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Introduction to turbulence

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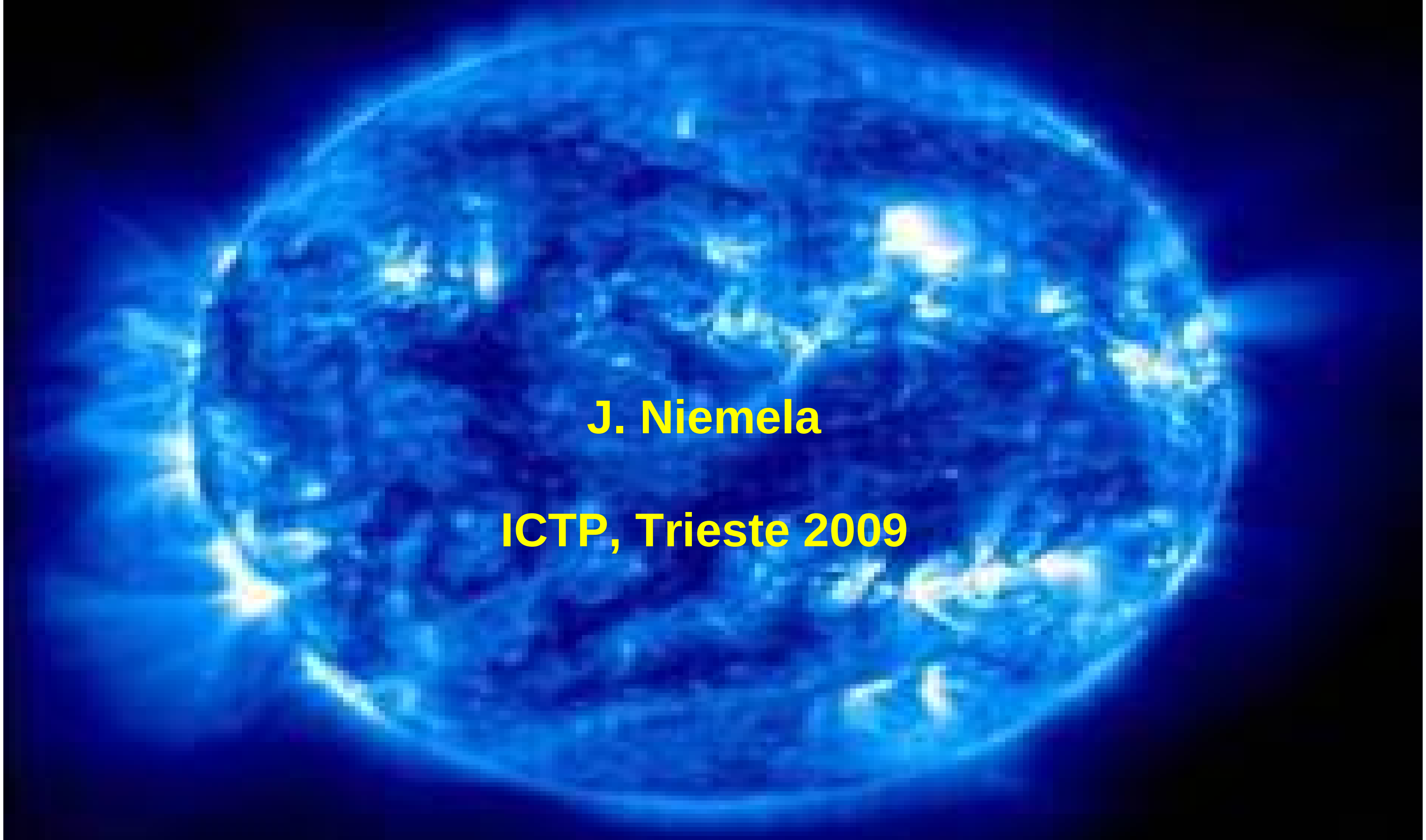


INTRODUCTION TO TURBULENCE

J. Niemela

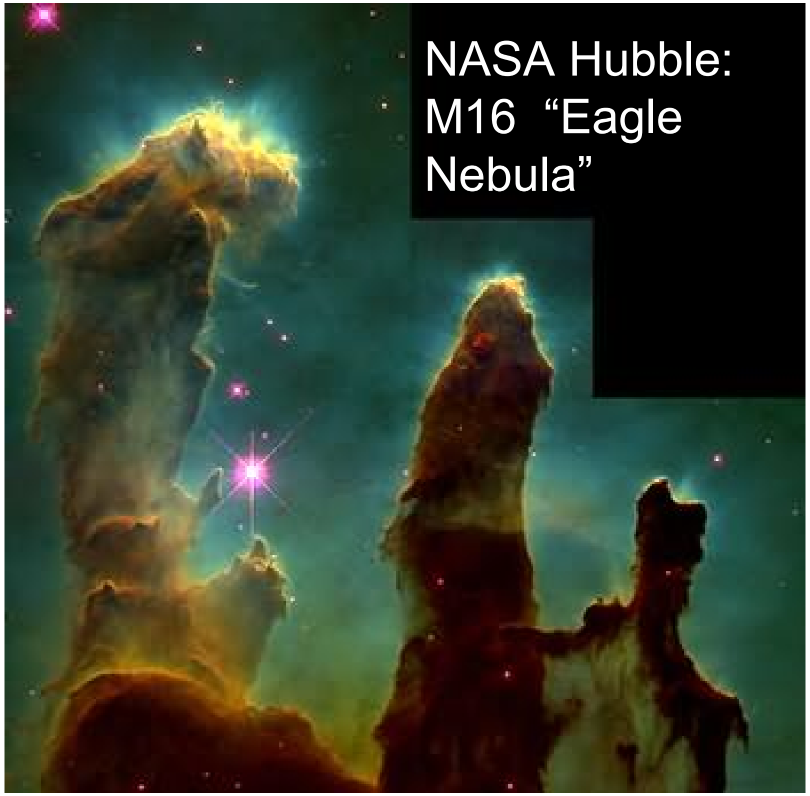
ICTP, Trieste 2009

2009 Winter College on Optics in Environmental Science

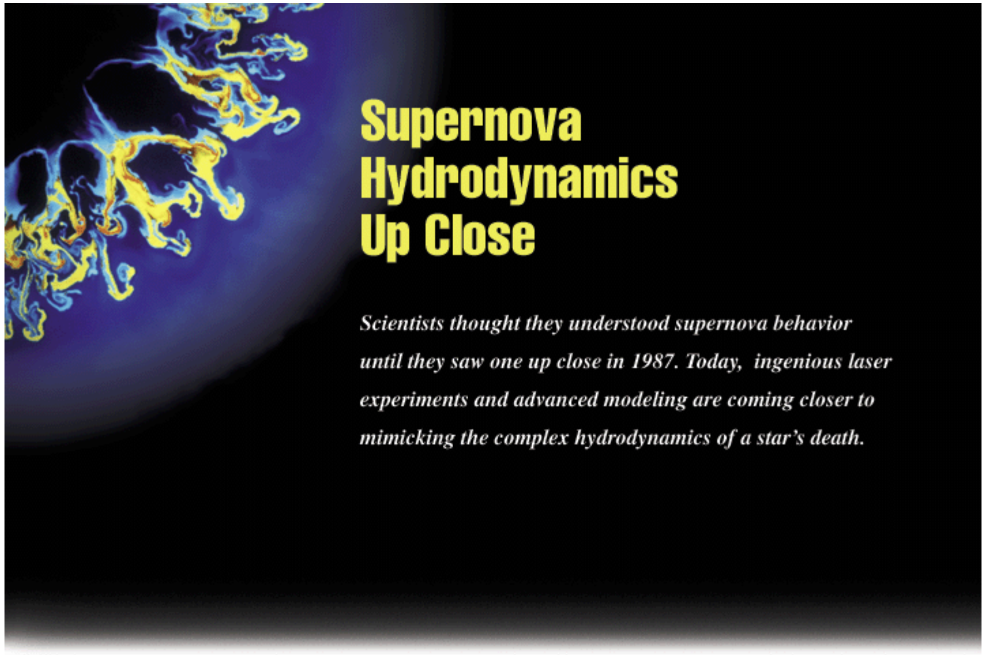
For some, the universe can be seen as a fluid system….

"Perhaps the fundamental equation that describes the swirling nebulae and the condensing, revolving, and exploding stars is ***just a simple equation for the hydrodynamic behavior*** *of nearly pure hydrogen gas".*-- Richard Feynman *Lectures on Physics*

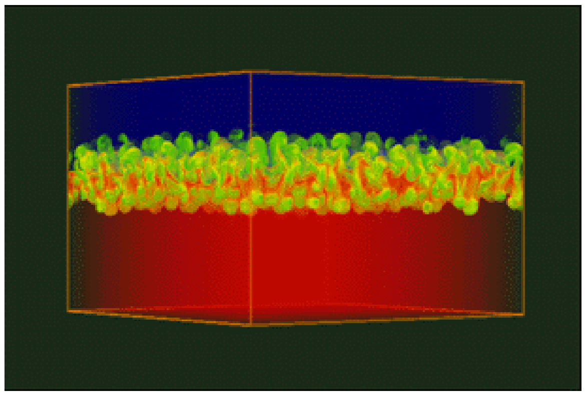
Emerging from the eroding tips of giant pillars of hydrogen and helium (roughly 9 trillion kilometers tall) in the Eagle Nebula are dense globules of gas as wide as our entire solar system. New stars are forming from gas condensing within these stellar cocoons. The columns generally are believed to evolve from a fluid instability related to Rayleigh-Taylor flow.



Consideration of fluid instabilities (again Rayleigh Taylor instability) was needed explain the first detailed observations of a supernova explosion: 1987a.

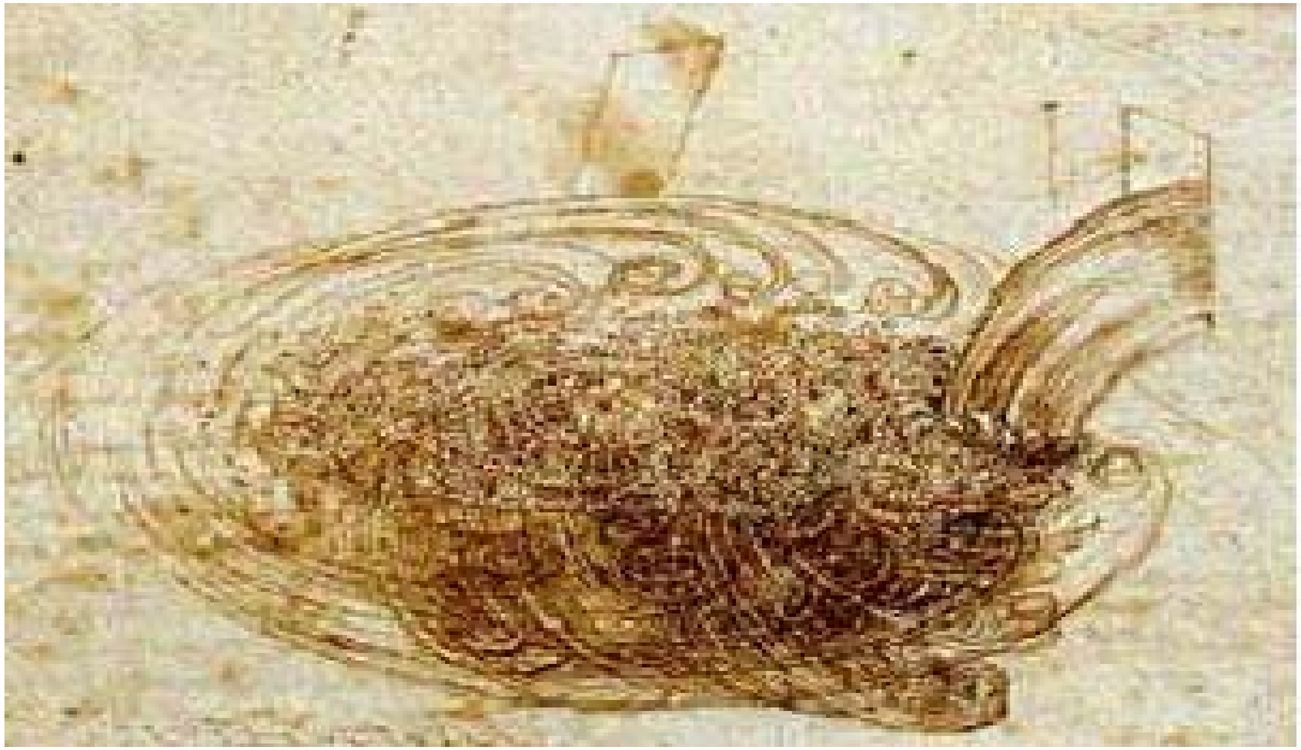


From Science and Technology Review (2000)



Simulation of Rayleigh Taylor instability when a dense fluid is initially placed on top of a lighter one. Here the dense fluid falls in spikes and the ligher fluid rises in “bubbles” to take its place.

