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Signaling the Arrival of the LHC Era

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Higgs Theory at the LHC
I & II

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Higgs Boson Theory

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Trieste, 8-13 December 2008
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L1: Motivations and Context for Standard Model Higgs
Precision Electroweak Constraints, part I

L2: Precision Electroweak Constraints, part II
Theory Constraints on Higgs Boson Mass

L3: Fragility of Higgs Boson Predictions at LHC

Standard Model

The definition of the Standard Model

- $SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetry with appropriate gauge coupling strengths
- Three generations of quarks, leptons and neutrinos with appropriate masses and mixing angles
- Higgs boson doublet that breaks gauge symmetry to $SU(3) \times U(1)_Y$

Standard Model is not Complete

Fundamental Theorem of Particle Physics Research: The Standard Model of particle physics is *not complete*, and it is this incompleteness that motivates almost all particle physicists to do particle physics.

How is the Standard Model incomplete? Indirectly, the Standard Model is incomplete because we do not know several important why questions that cannot be answered within the theory.

Incomplete Standard Model: Some Why Questions

- Why three generations?
- What is the dark matter?
- Why is matter so much more copious than anti-matter?
- Why is the strong CP violating angle so small?
- Why is the electroweak scale so much smaller than the Planck scale?
- Why is Quantum Mechanics valid?
- Why are there three spatial dimensions and one time dimension?
- Why is time so different?
- Why do the gauge couplings have their values, and appear to merge at high energy?
- Why are there large hierarchies in the quark and lepton masses and mixings?
- Why are neutrino masses so much smaller?
- Why does electroweak symmetry break?
- How do elementary particles get their masses?
- ...

Tests of the Standard Model

There are three main directions to go to test the Standard Model

- Rare/forbidden events ($\mu \to e\gamma$, p decay, etc.)
- Precision tests (Γ_Z, m_W/m_Z, etc.)
- Direct tests ($d\sigma/dm_{l^+l^-}$, $e^+e^- \to HZ$, etc.)

We shall forgo the first method of rare/forbidden events, partly because of time, and partly because it is a more uncertain exercise, or rather less direct probe, when it comes to Higgs boson physics.

Focus on the Higgs boson

When discussing tests of the SM there is no single issue that is more more important or more answerable in the near term by the LHC than ... the *question of the Higgs boson.*

There is no direct evidence for it. The SM is a *speculative theory*, like all other theories we have that involve electroweak symmetry breaking. Our tests of the SM discussion will have particular emphasis on testing the validity of the single Higgs boson assumption.

In other words, our primary interest to focus on observables sensitivity to the existence of the SM Higgs boson.

This leaves us with three main areas of discussion:

- **Precision Electroweak Analysis**
- **Higgs Hunting** (mostly in experimental lectures)
- **Robustness of SM Higgs Phenomenology**

All of these issues will be discussed in these lectures.

But first, a review of the SM Higgs sector.

Standard Model Higgs Sector I

The SM posits an $SU(2)_L$ complex doublet Φ (four degrees of freedom) with hypercharge $Y = 1/2$. The bosonic lagrangian is

$$\mathcal{L}_\Phi = |D_\mu \Phi|^2 - m^2|\Phi|^2 - \lambda|\Phi|^4 - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}$$

where

$$D_\mu \Phi = \left(\partial_\mu + ig\frac{\tau^a}{2}W^a_\mu + i\frac{g'}{2}B_\mu\right)\Phi$$

The Higgs potential is

$$V_\Phi = m^2|\Phi|^2 + \lambda|\Phi|^4$$

If $m^2 < 0$ and λ positive the minimum of the Higgs potential is away from the origin

$$H = \frac{-m^2}{\lambda} \equiv \frac{v}{\sqrt{2}}$$

This "vacuum expectation value" of the Higgs breaks the electroweak symmetry:

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

This gives rise to three broken generators, and therefore we have three (massless) Goldstone bosons (ϕ_i) and one physical state (h) making up the Higgs complex doublet:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2 - i\phi_1 \\ h - i\phi_3 \end{pmatrix}$$

Standard Model Higgs Sector II

The ϕ_i Goldstone fields can be absorbed as longitudinal components of the massive $W^\pm$ and Z bosons. (I.e., W and Z masses require electroweak symmetry breaking.)

The fermions get mass via $y_u Q H u_R$, $y_d Q H^* d_R$, etc. interactions, where y is the Yukawa coupling and $m_f = yv/\sqrt{2}$.

The lagrangian of remaining h interacting with itself and SM gauge fields and fermions is

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{m_h^2}{2}h^2 - \frac{\mu}{3!}h^3 - \frac{\eta}{4!}h^4 \\ & + \left[m_W^2 W_\mu W^\mu + \frac{m_Z^2}{2} Z_\mu Z^\mu\right]\left(1+\frac{h}{v}\right)^2 - m_f \bar{f} f \left(1+\frac{h}{v}\right)\end{aligned}$$

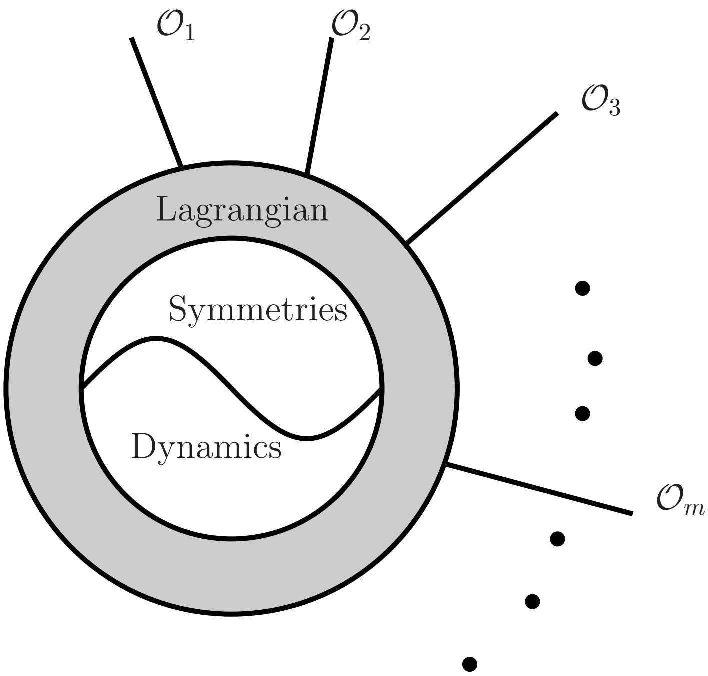
where $v = \sqrt{2}\langle H\rangle \simeq 246\,\mathrm{GeV}$, $m_h^2 = 2\lambda v^2$, $\mu = 3m_h^2/v$, and $\eta = 6\lambda$.

This lagrangian dictates all relevant Feynman rules we will need for the Higgs boson.

Why the SM is better than the Trivial Model

Finding a theory that matches observables is not hard at all. Give me any set of n observables $\{\mathcal{O}_i\}$ and I can give you this theory: For every $\mathcal{O}_i$ we posit the reason $\mathcal{R}_i$, which simply states that $\mathcal{O}_i$ is true.

Why is the SM better than this Trivial Model? The interrelations of symmetries and dynamics. The Lagrangian is the (usual) tool by which we can turn these into observables.



Observables in terms of Observables I

If there are n free parameters of the theory, and m computable and measurable observables, then we have $m - n$ predictions to test the theory (if observables are sufficiently "independent"):

$$\mathcal{O}_1, \mathcal{O}_2, \cdots, \mathcal{O}_n$$

which can be computed from the n parameters $\{P_i\}$

$$\mathcal{O}_i^{\rm expt} = \mathcal{O}_i^{\rm th}(P_1, P_2, \ldots, P_n)$$

These equations then can be inverted to obtain the parameters in terms of observables

$$P_i = F_i(\mathcal{O}_1^{\rm expt}, \mathcal{O}_2^{\rm expt}, \ldots, \mathcal{O}_n^{\rm expt}).$$

(Ignoring potential degeneracy issues.)

Now, for the remaining observables

$$\mathcal{O}_{n+1}, \mathcal{O}_{n+2}, \cdots, \mathcal{O}_m$$

we have the unambiguous predictions

$$\mathcal{O}_{n+j}^{\rm th}(P_1, P_2, \ldots, P_n).$$

Observables in terms of Observables II

Actually, we have expressed observables in terms of observables:

$$\mathcal{O}_{n+j}^{\rm th} = \mathcal{O}_{n+j}^{\rm th}(F_1(\vec{\mathcal{O}}^{\rm expt}), F_2(\vec{\mathcal{O}}^{\rm expt}), \ldots, F_n(\vec{\mathcal{O}}^{\rm expt}))$$

In practice, doing this analytically can be hard to do, and a χ^2 analysis is conducted, where

$$\chi^2 = \sum_i \frac{(\mathcal{O}_i^{\rm expt} - \mathcal{O}_i^{\rm th}(\vec{P}))^2}{(\Delta\mathcal{O}_i^{\rm expt})^2}.$$

One lets all the parameters $\vec{P}$ vary until the best χ^2 value is obtained. If $\chi^2/\text{d.o.f.} \lesssim 1$ the theory is compatible with the experimental measurements.

Nevertheless, in these lectures I will emphasize "observables in terms of observables" because it is pedagogically important and because it is possible to do this analytically in the examples I present.

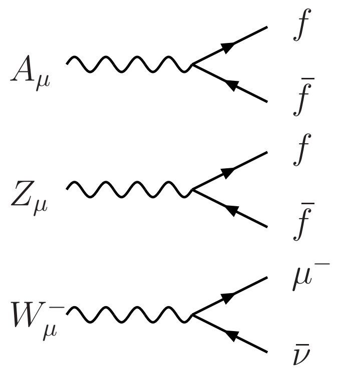
Tree-Level Analysis of SM

We analyze the electroweak at tree level, which can be described by three parameters (in addition to the fermion masses):

$$\mathcal{L} = \mathcal{L}(g, g', v)$$

where g is $SU(2)_L$ gauge coupling, g' is $U(1)_Y$ gauge coupling and v is the vev of the Higgs field.

The applicable feynman rules are



$i\bar{e}Q_f\gamma_\mu$

$\frac{ig}{\bar{c}_W}\gamma_\mu\left[(T^3_f - Q_f\bar{s}^2_W)P_L - Q_f\bar{s}^2_W P_R\right]$

$\frac{ig}{\sqrt{2}}\gamma_\mu P_L$

where $\bar{c}_W$, $\bar{s}_W$ and $\bar{e}$ are merely short-hand expressions for combinations of Lagrangian parameters:

$$\bar{e} \equiv \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad \bar{c}_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \bar{s}_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

Tree-Level Observables

Exchanging $\{g, g', v\}$ set for $\{e, s, v\}$ one finds

$$\begin{aligned}
\hat{\alpha} &= \frac{e^2}{4\pi} \quad \text{Coulomb potential} \\
\hat{G}_F &= \frac{1}{\sqrt{2}v} \quad \text{muon decay} \\
\hat{m}_Z^2 &= \frac{e^2 v^2}{4 s^2 c^2} \\
\hat{m}_W^2 &= \frac{e^2 v^2}{4 s^2} \\
\hat{s}_{\text{eff}}^2 &= s^2 \\
\hat{\Gamma}_{l^+l^-} &= \frac{v}{96\pi} \frac{e^3}{s^3 c^3} \left[\left(-\frac{1}{2} + 2s^2 \right)^2 + \frac{1}{4} \right]
\end{aligned}$$

The LHS are all observables/measurements.
The RHS are all theory predictions in terms of $\{e, s, v\}$.

s_{eff}^2 is determined from

$$\hat{A}_{LR} = \frac{\Gamma(Z \to e_L^+ e_L^-) - \Gamma(Z \to e_R^+ e_R^-)}{\Gamma(Z \to e_L^+ e_L^-) + \Gamma(Z \to e_R^+ e_R^-)} = \frac{(1/2 - \hat{s}_{\text{eff}}^2)^2 - \hat{s}_{\text{eff}}^4}{(1/2 - s_{\text{eff}}^2)^2 + \hat{s}_{\text{eff}}^4}$$

Measurements

$$
\begin{aligned}
\hat{\alpha} &= 1/137.035999911(46) \text{ (PDG)} \\
\hat{G}_F &= 1.16637(1) \times 10^{-5} \text{ (PDG)} \\
\hat{m}_Z^2 &= 91.1875 \pm 0.0021\,\text{GeV (LEP)} \\
\hat{m}_W^2 &= 80.410 \pm 0.032\,\text{GeV (LEP, Tevatron)} \\
\hat{s}_{\text{eff}}^2 &= 0.23098 \pm 0.00027 \text{ (SLAC)} \\
\hat{\Gamma}_{l^+l^-} &= 83.989 \pm 0.100\,\text{MeV (LEP)}
\end{aligned}
$$

$\Gamma_{l^+l^-}$ obtained from Γ_Z, $R_l = \Gamma_{\text{had}}/\Gamma_l$ and σ_{had} at LEP:

$$
\begin{aligned}
\sigma_{\text{had}} &= \frac{12\pi}{m_Z^2}\frac{\Gamma_{\text{had}}\Gamma_l}{\Gamma_Z^2} = \frac{12\pi}{m_Z^2} R_l \frac{\Gamma_l^2}{\Gamma_Z^2} \\
&\Longrightarrow \Gamma_l = \hat{m}_Z \hat{\Gamma}_Z \sqrt{\frac{\hat{\sigma}_{\text{had}}}{12\pi \hat{R}_l}}
\end{aligned}
$$

These are six measurements to be explained by only three parameters.

Pause for a Question

Does the Standard Model predict the correct W mass?

This is mostly a vacuous question in isolation. The answer is yes.

$$\hat{m}_W^2 = \frac{1}{4}\frac{e^2 v^2}{s^2}$$

An infinite number of combinations of e, s and v can solve this equation.

A more meaningful question: *Can the free parameters of the Standard Model be adjusted to predict all the observables compatible with observations?*

This question is to be answered by a χ^2 analysis

$$\chi^2(e,s,v) = \sum_{i=1}^{6} \frac{(\mathcal{O}_i^{\rm expt} - \mathcal{O}_i^{\rm th}(e,s,v))^2}{(\Delta\mathcal{O}_i^{\rm expt})^2}$$

where the three parameters are varied to see if values can be found that match all six observables under consideration.

A Simplified Analysis I

Of the six observables under consideration, historically three were measured extraordinarily well (α, G_F and m_Z) and served as inputs to the predictions of other observables. Let's do that here.

Inverting the lagrangian parameters and three well-measured observables, one finds

$$
\begin{aligned}
e^2 &= 4\pi\hat{\alpha} \\
v^2 &= \frac{\hat{G}_F^{-1}}{\sqrt{2}} \\
s^2 &= \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\hat{x}^2} \ \text{ where } \ \frac{\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z}
\end{aligned}
$$

A Simplified Analysis II

Now, use these to rewrite the remaining observables in terms of these first three:

$$
\begin{aligned}
\hat{m}_W^2 &= \pi\sqrt{2}\hat{G}_F^{-1}\hat{\alpha}\left(1-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^{-1} \\
\hat{s}_{\text{eff}}^2 &= \frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}} \\
\hat{\Gamma}_{l^+l^-} &= \frac{\sqrt{2}\hat{G}_F\hat{m}_Z^3}{12\pi}\left\{\left(\frac{1}{2}-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^2+\frac{1}{4}\right\}
\end{aligned}
$$

Substitute in experimental values to make predictions:

	Prediction	**Deviation**
$\hat{m}_W$	$80.939 \pm 0.003\,\text{GeV}$	$\sim 17\sigma$
$\hat{s}_{\text{eff}}^2$	0.21215 ± 0.00003	$\sim 70\sigma$
$\hat{\Gamma}_{l^+l^-}$	$84.834 \pm 0.012\,\text{MeV}$	$\sim 8\sigma$

Is Standard Model ruled out?

Of course not. But, we do not know that for sure without doing the next order computation. We can suspect that there are large corrections (compared to what? to experimental uncertainty) to the previous analysis.

$$\frac{\Delta\mathcal{O}}{\mathcal{O}} \sim \frac{g^2}{4\pi^2} \sim 1.2\%$$

and

$$\frac{m_W^{\rm tree} - \hat{m}_W}{\hat{m}_W} \sim 0.7\%$$

$$\frac{\hat{s}^2_{\rm eff} - s^2_{\rm eff,tree}}{\hat{s}^2_{\rm eff}} \sim 8.2\%$$

$$\frac{\Gamma^{\rm tree}_{l^+l^-} - \hat{\Gamma}_{l^+l^-}}{\hat{\Gamma}_{l^+l^-}} \sim 1.0\%$$

Why is $s^2_{\rm eff}$ deviation so large? Very sensitive to α corrections

$$\frac{\Delta\alpha}{\alpha} \simeq \frac{1/129 - 1/137}{1/137} = 6.2\%$$

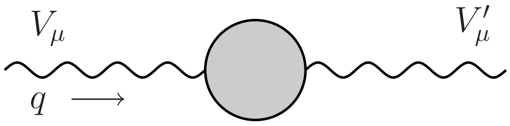
Methods of establishing Standard Model

In this lecture we focus on the class of corrections that arise solely from the self-energy corrections of the γ, $W^{\pm}$, and Z vector bosons. Restricting our analysis to this class of corrections enables us to do something complete and meaningful in the short time we have together.

A full-scale renormalization of the SM with all corrections explicitly calculated is a significantly more time-consuming project without significantly enhancing the conceptual learning.

Vector-Boson Self-Energies

By convention the one-loop corrections to the vector boson self-energies



is of the form

$$i[\Pi_{VV'}(q^2)g^{\mu\nu} - \Delta_{VV'}(q^2)q^\mu q^\nu].$$

Only the $\Pi_{VV'}$ piece of the self-energies matters for our analysis since the q^μ part of the second term is dotted into a light-fermion current and is zero by the Dirac equation, since the corresponding fermion masses is well-approximated to be zero:

$$q^\mu J_\mu^{\text{light fermion}} \to \bar{f}\gamma^\mu q_\mu f \to \bar{f} m f \to 0.$$

The way the self-energies are defined, they add to the vector boson masses by convention:

$$m_V^2 \to m_V^2 + \Pi_{VV}(q^2 = m_V^2)$$

Photon Self-Energies

Because the photon is massless we know that $\Pi_{\gamma\gamma}(0) = 0$ and $\Pi_{\gamma Z}(0) = 0$, and so we do not have to compute them.

Caveat: There is one subtlety to keep in mind. $\Pi_{\gamma Z}(0)$ is not zero when the $W^{\pm}$ bosons is included in the loop. This is special to the $W^{\pm}$ bosons (gauge degree of freedom partners of the W^3). In new physics scenarios (e.g., supersymmetry) there are no additional one-loop contributions to $\Pi_{\gamma Z}(0)$, and it is usually appropriate in analyses of beyond-the-SM contributions to precision EW observables to ignore it.

Our results when we are done will be of direct use for oblique analysis of physics beyond the SM, but as for SM a more complete analysis is needed, including taking into account vertex corrections.

