HEAVY QUARK JET FUNCTIONS

(THE TOP MASS IN EFT AND SOMETHING MORE)

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OUTLINE

- … Because top …
- Effective Field Theories in Jets: introduction and (some) results.
- A Transitive and Transportable jet mass .
- Connecting HQ mass definitions :

  The R-evolution.
- Some light on Tevatron top mass..
- Conclusion and Perspective.

TOP PHYSICS AT ILC/LHC

- **The top is the heaviest quark**
- **$\Gamma \approx 1.4$ GeV, it doesn't hadronize**

FERMILAB-TM-2380-E
TEVEWWG/top 2007/01
CDF Note 8735
DØ Note 5378
13th March 2007

Combination of CDF and DØ Results on the Mass of the Top Quark

The Tevatron Electroweak Working Group[1]
for the CDF and DØ Collaborations

Abstract

We summarize the top-quark mass measurements from the CDF and DØ experiments at Fermilab. We combine published Run-I (1992-1996) measurements with the most recent preliminary Run-II (2001-present) measurements using up to 1 fb^{-1} of data. Taking correlated uncertainties properly into account the resulting preliminary world average mass of the top quark is $M_t = 170.9 \pm 1.1(stat) \pm 1.5(syst)$ GeV/c^2, which corresponds to a total uncertainty of 1.8 GeV/c^2. The top-quark mass is now known with a precision of 1.1%.

$$M_t = 170.9 \pm 1.8 \text{ GeV}/c^2$$

172.6±1.4

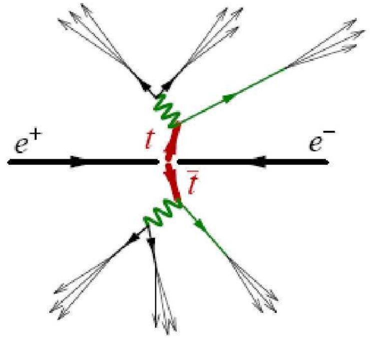
This is a 1% precision!

How should we judge this error?
Theoretical error?
What mass is it?

RECONSTRUCTION METHODS

Tev +LHC + ILC

Invariant mass reconstruction $Q \geq 2m_t$



- data available soon
- measures different top mass ?
- Uncertainties much more involved
- many different methods available
- error around 1 GeV challenging

Large p_T Events:

- select events with $p_T > 200$ GeV
- top pair back-to-back ⇒ decay products in different hemispheres
- large cone size around top/antitop jet axes: $\Delta R = 0.8 \ldots 1.8$

  M_t and $M_{\bar{t}}$ from in-cone momenta
- strong sensitivity to soft jets + Underlying Events
- Mass scale calibrations (W mass)

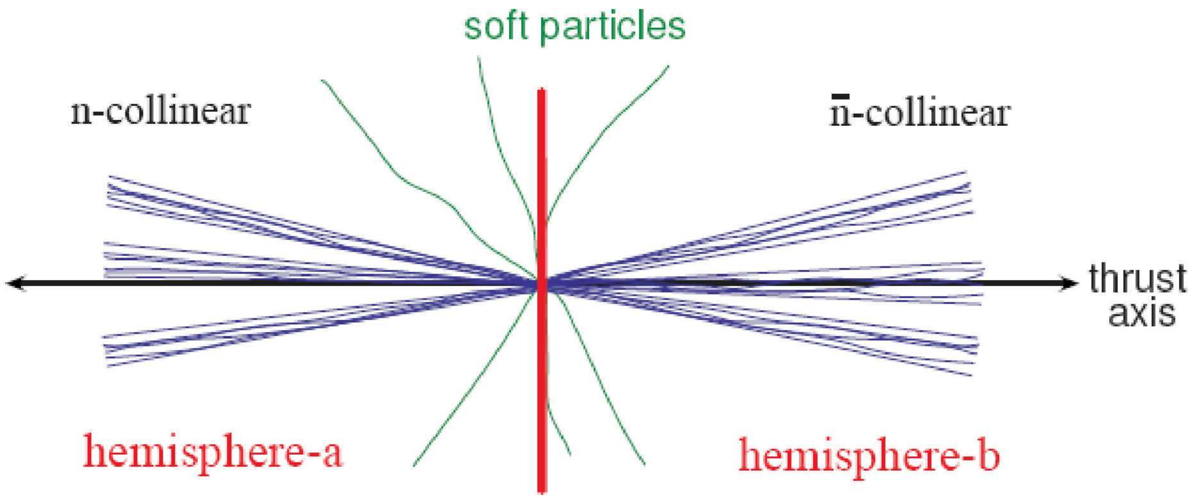
TARGETS AND USES OF TOP DISTRIBUTIONS

Extract the mass of the top from jet reconstruction with precision "in principle" less than Λ_{QCD}

