QCD ANALYSIS FOR HEAVY QUARK CONTRIBUTIONS

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QCD ANALYSIS FOR HEAVY QUARK CONTRIBUTIONS

- Motivation
- Why is HERA relevant for LHC?
- DIS Kinematics
- The Method of the QCD analysis
- The Wilson Coefficients in the Region Q^2 >> M^2
- QCD Analysis of Longitudinal Structure Function
- Results and conclusion

Motivation

▷ The deep-inelastic lepton–nucleon scattering is the source of important information about the nucleons structure.

▷ All calculations of high energy processes with initial hadrons, whether within the standard model or exploring new physics, require parton distribution functions (PDF's) as an essential input.

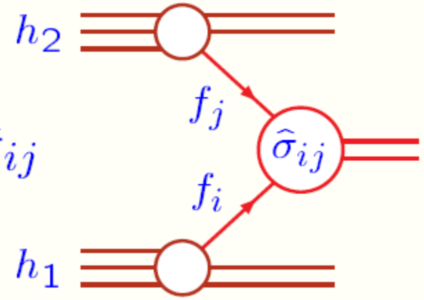
▷ The assessment of PDF's, their uncertainties and extrapolation to the kinematics relevant for future colliders such as the LHC is an important challenge to high energy physics in recent years.

▷ The PDF's are derived from global analysis of experimental data from a wide range of hard processes in the framework of perturbative quantum chromodynamics.

Why is HERA relevant for LHC?

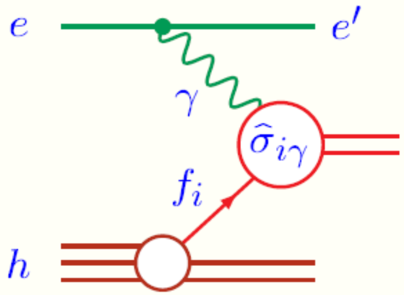
- Hard h-h cross sections (Tevatron, LHC):

$$\sigma_{hh} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij}$$



- PDFs cannot be calculated from theory and are obtained from fits to data
- DIS data are important: the structureless e, μ or ν directly probes the hadron:

$$\sigma_{eh} = \sum_{i} f_i \otimes \hat{\sigma}_{i\gamma}$$

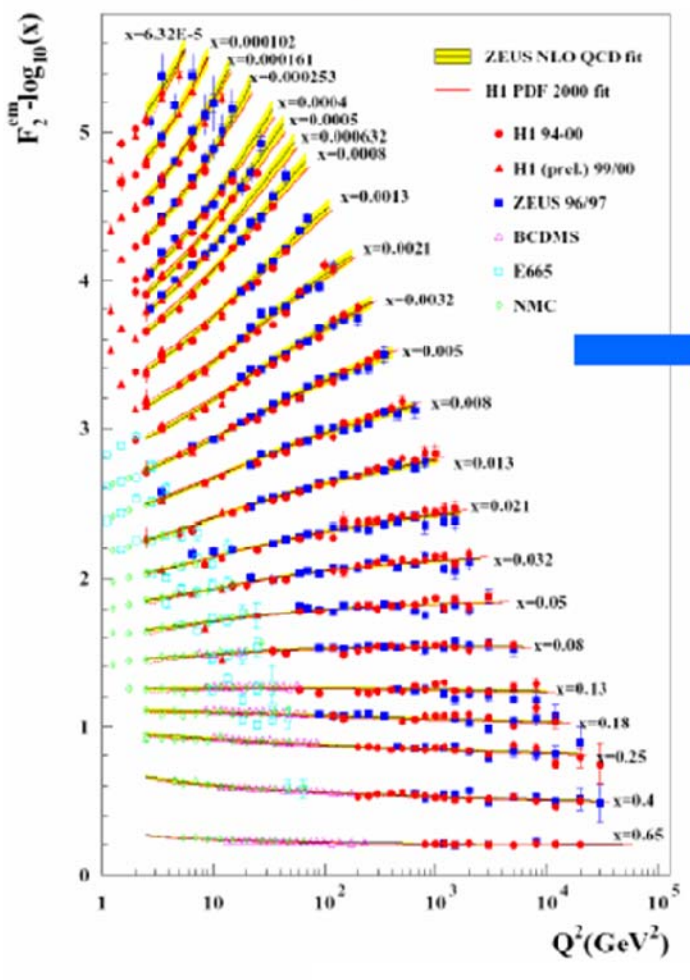


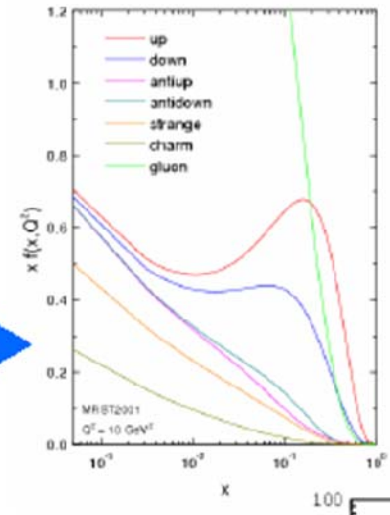
- Standard sets (GRV, MRST, CTEQ) are without PDF errors but we need these for a proper comparison of data and theory

… it is important to measure the proton structure at HERA and use this information for physics at the LHC

Why is HERA relevant for LHC?

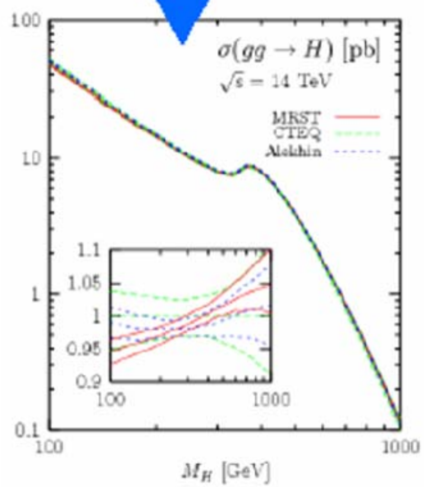
Structure of the Proton : PDFs from HERA



