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Propagation of optical radiation through atmospheric turbulence

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PROPAGATION OF OPTICAL RADIATION THROUGH ATMOSPHERIC TURBULENCE

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ATMOSPHERIC REFRACTIVE INDEX

describes propagation of optical radiation in the atmosphere

In clear air: absence of fog, smog and any other pollutant

ATMOSPHERIC TURBULENCE

is the main process affecting propagation of optical radiation because it produces fluctuations of refractive index.

Examples:

- Star twinkling (scintillation)

-Images over worm surface, or through jets stream

Turbulence description:

Refractive index n is a random function.

-For propagation purposes the **clear** atmosphere is completely described by a **real** refractive index (real = no absorption).

- To investigate this complex problem, it is useful to divide the refractive index of the atmosphere into two parts,

mean and fluctuating,

related to different processes.

1- The **mean part**, characterized by slow variations in time (long time-scale behaviour), is affected by large scale mean flows, due to wind and thermal convection, such as those during day and night;

2 - The **fluctuating part** is characterized by rapid temporal variations (short time-scale behaviour), due to small-scale turbulence motion.

In quasi-stationary conditions, that is when the mean value is roughly constant, the fluctuations are generally considered homogeneous and isotropic and the classical theories of turbulence can be applied.

WHERE FLUCTUATIONS COME FROM ?

In dry air refractive index **n** can be written as

$$\mathbf{n = 1 + b\ P/T}$$

T temperature and P pressure; b is dimensional constant.
b = 79 10^6 K millibar^{-1} for red light i.e. radiation of wavelength 600 nm.

Refractive index fluctuations, δn, are thus related to temperature fluctuations δT and pressure variations δP

$$\delta n = b\,(\delta P/P - \delta T/T)\,P/T$$

Positive fluctuations of pressure, e.g. due to wind, produce positive fluctuations of refractive index , while positive temperature fluctuations produce negative fluctuations.

At point Q in the atmosphere and time t, $n = n(Q,t)$ let us separate mean part and (rapidly) fluctuating part,

$$\mathbf{n = n_o + \delta\mu}$$

n_o average

$\delta\mu$ random fluctuation around the mean value

Average

$$n_o = 77.6\left(1 + \frac{\lambda^2}{7.52\ 10^{-3}}\right)\frac{P}{T}\ 10^{-6}$$

P pressure (in mbar), **T** temperature (Kelvin) , λ wavelength in μm.

Typical value in standard conditions $n_o = 1.00029$.

Quantities of interest are correlation functions of fluctuations $\delta\mu_1$ and $\delta\mu_2$ at two points P_1, P_2 , as we will see later.

INTUITIVE PRESENTATION OF CONSEQUENCES ON OPTICAL WAVES

In a homogeneous, and isotropic medium, eat ach point of a wavefront, phase velocity V is constant

V= c/n (c light velocity in the vacuum).

and a propagating wave, e.g. plane wave, keeps its wavefront plane.

When propagating. in a medium with small random fluctuations of the refractive index, the velocity of a wave at each point

$$V(P,t) = c / n(P,t)$$

randomly fluctuates in time, around an average value, producing fluctuations of phase around the unperturbed value. The wavefront becomes corrugated and fluctuates in time, with respect to an average wavefront, which is the wavefront of the unperturbed wave.

In the atmosphere, the refractive index fluctuations can. produce strong phase fluctuations even over very short paths, e.g. fractions of meter. This is due to the presence of the wavenumber, $k=2\pi/\lambda$,, in the phase. For instance, the complex amplitude of a TEM plane wave propagating in the z direction can be written as

$$v(P) = A \exp(iknz)$$

where k is wavenumber. Phase:

$$\phi = k n z$$

At optical frequencies $k \sim 10^7\ m^{-1}$ and strongly enhances even small fluctuations of n, e.g. in 1 m path length.