- ►Define an observable sensitive to the top mass
- ►Identify the physical scales of the problem
- ►parameterize soft gluons
- ►calculate perturbative pieces
- ►Including top's width effects, $\Gamma_t \sim 1.4$ GeV

For the moment we look at $e^+ e^- \to t\bar{t}$

OBSERVABLES

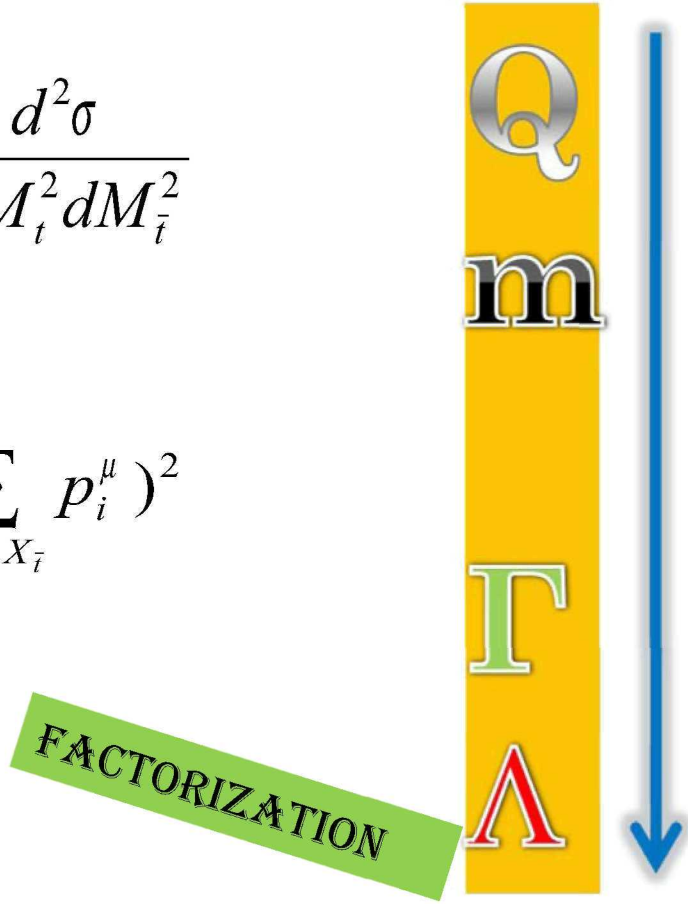


OBSERVABLES

The main observable is $\dfrac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}$

Where $M_t^2 = (\sum_{i\in X_t} p_i^\mu)^2,\ M_{\bar{t}}^2 = (\sum_{i\in X_{\bar{t}}} p_i^\mu)^2$

And $M_{t,\bar{t}}^2 - m^2 \sim m\Gamma << m^2$



$$\sigma(e^+e^- \to j_t j_t) = \sum_X^{res} (2\pi)^4 \delta(q - p_X) \sum_{i=a,v} L^{ij}_{\mu\nu} \langle 0|\mathcal{J}_j^{+\nu}(0)|X\rangle\langle X|\mathcal{J}_i^{\mu}(0)|0\rangle$$

SCET

Bauer, Fleming, Pirjol, Stewart, '00

SCET $[\lambda \sim m/Q \ll 1]$		
	n-collinear $(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft $(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

Light-cone coordinates

$$p^\mu = (+, -, \perp)$$

$$p_n^2 = p_n^+ p_n^- + p_\perp^2 \sim m^2 \ll Q^2$$

Leading order Lagrangian (n-collinear)

$$\mathcal{L}_{qn}^{(0)} = \bar{\xi}_n \left[in \cdot D_s + gn \cdot A_n + (i\not{D}_c^\perp - m) W_n \frac{1}{\bar{n} \cdot \mathcal{P}} W_n^\dagger (i\not{D}_c^\perp + m) \right] \frac{\not{\bar{n}}}{2} \xi_n$$

collinear-soft coupling

collinear Wilson line

$$iD_s^\mu = i\partial^\mu + gA_s^\mu \qquad W_n(x) = \sum_{\text{perms}} \exp\left(-\frac{g}{\bar{\mathcal{P}}} \bar{n} \cdot A_n(x) \right)$$

$$W_n(x) = \bar{P} \exp\left(-ig \int_0^\infty ds\ \bar{n} \cdot A_+ (\bar{n}s + x) \right)$$

