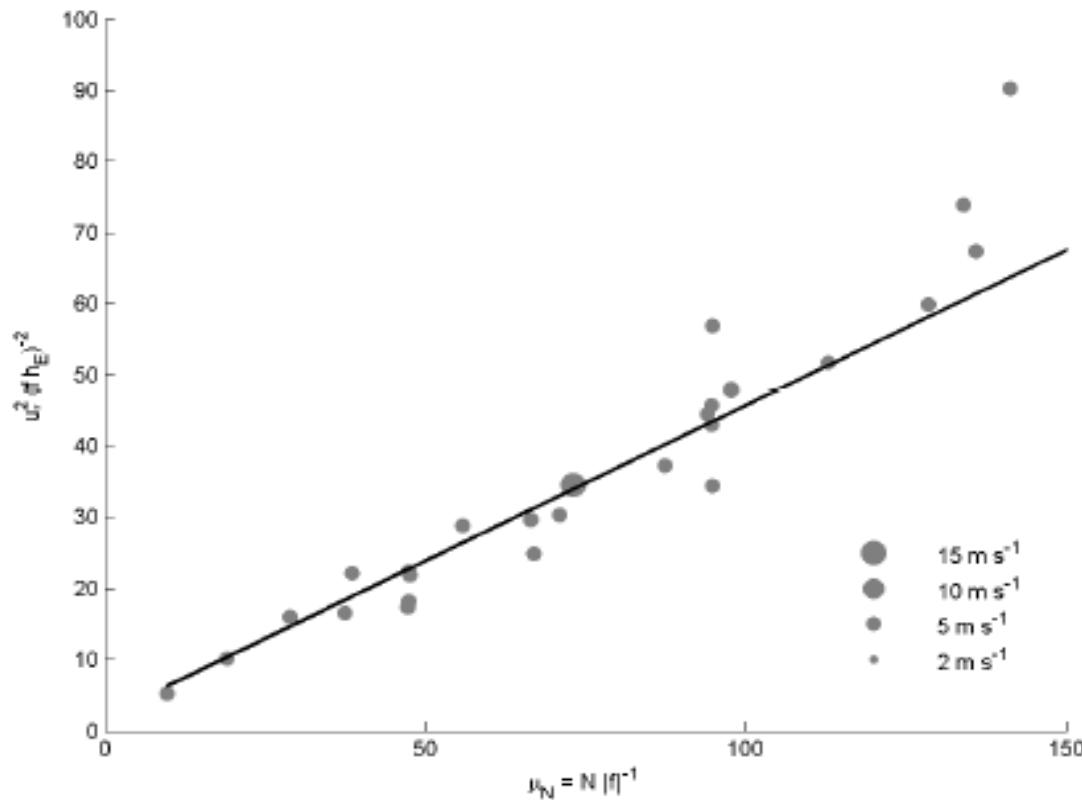
Stage I: Truly neutral ABL



Stage I TN ABL: C_R vs. $\text{Ro} = U_g (|f| z_{0u})^{-1}$ after LES

Bold line: $C_R = 0.7 \pm 0.1$. Dotted line: standard deviation

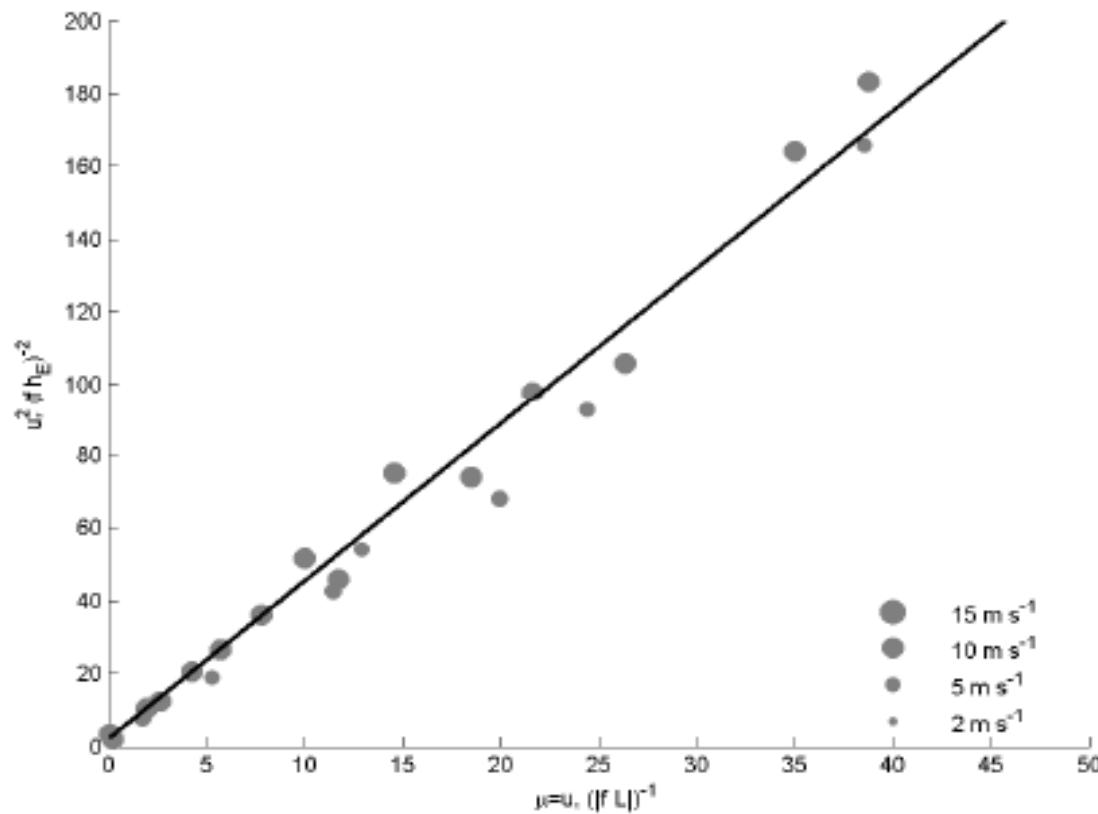
Stage I: Transition TN \rightarrow CN ABL



