Weak decays of Λ_b baryon in QCD

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Outline

1. Motivation and introduction

2. $\Lambda_b \to \Lambda\gamma, \Lambda l^+ l^-$ decays in LCSR

3. $\Lambda_b \to p\pi^-, pK^-$ decays in PQCD

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I. Motivation and introduction

- Heavy baryon decays offer a new platform to test the CKM sector of the SM and possibly open a door to physics beyond the SM.

- Heavy baryon decays allow the study of spin correlations, providing valuable information on the chirality of the short-distance transition.

- Heavy baryons containing a bottom quark will be copiously accumulated at the LHC experiment. The $b\bar{b}$ cross section is expected to be $\sim 500\mu b$ producing 10^{12} $b\bar{b}$ pairs in a standard (10^7 s) year of running at the LHCb operational luminosity of $2 \times 10^{32}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$ * .

*M. P. Altarelli and F. Teubert, arXiv:0802.1901 [hep-ph]

- The polarization asymmetries of Λ baryon in $\Lambda_b \to \Lambda + l^+ l^-$ are sensitive to right-handed couplings, which are suppressed in the standard model. Thus, these baryonic decays could be used to search for physics beyond the standard model.

- Nonleptonic two-body decay of $\Lambda_b \to p\pi^-, pK^-$ have been observed at Tevatron and their branching ratios and direct CP asymmetries are also measured by CDF collaboration †.

	BR ($\times 10^{-6}$)	CP
$\Lambda_b \to p\pi^-$	$3.5 \pm 0.6 \pm 0.9$	$-0.03 \pm 0.17 \pm 0.05$
$\Lambda_b \to pK$	$5.6 \pm 0.8 \pm 1.5$	$-0.37 \pm 0.17 \pm 0.03$

†CDF collaboration, arXiv: 0810.3258

2. $\Lambda_b \to \Lambda\gamma, \Lambda l^+ l^-$ decays in LCSR

- A short introduction to QCDSR/LCSR
- Effective Hamiltonian for $b \to s\gamma$ and $b \to s l^+ l^-$
- Distribution amplitudes of light Λ baryon
- LCSR for the form factors of $\Lambda_b \to \Lambda + \gamma$ and $\Lambda_b \to \Lambda + l^+ l^-$
- Numerical analysis of $\Lambda_b \to \Lambda + \gamma$ and $\Lambda_b \to \Lambda + l^+ l^-$ decays

A short introduction to QCDSR/LCSR

- The basic idea of QCD sum rules is calculating the correlation functions in the quark and hadron levels respectively, and then matching them with the assumption of quark-hadron duality (SVZ, 1979).

- The QCD representation of the correlation function is calculated in the framework of operator product expansion (OPE), where the short- and long-distance quark-gluon interactions are separated.

- The contribution from short distance can be computed using QCD perturbative theory, whereas the nonperturbative contributions can be parameterized by a few vacuum expectation values of composite operators (Colangelo and Khodjamirian, hep-ph/0010175).

- Problems of three-point SVZ sum rules [‡]:

  (a) One major problem of three-point SVZ sum rules is that OPE (short-distance expansion in condensates) upsets power counting in the large momentum or mass.

  (b)Another major problem of three-point SVZ sum rules is the contamination of the sum rules, at zero momentum transfer, by “nondiagonal” transition of the ground state to excited states.

- Light-cone sum rules (LCSR) were developed in late 80-th of last century in an attempt to solve or at least moderate the problems of three-point sum rules by making a partial resummation of the OPE to all orders and reorganizing the expansion in terms of twist of relevant operators rather than their dimension.

[‡]V. M. Braun, arXiv:hep-ph/9801222.

- The difference between LCSR and SVZSR is that the expansion at short distances is substituted by the expansion in the transverse distance between partons in the infinite momentum frame.

- Technically, the LCSR approach presents a marriage of QCD sum rules with the theory of hard exclusive processes.

- As a bonus, SVZ vacuum condensates are substituted by light-cone hadron distribution functions of increasing twist which have a direct physical significance.

- One advantage of LCSR approach is that the soft non-perturbative contribution to the transition form factor can be calculated quantitatively.

- LCSR has been widely used in the heavy-to-light decays,
  such as: $B \to (\pi, \rho) l \bar{\nu}$, $B \to K^* \gamma, K^* l^+ l^-$, $B \to \pi\pi$, $B \to KK$......
  $\Lambda_b \to p l \bar{\nu}$, $\Lambda_c \to \Lambda l^+ \nu$....

Effective Hamiltonian for $b \to s\gamma$ and $b \to sl^+l^-$

- In the SM the effective Hamiltonian for $b \to sl^+l^-$ can be given by

$$H_{eff}(b \to sl^+l^-)$$
$$= \frac{G_F}{2\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{\pi} \{ -\frac{2i}{q^2} C_7^{eff}(\mu, q^2)[m_b \bar{s}\sigma_{\mu\nu} q^\nu R b + m_s \bar{s}\sigma_{\mu\nu} q^\nu L b]\bar{l}\gamma^\mu l$$
$$+ C_9^{eff}(\mu) \bar{s}\gamma_\mu L b \bar{l}\gamma^\mu l + C_{10} \bar{s}\gamma_\mu L b \bar{l}\gamma^\mu \gamma_5 l \}.$$

- For $b \to s\gamma$, the effective Hamiltonian responsible for it can be written as:

$$H_{eff}(b \to s\gamma) \;=\; i\frac{G_F}{2\sqrt{2}} V_{tb} V_{ts}^* \frac{e}{4\pi^2} C_7^{eff}(\mu)[m_b \bar{s}\sigma_{\mu\nu} R b + m_s \bar{s}\sigma_{\mu\nu} L b] F^{\mu\nu}.$$

In the above the notations conventions are: $L = 1 - \gamma_5, R = 1 + \gamma_5, \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$.

- Parametrization of hadronic matrix elements:

$$\langle\Lambda(P)|\bar{s}\gamma_\mu b|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(g_1\gamma_\mu + g_2 i\sigma_{\mu\nu}q^\nu + g_3 q_\mu)\Lambda_b(P+q),$$

$$\langle\Lambda(P)|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(G_1\gamma_\mu + G_2 i\sigma_{\mu\nu}q^\nu + G_3 q_\mu)\gamma_5\Lambda_b(P+q),$$

$$\langle\Lambda(P)|\bar{b}i\sigma_{\mu\nu}q^\nu s|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(f_1\gamma_\mu + f_2 i\sigma_{\mu\nu}q^\nu + f_3 q_\mu)\Lambda_b(P+q),$$

$$\langle\Lambda(P)|\bar{b}i\sigma_{\mu\nu}q^\nu\gamma_5 s|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(F_1\gamma_\mu + F_2 i\sigma_{\mu\nu}q^\nu + F_3 q_\mu)\gamma_5\Lambda_b(P+q).$$

- The form factors f_1, f_3, F_1 and F_3 satisfy the following relations

$$f_1 = -\frac{q^2}{M_{\Lambda_b} - M} f_3, \qquad F_1 = \frac{q^2}{M_{\Lambda_b} - M} F_3.$$