SCET

- Factorization I

$$\sigma = \sum_{\vec{n}} \sum_{X_n X_{\bar{n}} X_s}^{res.} (2\pi)^4 \delta^4(q - P_{X_n} - P_{X_{\bar{n}}} - P_{X_s}) \sum_i L^{(i)}_{\mu\nu} \int d\omega\, d\bar{\omega}\, d\omega'\, d\bar{\omega}'$$

$$\times C(\omega,\bar{\omega}) C^*(\omega',\bar{\omega}') \langle 0 | \bar{\chi}_{\bar{n},\bar{\omega}'} \bar{\Gamma}^{\nu}_j \chi_{n,\omega'} | X_n X_{\bar{n}} X_s \rangle \langle X_n X_{\bar{n}} X_s | \bar{\chi}_{n,\omega} \Gamma^{\mu}_i \chi_{\bar{n},\bar{\omega}} | 0 \rangle$$

factorization of asymptotic final states

$$|X\rangle = |X_n X_{\bar{n}} X_s\rangle = |X_n\rangle \otimes |X_{\bar{n}}\rangle \otimes |X_s\rangle$$

Collinear: n Collinear: $\bar{n}$ Soft

HQET

Mass scheme choice → δm; Inclusive in decay products → Γ

$$\mathcal{L} = \bar{h}_{v_\pm}\left(iv_\pm . D - \delta m + \frac{i}{2}\Gamma\right)h_{v_\pm}$$

$$v_- = \left(\frac{Q}{m}, \frac{m}{Q}, 0\right)$$

$$v_+ = \left(\frac{m}{Q}, \frac{Q}{m}, 0\right)$$

bHQET ($\Gamma << m$)		
n-collinear	$(h,A)_+$	$\Gamma(\lambda, 1/\lambda, 1)$
n-collinear	$(h,A)_-$	$\Gamma(1/\lambda, \lambda, 1)$
Soft	$(q,A)_s$	$\Gamma(\Delta,\Delta,\Delta)$

and

$$\delta m = m_{pole} - m_j \sim \Gamma$$

Note that m_j cannot be Msbar otherwise power counting is violated

$$\delta m_{\overline{MS}} \sim \alpha_s m >> \Gamma$$

HQET

- Factorization II

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

- integrate out the top mass $\longrightarrow$ $\hat s_{t,\bar t} = \frac{M_{t,\bar t}^2 - m_t^2}{m_t} \sim \Gamma_t$
- Jet functions without large logs in peak region: $B_\pm(\hat s, \Gamma_t, \mu)$
- top decay "integrated out" (unstable particle EFT)$\longrightarrow i\,\Gamma_t$
- soft function S unchanged

EXPONENTIATTION OF ME

- The exponentiation theorems of Gatheral(1983), Frenkel and Taylor (1984) apply

$$m\tilde{B}(y,\mu) = \exp\left\{\frac{C_F\alpha_s(\mu)}{\pi}\left(\tilde{L}^2+\tilde{L}+\frac{\pi^2}{24}+1\right)+\frac{\alpha_s^2(\mu)C_F\beta_0}{\pi^2}\left[\frac{1}{6}\tilde{L}^3+\frac{2}{3}\tilde{L}^2+\frac{47}{36}\tilde{L}-\frac{\zeta(3)}{48}+\frac{5\pi^2}{576}+\frac{281}{216}\right]\right.$$
$$\left.+\frac{\alpha_s^2(\mu)C_FC_A}{\pi^2}\left[\left(\frac{1}{3}-\frac{\pi^2}{12}\right)\tilde{L}^2+\left(\frac{5}{18}-\frac{\pi^2}{12}-\frac{5\zeta_3}{4}\right)\tilde{L}-\frac{5\zeta_3}{8}-\frac{17\pi^4}{2880}+\frac{7\pi^2}{144}-\frac{11}{54}\right]\right\}.$$

This is one loop exact!.. And can be used to check for higher orders

MASS DEFINITIONS

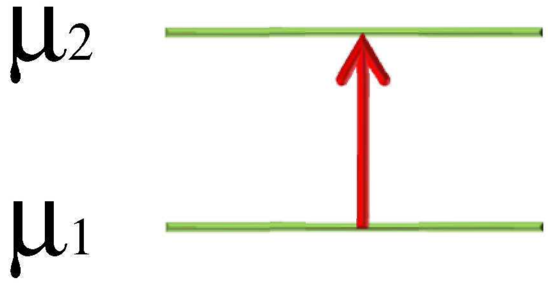
- Peak position mass $$\left.\frac{\partial B(\hat{s} - 2\delta m_{peak}, \Gamma, \mu)}{\partial \hat{s}}\right|_{\hat{s}=0} = 0$$