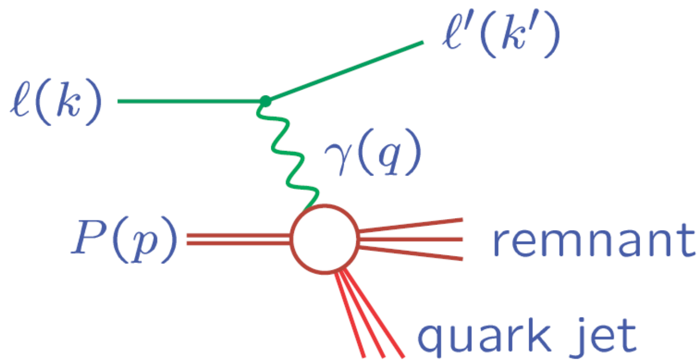


to

LHC

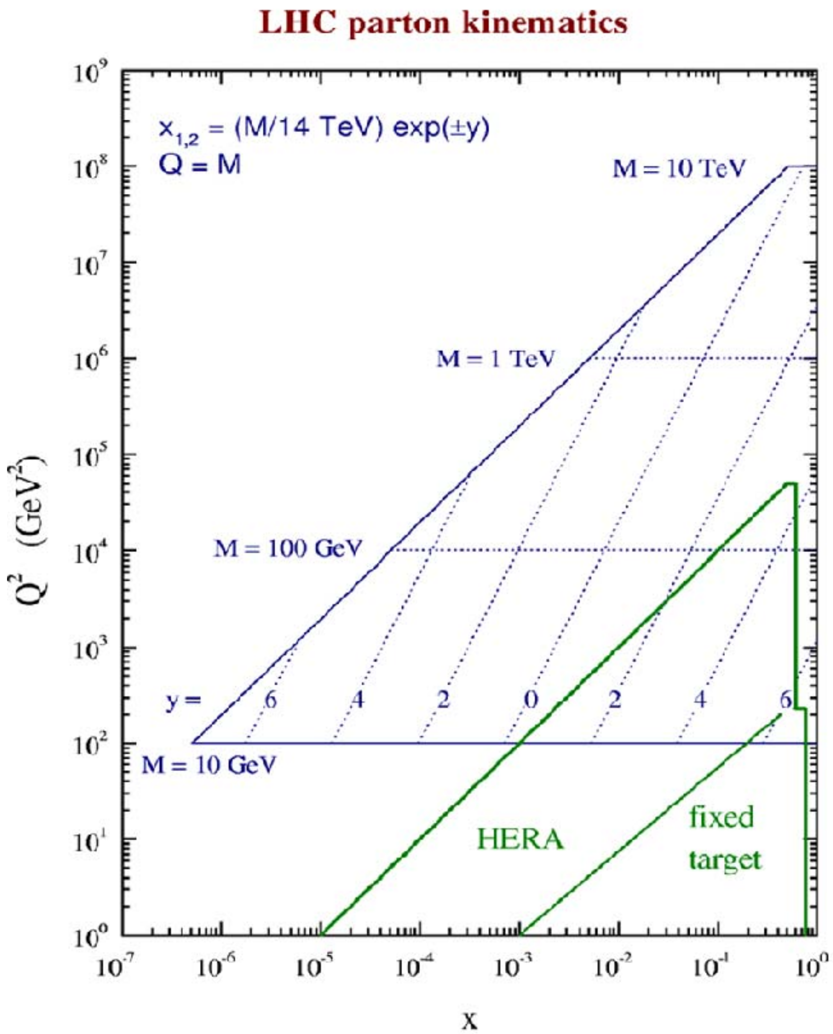


DIS Kinematics



s	$= (k+p)^2$	ℓP centre of mass energy squared
Q^2	$= -q^2$	Negative 4-momentum transfer squared
x	$= Q^2/(2p\cdot q)$	Parton momentum fraction in Breit frame
y	$= (p\cdot q)/(p\cdot k)$	E_γ/E_ℓ in proton rest frame (inelasticity)
W^2	$= (p+q)^2$	γP centre of mass energy squared

Why is HERA relevant for LHC?



Introduction

Deeply inelastic electron–nucleon scattering at large momentum transfer provides one of the cleanest possibilities to test the predictions of QCD.

In the case of pure photon exchange the structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$ describe the scattering cross section.

In lowest order in the coupling constant (α_s^0) and vanishing target–mass effects, the structure functions F_2 and F_L obey the Callan–Gross relation

$$F_2(x, Q^2) = 2xF_1(x, Q^2), \qquad F_L(x, Q^2) \equiv 0 \; . \tag{1}$$

C.G. Callan, and D.J. Gross, Phys. Rev. Lett. **22** (1969) 156.

Introduction

$F_L(x, Q^2)$ receives leading order contributions due to target mass effects. The Callan–Gross relation is further broken by QCD corrections. The corresponding Wilson coefficients for massless quarks were calculated in leading (LO), next-to-leading (NLO), and next-to-next-to-leading order (NNLO).

Since the longitudinal structure functions $F_L(x, Q^2)$ contains rather large heavy flavor contributions in the small x region, a consistent analysis has to account for these effects, which were calculated in leading and next-to-leading order.

The Method

The nucleon structure functions $F_i(x, Q^2)$ are described as Mellin convolutions between the parton densities $f_j(x, \mu^2)$ and the Wilson coefficients $C_i^j(x, Q^2/\mu^2)$

$$F_i(x, Q^2) = \sum_j C_i^j \left(x, \frac{Q^2}{\mu^2} \right) \otimes f_j(x, \mu^2) \qquad (2)$$

to all orders in perturbation theory.

Here μ^2 denotes the factorization scale and the Mellin convolution is given by the integral

$$[A \otimes B](x) = \int_0^1 dx_1 \int_0^1 dx_2 \;\; \delta(x - x_1 x_2) \; A(x_1) B(x_2) \; . \quad (3)$$

distributions f_j refer to massless partons, the heavy flavor effects are contained in the Wilson coefficients only.

Talks in PDF4LHC meeting at CERN, July 2008

The Method

The longitudinal structure function $F_L(x, Q^2)$ consists of the light and heavy flavor contributions

$$
\begin{aligned}
F_L(x,Q^2) &= F_L^{\text{light}}(x,Q^2) + F_L^{\text{heavy}}(x,Q^2) \\
&= C_L^{\text{NS}}\left(x, a_s, \frac{Q^2}{\mu^2}\right) \otimes q_{\text{NS}}(x,\mu^2) \\
&\quad + C_L^{\text{S}}\left(x, a_s, \frac{Q^2}{\mu^2}\right) \otimes q_{\text{S}}(x,\mu^2) \\
&\quad + C_L^{g}\left(x, a_s, \frac{Q^2}{\mu^2}\right) \otimes g(x,\mu^2) \ . \qquad (4)
\end{aligned}
$$

The Wilson Coefficients in the Region $Q^2 >> M^2$

We choose $Q^2 = \mu^2$ as uniform factorization scale. The Wilson coefficients in (4) are of the form

$$C_L^{(i)}\left(x, a_s, \frac{Q^2}{\mu^2}\right) = C_L^{(i),\text{light}}(x, a_s) + H_L^{(i)}\left(x, a_s, \frac{Q^2}{m^2}\right), \quad i = \text{S}, \text{NS}, \ g \ . \quad (5)$$

Not that in the above C_L^i given by

$$C_L^i\left(\frac{Q^2}{\mu^2}\right) = \sum_{l=l_0}^{\infty} a_s^l C_{L,i}^{(l)}\left(\frac{Q^2}{\mu^2}\right) , \quad (6)$$

In the limit $Q^2 \gg m^2$ the massive Wilson coefficients $H_{2,L,i}^{\text{S,NS}}$, are reported in

Isabella Bierenbaum, Johannes Blümlein and Sebastian Klein, hep-ph/0703285

The Wilson Coefficients in the Region $Q^2 >> M^2$

The $\overline{\text{MS}}$ coefficient functions, in the massless limit, are denoted by

$$\widehat{C}_{L,k}\left(\frac{Q^2}{\mu^2}\right) = C_{L,k}\left(\frac{Q^2}{\mu^2}, N_L + N_H\right) - C_{L,k}\left(\frac{Q^2}{\mu^2}, N_L\right) \quad , \qquad (7)$$

N_H, N_L: are the number of heavy and light flavors, respectively.

In the following we will consider the case of a single heavy quark, i.e. $N_H = 1$. The formalism is easily generalized to more than one heavy quark species.