Distribution amplitudes of light Λ baryon

- The distribution amplitudes of Λ barton up to leading Fock state can be defined by the non-local matrix element, similar to the case for the nucleon, as §

$$
\begin{aligned}
&4\langle 0|\epsilon^{ijk}u_\alpha^i(a_1z)d_\beta^j(a_2z)s_\gamma^k(a_3z)|\Lambda(P)\rangle \\
&= \mathcal{A}_1\,(\not{P}\gamma_5 C)_{\alpha\beta}\,\Lambda_\gamma + \mathcal{A}_2 M\,(\not{P}\gamma_5 C)_{\alpha\beta}\,(\not{z}\Lambda)_\gamma + \mathcal{A}_3 M\,(\gamma_\mu\gamma_5 C)_{\alpha\beta}\,(\gamma^\mu\Lambda)_\gamma \\
&+\mathcal{A}_4 M^2\,(\not{z}\gamma_5 C)_{\alpha\beta}\,\Lambda_\gamma + \mathcal{A}_5 M^2\,(\gamma_\mu\gamma_5 C)_{\alpha\beta}\,(i\sigma^{\mu\nu}z_\nu\Lambda)_\gamma + \mathcal{A}_6 M^3\,(\not{z}\gamma_5 C)_{\alpha\beta}\,(\not{z}\Lambda)_\gamma \,.
\end{aligned}
$$

- The invariant functions $\mathcal{S}_i$, $\mathcal{P}_i$,$\mathcal{V}_i$, $\mathcal{A}_i$,$\mathcal{T}_i$ can be related to the distribution amplitudes S_i, P_i, V_i, A_i and T_i,as

$$
\begin{aligned}
\mathcal{A}_1 &= A_1\,, & 2p\cdot z\mathcal{A}_2 &= -A_1 + A_2 - A_3\,, \\
2\mathcal{A}_3 &= A_3\,, & 4p\cdot z\mathcal{A}_4 &= -2A_1 - A_3 - A_4 + 2A_5\,, \\
4p\cdot z\mathcal{A}_5 &= A_3 - A_4\,, & (2p\cdot z)^2\mathcal{A}_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6\,.
\end{aligned}
$$

§V. Braun, R.J. Fries, N. Mahnke and E. Stein, Nucl. Phys. **B589** (2000) 381 [Erratum-ibid. B **607** (2001) 433] [arXiv:hep-ph/0007279].
Min-Qiu Huang and Dao-Wei Wang, arXiv: hep-ph/0608170.

- Up to leading conformal spin accuracy, the light-cone distribution amplitudes A_i can be expressed

$$\begin{aligned}
A_1(x_1,x_2,x_3) &= -120 x_1 x_2 x_3 \phi_3^0, \\
A_2(x_1,x_2,x_3) &= -24 x_1 x_2 \phi_4^0, \\
A_3(x_1,x_2,x_3) &= -12 x_3 (1-x_3) \psi_4^0, \\
A_4(x_1,x_2,x_3) &= -3(1-x_3)\phi_5^0, \\
A_5(x_1,x_2,x_3) &= -6 x_3 \phi_5^0, \\
A_6(x_1,x_2,x_3) &= -2\phi_6^0.
\end{aligned}$$

- At the case of Λ baryon, all the 6 parameters can be expressed in terms of 2 independent matrix elements of local operators:

$$\phi_3^0 = \phi_6^0 = -f_\Lambda, \ \phi_4^0 = \phi_5^0 = -\frac{1}{2}(\lambda_1 + f_\Lambda), \ \psi_4^0 = \psi_5^0 = -\frac{1}{2}(\lambda_1 - f_\Lambda),$$

with

$$f_\Lambda = 6.1 \times 10^{-3}\mathrm{GeV}^2, \ \lambda_1 = -1.2 \times 10^{-2}\mathrm{GeV}^2.$$

LCSR for the tensor current transition

- The correction function for the tensor current transition can be chosen as

$$z^\nu T_\nu(P,q) = i z^\nu \int d^4x e^{-iq\cdot x} \langle 0 | T\{j_{\Lambda_b}(0) j_\nu(x)\} | \Lambda(P)\rangle .$$

The current $j_{\Lambda_b}(0)$ and $j_\nu(x)$ are given by

$$\begin{aligned} j_{\Lambda_b}(0) &= \epsilon^{ijk}[u^i(0) C \gamma_5 \not{z} d^j(0)] \not{z} b^k(0) \ , \\ j_\nu(x) &= i\bar{b}(x)\sigma_{\mu\nu}(1-\gamma_5) q^\mu s(x) \ . \end{aligned}$$

- The coupling of the chosen current with Λ_b can be given by

$$\langle 0 | j_{\Lambda_b}(0) | \Lambda_b(P')\rangle = f_{\Lambda_b}(z\cdot P') \not{z} \Lambda_b(P').$$

- The correction function can be calculated in the hadron level as

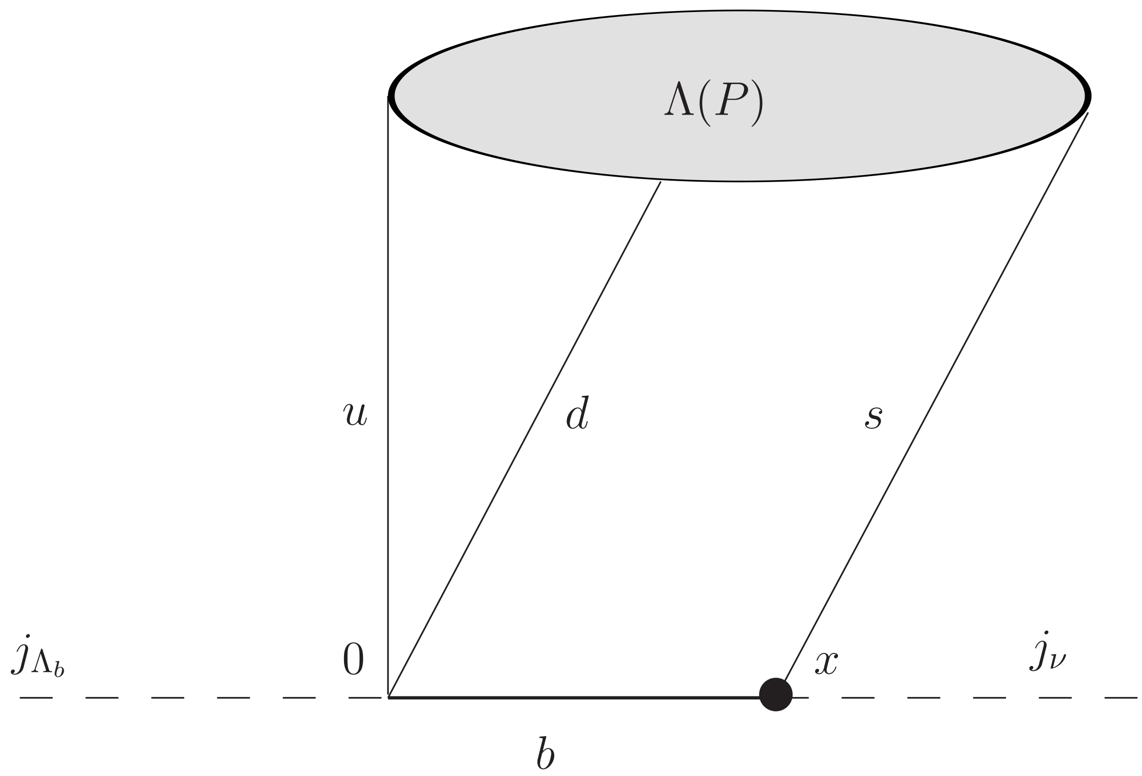
$$z^{\nu}T_{\nu} = 2f_{\Lambda_b}\frac{(z\cdot P')^2}{m_{\Lambda_b}^2 - P'^2}[f_1 \not{z} + f_2 \not{z} \not{q} + F_1 \not{z}\gamma_5 - F_2 \not{z} \not{q}\gamma_5]\Lambda(P) + ...,$$

where $P' = P + q$ and the ellipses denote the terms in proportion to the higher power of $1/P$ in the infinite momentum kinematics $P \sim \infty$, $q \sim const$, $z \sim 1/P$ and the contributions from the higher states of Λ_b channel.

