Signaling the Arrival of the LHC Era

The lightest Higgs boson mass in effective field theory with bulk and brane SUSY breaking

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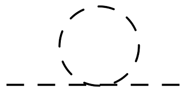
The value of the mass of Higgs boson

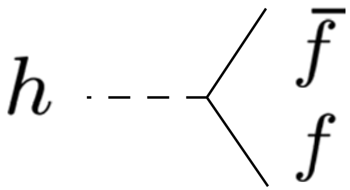
Radiative corrections

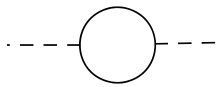


Λ^2

Contributions of heavy fields

$$\frac{i}{p^2 - m^2}$$ Large mass → Non-dynamical?





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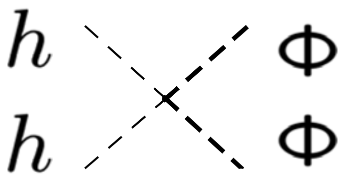

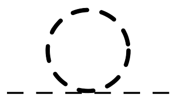

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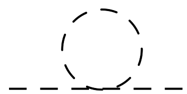 Λ^2

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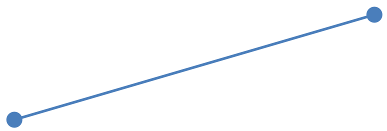

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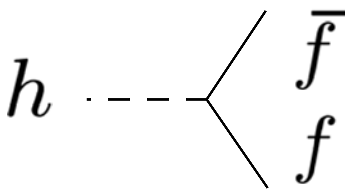
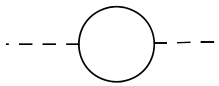
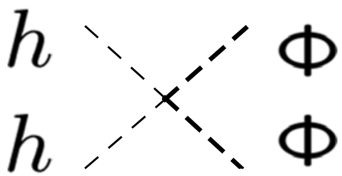
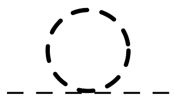




$$\Lambda^2 + m_f^2 \ln(\Lambda^2/m_f^2)$$





$$\Lambda^2 + m_\Phi^2 \ln(\Lambda^2/m_\Phi^2)$$

The effects of heavy fields are not decoupled

SUSY Cancellation between bosonic and fermionic contributions

SUSY breaking

$$\Delta m_h^2 = m_{\mathrm{sp}}^2 \left[\frac{\lambda}{16\pi^2} \ln\left(\frac{\Lambda}{m_{\mathrm{sp}}} \right) \right]$$

m_{sp} Largest mass scale associated with the mass splittings between bosons and fermions

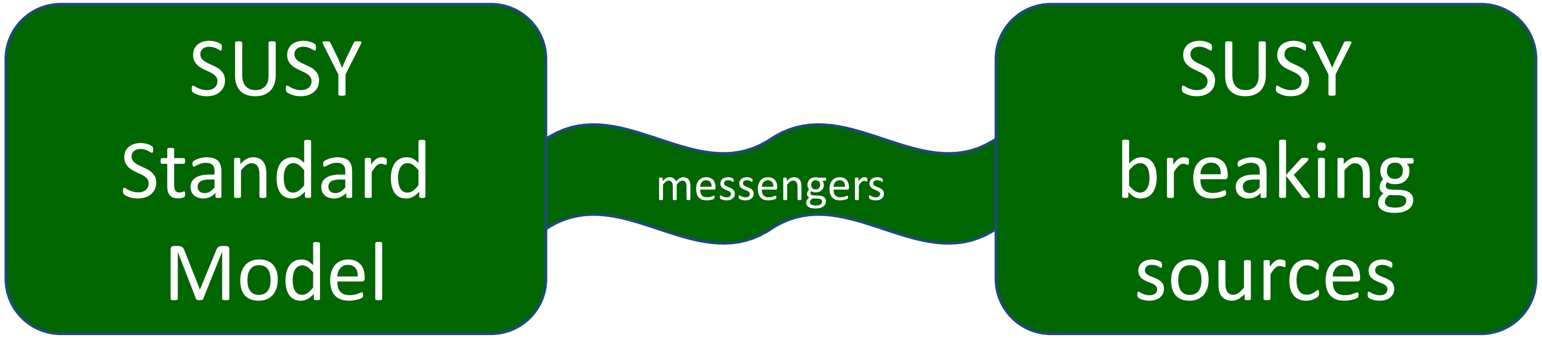
λ Dimensionless couplings

Λ Ultraviolet momentum cutoff

$\lambda \sim \mathcal{O}(1),\ \Lambda \sim M_P,$
$m_{\mathrm{sp}} \sim \mathrm{TeV}$ ⟹ $\Delta m_h^2 \sim m_W^2$

How is SUSY broken?