- First Momentum mass $$\int_{-\infty}^{L_m} d\hat{s}\ \hat{s}\ B^{\Gamma=0}(\hat{s} - 2\delta m_m, \mu) = 0$$

- Position space mass $$B^{\Gamma=0}(\hat{s}, \mu) = \mathrm{Im}\mathcal{B}(\hat{s}, \mu) \xrightarrow{FT} \widetilde{\mathcal{B}}(y, \mu)$$

$$\delta m_y(\mu) = \frac{R\, e^{\gamma}}{2} \frac{d}{d \ln iy} \ln \widetilde{\mathcal{B}}(y, \mu)\Big|_{iye^{\gamma} = \frac{1}{R}}$$

TRANSITIVE MASSES



- Transitivity: either we calculate the mass at 2 different scales or we evolve from one scale to the other we get the same result.
- The peak mass is not transitive
- The first momentum mass is no more transitive at 2 loops

The position space mass is always transitive

$$\frac{dm_y(\mu)}{d\ln\mu} = -R\Gamma^c[\alpha_s(\mu)]$$

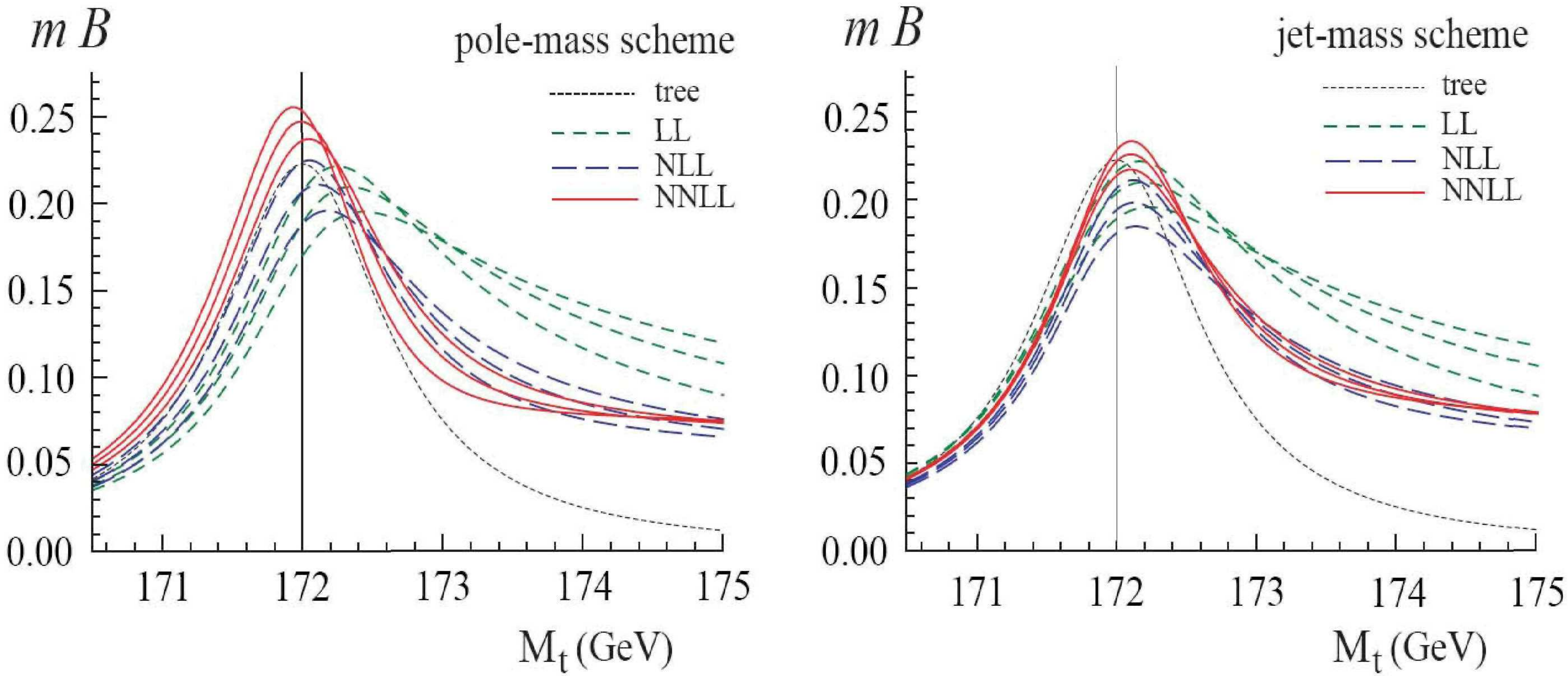
TRANSITIVITY TESTS

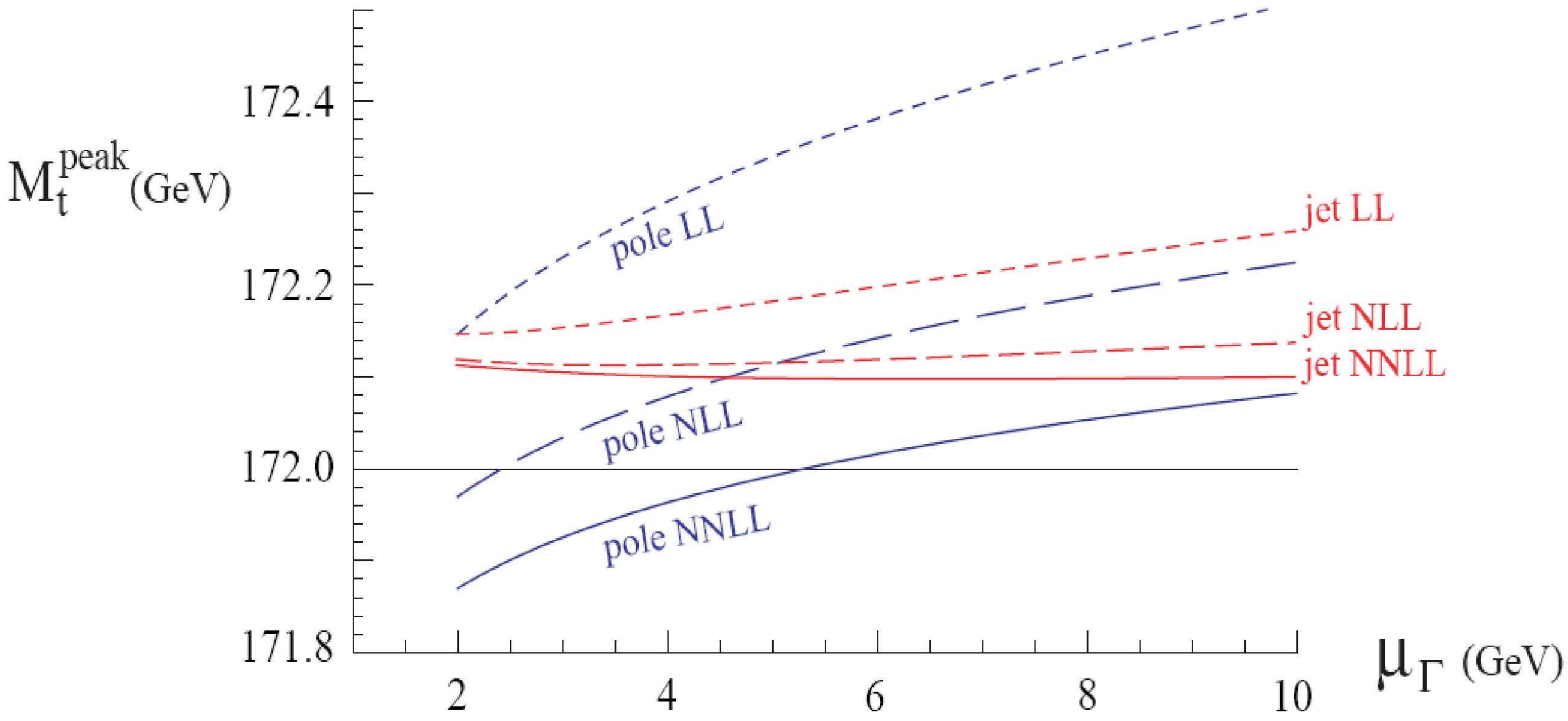
$$\delta m_J^{\rm abelian} = e^{\gamma_E} R \frac{C_F \alpha_s}{\pi}\left[\ln\frac{\mu}{R} + \frac{1}{2}\right]$$

True to all orders!