Turbulence is widespread, indeed almost the rule, in the flow of fluids. It is a complex phenomenon, for which the development of a satisfactory theoretical framework has been one of the greatest unsolved challenges of classical physics.



The first scientific investigations of fluid turbulence are generally attributed to Leonardo Da Vinci.

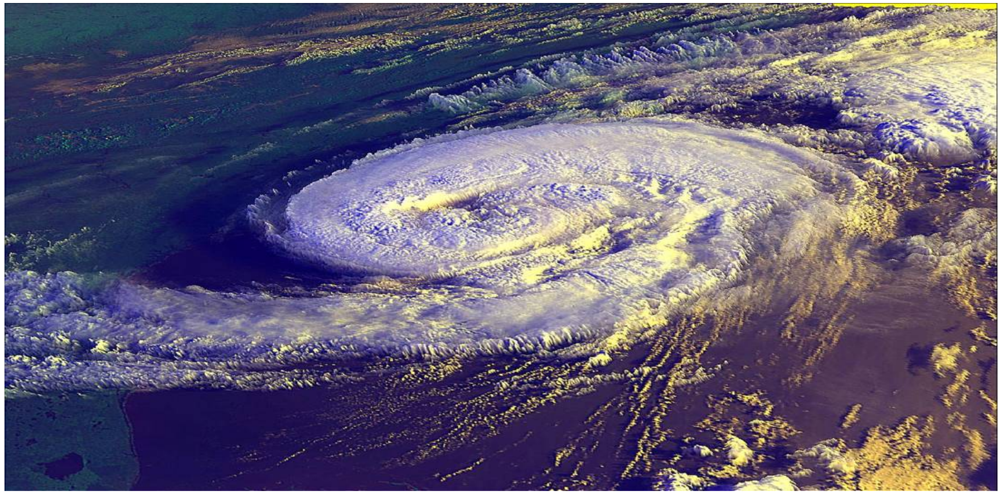
Fluid turbulence is characterized by:

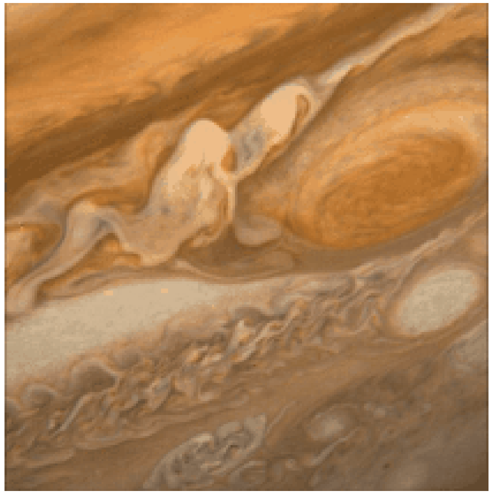
- Many interacting (temporal and spatial) degrees of freedom
- Non-equilibrium and nonlinear

Turbulence is exemplified by seemingly random motions of fluid, where it appears impossible to describe the motion in all details as a function of *both time and position*. However, it can be precisely characterized *statistically*, with reproducible *average* values of certain quantities.

Turbulence is a feature of the flow of fluid, not of the fluid itself.

Turbulence exists in a wide range of contexts such as the motion of submarines, ships and aircraft, pollutant dispersion in the earth's atmosphere and oceans, heat and mass transport in engineering applications as well as geophysics and astrophysics [See, e.g., D.J. Tritton, *Physical Fluid Dynamics.* Clarendon Press, Oxford (1988)].





The problem is also a paradigm for strongly nonlinear systems, distinguished by strong fluctuations and strong coupling among a large number of degrees of freedom. [G. Falkovich, K.R. Sreenivasan, *Phys. Today* 59, 43 (2006)].

Turbulence is particularly useful because the equations of motion are presumed to be known and can be simulated with precision. And so, even distant areas such as fracture [M.P. Marder, *Condensed Matter Physics.* Wiley, New York (2000)]--- perhaps even market fluctuations [B.B. Mandelbrot, *Scientific American* 280, 50 (1999)]---may benefit from a better understanding of it.

However, the complexity of the underlying equations has precluded much analytical progress, and the demands of computing power are such that routine simulations of large turbulent flows has not yet been possible [See: National Research Council Report: *Condensed Matter and Materials Physics: Basic Research for Tomorrow's Technology*, 308 pages, National Academy Press (1999)].

Thus, the progress in the field has depended more on experimental input. This experimental input in turn points in part to a search for optimal test fluids, and the development and utilization of novel instrumentation. That has lead to the work at ICTP’s turbulence laboratory at Elettra which utilizes low temperature helium as a test fluid

If this lecture were to be given a high school in the state of Kansas we would stop here….

CNN Headline Monday November 7, 2005:

“Kansas school board redefines science”

“Intelligent design holds that the universe is so complex that it must have been created by a higher power.”

Basic Fluid Equations

The equations of mass, momentum and energy conservation are written down in a coordinate system that is fixed in space ("Eulerian" description).

Considering a fluid of density ρ, moving at a velocity **u,** mass conservation takes the form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Assuming constant density (note: we will assume this even in the case where there is thermal expansion due to heating) this reduces to the divergence-less condition:

$$\nabla \cdot \mathbf{u} = 0$$

For a Newtonian fluid, in which the stress and strain tensors are linearly related, the momentum equation reduces to

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{F}_{ext}$$

in terms of the pressure p and kinematic viscosity $\nu=\mu/\rho$**,** where μ is the dynamic or shear viscosity. For convenience of notation, we have used the so-called substantive or convective derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \nabla\cdot\mathbf{u}$$

The body force term is represented by $\mathbf{F}_{ext}$ which could arise from sources such as gravity, rotation and magnetic field. Of particular interest is gravitational buoyancy in thermally driven flows, for which there is only one component in the direction of gravity and is given by $\mathbf{F}_{ext} = g\alpha\Delta T$ where g is the acceleration of gravity, α is the isobaric coefficient of thermal expansion and ΔT is the temperature difference across a layer of fluid in the direction of gravity.

The equation for energy conservation in the Boussinesq approximation [A. Oberbeck, *Annalen der Physik und Chemie* 7, 271 (1879)] is

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T - \mathbf{u} \cdot \nabla T$$

where $\kappa = k_f / \rho C_P$ is the thermal diffusivity of the fluid with thermal conductivity k_f and specific heat C_P.

These three equations above describe the fluid motion and determine the velocity, pressure and temperature.

They must, of course, be supplemented by the appropriate initial conditions and boundary conditions on rigid or``free" surfaces, conducting or insulating surfaces, rough or smooth surfaces, etc. as appropriate.

Under these conditions, the solutions of the equations of motion should indeed correspond to the observed flows, including turbulence (it is of course impossible to ever specify the initial conditions sufficiently to predict detailed trajectories in the space of the variables). It is not always clear if small deviations from the ideal boundary conditions produce only small effects.

Let us simplify our consideration to isothermal flows.

$$\frac{\partial \mathrm{v}}{\partial t} + (\mathrm{v} \cdot \nabla)\mathrm{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathrm{v}$$

$$\frac{\partial \mathrm{v}'}{\partial t} + (\mathrm{v}' \cdot \nabla')\mathrm{v}' = -\nabla' p' + \frac{1}{\mathrm{Re}}\nabla'^2 \mathrm{v}'$$

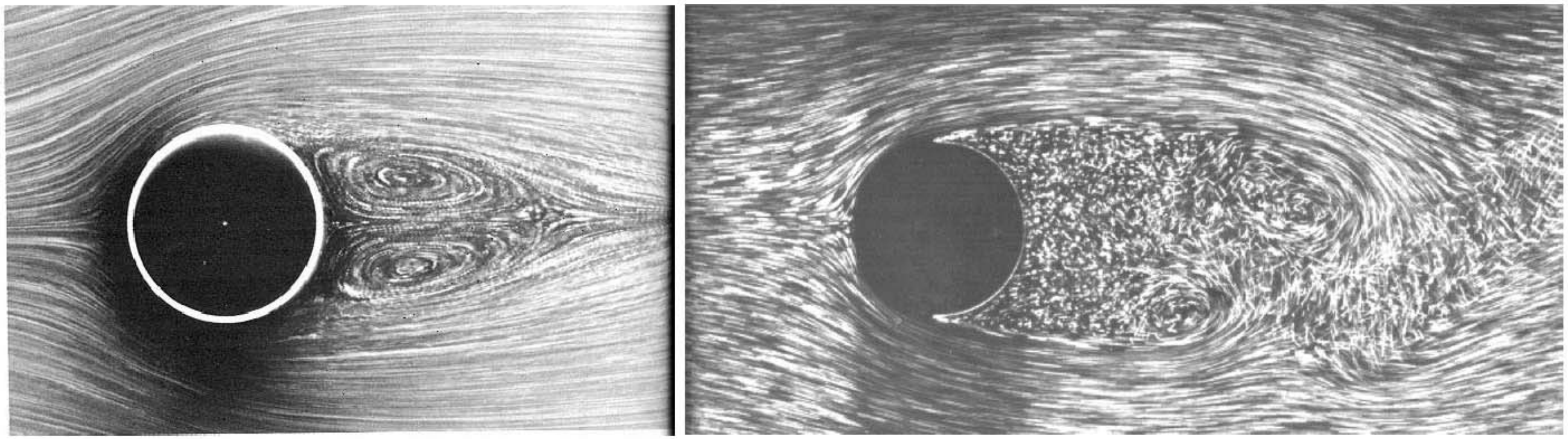
Re = UL/ν

$$\mathbf{v}' = \frac{\mathbf{v}}{\mathrm{U}}$$

$$\mathbf{r}' = \frac{\mathbf{r}}{\mathrm{L}}$$

$$\mathrm{t}' = \frac{\mathrm{tU}}{\mathrm{L}}$$

$$\mathrm{p}' = \frac{\mathrm{p}}{\rho \mathrm{U}^2}$$

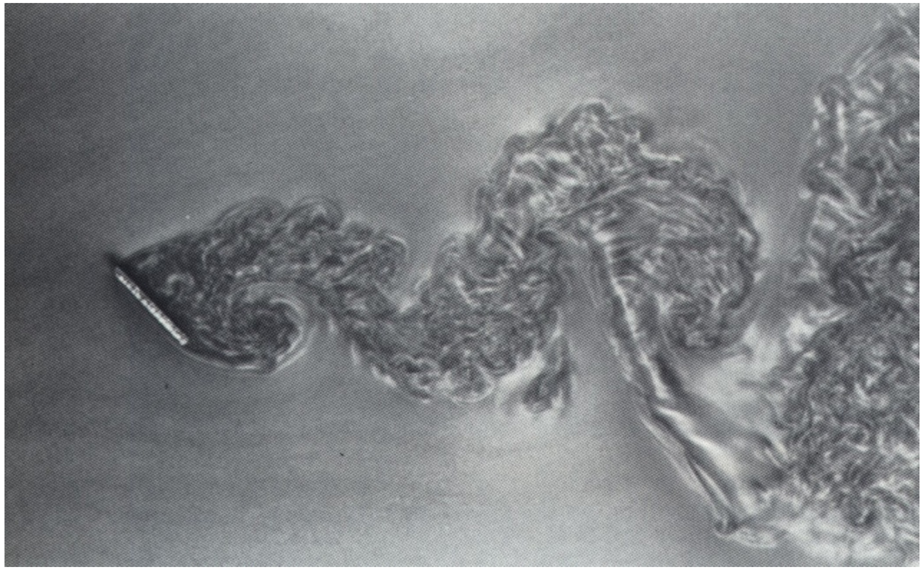


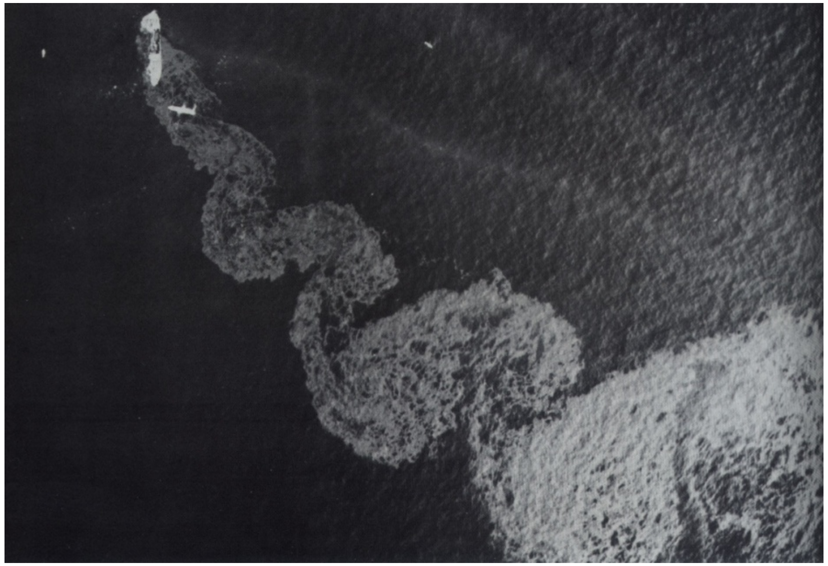
a b

Flow past a circular cylinder. (a) Re = 26 (LAMINAR). (b) Re = 2000 (TURBULENT).