SUSY must be broken in a way with no SUSY flavor problem



The mediation does not distinguish flavor

How is SUSY broken?

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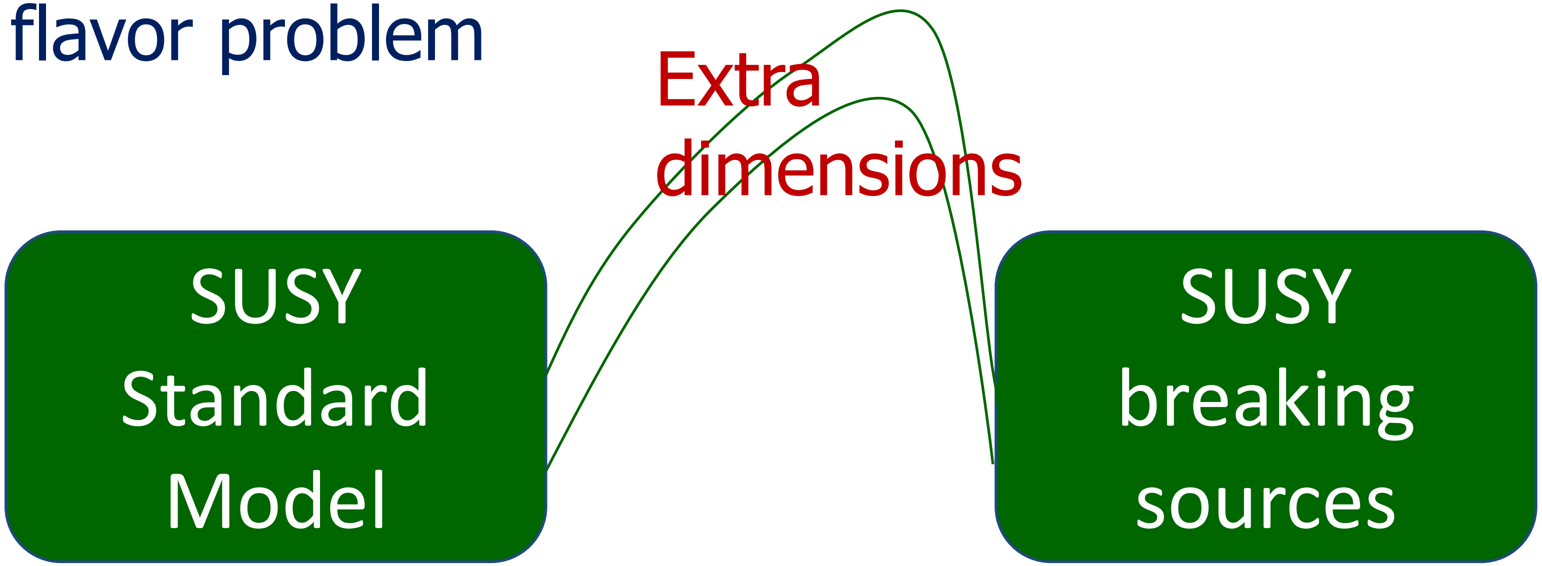
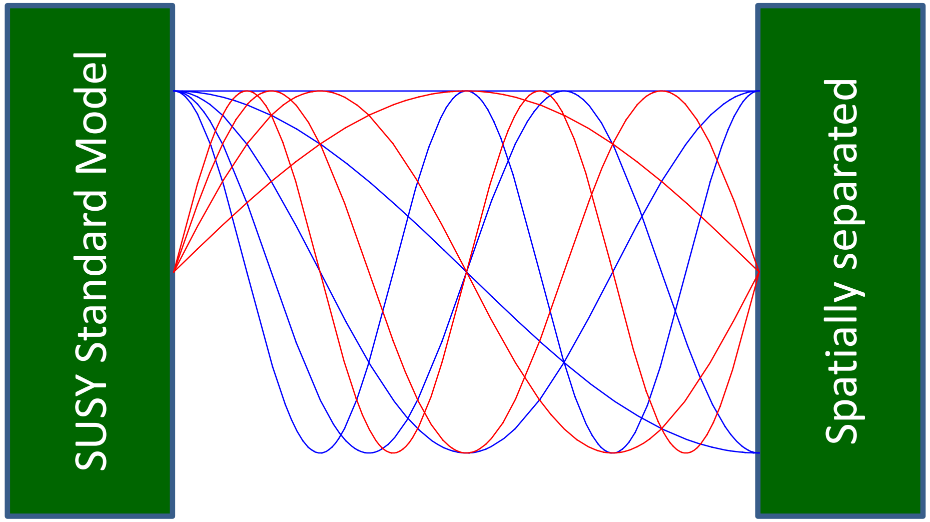


Mirabelli-Peskin 97, Randall- Sundrum 98, ..