$$\begin{aligned}
\delta m^{\rm peak}_{\rm abelian} &= \frac{\pi\Gamma_t}{4}\Bigg\{\frac{C_F\alpha_s}{\pi}\left[\ln\frac{\mu}{\Gamma_t}+\frac{3}{2}\right] + \frac{C_F^2\alpha_s^2}{\pi^2}\left[-\ln^2\frac{\mu}{\Gamma_t}+\left(\frac{\pi^2}{3}-5\right)\ln\frac{\mu}{\Gamma_t}-\frac{13}{4}+\frac{\pi^2}{2}-2\zeta_3\right] + \frac{C_F^3\alpha_s^3}{\pi^3}\Bigg[\left(1+\frac{\pi^2}{12}\right)\ln^3\frac{\mu}{\Gamma_t} \\
&\quad + \left(\frac{25}{2}-\frac{5\pi^2}{12}-4\zeta_3\right)\ln^2\frac{\mu}{\Gamma_t} + \left(\frac{75}{4}-\frac{151\pi^2}{48}+\frac{11\pi^4}{45}-8\zeta_3\right)\ln\frac{\mu}{\Gamma_t}+\frac{59}{8}-\frac{5\pi^2}{2}+\frac{11\pi^4}{30}+5\zeta_3-\pi^2\zeta_3-12\zeta_5\Bigg]\Bigg\}, \\
\delta m^{\rm mom}_{\rm abelian} &= R\Bigg\{\frac{C_F\alpha_s}{\pi}\left[\ln\frac{\mu}{R}+\frac{3}{2}\right] + \frac{C_F^2\alpha_s^2}{\pi^2}\left[\left(4-\frac{\pi^2}{3}\right)\ln\frac{\mu}{R}+8-\frac{\pi^2}{2}-2\zeta_3\right] + \frac{C_F^3\alpha_s^3}{\pi^3}\Bigg[(6-4\zeta_3)\ln^2\frac{\mu}{R} \\
&\quad + \left(46-\frac{8\pi^2}{3}-\frac{\pi^4}{45}-12\zeta_3\right)\ln\frac{\mu}{R}+\frac{159}{2}-\frac{16\pi^2}{3}-\frac{\pi^4}{30}-21\zeta(3)+\frac{4\pi^2\zeta_3}{3}-12\zeta(5)\Bigg]\Bigg\}. \qquad (6
\end{aligned}$$

Plot for B around the peak in pole scheme and jet mass scheme





172.4
172.2
172.0
171.8
M_t^{peak}(GeV)
pole LL
jet LL
jet NLL
jet NNLL
pole NLL
pole NNLL
2
4
6
8
10
μ_Γ (GeV)

CONNECTING MASSES

$$m_t^{pole} = m_y(\mu) + R(dm_{y1}\alpha_s + dm_{y2}\alpha_s^2 + ..)$$

$$m_t^{pole} = m_{\overline{MS}}(\mu)(1 + dm_{\overline{MS}1}\alpha_s + dm_{\overline{MS}2}\alpha_s^2 + ..)$$

- Both the MSb mass and the jet mass are renormalon free.
- The MSb mass is known at 3 loops. Now the jet mass at 2 loops.
- Numerically the loop corrections to mj are much smaller than the loop corrections to MSb-mass
- Subtleties due to the R dependence NEW!

MSR SCHEME AND R-EVOLUTION

$$m_{pole} = \bar{m}(\bar{m}) \left(1 + \sum_{n=1}^{\infty} \bar{a}_{n0} \left[\frac{\alpha_s(\bar{m})}{4\pi}\right]^n\right)$$

where $\bar{a}_{n0} = \bar{a}_n$

$$m_{pole} = m_R(R) + R \sum_{n=1}^{\infty} \bar{a}_n \left[\frac{\alpha_s(R)}{4\pi}\right]^n$$

Analogously to $\overline{\text{MS}}$ the MSR absorbs all UV interactions above R

MASSES AND SCALES

The masses of heavy quarks are chatacteriazed by 2 scales μ and R:
μ absorbs UV interactions and R the IR's.

Scheme	mass	R=f(m,μ)
1S	m_{1S}	$R = m_{1S} C_F \alpha_s(\mu)$
$\overline{MS}$	$\overline{m}(\mu)$	$R = \overline{m}(\mu)$
RGI	m_{RGI}	$R = m_{RGI}$
Kin	m_{kin}	$R = \mu_f^{kin}$
PS	m_{PS}	$R = \mu_f^{PS}$

Let's take for example the PS mass

$$m^{PS}(R) - m_{pole} = -\delta m^{PS}(R) \equiv \frac{1}{2} \int_{|q|<R} \frac{d^3 q}{(2\pi)^3} V(q)$$

And changing R, one adds potential energy to PS-mass

$$-\delta m^{PS}(R_1) \equiv \int_0^{R_0} + \int_{R_0}^{R_1} \frac{q^2 dq}{(2\pi)^2} V(q)$$

MATCHING TO OTHER SCHEMES

If μ>>R there are large logs in the matching

$$m_{pole} = m(R,\mu) + \delta m(R,\mu)$$

$$\delta m(R,\mu) = R \sum_{n=1} \left(\frac{\alpha_s(\mu)}{4\pi} \right)^n \sum_{k=0} a_{nk} \ln^k \left(\frac{\mu}{R} \right)$$

Some schemes are μ-independent (which implies $a_{11}=0$, a_{nk} fixed by $a_{n0}=a_n$),f.i. RGI, Kin, 1S, while others depend on both variables.