Z and W Masses

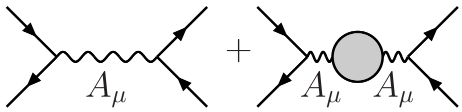
The computation of the Z and W masses is straightforward. The resulting theoretical prediction of m_Z and m_W in terms of the lagrangian parameters and the one-loop self-energy corrections is

$$(\hat{m}_Z)^{\mathrm{th}} = \frac{e^2 v^2}{4 s^2 c^2} + \Pi_{ZZ}(m_Z^2)$$

$$(\hat{m}_W)^{\mathrm{th}} = \frac{e^2 v^2}{4 s^2} + \Pi_{WW}(m_W^2)$$

Computing α

We next compute the theory prediction for α. It sounds odd to use the words "theory prediction of α" since we often are sloppy in our wording (or thinking) and view α as just a coupling. In reality, it is an observable defined in the Thomson limit of Compton scattering and probes the Coulomb potential at $q^2 \to 0$:



which is proportional to

$$-i\,\frac{4\pi\hat{\alpha}}{q^2}\bigg|_{q^2\to 0} = \frac{-ie^2}{q^2}\left[1 + \frac{\Pi_{\gamma\gamma}(q^2)}{q^2}\right]_{q^2\to 0}$$

If we define

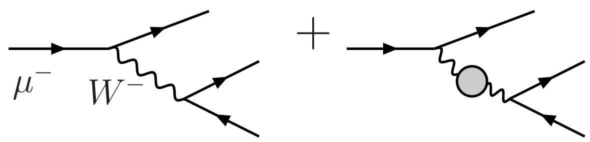
$$\Pi'_{\gamma\gamma}(0) \equiv \lim_{q^2\to 0}\frac{\Pi_{\gamma\gamma}(q^2)}{q^2}$$

then we can write the theory prediction for α as

$$(\hat{\alpha})^{\rm th} = \frac{e^2}{4\pi}\left(1 + \Pi'_{\gamma\gamma}(0)\right)$$

Muon Decay

The muon decay observable $\hat{G}_F$ is computed from the lifetime of the muon



which is proportional to $\hat{G}_F/\sqrt{2}$. This amplitude is then used to compute the muon lifetime

$$\tau_\mu^{-1} = \frac{\hat{G}_F^2 m_\mu^5}{192\pi^3} K(\alpha, m_e, m_\mu, m_W)$$

where the function K is mainly a kinematics function and can be obtained from the electroweak chapter in the PDG. The theory prediction for $\hat{G}_F$ is

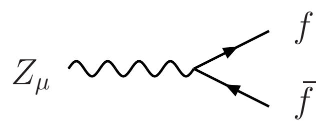
$$\begin{aligned} \frac{(\hat{G}_F)^{\rm th}}{\sqrt{2}} &= \frac{g^2}{8m_W^2}\left[1 + i\Pi_{WW}(q^2)\left(\frac{-i}{q^2 - m_W^2}\right)\right]_{q\to 0} \\ &= \frac{1}{2v^2}\left[1 - \frac{\Pi_{WW}(0)}{m_W^2}\right]. \end{aligned}$$

Computation of $\hat{s}^2_{\text{eff}}$ I

The observable associated with $\hat{s}^2_{\text{eff}}$ is a little trickier than the other ones. For one, there are many different types of $\hat{s}^2_{\text{eff}}$ observables, depending on the final state fermion. We have defined $\hat{s}^2_{\text{eff}}$ to be the observable associated with the left-right asymmetry of Z decays to leptons. We assume universality of the leptons.

$$A^l_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \equiv \frac{c_L^2 - c_R^2}{c_L^2 + c_R^2}$$

where at tree-level the c_L and c_R couplings are defined by



$$i\gamma_\mu(c_L P_L + c_R P_R)$$

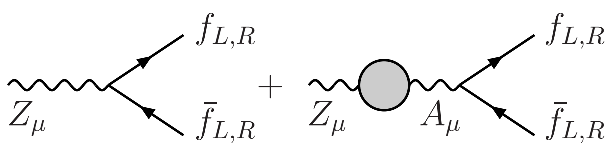
and

$$c_L = \frac{e}{sc}(T^3 - Qs^2) \quad \text{and} \quad c_R = -\frac{-eQs^2}{sc}$$

Computation of $\hat{s}^2_{\text{eff}}$ II

The definition of $\hat{s}^2_{\text{eff}}$ is chosen such that observable $\hat{A}^l_{LR}$ is written in terms of $\hat{s}^2_{\text{eff}}$ using the tree-level expression above with $s^2 \longrightarrow \hat{s}^2_{\text{eff}}$.

Compute the one-loop shifts in c_L and c_R. Neglect all Π_{ZZ} contributions since they will only affect the overall factor of c_L and c_R which cancels. On the other hand, the $Z - A$ mixing self-energy does contribute to the c_L and c_R couplings:



where

$$\begin{aligned} c_L &= \frac{e}{sc}(T^3 - Qs^2) + i\Pi_{\gamma Z}(m_Z^2)\left(\frac{-i}{m_Z^2}\right)(eQ) \\ &= \frac{e}{sc}\left[T^3 - Q\left(s^2 - sc\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}\right)\right] \end{aligned}$$

$$\begin{aligned} c_R &= \frac{-eQs^2}{sc} + i\Pi_{\gamma Z}(m_Z^2)\left(\frac{-i}{m_Z^2}\right)(eQ) \\ &= -\frac{eQ}{sc}\left[s^2 - sc\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}\right]. \end{aligned}$$

Computation $\hat{s}^2_{\text{eff}}$ III

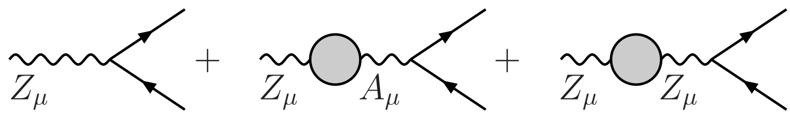
The above c_L and c_R expressions are exactly the same as the tree-level expressions except $s^2 \to s^2 - sc\Pi_{\gamma Z}(m_Z^2)/m_Z^2$ in the numerator. Thus, at the Z-pole

$$(\hat{s}^2_{\text{eff}})^{\text{th}} = s^2 - sc\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}$$

$$\text{where } \hat{A}_{LR} = \frac{(1/2 - \hat{s}^2_{\text{eff}})^2 - (\hat{s}^2_{\text{eff}})^2}{(1/2 - \hat{s}^2_{\text{eff}})^2 + (\hat{s}^2_{\text{eff}})^2}.$$

Computation of $\hat{\Gamma}_{l^+l^-}$ (I)

Now we compute $\hat{\Gamma}_{l^+l^-}$ from



The theoretical prediction for this observable in terms of independent lagrangian parameters and one-loop self-energies is

$$(\hat{\Gamma}_{l^+l^-})^{\rm th} = \frac{Z_Z}{48\pi}\frac{e^2}{s^2c^2}\hat{m}_Z\left[\left(-\frac{1}{2}+2(\hat{s}^2_{\rm eff})^{\rm th}\right)^2+\frac{1}{4}\right]$$

Recall that $\Pi_{\gamma Z}$ had the effect of just putting $s^2 \to (\hat{s}^2_{\rm eff})^{\rm th}$ into the numerator of the c_L and c_R expressions. The $\hat{m}_Z$ comes as a kinematical phase space mass of the Z decay.

Computation of $\hat{\Gamma}_{l^+l^-}$ (II)

Z_Z is a wavefunction residue piece. To compute it, start with $\Pi_{ZZ}(q^2)$, a self-energy that when resummed affects the Z boson propagator in a simple way

$$\text{Resummed Propagator} \quad \longrightarrow \quad P_Z^{\mu\nu}(q^2) = \frac{-ig^{\mu\nu}}{q^2 - m_Z^2 - \Pi_{ZZ}(q^2)}.$$

But,

$$\Pi_{ZZ}(q^2) = \Pi_{ZZ}(m_{\text{phys}}^2) + \Pi'_{ZZ}(m_{\text{phys}}^2)(q^2 - m_{\text{phys}}^2) + \cdots$$

The mass of the Z is defined to be the position of the real part of the pole of the propagator. In the neighborhood of $q^2 = m_{\text{phys}}^2$

$$\begin{aligned} q^2 - m_Z^2 - \Pi_{ZZ}(q^2) &= q^2 - m_Z^2 - \Pi_{ZZ}(m_{\text{phys}}^2) - \Pi'_{ZZ}(m_{\text{phys}}^2)(q^2 - m_{\text{phys}}^2) + \cdots \\ &= (q^2 - m_{\text{phys}}^2)(1 - \Pi'_{ZZ}(m_{\text{phys}}^2)) + \cdots \end{aligned}$$

Therefore, in the neighborhood of $q^2 = m_{\text{phys}}^2$ the Z propagator can be written as

$$\frac{-ig^{\mu\nu}}{(q^2 - m_{\text{phys}}^2)(1 - \Pi'_{ZZ}(m_{\text{phys}}^2)} = \frac{-iZ_Z g^{\mu\nu}}{(q^2 - m_{\text{phys}}^2)}$$

where

$$Z_Z = 1 + \Pi'_{ZZ}(\hat{m}_Z) + \text{ higher order terms}$$

Computation of $\hat{\Gamma}_{l^+l^-}$ (III)

At this point, we have all factors need to compute $\hat{\Gamma}_{l^+l^-}$.

Keep in mind a standard approximation for $\Pi'_{ZZ}(m_Z^2)$:

$$\Pi'_{ZZ}(m_Z^2) = \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

although it is not needed given the many good numerical tools available now. Nevertheless, I will use it at times.

Sometimes I will also utilize the variable δ_Z which is defined as $Z_Z = 1 + \delta_Z$, where

$$\delta_Z = \Pi'_{ZZ}(m_Z^2) \simeq \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} = \frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}$$

Reflection Pause

Where we are at: We now have written all of our observables in terms of lagrangian parameters and Π functions (one-loop corrections).

Usual next step: Construct χ^2 function of all relevant observables in terms of the input parameters, and fit.

Our next step: Let's now do analytic inversions write parameters in terms of observables, and ultimately observables in terms of observables. Often this step is not possible in practice to do analytically. Works here due to relative noncomplexity of one-looop self-energy corrections.

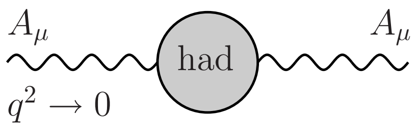
Why this analytic step? We will see infinities cancel automatically when we write observables in terms of observables, and perhaps provide a different perspective about the infinities that supposedly afflict our theories.

The troublesome case of $\hat{\alpha}$ (I)

Before we do those calculations, we need to say a few more things about the $\hat{\alpha}$ observable. It is an unusual observable among our list, because it is obviously incalculable. Recall from before that we found

$$e^2 = \frac{4\pi\hat{\alpha}}{1 + \Pi'_{\gamma\gamma}(0)}$$

The problem is with $\Pi'_{\gamma\gamma}(0)$, which requires us to know the result of the photon self energy as $q^2 \to 0$:



Of course we know from the beginning of this section that

$$\Pi_{\gamma\gamma}(q^2) \to q^2 B \quad \text{as } q^2 \to 0,$$

where B is some constant. There is no reason for B to be zero, and so there is no reason for the derivative of the self-energy $\Pi'_{\gamma\gamma}(0) \to B$ to be zero. Unfortunately, however, it is not calculable.

The troublesome case of $\hat{\alpha}$ (II)

The incalculability of $\Pi'_{\gamma\gamma}(0)$ threatens to derail our precision electroweak analysis. However, it has been known for some time now that we can get at this value by using a combination of theory tricks and experimental data. The first thing we do is to rewrite $\Pi'_{\gamma\gamma}(0)$ by adding and subtracting the self-energy at the higher scale $q^2 = m_Z^2$:

$$\Pi'_{\gamma\gamma}(0) = \mathrm{Re}\,\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} - \left[\frac{\mathrm{Re}\,\Pi_{\gamma\gamma}}{m_Z^2} - \Pi'_{\gamma\gamma}(0)\right]$$

The first term is calculable as computations are done at the scale $q^2 = m_Z^2$ where all interactions are perturbative in the SM. The two terms in the bracket are not calculable, but we will give it a name $\Delta\alpha(m_Z)$. There are three main contributions to $\Delta\alpha(m_Z)$:

$$\Delta\alpha(m_Z) = \Delta\alpha_l(m_Z) + \Delta\alpha_{\mathrm{top}}(m_Z) + \Delta\alpha^{(5)}_{\mathrm{had}}(m_Z)$$

where

$$\begin{aligned}
\Delta\alpha_l(m_Z) &= 0.03150 \;\; \text{with essentially no error} \\
\Delta\alpha_{\mathrm{top}}(m_Z) &= -0.0007(1) \;\; m_t \text{ dependent but negligible} \\
\Delta\alpha^{(5)}_{\mathrm{had}} &= \text{incalculable light hadrons contributions}
\end{aligned}$$

The troublesome case of $\hat{\alpha}$ (III)

Fortunately, there is a way to measure $\Delta\alpha^{(5)}_{\rm had}$. From the optical theorem and the methods of analytic continuation, one finds that

$$\Delta\alpha^{(5)}_{\rm had} = -\frac{\alpha m_Z^2}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{R_{\rm had}(q^2)dq^2}{q^2(q^2-m_Z^2)} \quad \text{where}$$

$$R_{\rm had}(q^2) = \frac{\sigma_{\rm had}(q^2)}{\sigma_{l^+l^-}(q^2)}.$$

Therefore, to get a numerical value for $\Delta\alpha^{(5)}_{\rm had}$ one must integrate over the experimental hadronic cross-section over a wide energy range.

As soon as q^2 is significantly above $\Lambda_{\rm QCD}$ the theoretical cross-section can be used without concern. However, for lower q^2 (lower than about 5 GeV in practice), only the experimental data can be used.

There are numerous experiments that contribute data for this integral in differing energy bins, and it is a challenge to understand all the systematics and statistical errors that go into the final number for $\Delta\alpha^{(5)}_{\rm had}$.

Experimental Determination of $\Delta\alpha_{\rm had}^{(5)}$

Many groups have gone through this difficult exercise and there are many different values obtained. The one the LEP Electroweak Working Group has been using is by Burkhardt and Pietrzyk, who conclude that

$$\Delta\alpha_{\rm had}^{(5)} = 0.02761 \pm 0.0036.$$

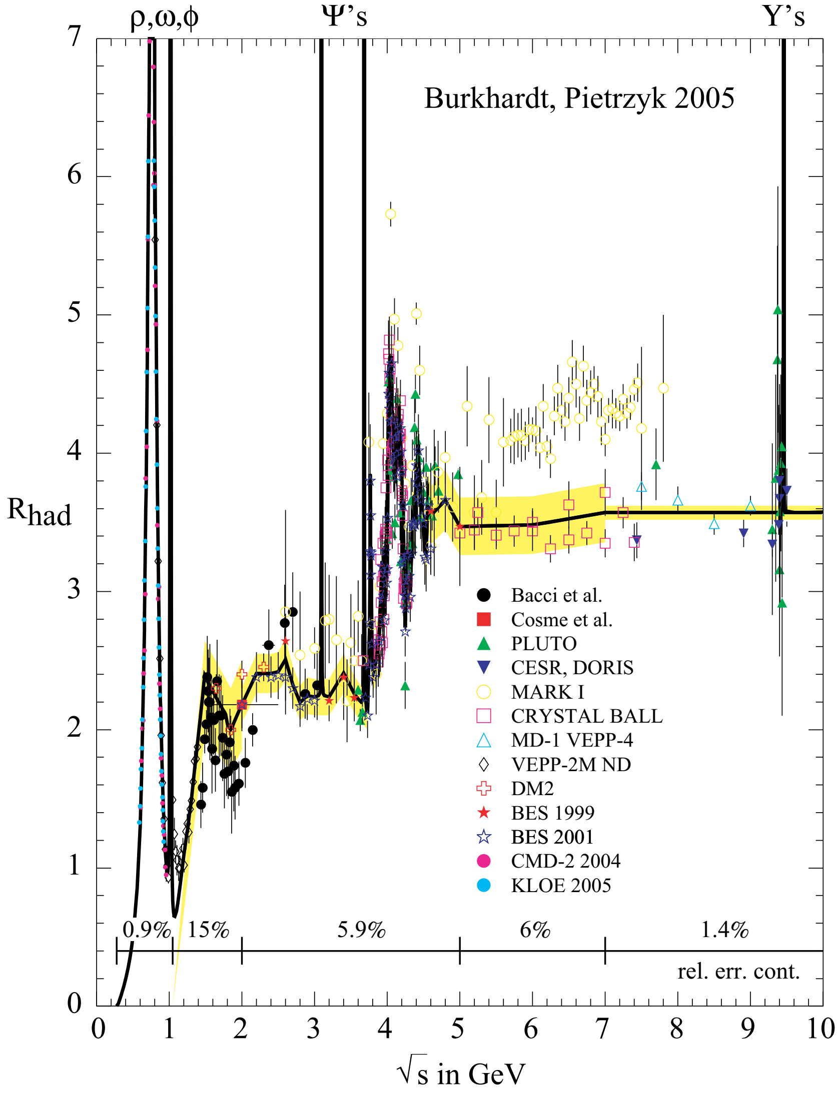
We will now trade in the incalculable $\hat{\alpha}$ for the calculable/measured $\hat{\alpha}(m_Z)$, which is related to the lagrangian parameters and Π's by

$$\hat{\alpha}(m_Z) = \frac{\hat{\alpha}}{1 - \Delta\alpha(m_Z)} = \frac{e^2}{4\pi}\left[1 + \frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2}\right]$$

Always remember, $\hat{\alpha}(m_Z)$ is an observable, which is a meaningful combination of many different experiments (Thomson scattering cross-section plus integration over $R_{\rm had}(q^2)$), and its experimental value is

$$\frac{1}{\hat{\alpha}(m_Z)} = 128.936 \pm 0.046.$$

Update of $\Delta\alpha$ data scan



Burkhardt, Pietrzyk, hep-ph/0506323

Parameters in terms of Observables (I)

As for determining v^2 from observables, we can get it directly and simply from the $\hat{G}_F$ equation

$$v^2 = \frac{1}{\sqrt{2}\hat{G}_F}\left[1 - \frac{\Pi_{WW}(0)}{m_W^2}\right].$$

At this point we have e^2 and v^2 in terms of $\hat{\alpha}(m_Z)$, $\hat{m}_Z$ and $\hat{G}_F$, but we still do not have the lagrangian parameter s^2 in terms of those three key observables. To do this, we need to go to the theory prediction equation for $\hat{m}_Z$ and solve for s^2.

$$\hat{m}_Z^2 = \frac{e^2}{4s^2c^2}v^2 + \Pi_{ZZ}(m_Z^2) \longrightarrow s^2c^2 = \frac{e^2v^2}{4}\left[\frac{1}{\hat{m}_Z^2 - \Pi_{ZZ}(m_Z^2)}\right].$$

After plugging in our previously obtained expressions for e^2 and v^2 in terms of observables we get after some algebra

$$s^2c^2 = \frac{\pi\hat{\alpha}(\hat{m}_Z^2)}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}(1+\delta_S)$$

where

$$\delta_S = \frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}.$$

Parameters in terms of Observables (II)

A convenient definition that I will sometimes use is

$$\hat{s}_0^2\hat{c}_0^2 = \frac{\pi\hat{\alpha}(\hat{m}_Z^2)}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}.$$

With this definition

$$s^2 = \hat{s}_0^2 + \frac{\hat{s}_0^2\hat{c}_0^2}{\hat{c}_0^2 - \hat{s}_0^2}\delta_S.$$

We now have expressions for each of the lagrangian parameters in terms of the three exceptionally well-measured observables

$$\{\hat{m}_Z, \hat{\alpha}(m_Z), \hat{G}_F\}$$

and the self-energy correction Π's and are ready to directly compute the theoretical prediction for each of the remaining observables.

