Satellite Remote Sensing: a Tool for Climate Studies

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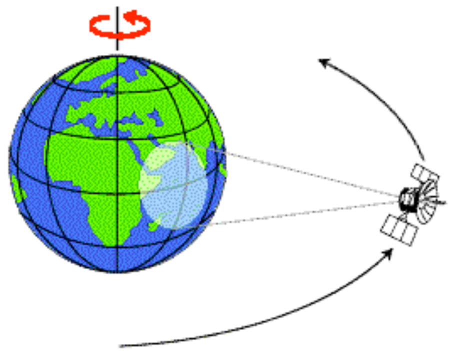

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Mechanisms, Monitoring, and Impact in Tropical Regions
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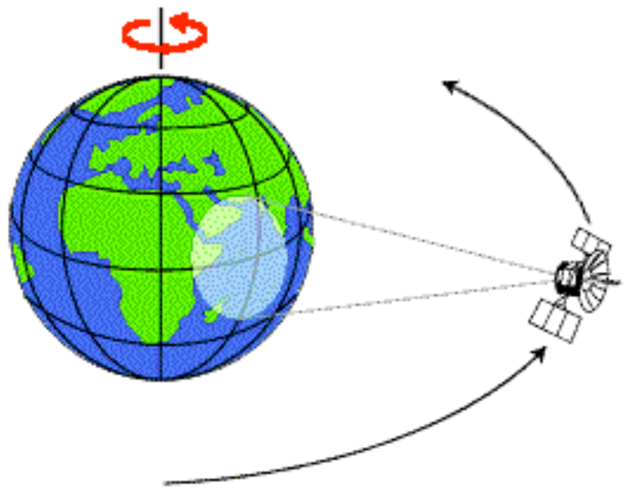
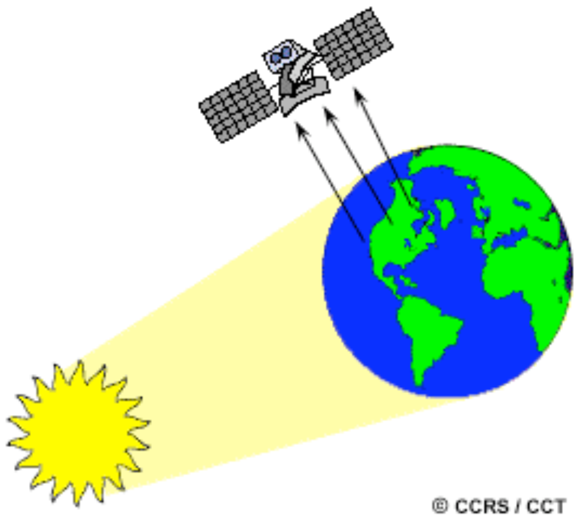
CONTENTS OF THE PRESENTATION

1. Satellite remote Sensing: definition & components
2. Electromagnetic wave propagation through surface and atmosphere.
3. Satellite platforms and sensor characteristics
4. Climate parameters retrieved from remote sensing.
5. Atmospheric profile retrieval
6. Data Assimilation
7. Conclusions

1. Satellite remote sensing: definition & components



What is Remote Sensing?



Remote sensing is the science (or the art) of acquiring information about the Earth's surface and atmosphere without actually being in contact with it.

This is done by:

◊ Measuring reflected or emitted radiation,

◊ Processing the measurements to correct for radiometric and geometric distortions

◊ Parameter retrieval (surface/ atmosphere) from the measurements

Remote Sensing: Retrieval of parameters of the medium at the end of the path (i.e. at the surface)

Remote Sounding: Retrieval of parameters of the medium along the path

Use of remote sensing in climate-related studies:

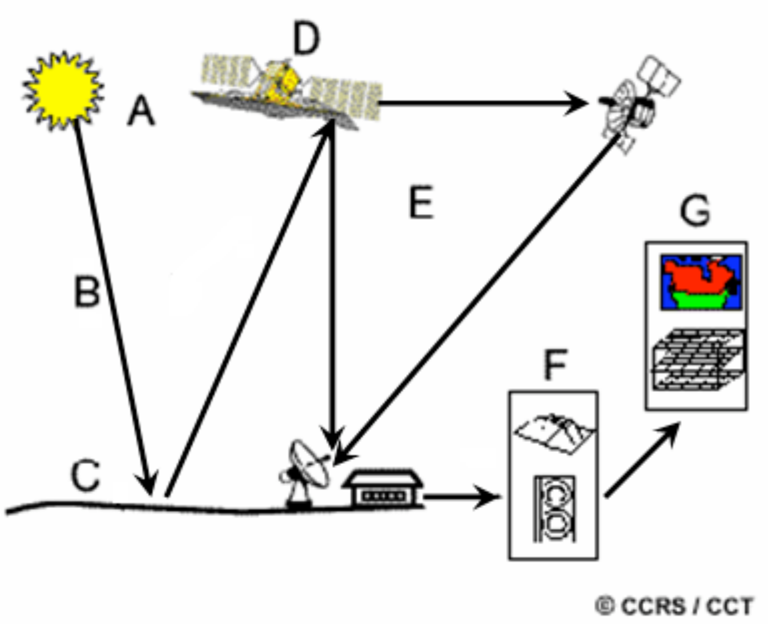
1. Climate monitoring (e.g. ice in polar regions, coastal erosion, sea and land surface temperature, etc.)

2. Model validation

3. Model initiation

4. Data assimilation (to correct/ adjust model output/ initial condition)

The Seven Components of Remote Sensing

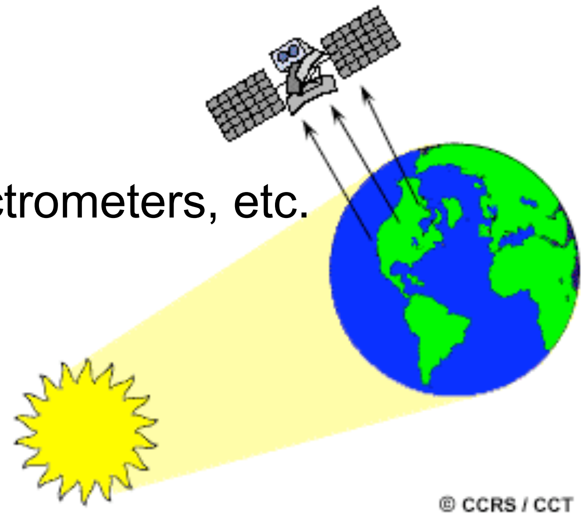


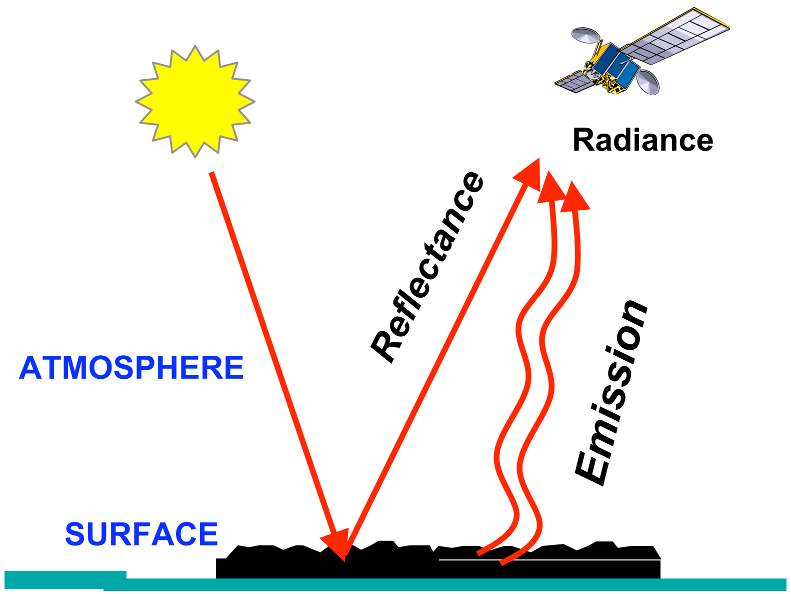
A ◊ Source of illumination (radiation)
B ◊ Interaction with the atmosphere
C ◊ Interaction with the surface target
D ◊ Recording radiation by the sensor
E ◊ Transmission, reception, and processing
F ◊ Interpretation and analysis (retrieval algorithms)
G ◊ Applications

Sources of illumination (radiation)

Natural sources: sun or stars

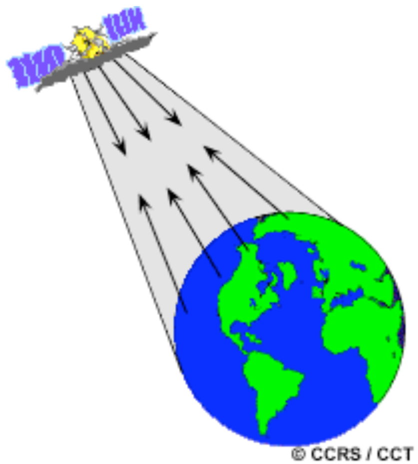
Passive sensors
Radiometers, spectrometers, etc.



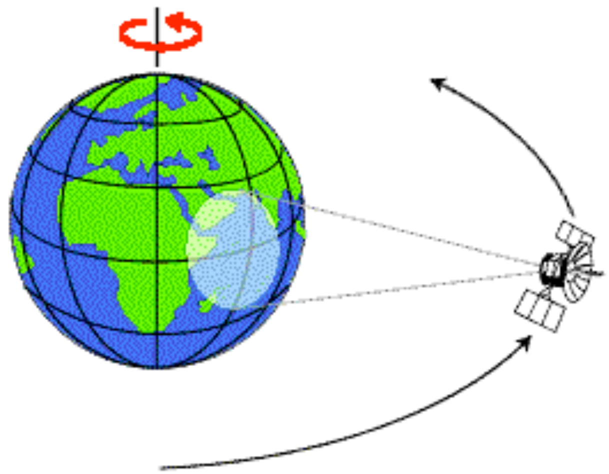


Self-illumination

Active sensors
Radar, Lidar



2. Electromagnetic wave propagation through surface and atmosphere.



Radiation from a Blackbody:

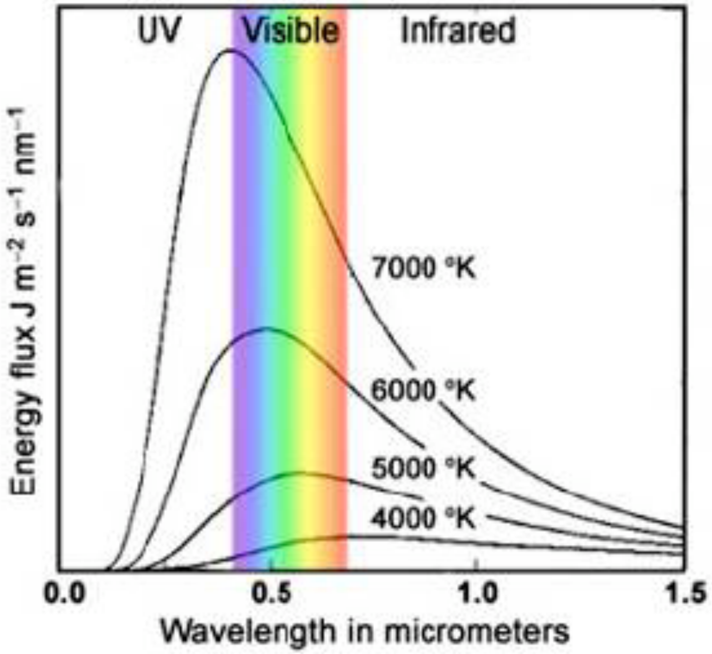
An important component in simulation what the satellite sensor receives; hence facilitates parameter retrieval (here we need the emissivity!)

A blackbody ◊ idealized material,
◊ perfectly opaque.
◊ absorbs all incident radiation and therefore reflects none.
◊ emits all of the received radiation (to maintain thermal equilibrium; i.e. its physical temperature)
◊ Hence, it is regarded as a perfect emitter.

A blackbody radiates uniformly in all directions at frequency *f* and temperature *T* (in K) according to the following expression:
Max Planck equation:

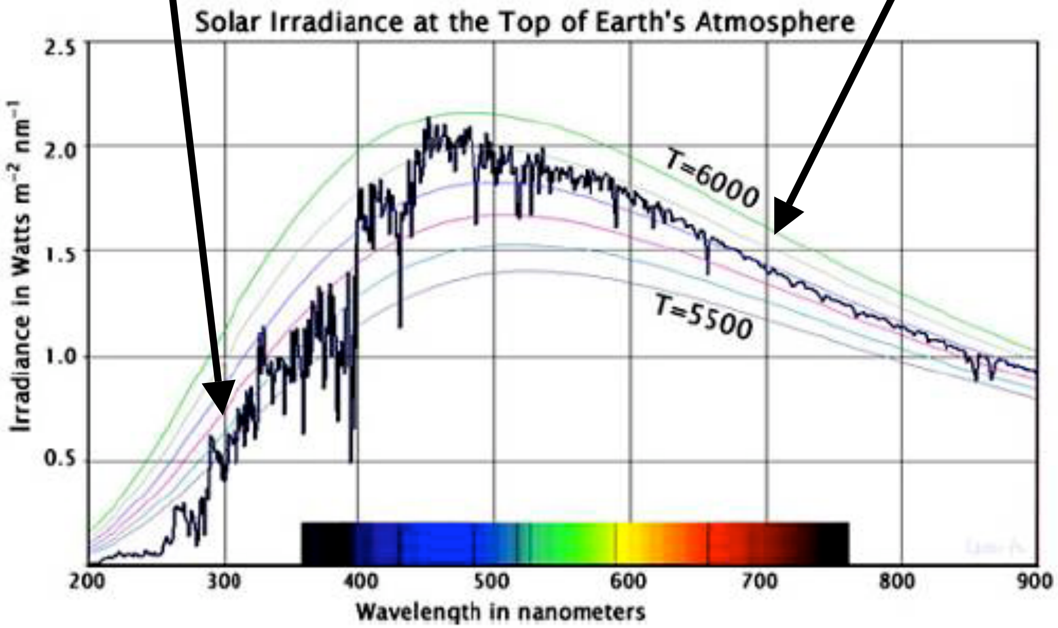
$$B_f = \frac{2hf^3}{c^2}\left(\frac{1}{e^{hf/kT}-1}\right)$$

B_f is the radiation flux density, *h* is Planck's constant, *k* is Boltzmann's constant, and *c* is the speed of light.



Gray-body behavior due to sun’s atmosphere

Blackbody behavior



Deviation from blackbody are more in the UV and visible but not in the IR

Source: http://earth.esa.int/landtraining07/D1LB1-Moreno.pd
ESA Optical theory Basics-1 Radiative Transfer course (Sep. 2007)

The *total emitted radiation* (*Ml*) from a blackbody:

◊ proportional to the fourth power of its absolute temperature.
◊ *Stefan-Boltzmann law*:

$$\boxed{M_\lambda = \sigma T^4}$$

σ ◊ the Stefan-Boltzmann constant= 5.6697 x 10^{-8} W m^{-2} K^{-4}.
Thus, the amount of energy emitted by an object such as the Sun or the Earth is a function of its temperature.

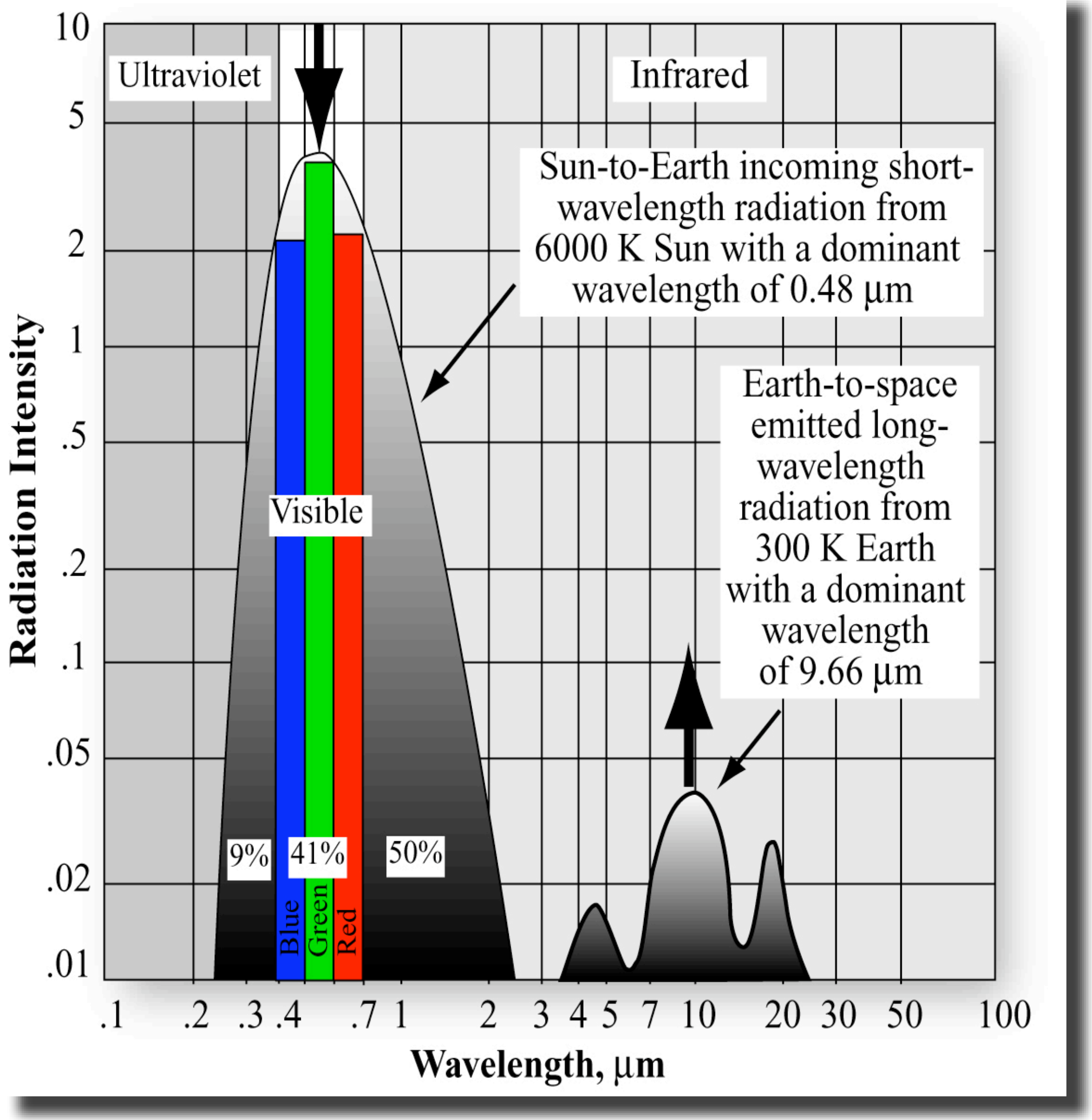
The dominant wavelength (λ*max*) of the Blackbody radiation:

◊ *Wein's displacement law*:

$$\boxed{\lambda_{\max} = \frac{k}{T}}$$

where *k* is a constant equaling 2898 μm K, and *T* is the absolute temperature in K.

Blackbody Radiation from the Sun and the Earth



The Sun approximates a 6,000° K blackbody:

Dominant wavelength of 0.48 µm (green light).

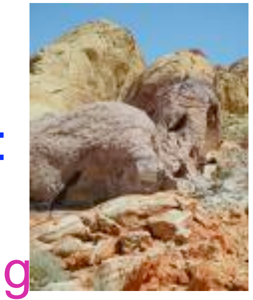
The Earth approximates a 300° K blackbody:

Dominant wavelength of 9.66 µm .

Overlap radiation between the sun and the earth: 3 µm 8 µm

The *dominant temperature* of the sensed object determines the thermal spectrum that has to be used.

◊If we are interested in soil, water, rock, etc. with ambient temperatures on the earth's surface of 300 K: (dominant wavelength is 9.67 μm),
Then: a sensor with thermal infrared detector operating in the **8 - 14** μm region might be most appropriate.



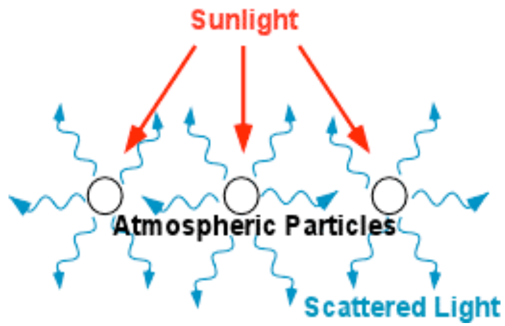
◊ If we are looking for 800 K forest fires**:**
Then: a dominant wavelength is approximately **3.62** μm
Then: a thermal infrared detector operating in the **3-5** μm region might be most appropriate

Electromagnetic wave processes:

Atmosphere	Surface / sub-surface
Scattering (molecular, particle)	Surface reflection/scattering
Transmission	scattering
Absorption	transmission
Emission	absorption
	emission

Electromagnetic Interaction with the Atmosphere

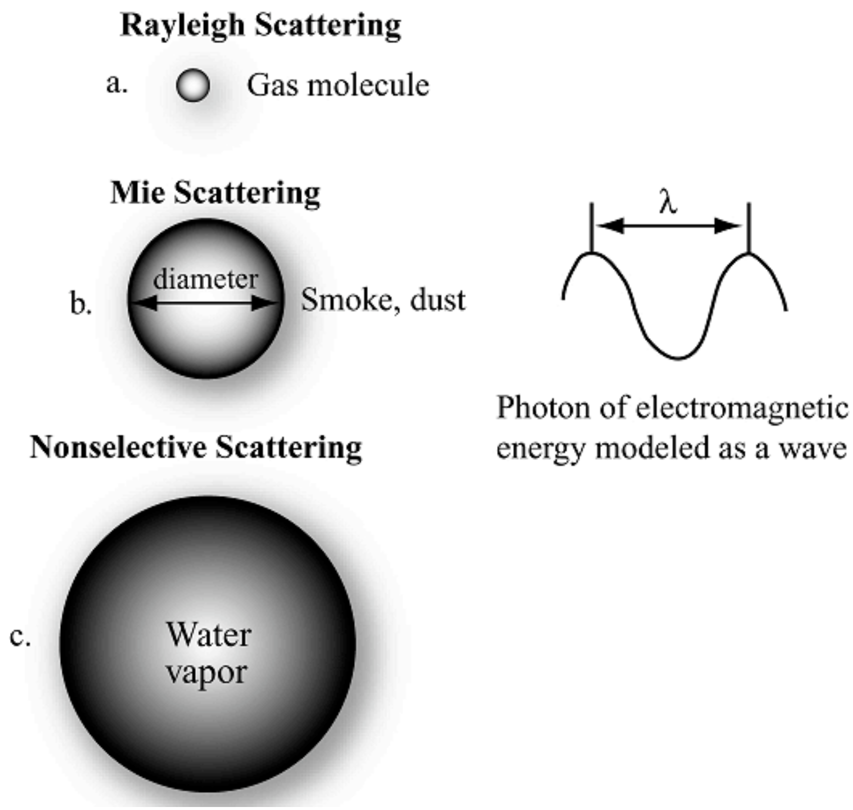
1. Scattering



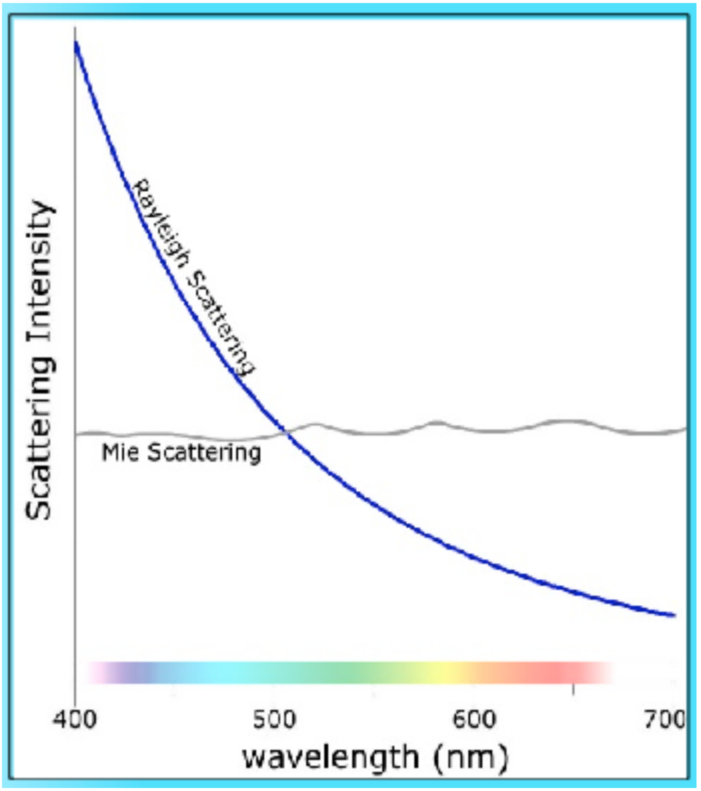
◊ Rayleigh: small particles against short wavelength (gas molecules: O_2, N_2, etc.)