Coupling constant up to 3-loops

We introduce the 3-loop evolution of the coupling via its β-function

$$\beta(\alpha_s) \equiv \frac{\partial \alpha_s(Q^2)}{\partial \log Q^2},$$

and its three-loop expansion is

$$\beta(\alpha_s) = -\frac{\beta_0}{4\pi}\alpha_s^2 - \frac{\beta_1}{16\pi^2}\alpha_s^3 - \frac{\beta_2}{64\pi^3}\alpha_s^4 + O(\alpha_s^5),$$

where

$$\begin{aligned}
\beta_0 &= \frac{11}{3}N_C - \frac{4}{3}T_f, \\
\beta_1 &= \frac{34}{3}N_C^2 - \frac{10}{3}N_C n_f - 2C_F n_f, \\
\beta_2 &= \frac{2857}{54}N_C^3 + 2C_F^2 T_f - \frac{205}{9}C_F N_C T_f - \\
&\quad \frac{1415}{27}N_C^2 T_f + \frac{44}{9}C_F T_f^2 + \frac{158}{27}N_C T_f^2,
\end{aligned}$$

are the coefficients of the beta function.
We have set

$$N_C = 3, \qquad C_F = \frac{N_C^2 - 1}{2N_C} = \frac{4}{3}, \qquad T_f = T_R n_f = \frac{1}{2}n_f,$$

where:
N_C is the number of colors,
n_f is the number of active flavors.

Coupling constant up to 3-loops

The solutions of the QCD β-function in the LO, NLO and NNLO are as following

$$A_s^{LO} = \frac{1}{\beta_0 \ln Q^2/\Lambda^2_{\overline{MS}}}$$

$$A_s^{NLO} = \frac{1}{\beta_0 \ln Q^2/\Lambda^2_{\overline{MS}}} - \frac{\beta_1 \ln(\ln Q^2/\Lambda^2_{\overline{MS}})}{\beta_0^3 (\ln Q^2/\Lambda^2_{\overline{MS}})^2}$$

$$\begin{aligned} A_s^{NNLO} &= \frac{1}{\beta_0 \ln Q^2/\Lambda^2_{\overline{MS}}} - \frac{\beta_1 \ln(\ln Q^2/\Lambda^2_{\overline{MS}})}{\beta_0^3 (\ln Q^2/\Lambda^2_{\overline{MS}})^2} + \\ & \frac{1}{\beta_0^5 (\ln Q^2/\Lambda^2_{\overline{MS}})^3} [\beta_1^2 \ln^2(\ln Q^2/\Lambda^2_{\overline{MS}}) - \\ & \beta_1^2 \ln(\ln Q^2/\Lambda^2_{\overline{MS}}) + \beta_2\beta_0 - \beta_1^2] \end{aligned}$$

$$A_s = \alpha_s/(4\pi)$$

QCD Analysis of Longitudinal Structure Function

One of the simplest and fastest possibilities in the structure function (SF) reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion. The Jacobi polynomials are especially suited since they allow one to factor out an essential part of the x-dependence of the SF into the weight function.

S. I. Alekhin, et al., Phys. Lett. B **452**, (1999) 402.
A. L. Kataev, et al., Nucl. Phys. B **573**, (2000) 405.
A. L. Kataev, et al., Phys. Part. Nucl. **34**, (2003) 20;

S. Atashabr Tehrani and Ali N. Khorramian, JHEP **0707**, 048 (2007);
Ali N. Khorramian , et al., Acta Phys. Polon. B **38**, 3551 (2007);
A. N. Khorramian and S. A. Tehrani, J. Phys. Conf. Ser. **110** (2008) 022022.

Ali N. Khorramian and S. Atashabr Tehrani, Phys. Rev. D **78**, (2008) 074019, arXiv:0805.3063 [hep-ph].

QCD Analysis of Longitudinal Structure Function

Thus, given the Jacobi moments $a_n(Q^2)$, a structure function $f(x, Q^2)$ may be reconstructed in a form of the series

$$xf(x, Q^2) = x^\alpha (1-x)^\beta \sum_{n=0}^{N_{max}} a_n(Q^2) \theta_n^{\alpha,\beta}(x), \qquad (8)$$

where N_{max} is the number of polynomials and the Jacobi polynomials

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) x^j, \qquad (9)$$

where $c_j^{(n)}(\alpha, \beta)$ are the coefficients that expressed through Γ-functions and satisfy the orthogonality relation with the weight $x^\alpha(1-x)^\beta$ as following

$$\int_0^1 dx x^\alpha (1-x)^\beta \Theta_k^{\alpha,\beta}(x) \Theta_l^{\alpha,\beta}(x) = \delta_{k,l} \ , \qquad (10)$$

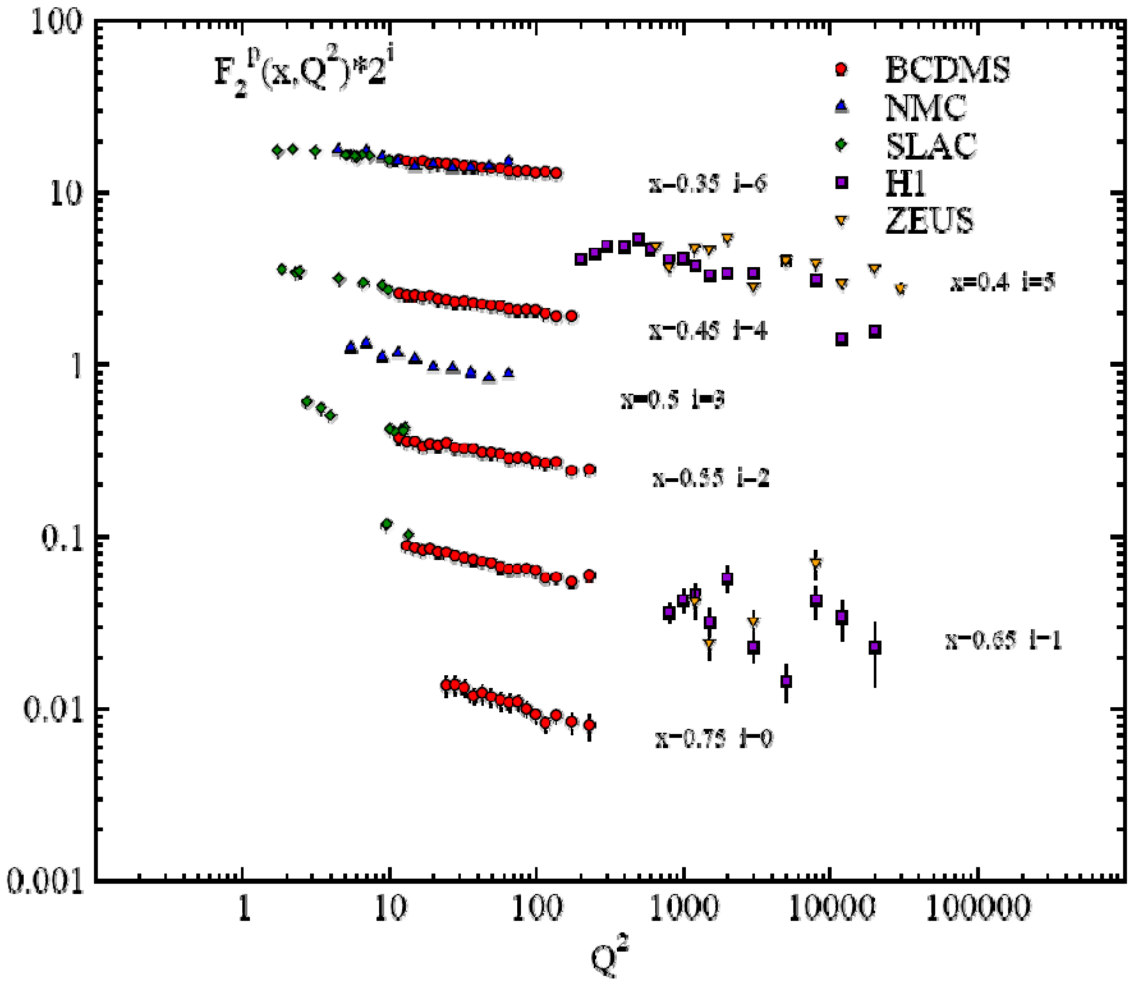
QCD Analysis of Longitudinal Structure Function