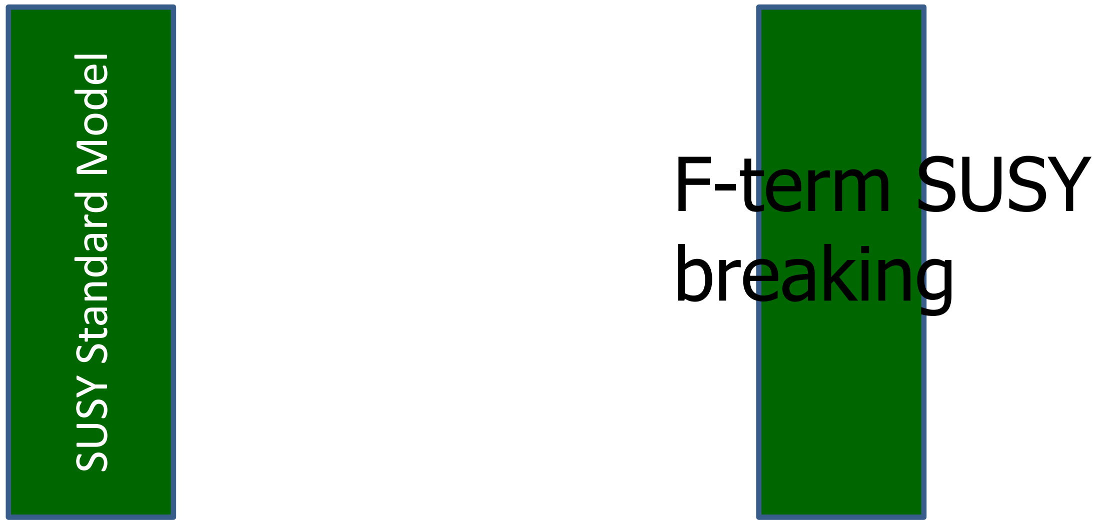
There are many even-function and odd-function modes

The mass splitting for each mode contributes to

$$\Delta m_h^2 = m_{\text{sp}}^2 \left[\frac{\lambda}{16\pi^2} \ln \left(\frac{\Lambda}{m_{\text{sp}}} \right) \right] \qquad \text{infinite?}$$

F-term SUSY breaking .. Mirabelli-Peskin 97, Marti-Pomarol 01, Clemente-King-Rayner 01, ..

Scherk-Schwarz SUSY breaking 79 .. Delgado-Pomarol-Quiros 98, Barbieri-Hall-Nomura 01, Bagger-Feruglio-Zwirner 01, Haba-Hosotani-Kawamura 03, ..



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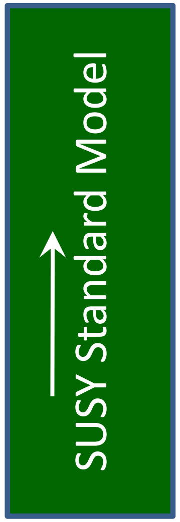
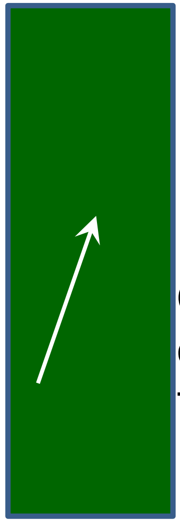


Scherk-Schwarz SUSY breaking



Change of the direction of SUSY transformation

There are many even-function and odd-function modes

The mass splitting for each mode contributes to

$$\Delta m_h^2 = m_{\text{sp}}^2 \left[\frac{\lambda}{16\pi^2} \ln \left(\frac{\Lambda}{m_{\text{sp}}} \right) \right]$$

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Introduction $\Lambda^2,\ m^2 \ln \Lambda,$

SUSY, extra dimensions

The Higgs bosons in mass eigenstates

Notation for the lightest Higgs boson

Radiative corrections in an effective 4D picture

Kaluza-Klein mode contributions

F-term and Scherk-Schwarz SUSY breaking

The Higgs boson mass corrections

The MSSM Higgs

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad (1, 2, -\tfrac{1}{2}),$$

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad (1, 2, +\tfrac{1}{2}).$$

$$V = (|\mu|^2 + m_1^2)|H_1|^2 + (|\mu|^2 + m_2^2)|H_2|^2 + [bH_1H_2 + \text{H.c.}]$$
$$+\frac{1}{8}g^2(H_2^\dagger\sigma^a H_2 + H_1^\dagger\sigma^a H_1)^2 + \frac{1}{8}g'^2(|H_2|^2 - |H_1|^2)^2$$