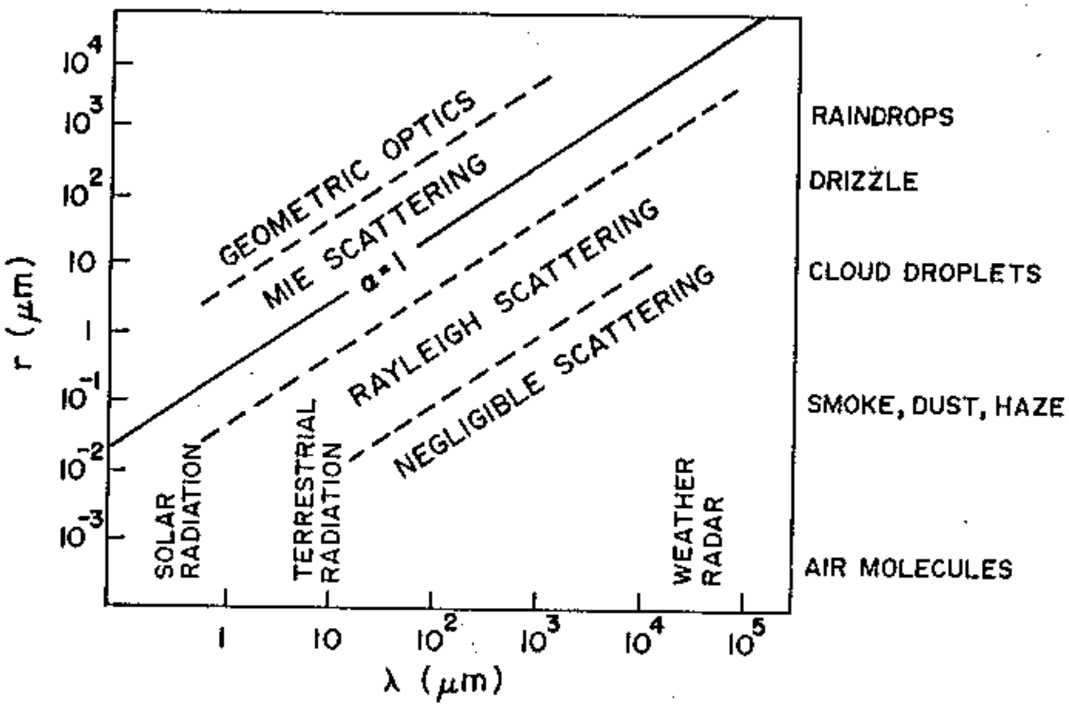
◊ Mei: medium-size particles against short and long wave (dust, pollen, smoke, ..)

◊ non-selective: large particles against all wavelengths (mainly water vapour)



Source: apollo.lsc.usc.edu/classes/met130


Scattering Intensity
Rayleigh Scattering
Mie Scattering
400
500
600
700
wavelength (nm)


10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
r (μm)
GEOMETRIC OPTICS
MIE SCATTERING
α=1
RAYLEIGH SCATTERING
NEGLIGIBLE SCATTERING
SOLAR RADIATION
TERRESTRIAL RADIATION
WEATHER RADAR
1
10
10^2
10^3
10^4
10^5
λ (μm)
RAINDROPS
DRIZZLE
CLOUD DROPLETS
SMOKE, DUST, HAZE
AIR MOLECULES

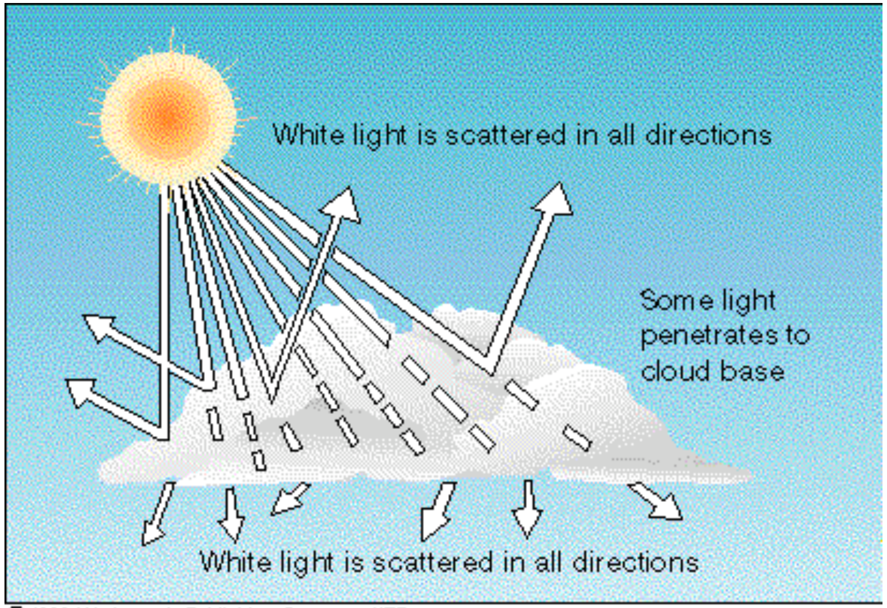
Nonselective Scattering

- produced when there are particles in the atmosphere several times the diameter of the radiation being transmitted

- Mainly due to water droplets found in clouds

- Not as dependent on wavelength; hence clouds appear white or gray

- Scattering can severely reduce the information contents of remotely sensed data.



Source: from apollo.lsc.vsc.edu/classes/met130

2. Absorption

◊ ozone: UV (outside typical spectrum of remote sensing)
◊ carbon dioxide: far IR (15μm)
◊ water vapour: 1 μm to 10 mm (long wave IR and short MW)

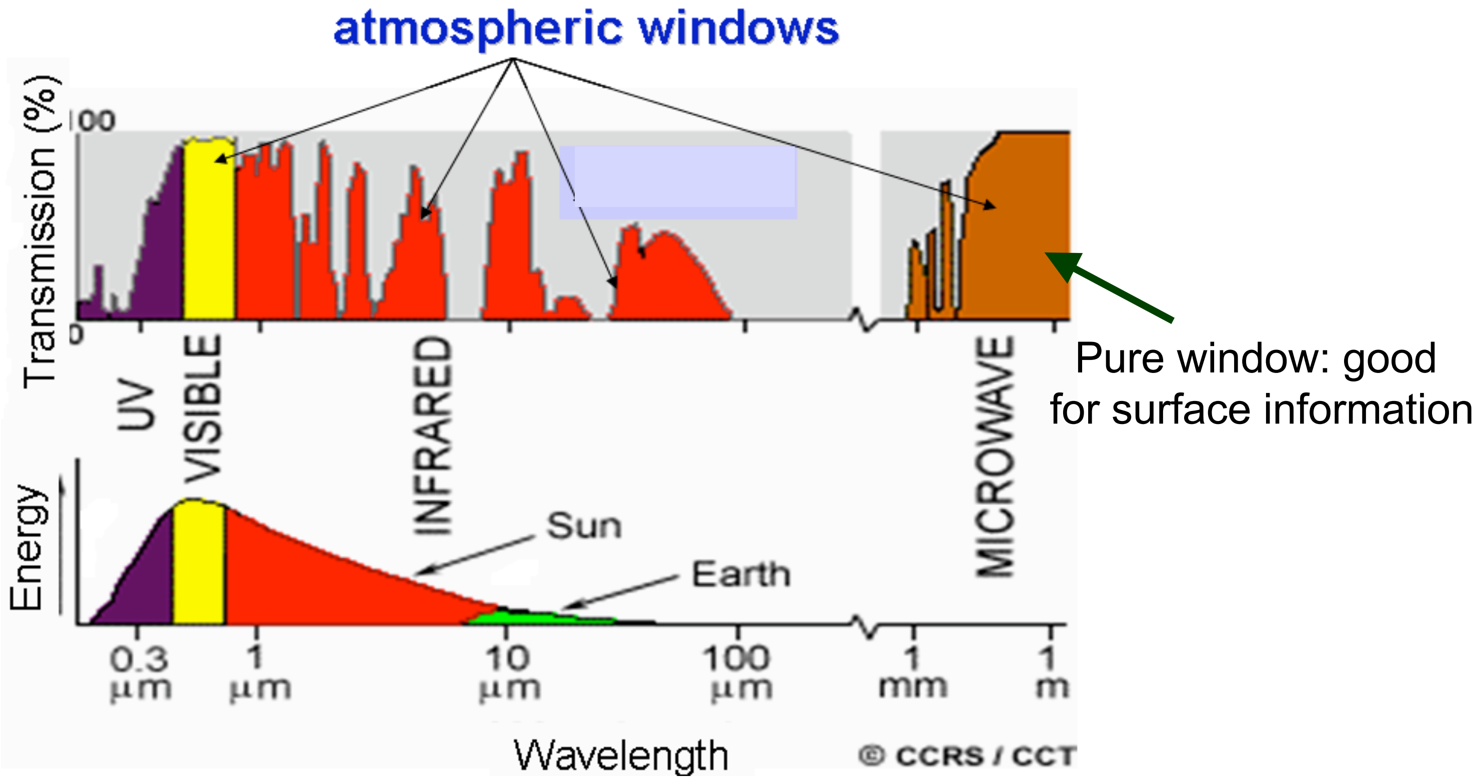
◊The cumulative effect of the absorption by the various constituents can cause the atmosphere to close down in certain regions of the spectrum. No energy is available to be sensed.

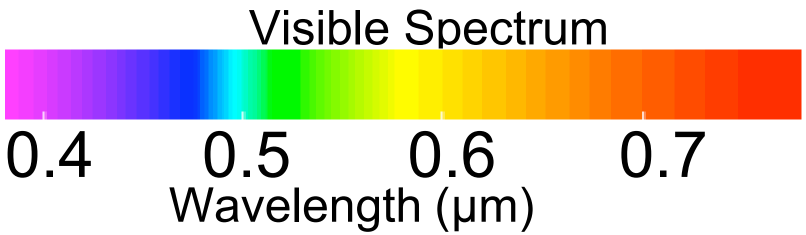
◊ Absorption and scattering are frequently combined into an extinction coefficient

3. Transmission

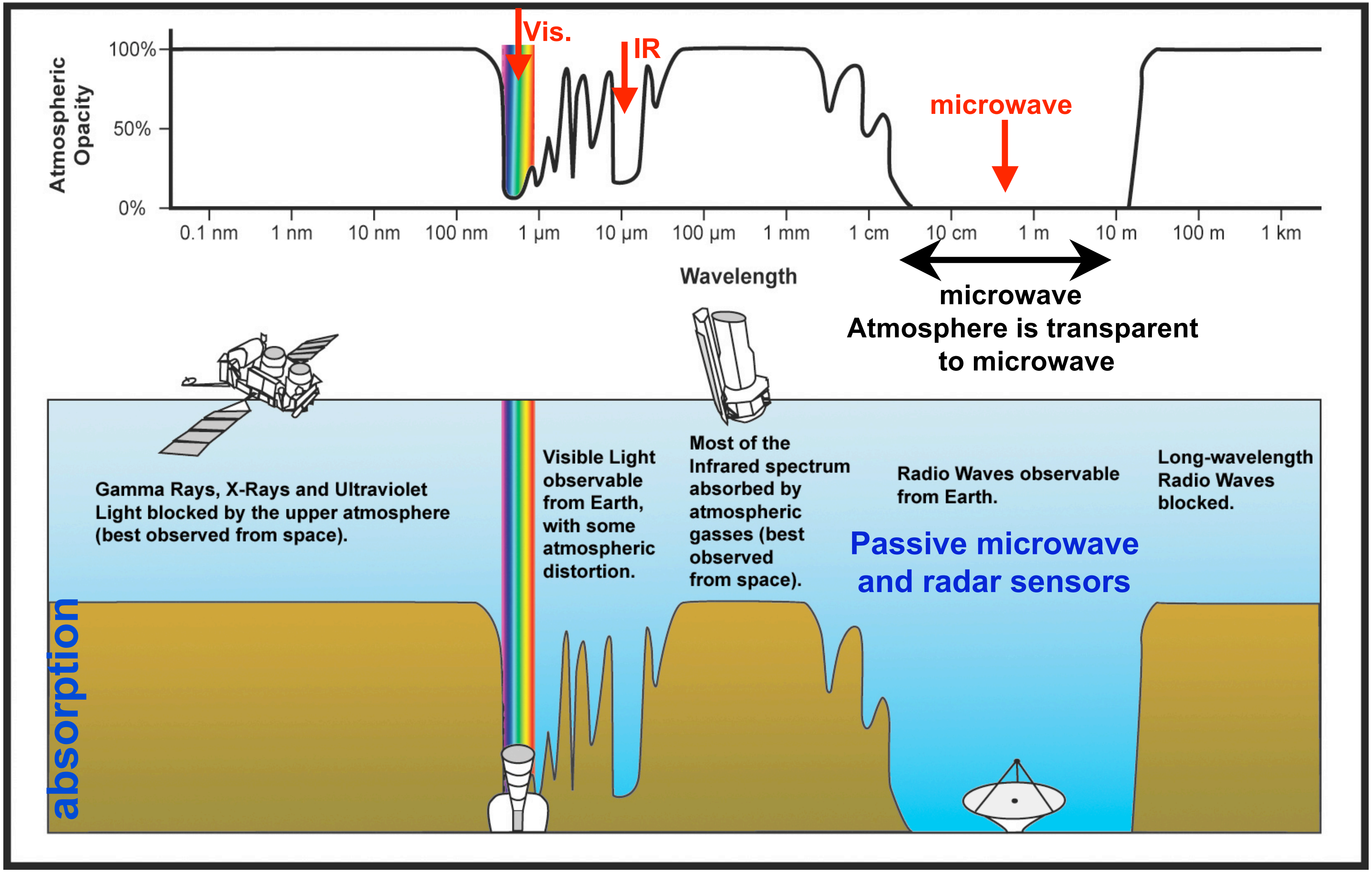
◊ The atmosphere transmits visible and some parts of the spectrum effectively.

◊ Parts of the spectrum that transmit energy effectively are called “atmospheric windows” (e.g. the visible region: 0.4 - 0.7 μm)

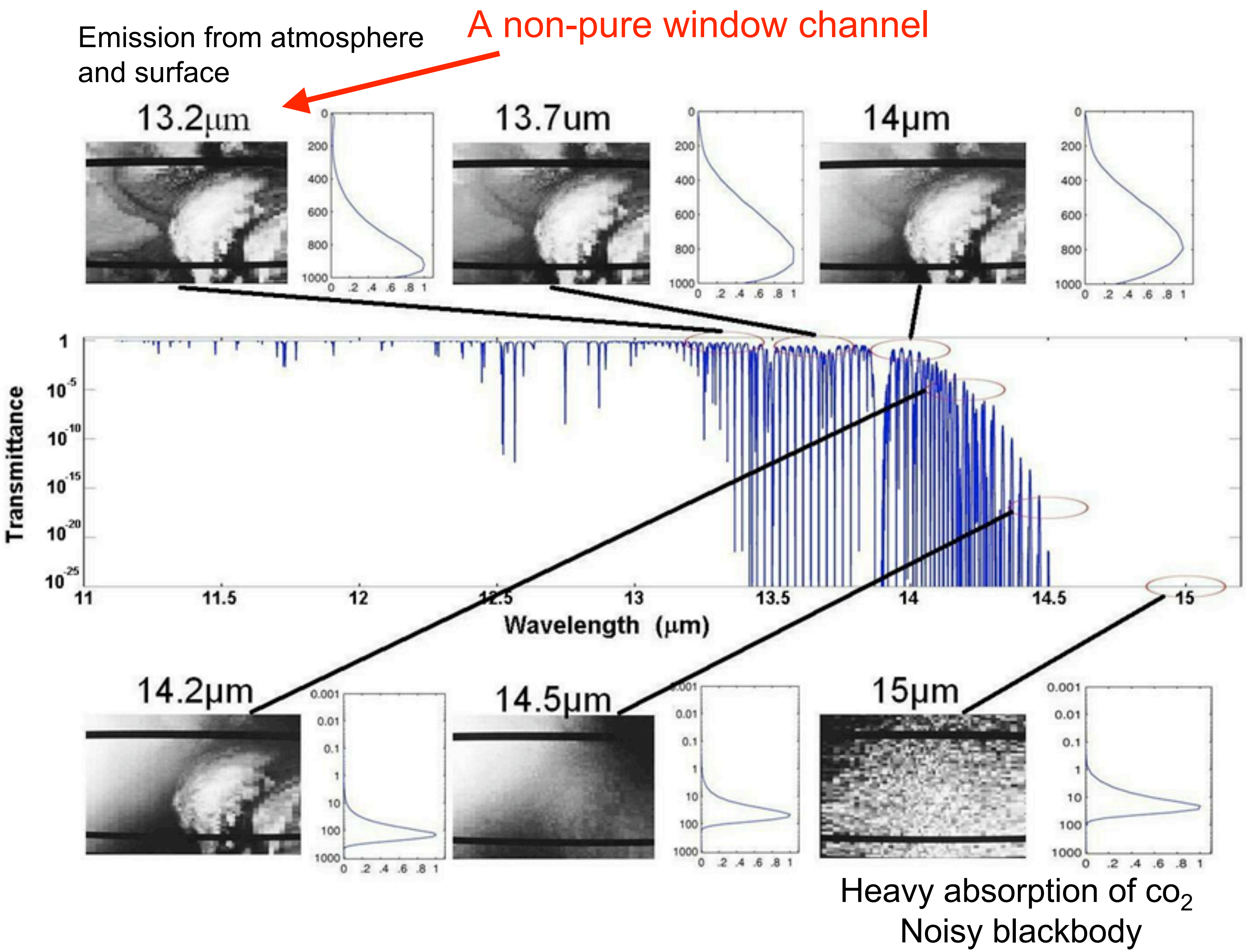




- Near-infrared region (NIR): 0.7-1.3 µm
- Middle-infrared (MIR): 1.3 - 3 µm
- Short-wave IR: 1.3 - 8 µm
- Thermal infrared region: 8 - 14 µm
- Microwave region (MW): >1 mm

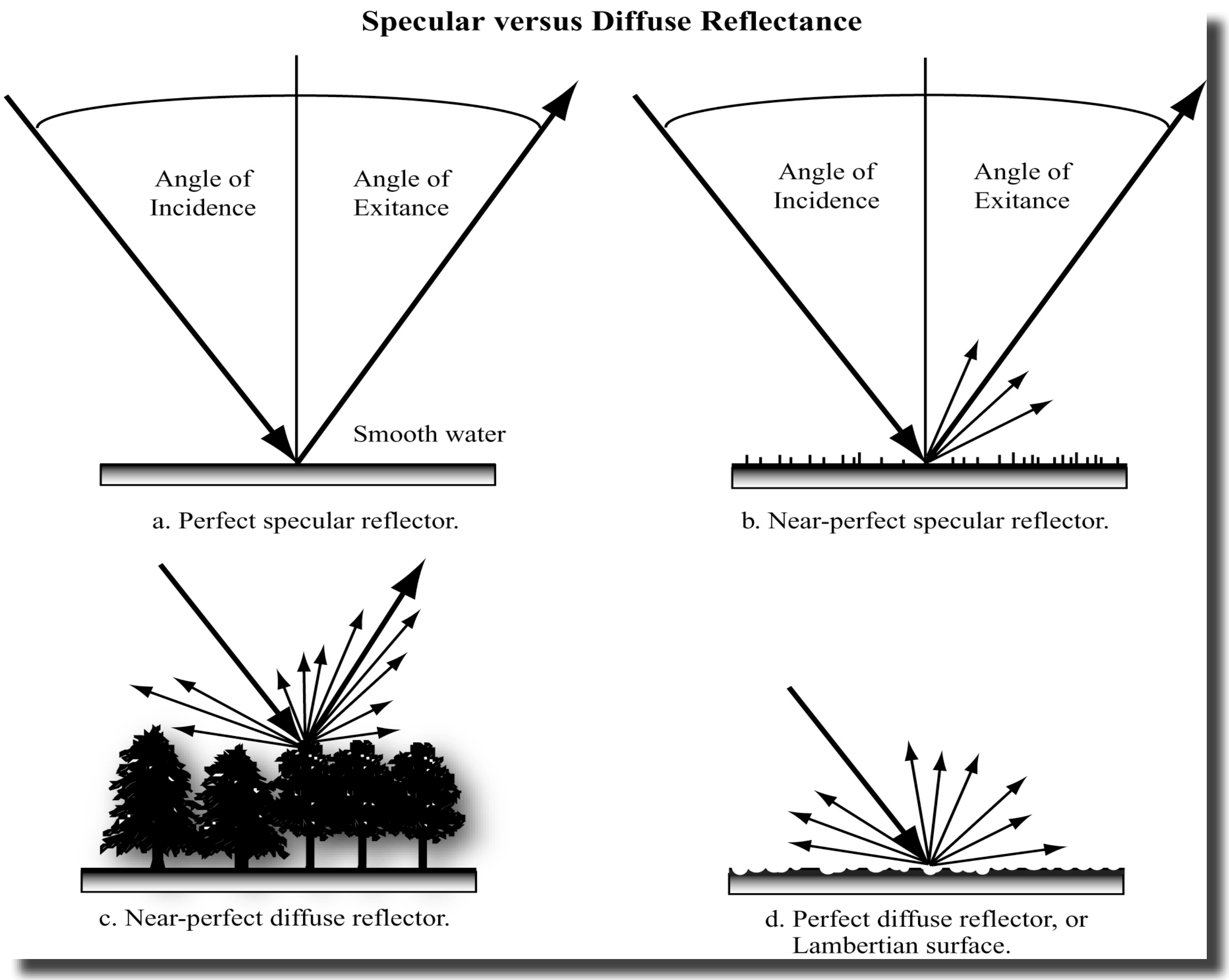


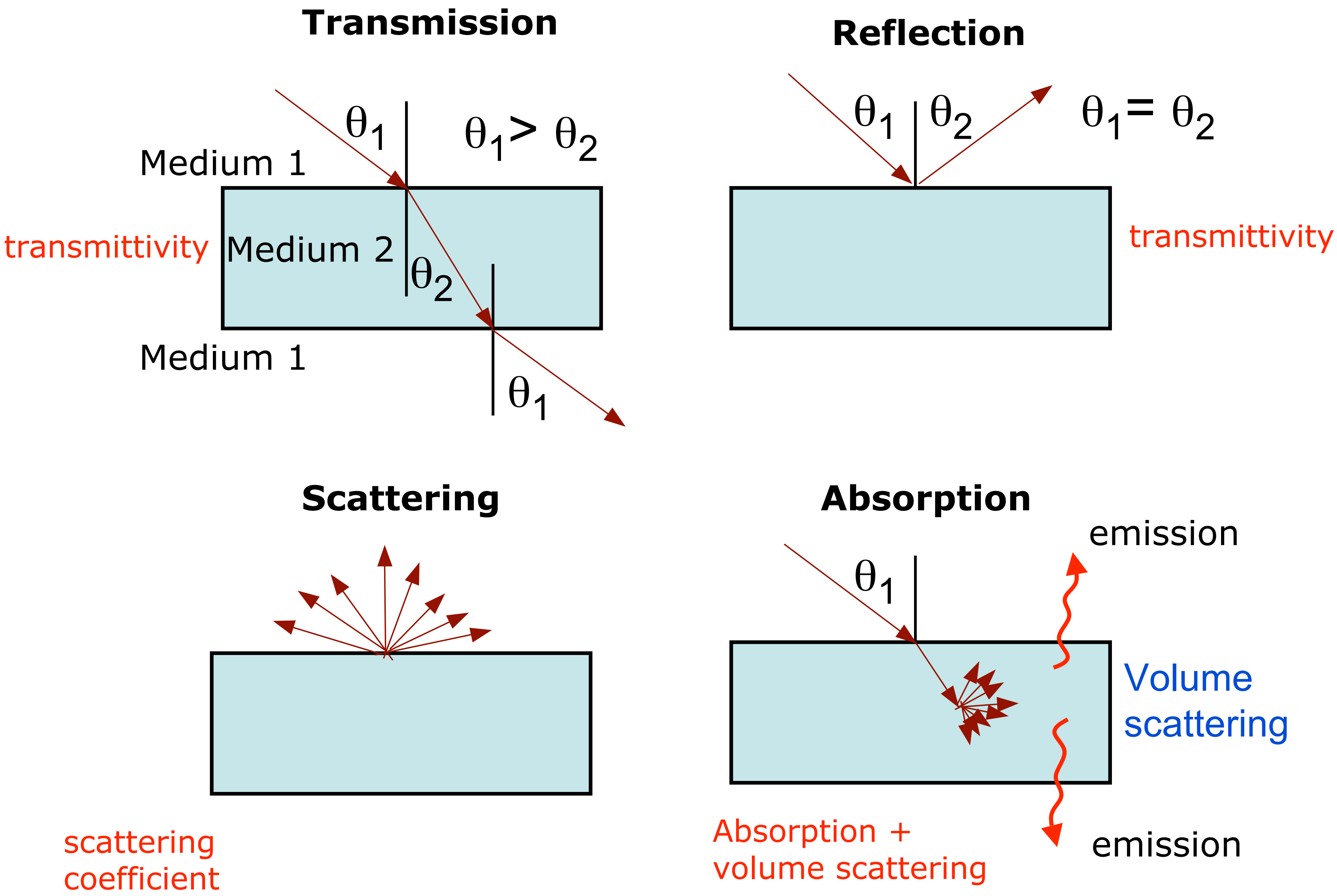
Satellite sensors are designed to measure radiation in wavelengths that can be reflected or emitted back up through the atmosphere to space