The Reynolds number is a manifestation of dynamical similarity. That is, if we consider two simple flows that are geometrically similar, then they are dynamically identical if the corresponding Re is the same for both, regardless of the specific velocities, lengths and fluid viscosities involved.

Matching such parameters between laboratory testing of a model and the actual full-scale object (the prototype) is the principle upon which aerodynamic model-testing is based.





Above left: wake behind a small flat plate inclined 45 degrees to the direction of the flow (left to right) in the laboratory. Above right: A foundered ship in the sea happens to be inclined 45 degrees to the direction of the current.

Some dimensional considerations

The molecular time scale for momentum diffusion is (dimensionally):

$$t_M \sim \frac{L^2}{\nu}$$

A characteristic time scale for turbulent flows can be estimated from the largest scale flows, of characteristic length L and velocity u, which are the most effective at mixing (again dimensionally):

$$t_T \sim L / u$$

Then $$\frac{t_M}{t_T} \sim \frac{uL}{\nu} = \mathrm{Re}$$

Re of a turbulent flow then can be interpreted as the ratio of a characteristic molecular time scale to a turbulent time scale in the case that the former is evaluated over the same length scale.

It is tempting, but not necessarily useful to think in terms of an effective "eddy" viscosity or diffusivity K so that there is a simple diffusion time for our effective "fluid"

$$t \sim \frac{L^2}{K}$$

Setting this turbulent diffusion time equal to the actual turbulent scale we have (for diffusion of heat)

$$\frac{L}{u} = \frac{L^2}{K} \rightarrow K = uL$$

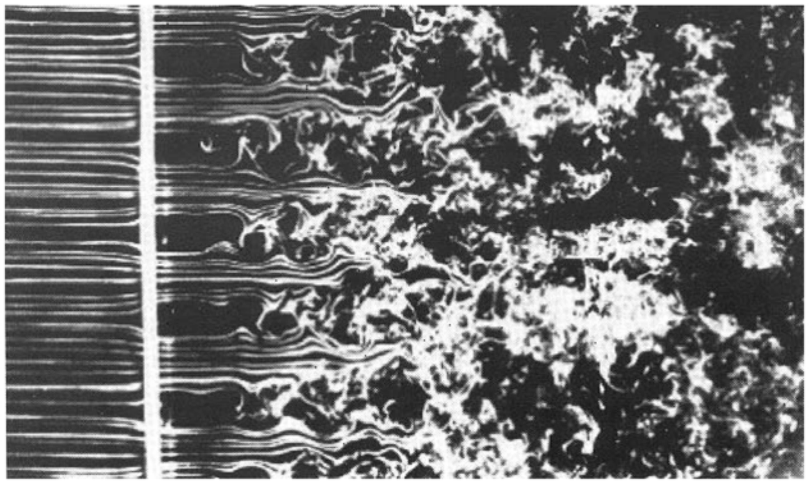
$$\frac{K}{\kappa} \cong \frac{K}{\nu} \sim \frac{uL}{\nu} = \mathrm{Re} \qquad \text{(Valid for gases)}$$

So that Re also appears as a ratio of this turbulent diffusion to molecular diffusion

Approximating homogeneous turbulence in the laboratory by placing a large array of crossed "cylinders in a grid pattern in the flow.

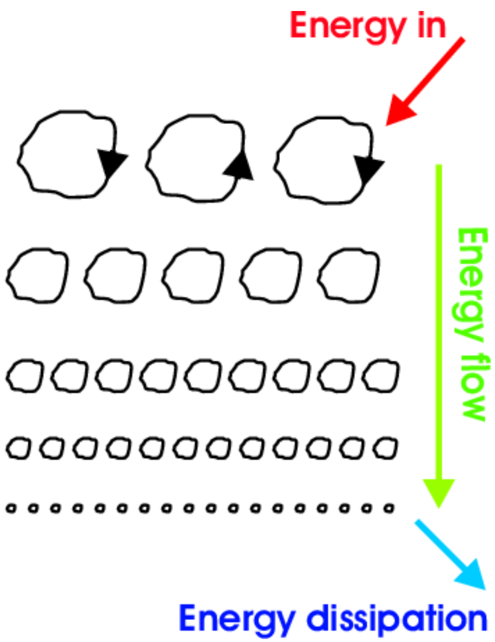
grid

steady flow



Initial turbulent forcing at scale of the mesh size M.

turbulent flow



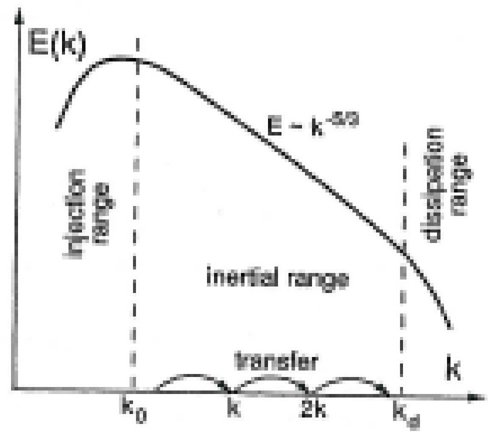
Eddies produced by flow (e.g., through a grid at length scale M= mesh size)

↓

Cascade of energy in inertial range (*local* transfer of energy due to nonlinear inertial term in NS eqn.).

↓

Viscous dissipation at small scales for which local Re~1.



The energy spectrum is derived (Kolmogorov) from dimensional arguments and has the form

$$E(k)=C\varepsilon^{2/3}k^{-5/3}$$

where k is the wavenumber and $E(k)\,dk$ is the kinetic energy in the wavenumber shell between k and $k+dk$ *and* ε is the energy dissipation rate per unit mass. The proportionality constant C is empirically determined to be about 1.5.

The rate of energy dissipation per unit mass, ε, is the same as the rate of energy input.

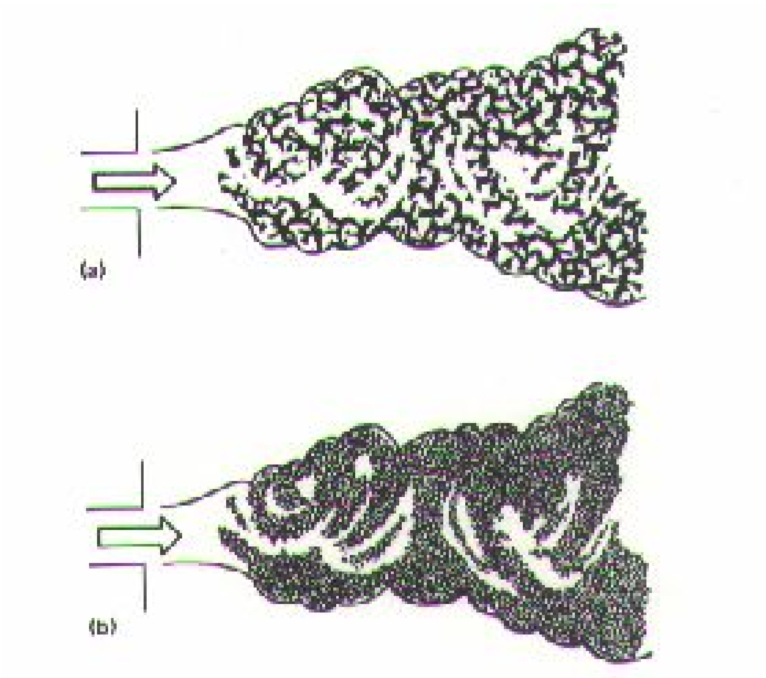
Reducing viscosity simply allows the cascade to continue to even smaller scales until the characteristic Reynolds number of the smallest eddies becomes of the order unity, or when the eddy turnover time is equal to its characteristic diffusion time. The result for the dissipation length or “inner” length scale is:

$$\eta=(\nu^3/\epsilon)^{1/4}=\ell\mathrm{Re}_\ell^{-3/4}$$

$Re_\ell = u\ell/\nu$: characteristic Reynolds number of the large (integral or “outer”) scale ℓ

The difference between two flows with the same integral scale but different Re is the size of the smallest eddies. Index of refraction gradients are steep for the smallest eddies and hence shimmering seen on hot days.

A turbulent jet



(a) “Low” Re. (b) “High” Re

“inner scales” of length, time, and velocity:

$$\eta \equiv \left(\nu^3 / \varepsilon\right)^{1/4}$$

$$\tau \equiv \left(\nu / \varepsilon\right)^{1/2}$$

$$\mathrm{v} \equiv \left(\nu\varepsilon\right)^{1/4}$$

Scale relations

$\eta / \ell \sim (u\ell/\nu)^{-3/4} = Re^{-3/4}$

$\tau/t_T \sim (u\ell/\nu)^{-1/2} = Re^{-1/2}$

$v/u \sim (u\ell/\nu)^{-1/4} = Re^{-1/4}$

Note that $\dfrac{\eta \mathrm{v}}{\nu} = 1$

Given that the analytical approach is difficult due to the nonlinearity of the NS equations, what about direct numerical simulations (DNS) in which the appropriate equations are solved on a computer without making any approximation?

As we just saw, the range of scales needing to be well resolved, ℓ/η, grows as $Re^{3/4}$. If the flow is to be followed in a numerical simulation with a uniform grid, the minimum number of necessary grid points (in three dimensions) is then proportional to $Re^{9/4}$, thus hampering DNS efforts.

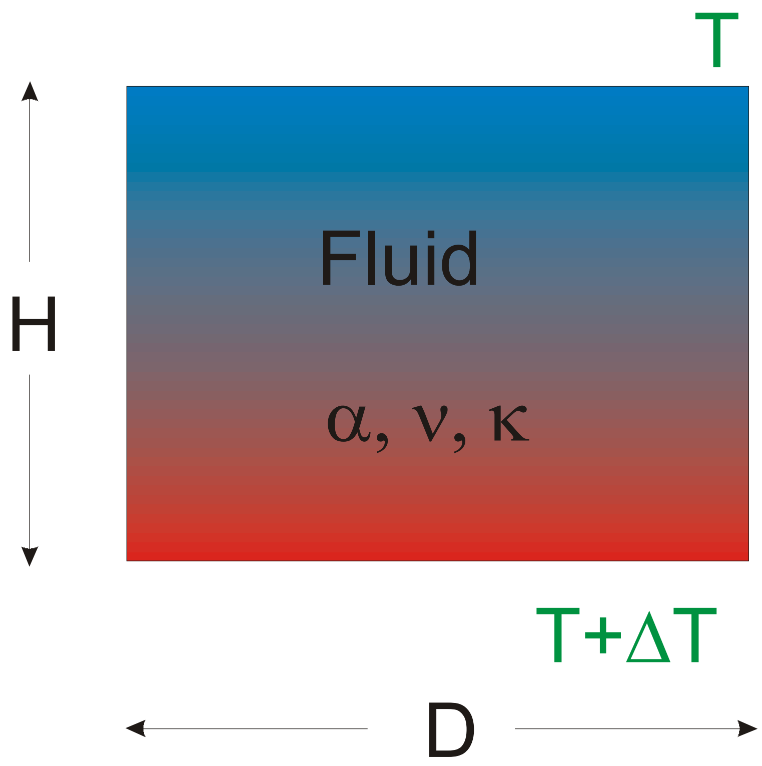
Under certain circumstances, large eddy simulations (which compute only the large scales but model the small scales) and turbulence models (which compute only average information) do better in terms of providing useful information, but they are not satisfactory as universal recipes. The state of the art in computer hardware is years away from allowing us to address the most important problems in natural and engineering fluid turbulence.