Using now Eqs. (8-9), one can relate the SF with its Mellin moments

$$F_L^{N_{max}}(x,Q^2) = x^{\alpha}(1-x)^{\beta}\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta) \times M_{F_L}(j+2,Q^2), \quad (11)$$

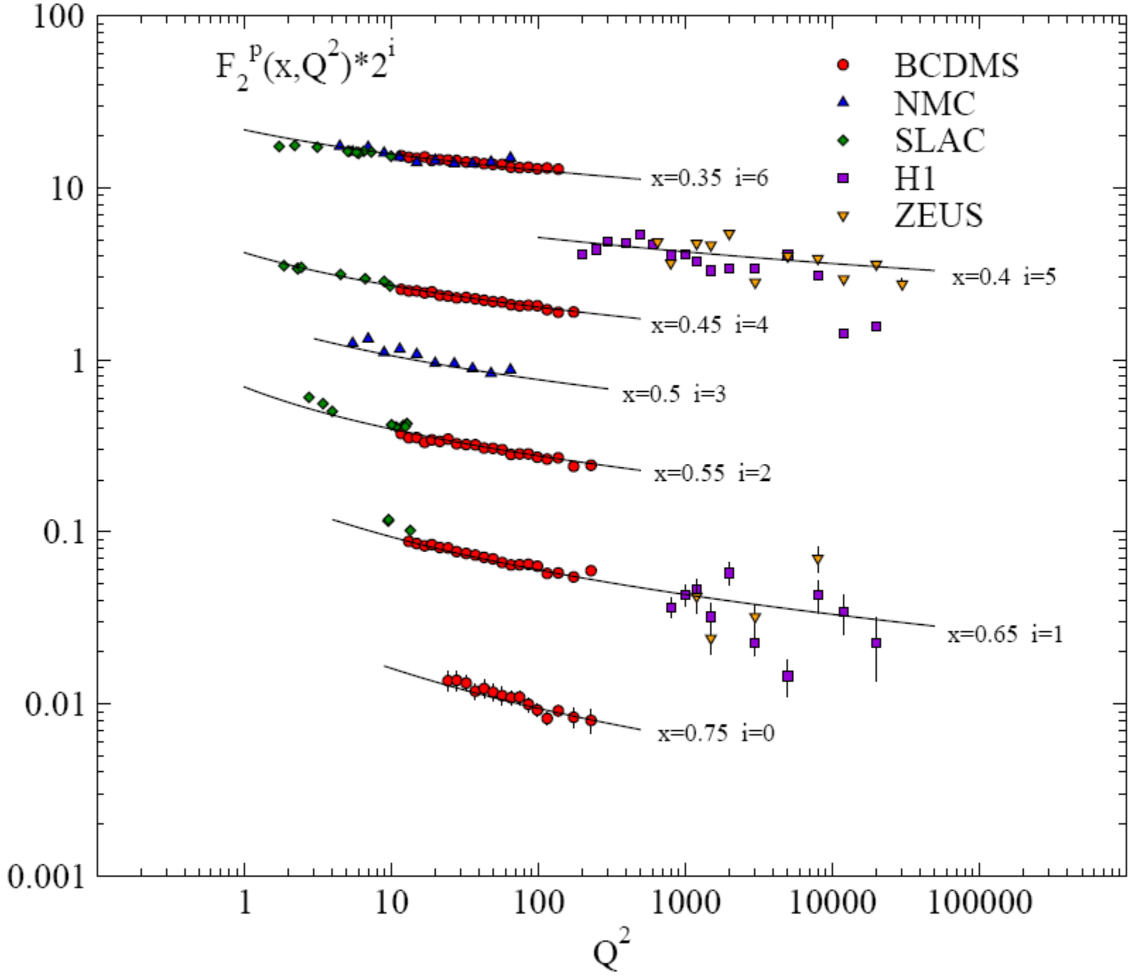
where $M_{F_L}(j+2,Q^2)$ are the moments of longitudinal structure function. It is obvious that the Q^2-dependence of the structure function is defined by the Q^2-dependence of the moments. The equation (11) is valid both for the singlet and non singlet SF.

QCD Fits for Nonsinglet Structure Function



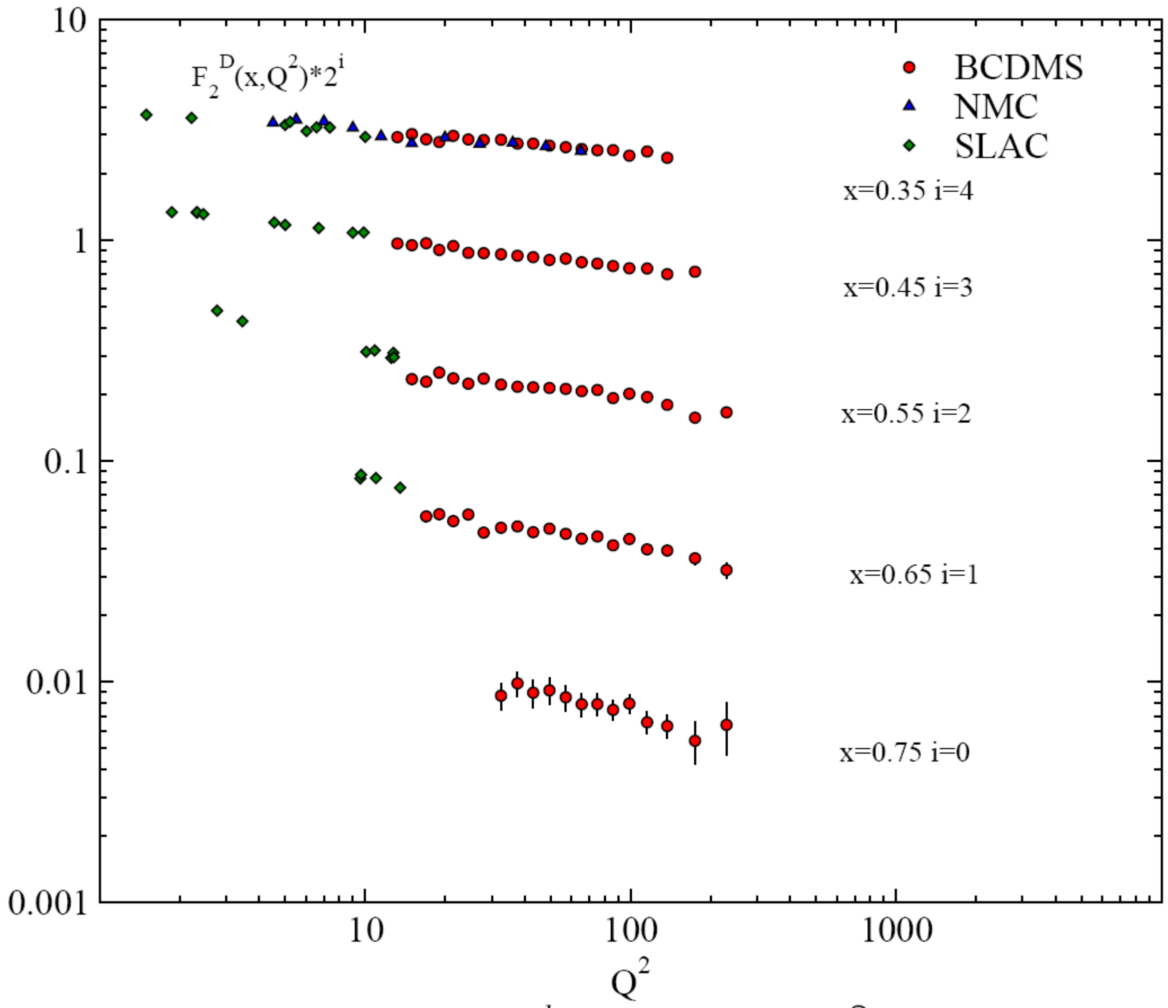
The structure function F_2^p as function of Q^2 in intervals of x.

