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# Resolving The Sign Ambiguity in $\Delta\Gamma_s$

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Based on:

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# Plan of Talk

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- Introduction and Motivation
- Resolving the sign ambiguity with  $B_s \rightarrow D_s K$
- Concluding remarks

# B physics: Mixing and decay

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- Apart from the direct search of BSM physics at colliders low energy observables in flavor physics plays an important role for an indirect search of NP
- NP in B: look in CP asymmetries, branching fractions, mass and lifetime differences
- FCNC processes play an important role for the detection of NP effects
- It is obvious that  $B_s - \overline{B}_s$  mixing will play an important role in the search for NP in  $b \rightarrow s$  FCNC processes

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$\Delta M_q$  and  $\Delta \Gamma_q$

# Basic definition

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Effective Hamiltonian for  $B_q - \bar{B}_q$  mixing

$$H_{eff} = \begin{pmatrix} M_{11q} - \frac{i}{2}\Gamma_{11q} & M_{12q} - \frac{i}{2}\Gamma_{12q} \\ M_{12q}^* - \frac{i}{2}\Gamma_{12q}^* & M_{11q} - \frac{i}{2}\Gamma_{11q} \end{pmatrix}$$

Mass eigenstates

$$|B_L\rangle = p|B_q\rangle + q|\bar{B}_q\rangle$$

$$|B_H\rangle = p|B_q\rangle - q|\bar{B}_q\rangle$$

Eigenvalues  $\mu_{L,H} = M_{L,H} - \frac{i}{2}\Gamma_{L,H}$

$M_{L,H} \implies$  Masses of  $B_{L,H}$ ,  $\Gamma_{L,H} \implies$  Decay widths of  $B_{L,H}$

In general  $M_H > M_L$  And within SM  $\Gamma_L > \Gamma_H$

# Basic definition

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Oscillation frequency :  $\Delta M_s = M_{H_s} - M_{L_s} = 2|M_{12s}|$

Width difference :  $\Delta\Gamma_s = \Gamma_{L_s} - \Gamma_{H_s} = \frac{4\text{Re}(M_{12s}\Gamma_{12s}^*)}{\Delta M_s}$

$= 2|\Gamma_{12s}| \cos \phi_s$

These quantities are defined from:

$\phi_s = \text{arg}\left(-\frac{M_{12}^s}{\Gamma_{12}^s}\right) = \phi_s^{SM} + \phi_s^\Delta - 2\beta_s$  with  $\phi_s^{SM} = 0.24^\circ \pm 0.08^\circ$

and  $\beta_s$  is the phase from  $b \rightarrow c\bar{c}s$  decay amplitude

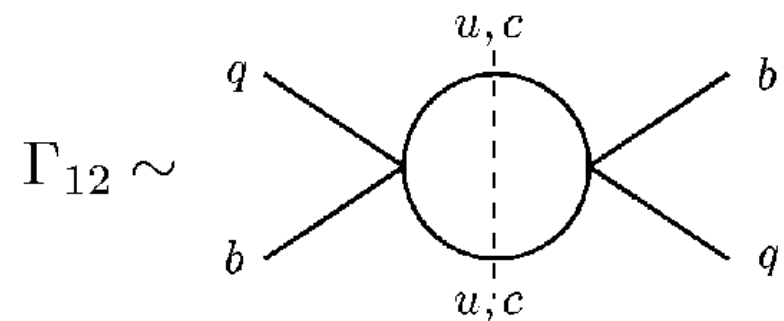
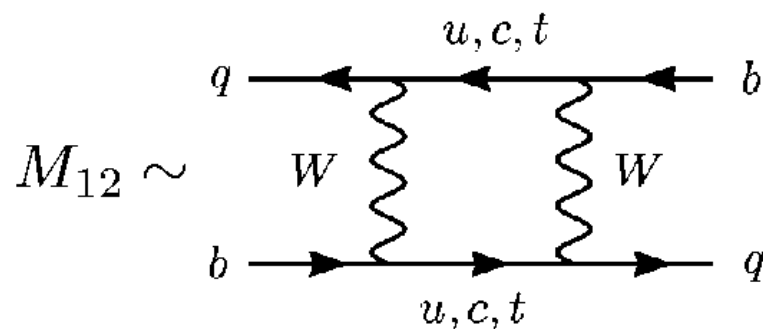
$$2M_{Bq} \langle B_q^0 | H_{eff}^{\Delta B=2} | \bar{B}_q^0 \rangle = M_{12q} - \frac{i}{2}\Gamma_{12q}$$