Emission from atmosphere
and surface
A non-pure window channel
13.2μm
13.7um
14μm
Transmittance
Wavelength (μm)
14.2μm
14.5μm
15μm
Heavy absorption of co_2
Noisy blackbody

Surface and sub-surface processes




Transmission
θ1
θ1 > θ2
Medium 1
transmittivity
Medium 2
θ2
Medium 1
θ1
Reflection
θ1
θ2
θ1 = θ2
transmittivity
Scattering
scattering
coefficient
Absorption
emission
θ1
Volume
scattering
Absorption +
volume scattering
emission

A major difference between microwave and optical remote sensing
Microwave penetrates the surface (hence holds sub-surface information)

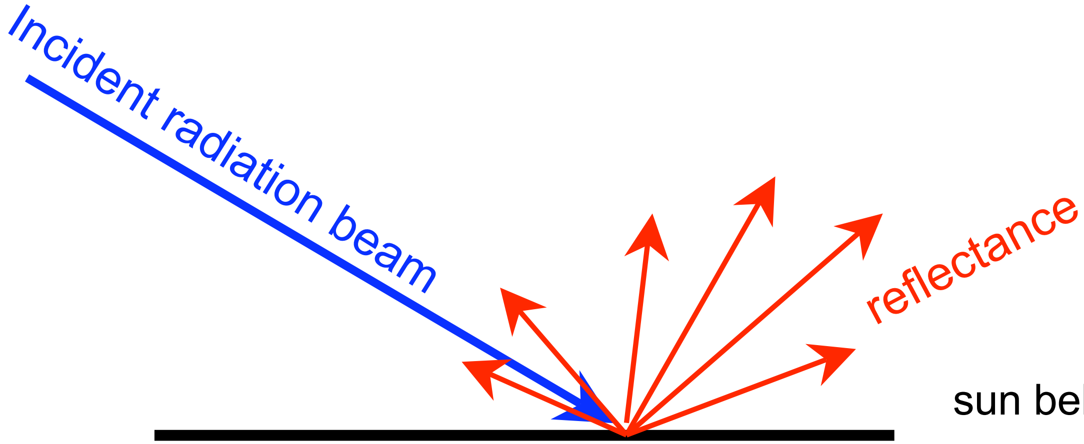
Aerial photograph of Amundsen-Scott research Station, Antarctic

Same area as seen by Radarsat
The image reveals an abandoned cluster of buildings that are now buries under ice and snow

The Bidirectional Reflectance Distribution Function (BRDF)

Objects look differently when illuminated or viewed from different angles

◊ Reflectance of a target as a function of illumination & viewing geometry.
◊ Depends on wavelength & structural and optical properties of the surface.



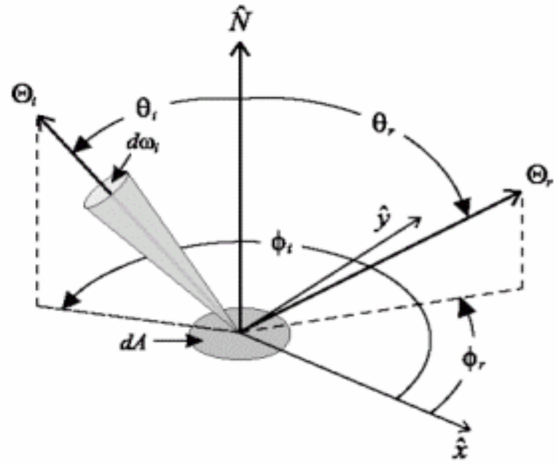
But what if the surface is not flat?
That is one reason that makes retrieval over land difficult

sun behind observer sun opposite to observer

Source: http://www-modis.bu.edu/brdf/brdfexpl.html

The BRDF function:

$$\rho(\theta_i,\phi_i,\theta_r,\phi_r,\lambda)=\frac{dL_r(\theta_i,\phi_i,\theta_r,\phi_r,\lambda)}{dE_i(\theta_i,\phi_i,\lambda)}$$



L_r ◊ reflected radiation in direction (θ_r,ϕ_r)

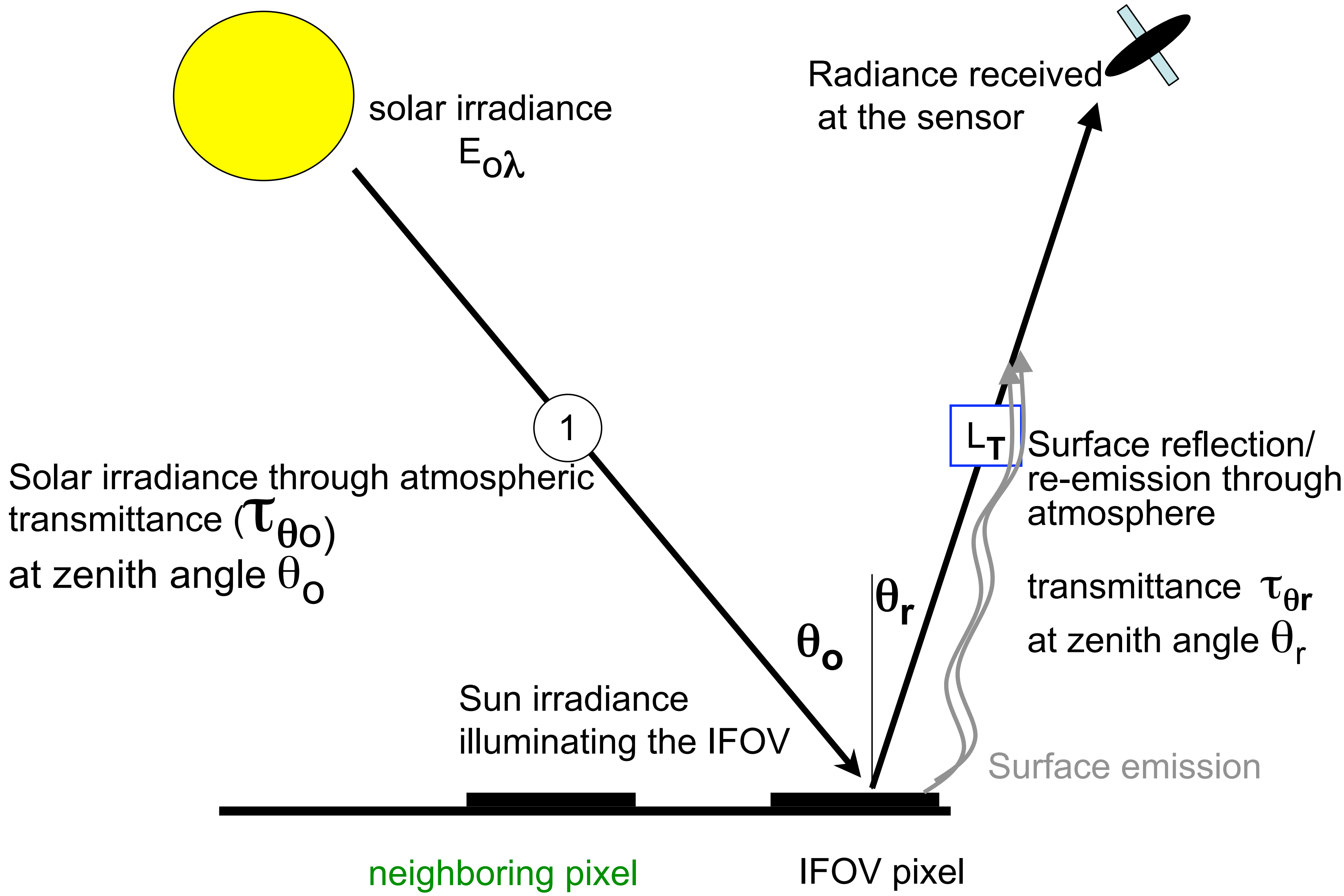
E_i ◊ incident radiation in direction (θ_i,ϕ_i)

The BRDF is needed in remote sensing for:

◊ Correction of view and illumination angle effects (for image mosaic);
◊ Deriving albedo,
◊ Land cover classification,
◊ Cloud & aerosol detection,
◊ Atmospheric correction
◊ Radiometric boundary condition for any radiative transfer problem
hence of relevance for climate modeling and energy budget investigations.

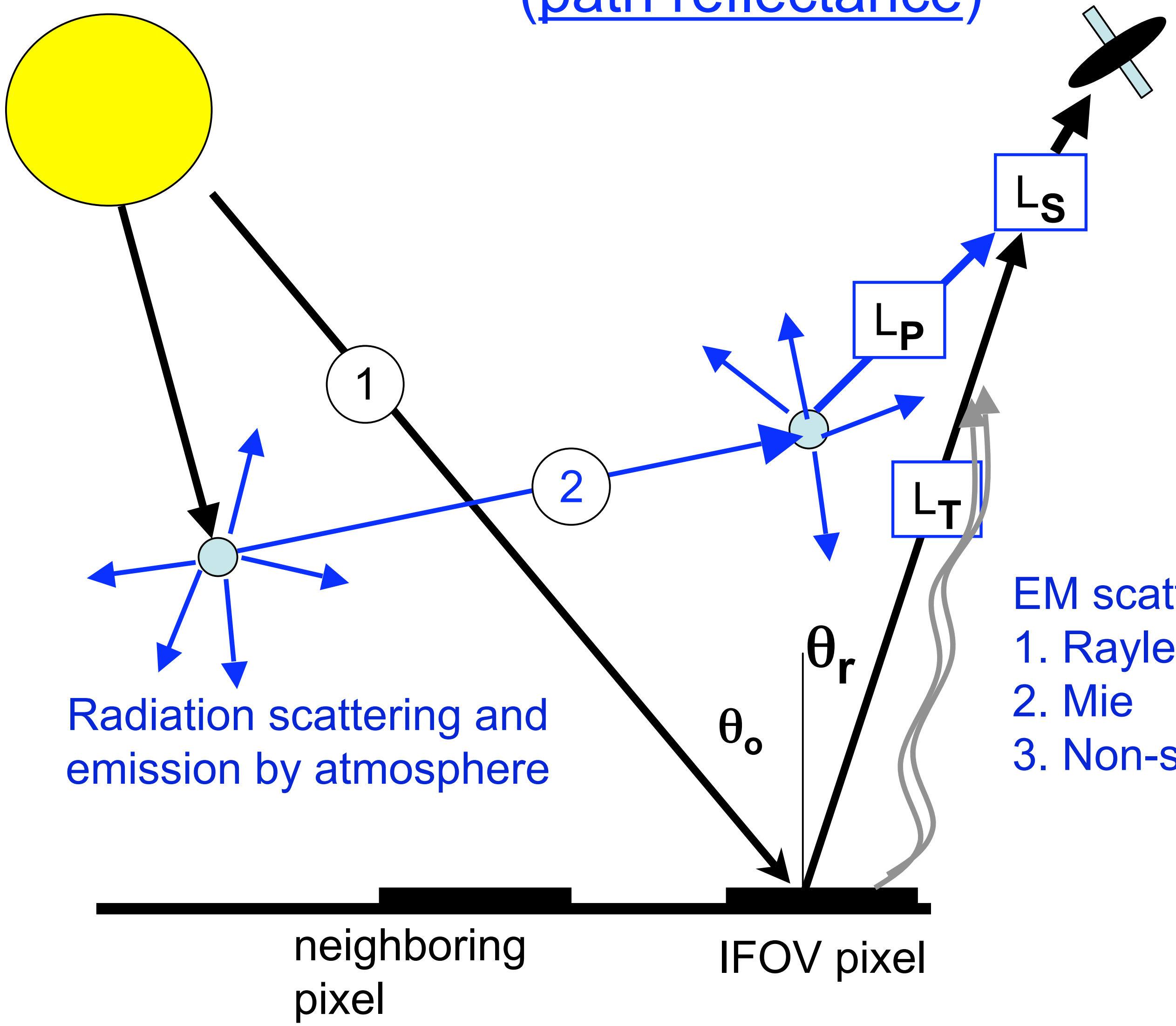
What Does a Satellite Sensor Measure?

1. solar reflectance from the IFOV (<u>surface function</u>)



What Does a Satellite Sensor Actually Measure?

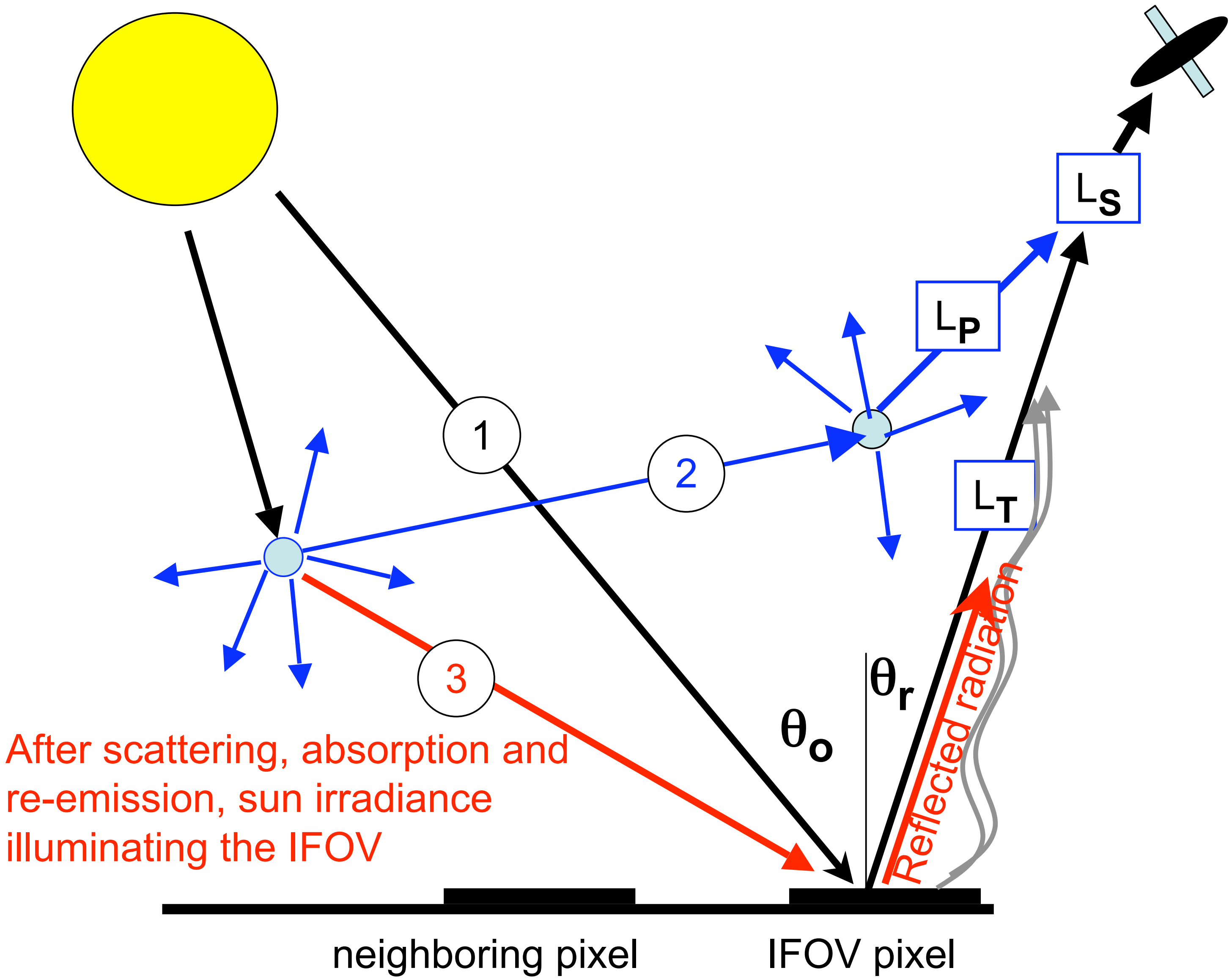
2. Scattering of radiation within the atmosphere (path reflectance)



EM scattering mechanisms:
1. Rayleigh
2. Mie
3. Non-selective

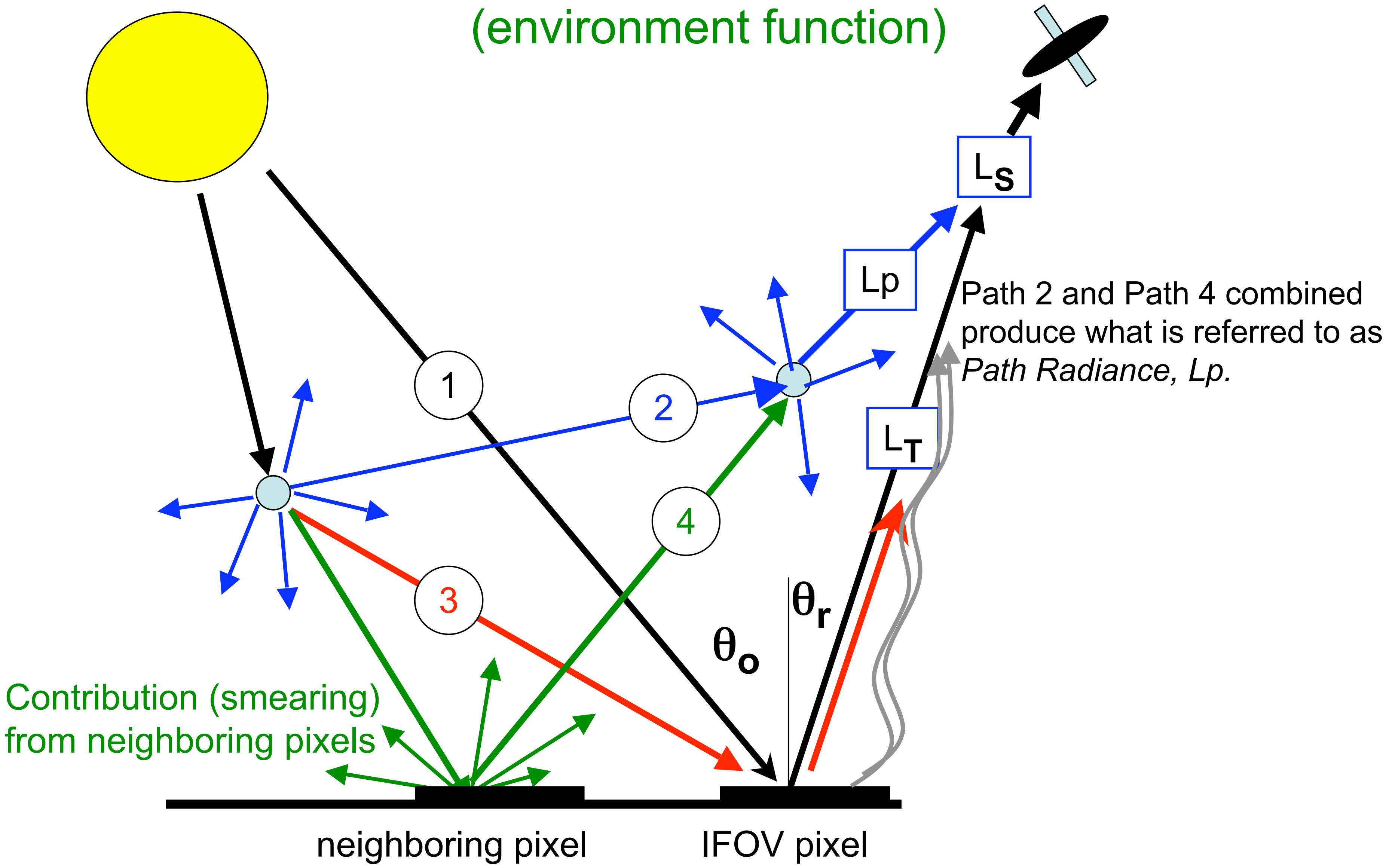
What Does a Satellite Sensor Actually Measure?

3. Downward reflectance of the atmosphere

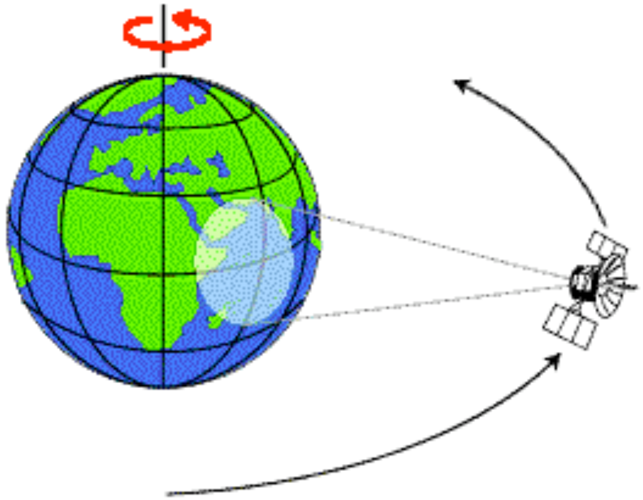


What Does a Satellite Sensor Actually Measure?