QCD Fits



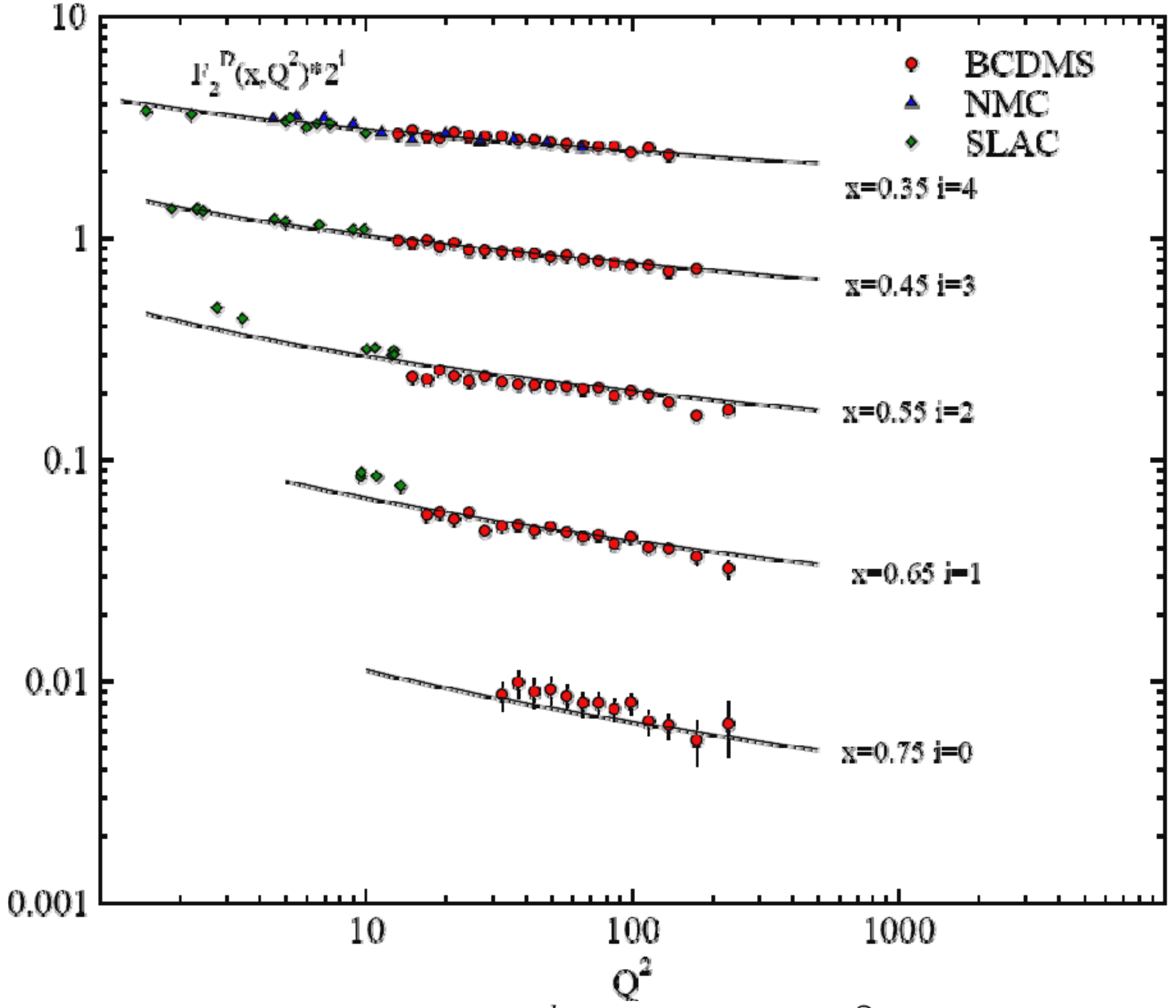
The structure function F_2^p as function of Q^2 in intervals of x.

QCD Fits



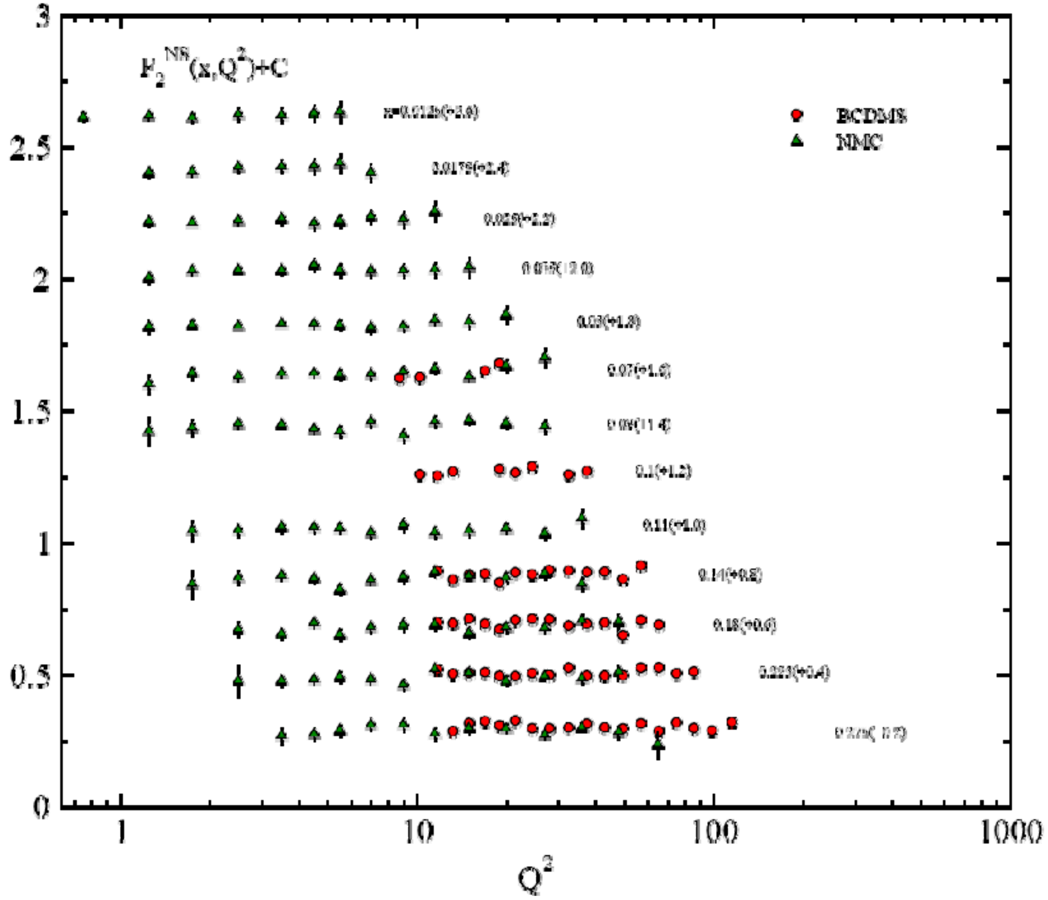
The structure function F_2^d as function of Q^2 in intervals of x.