However it is always true

$$\frac{d}{d\ln\mu} m(R,\mu) = -R\,\gamma_\mu[\alpha_s(\mu)]$$

In this case all the a's with k>2 are fixed with a_{n0} and a_{n1}

The μ–running of the mass at the end depends only on $\alpha_s(\mu)$

THE R-RGE

The pole mass does not depend on R.. So for μ-independent schemes

$$\frac{d}{d\ln R} m(R) = -\frac{d}{d\ln R} \delta m(R) = R\,\gamma_R\left[\alpha_s(R)\right]$$

For μ-dependent scheme the evolution in the 2 variables must be correlated otherwise large logs arise in the evolution.
We can set $\mu=\kappa R$ and vary $1/2<\kappa<2$ to estimate the error on the RGE.

THE POLE MASS CONNECTION

$$m(R_1) - m_{pole} = \Lambda_{QCD} \int_{t_1}^{\infty} dt\, \gamma_R(t) \frac{d}{dt} e^{-G(t)}$$

The pole mass is obtained in the limit $R_0 \longrightarrow 0$

It is interesting to see what happens with the Borel transform of this

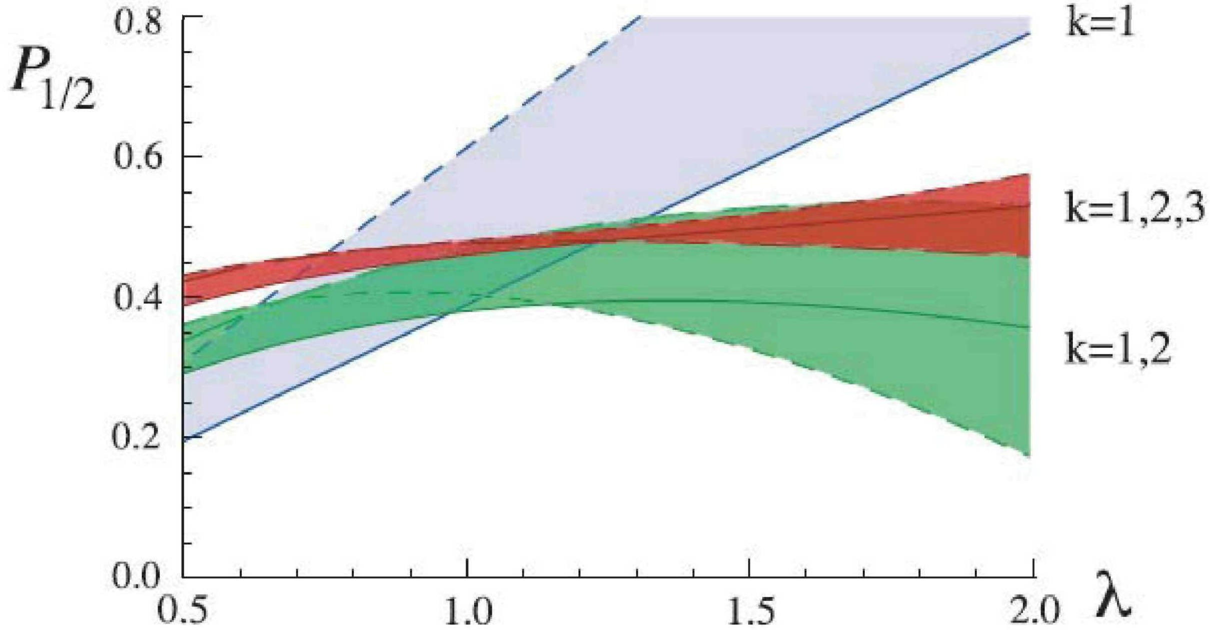
$$B(u) = 2R \left[\sum_{\ell=0} g_\ell Q_\ell(u) - P_{1/2} \sum_{\ell=0} g_\ell \frac{\Gamma(1+\hat{b}_1-\ell)}{(1-2u)^{1+\hat{b}_1-\ell}} \right]$$

$$P_{1/2} = \sum_{k=0} \frac{S_k}{\Gamma\left(1+\hat{b}_1+k\right)}$$

We have a sum rule to fix renormalons! No need of bubble resummation!