4. Radiation reflected or scattered by nearby terrain (environment function)



3. Satellite Platforms and Sensor Characteristics

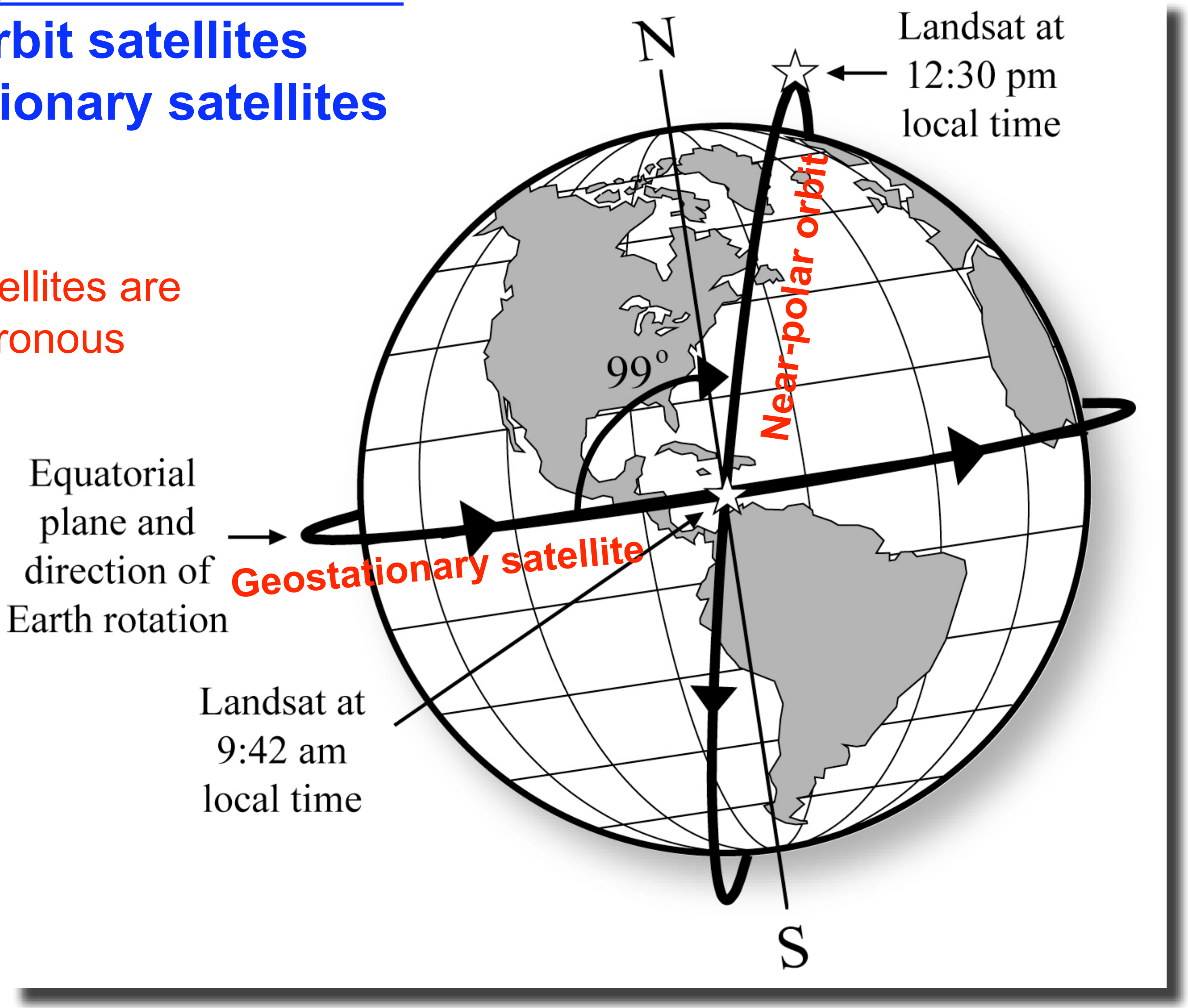


Two categories of satellites:

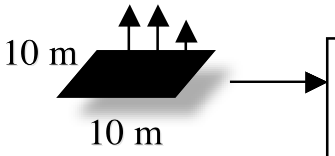
◊ **Polar Orbit satellites**

◊ **Geostationary satellites**

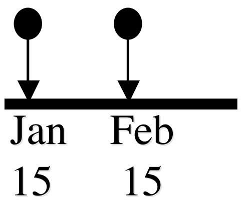
Polar-orbit satellites are Sun Synchronous



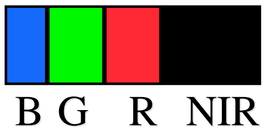
Remote Sensor Resolutions



- **Spatial** : the size of the field-of-view, e.g. 10 x 10 m.



- **Temporal** : how often the sensor acquires data, e.g. every 30 days

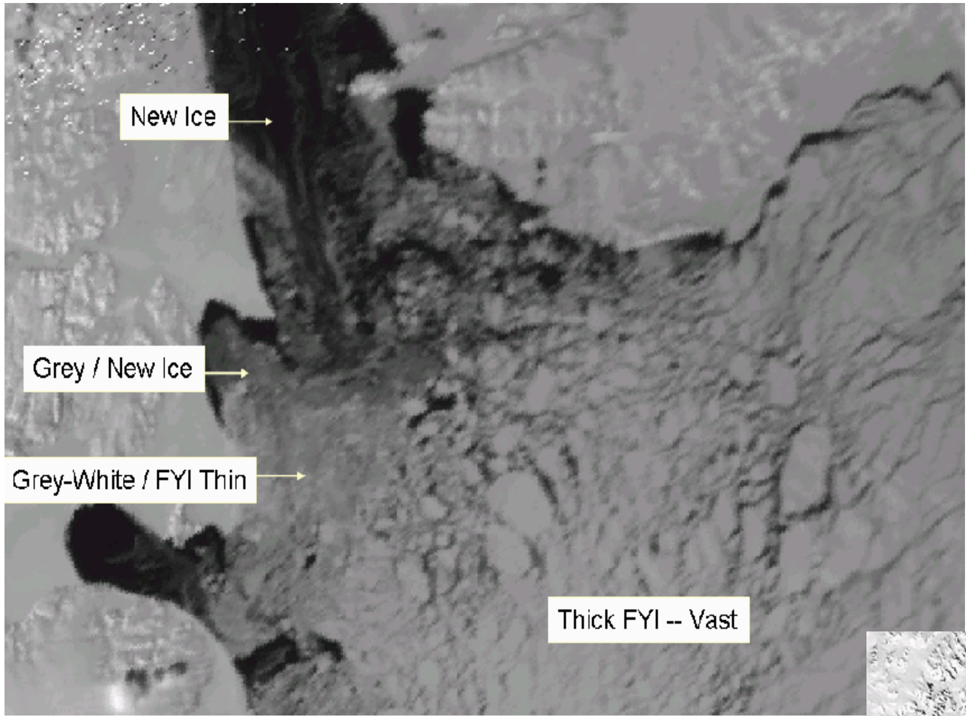


- **Spectral** : the number and size of spectral regions the sensor records data in, e.g. blue, green, red, near-infrared thermal infrared, microwave (radar).

- **Radiometric** : the sensitivity of detectors to small differences in electromagnetic energy.

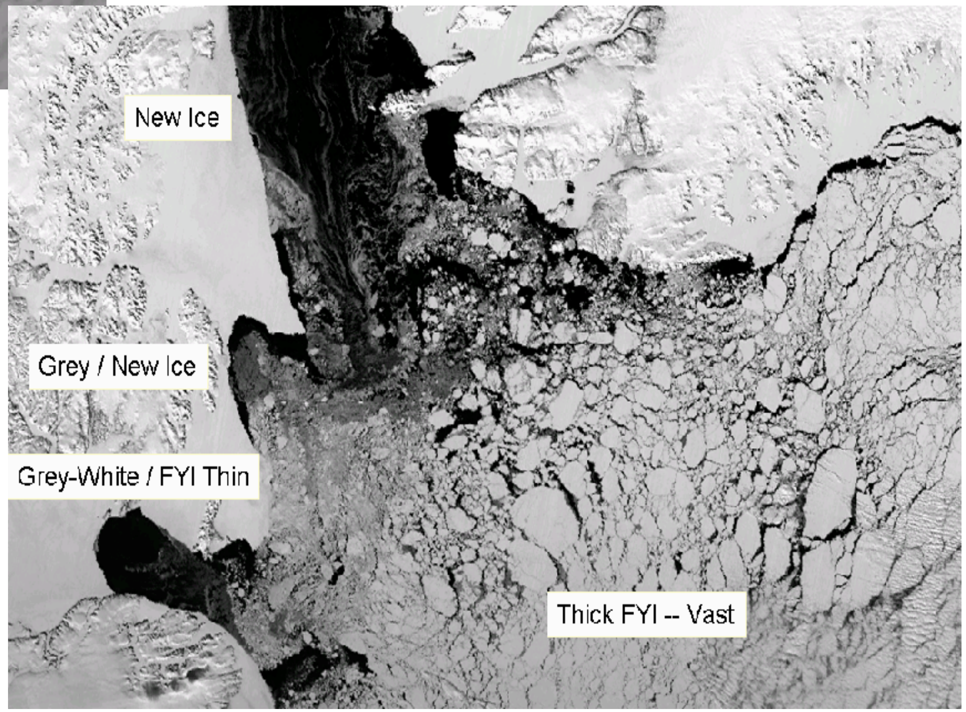
Spatial resolution

May 15, 2001
North Water Polynya



AVHRR (res. 1.1 – 6.5 km)

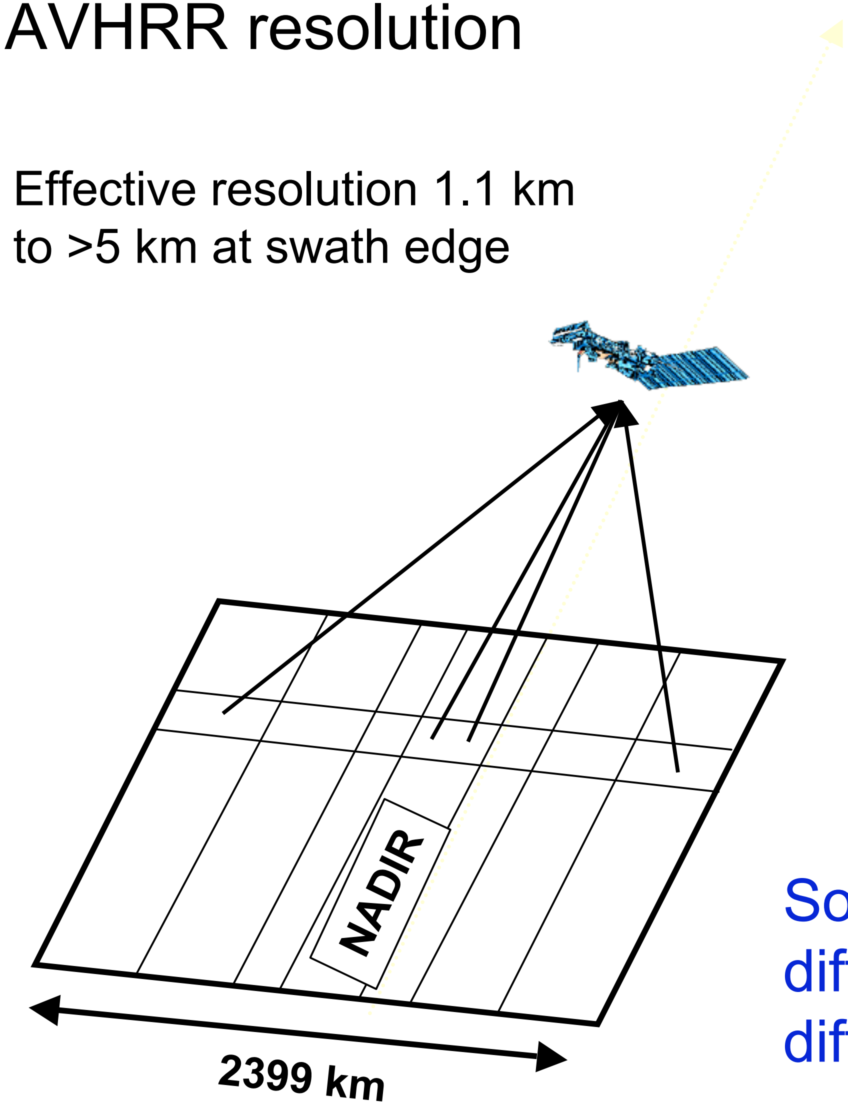
OLS (res. 500m)



Spatial resolution varies across the swath

AVHRR resolution

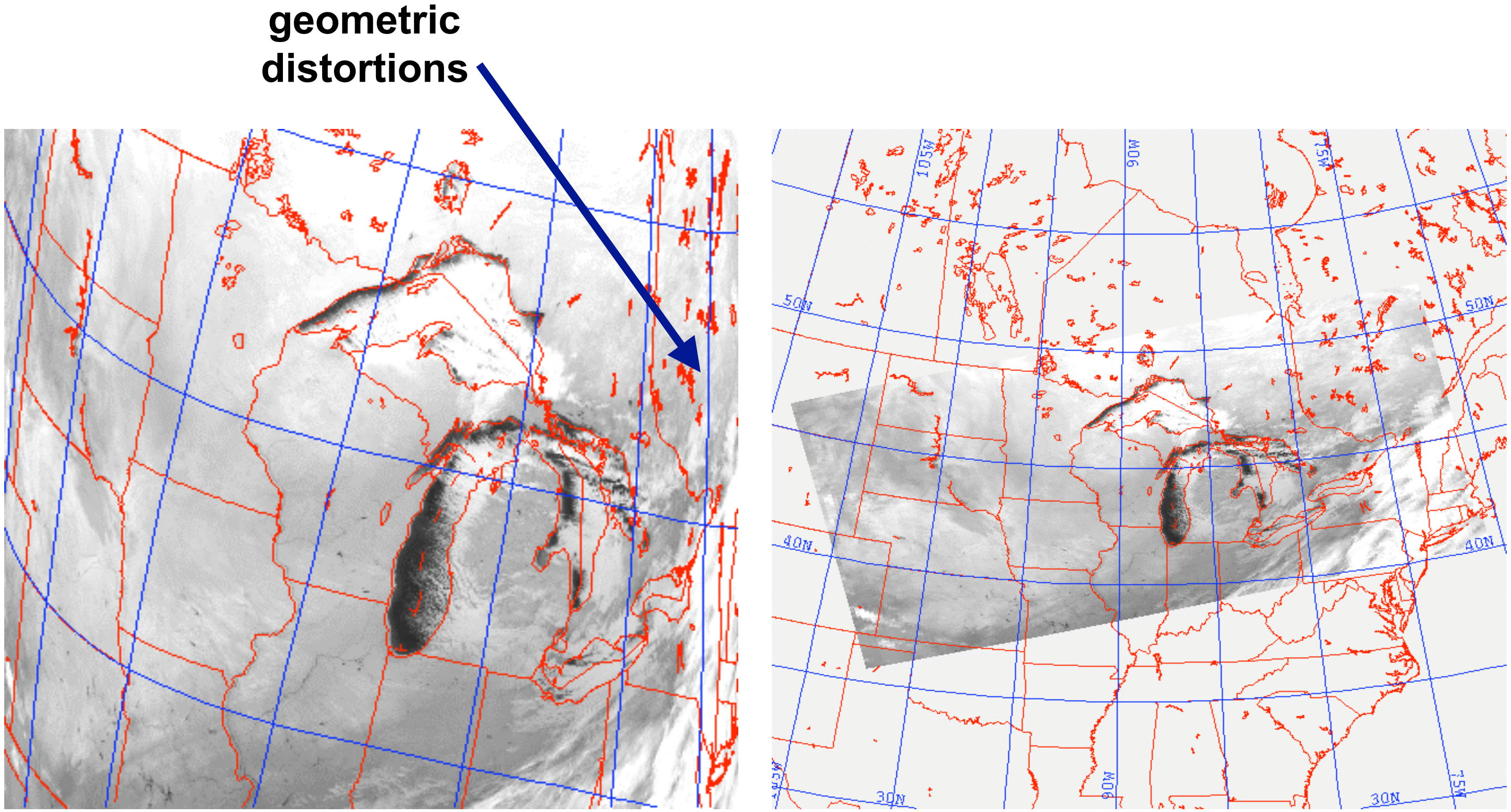
- Effective resolution 1.1 km to >5 km at swath edge



Channel	Spectral Range (μ m)
1	0.58-0.68
2	0.73-1.00
3A	1.58-1.64
3B	3.55-3.93
4	10.3-11.3
5	11.5-12.5

Some multi-channel sensors have different spatial resolution for different channels

Spatial resolution varies across the swath causing geometrical distorsion

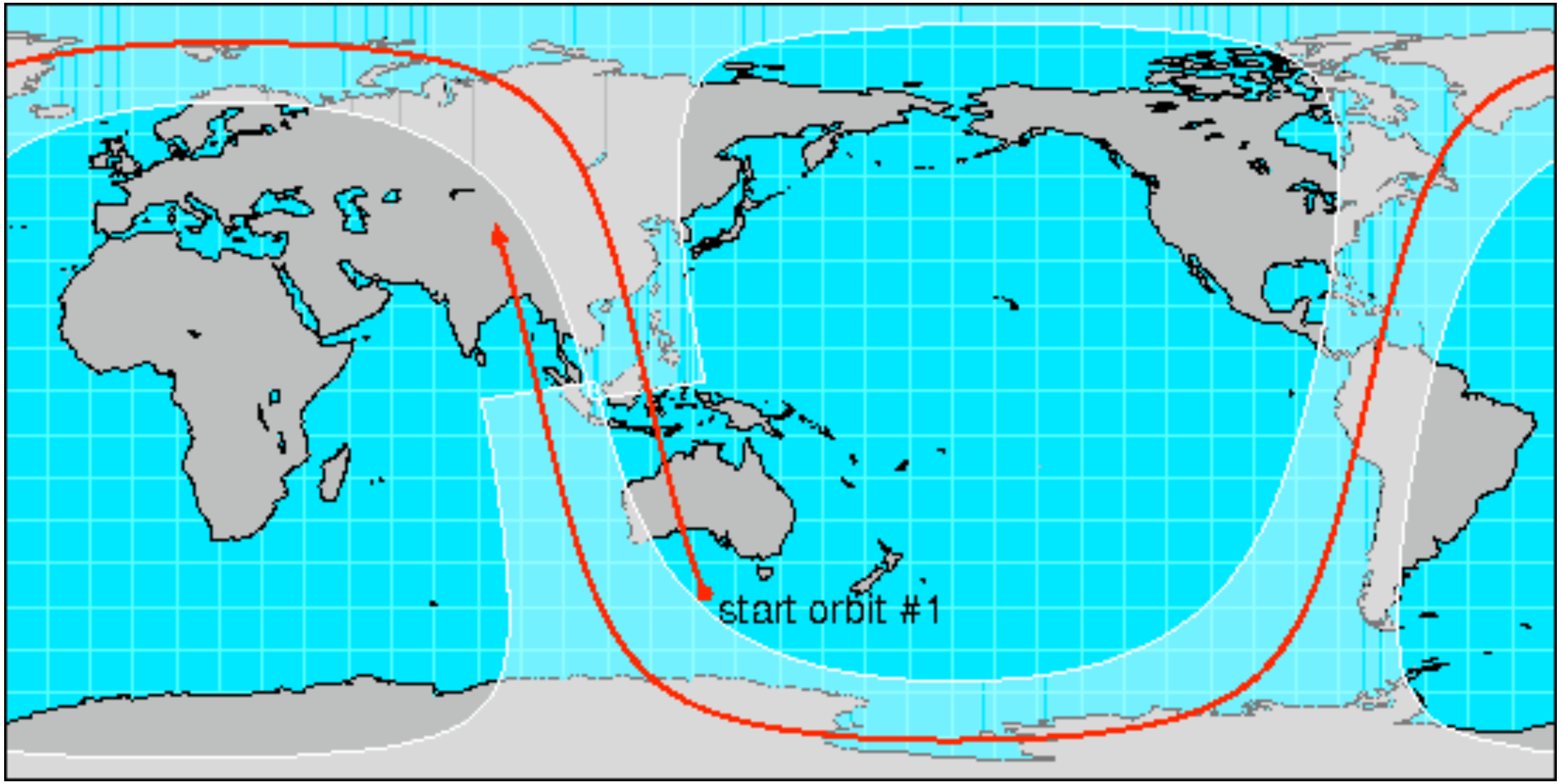


Original image (non-rectified) Rectified image

As a result, the spatial resolution varies from 1.1 km directly beneath the satellite, to 5.5 km at the edge of the swath. This causes geometric distortions to occur.

Temporal resolution

determined by the repeat cycle, swath, and location on earth's surface

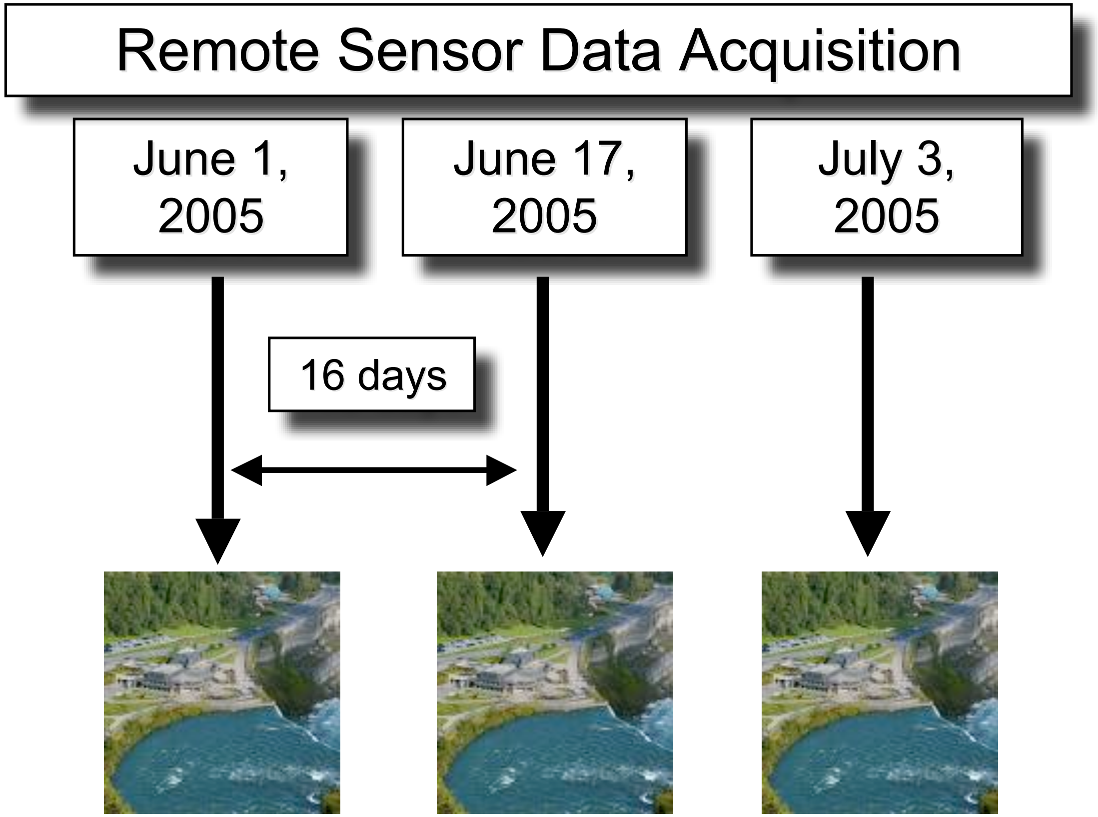


Map of the ground path of one revolution of a typical near-polar orbit

larger spatial coverage in polar regions

Consequently better temporal resolution

Temporal resolution

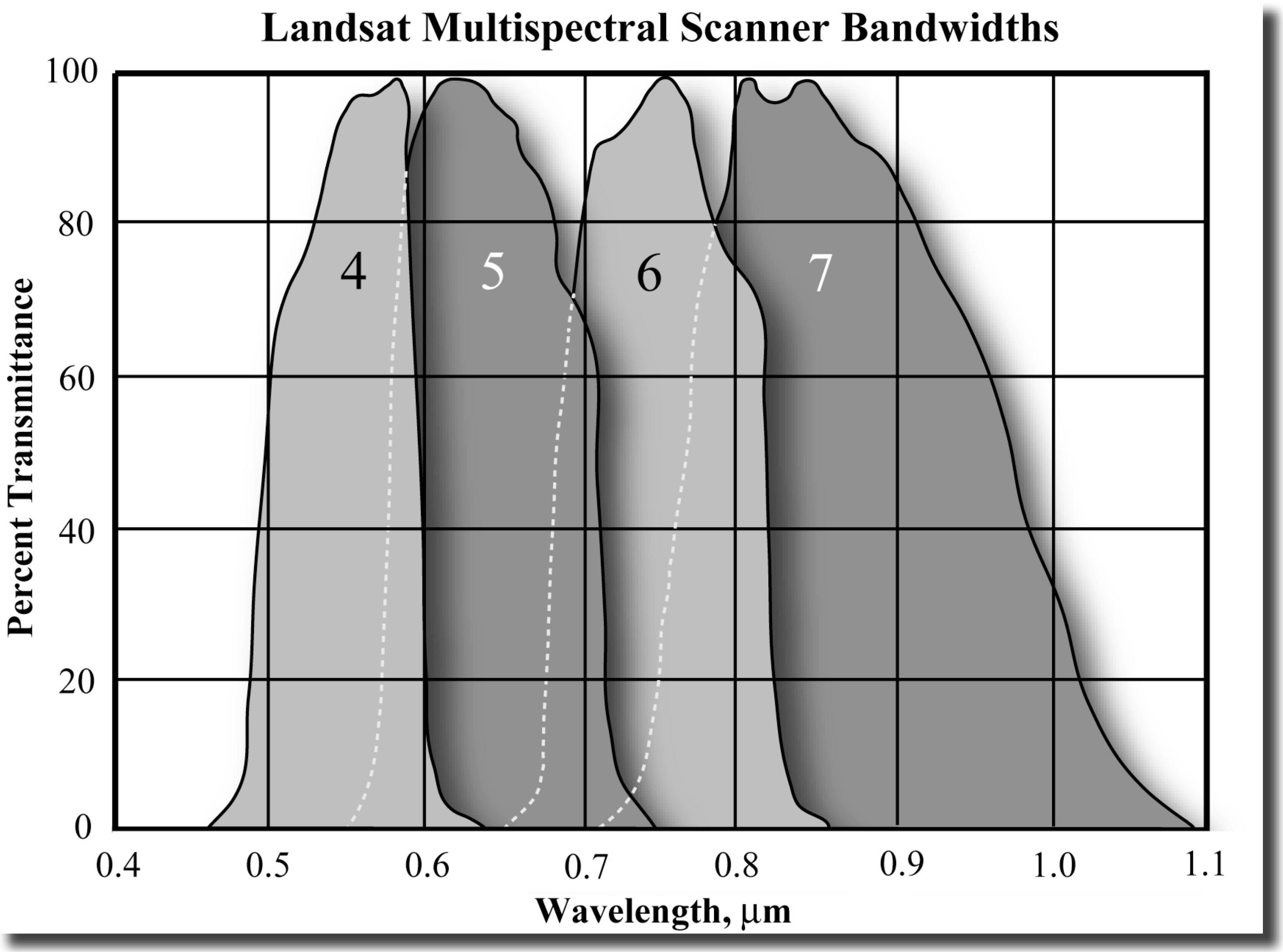


Satellite remote sensing:

◊ Good in monitoring slow varying phenomenon (crop growth, sand dune motion, coastal erosion, etc.)