- To the leading order of α_s and making use of the OPE technique, the correlation function can be computed as

$$z^{\nu}T_{\nu} = -2(C\gamma_5 \not{z})_{\alpha\beta}[\not{z}(1-\gamma_5)]_{\gamma} \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(k-q)\cdot x} \frac{z\cdot k}{k^2 - m_b^2} \langle 0|\epsilon^{ijk} u_{\alpha}^i(0) d_{\beta}^j(0) s_{\gamma}^k(x)|\Lambda(P)\rangle,$$

which can be represented by the following figure intuitively.



$\Lambda(P)$
u
d
s
j_{Λ_b}
0
x
j_ν
b

- Matching the correlation function obtained in the hadron level and quark representation, we can derive the sum rules for the form factors as

$$\begin{aligned} f_{\Lambda_b} f_1(q^2) e^{-m_{\Lambda_b}^2/M_B^2} &= q^2 M \Big\{ \int_{x_0}^1 \frac{dx_3}{x_3} e^{-s/M_B^2} \Big[\frac{1}{M_B^2} \Big(-\tilde{A}_1(x_3) + \tilde{A}_2(x_3) - \tilde{A}_3(x_3) \Big) \\ &\quad - \frac{M^2}{M_B^4} \Big(\tilde{\tilde{A}}_1(x_3) - \tilde{\tilde{A}}_2(x_3) + \tilde{\tilde{A}}_3(x_3) + \tilde{\tilde{A}}_4(x_3) - \tilde{\tilde{A}}_5(x_3) + \tilde{\tilde{A}}_6(x_3) \Big) \Big] \\ &+ \frac{x_0 e^{-s_0/M_B^2}}{m_b^2 - q^2 + x_0^2 M^2} \Big[\Big(-\tilde{A}_1(x_3) + \tilde{A}_2(x_3) - \tilde{A}_3(x_3) \Big) \\ &\quad - \frac{M^2}{M_B^2} \Big(\tilde{\tilde{A}}_1(x_0) - \tilde{\tilde{A}}_2(x_0) + \tilde{\tilde{A}}_3(x_0) + \tilde{\tilde{A}}_4(x_0) - \tilde{\tilde{A}}_5(x_0) + \tilde{\tilde{A}}_6(x_0) \Big) \\ &\quad + M^2 x_0 \frac{d}{dx_0} \Big(\frac{x_0 \Big(\tilde{\tilde{A}}_1(x_0) - \tilde{\tilde{A}}_2(x_0) + \tilde{\tilde{A}}_3(x_0) + \tilde{\tilde{A}}_4(x_0) - \tilde{\tilde{A}}_5(x_0) + \tilde{\tilde{A}}_6(x_0) \Big)}{m_0^2 - q^2 + x_0^2 M^2} \Big) \Big] \Big\} . \end{aligned}$$

$$\begin{aligned} f_{\Lambda_b} f_2(q^2) e^{-m_{\Lambda_b}^2/M_B^2} &= \int_{x_0}^1 dx_3 e^{-s/M_B^2} \Big[\Big(\int_0^{1-x_3} dx_3 A_1(x_1, 1-x_1-x_3, x_3) \Big) \\ &\quad - \frac{M^2}{M_B^2} \Big(-2\tilde{A}_1(x_3) + \tilde{A}_2(x_3) - \tilde{A}_3(x_3) - \tilde{A}_4(x_3) + \tilde{A}_5(x_3) \Big) \\ &\quad + \frac{M^4}{M_B^4} \Big(\tilde{\tilde{A}}_1(x_3) - \tilde{\tilde{A}}_2(x_3) + \tilde{\tilde{A}}_3(x_3) + \tilde{\tilde{A}}_4(x_3) - \tilde{\tilde{A}}_5(x_3) + \tilde{\tilde{A}}_6(x_3) \Big) \Big] \end{aligned}$$

$$-\frac{M^2 x_0^2 e^{-s_0/M_B^2}}{m_b^2 - q^2 + x_0^2 M^2}\Big[\Big(-2\tilde{A}_1(x_0) + \tilde{A}_2(x_0) - \tilde{A}_3(x_0) - \tilde{A}_4(x_0) + \tilde{A}_5(x_0)\Big)$$
$$-\frac{M^2}{M_B^2}\Big(\tilde{\tilde{A}}_1(x_0) - \tilde{\tilde{A}}_2(x_0) + \tilde{\tilde{A}}_3(x_0) + \tilde{\tilde{A}}_4(x_0) - \tilde{\tilde{A}}_5(x_0) + \tilde{\tilde{A}}_6(x_0)\Big)$$
$$+M^2 \frac{d}{dx_0}\Big(\frac{x_0^2\big(\tilde{\tilde{A}}_1(x_0) - \tilde{\tilde{A}}_2(x_0) + \tilde{\tilde{A}}_3(x_0) + \tilde{\tilde{A}}_4(x_0) - \tilde{\tilde{A}}_5(x_0) + \tilde{\tilde{A}}_6(x_0)\big)}{m_0^2 - q^2 + x_0^2 M^2}\Big)\Big],$$

where

$$s = (1 - x_3)M^2 + \frac{m_b^2 + (1 - x_3)Q^2}{x_3},$$

and

$$x_0 = \frac{\sqrt{(Q^2 + s_0 - M^2)^2 + 4M^2(Q^2 + m_b^2)} - (Q^2 + s_0 - M^2)}{2M^2}.$$

- The form factors $F_1(q^2)$ and $F_2(q^2)$ satisfy the following relations

$$F_1(q^2) = f_1(q^2), \;\; F_2(q^2) = f_2(q^2).$$

LCSR for the vector current transition

- The correction function for the vector current transition can be selected as

$$z^\nu \tilde{T}_\nu(P,q) = i z^\nu \int d^4x e^{-iq\cdot x} \langle 0|T\{j_{\Lambda_b}(0)\tilde{j}_\nu(x)\}|\Lambda(P)\rangle,$$

where the current $\tilde{j}_\nu(x)$ are given by

$$\tilde{j}_\nu(x) = \bar{b}(x)\gamma_\nu(1-\gamma_5)s(x) \ .$$

- Comparing the sum rules for $g_1(q^2)$, $g_2(q^2)$ and $G_1(q^2)$, $G_2(q^2)$, we can find the following relations

$$\begin{aligned} f_1(q^2) &= q^2 g_2(q^2), \ \ f_2(q^2) = g_1(q^2), \\ G_1(q^2) &= g_1(q^2), \ \ G_2(q^2) = g_2(q^2). \end{aligned}$$

Numerical analysis of $\Lambda_b \to \Lambda + \gamma$ and $\Lambda_b \to \Lambda + l^+ l^-$

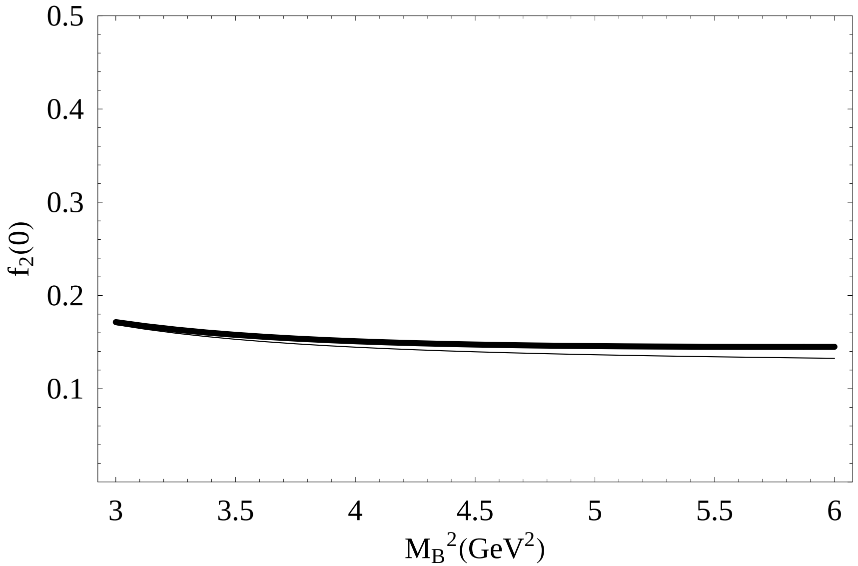
- The input parameters used in this work can be grouped as below