- Mass difference can be calculated from the dispersive part of the box diagram
- Width difference can be calculated from the absorptive part of the box diagram

# Basic definition

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- Only the box diagrams with internal  $c$  and  $u$  quarks contribute to decay width difference
- For  $B_s$  systems the intermediate  $c\bar{c}$  contribution is the dominating one
- Decay width for  $B_d$  system have contributions from intermediate  $c\bar{c}, u\bar{u}, u\bar{c}$  and  $c\bar{u}$  box diagrams



# Importance of measurement of $\Delta\Gamma_s$

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- $\Delta\Gamma_s$  is positive in SM  $\Rightarrow \Gamma_L > \Gamma_H$   
 $\Rightarrow$  A negative sign of  $\Delta\Gamma_s$  will be a clear signal of NP
- Sign of  $\cos \phi_s^\Delta$  will determine the sign of  $\Delta\Gamma_s$   
( $\Delta\Gamma_s = 2|\Gamma_s^{12}| \cos \phi_s^\Delta$ )
- CDF has measured  
 $\phi_s^\Delta - 2\beta_s \in [-1.36, -0.24] \cup [-2.90, -1.78]$  at 68% CL
- $\Delta\Gamma_s/\Gamma_s$  is expected to be large,  $(15 \pm 6)\%$  [SM], is large enough to be accessible by experiment in near future
- If this is outside the SM range, NP!!



# Sign ambiguities in $\Delta\Gamma_s$

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- The lifetime measurement in an untagged decay to a  $CP$  eigenstate determine  $\Gamma_s^{12} \cos(\phi_s^\Delta - 2\beta_s) \cos(\phi_s^\Delta + \phi_{SM})$   
→ determines  $\phi_s^\Delta$  with four-fold ambiguity

$$\phi_s^\Delta \leftrightarrow \pi \pm \phi_s^\Delta$$

$$\phi_s^\Delta \leftrightarrow -\phi_s^\Delta - \phi_s^{SM} + 2\beta_s^{SM}$$

$$\phi_s^\Delta \leftrightarrow -\phi_s^\Delta - \phi_s^{SM} + 2\beta_s^{SM} + \pi$$

- Constrain on  $\phi_s^\Delta$  from the  $CP$  asymmetry  
( $a_{f_s}^s \propto \sin(\phi_s^\Delta + \phi_s^{SM})$ ) measurement of untagged flavour specific  $B_s$  decays, has a two-fold ambiguity  
( $\phi_s^\Delta \leftrightarrow \pi - \phi_s^\Delta$ )

# How to resolve this ambiguity?

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It is possible to resolve it with  $B_s \rightarrow D_s K$

$$\lambda_{D_s^- K^+} = \frac{q \langle D_s^- K^+ | \bar{B}_s \rangle}{p \langle D_s^- K^+ | B_s \rangle} = |\lambda_{D_s^- K^+}| e^{-i(\gamma_s + \phi_s^\Delta - \delta)}$$
$$\lambda_{D_s^+ K^-} = \frac{q \langle D_s^+ K^- | \bar{B}_s \rangle}{p \langle D_s^+ K^- | B_s \rangle} = \frac{1}{|\lambda_{D_s^- K^+}|} e^{-i(\gamma_s + \phi_s^\Delta + \delta)}$$

Tagged  $B_s \rightarrow D_s^\pm K^\mp$  decay will determine  $\phi_s^\Delta - 2\beta_s + \gamma$

# Time dependent decay rate

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$$\begin{aligned} & \Gamma (B_s(t) \rightarrow D_s^\mp K^\pm) \\ & = N e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) \pm (1 - b |\lambda_{D_s^- K^+}|) \cos(\Delta M_s t) \right. \\ & \quad \left. - b \cos(\gamma_s + \phi_s^\Delta \mp \delta) \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + b \sin(\gamma_s + \phi_s^\Delta \mp \delta) \sin(\Delta M_s t) \right] \end{aligned}$$

$$\begin{aligned} & \Gamma (\bar{B}_s(t) \rightarrow D_s^\mp K^\pm) \\ & = N e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) \mp (1 - b |\lambda_{D_s^- K^+}|) \cos(\Delta M_s t) \right. \\ & \quad \left. - b \cos(\gamma_s + \phi_s^\Delta \mp \delta) \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - b \sin(\gamma_s + \phi_s^\Delta \mp \delta) \sin(\Delta M_s t) \right] \end{aligned}$$