- $g,\ g'$ $\quad SU(2)_L,\ U(1)_Y$ gauge couplings
- μ $\quad$ a supersymmetric mass
- $m_1,\ m_2,\ b$ $\quad$ SUSY-breaking coupling constants

$$= (|\mu|^2 + m_1^2)(|H_1^0|^2 + |H_1^-|^2) + (|\mu|^2 + m_2^2)(|H_2^+|^2 + |H_2^0|^2)$$
$$+ \left[b(H_1^- H_2^+ - H_1^0 H_2^0) + \text{H.c.}\right] + \frac{1}{2}g^2|H_2^+ H_1^{0*} + H_2^0 H_1^{-*}|^2$$
$$+\frac{1}{8}(g^2 + g'^2)(|H_2^+|^2 + |H_2^0|^2 - |H_1^0|^2 - |H_1^-|^2)^2.$$

$$V^{\text{neutral}} = (|\mu|^2 + m_1^2)|H_1^0|^2 + (|\mu|^2 + m_2^2)|H_2^0|^2 - \left[b H_1^0 H_2^0 + \text{H.c.} \right] + \frac{1}{8}(g^2 + g'^2)(|H_2^0|^2 - |H_1^0|^2)^2.$$

$$V = (|\mu|^2 + m_1^2)(|H_1^0|^2 + |H_1^-|^2) + (|\mu|^2 + m_2^2)(|H_2^+|^2 + |H_2^0|^2) + \left[b(H_1^- H_2^+ - H_1^0 H_2^0) + \text{H.c.} \right] + \frac{1}{2} g^2 |H_2^+ H_1^{0*} + H_2^0 H_1^{-*}|^2 + \frac{1}{8}(g^2 + g'^2)(|H_2^+|^2 + |H_2^0|^2 - |H_1^0|^2 - |H_1^-|^2)^2.$$

$$V^{\rm neutral} = (|\mu|^2 + m_1^2)|H_1^0|^2 + (|\mu|^2 + m_2^2)|H_2^0|^2 - \left[b H_1^0 H_2^0 + {\rm H.c.} \right] + \frac{1}{8}(g^2 + g'^2)(|H_2^0|^2 - |H_1^0|^2)^2$$

$$= \frac{1}{2} m_{h^0}^2 h^{0 2} + \frac{1}{2} m_{H^0}^2 H^{0 2} + \frac{1}{2} m_{G^0}^2 G^{0 2} + \frac{1}{2} m_{A^0}^2 A^{0 2}.$$

$$m_{h^0}^2 = \frac{1}{2}\left[m_{A^0}^2 + m_Z^2 - \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4 m_{A^0}^2 m_Z^2 \cos^2 2\beta} \right]$$

$$m_{H^0}^2 = \frac{1}{2}\left[m_{A^0}^2 + m_Z^2 + \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4 m_{A^0}^2 m_Z^2 \sin^2 2\beta} \right].$$

$$m_{A^0}^2 = \frac{2b}{\sin 2\beta}, \qquad \tan\beta = \frac{v_2}{v_1}, \qquad m_{G^0}^2 = 0.$$

$$m_{h^0}^2 \le m_{h^0}^2(m_{A^0}^2 \to \infty) = m_Z^2 \cos^2 2\theta.$$ The lightest Higgs

charged $m_{H^\pm}^2 = m_{A^0}^2 + m_W^2, \quad m_{G^\pm}^2 = 0.$

Stop and top contributions to the eff potential

$$\frac{3}{(4\pi)^2}(m_{\tilde t}^2+|y_t H_2^0|^2)^2\left(\ln\frac{m_{\tilde t}^2+|y_t H_2^0|^2}{\Lambda^2}-\frac{1}{2}\right)$$
$$-\frac{3}{(4\pi)^2}|y_t H_2^0|^4\left(\log\frac{|y_t H_2^0|^2}{\Lambda^2}-\frac{1}{2}\right).$$