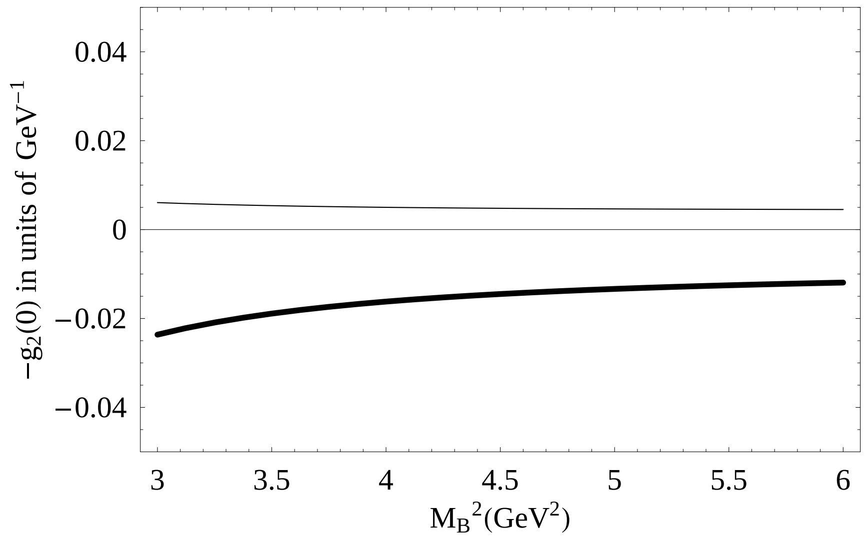
$$
\begin{array}{ll}
m_b = (4.68 \pm 0.03)\mathrm{GeV}, & m_s(1\mathrm{GeV}) = 142\mathrm{MeV}, \\
m_{\Lambda_b} = 5.62\mathrm{GeV}, & m_{\Lambda} = 1.12\mathrm{GeV}, \\
f_{\Lambda_b} = 3.9 \times 10^{-3}\mathrm{GeV}^2, & f_{\Lambda} = 6.1 \times 10^{-3}\mathrm{GeV}^2 \\
\lambda_1 = -1.2 \times 10^{-2}\mathrm{GeV}^2, & s^0_{\Lambda_b} = 39 \pm 1\mathrm{GeV}^2.
\end{array}
$$

- We require the contributions from the higher states to be less than 30 % and the value of the form factors f_i and g_i does not vary drastically within the selected region for the Borel masses. In this way, we indeed find a Borel platform $M_B^2 \in [3.0, 6.0]\mathrm{GeV}^2$ as showed in the below figure.

Form factor for $\Lambda_b \to \Lambda$ transition

- The numbers of $f_2(0)$ and $g_2(0)$ within the Borel window.





- Numerical results for the parameters for $\xi_i(0), a_i$ and b_i.

parameter	COZ	FZOZ	QCDSR	twist-3	up to twist-6
$f_2(0)$	$0.74^{+0.06}_{-0.06}$	$0.87^{+0.07}_{-0.07}$	0.45	$0.14^{+0.02}_{-0.01}$	$0.15^{+0.02}_{-0.02}$
a_1	$2.01^{+0.17}_{-0.10}$	$2.08^{+0.15}_{-0.09}$	0.57	$2.91^{+0.10}_{-0.07}$	$2.94^{+0.11}_{-0.06}$
a_2	$1.32^{+0.14}_{-0.08}$	$1.41^{+0.11}_{-0.08}$	-0.18	$2.26^{+0.13}_{-0.08}$	$2.31^{+0.14}_{-0.10}$
$g_2(0)(10^{-2}\text{GeV}^{-1})$	$-2.4^{+0.3}_{-0.2}$	$-2.8^{+0.4}_{-0.2}$	-1.4	$-0.47^{+0.06}_{-0.06}$	$1.3^{+0.2}_{-0.4}$
a_1	$2.76^{+0.16}_{-0.13}$	$2.80^{+0.16}_{-0.11}$	2.16	$3.40^{+0.06}_{-0.05}$	$2.91^{+0.12}_{-0.09}$
a_2	$2.05^{+0.23}_{-0.13}$	$2.12^{+0.21}_{-0.13}$	1.46	$2.98^{+0.09}_{-0.08}$	$2.24^{+0.17}_{-0.13}$

In the COZ model [¶], the leading twist DAs A_1 can be written as

$$\begin{aligned} A_1^{COZ}(x_1,x_2,x_3) &= -42\phi_{as}(x_1,x_2,x_3) \\ &\quad \times[0.26(x_3^2+x_2^2)+0.34x_1^2-0.56x_2x_3-0.24x_1(x_2+x_3)], \\ \phi_{as}(x_1,x_2,x_3) &= 120x_1x_2x_3. \end{aligned}$$

While in the FZOZ model [‖], the manifest expression of A_1 can be written as

$$\begin{aligned} A_1^{FZOZ}(x_1,x_2,x_3) &= -42\phi_{as}(x_1,x_2,x_3) \\ &\quad \times[0.093(x_3^2+x_2^2)+0.376x_1^2-0.194x_2x_3-0.207x_1(x_2+x_3)], \end{aligned}$$

[¶] V. L. Chernyak, A. A. Ogloblin and I. R. Zhitnitsky, Z. Phys. **C 42**, 569 (1989).

[‖] Ref: G. R. Farrar, H. Zhang, A. A. Ogloblin and I. R. Zhitnitsky, Nucl. Phys. **B 311** (1989) 585.

- In the above, we can see that the transition form factors f_2 and g_2 in the COZ and FZOZ models are approximately five times larger than that using the LCDAs of Λ baryon based on the conformal spin expansion.

- This is similar to that observed in the studies of pion form factor , nucleon form factor and also $\Lambda_c \to \Lambda$ transition that the transition form factors are likely to overshoot the experimental data, if one uses the COZ and FZOZ models.

- The sum rules of the form factors are quite stable in the region $0 < q^2 < m_b^2 - 2m_b\Lambda_{QCD}$ with the variation of M_B^2. Both the form factors f_i and g_i can be fitted in the double-pole form as

$$\xi_i(q^2) = \frac{\xi_i(0)}{1 - a_i q^2/m_{\Lambda_b}^2 + b_i q^4/m_{\Lambda_b}^4},$$

  with ξ_i being f_i and g_i, which can be extrapolated to the whole kinematical region $0 < q^2 < (m_{\Lambda_b} - m_\Lambda)^2$.

- The decay width of $\Lambda_b \to \Lambda + \gamma$ in the light-cone sum rules.

Model of DAs	twist-3	up to twist-6	COZ	FZOZ
BR	$0.63^{+0.17}_{-0.12} \times 10^{-5}$	$0.73^{+0.15}_{-0.15} \times 10^{-5}$	$1.8^{+0.3}_{-0.3} \times 10^{-4}$	$2.6^{+0.4}_{-0.4} \times 10^{-4}$

- As a comparison, the number of decay rate for $\Lambda_b \to \Lambda + \gamma$ calculated in other approaches are collected as follows.