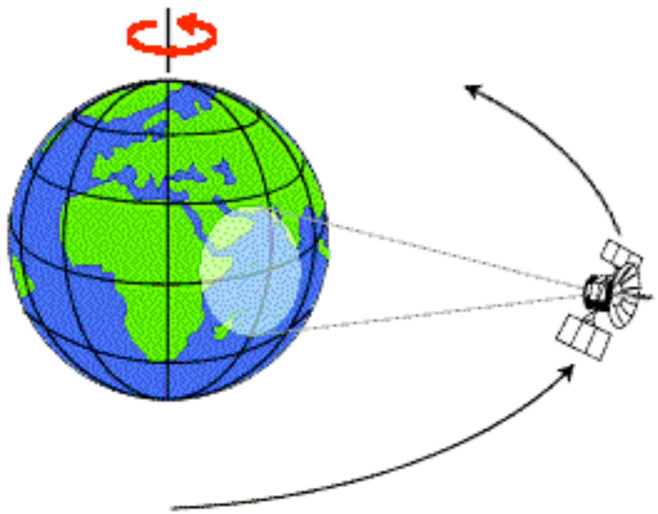
◊ Not good in monitoring highly dynamic phenomenon such as aerosol transportation (unless with very coarse resolution),

Spectral resolution



A radiometer measures the radiation integrated over a band, not from a line spectrum

4. Climate parameters retrieved from remote sensing.



Climate Parameters Retrieved from Remote Sensing Observations:

1. <u>Land surface parameters:</u>

• Terrain height	4
• Vegetation cover (NDVI-LAI) / height	5
• Soil texture/ moisture	3
• Land use (urban, desert, vegetation, etc.)	5
• Surface temperature	5
• Soil temperature and moisture profile	2
• Surface albedo (under cloud-free sky)	4
• Surface/ ground runoff	3
• Surface IR emission	3

Climate Parameters Retrieved from Remote Sensing Observations:

2. Ocean surface parameters:

• Sea Surface Temperature (SST)	5
• Land sea mask	5
• Ocean surface wind	5
• Floating sea ice motion	4

Climate Parameters Retrieved from Remote Sensing Observations:

3. Gas/ Aerosol emission:

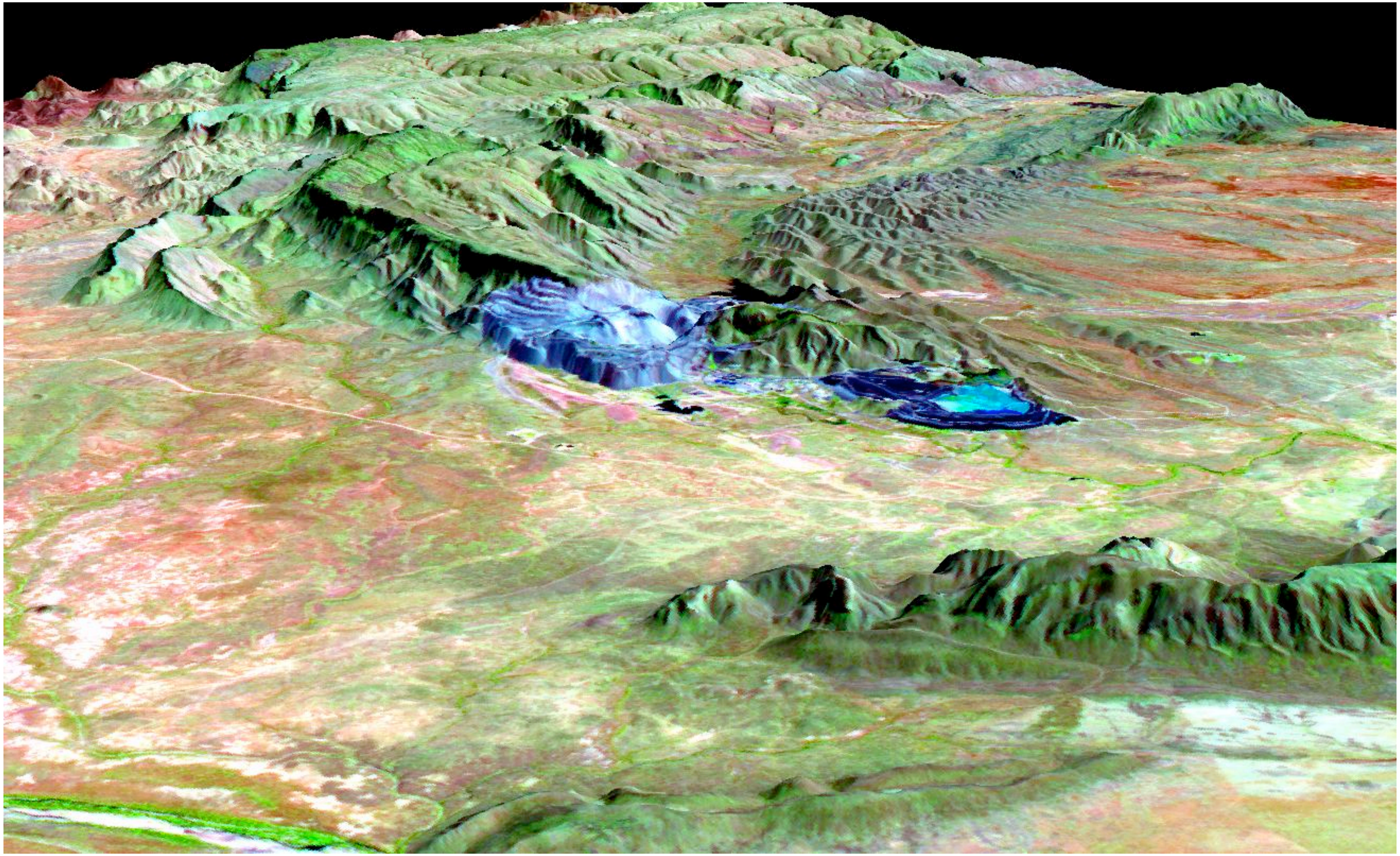
• Ozone (stratospheric/ tropospheric)	?
• Methane	?
• CFCs	?
• Carbons	?
• aerosol (size, optical depth)	3
• water vapour (column, profile)	5
• nitrogen dioxide	?

Climate Parameters Retrieved from Remote Sensing Observations:

4. Atmospheric Parameter:

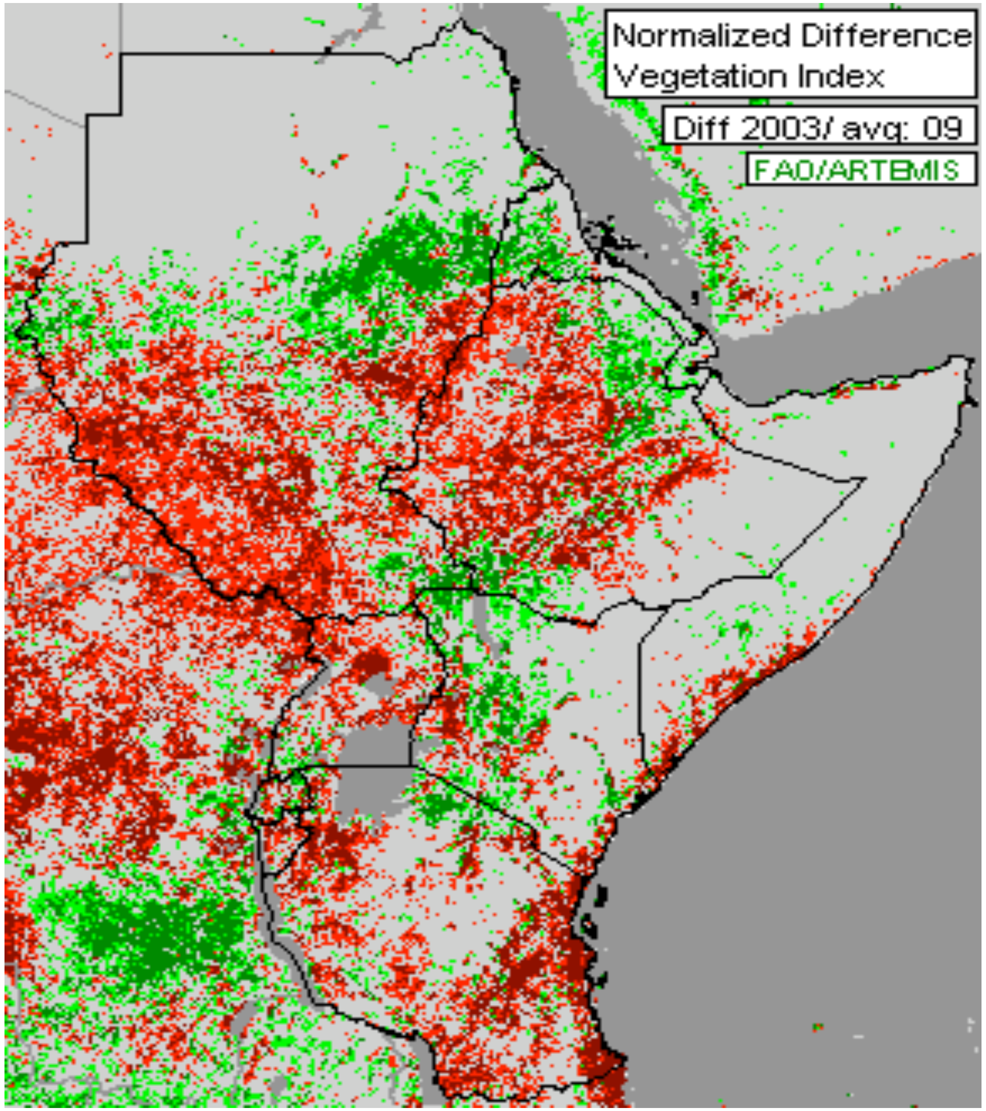
• Cloud height / cloud top temperature	4
• Evapotranspiration	3
• Temperature profile	5
• Relative humidity	5
• Wind speed	4
• Geopotential height	4
• Precipitation rate	4

Terrain height



ASTER 3D view of the Argyle diamond mine, western Australia. It is the world's largest single producer of diamonds. Image was created by draping an ASTER 4-3-2 RGB image over an ASTER-derived DEM. Vertical exaggeration is 2x. The scene was acquired August 20, 2000.

Normalized Difference Vegetation Index in Eastern Africa: Last satellite image (Difference between 2003 and Average)



NDVI is a measure of how green the canopy is.

It is the difference between the emitted infrared and the reflected green visible light.

It is measured by any visible/ infrared sensor: AVHRR/ MODIS/ MERIS, …etc.

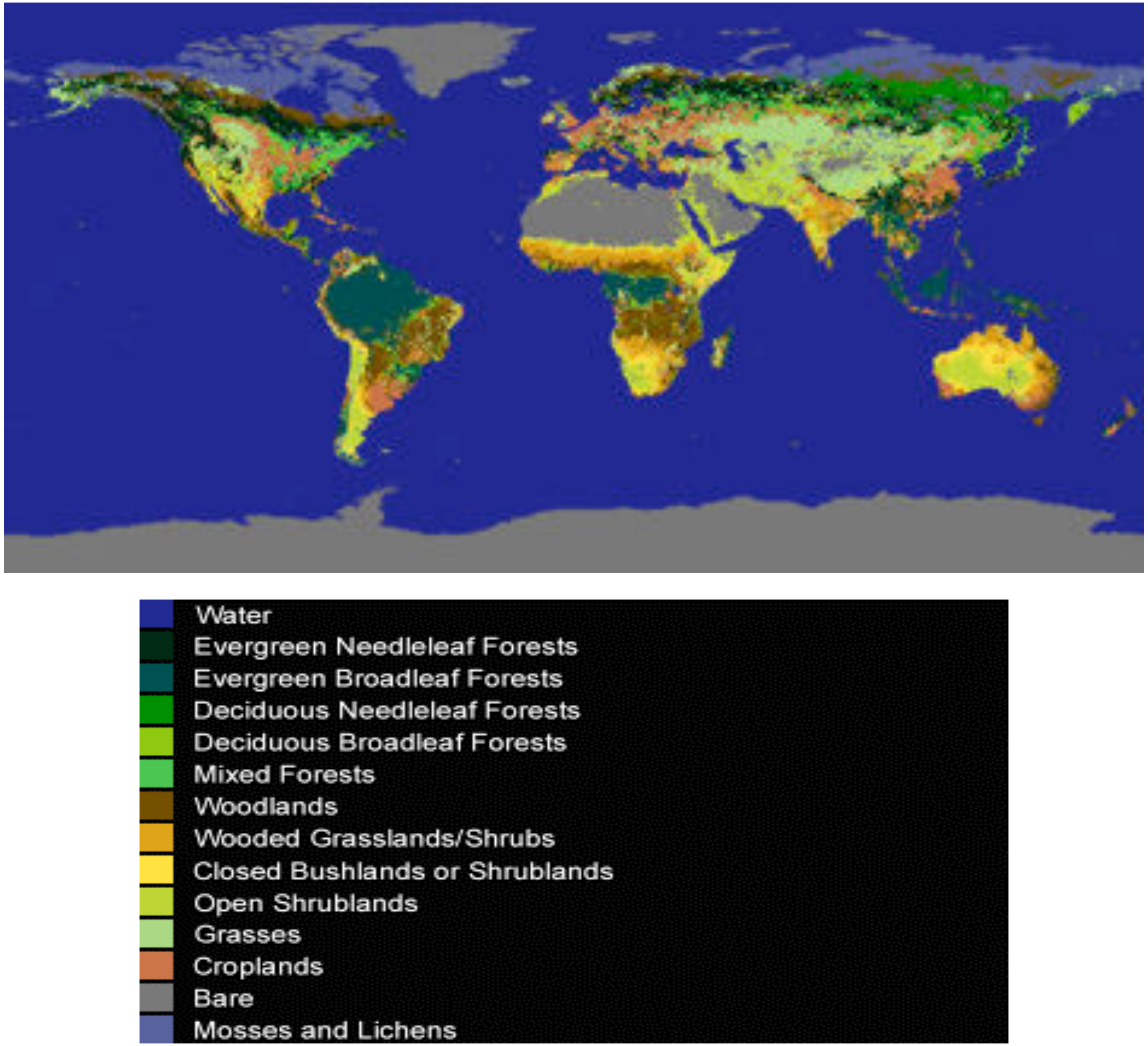
SeaWiFS image

The patterns visible to the south of the Greek islands are possible caused by variations in the specular reflection of sunlight from the water's surface.

Land masses perturb the wind field which in turn results in bands of differing roughness and wave orientation at the sea surface which in turn affects how much sun glint the sensor sees.

land use (urban, desert, vegetation, etc.)

Land Cover Classification from MODIS data



Land and sea surface temperature
The Split Window technique

Two neighbouring bands are selected:

◊ One in the strong water vapor absorption region (e.g. 10.5 mm from AVHRR),

◊ The other is outside the absorption region (e.g. 12.4 mm from AVHRR).

$$T_s = a_o + a_1 T_i + a_2 T_j$$

where

a_i *are coefficients that depend on emissivity and atmospheric transmittence*

T_i *and* T_j *are brightness temperatures at bands i and j*

There is a proof that this form combines LST and atmospheric correction into one process (see Liang, p. 369).

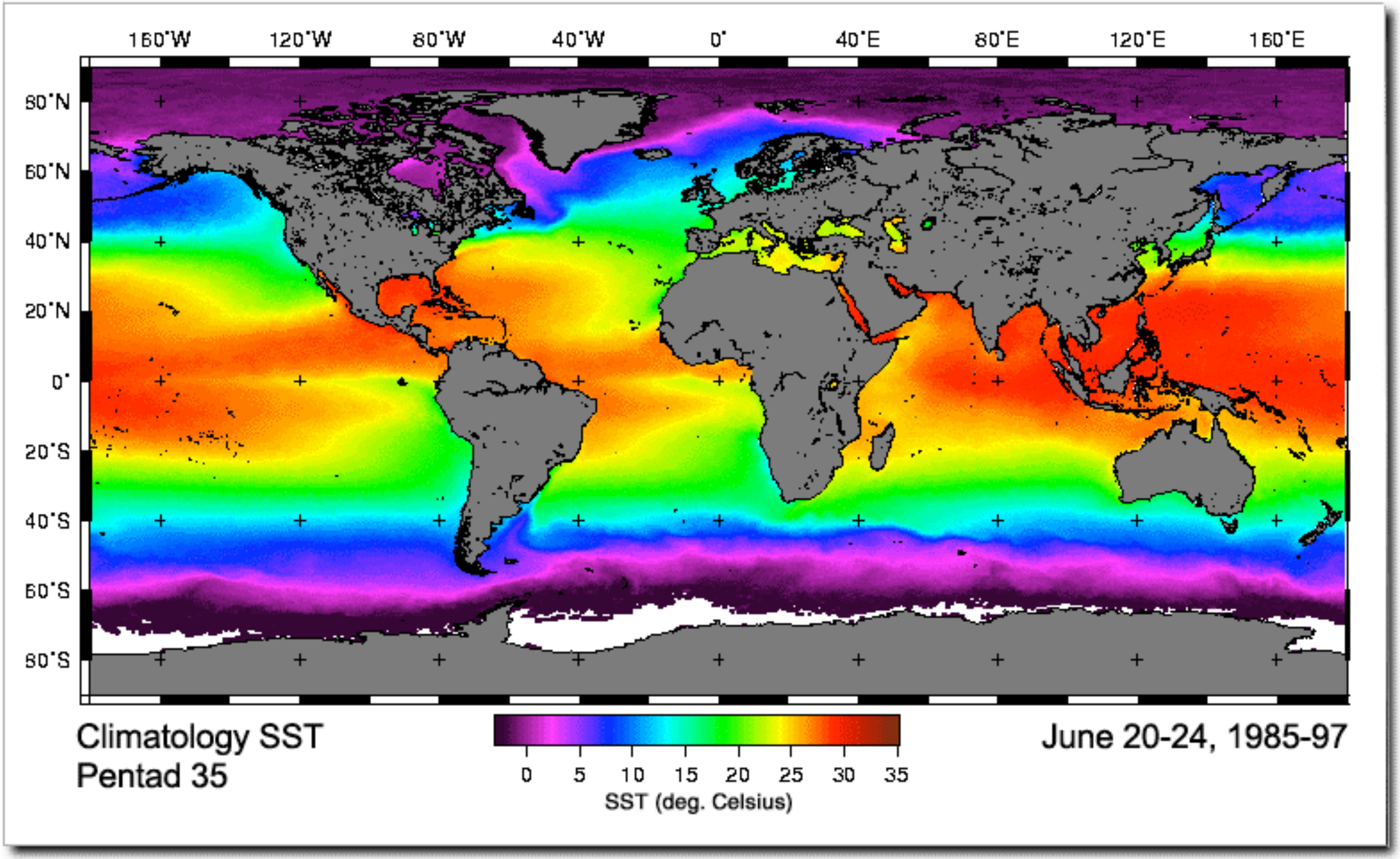
$$T_s = a + bT_{11} + c(T_{11} - T_{12}) + d[(T_{11} - T_{12})(\sec\theta - 1)]$$

Difference in brightness channels account for atmospheric effect

Accounts for viewing geometry (path length variations)

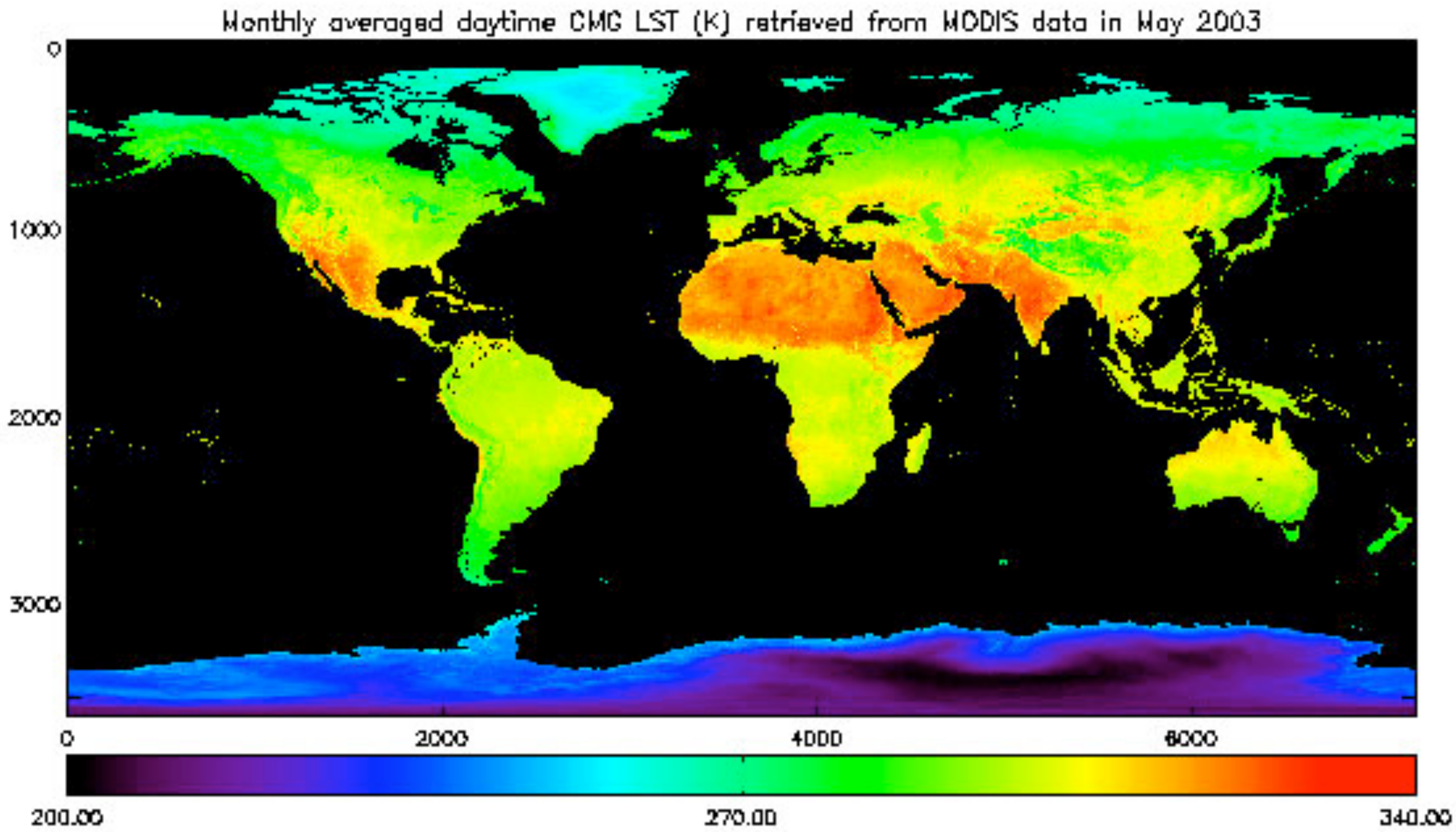
a, b, c, d *are coefficients determined by radiative transfer modelling and surface observations*