$P_{1/2}$ is the normalization of the first renormalon and we can show that its series is absolutely convergent.

THE POLE MASS CONNECTION



$$P_{1/2} = 0.47 \pm 0.10$$

TOP MASS IN MC AND TEVATRON TOP MASS

Transvers scales ~ momentum transfer of primary partons

MC

For the top quark the width ~1.5 GeV is another cutoff. In EFT the same phsyics is described with the evolution of H_Q and H_m, the partonic contributions to the S and the JET FUNCTIONS B.

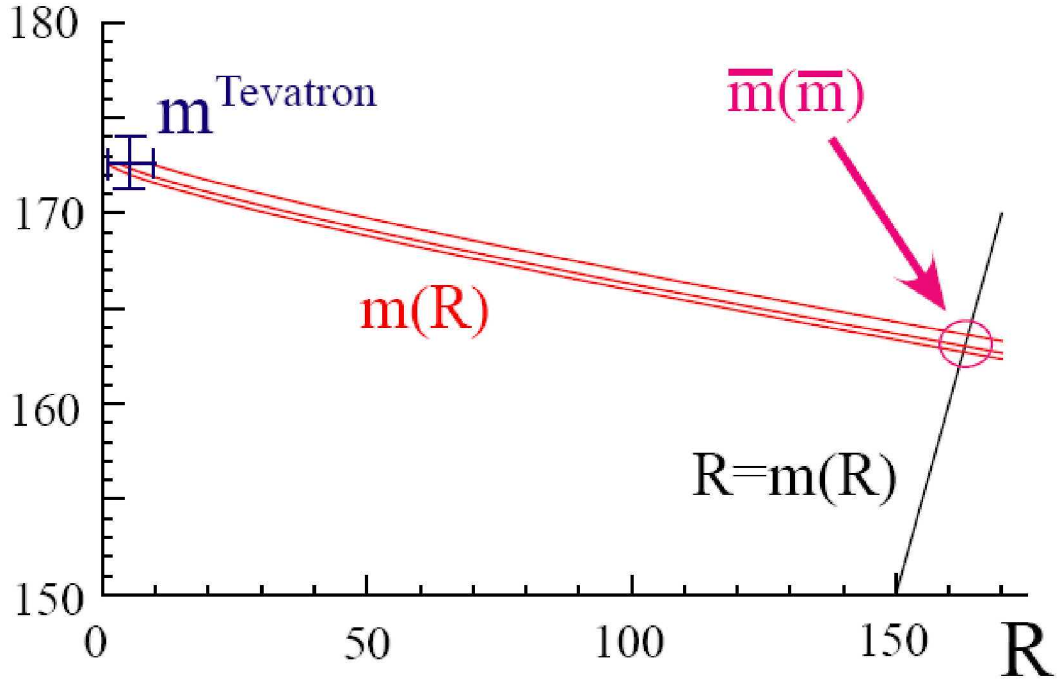
Mass scheme in MC <-> δm in B- functions (low energy fluctuation traded bet'n jet func. and mass definition). So it is the size of R_{sc} which fixes the mass.

Evolution to shower cut-off R_{sc}~1 GeV

NB. In hadronic colliders initial state interact.'s may not lead at a positive shift of the mass

Hadronization models <-> Soft function

TOP MASS AT TEVATRON



Vertical error is experimental
Horiz. error is theoretical:

$$m_t^{MC}(R_{SC}) = m_t^{MSR}(3^{+6}_{-2})$$

The black line is $m_t^{MSR}(R) = R$

3-loop R-evolution with MSR

Final Result: $\overline{m}_t(\overline{m}_t) = 163 \pm 1.3^{+0.6}_{-0.3}$

Where the first error is the combined experimental one and the second is from the scheme uncertainty in the matching with Tevatron.

CONCLUSIONS AND PRESPECTIVES

- Effective field theories represent a useful instrument also in LHC era: They allow to incorporate higher logs in the amplitudes and in MC's, to prove factorization, to separate the relevant scales of the physical problems and resum logs.
- We have specialized in treating jet massive particles. We are at the point of completing the first NNLL analysis of top-(anti-top) event shapes.
- We (start to) understand what is going on in MC's and we can also start to improve these in order to include the higher logs effects.

CONCLUSIONS AND PERSPECTIVES

- We can include EW effects in the analysis of massive particles
- We have prepared a new instrument to analyze and detect renormalon singularities independent of large-β_0 limit. To the moment we have applied it just to masses (u=1/2 renormalons). Can it be extended to higher order renormalons? (Work in Progress)