So experiments are really needed. But you could ask this question: why not use nature's "laboratories", such as the atmosphere and oceans, which we know can be instrumented and studied for turbulence dynamics? This is certainly less costly and perhaps more relevant to the actual situations that we may be interested in.

Of course, this is done, but is not a substitute for controlled laboratory studies when questions become sharp and a deeper understanding is required.

The majority of large-scale turbulent flows in nature have buoyancy as their major driving force from atmospheric and oceanic flows to turbulent motion in stars

Rayleigh-Benard Convection (RBC)



Viscous diffusion time scale: $\tau_{\mathrm{v}} = \dfrac{\mathrm{H}^2}{\nu}$

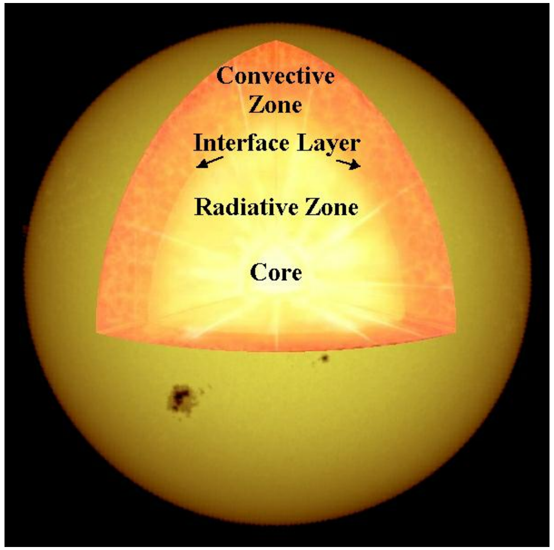
Thermal diffusion time scale: $\tau_{\theta} = \dfrac{\mathrm{H}^2}{\kappa}$

Buoyancy time scale: $\tau_B = \left(\dfrac{H}{g\alpha\Delta T}\right)^{1/2}$

Rayleigh number: $\mathrm{Ra} = \dfrac{\mathrm{g}\alpha\Delta\mathrm{TH}^3}{\nu\kappa}$

Prandtl number Pr: $\mathrm{Pr} = \dfrac{\nu}{\kappa}$

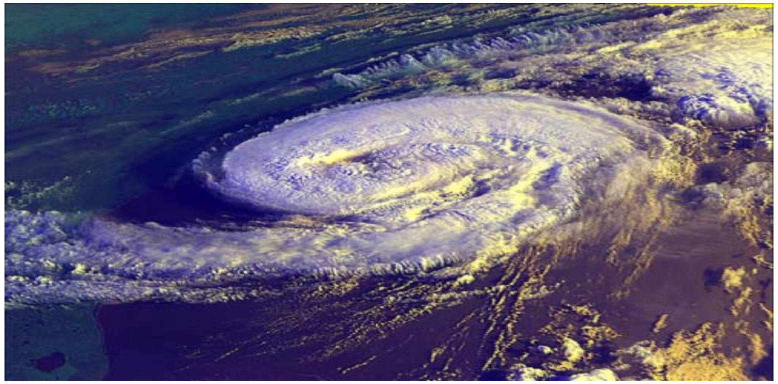
Some of the motivating examples of thermal convection at limiting values of the control parameters in nature



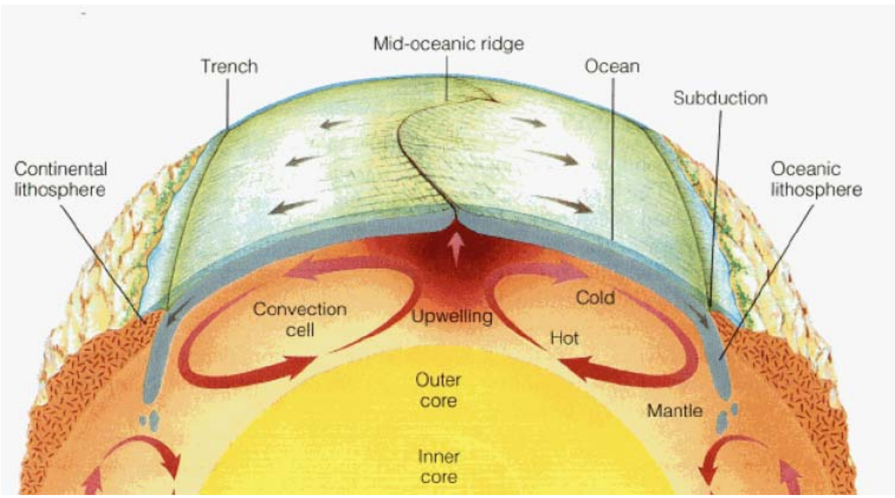
Sun

Ra $\sim 10^{22}$

$10^{-3} < Pr < 10^{-10}$



Atmosphere Ra$\sim 10^{17}$ Pr~ 0.7



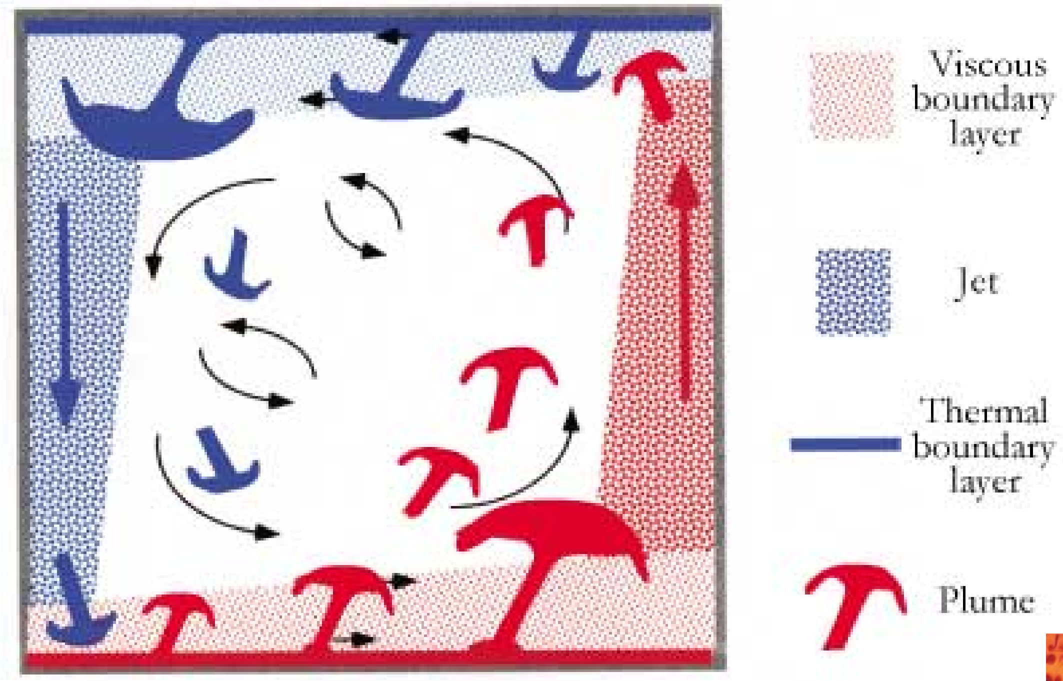
Mantle

Ra $\sim 10^{6}$

Pr$\sim 10^{21}$

(Pr$\sim 10^{3}$,magma)

Cartoon of turbulent convection



(from L. Kadanoff, *Physics Today*, August 2001)

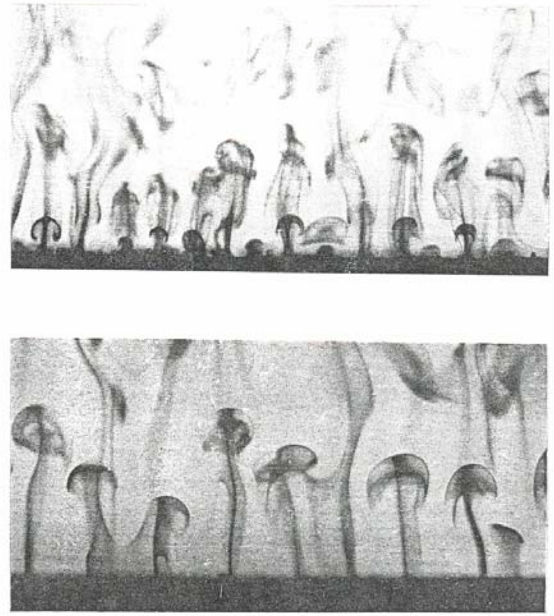
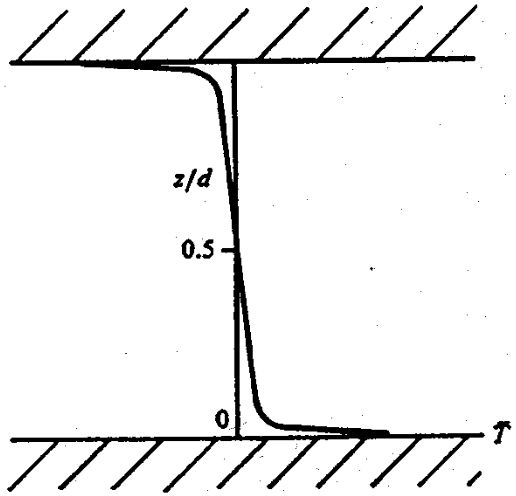
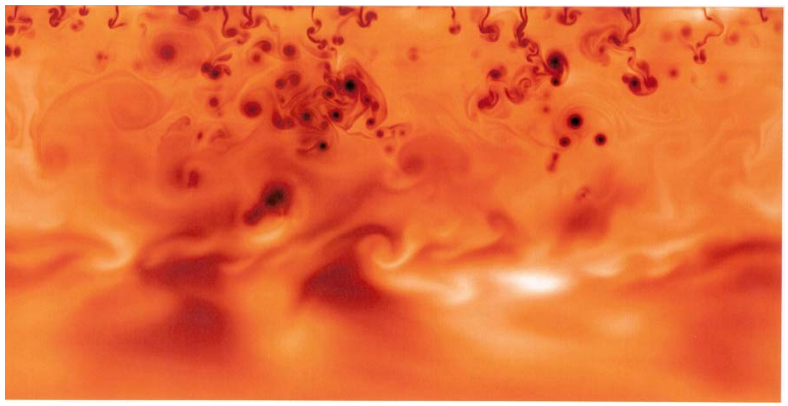


FIGURE 1. Photographs of thermals rising from a heated horizontal surface.