With an expansion by powers of $|y_t H_2^0|^2/m_{\tilde t}^2$,

$$-\frac{3}{16\pi^2}|y_t H_2^0|^4\left(\ln\frac{|y_t H_2^0|^2}{m_{\tilde t}^2+|y_t H_2^0|^2}-\frac{3}{2}\right)\equiv U_t(|H_2^0|^2).$$

$$m_{h^0}^2=\frac{1}{2}\left[m_{A^0}^2+m_Z^2+\Delta_t\right.$$
$$\left.-\sqrt{((m_{A^0}^2-m_Z^2)\cos 2\beta+\Delta_t)^2+(m_{A^0}^2+m_Z^2)^2\sin^2 2\beta}\right].$$

$$\Delta_t=2v_2^2 U_t''(v_2^2)=2v_2^2\left(-\frac{3}{8\pi^2}|y_t|^4\ln\frac{|y_t|^2 v_2^2}{m_{\tilde t}^2}\right)=\frac{3\sqrt{2}m_t^4 G_F}{2\pi^2\sin^2\beta}\ln\left(\frac{m_{\tilde t}^2}{m_t^2}\right).$$

$$G_F=\frac{1}{2\sqrt{2}}(v_1^2+v_2^2)^{-1}=1.17\times 10^{-5}\ \text{GeV}^{-2}.$$

The upper bound to the lightest Higgs boson mass

$$m_{h^0}^2 \le m_{h^0}^2(m_{A^0}^2 \to \infty) = m_Z^2 \cos^2 2\beta + \Delta_t \sin^2 \beta$$
$$= m_Z^2 \cos^2 2\beta + \frac{3\sqrt{2} m_t^4 G_F}{2\pi^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right).$$

m_t 181GeV $m_{\tilde{t}}$ 1TeV ➡ $\frac{3\sqrt{2} m_t^4 G_F}{2\pi^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$ (100GeV)^2

$$m_{h^0}^2 = \frac{1}{2}\Big[m_{A^0}^2 + m_Z^2 + \Delta_t$$
$$- \sqrt{((m_{A^0}^2 - m_Z^2)\cos 2\beta + \Delta_t)^2 + (m_{A^0}^2 + m_Z^2)^2 \sin^2 2\beta}\Big].$$

$$\Delta_t = 2v_2^2 U_t''(v_2^2) = 2v_2^2\left(-\frac{3}{8\pi^2}|h_t|^4 \ln\frac{|h_t|^2 v_2^2}{m_{\tilde{t}}^2}\right) = \frac{3\sqrt{2} m_t^4 G_F}{2\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right).$$

$$G_F = \frac{1}{2\sqrt{2}}(v_1^2 + v_2^2)^{-1} = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}.$$

Kaluza-Klein mode contributions

$$\frac{3}{(4\pi)^2}\left(m_{\tilde{t}}^2+|y_tH_2|^2+\frac{n^2}{R^2}\right)^2\left(\ln\frac{m_{\tilde{t}}^2+|y_tH_2^0|^2+n^2/R^2}{\Lambda^2}-\frac{1}{2}\right)$$

$$-\frac{3}{(4\pi)^2}\left(|y_tH_2^0|^2+\frac{n^2}{R^2}\right)^2\left(\ln\frac{|y_tH_2^0|^2+n^2/R^2}{\Lambda^2}-\frac{1}{2}\right).$$

For a detail in MSSM + UED, Bhattacharyya-Majee-Raychaudhuri 07, ..

With an expansion by powers of $(m_{\tilde{t}}/(nR^{-1}))$,

$$\frac{3}{16\pi^2}\frac{R^2}{n^2}m_{\tilde{t}}^2|y_tH_2^0|^4=U_t^{(n)}(|H_2^0|^2).$$

$$m_{h^0}^2=\frac{1}{2}\Bigg[m_{A^0}^2+m_Z^2+\Delta_t+\sum_{n=1}^{\infty}\Delta_t^{(n)}$$

$$-\sqrt{\left((m_{A^0}^2-m_Z^2)\cos 2\beta+\Delta_t+\sum_{n=1}^{\infty}\Delta_t^{(n)}\right)^2+(m_{A^0}^2+m_Z^2)^2\sin^2 2\beta}\;\Bigg].$$

$$\Delta_t^{(n)}=2v_2^2U_t^{(n)''}(v_2^2)=\frac{3}{4\pi^2}\frac{R^2}{n^2}m_{\tilde{t}}^2|y_t|^4v_2^2=\frac{3\sqrt{2}m_t^4G_FR^2}{2\pi^2\sin^2\beta\, n^2}m_{\tilde{t}}^2.$$

$$m_{h^0}^2 \leq m_{h^0}^2(m_{A^0}^2 \to \infty) = m_Z^2 \cos^2 2\beta + \left(\Delta_t + \sum_{n=1}^{\infty} \Delta_t^{(n)} \right) \sin^2 \beta$$

$$= m_Z^2 \cos^2 2\beta + \frac{3\sqrt{2} m_t^4 G_F}{2\pi^2} \ln \left(\frac{m_{\tilde{t}}^2}{m_t^2} \right) + \frac{\sqrt{2}}{4} m_t^4 G_F (R m_{\tilde{t}})^2.$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \simeq 1.6.$$