QCD Fits



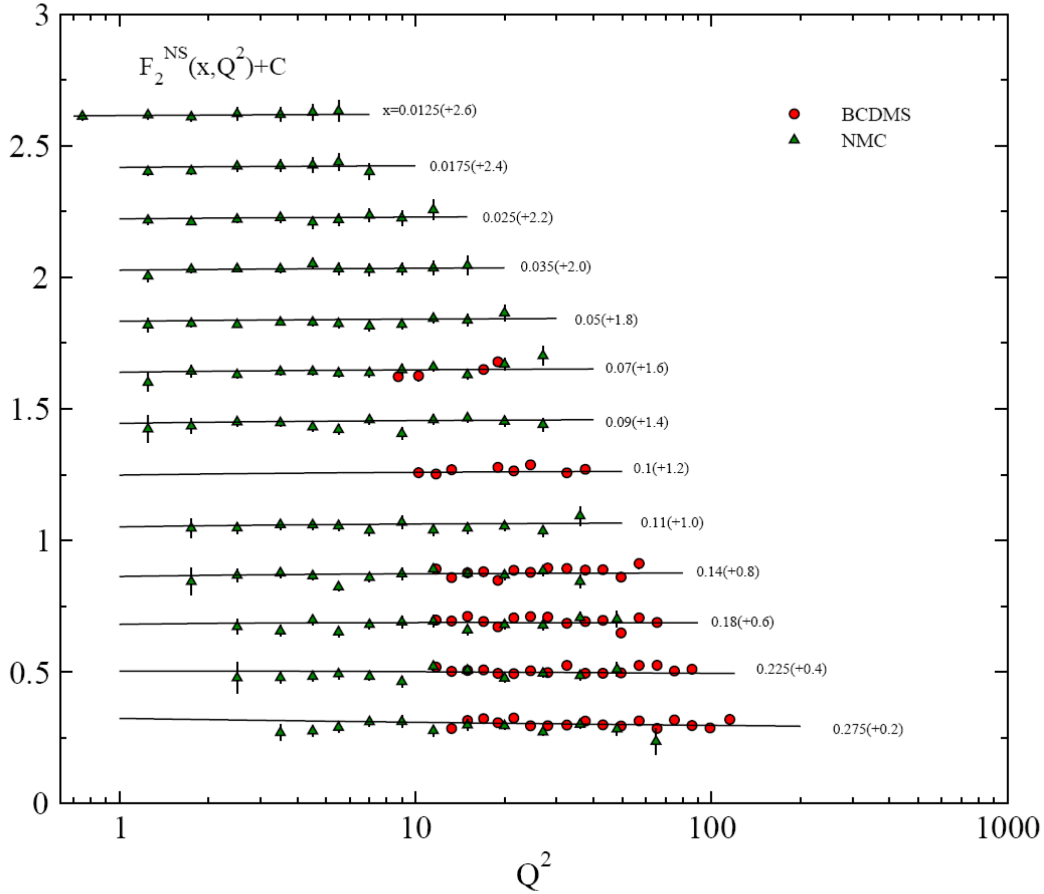
The structure function F_2^d as function of Q^2 in intervals of x.

QCD Fits



The structure function F_2^{NS} as function of Q^2 in intervals of x.

QCD Fits



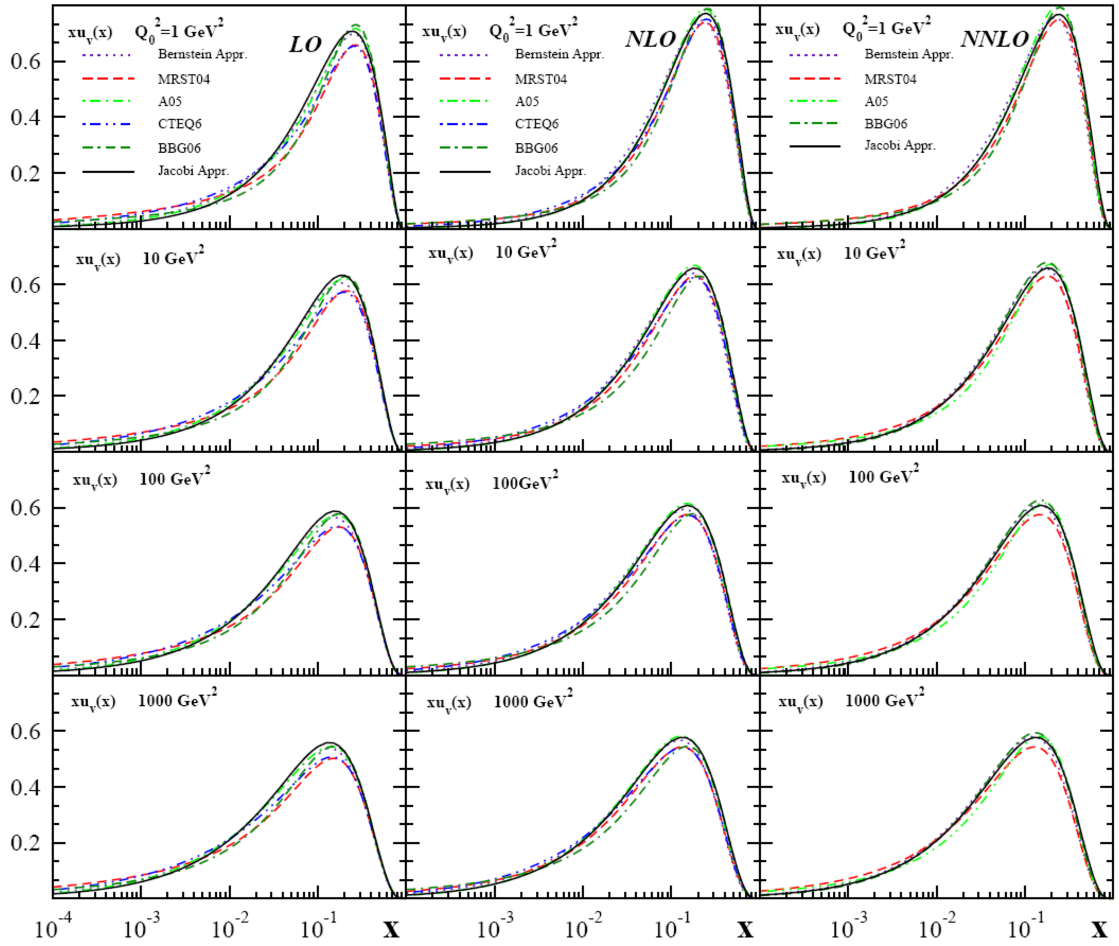
The structure function F_2^{NS} as function of Q^2 in intervals of x.