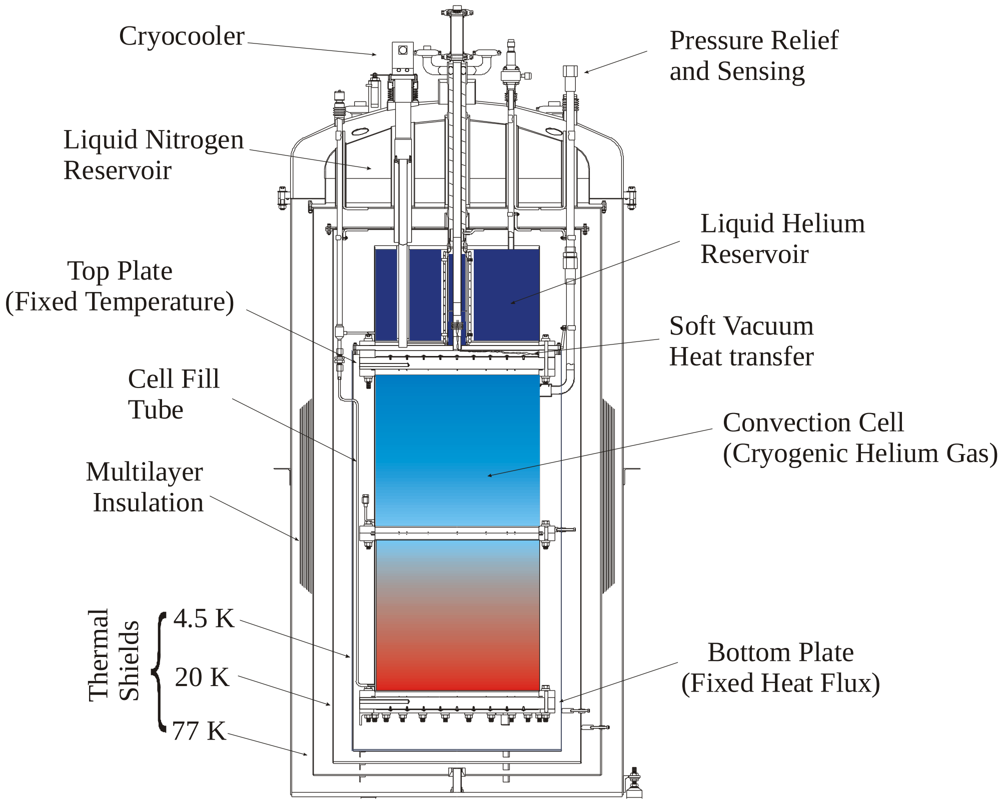




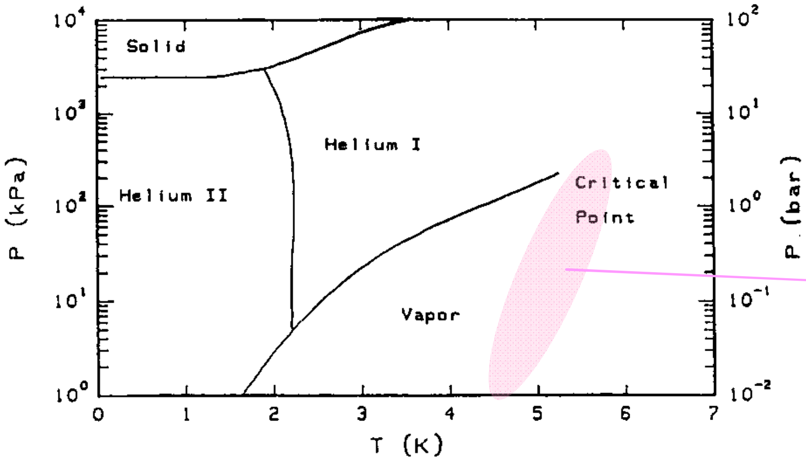
Solar plumes (simulation, U. chicago)

Atmospheric plumes (Leonardo Da Vinci)

The ICTP apparatus for turbulent convection studies



Elettra Synchrotron Laboratory



$$Ra = g \cdot \left(\frac{\alpha}{\nu\kappa} \right) \cdot \Delta T \cdot H^3$$

$$\downarrow$$

$$10^{-3} < \frac{\alpha}{\nu\kappa} < 10^{9}$$

Water: $\frac{\alpha}{\nu\kappa} \cong 10$

Air: $\frac{\alpha}{\nu\kappa} \cong 0.1$

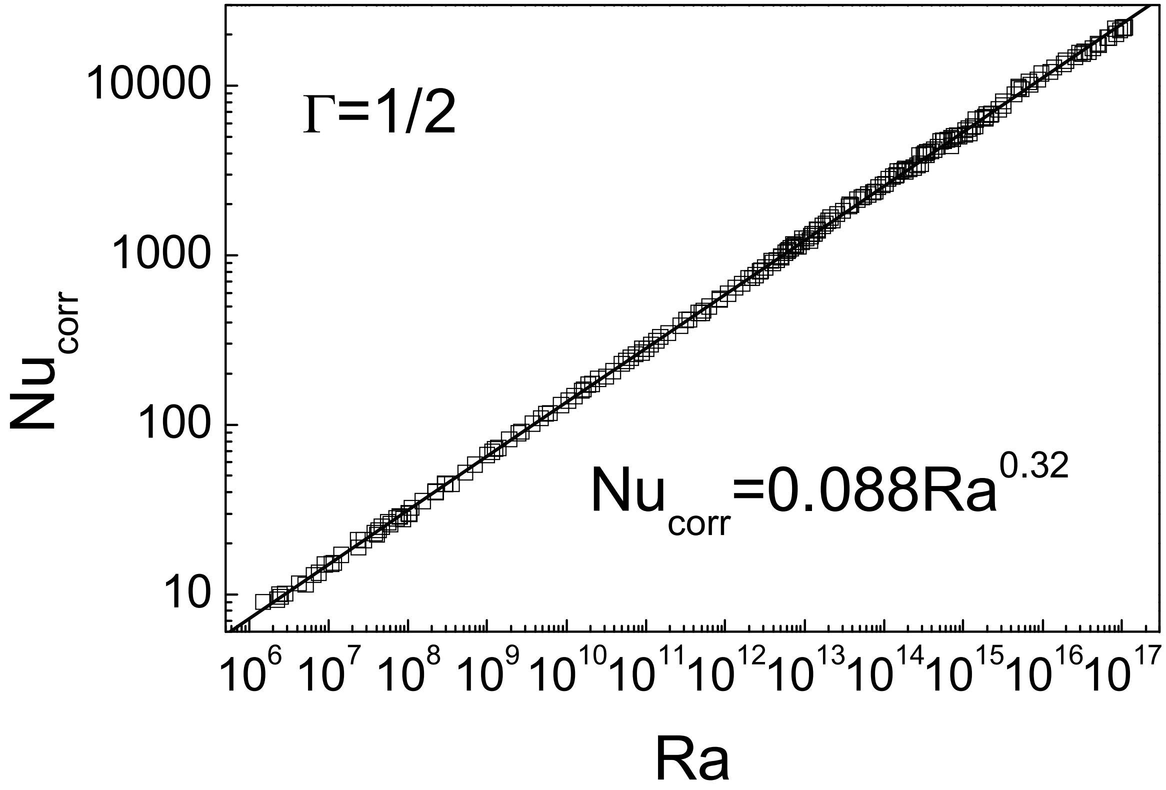
12 orders of magnitude variation of Ra!

Plus: ultra high Ra ($>10^{17}$)

Turbulent transfer of heat energy

Nu= dimensionless measure of heat transfer
(= *ratio of turbulent to molecular thermal conductivity*)

Conductivity enhancement by 20,000!



Log-log plot of the Nusselt number versus Rayleigh number

J.J. Niemela, L. Skrbek, K.R. Sreenivasan & R.J. Donnelly, *Nature,* **404**, 837 (2000)

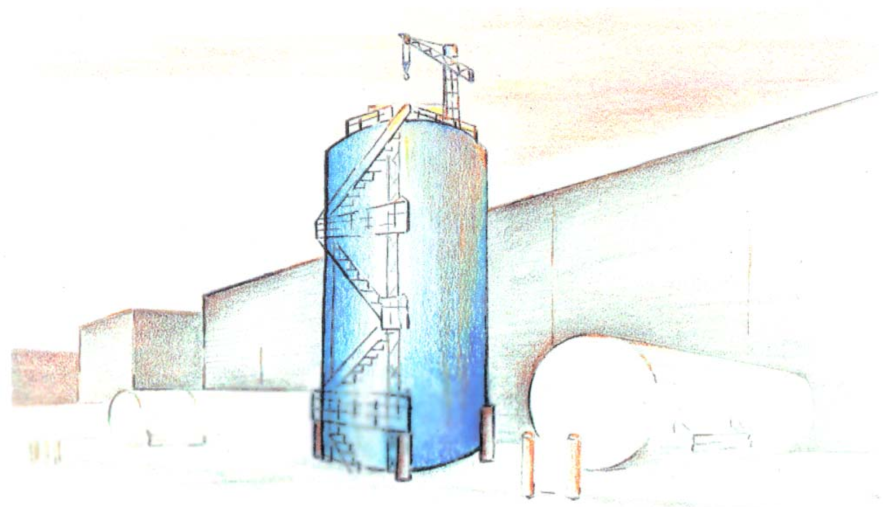
J.J. Niemela & K.R. Sreenivasan*, J. Fluid Mech.,* **557** 411-422 (2006).

Proposals to use large cryogenic facilities….

Right: a proposed convection cell capable of Ra~10^{21}

20 m high (outside), 7m diameter

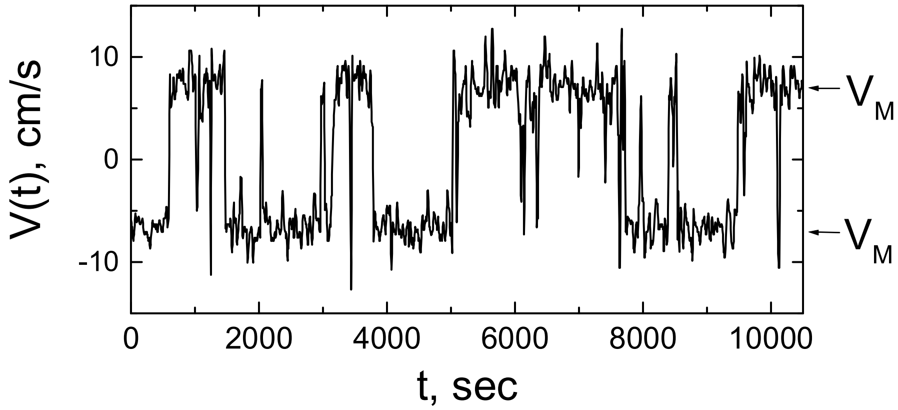
<u>Refrigeration needed</u>
< 200 W



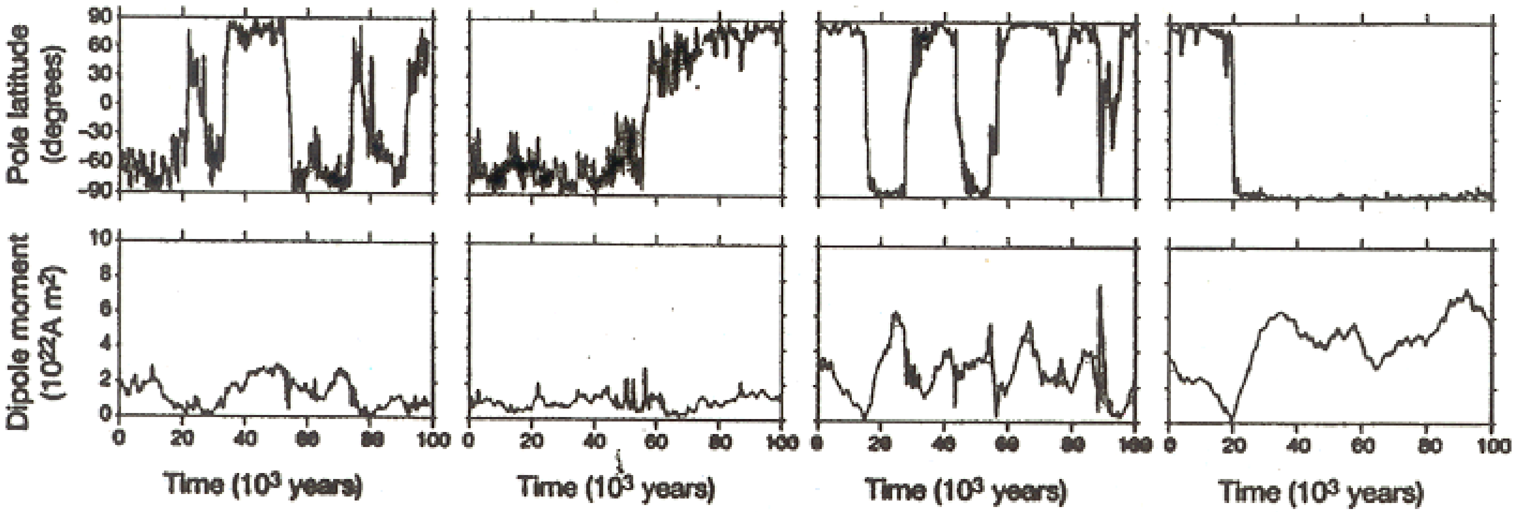
RHIC, BNL

Huge accelerator facilities like CERN or BNL would have plenty of liquid helium on hand, used to cool superconducting magnets.