Computation of Observables (analytic)

After some more algebra, which the student should do him/herself, here are the answers:

$$(\hat{m}_W)^{\rm th} = \frac{\pi\hat{\alpha}(\hat{m}_Z^2)}{\sqrt{2}\hat{G}_F\hat{s}_0^2}\left[1-\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}-\frac{c_0^2}{c_0^2-s_0^2}\delta_S-\frac{\Pi_{WW}(0)}{m_W^2}+\frac{\Pi_{WW}(m_W^2)}{m_W^2}\right]$$

$$(\hat{s}_{\rm eff}^2)^{\rm th} = \hat{s}_0^2+\frac{s_0^2c_0^2}{c_0^2-s_0^2}\left[\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2}-\frac{\Pi_{WW}(0)}{m_W^2}-\frac{(c_0^2-s_0^2)}{s_0c_0}\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}-\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right]$$

$$\begin{aligned}(\hat{\Gamma}_{l^+l^-})^{\rm th} \;=\; \hat{\Gamma}^0_{l^+l^-}\Bigg[1&-\frac{as_0^2c_0^2}{c_0^2-s_0^2}\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2}+\left(1+\frac{as_0^2c_0^2}{c_0^2-s_0^2}\right)\frac{\Pi_{WW}(0)}{m_W^2}\\ &+as_0c_0\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}-\frac{\Pi_{ZZ}(0)}{m_Z^2}+a\frac{s_0^2c_0^2}{c_0^2-s_0^2}\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}\Bigg]\end{aligned}$$

where

$$a=\frac{-8(-1+4s_0^2)}{(-1+4s_0^2)^2+1}\simeq 0.636.$$

Computation of Observables (numerical)

In summary, the theoretical predictions for $\hat{s}^2_{\rm eff}$, $\hat{m}_W$ and $\hat{\Gamma}_{l^+l^-}$ can be rewritten as

$$
\begin{aligned}
(\hat{s}^2_{\rm eff})^{\rm th} &= \hat{s}^2_0 - (0.328)\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} - (0.421)\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} \\
&\quad -(0.328)\frac{\Pi_{WW}(0)}{m_W^2} + (0.328)\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2}
\end{aligned}
$$

$$
\begin{aligned}
(\hat{m}_W)^{\rm th} &= \hat{m}_W^0 + (17.0\,{\rm GeV})\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} + (17.0\,{\rm GeV})\frac{\Pi_{WW}(0)}{m_W^2} \\
&\quad +(40.0\,{\rm GeV})\frac{\Pi_{WW}(m_W^2)}{m_W^2} - (57.1\,{\rm GeV})\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2}
\end{aligned}
$$

$$
\begin{aligned}
(\hat{\Gamma}_{l^+l^-})^{\rm th} &= \hat{\Gamma}^0_{l^+l^-} + (17.5\,{\rm MeV})\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} + (22.5\,{\rm MeV})\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} \\
&\quad +(101\,{\rm MeV})\frac{\Pi_{WW}(0)}{m_W^2} - (83.9\,{\rm MeV})\frac{\Pi_{ZZ}(0)}{m_Z^2} - (17.5\,{\rm MeV})\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2}
\end{aligned}
$$

where

$$
\hat{c}_0^2\hat{s}_0^2 = \frac{\pi\hat{\alpha}(m_Z^2)}{\sqrt{2}\hat{G}_F\hat{m}_Z}, \quad (\hat{m}_W^0)^2 = \frac{\pi\hat{\alpha}(m_Z^2)}{\sqrt{2}\hat{G}_F\hat{s}_0^2}
$$

$$
\hat{\Gamma}^0_{l^+l^-} = \frac{\hat{\alpha}(m_Z^2)\hat{m}_Z}{12\hat{s}_0^2\hat{c}_0^2}\left[\left(-\frac{1}{2}+2\hat{s}_0^2\right)^2 + \frac{1}{4}\right]
$$

Example of Cancelation of Infinities

Let's compute the contributions to precision EW observables from one-loop fermion self-energies to the vector bosons.

Repeatedly come across what are usually called Passarino-Veltman functions. There are several conventions for Passarino-Veltman functions in use. We'll use (D. Pierce et al.):

$$16\pi^2\mu^{4-n}\int\frac{d^nq}{i(2\pi)^n}\frac{1}{q^2-m^2+i\varepsilon}=A_0(m^2)$$

$$16\pi^2\mu^{4-n}\int\frac{d^nq}{i(2\pi)^n}\frac{1}{[q^2-m_1^2+i\varepsilon][(q-p)^2-m_2^2+i\varepsilon]}=B_0(p^2,m_1^2,m_2^2)$$

$$16\pi^2\mu^{4-n}\int\frac{d^nq}{i(2\pi)^n}\frac{q_\mu}{[q^2-m_1^2+i\varepsilon][(q-p)^2-m_2^2+i\varepsilon]}=p_\mu B_1(p^2,m_1^2,m_2^2)$$

$$16\pi^2\mu^{4-n}\int\frac{d^nq}{i(2\pi)^n}\frac{q_\mu q_\nu}{[q^2-m_1^2+i\varepsilon][(q-p)^2-m_2^2+i\varepsilon]}$$

$$=p_\mu p_\nu B_{21}(p^2,m_1^2,m_2^2)+g_{\mu\nu}B_{22}(p^2,m_1^2,m_2^2)$$

Poles of Passarino-Veltman Functions (I)

Some of these functions have poles at $n = 4$, and thus have an "infinite" piece proportional to

$$\Delta \equiv \frac{1}{4-n} - \gamma_E + \ln 4\pi$$

where $\gamma_E \simeq 0.5772$ is the Euler-Mascheroni constant that always accompanies the $1/(4-n)$ pole term just as the $\ln 4\pi$ factor does.

The primitive one-point and two-point functions have analytic solutions

$$A_0(m^2) = m^2 \left(\Delta + 1 - \ln m^2/\mu^2\right)$$

$$\begin{aligned} B_0(p^2, m_1^2, m_2^2) &= \Delta - \int_0^1 \ln \frac{(1-x)m_1^2 + x m_2^2 - x(1-x)p^2 - i\varepsilon}{\mu^2} \\ &= \Delta - \ln(p^2/\mu^2) - I(x_+) - I(x_-) \end{aligned}$$

where

$$x_\pm = \frac{(p^2 - m_2^2 + m_1^2) \pm \sqrt{(p^2 - m_2^2 + m_1^2)^2 - 4p^2(m_1^2 - i\varepsilon)}}{2p^2}, \quad \text{and}$$

$$I(x) = \ln(1-x) - x\ln(1-x^{-1}) - 1.$$

Poles of Passarino Veltman Functions (II)

Computing the other two-point functions, we can summarize the results in terms of the of their Δ-dependent "infinite pieces" and their finite function pieces (written as lower-case):

$$A_0(m^2) = m^2\Delta + a_0(m^2)$$

$$B_0(p^2, m_1^2, m_2^2) = \Delta + b_0(p^2, m_1^2, m_2^2)$$

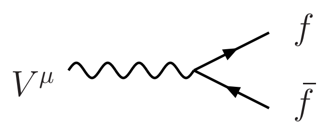
$$B_1(p^2, m_1^2, m_2^2) = \frac{1}{2}\Delta + b_1(p^2, m_1^2, m_2^2)$$

$$B_{21}(p^2, m_1^2, m_2^2) = \frac{1}{3}\Delta + b_{21}(p^2, m_1^2, m_2^2)$$

$$B_{22}(p^2, m_1^2, m_2^2) = \left(\frac{m_1^2 + m_2^2}{4} - \frac{p^2}{12}\right)\Delta + b_{22}(p^2, m_1^2, m_2^2)$$

Fermion Loop Computation

Now we come to our example. We compute the general one-loop contribution using Feynman rule



$iA\gamma^\mu(v - a\gamma_5)$

where A, v and a are parametrizations of the coupling. The fermion couplings to a V' vector boson are A', v', and a'. With these basic rules we are ready to compute the one-loop function $\Pi_{VV'}(p^2)$:

$$i\Pi^{\mu\nu}_{VV'} = -\int \frac{d^n q}{(2\pi)^n} \text{Tr}\left[iA\gamma^\mu(v - a\gamma_5) i\frac{[(\not{q} - \not{p}) + m_2]}{(q-p)^2 - m_2^2} iA'\gamma^\nu(v' - a'\gamma_5)\frac{i(\not{q} + m_1)}{q^2 - m_1^2}\right]$$

After some manipulations one finds that

$$\begin{aligned}\Pi^{\mu\nu}_{VV'} &= \frac{AA'}{4\pi^2}\Big\{(vv' + aa')\left[2p^\mu p^\nu(B_{21} - B_1) + g^{\mu\nu}(-2B_{22} - p^2 B_{21} + p^2 B_1)\right] \\ &\quad + m_1 m_2(vv' - aa')g^{\mu\nu}B_0\Big\}(p^2, m_1^2, m_2^2).\end{aligned}$$

Divergent Parts of Self-Energy Correction (I)

Only the transverse piece of the self-energy proportional to $g^{\mu\nu}$ plays a role here: $\Pi^{\mu\nu}(p^2) = \Pi(p^2)g^{\mu\nu} + \cdots$.

If our calculations are correct and the theory makes sense, all divergences that arise in Π corrections should cancel out in the observables.

For fermion self-energies, we can check for finiteness of the theory predictions given the expressions above. The Δ-divergence part of $\Pi_{VV'}$ is

$$\Pi^{\Delta}_{VV'}(p^2) = \frac{AA'}{4\pi^2}\left\{(vv' + aa')\left(-\frac{1}{2}(m_1^2 + m_2^2) + \frac{p^2}{3}\right) + (vv' - aa')m_1 m_2\right\}\Delta$$

Divergent Parts of Self-Energy Correction (II)

For the top-bottom quark doublet, we can compute these Δ-divergence pieces. The nonzero contributions are

$$\Pi^{\Delta}_{ZZ}(m_Z^2) = m_Z^2 \sum_{i=t,b} \frac{e^2}{4s^2c^2} \left[(T_i^3 - 2Q_i s^2)^2 + (T_i^3)^2 \right] \Delta$$

$$\Pi^{\Delta}_{\gamma\gamma}(m_Z^2) = m_Z^2 \sum_{i=t,b} (eQ_i)^2 \, \Delta$$

$$\Pi^{\Delta}_{\gamma Z}(m_Z^2) = m_Z^2 \sum_{i=t,b} \frac{e^2 Q_i}{2sc} (T_i^3 - 2Q_i s^2) \, \Delta$$

$$\Pi^{\Delta}_{WW}(m_W^2) = m_W^2 \frac{e^2}{4s^2} \, \Delta$$

Substituting these expressions into our earlier equations for the observables, one finds that all Δ-divergent terms cancel identically, as they should.

You can check this as an exercise.

Higgs Contributions to Self-Energies

The Higgs boson contributes to the self energies of the vector bosons.

Two good exercises are to

... compute the Higgs-mediated vector boson self energies

$$\begin{aligned}\Pi_{VV}(p^2) &= -\frac{1}{2\pi^2}\frac{m_V^4}{v^2}\left[B_0(p^2, m_V^2, m_H^2) + \frac{1}{m_V^2}B_{22}(p^2, m_V^2, m_H^2)\right] \\ &\quad -\frac{1}{16\pi^2}\frac{m_V^2}{v^2}A_0(m_H^2)\end{aligned}$$

where $V = W, Z$.

... and show that all m_H^2-dependent infinities cancel in the observables.

Standard Model Global Fits

We now go beyond our oblique subset and discuss a complete global fit to the SM observables.

Introduce a χ^2 function that is (approximately)

$$\chi^2 = \sum \frac{(\mathcal{O}_i^{\rm th}(\vec{P}) - \mathcal{O}_i^{\rm expt})^2}{(\Delta\mathcal{O}_i^{\rm expt})^2}$$

where the parameters are

$$\vec{P} = \{m_H, m_t, \alpha_S, \alpha, \ldots\}$$

and the observables are

$$\mathcal{O}_i^{\rm expt} = \{\hat{s}^2_{\rm eff}, \hat{m}_W, \hat{\Gamma}_l, \hat{m}_t, \hat{\alpha}_S, \Delta\alpha^{(5)}_{\rm had}, \ldots\}$$

In the next few slides we review the results of LEP Electroweak Working Group, LEP Collaborations, hep-ex/0511027. More recent updates are very similar to these results. See LEPEWWG web page

`http://lepewwg.web.cern.ch/LEPEWWG/`

$$\text{Pull}_i = \frac{\mathcal{O}_i^{\rm th}(\vec{P}_0) - \mathcal{O}_i^{\rm expt}}{\Delta\mathcal{O}_i^{\rm expt}}, \quad \text{where} \quad \vec{P}_0 = \vec{P} \text{ at best fit point.}$$

		Measurement with Total Error	Systematic Error	Standard Model fit	Pull
	$\Delta\alpha^{(5)}_{\mathrm{had}}(m^2_{\mathrm{Z}})$	0.02758 ± 0.00035	0.00034	0.02767	-0.3
a)	<u>LEP-I</u>				
	line-shape and				
	lepton asymmetries:				
	m_{Z} [GeV]	91.1875 ± 0.0021	$^{(a)}0.0017$	91.1874	0.0
	Γ_{Z} [GeV]	2.4952 ± 0.0023	$^{(a)}0.0012$	2.4959	-0.3
	σ^0_{had} [nb]	41.540 ± 0.037	$^{(b)}0.028$	41.478	1.7
	R^0_ℓ	20.767 ± 0.025	$^{(b)}0.007$	20.743	1.0
	$A^{0,\,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	$^{(b)}0.0003$	0.0164	0.7
	+ correlation matrix				
	τ polarisation:				
	$\mathcal{A}_\ell$ ($\mathcal{P}_\tau$)	0.1465 ± 0.0033	0.0016	0.1480	-0.5
	$\mathrm{q\overline{q}}$ charge asymmetry:				
	$\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}(Q^{\mathrm{had}}_{\mathrm{FB}})$	0.2324 ± 0.0012	0.0010	0.23140	0.8
b)	<u>SLD</u>				
	$\mathcal{A}_\ell$ (SLD)	0.1513 ± 0.0021	0.0010	0.1480	1.6
c)	<u>LEP-I/SLD Heavy Flavour</u>				
	R^0_{b}	0.21629 ± 0.00066	0.00050	0.21579	0.8
	R^0_{c}	0.1721 ± 0.0030	0.0019	0.1723	-0.1
	$A^{0,\,\mathrm{b}}_{\mathrm{FB}}$	0.0992 ± 0.0016	0.0007	0.1038	-2.8
	$A^{0,\,\mathrm{c}}_{\mathrm{FB}}$	0.0707 ± 0.0035	0.0017	0.0742	-1.0
	$\mathcal{A}_{\mathrm{b}}$	0.923 ± 0.020	0.013	0.935	-0.6
	$\mathcal{A}_{\mathrm{c}}$	0.670 ± 0.027	0.015	0.668	0.1
	+ correlation matrix				
d)	<u>LEP-II and Tevatron</u>				
	m_{W} [GeV] (LEP-II, Tevatron)	80.410 ± 0.032		80.378	1.0
	Γ_{W} [GeV] (LEP-II, Tevatron)	2.123 ± 0.067		2.092	0.5
	m_{t} [GeV] (Tevatron)	172.7 ± 2.9	2.4	173.3	-0.2

	- 1 - all Z-pole data	- 2 - all Z-pole data plus m_t	- 3 - all Z-pole data plus m_W, Γ_W	- 4 - all Z-pole data plus m_t, m_W, Γ_W
m_t [GeV]	173^{+13}_{-10}	$172.7^{+2.8}_{-2.8}$	179^{+12}_{-9}	$173.3^{+2.7}_{-2.7}$
m_H [GeV]	111^{+190}_{-60}	112^{+62}_{-41}	148^{+248}_{-83}	91^{+45}_{-32}
$\log(m_H/\text{GeV})$	$2.05^{+0.43}_{-0.34}$	$2.05^{+0.19}_{-0.20}$	$2.17^{+0.43}_{-0.36}$	$1.96^{+0.18}_{-0.19}$
$\alpha_S(m_Z^2)$	0.1190 ± 0.0027	0.1190 ± 0.0027	0.1190 ± 0.0028	0.1186 ± 0.0027
χ^2/d.o.f. (P)	16.0/10 (9.9%)	16.0/11 (14%)	17.3/12 (14%)	17.8/13 (17%)
$\sin^2\theta_{\text{eff}}^{\text{lept}}$	0.23149 $\pm$0.00016	0.23149 $\pm$0.00016	0.23143 $\pm$0.00014	0.23140 $\pm$0.00014
$\sin^2\theta_W$	0.22321 $\pm$0.00062	0.22331 $\pm$0.00041	0.22285 $\pm$0.00043	0.22304 $\pm$0.00033
m_W [GeV]	80.363 ± 0.032	80.364 ± 0.021	80.387 ± 0.022	80.377 ± 0.017

Table 1: Results of the fits to: (1) all Z-pole data (LEP-I and SLD), (2) all Z-pole data plus direct m_t determination, (3) all Z-pole data plus direct m_W and Γ_W determinations, (4) all Z-pole data plus direct m_t, m_W, Γ_W determinations (i.e., all high-Q^2 results). As the sensitivity to m_H is logarithmic, both m_H as well as $\log(m_H/\text{GeV})$ are quoted. The bottom part of the table lists derived results for $\sin^2\theta_{\text{eff}}^{\text{lept}}$, $\sin^2\theta_W$ and m_W. See text for a discussion of theoretical errors not included in the errors above.

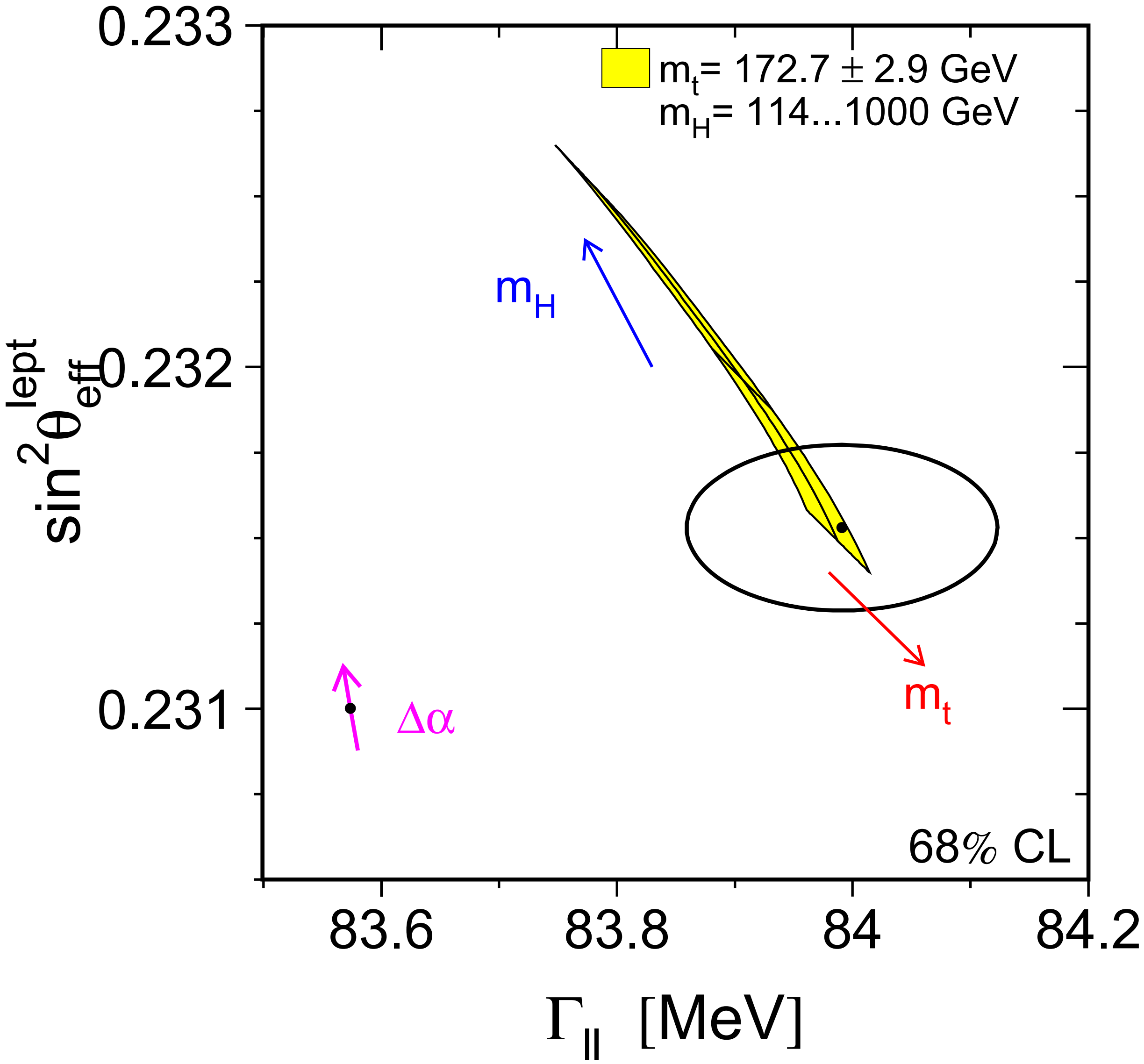


Figure 1: LEP-I+SLD measurements of $\sin^2\theta_{\rm eff}^{\rm lept}$ and $\Gamma_{\ell\ell}$ and the SM prediction. The point shows the predictions if among the electroweak radiative corrections only the photon vacuum polarisation is included. The corresponding arrow shows variation of this prediction if $\alpha(m_Z^2)$ is changed by one standard deviation. This variation gives an additional uncertainty to the SM prediction shown in the figure.