Model	pole model	QCDSR	quark model	HQET	bag model
BR ($\times 10^{-5}$)	$0.10 \sim 0.45$	3.7 ± 0.5	0.23	$0.5 \sim 1.5$	0.4

- The decay width of $\Lambda_b \to \Lambda + l^+ l^-$ in the light-cone sum rules.

$BR(\times 10^{-6})$	$\Lambda_b \to \Lambda + \mu^+\mu^-$		$\Lambda_b \to \Lambda + \tau^+\tau^-$	
	without LD	with LD	without LD	with LD
LCSR (this work)	$6.1^{+5.8}_{-1.7}$	39^{+23}_{-11}	$2.1^{+2.3}_{-0.6}$	$4.0^{+3.7}_{-1.1}$
QCDSR	2.1	53	0.18	11
PM	1.2	36	0.26	9.0

3. $\Lambda_b \to p\pi^-, pK^-$ decays in PQCD

- A brief review of PQCD approach
- Effective Hamiltonian for nonleptonic two-body decays of Λ_b
- Hadronic distribution amplitudes
- Factorization formulae of $\Lambda_b \to p\pi^-, pK^-$ decays
- Numerical analysis of $\Lambda_b \to p\pi^-, pK^-$ decays

A brief review of PQCD approach

- Factorization is an important concept in QCD perturbation theory. Factorization amounts to the separation of physics at different energy scales.

- Collinear factorization and k_T factorization are two fundamental tools to analyze the exclusive heavy flavor hadron decays.

  Collinear factorization:
  QCD factorization (Beneke, Buchalla, Neubert, Sachajda, ...),
  Light-cone sum rules(Chernyak, Zhitnitsky, Balitsky, Braun, ...),
  Soft-collinear effective theory(Bauer, Pirjol, Rothstein, Stewart,..).

  k_T factorization:
  PQCD approach(Keum, Li, Sanda, Lü, Ukai, Yang, ...).

- The transition form factors in PQCD approach are dominated by the hard gluon exchange corresponding to the hard rescattering mechanism. Soft contribution, though indeed playing a role, is less important because of suppression from the Sudakov mechanism.

- Unlike QCD sum rules, soft contribution can not be included into the PQCD formalism in a consistent way: if there is no hard gluon exchange to provide a large characteristic scale, twist expansion does not hold. Therefore, soft contribution can not be estimated using the same meson distribution amplitudes resulting from twist expansion.

- Factorization formulae in PQCD approach**:

Taking the decay of $B \to M_1 M_2$ as an example, the amplitude can be factorized into the convolution of the six-quark hard kernel, the Wilson coefficient, the jet function and the Sudakov factor with the bound-state wave functions.

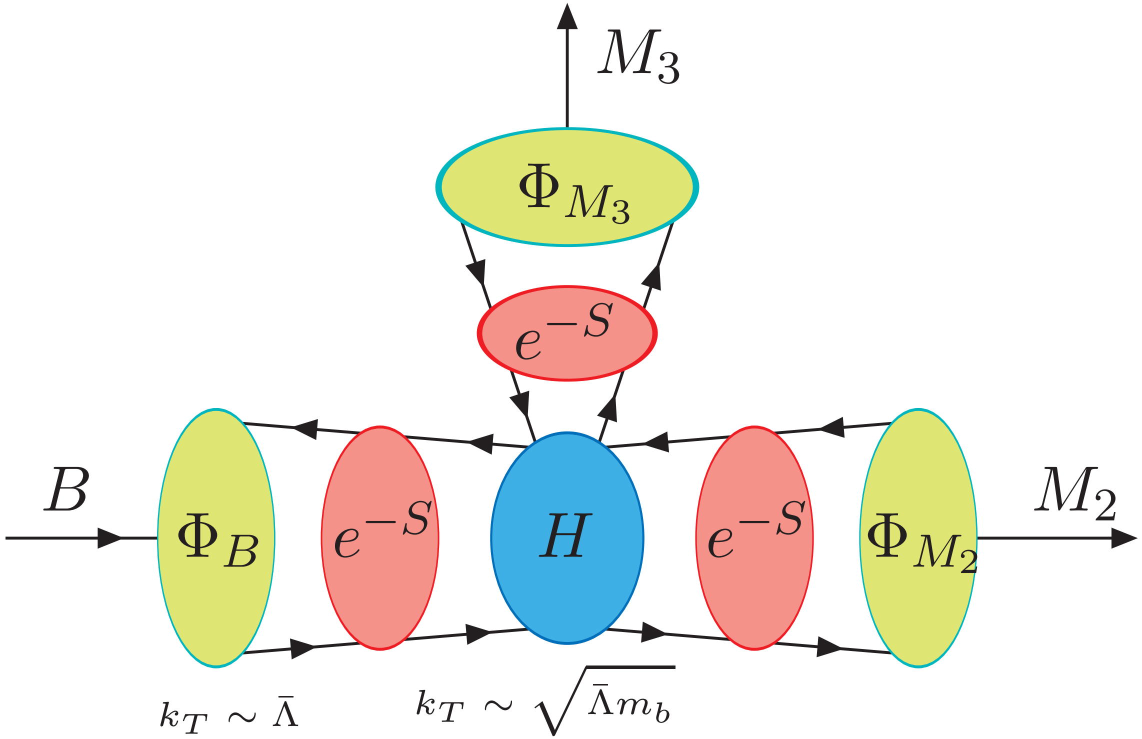
$$A = \phi_B \otimes H^{(6)} \otimes J \otimes S \otimes \phi_{M_1} \otimes \phi_{M_2} \,,$$

all of which are well-defined and gauge-invariant.

J denotes the jet function from threshold resummation , which organizes the double logarithms $ln^2 x$ due to the radiative corrections to the hard kernel. S denotes the Sudakov factor from k_T resummation, which organizes the double logarithms $ln^2 k_T$ due to the radiative corrections to the meson wave function.

H. n. Li, Prog. Part. Nucl. Phys. **51 (2003) 85.

Graphics representation of PQCD factorization theorem:



Effective Hamiltonian for nonleptonic two-body decays of Λ_b

- The weak effective Hamiltonian are specified as below:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\Big\{V_{ub}V_{uq}^*[C_1(\mu)Q_1^u(\mu)+C_2(\mu)Q_2^u(\mu)] - V_{tb}V_{tq}^*[\sum_{i=3}^{10}C_i(\mu)Q_i(\mu)]\Big\} + \text{H.c.}.$$

- The functions Q_i ($i = 1, ..., 10$) are the local four-quark operators:
  - current–current (tree) operators

$$Q_1^u = (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \qquad Q_2^u = (\bar{u}_\alpha b_\alpha)_{V-A}(\bar{q}_\beta u_\beta)_{V-A},$$

  - QCD penguin operators

$$Q_3 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, \qquad Q_4 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V-A},$$

$$Q_5 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, \qquad Q_6 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V+A},$$

– electro-weak penguin operators

$$Q_7 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad Q_8 = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A},$$

$$Q_9 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad Q_{10} = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}.$$

Hadronic distribution amplitudes

- Distribution amplitude of Λ_b:

$$
\begin{aligned}
&(Y_{\Lambda_b})_{\alpha\beta\gamma}(k_i,\nu) \\
&= \frac{1}{2\sqrt{2}N_c}\int\prod_{l=2}^{3}\frac{dw_l^+ d\mathbf{w_l}}{(2\pi)^3}e^{ik_l w_l}\varepsilon^{abc}\langle 0|T[b^a_\alpha(0)u^b_\beta(w_2)d^c_\gamma(w_3)]|\Lambda_b(p)\rangle \\
&= \frac{f_{\Lambda_b}}{8\sqrt{2}N_c}[(\not p+M_{\Lambda_b})\gamma_5 C]_{\beta\gamma}[\Lambda_b(p)]_\alpha\Psi(k_i,\nu).
\end{aligned}
$$