QCD Fit results

		LO	NLO	NNLO
u_v	A_u	2.411	3.565	3.724
	a_u	0.599	0.698	0.7060
	b_u	3.644	3.539	3.532
	ρ_u	$-.0200$	0.162	0.100
	γ_u	3.200	1.210	1.140
d_v	A_d	1.759	2.172	2.344
	a_d	0.649	0.703	0.711
	b_d	2.983	3.049	4.211
	ρ_d	0.539	0.389	0.016
	γ_d	-1.400	-1.370	0.429
$\Lambda_{QCD}^{N_f=4}, MeV$		207	256	230
χ^2/ndf		$580.467/534 = 1.087$	$539.078/534 = 1.009$	$535.856/534 = 1.003$

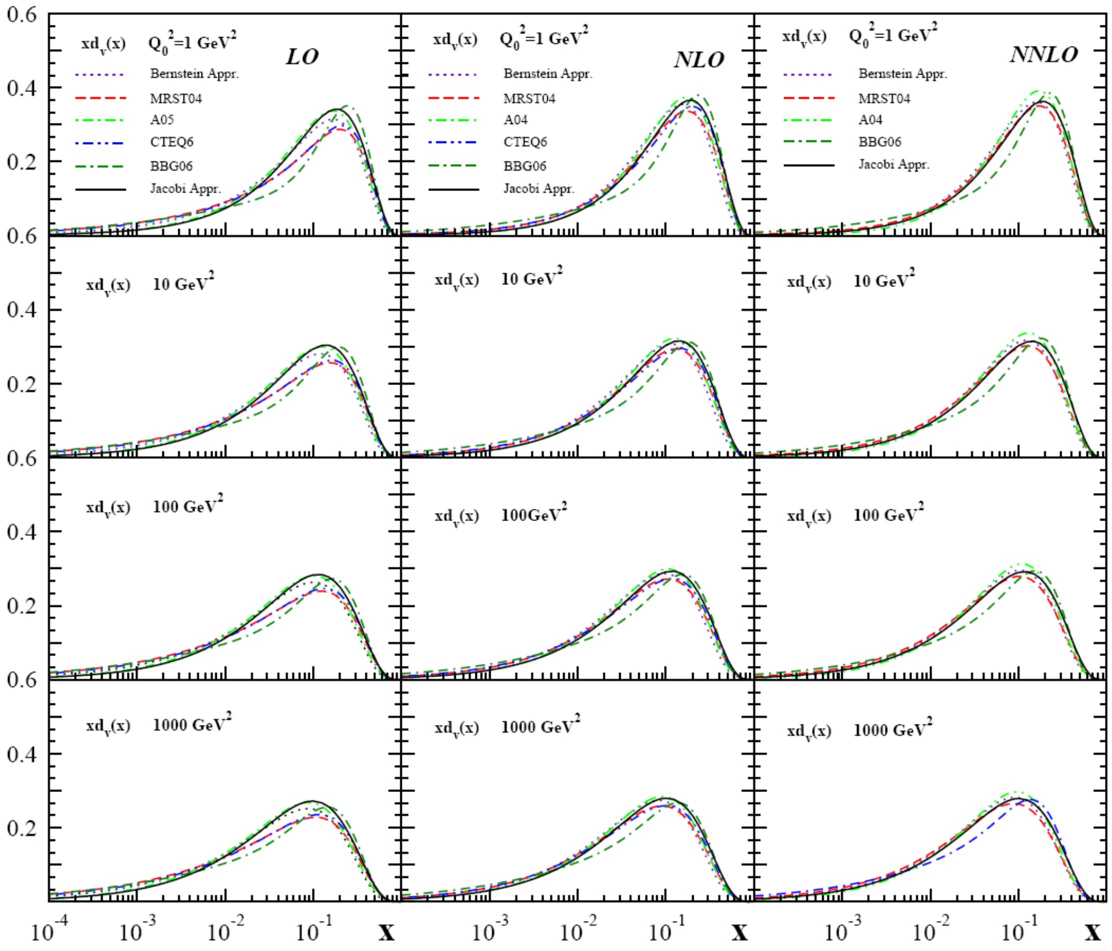
Parameter values of the LO, NLO, and NNLO non-singlet QCD fit at $Q_0^2 = 1$ GeV^2.

QCD Fit results



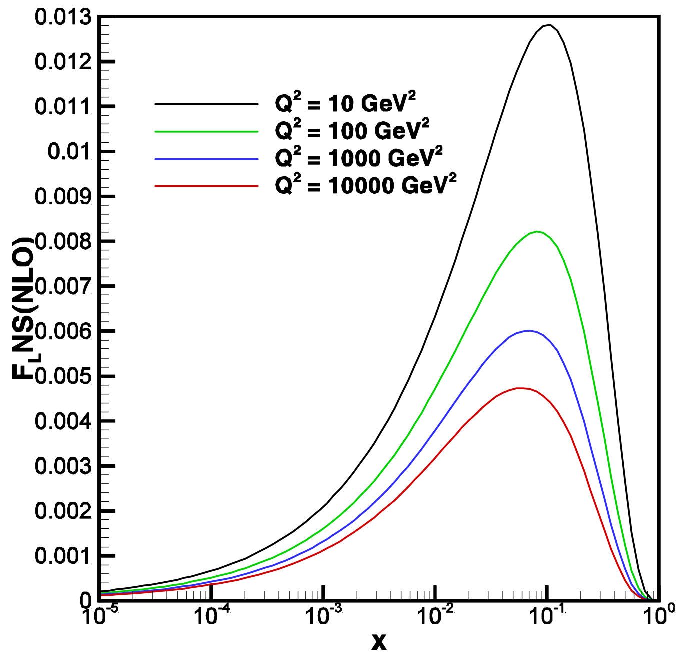
The parton distribution xu_v at some different values of Q^2.

QCD Fit results



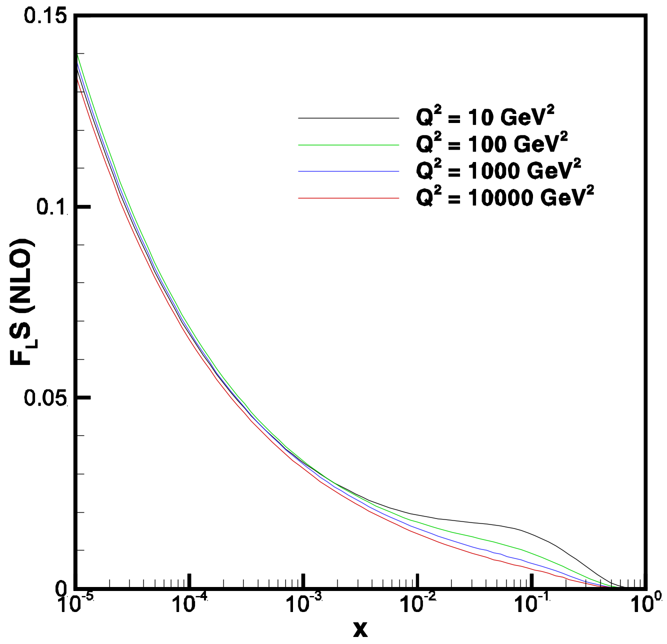
The parton distribution xd_v at some different values of Q^2.

