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Random functions for atmospheric wave propagation

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RANDOM FUNCTIONS

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RANDOM FUNCTIONS

No predictable functions.

Examples of our interest:

1 - temperature at a point in the atmosphere

2 - wind velocity in the atmosphere

In general instruments measure averages over a given time and space area, depending on the characteristics of the instrument. If the sensitivity is sufficient on measure "instantaneous values at a given point "

One should use probability. Difficult.

Quantities used: Average, Fluctuations with respect to the average, and Correlations

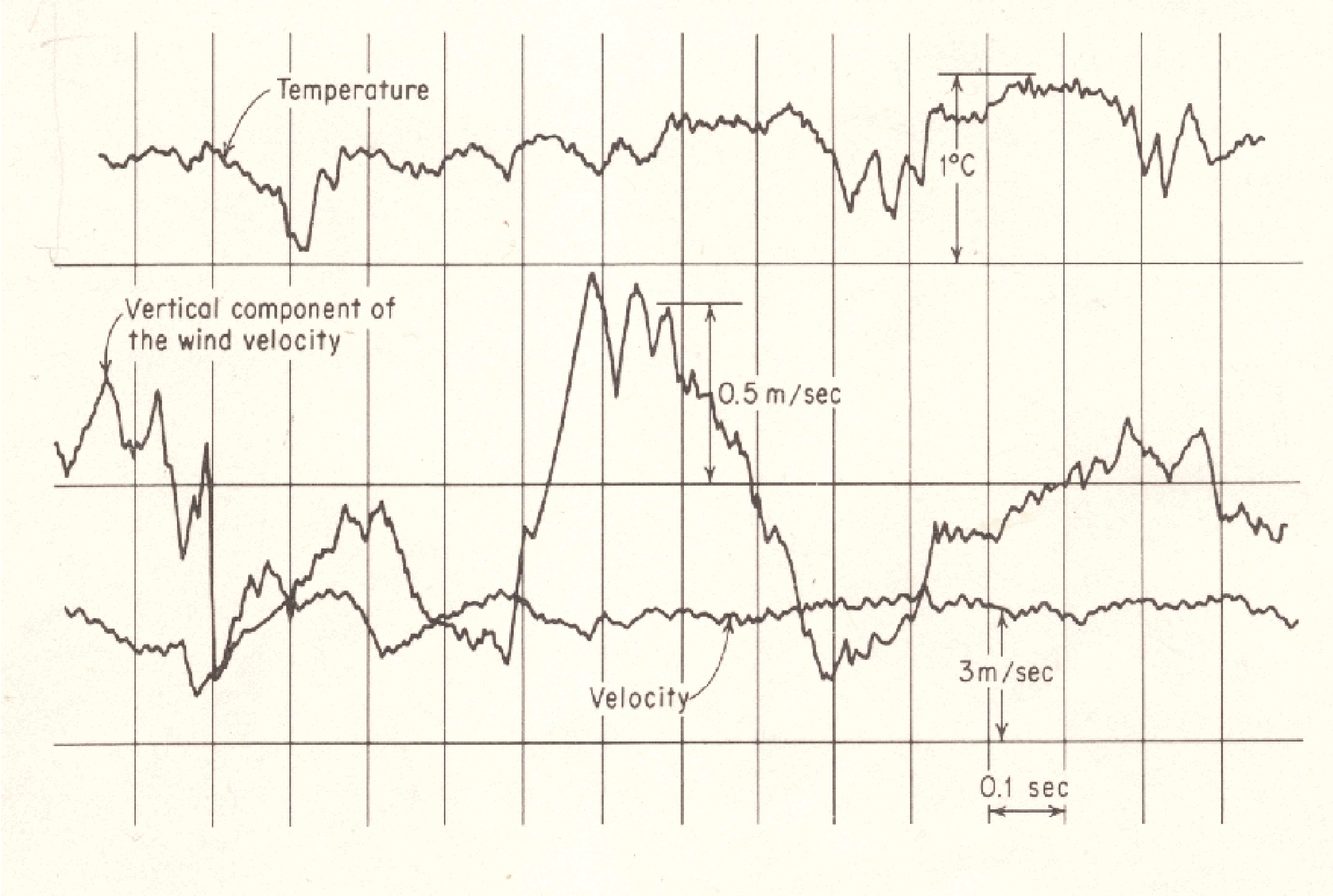

Fig. 1 Simultaneous record of temperature and wind velocity.