Stage I Transition TN \rightarrow CN: $u_*^2(fh_E)^{-2}$ vs. $\mu_N = N / |f|$, after LES:

Theory: $u_*^2(fh_E)^{-2} = C_R^{-2} + C_{CN}^{-2}\mu_N$. Empirical $C_R=0.6$, $C_{CN}=1.36$

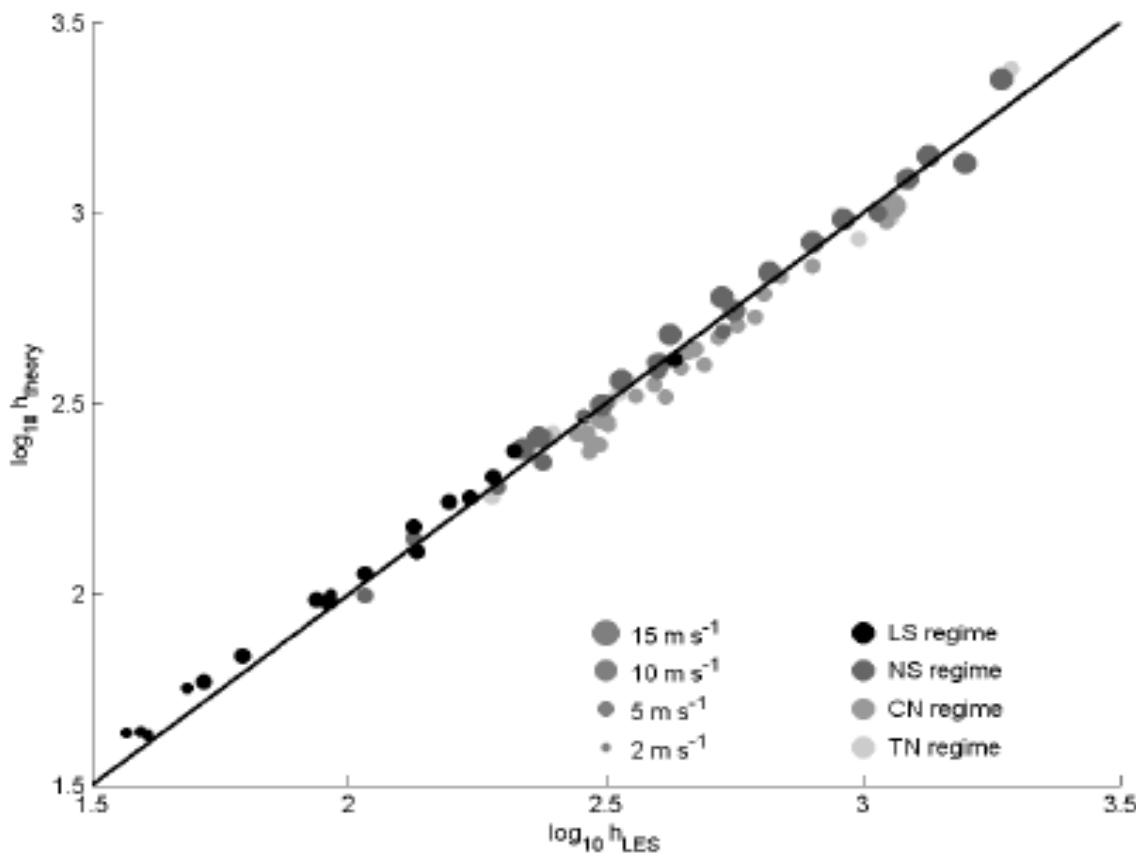
Stage I: Transition TN→NS ABL



Stage I Transition TN → NS: $u_*^2(fh_E)^{-2}$ vs. $\mu = u_* |fL|^{-1}$, after LES.