A mean wind and its reversals

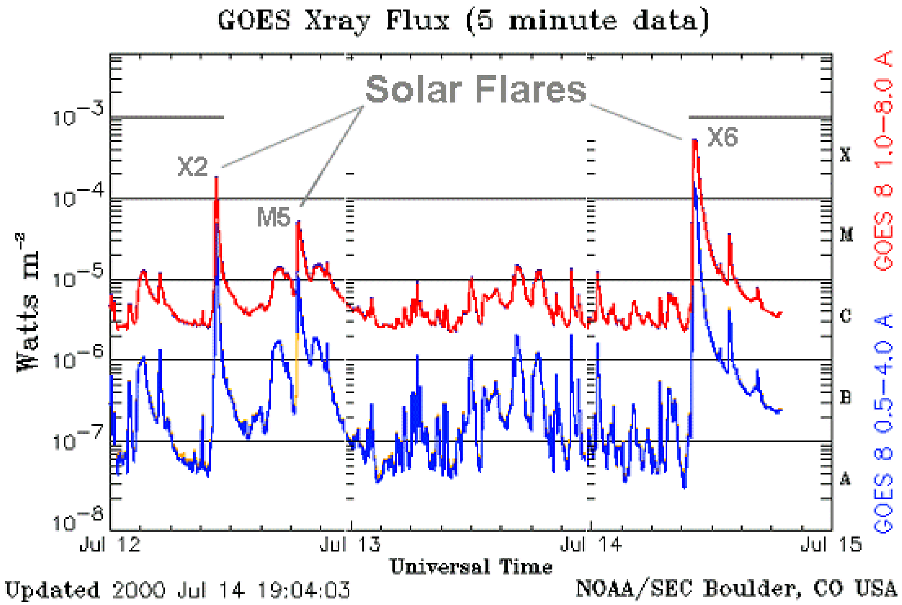


Geomagnetic polarity reversals: range of time scales~ 10^3-10^5 years.

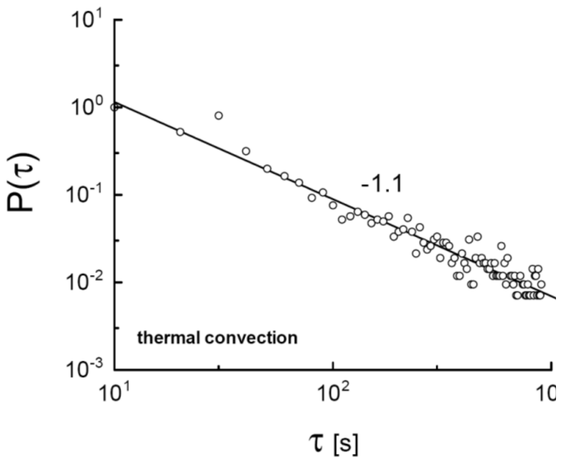


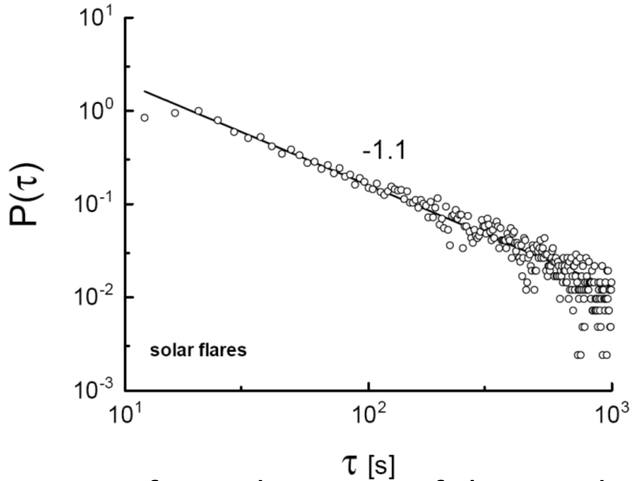
Glatzmaier, Coe, Hongre and Roberts *Nature* **401**, p. 885-890, 1999

Lifetimes of solar flares depend on convective motions



Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI)

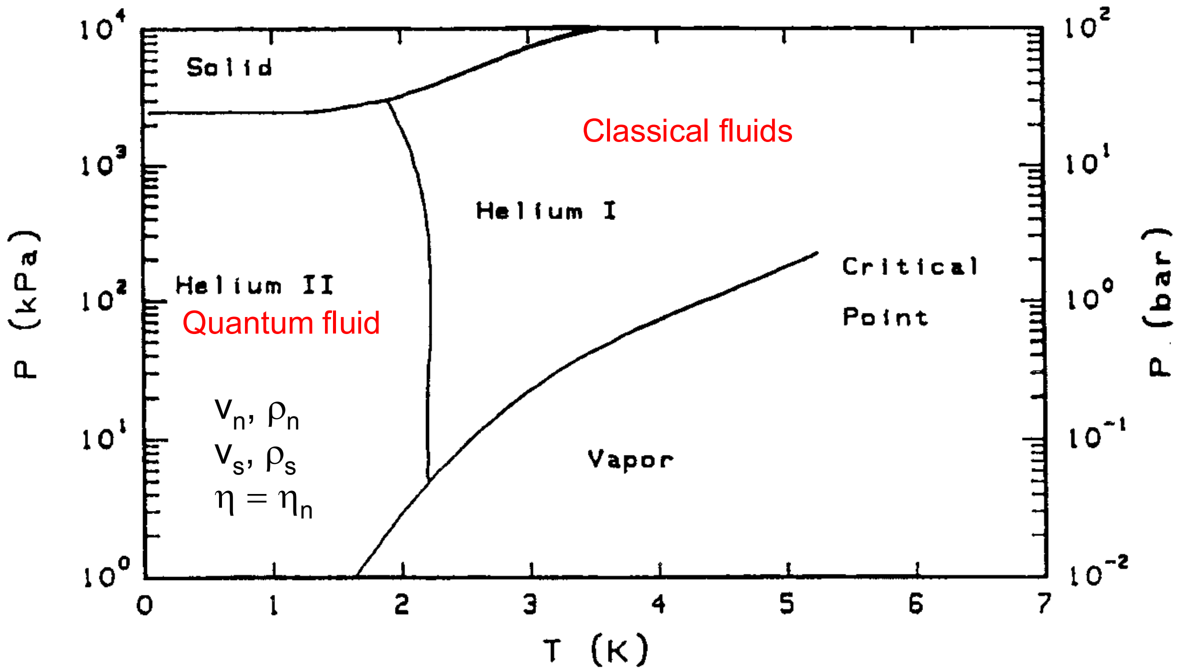




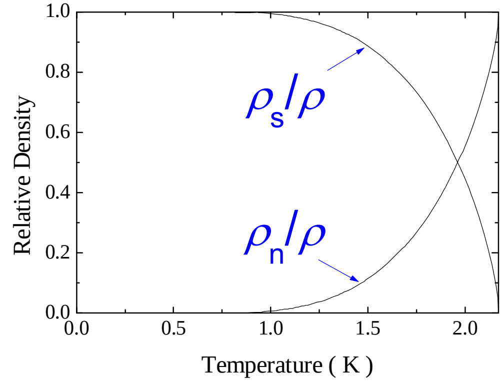
Bershadskii, Niemela and Sreenivasan (2004)

PDFs of duration times for (left) the maintenance of one direction of the wind in confined thermal convection and (right) medium energy solar flares.

Superfluid turbulence



Below T_λ: two fluid model



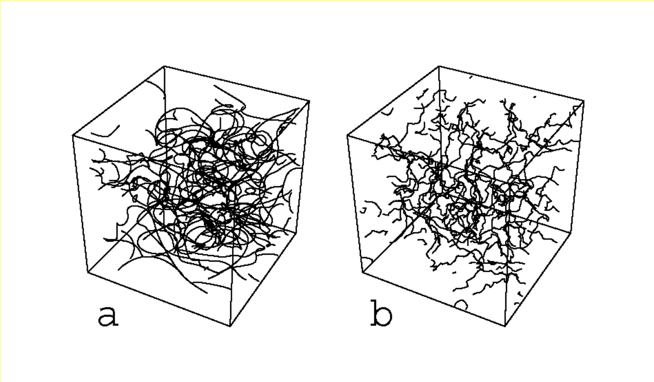
irrotational (curl v_s=0)!

A solution in cyl. coordinates of the NLSE for an ideal Bose gas $\Psi_0 = f(r)e^{i\phi}$ describes a quantized vortex, where f(r) represents the superfluid density varying from *zero* as r goes to zero to a constant value at vortex core radius of approximately one Angstrom (see Koplik and Levine, PRL 1993).

$$\mathbf{v_s} = \frac{\hbar}{m_4}\nabla\varphi \longrightarrow \text{circulation: } \kappa = \oint \mathbf{v_s}\cdot d\mathbf{l} = \frac{\hbar}{m_4}\Delta\varphi = n\frac{h}{m_4}$$

Feynmann (PLTP, 1955) envisioned turbulence as a tangle of such quantized vortices

Turbulent tangles of quantized vortices

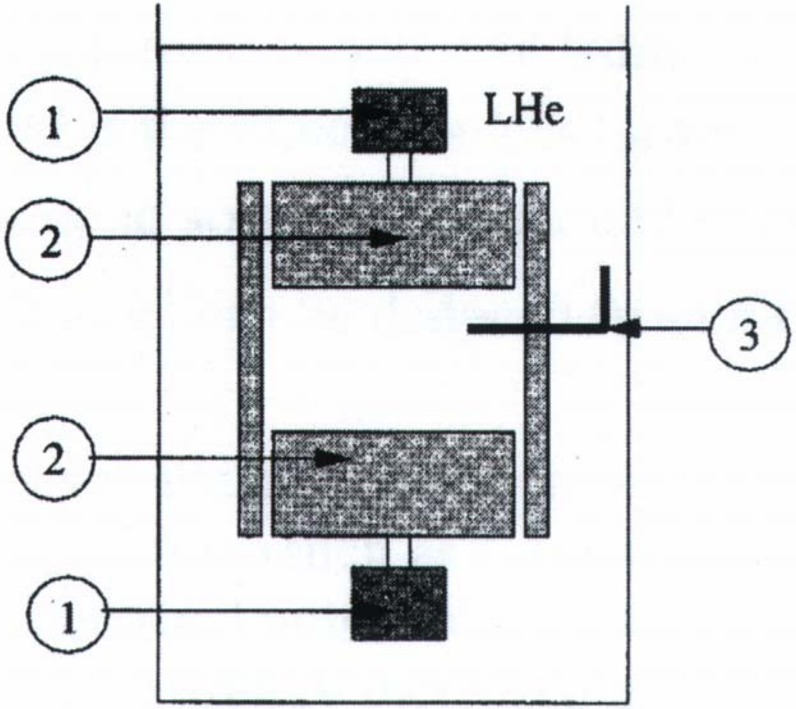


The simulated tangle of quantized vortices on the left corresponds to 1.6 K, while that on the right is at 0 K. After Tsubota, et al (2000).