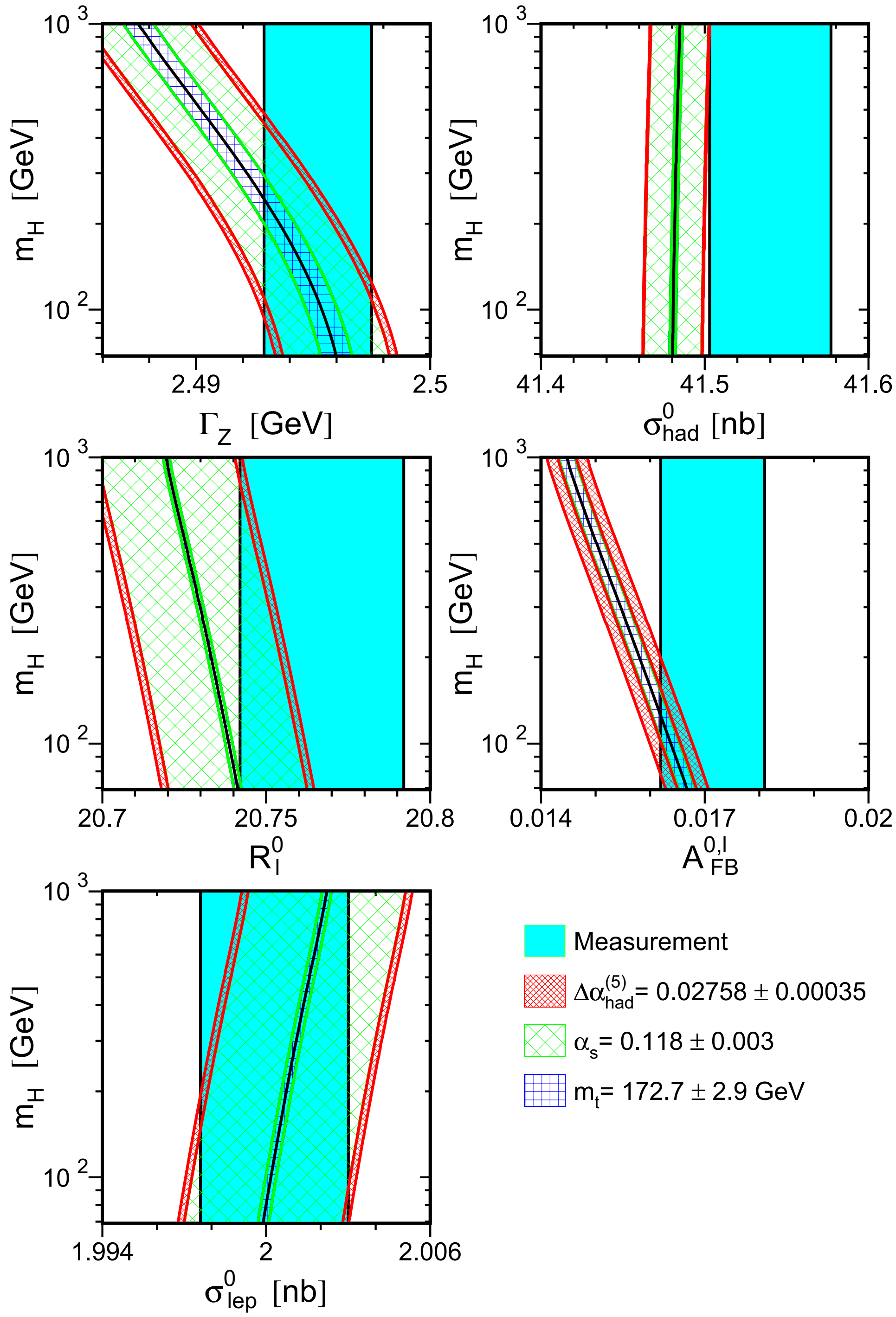


Figure 2: Comparison of LEP-I measurements with the SM prediction as a function of m_{H}. The measurement with its error is shown as the vertical band. The width of the SM band is due to the uncertainties in $\Delta\alpha^{(5)}_{\mathrm{had}}(m_{\mathrm{Z}}^2)$, $\alpha_{\mathrm{S}}(m_{\mathrm{Z}}^2)$ and m_{t}. The total width of the band is the linear sum of these effects.

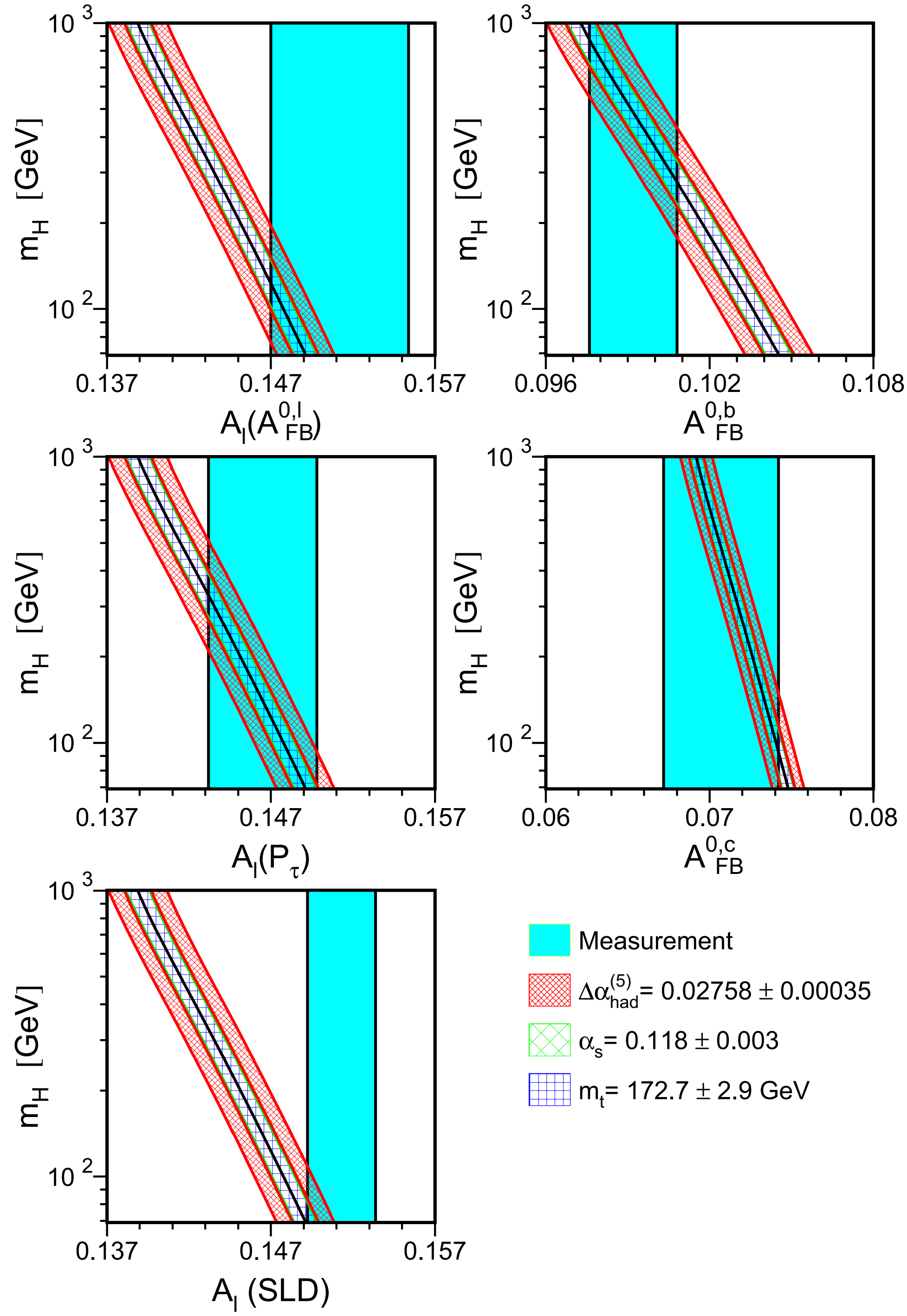


Figure 3: Comparison of LEP-I measurements with the SM prediction as a function of m_{H}. The measurement with its error is shown as the vertical band. The width of the SM band is due to the uncertainties in $\Delta\alpha^{(5)}_{\mathrm{had}}(m_{\mathrm{Z}}^2)$, $\alpha_{\mathrm{S}}(m_{\mathrm{Z}}^2)$ and m_{t}. The total width of the band is the linear sum of these effects. Also shown is the comparison of the SLD measurement of $\mathcal{A}_\ell$, dominated by A^0_{LR}, with the SM.

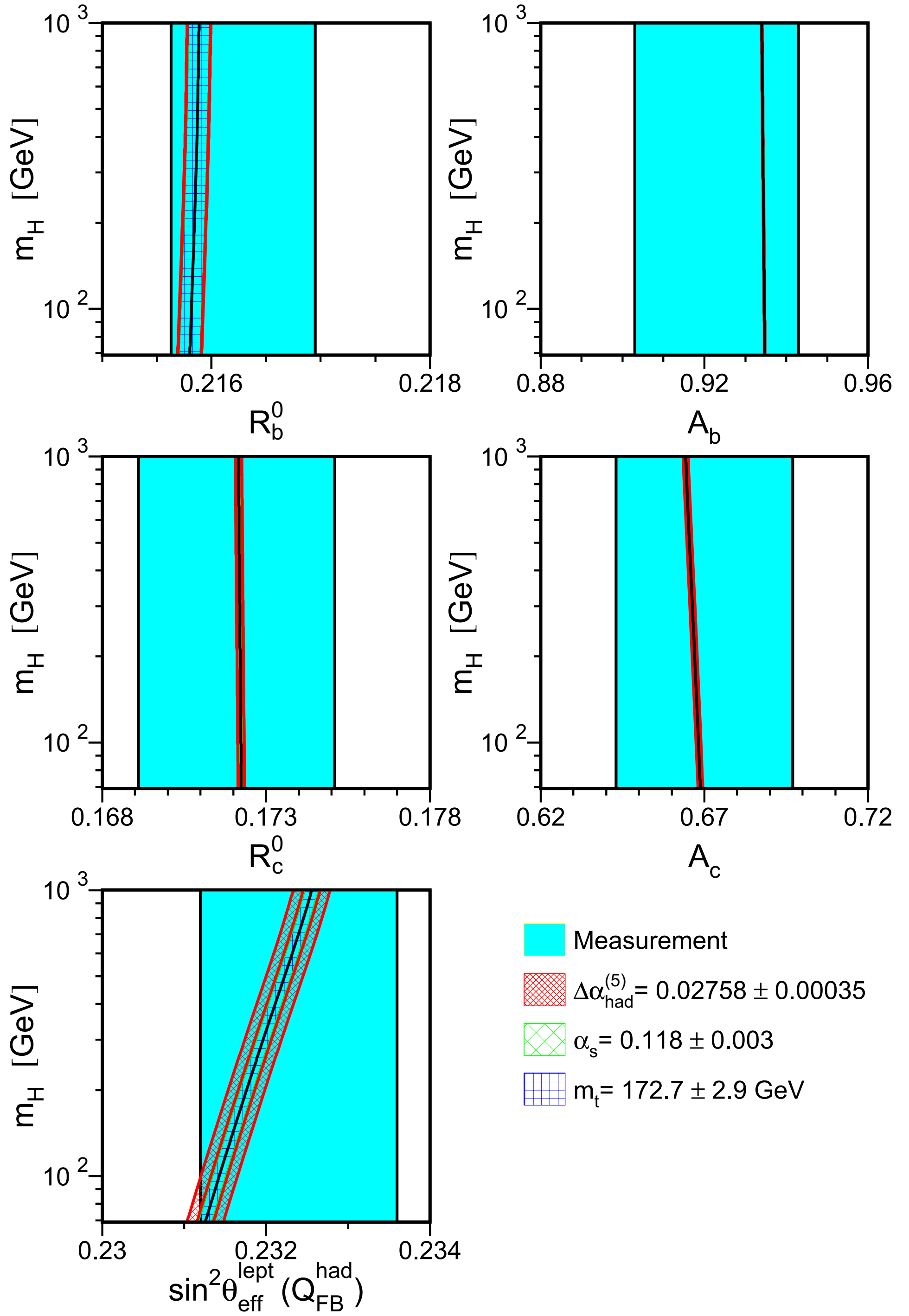


Figure 4: Comparison of LEP-I and SLD heavy-flavour measurements with the SM prediction as a function of m_{H}. The measurement with its error is shown as the vertical band. The width of the SM band is due to the uncertainties in $\Delta\alpha^{(5)}_{\mathrm{had}}(m^2_{\mathrm{Z}})$, $\alpha_{\mathrm{S}}(m^2_{\mathrm{Z}})$ and m_{t}. The total width of the band is the linear sum of these effects. Also shown is the comparison of the LEP-I measurement of the inclusive hadronic charge asymmetry $Q^{\mathrm{had}}_{\mathrm{FB}}$ with the SM.

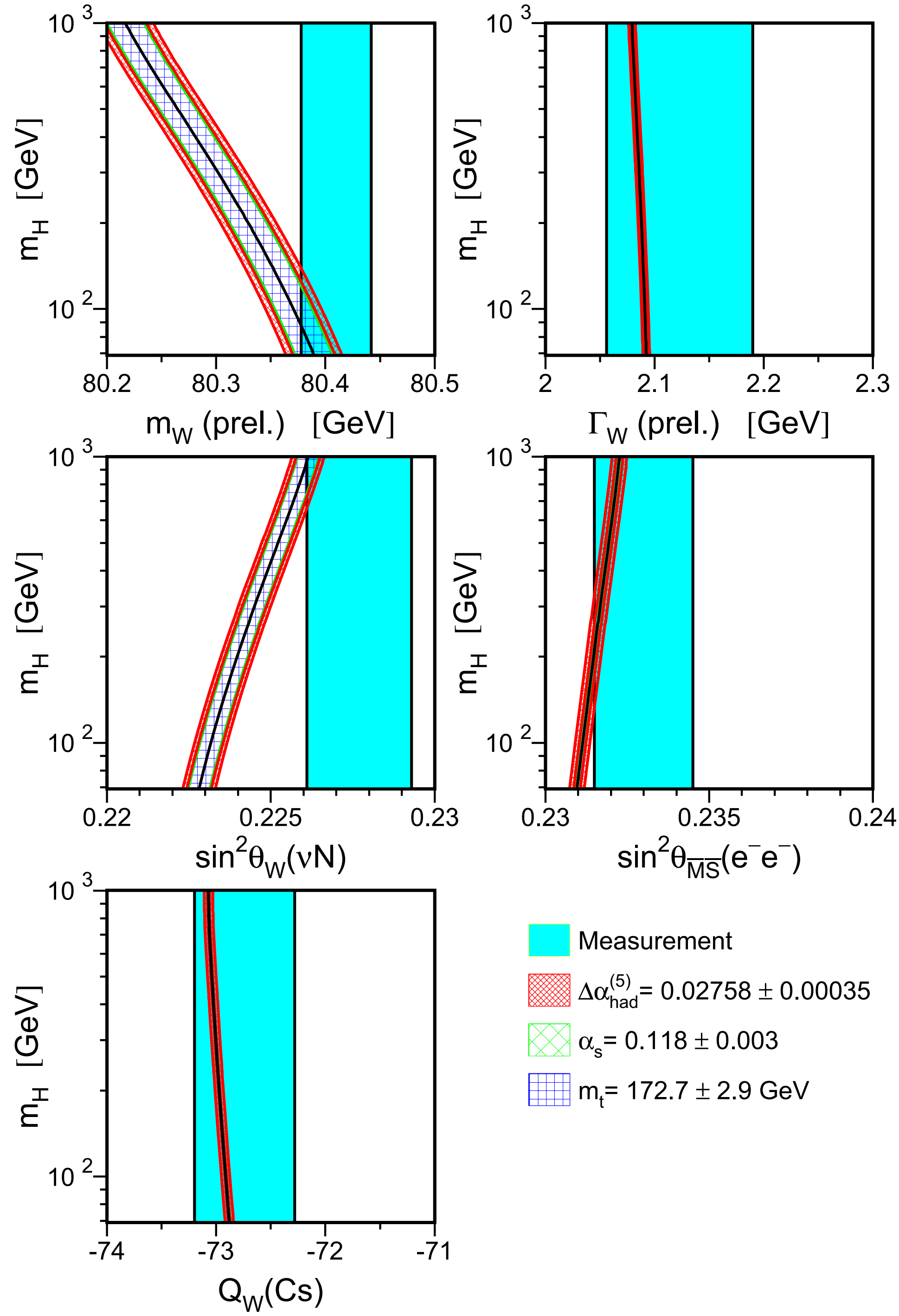


Figure 5: Comparison of m_{W} and Γ_{W} measured at LEP-II and $\mathrm{p\overline{p}}$ colliders, of $\sin^2\theta_{\mathrm{W}}$ measured by NuTeV and of APV in caesium with the SM prediction as a function of m_{H}. The measurement with its error is shown as the vertical band. The width of the SM band is due to the uncertainties in $\Delta\alpha^{(5)}_{\mathrm{had}}(m^2_{\mathrm{Z}})$, $\alpha_{\mathrm{S}}(m^2_{\mathrm{Z}})$ and m_{t}. The total width of the band is the linear sum of these effects.