$$\text{with, } b = \frac{2|\lambda_{D_s^- K^+}|}{1+|\lambda_{D_s^- K^+}|^2}$$

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## Untagged analysis

# Untagged decay rate

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$$\begin{aligned}\Gamma [D_s^\mp K^\pm, t] &\equiv \Gamma (B_s(t) \rightarrow D_s^\mp K^\pm) + \Gamma (\bar{B}_s(t) \rightarrow D_s^\mp K^\pm) \\ &= 2N e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - b \cos(\gamma_s + \phi_s^\Delta \mp \delta) \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) \right]\end{aligned}$$

- We can determine branching fraction and lifetime from the untagged decay rate
- The overall normalization constant is related to *CP*-averaged branching fraction

# Branching fractions

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$$\mathcal{B}(B_s^0 \rightarrow D_s^\mp K^\pm) = \frac{N}{\Gamma_s} \left[ 1 - b \cos(\gamma_s - \phi_s^\Delta \mp \delta) \frac{\Delta\Gamma_s}{\Gamma_s} \right]$$

Branching ratios will not be useful to remove the sign ambiguity

Ratio of branching fractions:

$$\frac{\mathcal{B}(B_s \rightarrow D_s^+ K^-) - \mathcal{B}(B_s \rightarrow D_s^- K^+)}{\mathcal{B}(B_s \rightarrow D_s^+ K^-) + \mathcal{B}(B_s \rightarrow D_s^- K^+)} = b \frac{\sin(\gamma_s + \phi_s^\Delta) \sin \delta}{1 - \cos(\gamma_s + \phi_s^\Delta) \cos \delta} \frac{\Delta\Gamma_s}{2\Gamma_s}$$

The ratio of branching fractions will be useful to place tighter bound on  $|\sin(\gamma_s + \phi_s^\Delta) \sin \delta|$

# Lifetime informations

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Maximum likelihood fit of untagged decay rate to a single exponential  $\propto \exp[-\Gamma_{D_s^\mp K^\pm} t]$  determines

$$\Gamma_{D_s^\mp K^\pm} = \Gamma_s + b \cos(\gamma_s + \phi_s^\Delta \mp \delta) \frac{\Delta\Gamma_s}{2}$$
$$\Gamma_s + b \cos(\gamma_s + \phi_s^\Delta \mp \delta) \cos \phi_s^\Delta |\Gamma_{12}^s|$$

After addition:  $\frac{\Gamma_{D_s^+ K^-} + \Gamma_{D_s^- K^+}}{2} - \Gamma_s$

$$= b \cos \delta \cos(\gamma_s + \phi_s^\Delta) \cos \phi_s^\Delta |\Gamma_{12}^s| \equiv L |\Gamma_{12}^s|$$

# Inputs

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- We assume the fact that  $\gamma$  is well known from the B-factories  $\rightarrow$  Measured  $CP$  asymmetries in  $B_d \rightarrow \rho^+ \rho^-$  and  $B_d \rightarrow J/\psi K_s$  are combined to solve :  
$$\gamma = \pi - \alpha - \beta = 71^\circ \pm 5^\circ$$
- $\beta_s = \arg[V_{ts} V_{tb}^* / V_{cs} V_{cb}^*] = 1.1^\circ \pm 0.3^\circ$
- We use the realistic value for  $|\lambda_{D_s^- K^+}| \approx 0.4$
- We have taken the strong phase  $|\delta| < 0.2$