T_{11} ***and*** T_{12} ***– TOA brightness temperatures at 11 μm and 12 μm (AVHRR channels 4 and 5)***



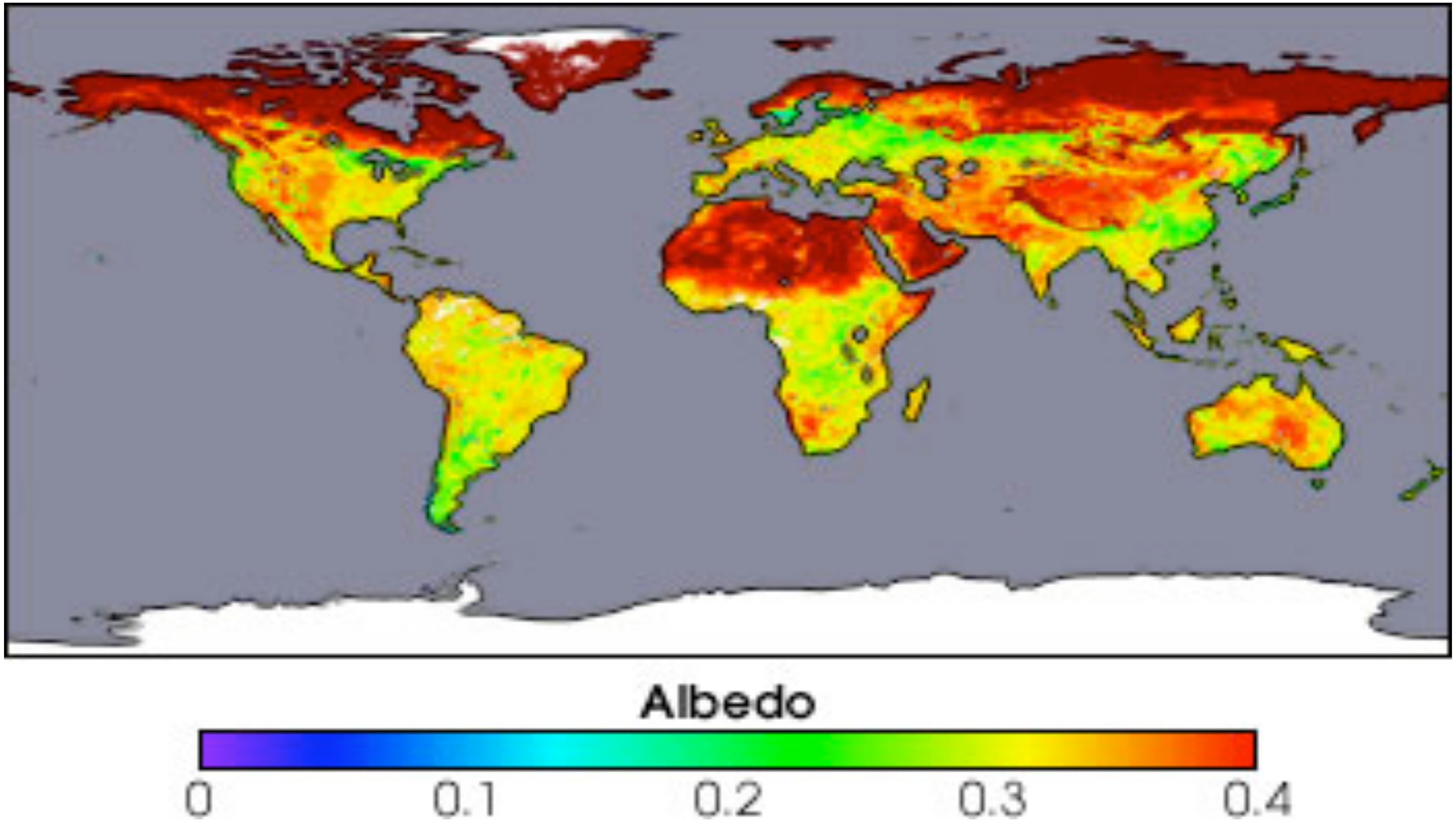
AVHRR daily-averaged SST at 54 km resolution.
Also available at 18 km and 9 km resolution

Land surface temperature



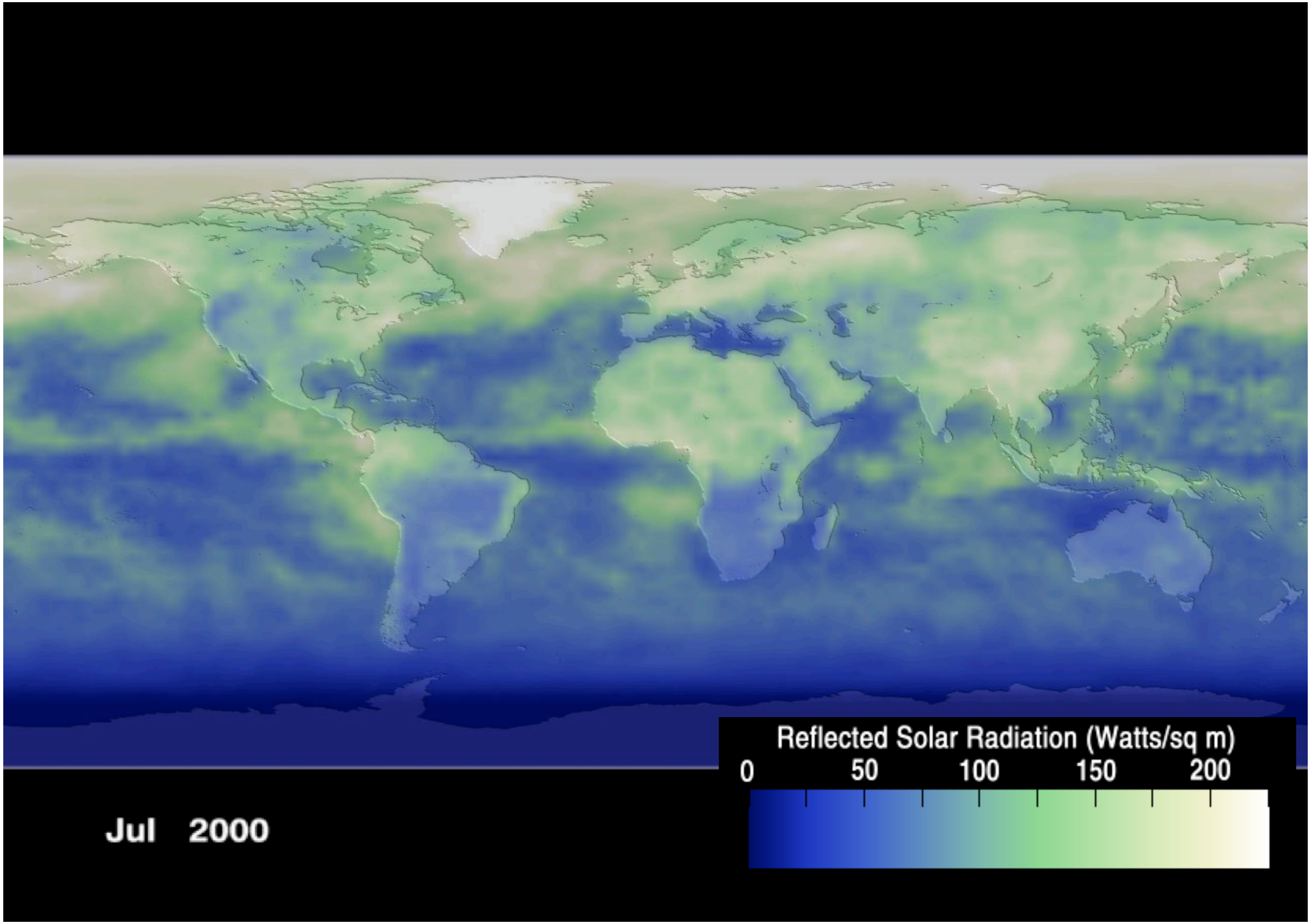
MODIS Land Surface Temperature
Monthly averaged, at 0.05 deg. Lat. Long. resolution

Surface albedo from MODIS (Moderate Resolution Imaging Spectroradiometer)

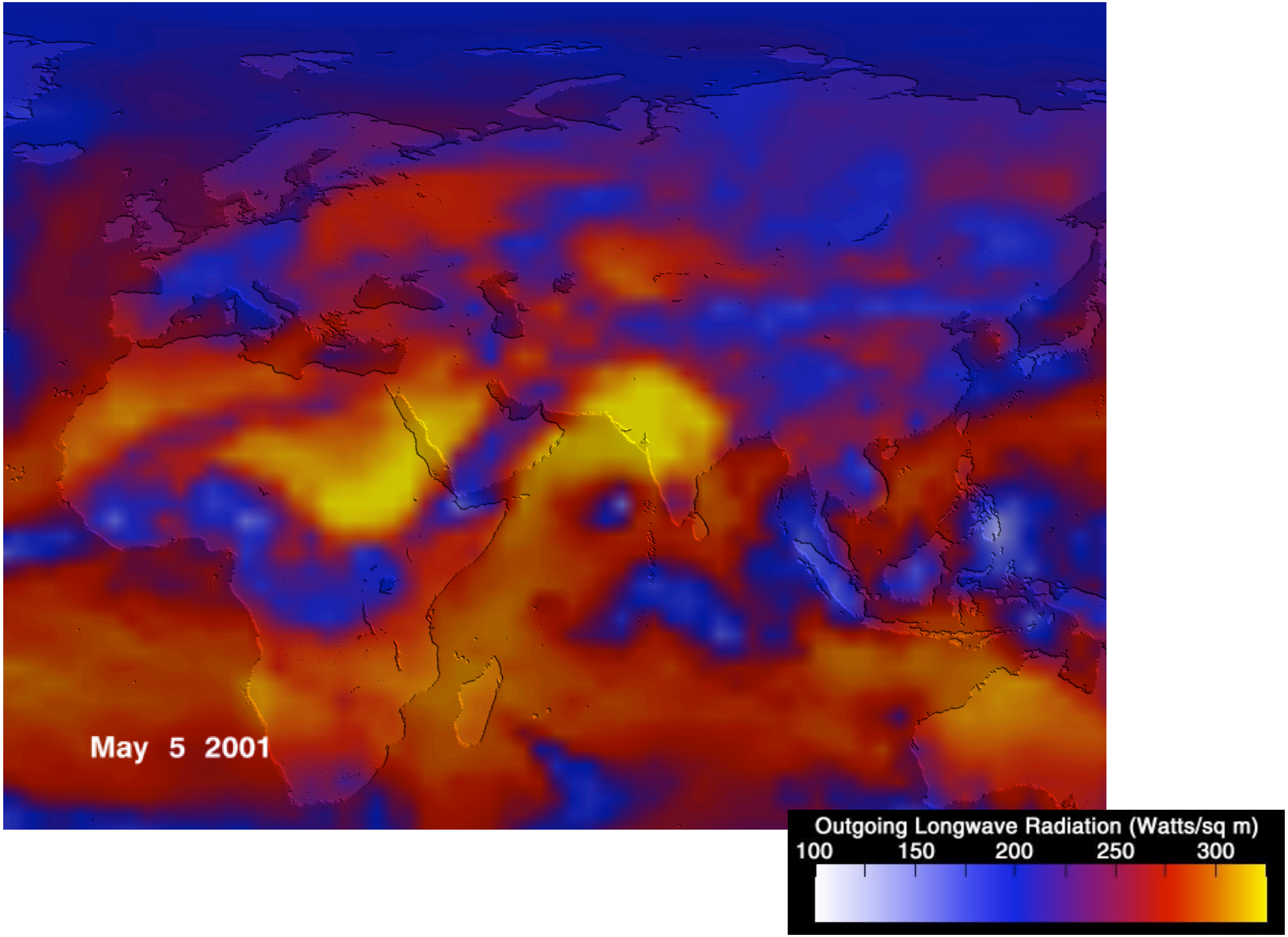


Global surface albedo over land from MODIS. This image was produced using data composite over a 16-day period, from April 7-22, 2002.

Reflected solar radiation



surface IR emission (long wave radiation)



Nadir versus limb radiation

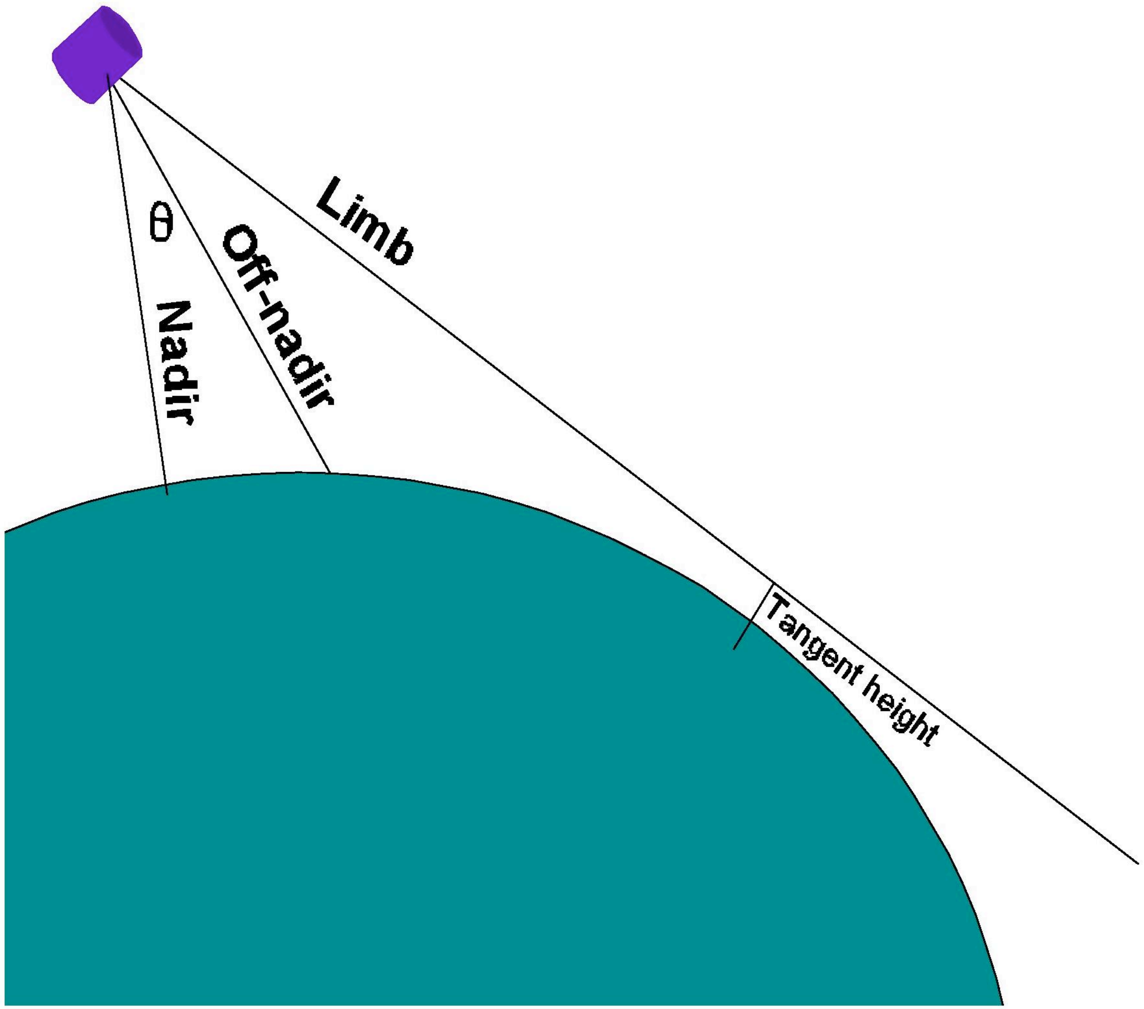
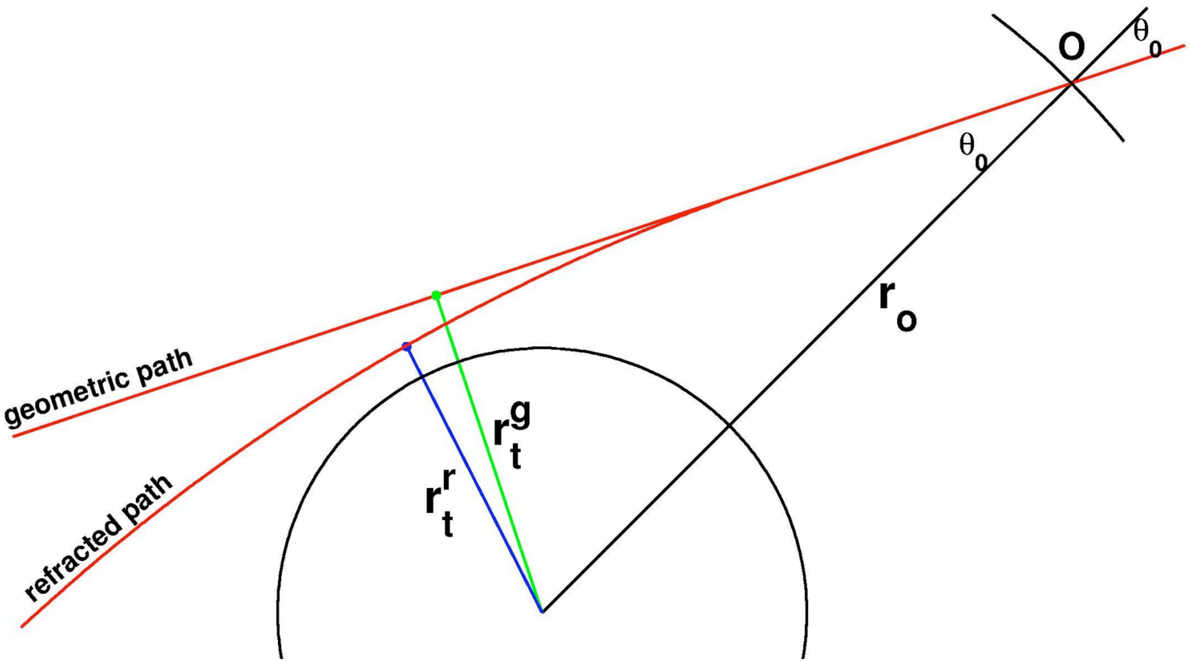


Figure 1: Viewing geometries.