- Distribution amplitudes of proton:

$$
\begin{aligned}
&(Y_p)_{\alpha\beta\gamma}(k_i',\mu) \\
&= \frac{1}{2\sqrt{2}N_c}\int\prod_{l=2}^{3}\frac{dw_l^+ d\mathbf{w}_l}{(2\pi)^3}e^{ik_l'\cdot w_l}\epsilon^{abc}\langle 0|T[u^a_\alpha(0)u^b_\beta(w_2)d^c_\gamma(w_3)]|P(p')\rangle\ , \\
&= \frac{f_N}{8\sqrt{2}N_c}\{(\not p' C)_{\alpha\beta}[\gamma_5 N(p')]_\gamma\Phi^V(k_i',\mu)+(\not p'\gamma_5 C)_{\alpha\beta}[N(p')]_\gamma\Phi^A(k_i',\mu) \\
&\quad -(\sigma_{\mu\nu}p'^{\nu}C)_{\alpha\beta}[\gamma^\mu\gamma_5 N(p')]_\gamma\Phi^T(k_i',\mu)\}\ .
\end{aligned}
$$

- The light-cone distribution amplitudes for the pseudoscalar meson:

$$
\begin{aligned}
&\langle P(P)|\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle \\
&= -\frac{i}{\sqrt{6}}\int_0^1 dx e^{ixP\cdot z}\left[\gamma_5 \not{P}\Pi^A(y,\mu) + m_0\gamma_5\Pi^P(y,\mu) - m_0\sigma^{\mu\nu}\gamma_5 P_\mu z_\nu \frac{\Pi^\sigma(y,\mu)}{6}\right]_{\alpha\beta} \\
&= -\frac{i}{\sqrt{6}}\int_0^1 dx e^{ixP\cdot z}\left[\gamma_5 \not{P}\Pi^A(y,\mu) + \gamma_5 m_0\Pi^P(y,\mu) + m_0\gamma_5(\not{n}\,\not{v} - 1)\Pi^T(y,\mu)\right]_{\alpha\beta} .
\end{aligned}
$$

- Evolution of hadronic wavefunctions :

$$
\begin{aligned}
\Psi(x_i,b_i,p,\nu) &= \exp[-\sum_{l=2}^{3} s(\omega, x_l p^+) - 3\int_\omega^\nu \frac{d\overline{\mu}}{\overline{\mu}}\gamma_q(\alpha_s(\overline{\mu}))]\Psi(x_i), \\
\Phi^j(x_i',b_i',p',\nu) &= \exp[-\sum_{l=1}^{3} s(\omega', x_l' p^-) - 3\int_{\omega'}^\nu \frac{d\overline{\mu}}{\overline{\mu}}\gamma_q(\alpha_s(\overline{\mu}))]\Phi^j(x_i'), \\
\Pi^j(y,b_q,q,\nu) &= \exp[-\sum_{l=1}^{2} s(\omega_q, q_l^+) - 2\int_{\omega_q}^\nu \frac{d\overline{\mu}}{\overline{\mu}}\gamma_q(\alpha_s(\overline{\mu}))]\Pi^j(y),
\end{aligned}
$$

- The choice of factorization scales

$$\begin{aligned} \omega &= min(1/\tilde{b}_1, 1/\tilde{b}_2, 1/\tilde{b}_3), \\ \omega' &= min(1/\tilde{b}'_1, 1/\tilde{b}'_2, 1/\tilde{b}'_3) \\ \omega_q &= 1/b_q, \end{aligned}$$

with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$.

- Evolution of hard kernels:

$$H_{l,\mu}^{\alpha'\beta'\gamma'\rho'\alpha\beta\gamma\rho}(x, x', b, b', M_{\Lambda_b}, \nu)$$
$$= \exp[-n \int_{\nu}^{t} \frac{d\overline{\mu}}{\overline{\mu}} \gamma_q(\alpha_s(\overline{\mu}))] \times H_{l,\mu}^{\alpha'\beta'\rho'\gamma'\alpha\beta\gamma\rho}(x, x', b, b', M_{\Lambda_b}).$$

with the integer $n = 6$ for the factorizable diagrams and $n = 8$ for the nonfactorizable diagrams.

Factorization formula of the decay amplitude

- The hadronic matrix elements can be written as:

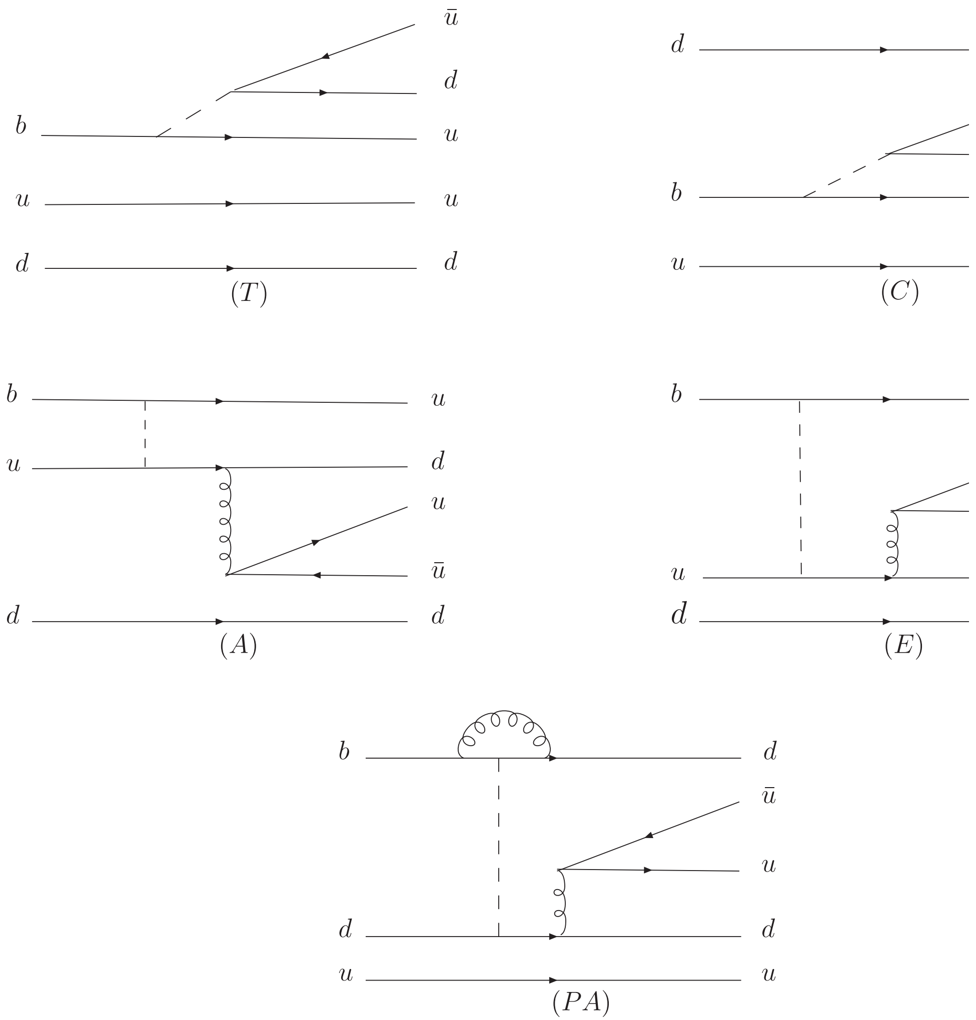
$$M_{l,\mu} = \int [Dx] \int [Db] (\overline{Y}_p)_{\alpha'\beta'\gamma'}(x_i', b_i', p', \nu)(Y_M)^{\dagger}_{\rho\rho'}(q, y, b_q, \nu)$$
$$H_{l,\mu}^{\alpha'\beta'\gamma'\rho'\alpha\beta\gamma\rho}(x_i, x_i', b_i, b_i', M_{\Lambda_b}, \nu)(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i, b_i, p, \nu),$$