If $m_{\tilde{t}} \propto R^{-1}$, Kaluza-Klein mode contributions are independent of the compactification scale

$$m_{h^0}^2 = \frac{1}{2} \left[m_{A^0}^2 + m_Z^2 + \Delta_t + \sum_{n=1}^{\infty} \Delta_t^{(n)} \right.$$

$$\left. - \sqrt{ \left((m_{A^0}^2 - m_Z^2) \cos 2\beta + \Delta_t + \sum_{n=1}^{\infty} \Delta_t^{(n)} \right)^2 + (m_{A^0}^2 + m_Z^2)^2 \sin^2 2\beta } \right].$$

$$\Delta_t^{(n)} = 2 v_2^2 U_t^{(n)''}(v_2^2) = \frac{3}{4\pi^2} \frac{R^2}{n^2} m_{\tilde{t}}^2 |y_t|^4 v_2^2 = \frac{3\sqrt{2} m_t^4 G_F R^2}{2\pi^2 \sin^2 \beta \, n^2} m_{\tilde{t}}^2.$$

Zero mode SUSY Standard model

The SM gauge bosons, 3rd generation of quarks and leptons, 2 Higgs fields and their superpartners

F term

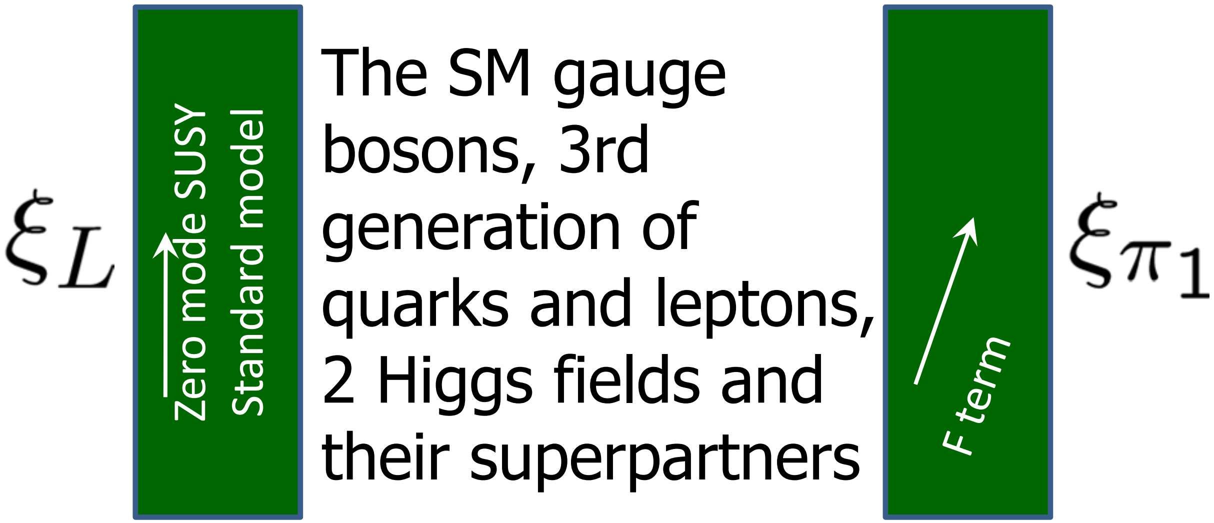
$$\begin{pmatrix}\tilde\psi^1\\ \tilde\psi^2\end{pmatrix}(x,y)=\sqrt{\frac{2}{\pi R}}\begin{pmatrix}\cos\frac{\beta y}{2\pi R} & \sin\frac{\beta y}{2\pi R}\\ -\sin\frac{\beta y}{2\pi R} & \cos\frac{\beta y}{2\pi R}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\tilde\psi^{1,0}(x)+\tilde\psi^{1,n}(x)\cos\frac{ny}{R}\\ \tilde\psi^{2,n}(x)\sin\frac{ny}{R}\end{pmatrix}.$$

$$\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}(x,y)=\sqrt{\frac{2}{\pi R}}\begin{pmatrix}\frac{1}{\sqrt{2}}\psi^{1,0}(x)+\psi^{1,n}(x)\cos\frac{ny}{R}\\ \psi^{2,n}(x)\sin\frac{ny}{R}\end{pmatrix}.$$