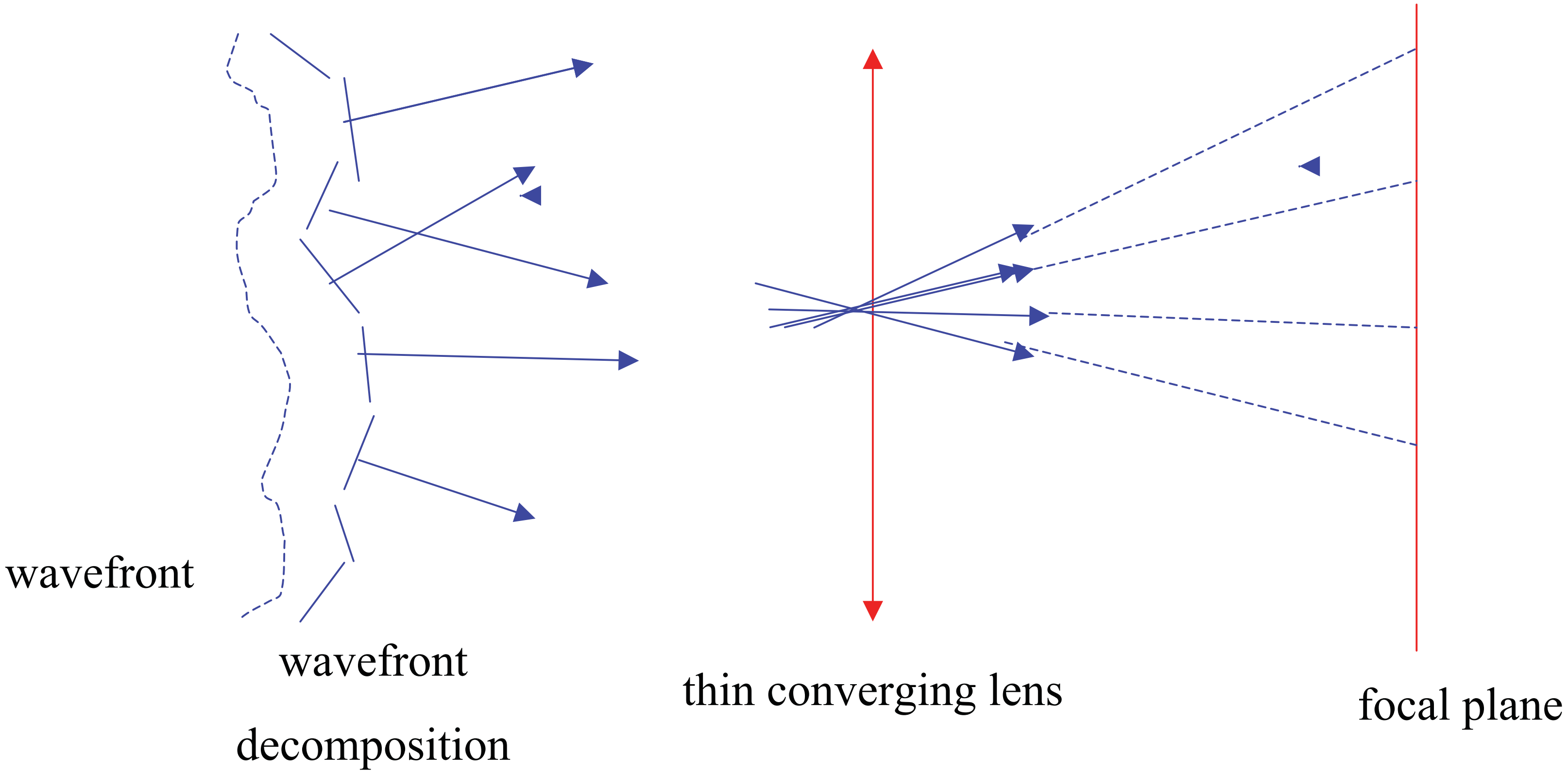
CONSEQUENCE OF WAVEFRONT DETERIORATION

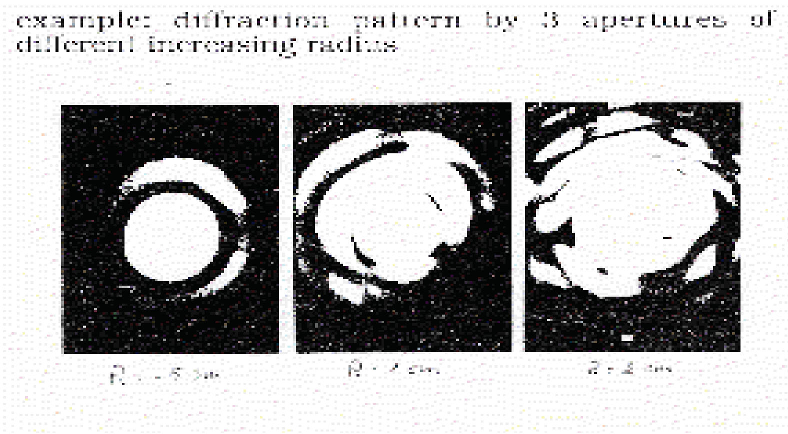
performance deterioration of systems operating in the atmosphere, such as image producing systems.

SIMPLE EXAMPLE: LENS ILLUMINATED BY PLANE WAVE

If no turbulence Airy pattern. Decompose the deteriorated wavefront, impinging, at a given time, in small pieces of plane wave. Each one is focused by the lens as an Airy pattern at different points of the focal plane. All patterns rapidly fluctuate in time.

Result: enlargement and blurring of the image.





a **b** **c**

DIFFRACTION PATTERN

From P. Burlamacchi, A. Consortini: Study of Atmospheric Turbulence by Means of a Laser beam - Optica Acta, 14, 17 (1967)

L ~ 50 m

h ~ 1.5 m