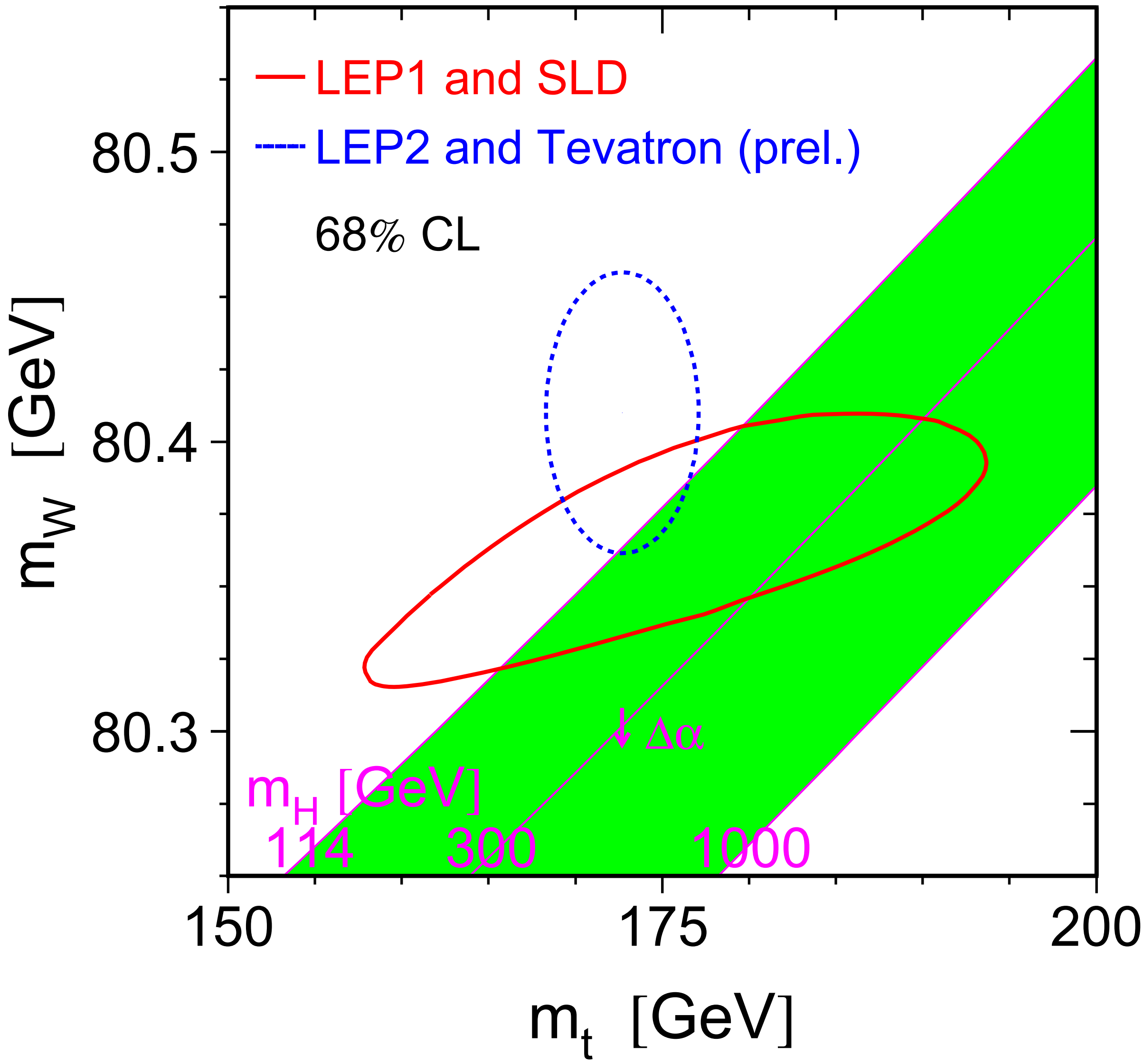


Figure 6: The comparison of the indirect measurements of m_{W} and m_{t} (LEP-I+ SLD data) (solid contour) and the direct measurements ($\mathrm{p\overline{p}}$ colliders and LEP-II data) (dashed contour). In both cases the 68% CL contours are plotted. Also shown is the SM relationship for the masses as a function of the Higgs mass. The arrow labelled $\Delta\alpha$ shows the variation of this relation if $\alpha(m_{\mathrm{Z}}^2)$ is changed by one standard deviation. This variation gives an additional uncertainty to the SM band shown in the figure.

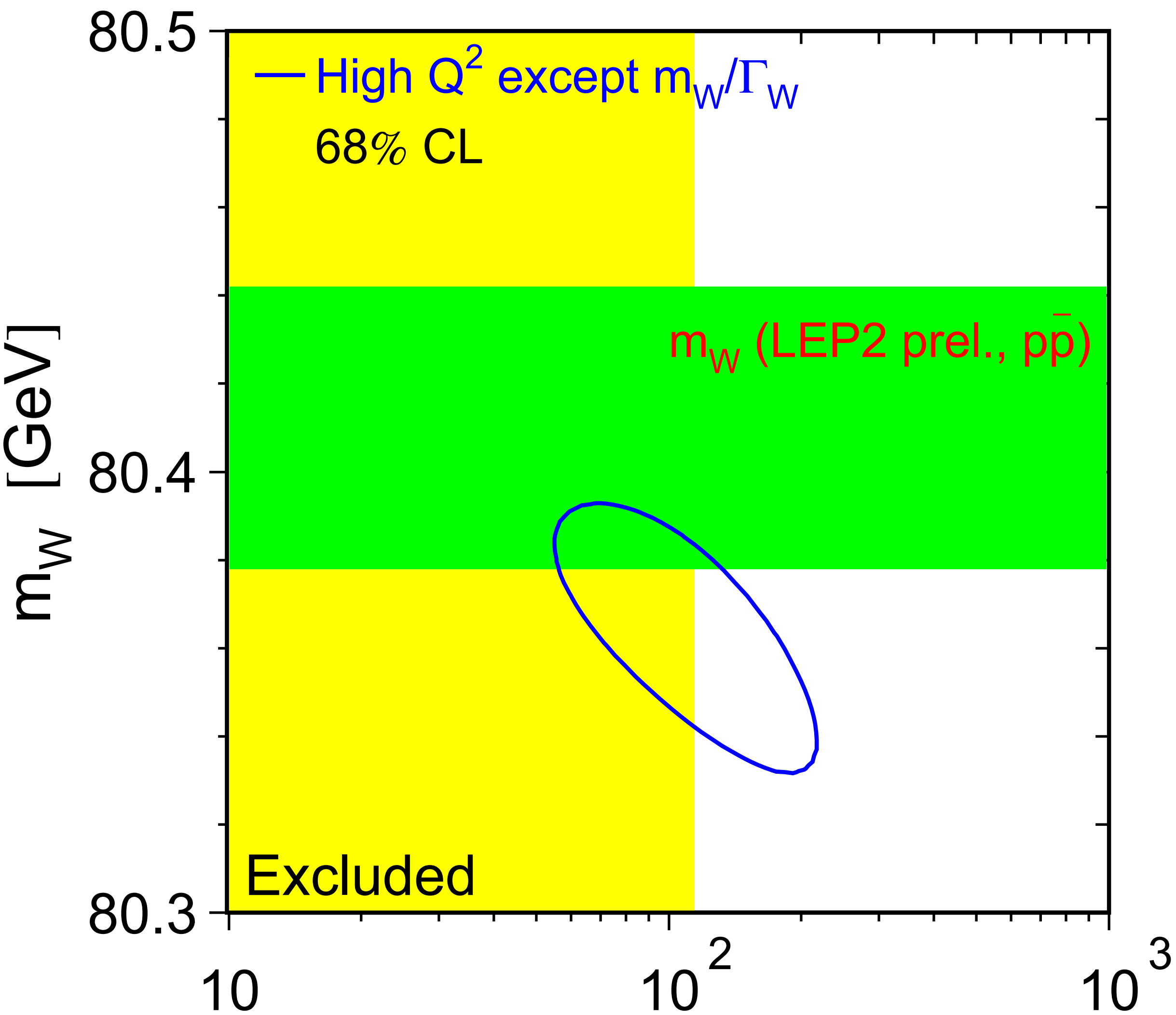


Figure 7: The 68% confidence level contour in m_{W} and m_{H} for the fit to all data except the direct measurement of m_{W}, indicated by the shaded horizontal band of ± 1 sigma width. The vertical band shows the 95% CL exclusion limit on m_{H} from the direct search.

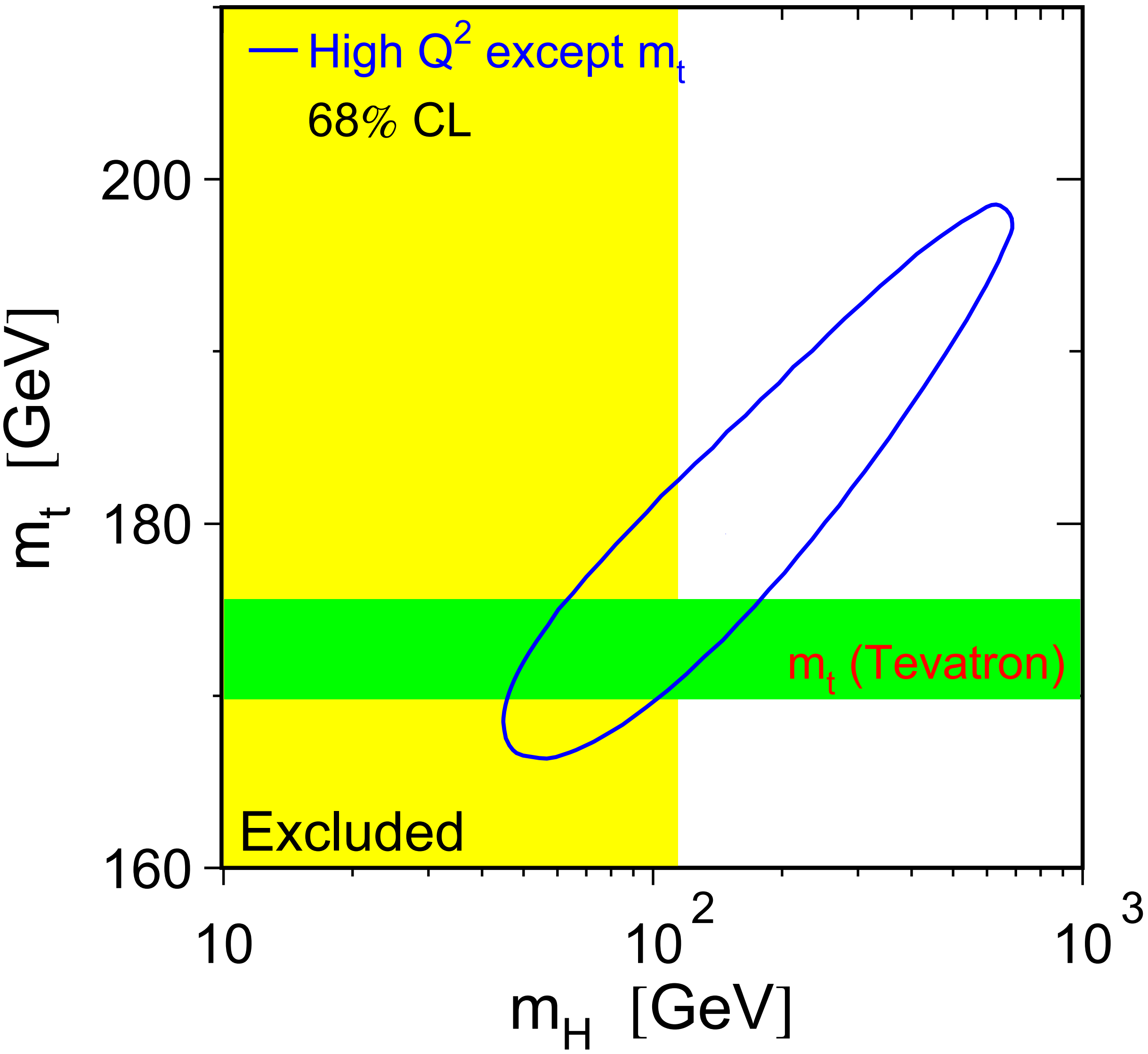


Figure 8: The 68% confidence level contour in m_{t} and m_{H} for the fit to all data except the direct measurement of m_{t}, indicated by the shaded horizontal band of ± 1 sigma width. The vertical band shows the 95% CL exclusion limit on m_{H} from the direct search.

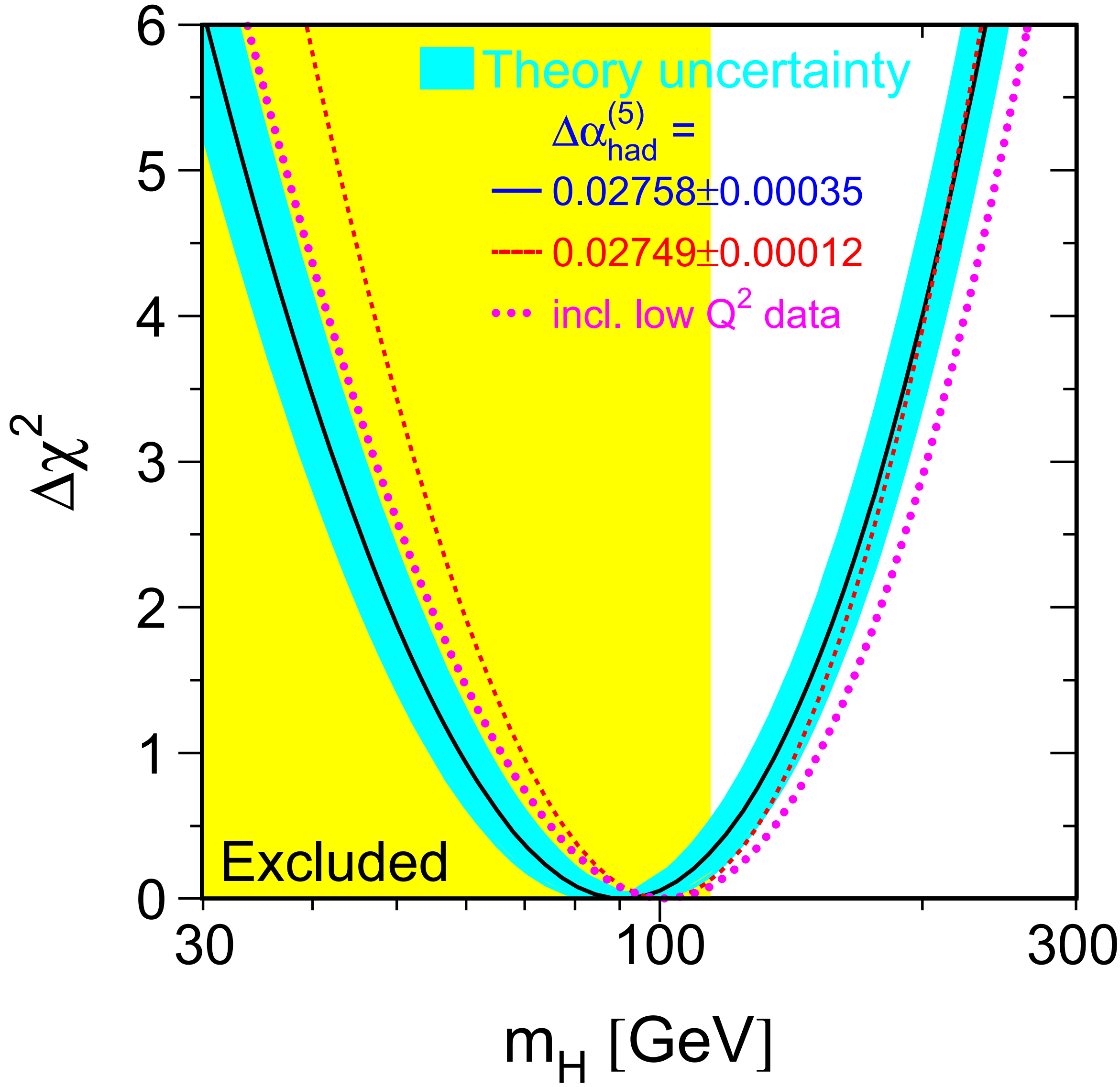


Figure 9: $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ *vs.* m_H curve. The line is the result of the fit using all data (last column of Table 1); the band represents an estimate of the theoretical error due to missing higher order corrections. The vertical band shows the 95% CL exclusion limit on m_H from the direct search. The dashed curve is the result obtained using the evaluation of $\Delta\alpha^{(5)}_{\mathrm{had}}(m^2_\mathrm{Z})$ from Troconiz, Yndurain, 2004.

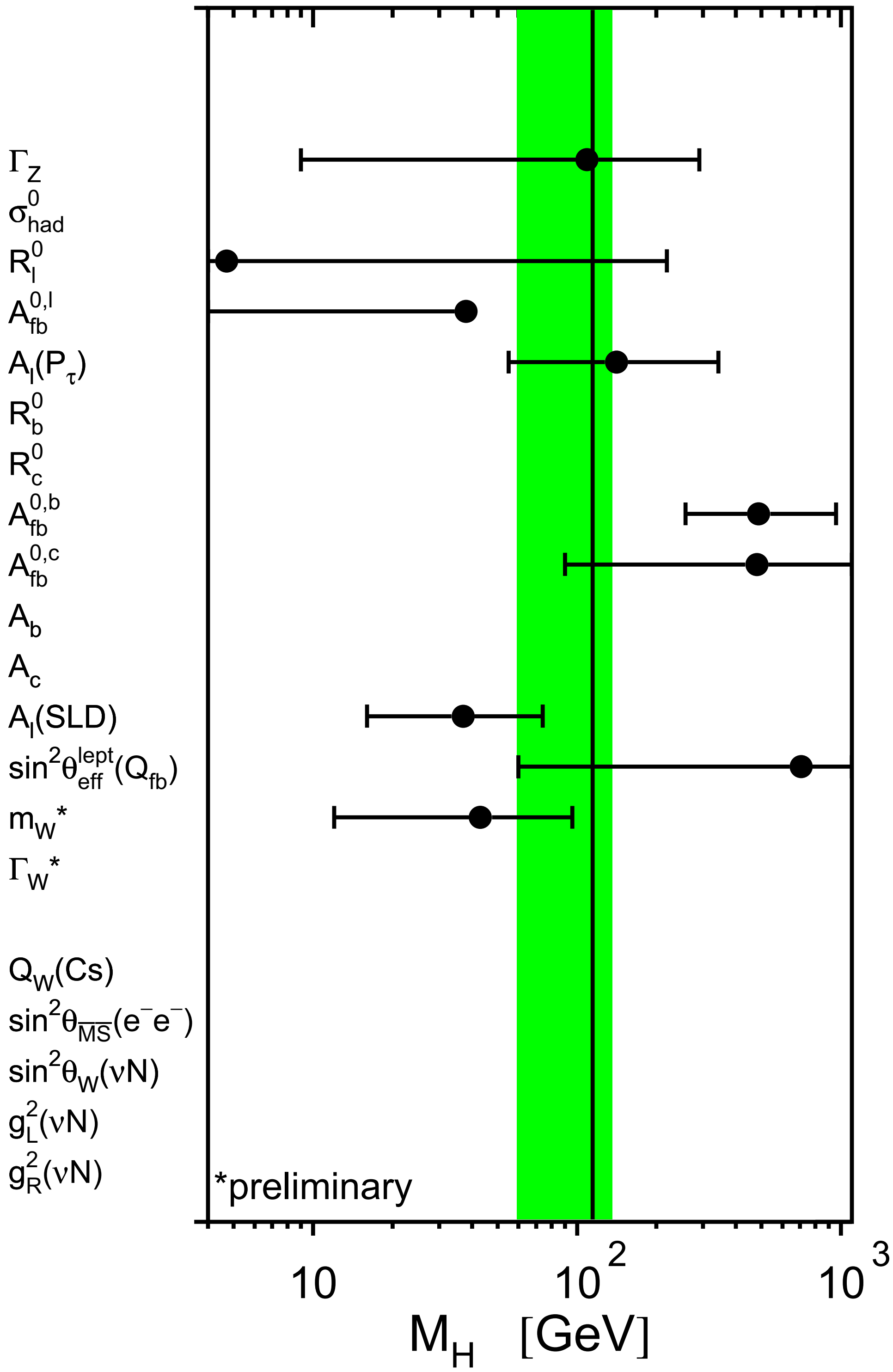


Figure 10: Constraints on the mass of the Higgs boson from each pseudo-observable. The Higgs-boson mass and its 68% CL uncertainty is obtained from a five-parameter SM fit to the observable, constraining $\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = 0.02761 \pm 0.00036$, $\alpha_S(m_Z^2) = 0.118 \pm 0.003$, $m_Z = 91.1875 \pm 0.0021$ GeV and $m_t = 172.7 \pm 2.9$ GeV. Because of these four common constraints the resulting Higgs-boson mass values are highly correlated. The shaded band denotes the overall constraint on the mass of the Higgs boson derived from all pseudo-observables including the above four SM parameters as reported in the last column of Table 1.