with

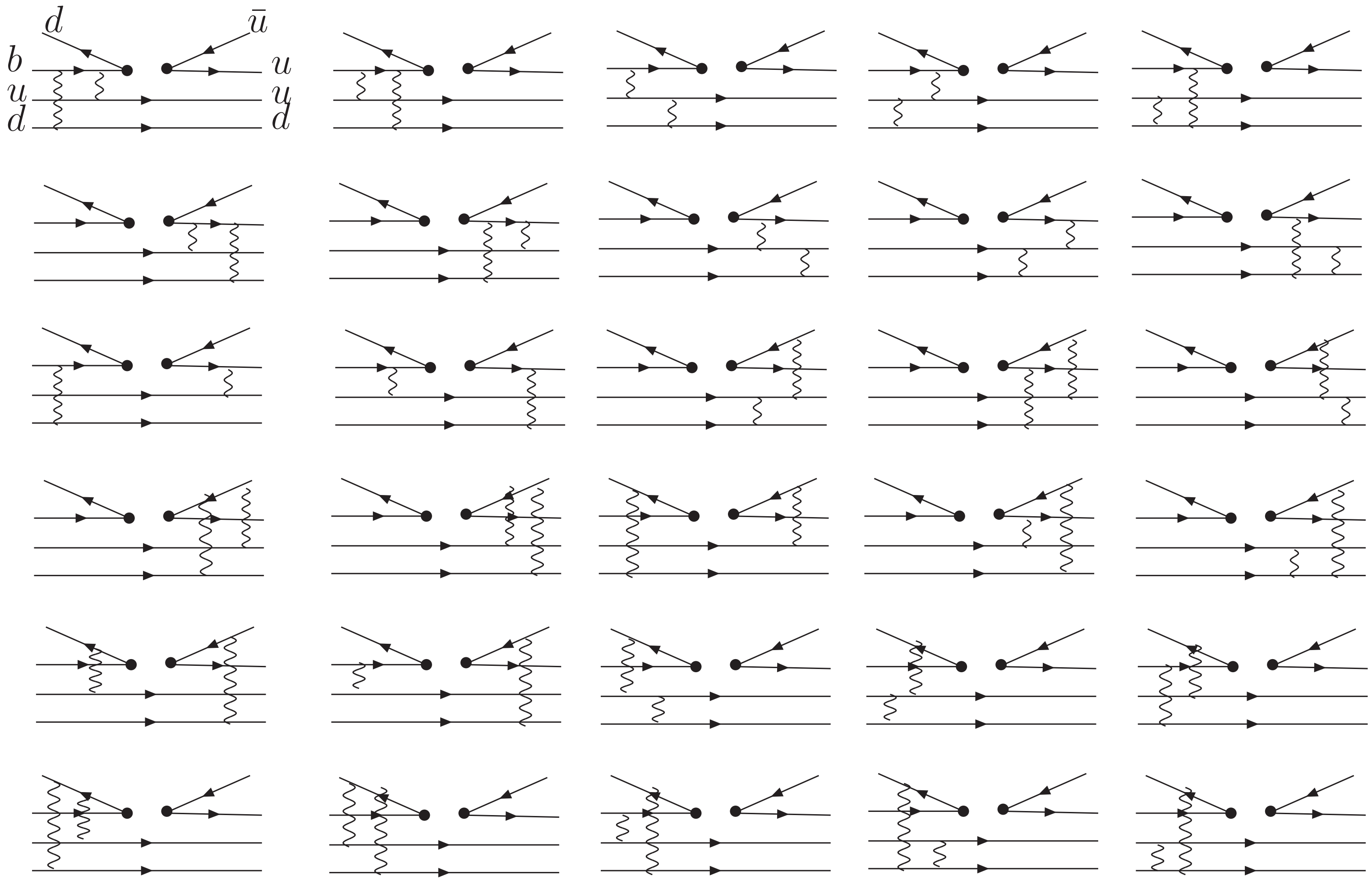
$$[Dx] = [dx][dx']dy, \quad [dx] = dx_1 dx_2 dx_3 \delta(1 - \sum_{l=1}^{3} x_l),$$

$$[dx'] = dx_1' dx_2' dx_3' \delta(1 - \sum_{l=1}^{3} x_l').$$

Typical Feynman diagrams in the Λ_b decays



Lowest-order hard amplitudes for Λ_b decays in PQCD



Numerical analysis of $\Lambda_b \to p\pi^-, pK^-$ decays

- The Λ_b baryon distribution amplitude Ψ(Ref: F. Schlumpf, hep-ph/9211255):

$$\Psi(x_1, x_2, x_3) = N x_1 x_2 x_3 \exp[-\frac{M_{\Lambda_b}^2}{2\beta^2 x_1} - \frac{m_q^2}{2\beta^2 x_2} - \frac{m_q^2}{2\beta^2 x_3}].$$

- The distribution amplitudes of proton up to next-to-leading conformal spin [††]:

$$\begin{aligned}
\phi^V(x_i,\mu) &= 120 x_1 x_2 x_3 \left[\phi_3^0(\mu) + \phi_3^+(\mu)(1-3x_3)\right], \\
\phi^A(x_i,\mu) &= 120 x_1 x_2 x_3 (x_2 - x_1)\phi_3^-(\mu), \\
\phi^T(x_i,\mu) &= 120 x_1 x_2 x_3 [\phi_3^0(\mu) - \frac{1}{2}\left(\phi_3^+ - \phi_3^-\right)(\mu)(1-3x_3)].
\end{aligned}$$

[††] V.M. Braun, A. Lenz, and M. Wittmann, Phys. Rev. D **73**, 094019 (2006).

- The distribution amplitudes of pseudoscalar meson :

$$\begin{aligned}
\Pi_\pi^A(x) &= \frac{3f_\pi}{\sqrt{6}}x(1-x)[1+0.44C_2^{3/2}(t)], \\
\Pi_\pi^P(x) &= \frac{f_\pi}{2\sqrt{6}}[1+0.43C_2^{1/2}(t)], \\
\Pi_\pi^T(x) &= -\frac{f_\pi}{2\sqrt{6}}[C_1^{1/2}(t)+0.55C_3^{1/2}(t)], \\
\Pi_K^A(x) &= \frac{3f_K}{\sqrt{6}}x(1-x)[1+0.17C_1^{3/2}(t)+0.2C_2^{3/2}(t)], \\
\Pi_K^P(x) &= \frac{f_K}{2\sqrt{6}}[1+0.24C_2^{1/2}(t)], \\
\Pi_K^T(x) &= -\frac{f_K}{2\sqrt{6}}[C_1^{1/2}(t)+0.35C_3^{1/2}(t)],
\end{aligned}$$

with C_n^m being Gegenbauer polynomials and $t = 2x - 1$.