Results



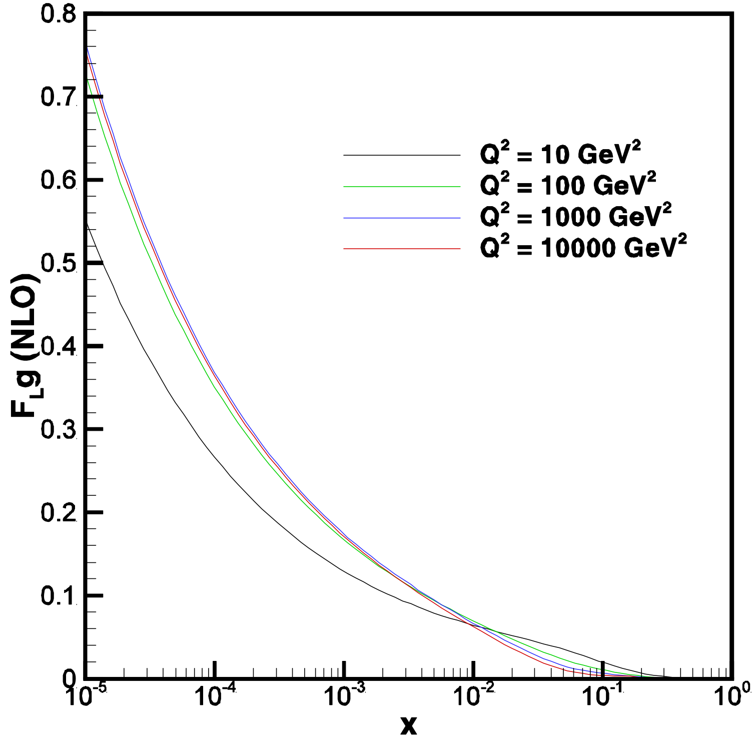
The light flavor and asymptotic heavy flavor non-singlet contributions due to charm to $F_L(x, Q^2)$

Results



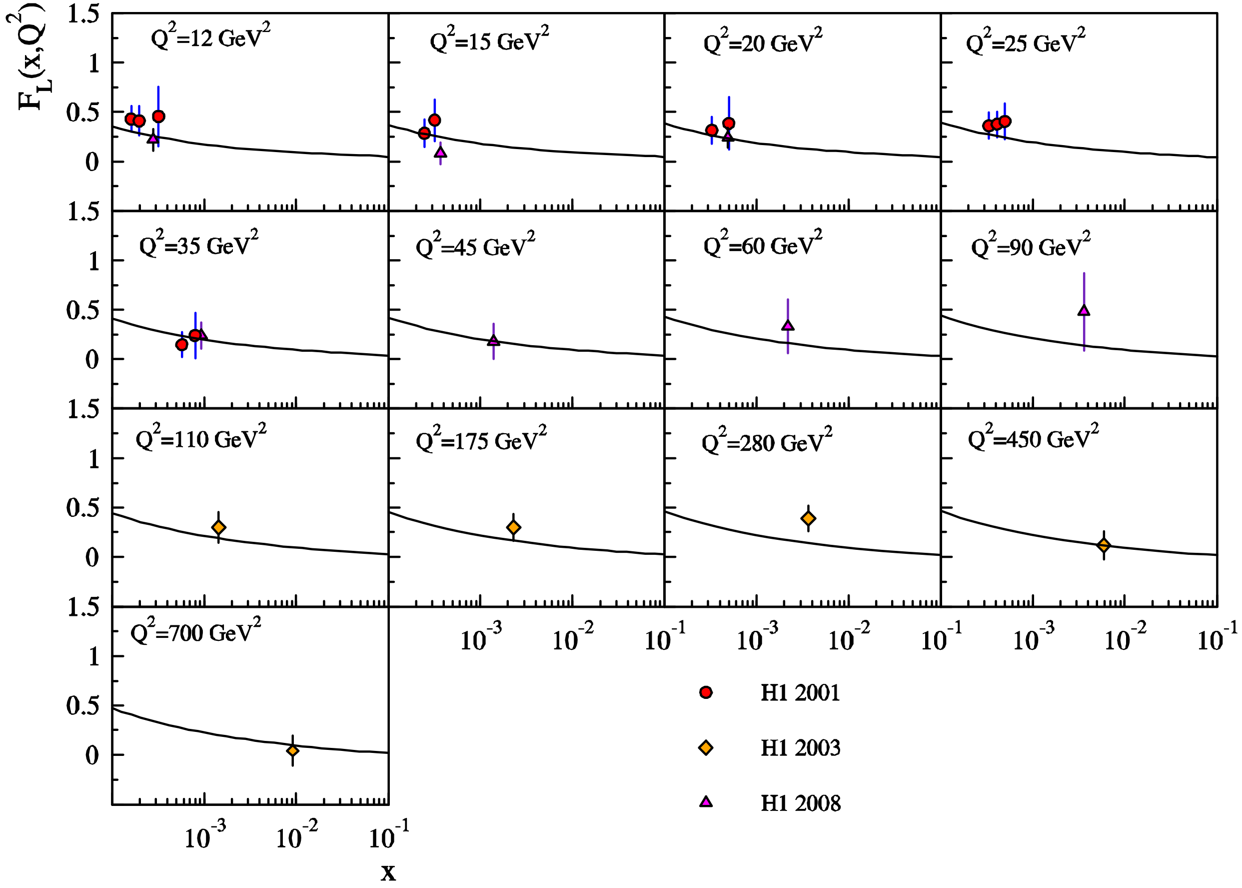
The light flavor and asymptotic heavy flavor singlet contributions due to charm to $F_L(x, Q^2)$

Results



The light flavor and asymptotic heavy flavor gluon contributions due to charm to $F_L(x, Q^2)$

Results



F. D. Aaron *et al.* [H1 Collaboration], Phys. Lett. B **665** (2008) 139

The results for $F_L(x, Q^2)$ as a function of x at fixed Q^2 values. Data are from the H1 Collaboration.

Conclusion

▷ In the region of large hadronic masses $W^2 \simeq Q^2(1-x)/x$ the sea–quark distribution receives substantial contributions due to heavy flavor (charm and beauty) pair production.

▷ Depending on the range of the Bjorken variable x and the gauge boson virtuality Q^2, these contributions can amount to 20–40% of the structure functions.

▷ Both for the measurement of the QCD scale Λ_{QCD} and for the extraction of the light parton densities a correct description of the heavy flavor contributions is therefore required.

▷ In case of the structure function $F_2(x,Q^2)$ the asymptotic heavy flavor terms describe the complete contributions very well already for scales $Q^2 \gtrsim 30\ \mathrm{GeV}^2$, whereas for $F_L(x,Q^2)$ this applies only at much higher scales, $Q^2 \gtrsim 800\ \mathrm{GeV}^2$.