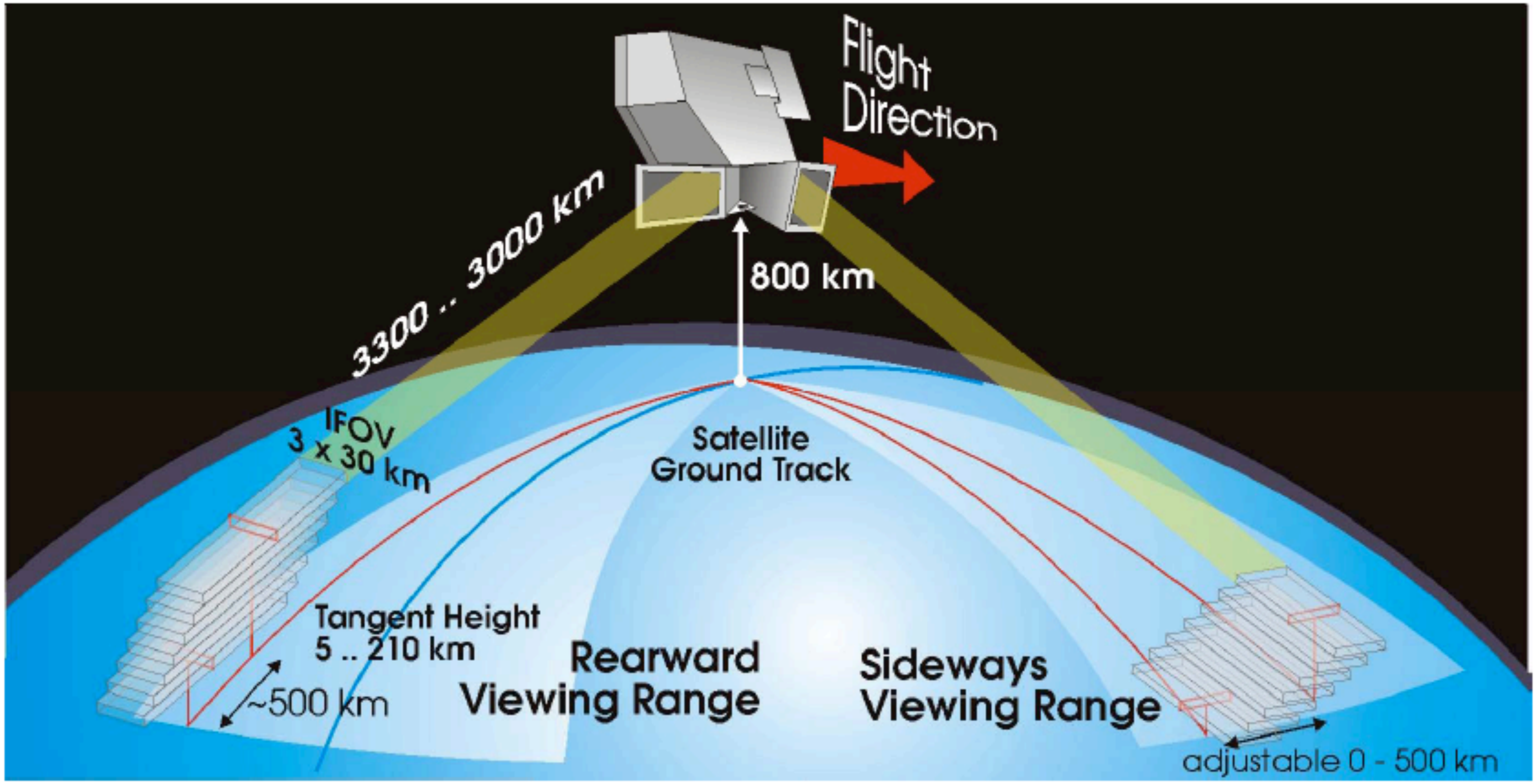


Tangent point tangent height =limb path

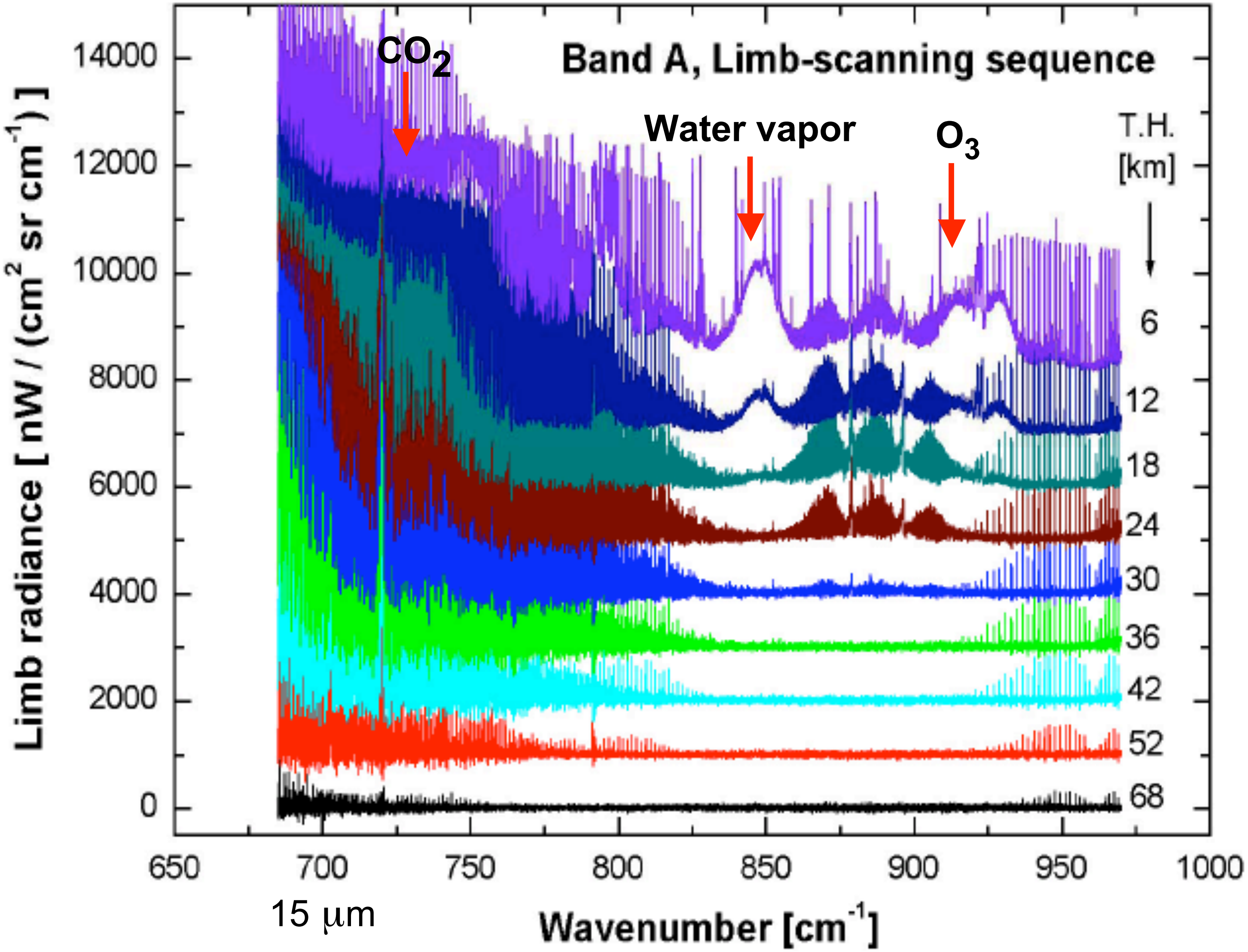
Vertical resolution ~ 1.5 km

Horizontal ~ 200km

MIPAS Observation Principle



Source: http://earth.esa.int/dragon/D2_L5_Carli.pdf

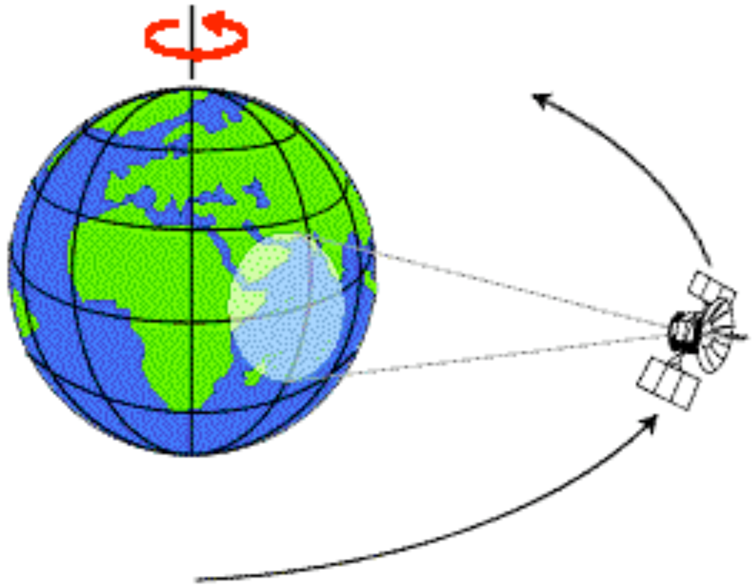


The higher the tangent height the less gases, therefore the more flat the spectrum

Source: http://earth.esa.int/dragon/D2_L5_Carli.pdf

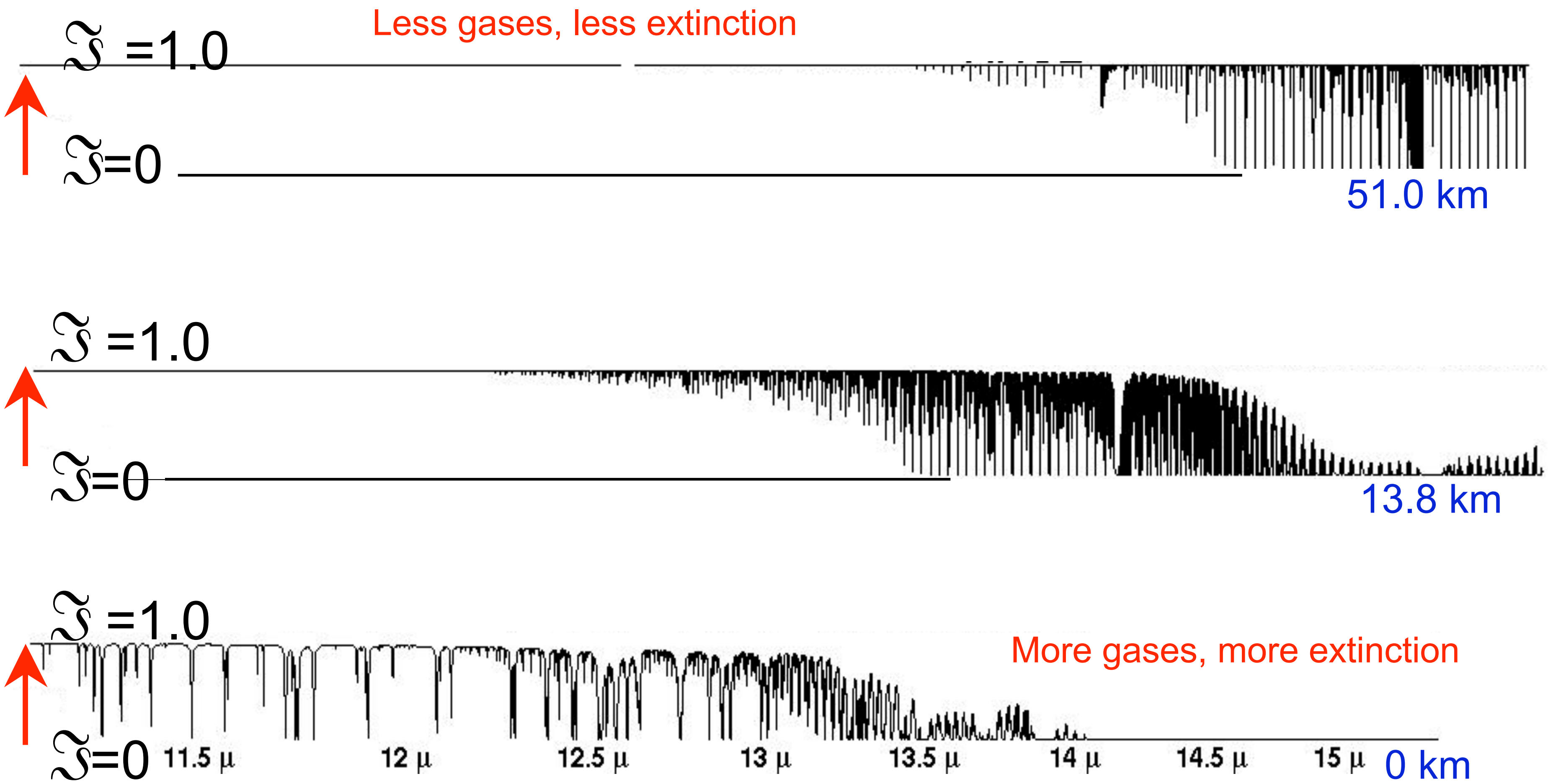
5. Atmospheric profile retrieval

(Temperature, H2O, O3, wind, etc.)

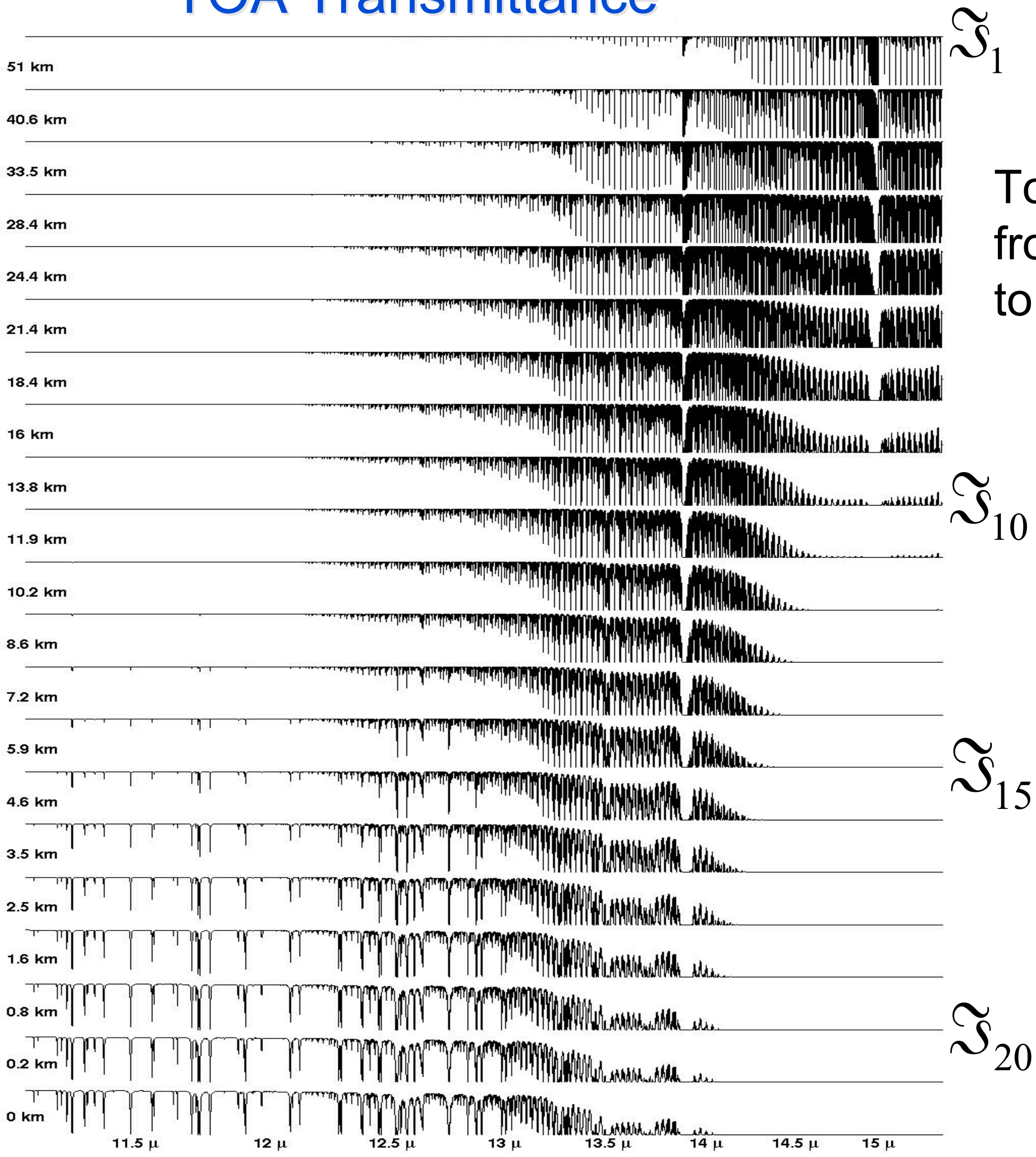


Think of atmosphere as layers:

Atmospheric “path” transmittance ($\Im$) from the bottom of a given layer to the TOA; for the mid infrared spectral region.



TOA Transmittance



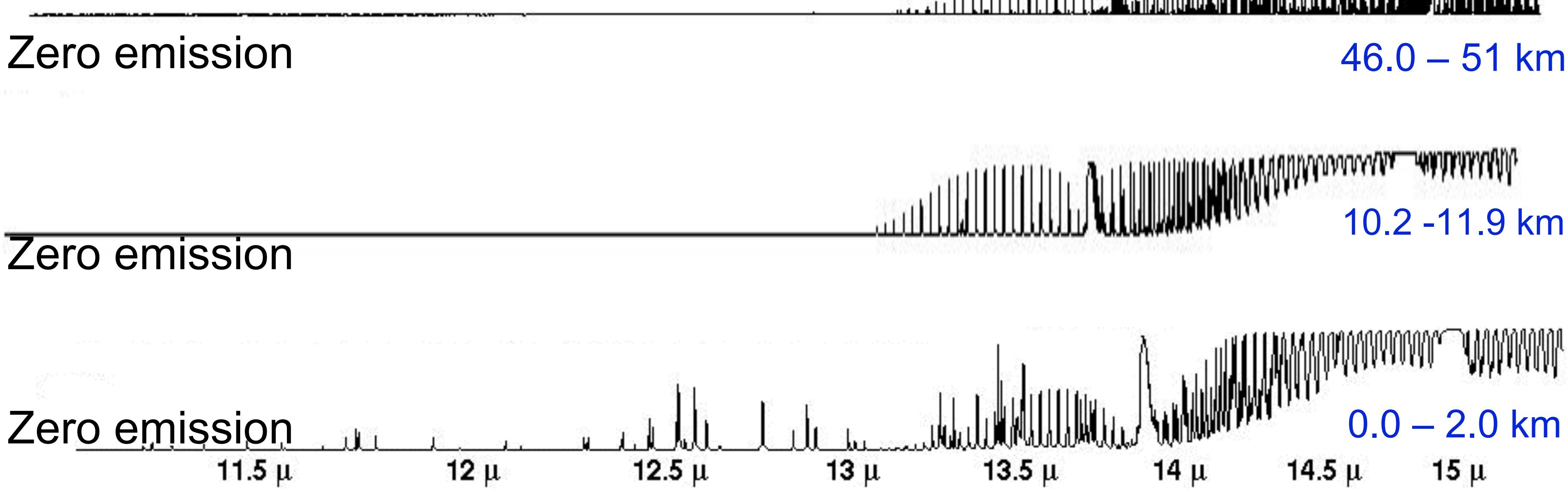
Total transmittance $\mathfrak{I}$ from a layer at z altitude to the TOA.

Layer Atmospheric Emission

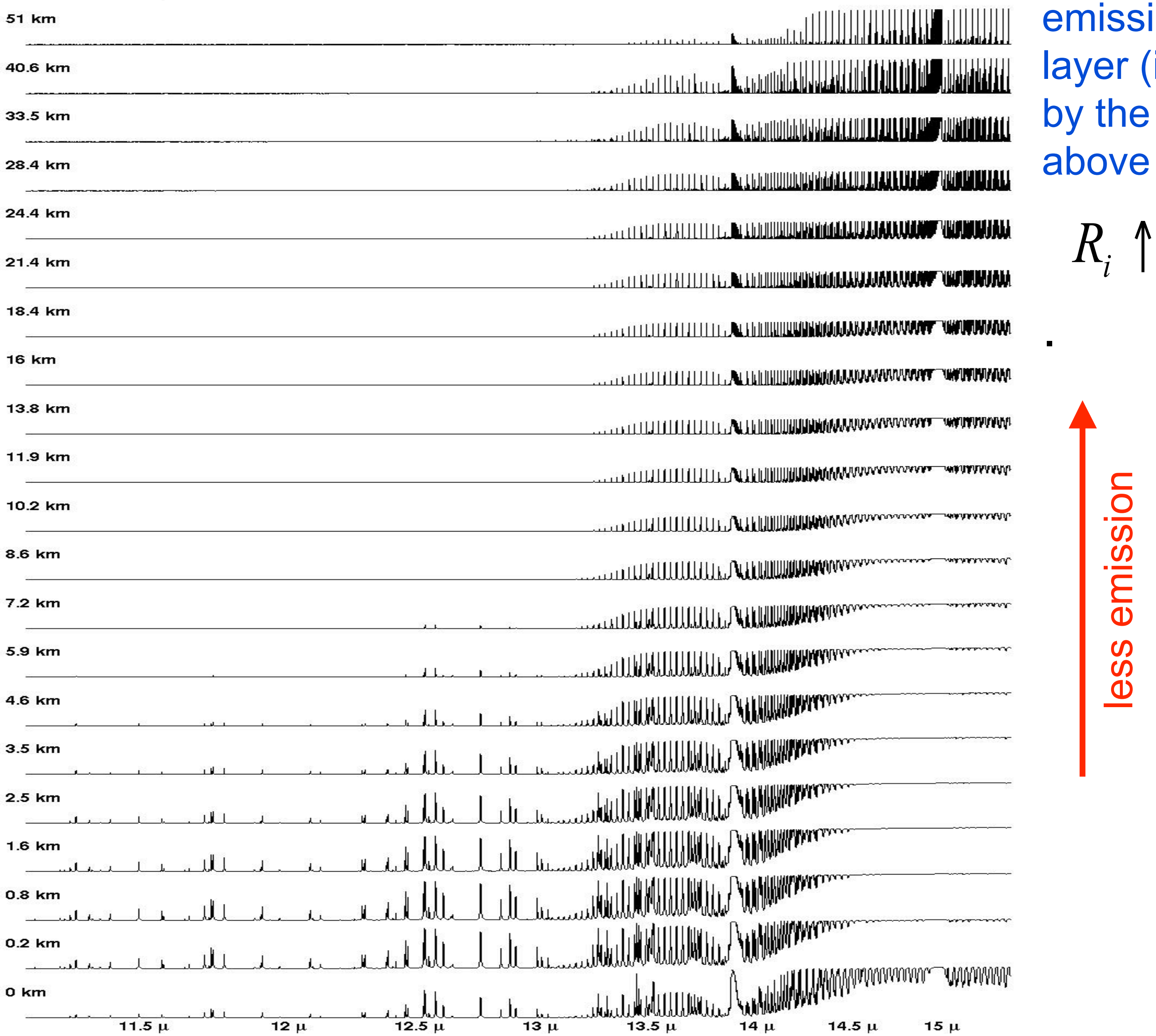
τ_i **<u>is the “layer” transmittance (at later i)</u>**

emissivity from this layer   Blank’s function from this layer

$$layer\ (i)\quad R_i \uparrow = (1-\tau_i)\,B_i$$



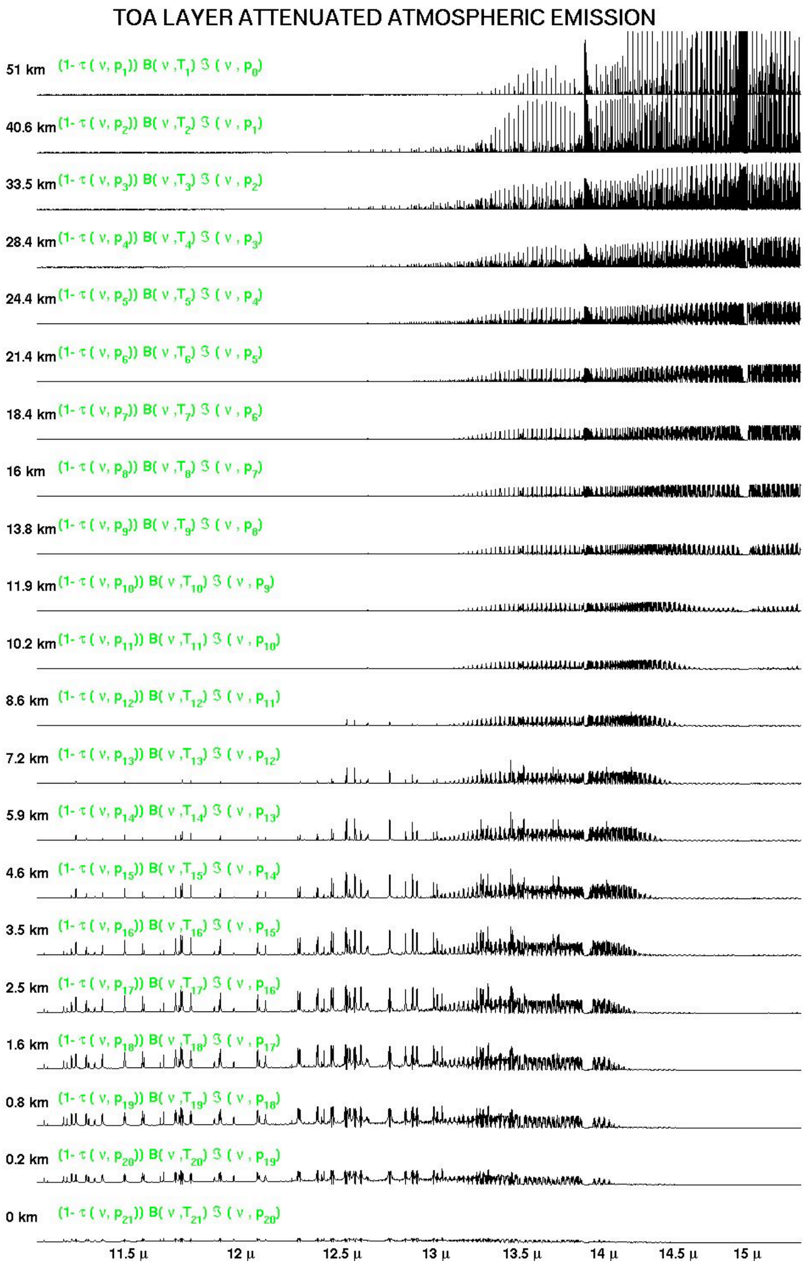
layer atmospheric emission



emission from each layer (i), not attenuated by the atmosphere above it

$$R_i \uparrow = (1 - \tau_i) B_i$$

.