Diffraction patterns of circular apertures illuminated by a plane wave (expanded and collimated laser beam) after about 50 m horizontal path in the atmosphere at a height of about 1.5 m above ground. Aperture radius: a) 0.5 cm, b) 1 cm and c) 2 cm. These patterns compare to Airy pattern.

General effects of turbulence on images: it lowers maxima, increases minima and makes zeros disappear. Blurring.

QUALITY OF IMAGES: STREHL RATIO

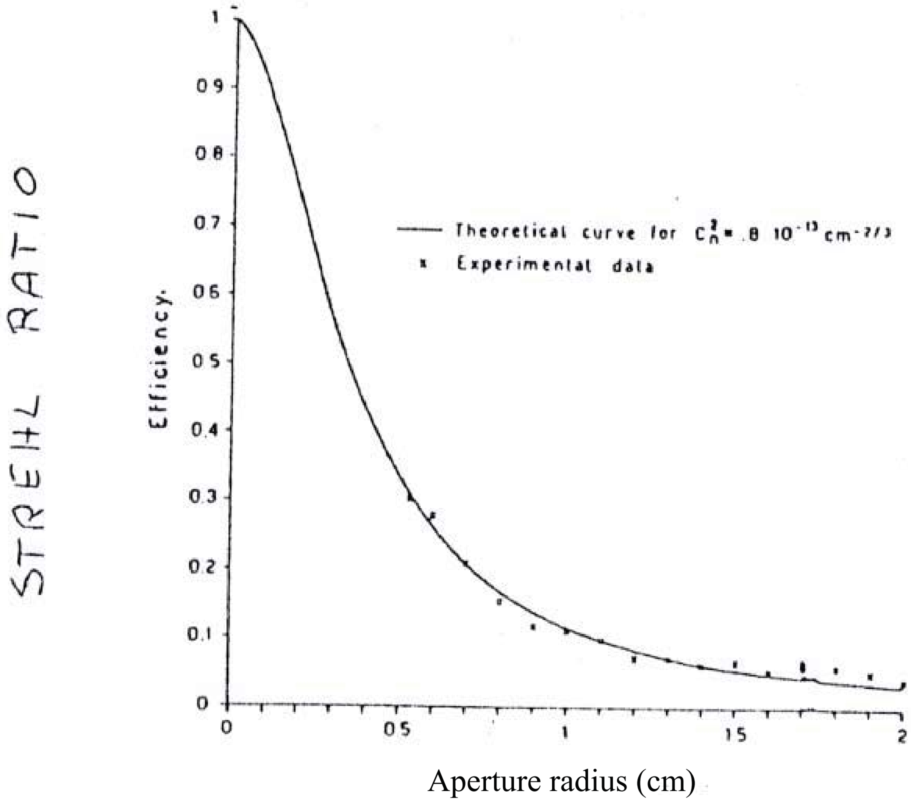
Strehl ratio, S, previously called Efficiency, of an optical system is defined as the ratio between the maximum Intensity in the centre of the image and the Intensity value theoretically expected. S=1 means a perfect image. If S is larger than 0.80 the image is considered good.