$$\begin{pmatrix}\xi_L(x,y=0)\\ \xi_R(x,y=0)\end{pmatrix}_{\text{zero mode}}=\sqrt{\frac{1}{\pi R}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\xi^{1,0}(x)\\ 0\end{pmatrix}\ \text{at}\ \ y=0.$$

$$\begin{pmatrix}\xi_L(x,y=\pi R)\\ \xi_R(x,y=\pi R)\end{pmatrix}_{\text{zero mode}}=\sqrt{\frac{1}{\pi R}}\begin{pmatrix}\cos\frac{\beta}{2} & \sin\frac{\beta}{2}\\ -\sin\frac{\beta}{2} & \cos\frac{\beta}{2}\end{pmatrix}\begin{pmatrix}\xi^{1,0}(x)\\ 0\end{pmatrix}\ \text{at}\ y=\pi R.$$

$$\begin{pmatrix}\xi_{\pi_1}\\ \xi_{\pi_2}\end{pmatrix}=\begin{pmatrix}\cos\frac{\beta}{2} & -\sin\frac{\beta}{2}\\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2}\end{pmatrix}\begin{pmatrix}\xi_L\\ \xi_R\end{pmatrix}.$$



$$\mathcal{L}_{\pi R} = \delta(y-\pi R)\frac{1}{2}\left[-\frac{c_l}{\Lambda^3}T^\dagger_{L\pi_1}T_{L\pi_1}S^\dagger S-\frac{c_r}{\Lambda^3}T^\dagger_{R\pi_1}T_{R\pi_1}S^\dagger S\right]_D$$

$$-\mathcal{L}_4^{\text{mass}} = \frac{1}{R^2}\left[\frac{\beta^2}{4\pi^2}\tilde{t}^{1*}_{L,0}\tilde{t}^1_{L,0}+\sum_{n=1}^{\infty}\left(n^2+\frac{\beta^2}{4\pi^2}\right)(\tilde{t}^{1*}_{L,n}\tilde{t}^1_{L,n}+\tilde{t}^{2*}_{L,n}\tilde{t}^2_{L,n})\right.$$
$$-\sum_{n=1}^{\infty}\frac{n\beta}{\pi}(\tilde{t}^{1*}_{L,n}\tilde{t}^2_{L,n}+\tilde{t}^{2*}_{L,n}\tilde{t}^1_{L,n})+\sum_{n,m=1}^{\infty}\frac{2\alpha}{\pi^2}(-1)^{n+m}\tilde{t}^{1*}_{L,n}\tilde{t}^1_{L,m}$$
$$\left.+\sum_{k=n}^{\infty}\frac{\sqrt{2}\alpha}{\pi^2}(-1)^n(\tilde{t}^{1*}_{L,n}\tilde{t}^1_{L,0}+\tilde{t}^{1*}_{L,0}\tilde{t}^1_{L,n})+\frac{\alpha}{\pi^2}\tilde{t}^{1*}_{L,0}\tilde{t}^1_{L,0}+(L\leftrightarrow R)\right].$$

$$\alpha = c_{\tilde{t}}\,\pi\left(\frac{|\langle F_S\rangle|^2}{\Lambda^4}\right)\Lambda R.$$

$c_l = c_r = c_{\tilde{t}}$ for simplicity

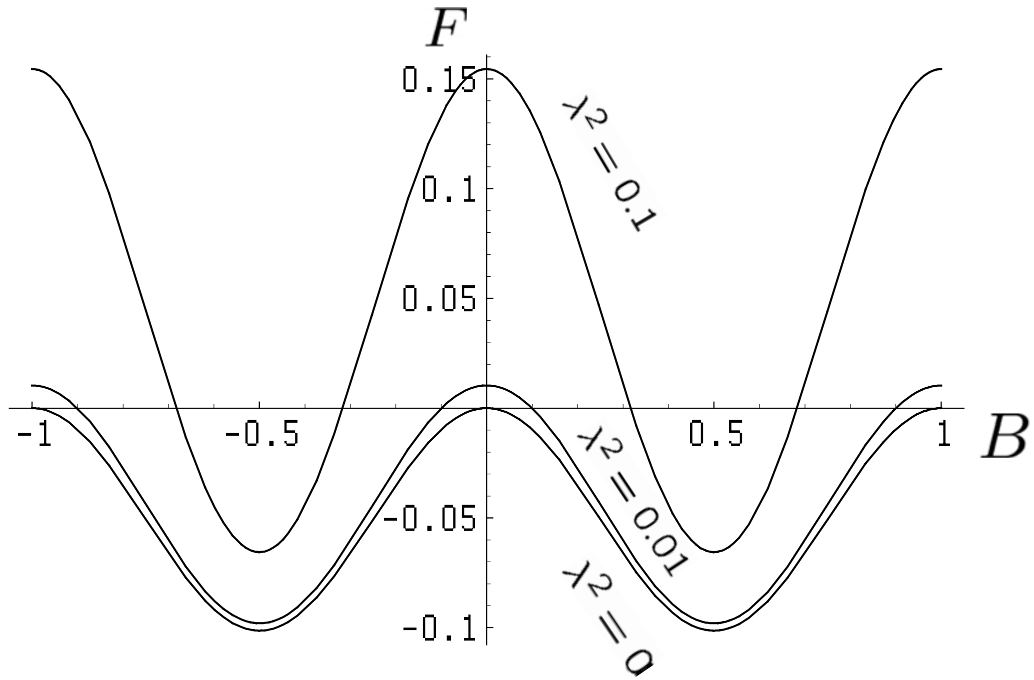
Up to the overall R^{-2}, L part of mass term is written in a basis of $(\tilde{t}^1_{L,0}, \tilde{t}^1_{L,1}, \tilde{t}^1_{L,2}, \cdots | \tilde{t}^2_{L,1}, \tilde{t}^2_{L,2}, \cdots)^T$ as