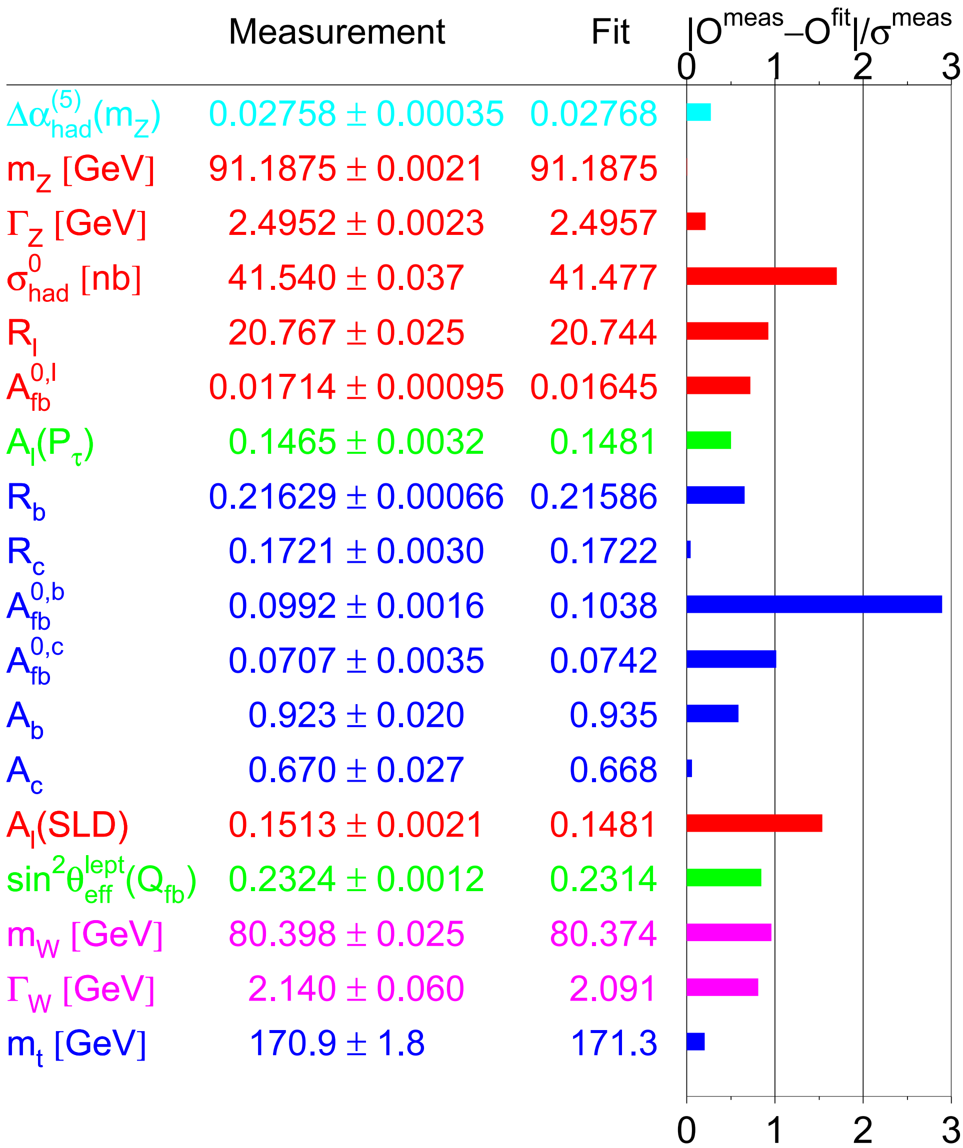


Figure 11: [LATEST FROM WINTER 2007] Constraints on the mass of the Higgs boson from each pseudo-observable. The Higgs-boson mass and its 68% CL uncertainty is obtained from a five-parameter SM fit to the observable, constraining $\Delta\alpha_{\rm had}^{(5)}(m_Z^2) = 0.02761 \pm 0.00036$, $\alpha_S(m_Z^2) = 0.118 \pm 0.003$, $m_Z = 91.1875 \pm 0.0021$ GeV and $m_t = 172.7 \pm 2.9$ GeV. Because of these four common constraints the resulting Higgs-boson mass values are highly correlated. The shaded band denotes the overall constraint on the mass of the Higgs boson derived from all pseudo-observables including the above four SM parameters as reported in the last column of Table 1.

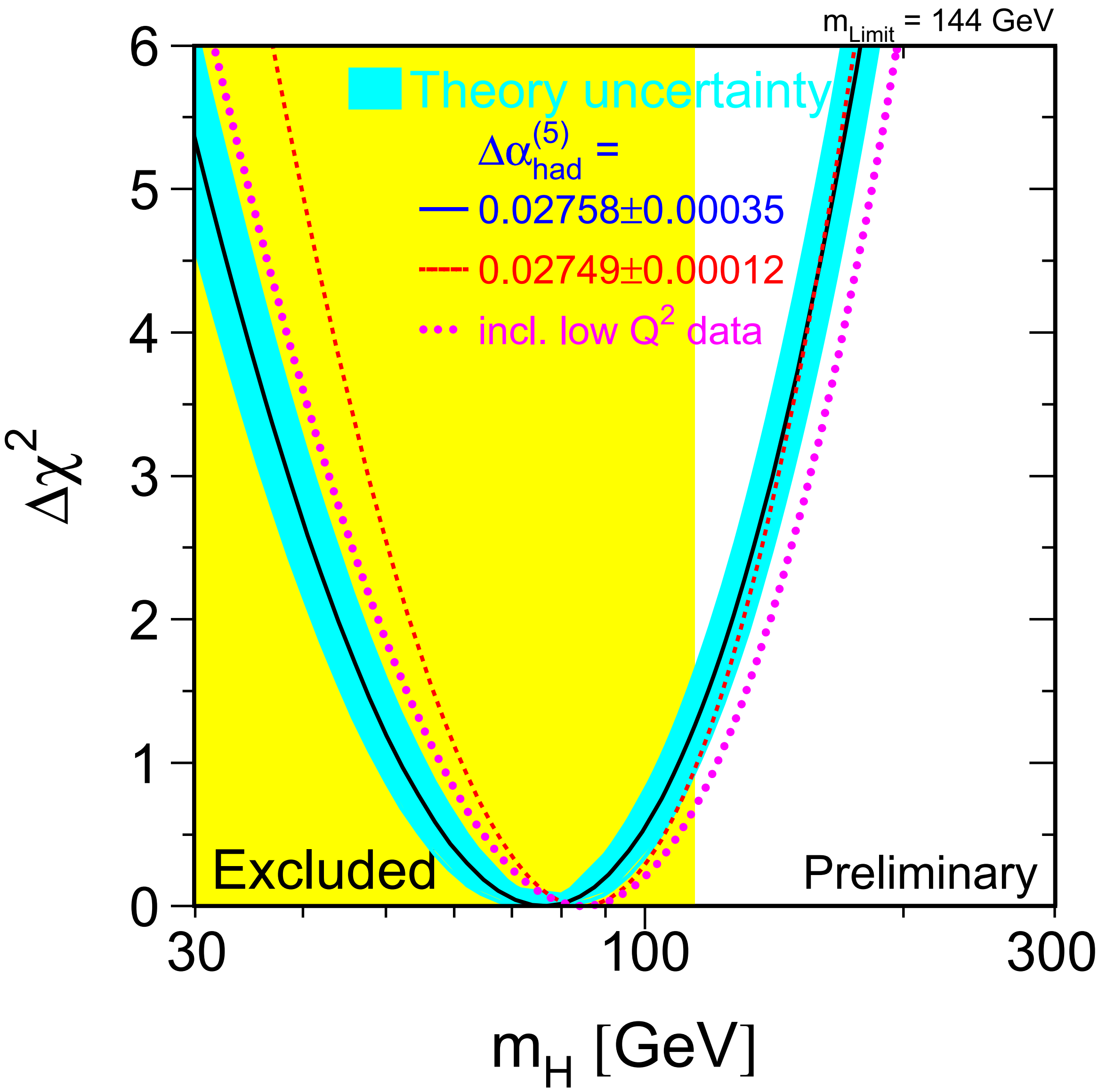


Figure 12: [LATEST FROM WINTER 2007] $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ *vs.* $m_{\rm H}$ curve. The line is the result of the fit using all data (last column of Table 1); the band represents an estimate of the theoretical error due to missing higher order corrections. The vertical band shows the 95% CL exclusion limit on $m_{\rm H}$ from the direct search. The dashed curve is the result obtained using the evaluation of $\Delta\alpha^{(5)}_{\rm had}(m^2_{\rm Z})$ from Troconiz, Yndurain, 2004.

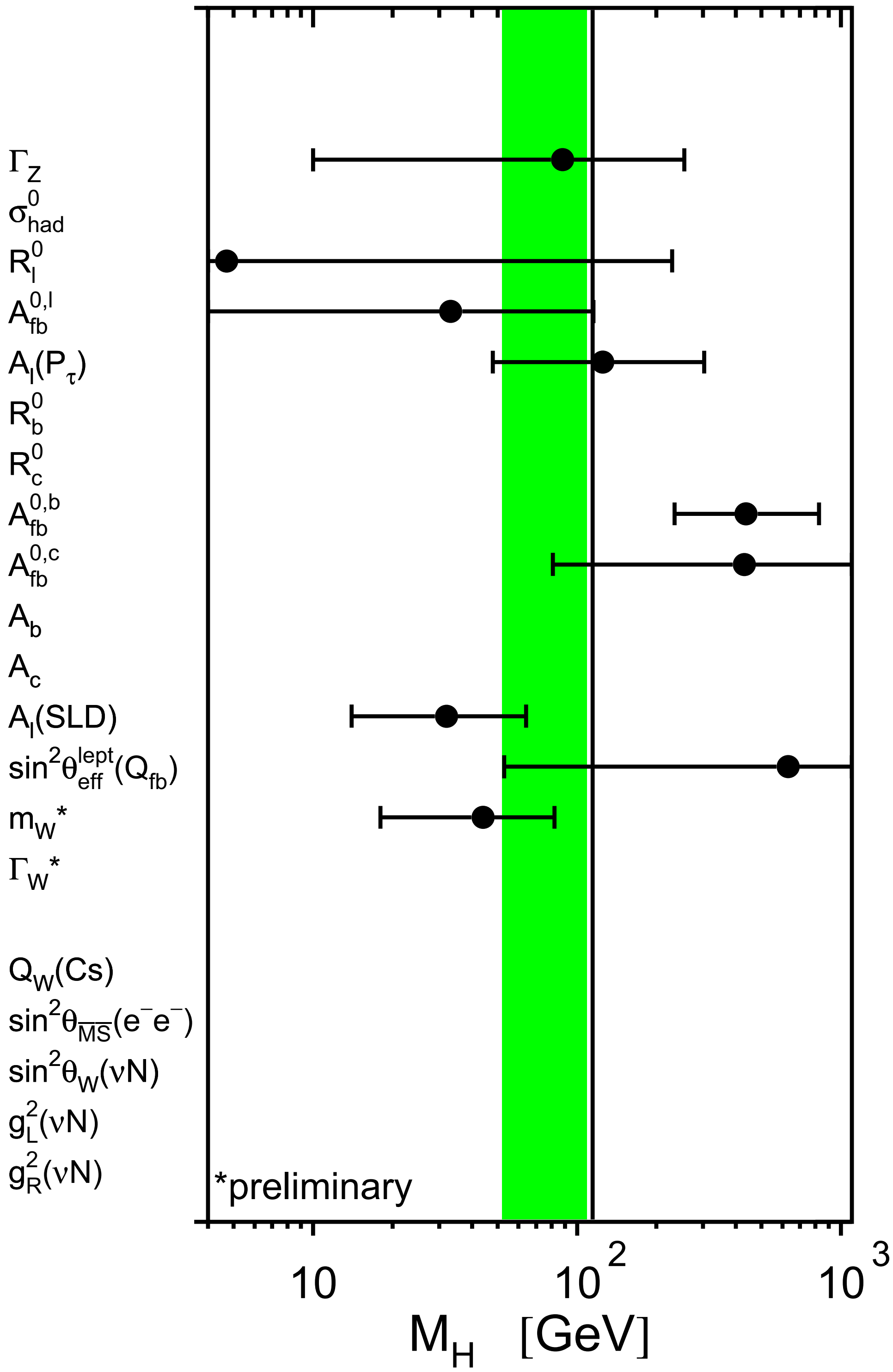


Figure 13: [LATEST FROM WINTER 2007] Constraints on the mass of the Higgs boson from each pseudo-observable. The Higgs-boson mass and its 68% CL uncertainty is obtained from a five-parameter SM fit to the observable, constraining $\Delta\alpha^{(5)}_{\mathrm{had}}(m^2_Z) = 0.02761 \pm 0.00036$, $\alpha_S(m^2_Z) = 0.118 \pm 0.003$, $m_Z = 91.1875 \pm 0.0021$ GeV and $m_t = 172.7 \pm 2.9$ GeV. Because of these four common constraints the resulting Higgs-boson mass values are highly correlated. The shaded band denotes the overall constraint on the mass of the Higgs boson derived from all pseudo-observables including the above four SM parameters as reported in the last column of Table 1.

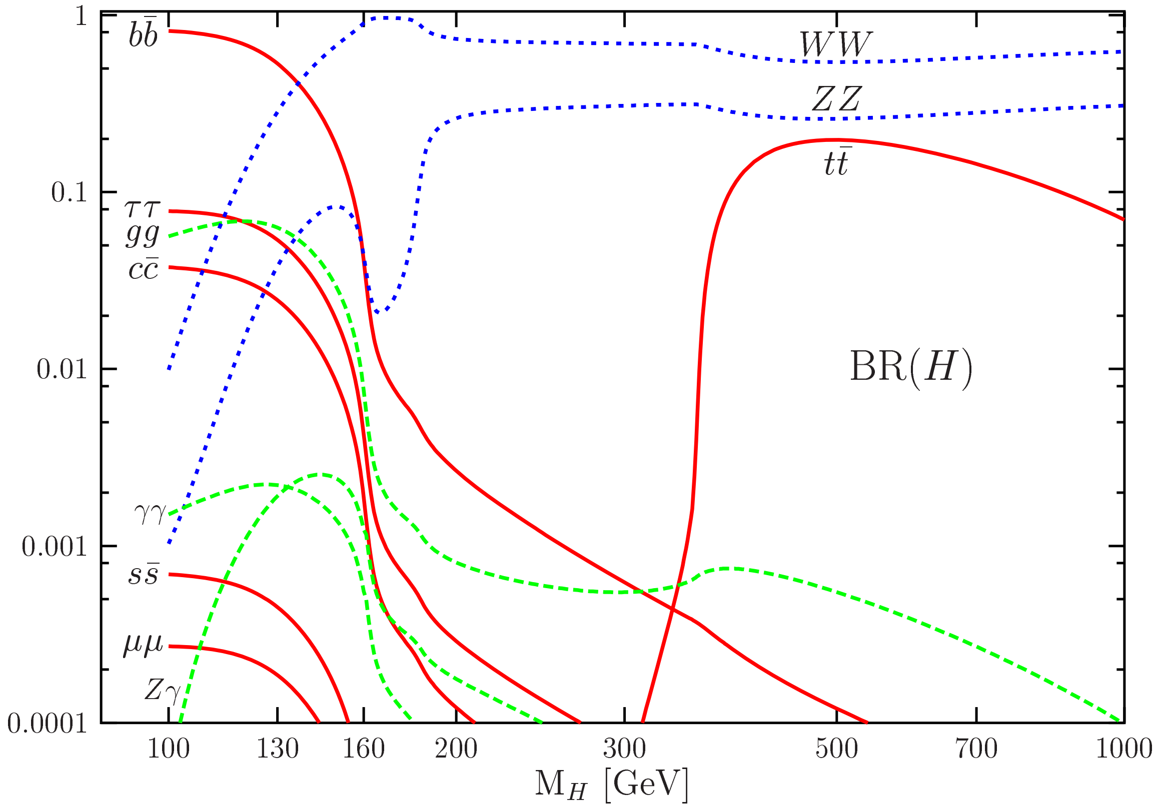


Figure 14: SM Higgs decay branching ratios as a function of M_H. (Djouadi et al., 2005)

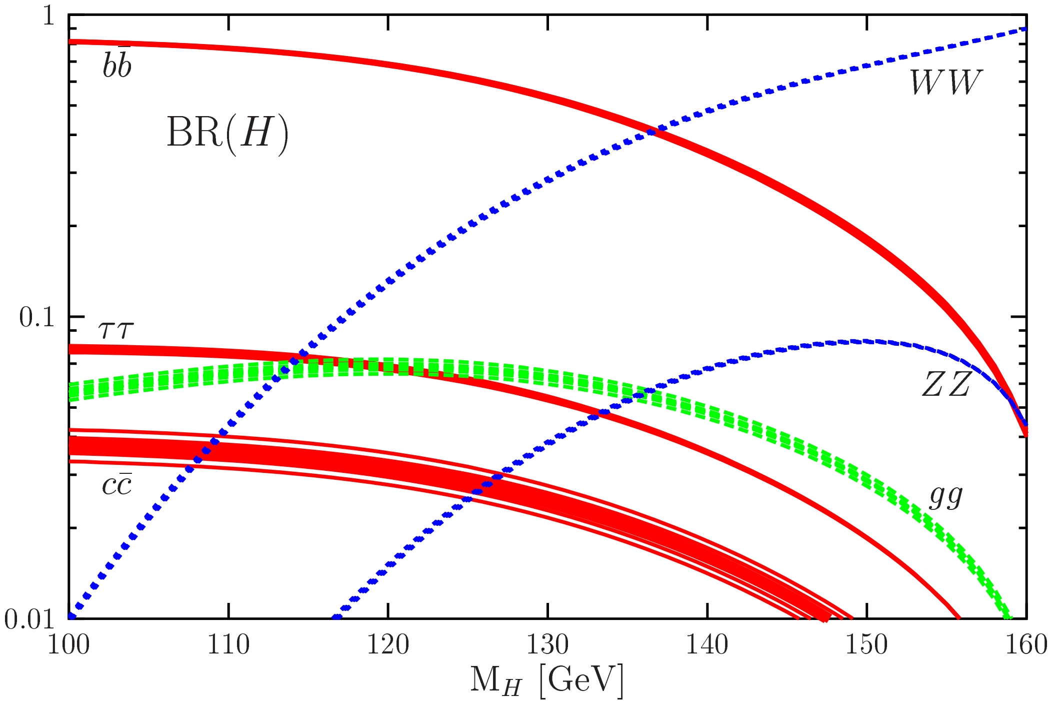


Figure 15: SM Higgs boson decay branching ratios in the low and intermediate Higgs boson mass range. (Djouadi et al., 2005)