Preliminary results of decay rate and direct CP asymmetry

- Predictions in PQCD approach (only central value):

$$BR(\Lambda_b \to p\pi) = 2.1 \times 10^{-6}, \qquad A_{CP}(\Lambda_b \to p\pi) = 28\%,$$
$$BR(\Lambda_b \to pK) = , \qquad A_{CP}(\Lambda_b \to pK) = .$$

- Predictions in generalized factorization approach ‡‡:

$$BR(\Lambda_b \to p\pi) = (0.85 - 1.16) \times 10^{-6},$$
$$BR(\Lambda_b \to pK) = (1.38 - 1.87) \times 10^{-6}.$$

- Experimental data (Ref: CDF collaboration, arXiv: 0810.3258):

	BR ($\times 10^{-6}$)	CP
$\Lambda_b \to p\pi^-$	$3.5 \pm 0.6 \pm 0.9$	$0.03 \pm 0.17 \pm 0.05$
$\Lambda_b \to pK$	$5.6 \pm 0.8 \pm 1.5$	$0.37 \pm 0.17 \pm 0.03$

‡‡R. Mohanta, A. K. Giri and M. P. Khanna, Phys. Rev. D **63** (2001) 074001

4. Discussions and conclusions

- PQCD predictions of non-leptonic two-body decays of $\Lambda_b \to p\pi$ are consistent with the experimental data. Its application to other channels like $\Lambda_b \to (\Lambda, \Sigma, \Xi) + (\pi, K, \eta, \eta')$ are in processes now.

- Theoretical predictions presented here can be systemically improved by including the higher conformal spin contributions as well as radiative corrections to the hard amplitudes.

- A systematic study of light baryon distribution amplitudes is mandatory, which are essential to investigate the exclusive processes involving light baryons using the factorization theorem.

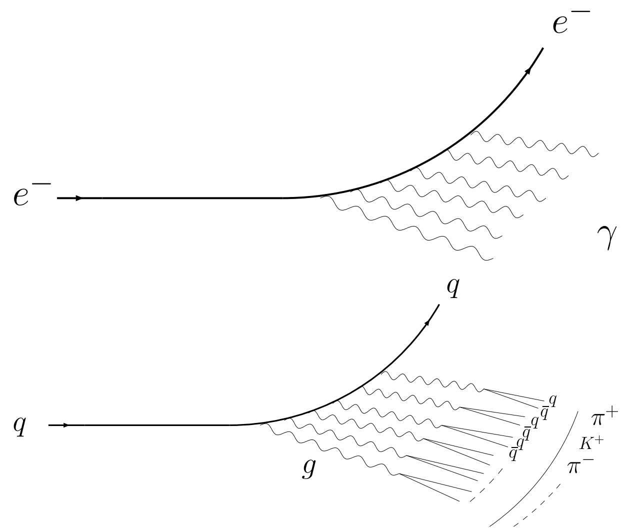
Thanks very much for your attentions!

BACKUP!

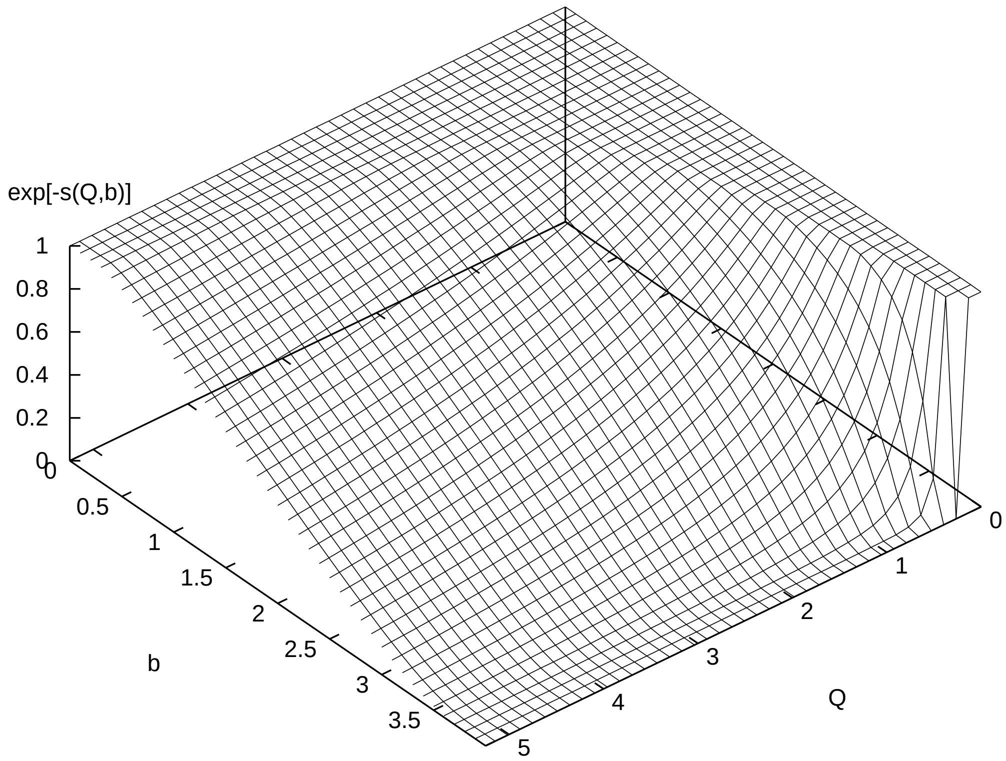
Physical interpretation of Sudakov factor

- In order to understand the Sudakov factor physically (Ref: Y. Y. Keum, M. Matsumori and A. I. Sanda,, Phys. Rev. D **72** (2005) 014013), first we consider QED. When a charged particle is accelerated, infinitely many photons must be emitted by the bremsstrahlung.

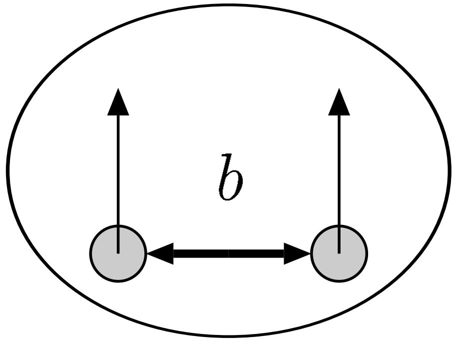
  A similar phenomenon occurs when a quark is accelerated: infinitely many gluons must be emitted. According to the feature of strong interaction, gluons cannot exist freely, so hadronic jet is produced. Then we observe many hadrons in the end if gluonic bremsstrahlung occurs.



- The amplitude for an exclusive decay $H_b \to h_1 h_2$ is proportional to the probability that no bremsstrahlung gluon is emitted. This is the Sudakov factor and it is depicted in the following figure.



- As can be seen, the Sudakov factor is large for small b and Q. Large b implies that the quark and antiquark pair is separated, which in turn implies less color shielding (see the figure below). Similar absence of shielding occurs when b quark carries most of the momentum while the momentum fraction of spectator quark x in the B meson is small.



Then the Sudakov factor suppresses the long distance contributions for the decay process and gives the effective cutoff about the transverse direction. In short, the Sudakov factor corresponds to the probability for emitting no photons. According to this factor, the property of short distance is guaranteed.

- The explicit form for the function $s(Q,b)$ is:

$$s(Q,b) = \frac{A^{(1)}}{2\beta_1}\hat{q}\,\text{ln}\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}\left(\hat{q}-\hat{b}\right) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\,\text{ln}\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\text{ln}\left(\frac{\hat{q}}{\hat{b}}\right)$$
$$+\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\text{ln}(2\hat{q})+1}{\hat{q}} - \frac{\text{ln}(2\hat{b})+1}{\hat{b}}\right] + \frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\text{ln}^2(2\hat{q}) - \text{ln}^2(2\hat{b})\right],$$

where the variables are defined by

$$\hat{q} \equiv \text{ln}[Q/(\sqrt{2}\Lambda)], \quad \hat{b} \equiv \text{ln}[1/(b\Lambda)],$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1 = \frac{33-2n_f}{12}, \quad \beta_2 = \frac{153-19n_f}{24},$$
$$A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1\text{ln}(\frac{1}{2}e^{\gamma_E}),$$

n_f is the number of the quark flavors and γ_E is the Euler constant. We will use the one-loop running coupling constant, i.e. we pick up the four terms in the first line of the expression for the function $s(Q,b)$.