# L vs $\sin(\phi_s^\Delta - 2\beta_s)$

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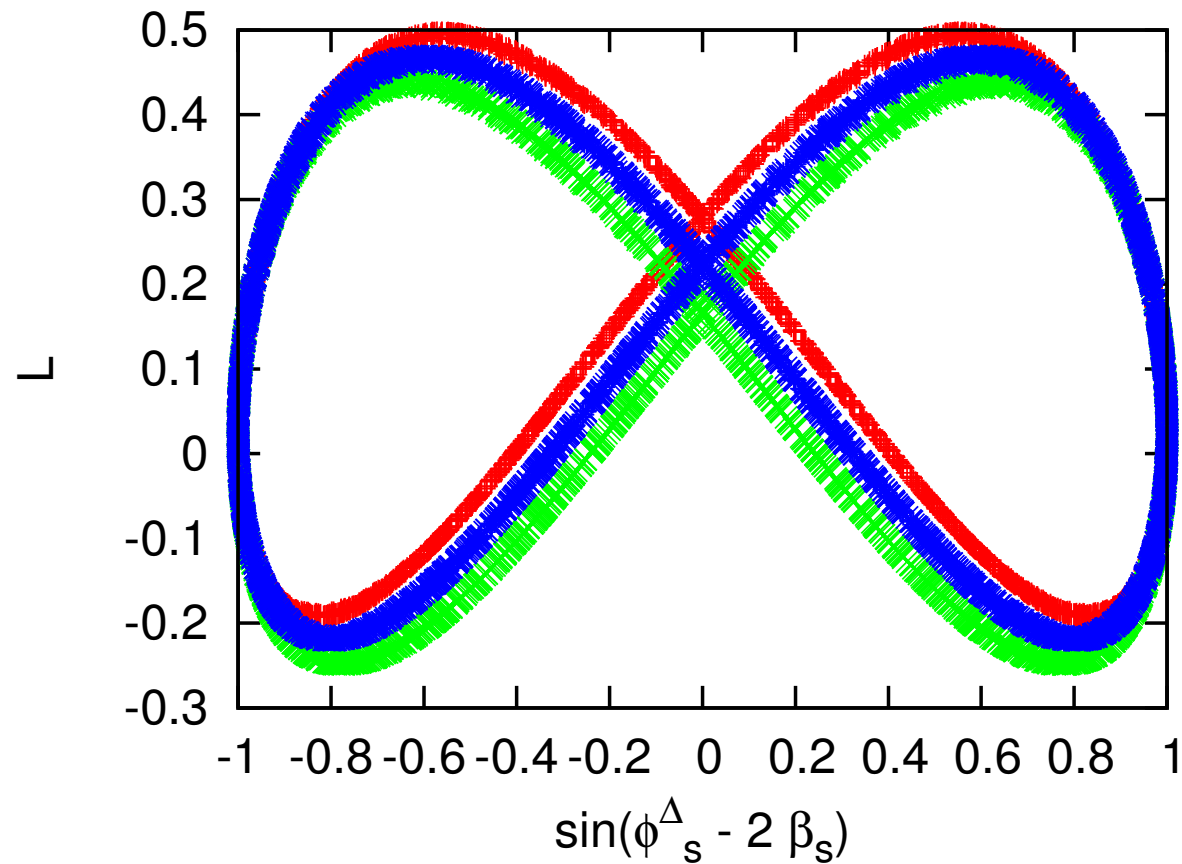


Figure 1: For  $\sin(\phi_s^\Delta - 2\beta_s) = -0.72$ ,  $L > 0 \rightarrow \Delta\Gamma_s > 0$  and  $L < 0 \rightarrow \Delta\Gamma_s < 0$

# Tagged analysis

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With :

$$\Gamma(B_s(t) \rightarrow D_s K) = [\Gamma(B_s(t) \rightarrow D_s^- K^+) + \Gamma(B_s(t) \rightarrow D_s^+ K^-)]/2$$

The  $CP$  asymmetry can be defined as :

$$\frac{\Gamma(B_s(t) \rightarrow D_s K) - \Gamma(\bar{B}_s(t) \rightarrow D_s K)}{\Gamma(B_s(t) \rightarrow D_s K) + \Gamma(\bar{B}_s(t) \rightarrow D_s K)} = \frac{b \cos \delta \sin(\gamma_s + \phi_s^\Delta) \sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s/2) - b \cos \delta \cos(\gamma_s + \phi_s^\Delta) \sinh(\Delta\Gamma_s/2)}$$

The coefficient of the oscillating term:

$S \equiv b \cos \delta \sin(\gamma_s + \phi_s^\Delta) \rightarrow$  This term will be useful to remove the discrete ambiguity  $\phi_s^\Delta \leftrightarrow \pi - \phi_s^\Delta + 2\beta_s$