Example of random functions in the atmosphere, from book Tatarskii V. I. “The effects of the turbulent atmosphere on wave propagation”. Springfield, Va, 1971

Statistical average of a function f(t) = ensemble average

Ergodic theorem: ensemble average = time average over infinite time.

In practice average over a sufficiently long time period.

In general: f(P,t)

Average of function at a given point:

<f(P,t)>

or simply <f(P)> or <f>

If average does not depend:

1) on the period of time chosen for averaging f(P, t) is a stationary random function,
2) on the position of the point the process is called homogeneous.

FLUCTUATIONS and CORRELATION FUNCTION

Fluctuation at a given time at point P:

$$\delta f(P, t) = f(P,t) - \langle f \rangle$$

Correlation, of fluctuations, of function f at two points P_1, P_2, at the same time t

$$B_f(P_1, P_2) = \langle \delta f(P_1, t)\, \delta f(P_2, t) \rangle$$

Time could be different, more general.

When the two points coincide $P_1 = P_2 = P$

$$B_f(P, P) = \langle \delta f^2(P, t) \rangle$$

mean square fluctuation at point P, Variance $\sigma^2(P)$

Stationary random functions

Cannot be Fourier transformed. Their correlation functions can be

Homogeneity and Isotropy

Homogeneity: Correlation function $B_f(P_1, P_2)$ only depends on the distance (vector) between the two points,

Isotropy: $B_f(P_1, P_2)$ only depends on the modulus r of the distance between the two points

$$B_f(P_1, P_2) = B_f(r)$$

Properties:

$$B_f(0) = \sigma^2$$

$$B_f(0) \geq | B_f(r) |$$

Correlation function $B_f(r)$ can be Fourier transformed; 3d transform.

STRUCTURE FUNCTION

$$D_f(P_1, P_2) = \langle [\delta f(P_1) - \delta f(P_2)]^2 \rangle$$

Structure function of function f at two different points, and same time not shown. Difference of fluctuations squared. Allows study of processes with slowly varying average.

Useful for experiments

In the case of homogeneity and isotropy:

$$D_f(P_1, P_2) = D_f(r)$$

and structure and correlation are related by:

$$D_f(r) = 2\,[B_f(0) - B_f(r)]$$

$$B_f(0) = \tfrac{1}{2}\, D_f(\infty)$$

FOURIER TRASFORM OF CORRELATION FUNCTION

Correlation function $B_f(r)$ of homogeneous and isotropic random function can be Fourier transformed, three dimensional transform.

By taking into account the symmetry, two of the integrals can be performed and one obtains a transform $\Phi_f(\kappa)$ as function of one spatial wave number κ :

$$\Phi_f(\kappa) = 1/(2\pi\kappa^2) \int_0^\infty r\, B_f(r) \sin(\kappa r)\, dr$$

$$B_f(r) = (4\pi/r) \int_0^\infty \kappa\, \Phi_f(\kappa) \sin(\kappa r)\, d\kappa$$

Function $\Phi_f(\kappa)$ is called the power spectrum, or simply spectrum of the process. The name power spectrum originates from the power dissipated in a resistance .

ATMOSPHERIC REFRACTIVE INDEX

Optical propagation in atmosphere: random quantity of interest is atmospheric refractive index, n.

More precisely: Correlation function, or Structure function or Spectrum (if any), of the refractive index

$B_n(P_1, P_2)$, $D_n(P_1, P_2)$, $\Phi_n(\underline{\kappa})$

In clear air, absence of pollutants, haze and so on, main effect is turbulence.

In homogeneous isotropic case

$B_n(r)$, $D_n(r)$, $\Phi_n(\kappa)$

A given form of the correlation function, or of the structure function or of the corresponding spectrum is called a model of the atmospheric turbulence.

MAIN PROBLEM IN PROPAGATION OF

E. M. RADIATION IN RANDOM MEDIA

and

MAINLY ATMOSPHERIC TURBULENCE

Find relationships between correlation and or structure functions of quantities of the e.m. field and the function(s) describing the medium.

Example: Phase correlation function at two points of a wavefront of a plane wave after a path in the atmosphere and the correlation function of the refractive index.

Depending on the problem one could need higher order correlations.

For our purposes no need of higher order correlations.