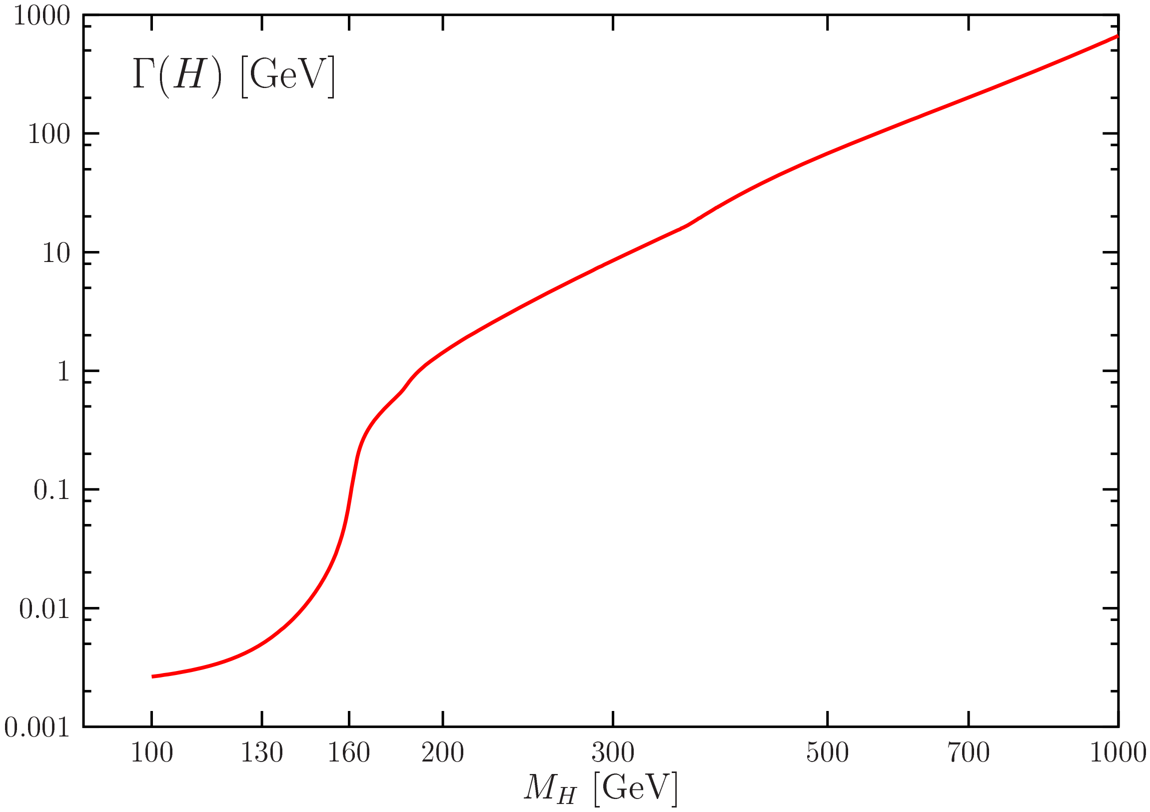


Figure 16: SM Higgs total decay width as a function of M_H. Note how narrow it is for $M_H < m_W/2$. (Djouadi et al., 2005).

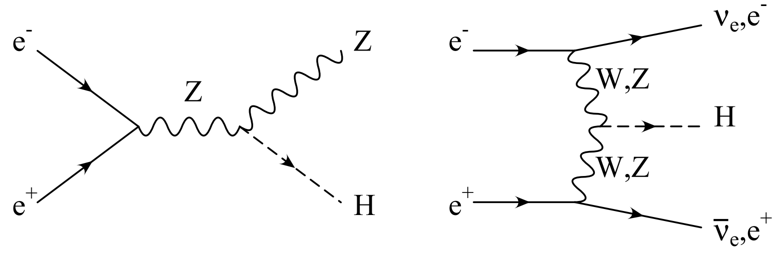


Figure 17: Higgs boson production channels at LEP2.

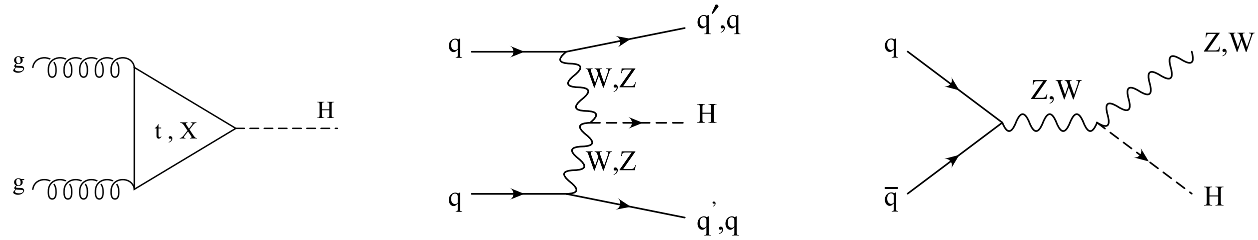


Figure 18: Leading Higgs production processes at hadron colliders: $gg \to H$, $qq \to qqH$, and $q\bar{q} \to WH, ZH$.

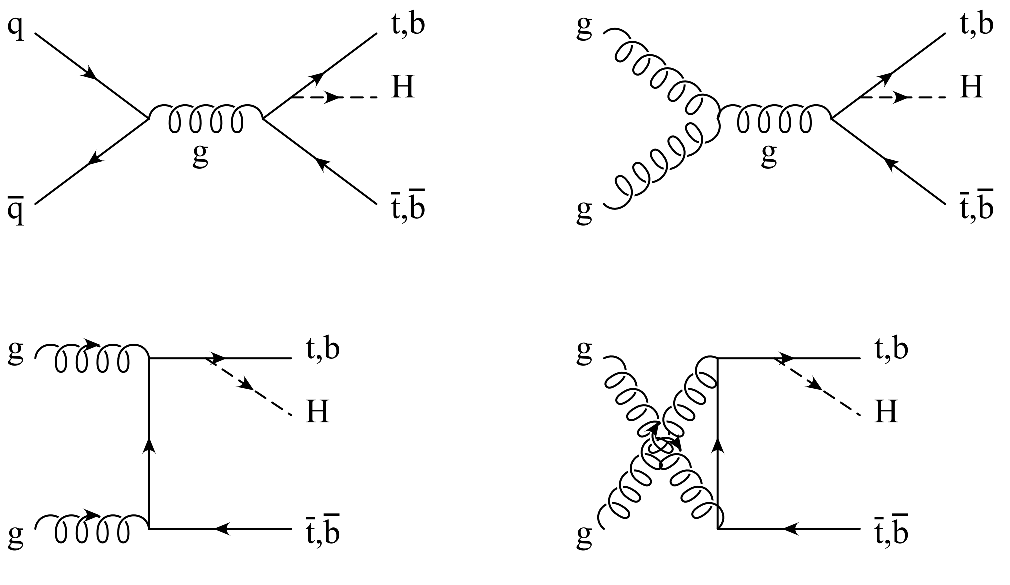


Figure 19: Higgs production with heavy quarks: sample of Feynman diagrams illustrating the two corresponding parton level processes $q\bar{q}, gg \to t\bar{t}H, b\bar{b}H$. Analogous diagrams with the Higgs boson leg attached to the remaining top(bottom)-quark legs are understood. (Figs from L. Reina, 2004)

LEP II Searches for the Higgs Boson

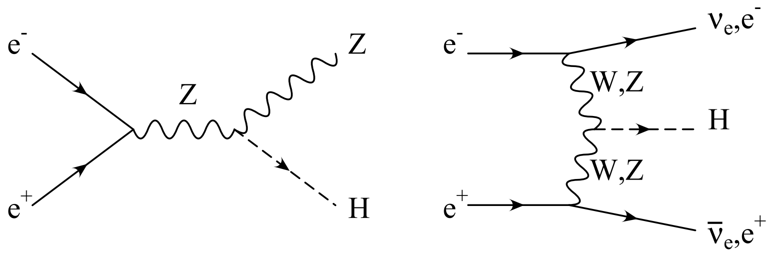


Figure 20: Higgs boson production channels at LEP2. (fig. from L. Reina, 2004)

The LEP searches for the Higgs boson concluded with runs at about $\sqrt{s} \leq 209\,\text{GeV}$, and ultimately reached the limit of

$$M_H \geq 114.4\,\text{GeV}$$

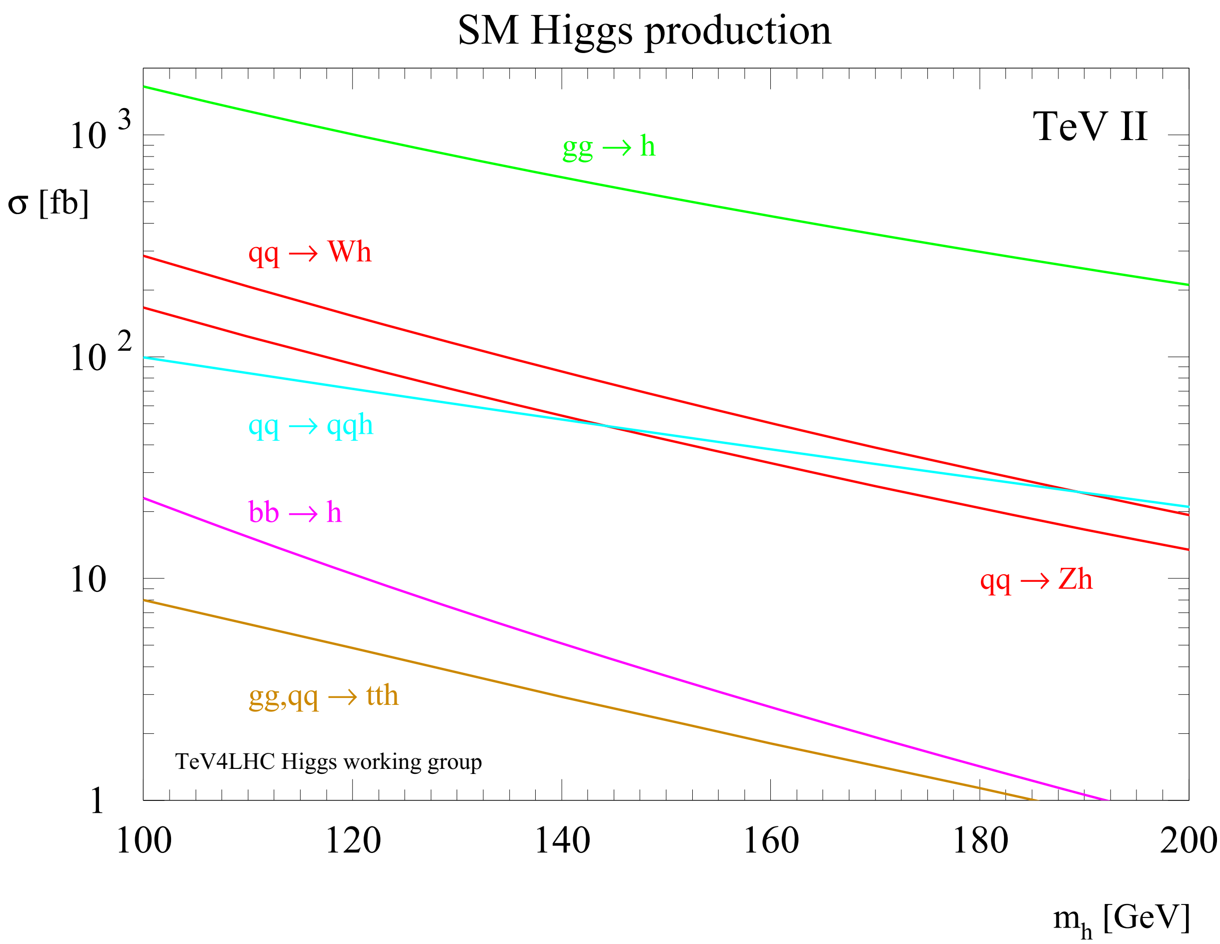


Figure 21: Tevatron cross section, $\sqrt{s} = 1.96\,\text{TeV}$. (Tev 4 LHC Workshop)

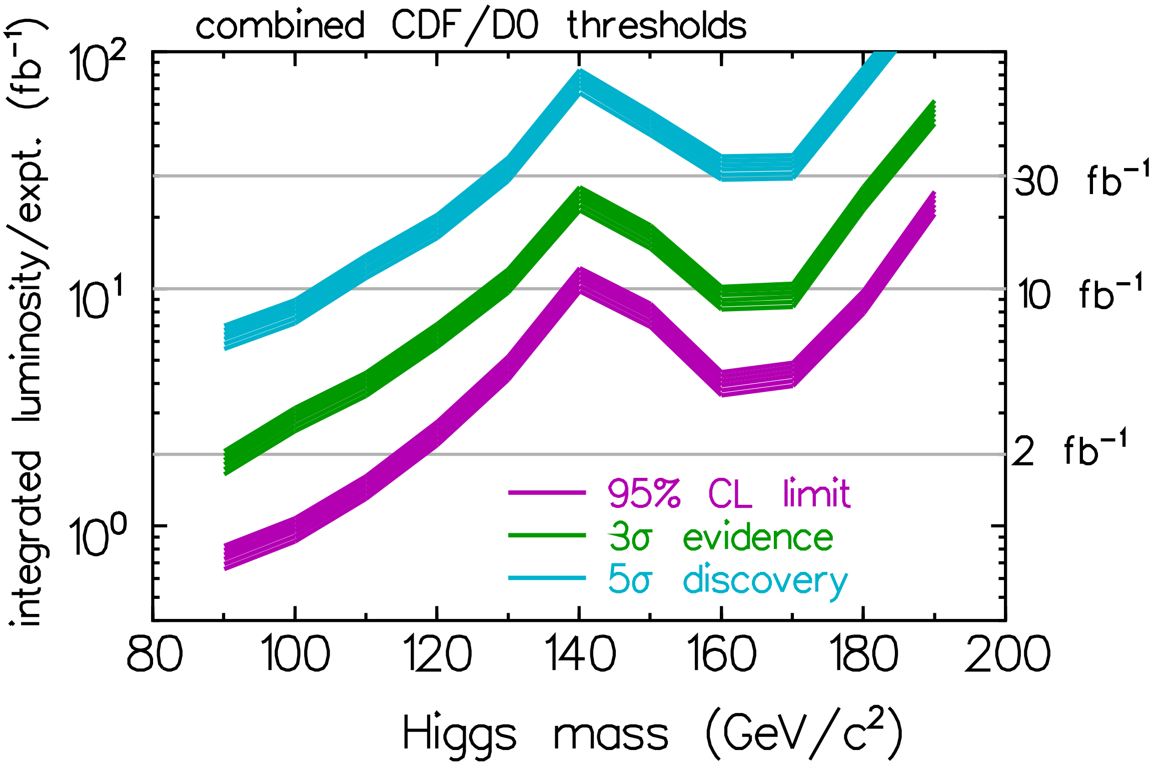


Figure 22: Integrated luminosity required for each experiment at the Tevatron, Run II, to exclude a SM Higgs boson at 95% CL or to observe it at the 3σ or 5σ level. (Tevatron Run 2 Workshop)

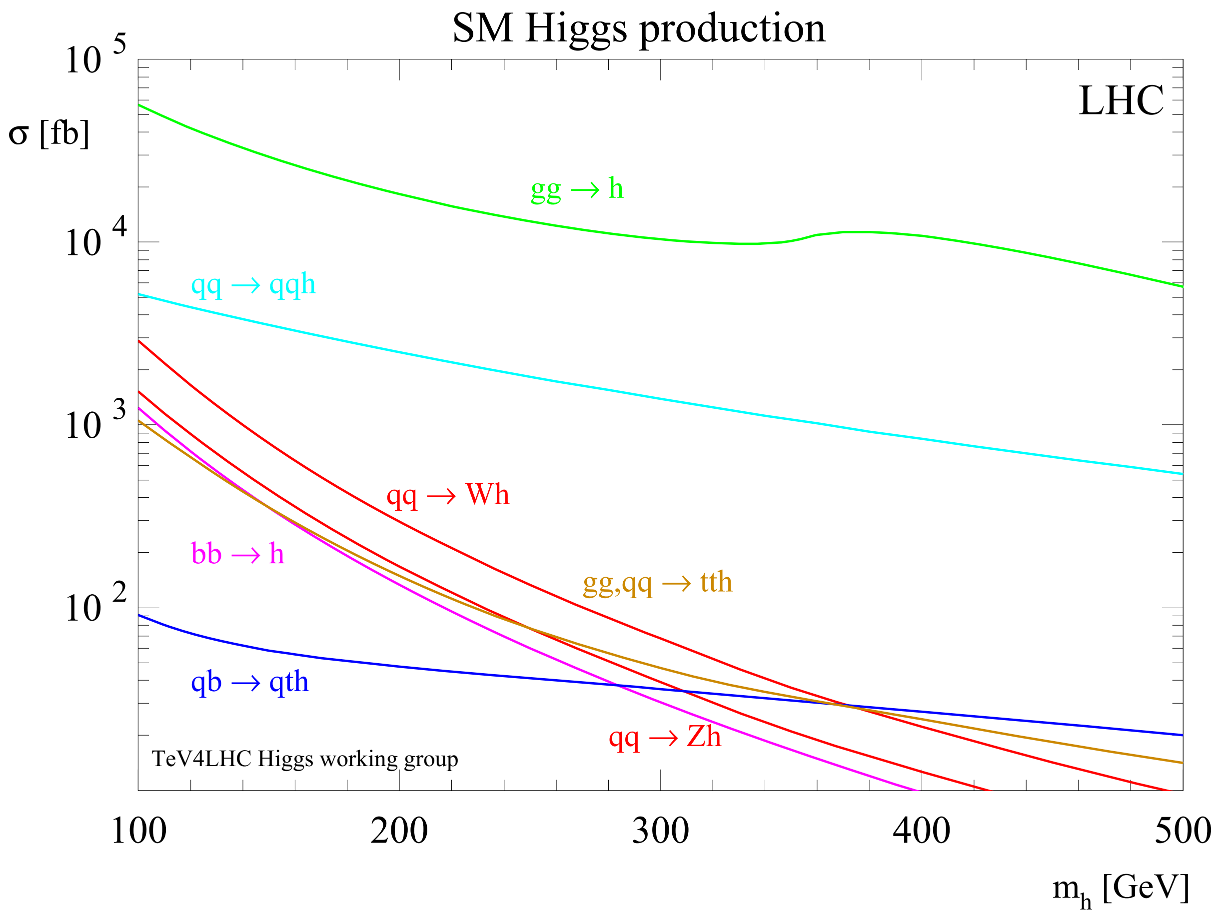


Figure 23: Cross sections for SM Higgs boson production processes at the LHC at $\sqrt{s} = 14\,\text{TeV}$. (TeV 4 LHC Workshop)