For a random tangle of vortex lines there is only one length scale, the average inter-vortex line spacing

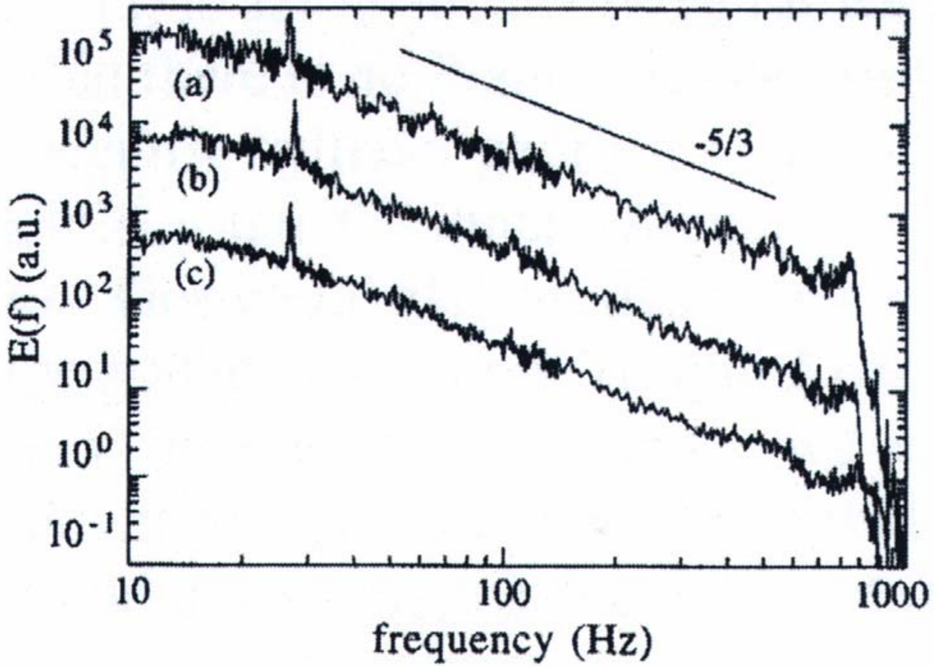
Turbulent flows in the Kolmogov sense must mimic eddies on all scales through *partial polarization* of vortex bundles.

Superfluid “washing machine” (Maurer and Tabeling 1998)



Counter-rotating disks

Strong evidence of classical energy cascade

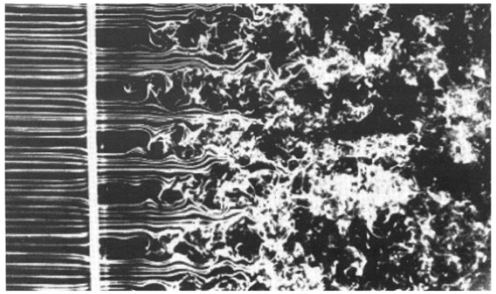


(a) Helium I

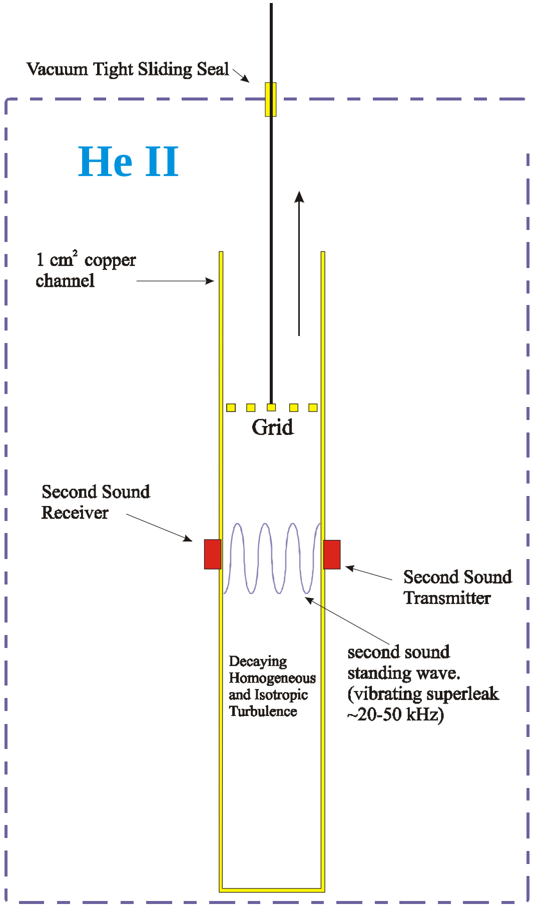
(b) Helium II $\rho_n \sim \rho_s$

(c) Helium II $\rho_s/\rho \sim 1$

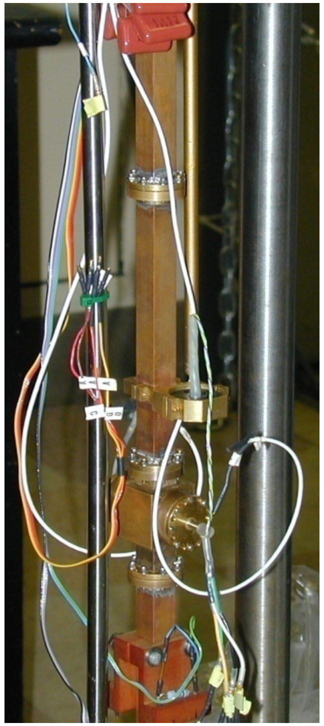
Grid flow in a superfluid: quasi-classical turbulence



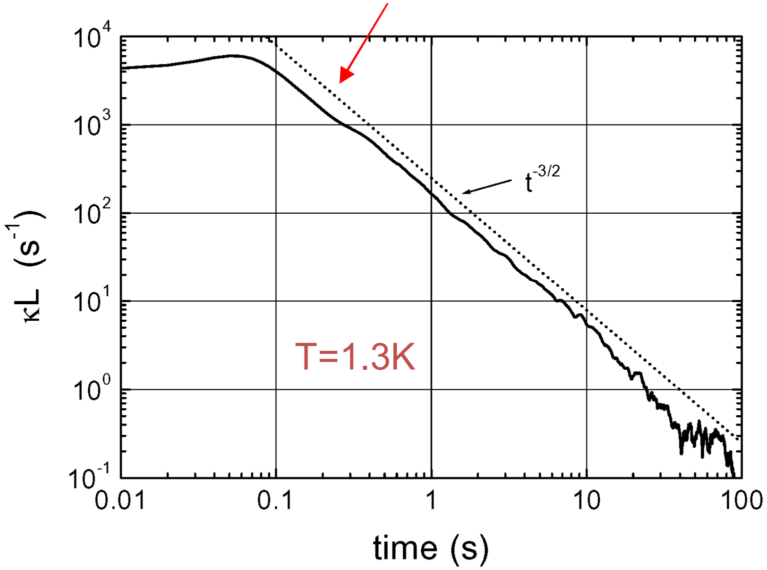
Dissertations: M.R. Smith, S.R. Stalp
Stalp et al (PRL, 2000; Phys. Fluids, 2002)
Skrbek, et al (PRL, 2001);
Niemela, et al (JLTP, 2005)



Pocket-size! 1-cm square channel



Classical decay law

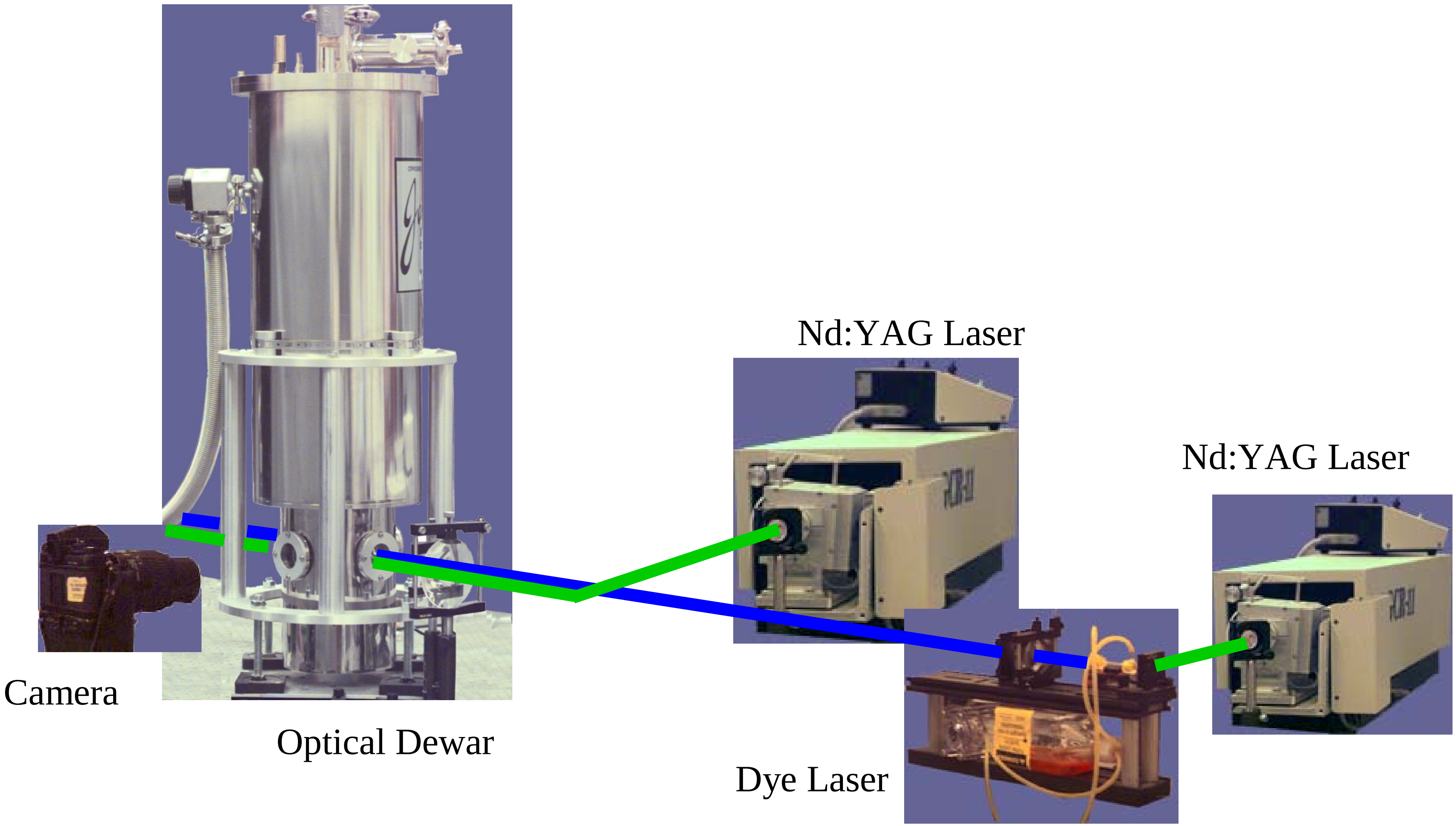


$$L(t) = \frac{(3C)^{3/2} d}{\kappa \nu'^{1/2}} \cdot t^{-3/2}$$

Measure decay of L = length of vortex line per unit volume

Better arrangement using dedicated optical cryostat:

Cryogenic PIV (particle image velocimetry) experiment (shown here at Yale University)

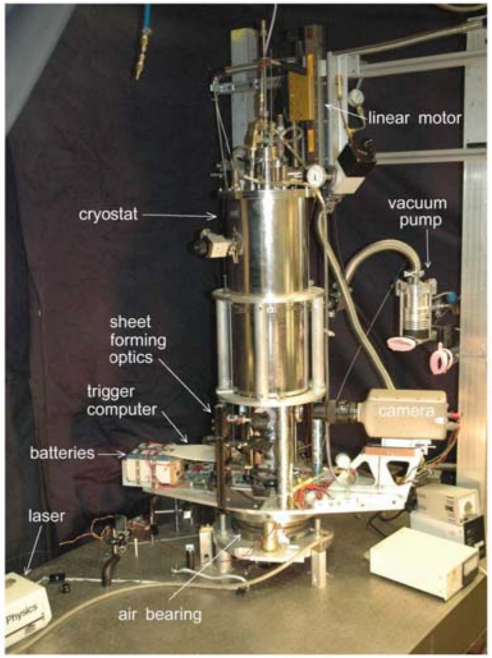


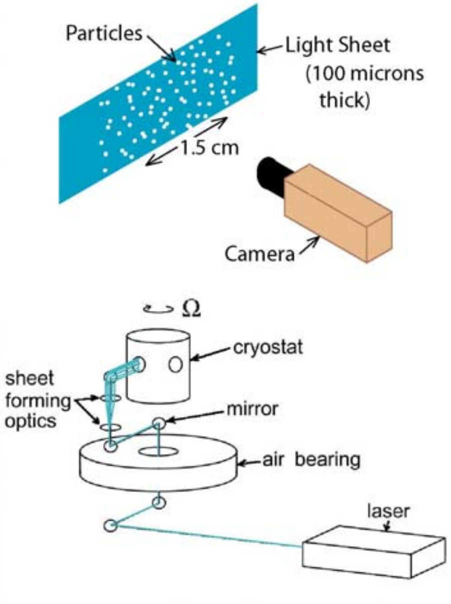
Visualizing quantized vortices

Dissertation: Bewley
Bewley, et al (*Nature* 2006)
Paoletti, et al (*PRL* 2008)