# $S$ vs $\sin(\phi_s^\Delta - 2\beta_s)$

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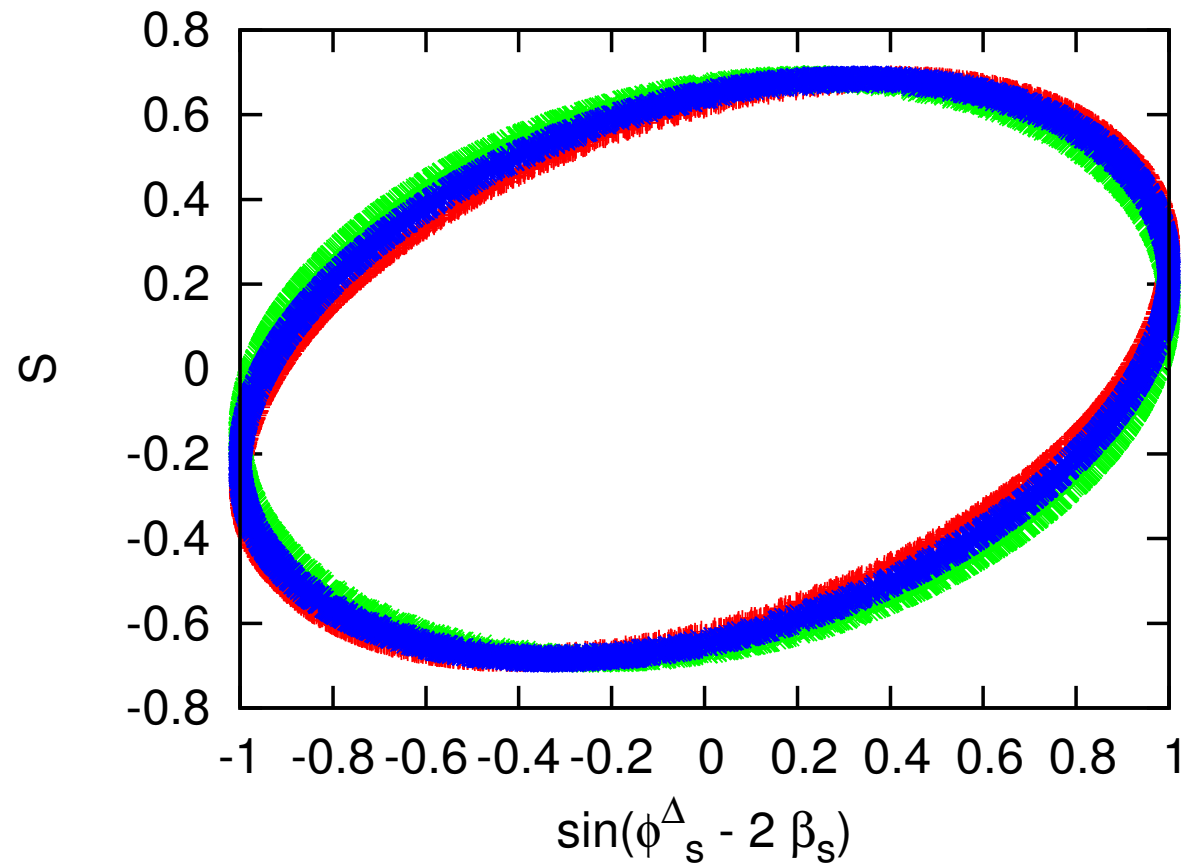


Figure 2: Two solutions with  $\sin(\phi_s^\Delta - 2\beta_s) = -0.72$   
 $\rightarrow S \approx 0.3$  and  $S \approx -0.6$

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# Tagged + Untagged

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What we will get if we combine tagged and untagged analysis?

Combining tagged and untagged analysis we can define :

$$\tan(\gamma_s + \phi_s^\Delta) = \frac{S}{L} \cos \phi_s^\Delta$$

- Using above equation we can resolve the ambiguity even if we don't have any knowledge about  $\delta$
- Above equation has four solutions for  $\phi_s^\Delta \rightarrow$  Two of them can be eliminated with the information on the sign of "S"
- Remaining two solutions are not related by

$$\phi_s^\Delta - 2\beta_s \leftrightarrow \pi - \phi_s^\Delta + 2\beta_s$$

# Conclusions

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- It is important to determine the sign of  $\Delta\Gamma_s \rightarrow$  negative  $\Delta\Gamma_s$  will be a clear signal for New Physics
- In combination with  $B_s \rightarrow J/\psi\phi$ , the discrete ambiguity can be resolved from tagged and untagged analysis of  $B_s \rightarrow D_s K$
- Lifetime measurement with an accuracy of roughly 2% requiring at least 2500 events
- Tagged measurement looks better  $\Rightarrow$  a fairly small data sample should permit to discriminate between the two solutions