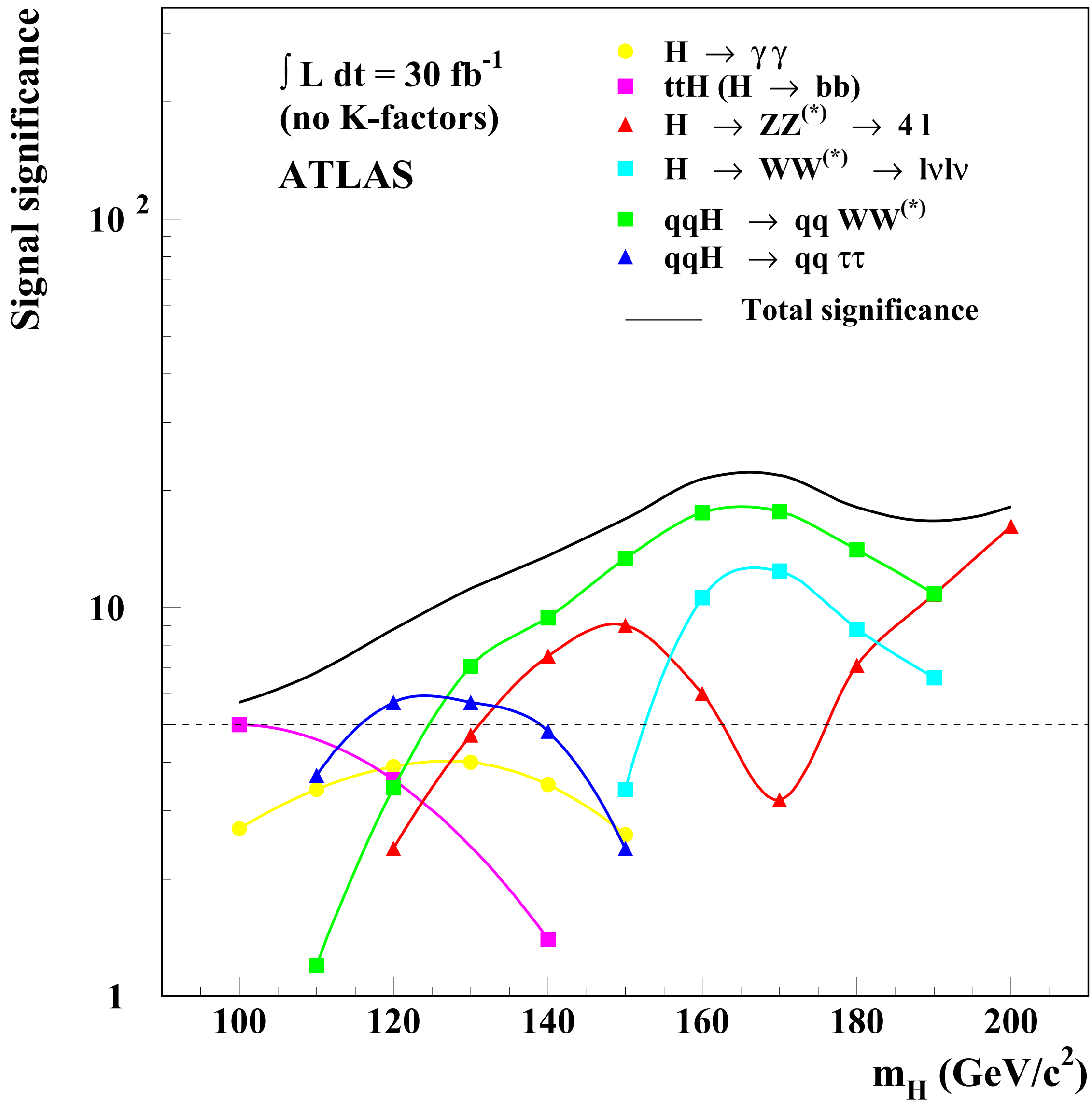


Figure 24: Significance plots for the SM Higgs boson discovery at 30 fb^{-1} integrated luminosity. (ATLAS)

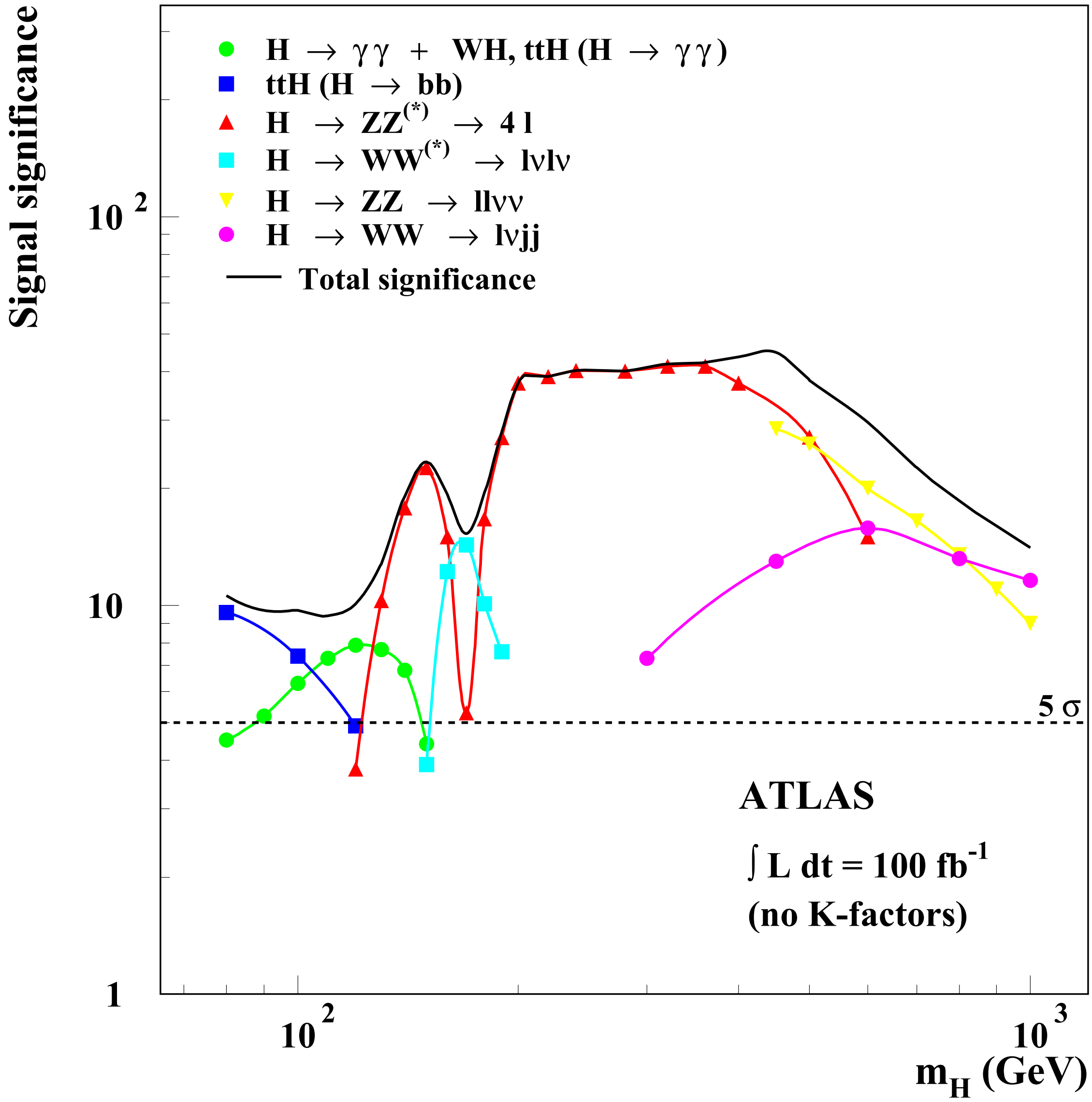


Figure 25: Significance plots for the SM Higgs boson discovery at 100 fb^{-1} integrated luminosity. (ATLAS)

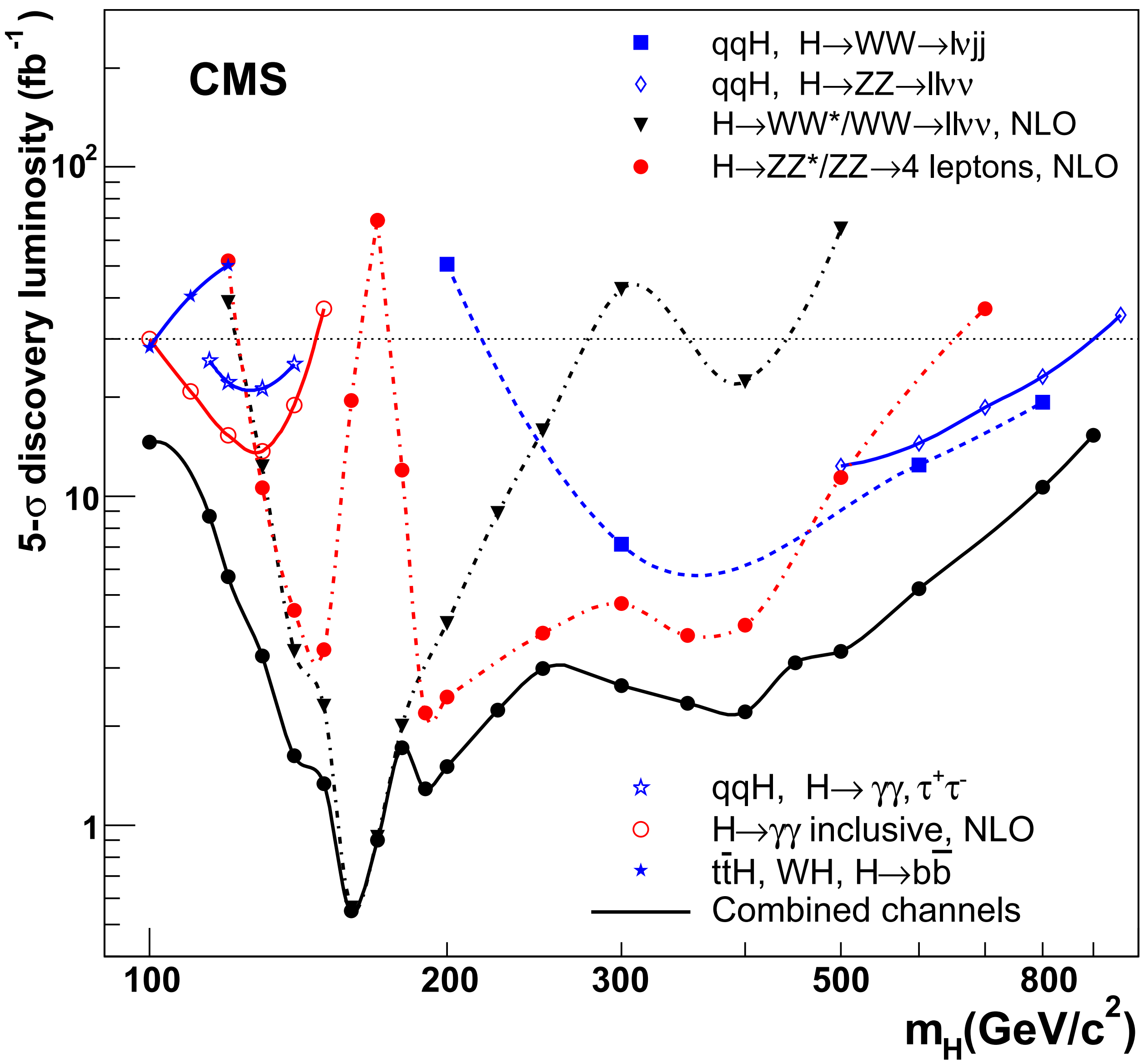


Figure 26: Luminosity required to reach a 5σ discovery signal in various channels at CMS (CMS).

“Theory Constraints” on the Higgs Boson Mass

The Higgs boson mass in the SM is a free parameter determined by

$$m_h^2 = 2\lambda v^2$$

where λ is the free (dimensionless) parameter.

From *data* we have determined that

$$114 \leq m_H \lesssim 200\,\mathrm{GeV}$$

where the lower bound comes from direct searches and the upper bound from precision EW studies.

There are three main “theory constraints” of the SM Higgs boson:
1. Perturbative Unitarity
2. Triviality/Perturbative scaling
3. Vacuum Stability

Perturbative Unitarity

The Goldstone equivalence theorem (see, e.g., Peskin-Schroeder):

$$A(V_L^1 \cdots V_L^n \to V_L^1 \cdots V_L^m) = (i)^n(-i)^m A(\omega^1 \cdots \omega^n \to \omega^1 \cdots \omega^m) + \mathcal{O}\left(\frac{m_V^2}{s}\right)$$

Let look at $W_L^+ W_L^- \to W_L^+ W_L^-$:

$$A(\omega^+\omega^- \to \omega^+\omega^-) = -\frac{m_H^2}{v^2}\left(\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2}\right)$$

Partial wave amplitude decomposition of the amplitude is

$$A = 16\pi \sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)a_l$$

Using orthogonality of the Legendre polynomials, cross-section is

$$\sigma = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2$$

From the optical theorem

$$\begin{aligned}\sigma &= \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2 = \frac{1}{s}\mathrm{Im}\,|A(\theta=0)| \\ &= \frac{16}{\pi}\sum_{l=0}^{\infty}(2l+1)\,\mathrm{Im}\,a_l\end{aligned}$$

Thus

$$|a_l|^2 = \mathrm{Re}(a_l)^2 + \mathrm{Im}(a_l)^2 = \mathrm{Im}(a_l)$$

which implies that

$$|\mathrm{Re}(a_l)| \leq \frac{1}{2}$$

since

$$x^2 + y^2 = y \implies x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}.$$

The $J = 0$ partial wave a_0 is

$$a_0 = \frac{1}{16\pi s}\int_{-s}^{0} A dt = -\frac{m_H^2}{16\pi v^2}\left[2 + \frac{m_H^2}{s - m_H^2} - \frac{m_H^2}{s}\log\left(1 + \frac{s}{m_H^2}\right)\right]$$

For $s \gg m_H^2$, we have

$$a_0 \to -\frac{m_H^2}{8\pi v^2}$$

This needs to be less than 1/2:

$$\left|-\frac{m_H^2}{8\pi v^2}\right| \leq \frac{1}{2} \implies m_H < 2v\sqrt{\pi} = 870\,\mathrm{GeV}$$

Considering other channels in addition one finds

$$m_H < 710\,\mathrm{GeV}$$

What does this number mean? If $m_H \gtrsim 710\,\mathrm{GeV}$ a tree-level calculation is not sufficient.

Triviality Bound

The Higgs self-coupling is subject to quantum corrections. The RGE is

$$32\pi^2 \frac{d\lambda}{d\log Q} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^4)\lambda$$
$$+\frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

If λ is large (i.e., large $m_H = 2\lambda v^2$) the first term dominates

$$\frac{d\lambda}{d\log Q} = \frac{3}{4\pi^2}\lambda^2$$

Easy to solve this

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0)\ln\frac{Q^2}{Q_0^2}}$$

This is a rapidly diverging function of Q. λ diverges at the Landau Pole scale, Q_{LP}.:

$$1 - \frac{3}{4\pi^2}\lambda_0 \ln\frac{Q_{LP}^2}{Q_0^2} \implies Q_{LP} = Q_0 \exp\left[\frac{2\pi^2}{3\lambda_0}\right] = m_H \exp\left[\frac{4v^2\pi^2}{3m_H^2}\right]$$

Why is this important?
1. Given a Higgs mass we can say at what scale Q_{LP} the theory breaks down.
2. Given an interesting scale (M_{GUT}, $M_{neutrino}$, etc.) you can determine what low-energy Higgs mass keeps the theory perturbative up to that scale.

Vacuum stability

If λ is very small, then the term that dominates is y_t^4 term:

$$\frac{d\lambda}{d\log Q} = -\frac{3y_t^4}{4\pi^2}$$

which drives λ smaller and smaller as we run up to the UV.

If $\lambda(Q) < 0$ at some scale, there is worry that the vacuum is not stable.

One can approximately say that at the lowest scale Q_V such that

$$\lambda(Q_V) < 0$$

one expects new physics to lift the vacuum. Thus, the SM "breaks down" for $Q > Q_V$.

Vacuum stability puts a lower bound on m_H for any given scale Q_V up to which we require $\lambda(Q < Q_P) > 0$.

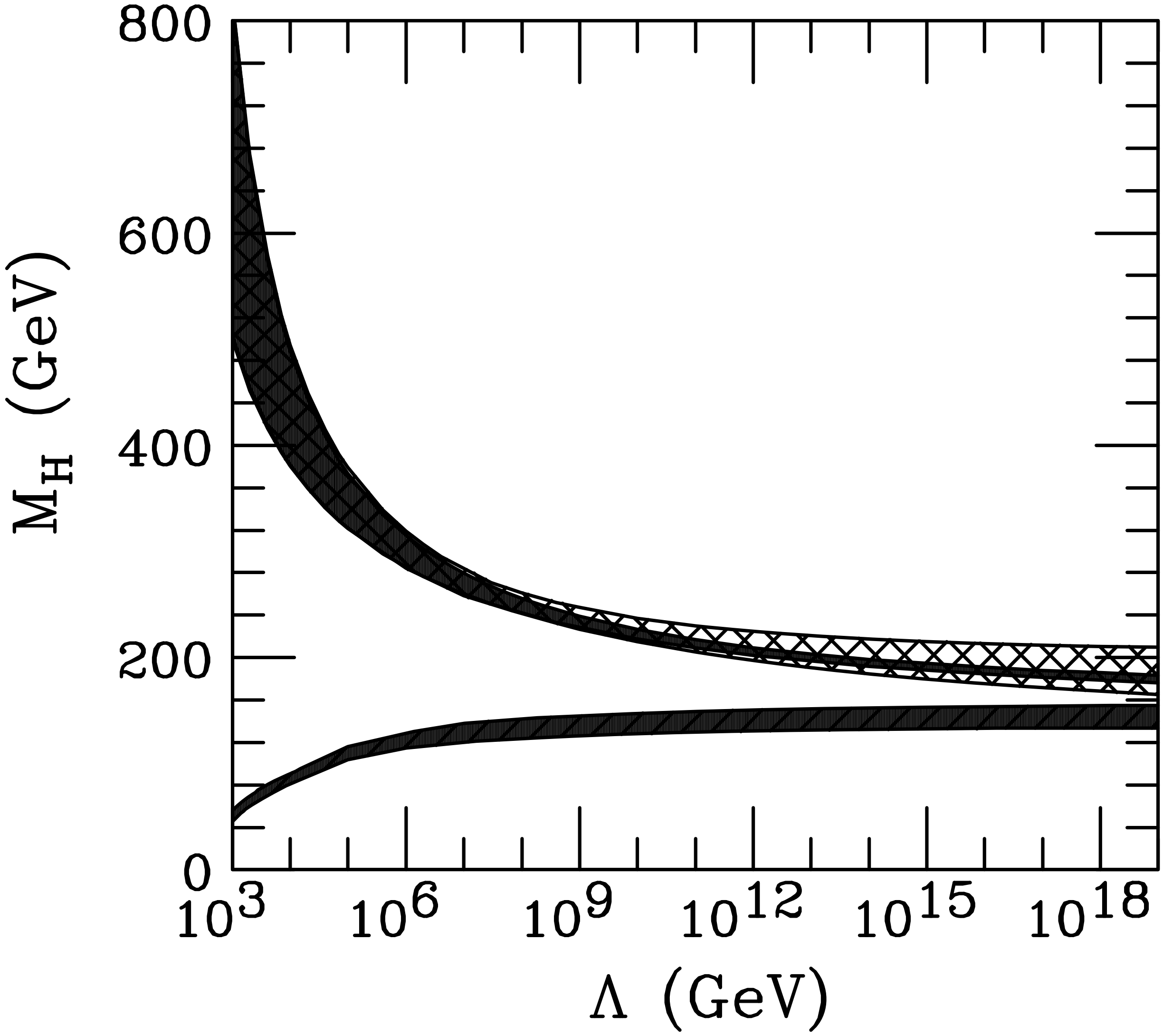


Figure 27: Hambye, Riesselmann, '97