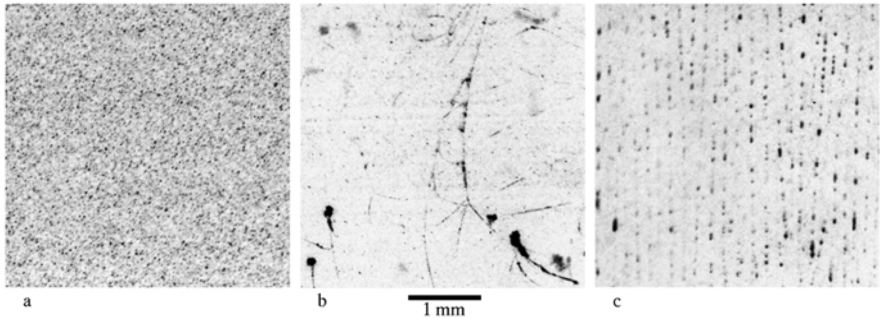
JJN: “Reconnecting to superfluid turbulence”
http://physics.aps.org/articles/v1/26

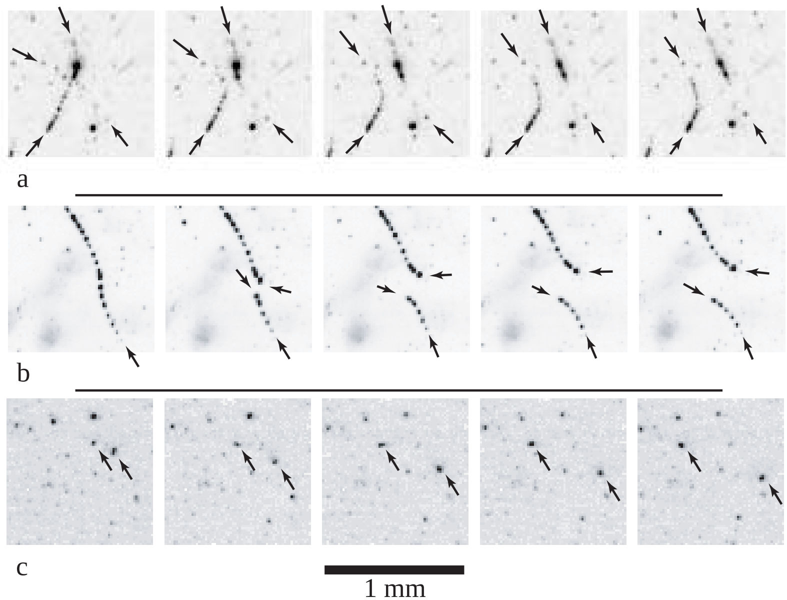
Apparatus





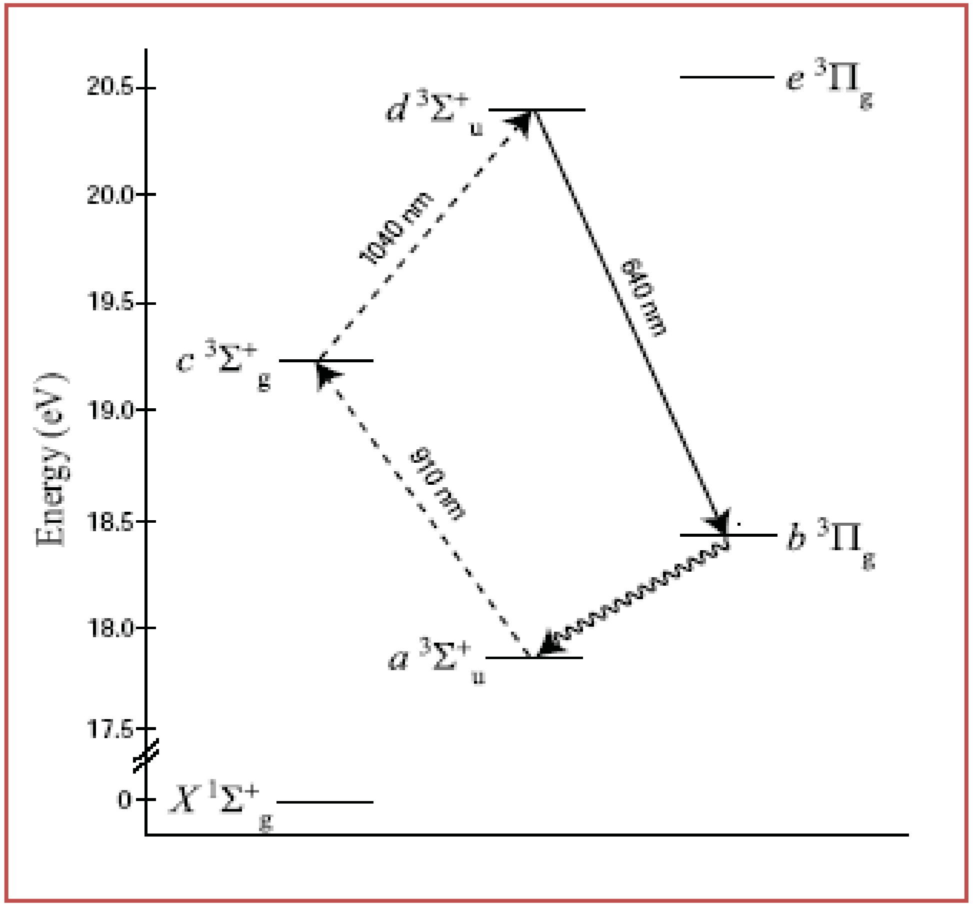
Laser Beam routing





Future work: Laser induced fluorescence of helium molecules (created by electron bombardment)

From Dan McKinsey, Yale U.



- Illuminate with pulsed i/r laser at 910 nm (modest power).
- Immediately after, illuminate with pulsed i/r laser at 1040 nm.
- Observe decay of $d^3\Sigma^+_u$ to $b^3\Pi_g$ with emission at 640 nm (lifetime 25 ns).
- The $b^3\Pi_g$ returns to $a^3\Sigma^+_u$ by non-radiative processes (may need to be accelerated by optical means)
- Process recycles.
- $\rightarrow$ ~ $4\mathrm{x}10^7$ photons/s at 640 nm.

Shadowgraphy/Schlieren methods for thermal flows:

Shadowgraphy
(from Woodcraft et al)

Integrates along optical path

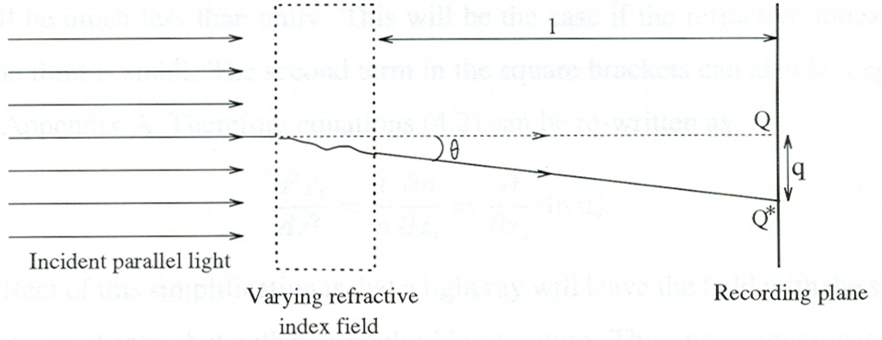
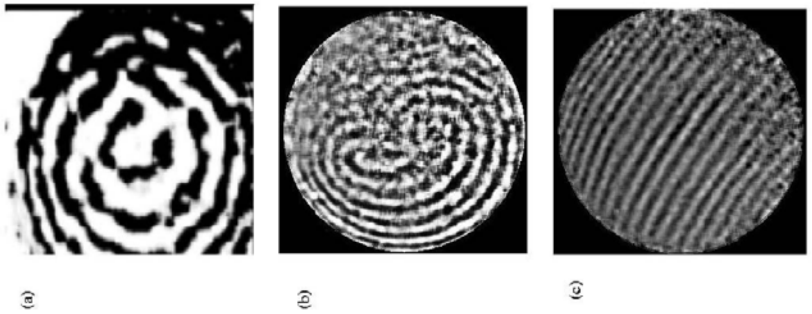


Figure 4.4: *Deflection of light by an arbitrarily varying refractive index field*



From Lucas group, Manchester near convective onset in liquid.

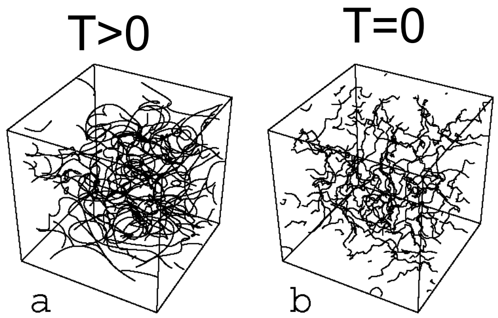
Refractive index *n* and density ρ are related through the Clausius-Mossotti relation:

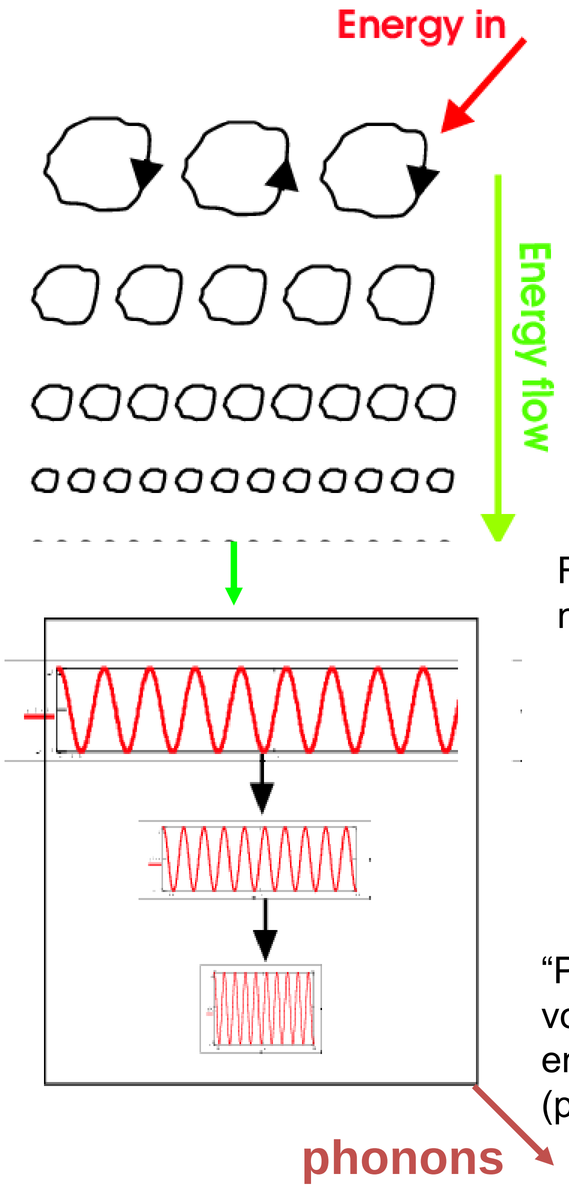
$$\frac{n^2 - 1}{n^2 + 2} = \rho \frac{4\pi}{3M}\gamma$$

M=molecular weight, γ = polarizability

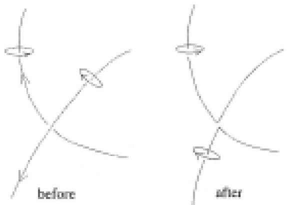
Visibility depends on (n-1) $\alpha\Delta T$—–—even though n ~1 for both gas and liquid, α can be rather large especially near the critical point.

Where does the turbulent energy go at T=0 where there is no viscosity?

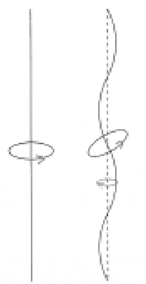




Reconnections are need to bridge the gap



“Plucked” vortex cores emit sound (phonons)



Classical Richardson cascade on scales greater than vortex line spacing.

↓ energy flow

Kelvin wave cascade on scales less than ℓ .

↓ energy flow

Phonons