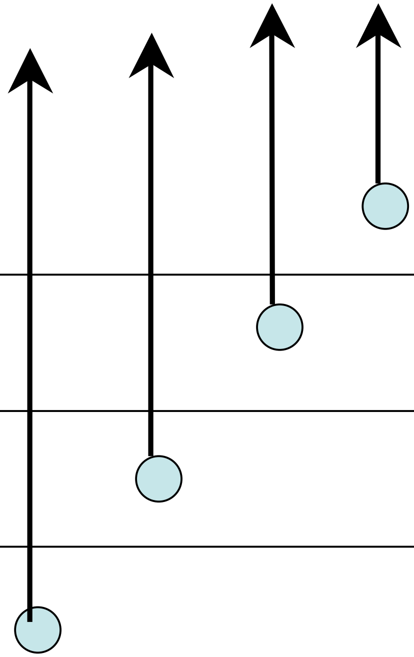


emission at TOA, received from each layer after attenuation through next (upper) layers.

$$R_i = (1 - \tau_i) B_i \Im_{(i-1)}$$

$$R_{22} = (1 - \tau_{22}) B_{22} \Im_{21}$$

Example of the summation of attenuated emission from 4 layers:



	Transmittance from each layer	Emission from each layer	Radiation from each layer to TOA
1	τ_1	$(1-\tau_1)B_1$	$(1-\tau_1)B_1$
2	τ_2	$(1-\tau_2)B_2$	$(1-\tau_2)\tau_1 B_2$
3	τ_3	$(1-\tau_3)B_3$	$(1-\tau_3)\tau_2\tau_1 B_3$
4	τ_4	$(1-\tau_4)B_4$	$(1-\tau_4)\tau_3\tau_2\tau_1 B_4$

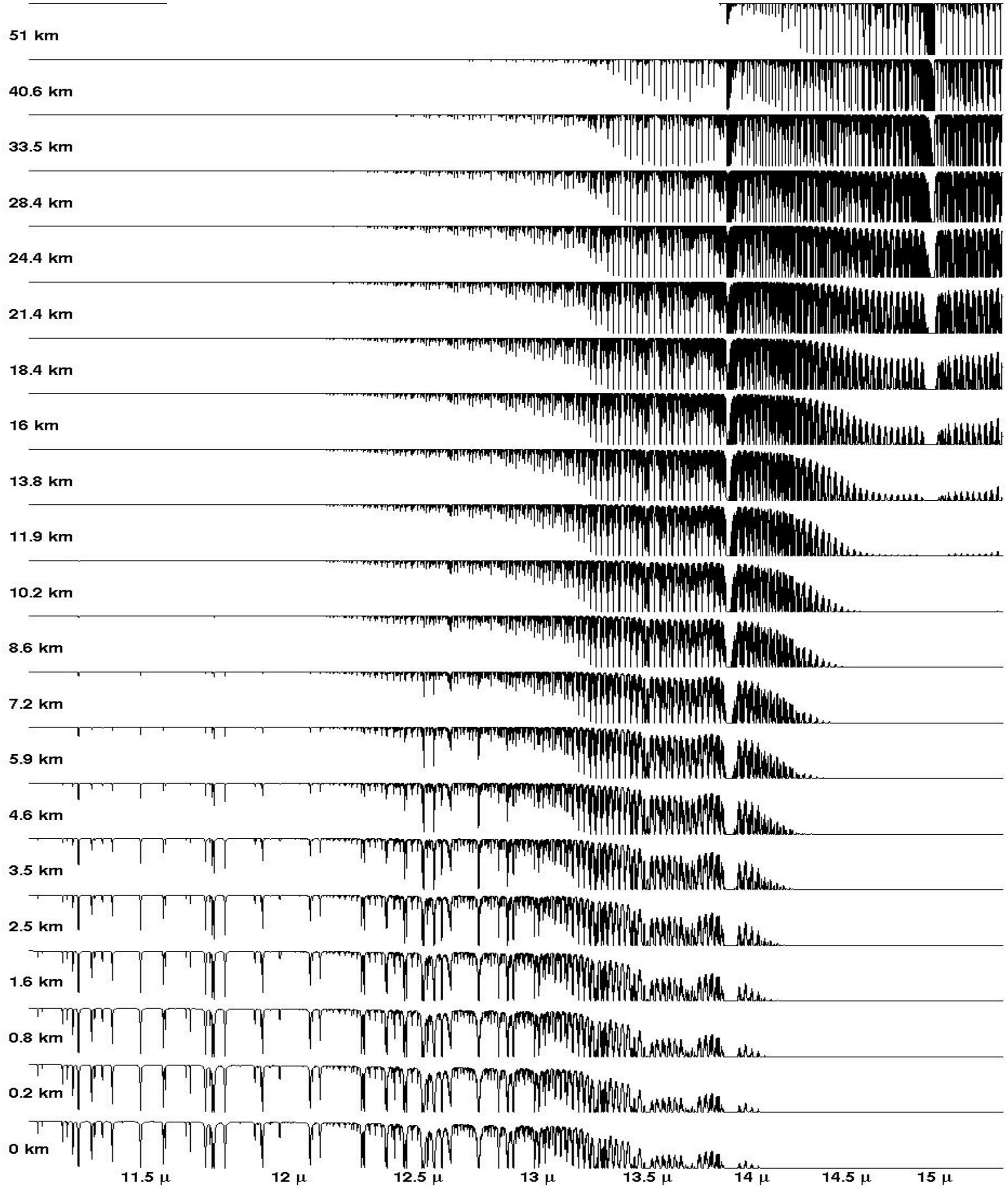
Example of decomposition of one term

$$(1-\tau_3)\tau_2\tau_1 B_3 = (\tau_2\tau_1 - \tau_3\tau_2\tau_1)B_3 = (\Im_2 - \Im_3)B_3$$

Hence, the radiance at the TOA integrated over the atmospheric column is:

$$R\uparrow = \sum_1^n (\Im_{i-1}(\upsilon, p) - \Im_i(\upsilon, p))\, B_i\,(\upsilon, p) \equiv \int B_i\, d\Im_i \equiv \int B_i \frac{d\Im_i}{dp} dp$$

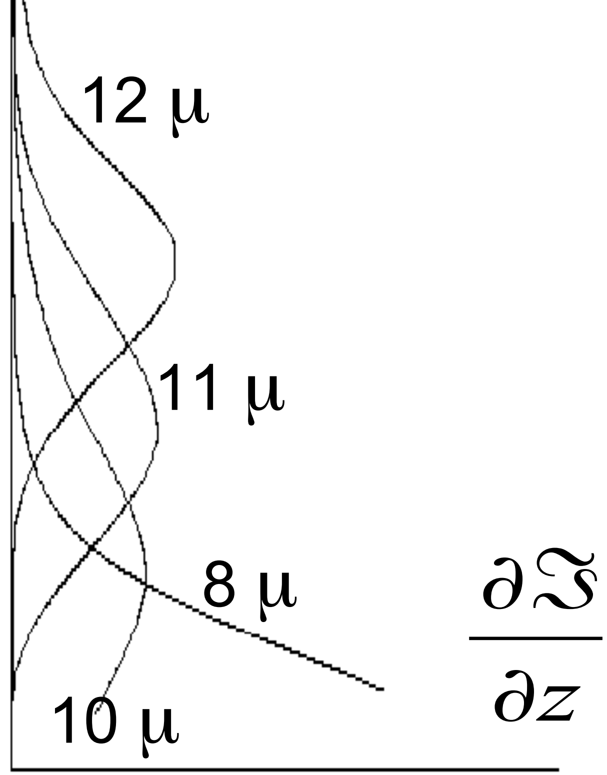
TOA Transmittance



$$R \uparrow = \int B_i \frac{d\Im_i}{dp} dp$$

$$\frac{\partial \Im}{\partial z} \; or \; \frac{\partial \Im}{\partial p}$$

is the weighting function of layer radiation



How to retrieve temperature profile from satellite observations:

1. Need a number of channels equal the number of the steps in the atmospheric (temperature) profile.

2. Use the discrete form of the TOA radiance equation:

$$R\uparrow = \sum_{1}^{n} B_i \left(\frac{\Delta\Im}{\Delta p}\right)_i (\Delta p)_i$$

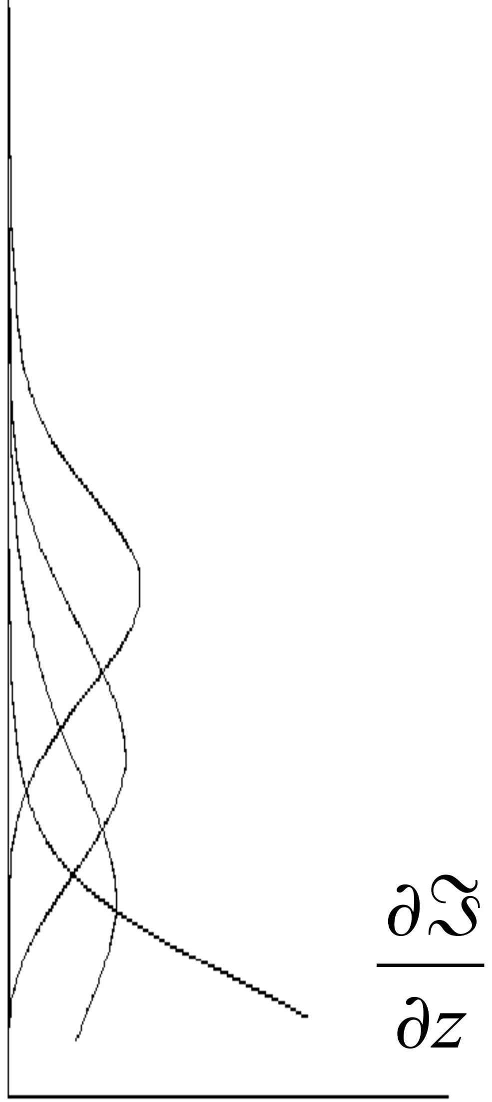
The unknown *T* (temperature) at the discrete points 1 to n is implied in the terms

B_i *and* $\left(\frac{\Delta\Im}{\Delta p}\right)_i$

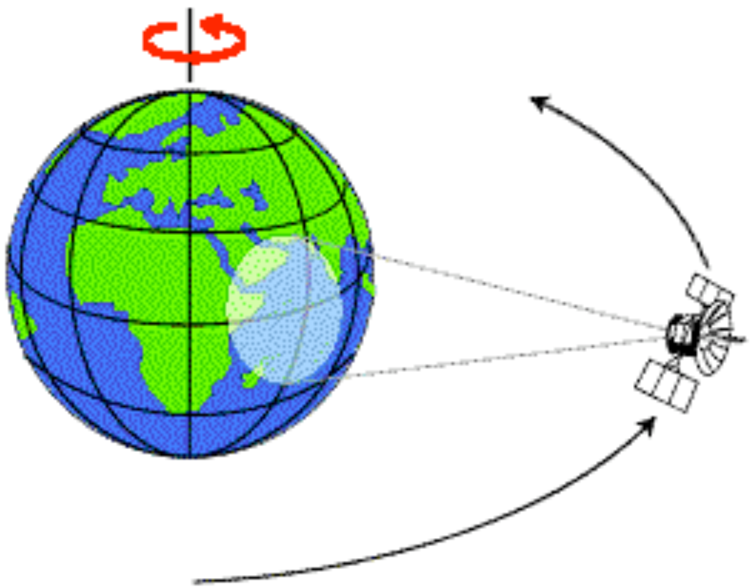
Solve these non linear equation to obtain *T* at each height step.

This is, however, <u>not the method used in actual operational temperature sounding retrieval</u>. A data assimilation method is used.

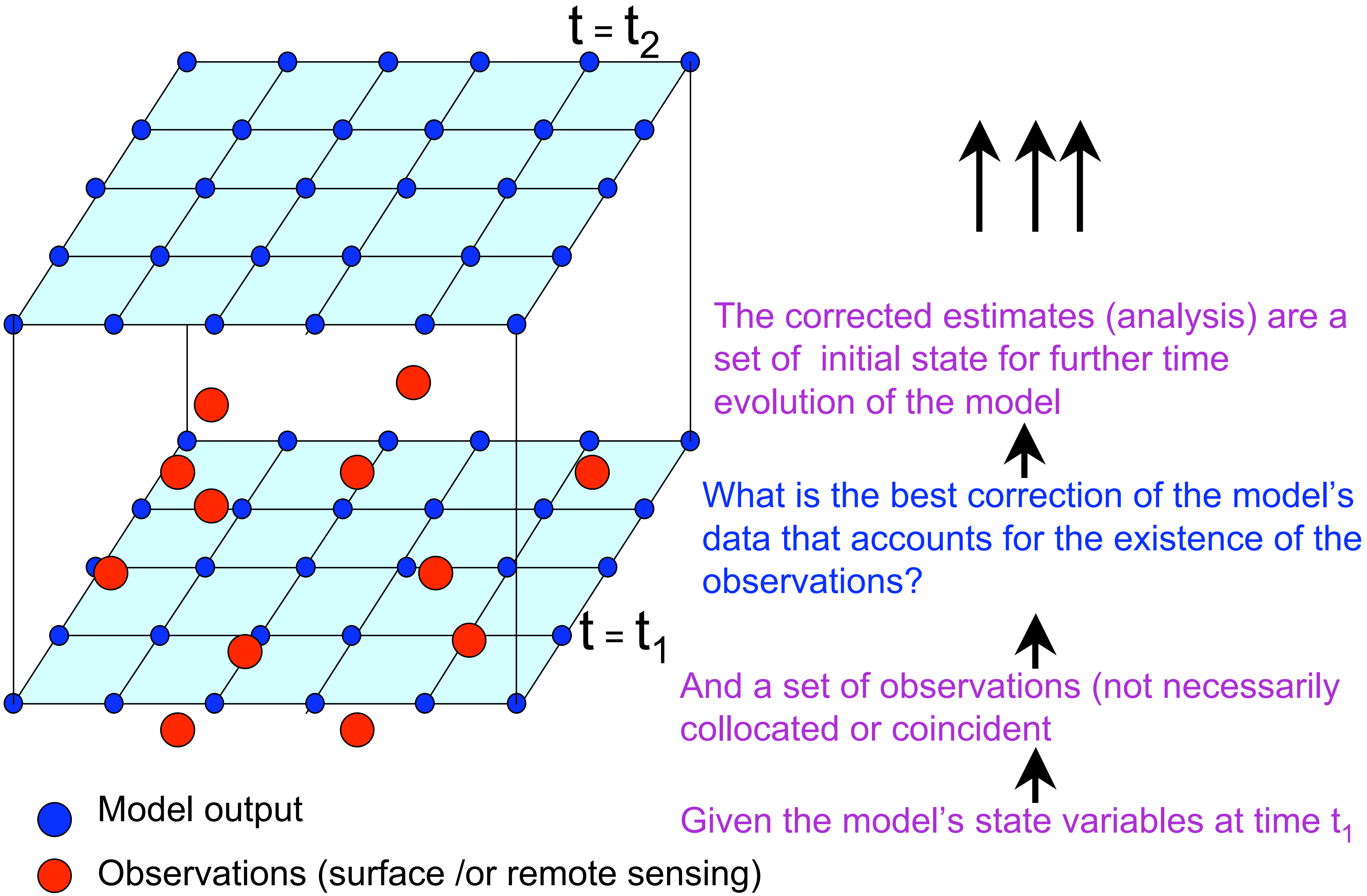
$$R\uparrow = \int B_i \left(\frac{d\Im}{dp}\right)_i dp$$



6. Data Assimilation



What is Data Assimilation?

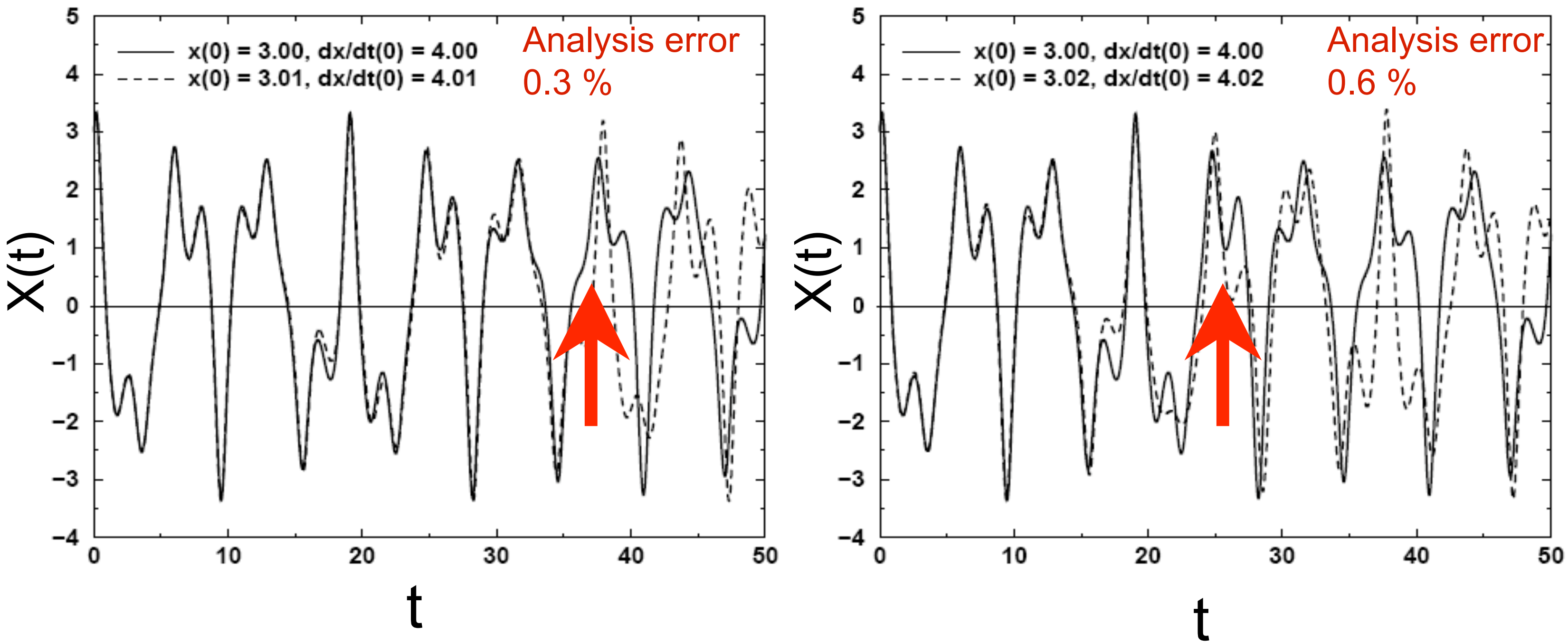


Why Data assimilation?

◊ We need to predict the state variables in future, using the physical model …

◊ Models have to be constrained periodically by reality …

◊ Therefore, it is necessary to fit the model state as closely as possible to observations from the real world.



Integration of Duffing's equation: $x^{\bullet\bullet} + 0.05x^{\bullet} + x^3 = 7.5\cos(t)$

From slightly different initial conditions

Two Criteria of Data Assimilation

- Minimization of Variance

  This is the criterion for the Optimal Interpolation Technique

- Maximization of the conditional probability of the corrected output (analysis), given a set of observations.

  This is the criterion for the variational analysis techniques

Here is a simple question:

If the model output at one grid point = 10, and
the observation at the same point = 8

What would be the "corrected" value of the model output?

The answer to this question reflects the level of understanding the concept of data assimilation.

Data Assimilation is NOT regression analysis!

It is about the Truth!

Model data and observations are co-located

The analysis equation is: (corrected model results)

$$\boxed{x^a = x^b + W(x^{\text{obs}} - x^b).} \qquad (3)$$

Subtract the truth from both sides:

$$x^a - x^t = x^b - x^t + W(x^{\text{obs}} - x^t - x^b + x^t)$$

Analysis error	$\epsilon^a = x^a - x^t$	Analysis ≡ corrected model results
Background error	$\epsilon^b = x^b - x^t$	Background ≡ model results
Observation error	$\epsilon^{\text{obs}} = x^{\text{obs}} - x^t$	Observations ≡ remote sensing data

The analysis equation in terms of errors is:

$$\boxed{\epsilon^a = \epsilon^b + W(\epsilon^{\text{obs}} - \epsilon^b)} \qquad (4)$$

Model data and observations are co-located (cont.)

Take an ensemble average:

$$< \epsilon^a >=< \epsilon^b > +\mathrm{W}(< \epsilon^{\mathrm{obs}} > - < \epsilon^b >). \qquad (4)$$

If $< \epsilon^b >=< \epsilon^{\mathrm{obs}} >= 0$, then $< \epsilon^a >= 0$.

Square (4) and take an ensemble average:

$$< (\epsilon^a)^2 >=< (\epsilon^b)^2 > +\mathrm{W}^2 < (\epsilon^{\mathrm{obs}} - \epsilon^b)^2 > +2\mathrm{W} < \epsilon^b(\epsilon^{\mathrm{obs}} - \epsilon^b) > .$$

Minimize $< (\epsilon^a)^2 >$ with respect to W assuming $< \epsilon^b \epsilon^{\mathrm{obs}} >= 0$:

$$d < (\epsilon^a)^2 > /d\mathrm{W} = 2\mathrm{W} < (\epsilon^{\mathrm{obs}})^2 + (\epsilon^b)^2 > -2 < (\epsilon^b)^2 >= 0$$

Let

$$(\sigma^{\mathrm{obs}})^2 =< (\epsilon^{\mathrm{obs}})^2 >, \quad (\sigma^b)^2 =< (\epsilon^b)^2, \quad (\sigma^a)^2 =< (\epsilon^a)^2 > .$$

so that

$$\boxed{\mathrm{W} = \frac{(\sigma^b)^2}{(\sigma^b)^2 + (\sigma^{\mathrm{obs}})^2}.} \qquad (5)$$

Variational Methods: 1DVAR, 2DVAR, 3DVAR

Given a set of observations (Z) and a model state (X), what does knowledge of the observations tell about the model state?
The information is contained in the conditional p.d.f. (probability density function)

$$P_{X|Z}(X \mid Z) = \frac{p_{zx}(Z, X)}{p_Z(Z)} = \frac{p_{Z|X}(Z \mid X)\, p_X(X)}{p_Z(Z)}$$

In the variational analysis, we are looking for the value of *X* that maximizes the above conditional p.d.f.

But usually, we need an assumption about p.d.f. ◊ assume that is Gaussian

Assume that the errors from the model and observations are both Gaussian

$$p(\varepsilon) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{\varepsilon^2}{2\sigma^2})$$

Then the joint p.d.f. of the observation and the model output errors (using the fact that they are independent because they are Gaussian and uncorrelated:

$$p(\varepsilon^r, \varepsilon^b) = p(\varepsilon^r)p(\varepsilon^b) = \frac{1}{2\pi\sigma^b\sigma^r} \exp[-\frac{(\varepsilon^r)^2}{2(\sigma^o)2} - \frac{(\varepsilon^b)^2}{2(\sigma^b)2}]$$

We are looking for x that minimizes

$$J(x) = [\frac{(x^r - x)^2}{2(\sigma^r)^2} \frac{(x^b - x)^2}{(2\sigma^b)^2}]$$

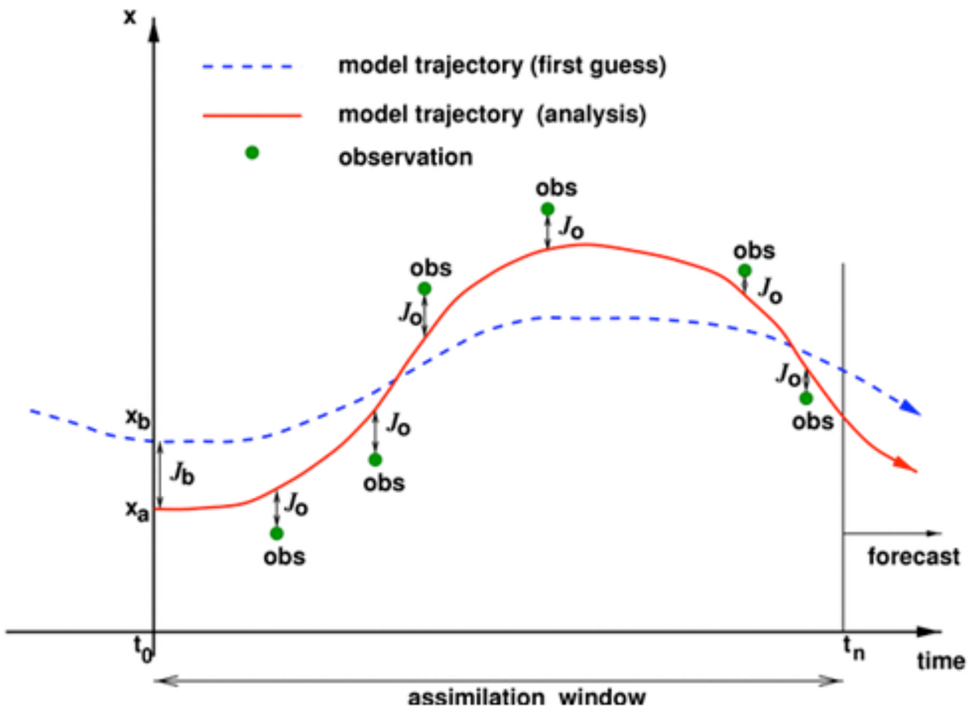
This is called a cost function

$$p_{zx}(Z,X)=\frac{1}{(2\pi)^{n/2}\,|P|^{1/2}}\frac{1}{(2\pi)^{m/2}\,|R|^{1/2}}\ \exp\{-\frac{1}{2}(Z-HX)^T R^{-1}(Z-HX)-\frac{1}{2}(X-\mu)^T P^{-1}(X-\mu)\}$$

H is the observation operator
(model space → observation space)

The maximum a posteriori estimate is obtained by minimizing:

$$J(X)=\frac{1}{2}(Z-HX)^T R^{-1}(Z-HX)-\frac{1}{2}(X-\mu)^T P^{-1}(X-\mu)$$



Thank You !

Nudging is planned to be used rather than a statistical method because of the difficulty obtaining meaningful error statistics for the concentration product.
[Status: Completion - Winter 2001]

finding the model state (at the initial time t_0) that minimizes the cost-function ϑ:

$$\vartheta(\mathbf{x}_0) = \frac{1}{2}\sum_{i=0}^{n}\left(H_i(\mathbf{x}_i) - \mathbf{y}_i^o\right)^{\mathbf{T}}\mathbf{R}_i^{-1}\left(H_i(\mathbf{x}_i) - \mathbf{y}_i^o\right) + \frac{1}{2}\left(\mathbf{x}_0 - \mathbf{x}_0^{\boldsymbol{b}}\right)^{\mathbf{T}}\mathbf{B}^{-1}\left(\mathbf{x}_0 - \mathbf{x}_0^{\boldsymbol{b}}\right)$$