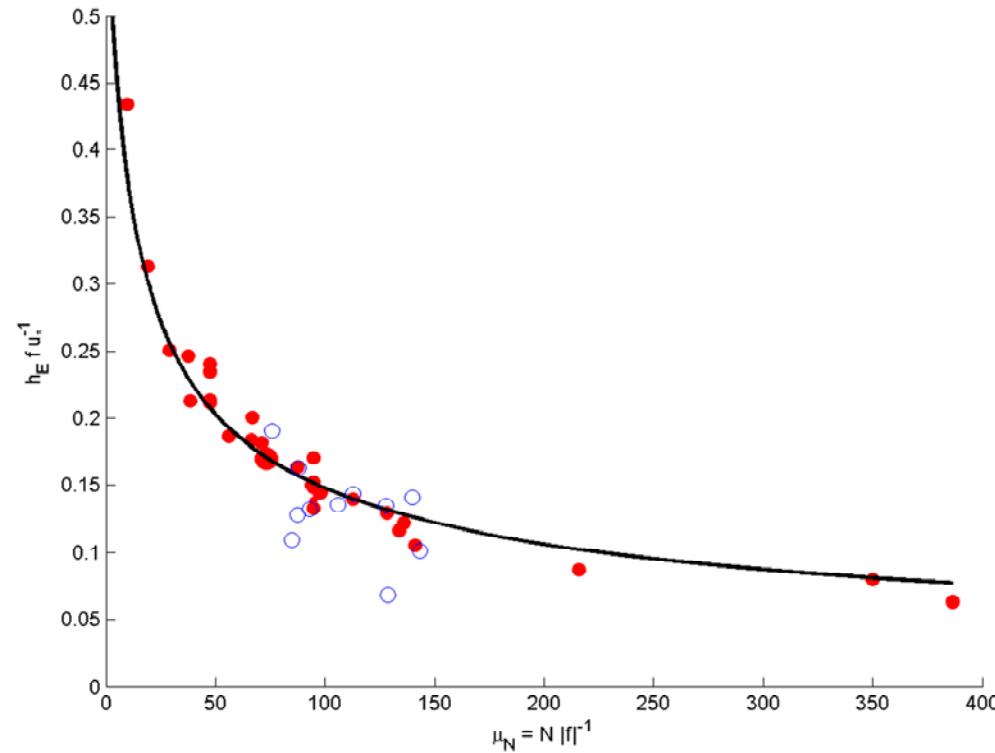
Theory: $u_*^2(fh_E)^{-2} = C_R^{-2} + C_{NS}^{-2}\mu$, empirical $C_R = 0.6$, $C_{NS} = 0.51$

Stage II: General case



Stage II: Correlation: h_{theory} vs. h_{LES} after all available LES data

The height of the conventionally neutral (CN) ABL



Z & Esau, 2002, 2007: the effect of free-flow stability (N) on CN ABL height, h_E , (LES – red; field data – blue; theory – curve). Traditional theory overlooks this dependence and overestimates h_E up to an order of magnitude.

Conclusions (SBL height)

- h_E , depends on many factors → multi-limit analysis / complex formulation
- difficult to measure: baroclinic shear (Γ), vertical velocity (w_h), h_E itself
- hence use LES, DNS and lab experiments
- baroclinic ABL: substitute $u_T = u_*(1 + C_0 \Gamma/N)^{1/2}$ for u_* in the 2nd term of

$$\frac{1}{h_E^2} = \frac{f^2}{C_R^2 \tau_*} + \frac{N |f|}{C_{CN}^2 \tau_*} + \frac{|f\beta F_*|}{C_{NS}^2 \tau_*^2} \quad (C_R = 0.6, C_{CN} = 1.36, C_{NS} = 0.51)$$

- account for vertical motions: $h_{E-\text{corr}} = h_E + w_h t_T$, where $t_T = C_t h_E / u_*$
- generally prognostic (relaxation) equation (Z. and Baklanov, 2002):

$$\frac{\partial h}{\partial t} + \vec{U} \cdot \nabla h - w_h = K_h \nabla^2 h - C_t \frac{u_*}{h_E} (h - h_E) \quad (C_t = 1)$$



Thank you for your attention