$$\left(\begin{array}{ccccc|cccc}
B^2+F & -\sqrt{2}F & \sqrt{2}F & -\sqrt{2}F & \cdots & 0 & 0 & 0 & \cdots \\
-\sqrt{2}F & 1+B^2+2F & -2F & 2F & \cdots & -2B & 0 & 0 & \cdots \\
\sqrt{2}F & -2F & 4+B^2+2F & -2F & \cdots & 0 & -4B & 0 & \cdots \\
-\sqrt{2}F & 2F & -2F & 9+B^2+2F & \cdots & 0 & 0 & -6B & \cdots \\
 & & & & \cdots & & & & \cdots \\
\hline
0 & -2B & 0 & 0 & \cdots & 1+B^2 & 0 & 0 & \cdots \\
0 & 0 & -4B & 0 & \cdots & 0 & 4+B^2 & 0 & \cdots \\
0 & 0 & 0 & -6B & \cdots & 0 & 0 & 9+B^2 & \cdots \\
 & & & & \cdots & & & & \cdots
\end{array}\right).$$

$$B = \frac{\beta}{2\pi}, \qquad F = \frac{\alpha}{\pi^2} = \frac{c_{\tilde{t}} |\langle F_S \rangle|^2 R}{\pi \Lambda^3}.$$

The eigenvalue equation

$$\frac{1}{\lambda^2 - B^2} + \sum_{n=1}^{\infty} \left(\frac{1}{\lambda^2 - (n+B)^2} + \frac{1}{\lambda^2 - (n-B)^2} \right) = \frac{1}{F}$$



Invariance under $B \to B+1,\ B \leftrightarrow -B.$

$$B = \frac{\beta}{2\pi}, \qquad F = \frac{\alpha}{\pi^2} = \frac{c_{\tilde{t}}|\langle F_S\rangle|^2 R}{\pi\Lambda^3}.$$

The eigenvalue equation

$$\frac{1}{\lambda^2 - B^2} + \sum_{n=1}^{\infty}\left(\frac{1}{\lambda^2-(n+B)^2} + \frac{1}{\lambda^2-(n-B)^2}\right) = \frac{1}{F}$$

For $B = 1/2$,

$$\frac{\pi \tan \pi\lambda}{\lambda} = -\frac{1}{F}.$$

The lightest top and stop masses

$$m_t^2 = |y_t|^2 v_2^2, \quad m_{\tilde{t}}^2 = \frac{3}{R^2 \pi^4} \left(\frac{1}{F} + \pi^2 \right).$$

The n -th Kaluza-Klein top and stop masses

$$m_{t,n}^2 = \frac{n^2}{R^2}, \quad m_{\tilde{t},n}^2 \approx \frac{(n + \frac{1}{2})^2}{R^2}.$$

The upper bound to the lightest Higgs boson mass

$$m_{h^0}^2 \leq m_Z^2 \cos^2 2\beta + \frac{3\sqrt{2} m_t^4 G_F}{2\pi^2} \ln \frac{(2m_{\tilde{t}})^2}{(\pi m_t)^2}.$$

The summation over infinite numbers of Kaluza-Klein mode contributions converges.

The corrections of Kaluza-Klein modes to the lightest Higgs boson mass are independent of the compactification scale.

The lightest stop mass is R dependent. The eigenvalue can have a small value. Thus the mass splitting between zero-mode stop and top can be around 1 TeV even for a large R^{-1}.