In our previous example, Strehl ratio goes from ~ 0.30 and down, see next page.

In 1960/70 theoretical and experimental results let the scientists realize that atmospheric **turbulence puts severe limits** to the Resolving Power **R**, of an optical system, of diameter D. In the absence of turbulence:

$$\mathbf{R = \lambda/D} \quad \text{definition (Rayleigh).}$$

Due to turbulence, increasing the system diameter does not improve the resolving power. **Turbulence puts a limit to the useful diameter** and, in practical situations, also very large astronomical telescopes have the resolving power of a lens with a diameter of a few tens of centimetres. A measure of this diameter is the so called Fried parameter, r_o, theoretically introduced by D. Fried.



Example of Strehl ratio versus the aperture radius, from the above diffraction measurements (Optica Acta)

Refractive index relevant quantities are: **Correlation function**, or **Structure function** or **Spectrum** (if any), of the refractive index fluctuations at two different points P_1 and P_2

$$B_n(P_1, P_2) = \langle \delta\mu_1 \, \delta\mu_2 \rangle$$

$$D_n(P_1, P_2) = \langle (\delta\mu_1 - \delta\mu_2)^2 \rangle$$

$\langle \dots \rangle$ ensemble average. Ergodic theorem: time average.

$$B_n(P_1, P_1) = \langle \delta\mu_1^{\,2} \rangle \qquad \text{variance at } P_1$$

HOMOGENEOUS ISOTROPIC CASE

$$B_n(r), \quad D_n(r), \quad \Phi_n(\kappa)$$

Spectrum: Fourier Transform of the Correlation Function.

$$D_n(r) = 2\,[B_n(0) - B_n(r)]$$

r denotes distance between the two points, and κ spatial frequency.

MODELS OF ATMOSPHERIC TURBULENCE

A **given form of the correlation function** of refractive index, or of **the structure function** or of the **corresponding spectrum**, is called a model of the atmospheric turbulence.

No universal model.

Recall Turbulence: from large eddies (vortices) cascade towards smaller and smaller ones. (See Lecture by Prof. Niemela)

Outer scale L_o and inner scale l_o

Input range, inertial range, dissipation range

THE PROPAGATION PROBLEM

Find:

1 - **Phase Correlation function and**

2 - Amplitude Correlation function

(or corresponding structure functions or spectra)

- at the end of a path
- for a **given wave** and a **given TURBULENCE MODEL**

The first case investigated was propagation of a plane wave; then spherical waves and later laser beams were investigated.

KOLMOGOROV MODEL

Generalization to atmosphere of Kolmogorov model of turbulence

Structure function of the refractive index between two points at distance r

1) $D_n(r) = C_n^2 \, r^{2/3}$ for $l_o < r < L_o$

2) $D_n(r) = C_n^2 \, l_o^{2/3} \, (r/ l_o)^2$ for $r < l_o$

C_n^2 Structure Constant

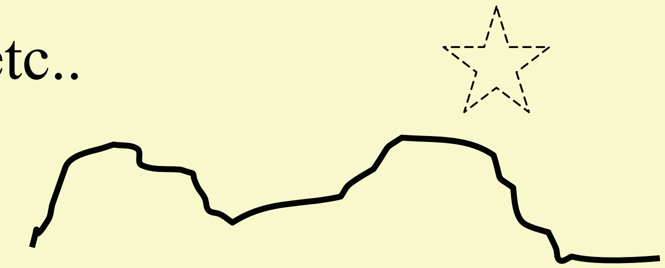
Eq. 1 well known **two third law**

Interval $l_o < r < L_o$ **inertial range**

Interval $r < l_o$ **dissipation range**

For $r > L_o$ no homogeneity and correlation function vanishes.

TURBULENCE PARAMETERS



C_n^2 varies during day, season, level etc..

At ground level:

C_n^2 from ~ 10^{-13} $m^{-2/3}$ to ~ 10^{-17} $m^{-2/3}$

At higher levels

C_n^2 from ~ 10^{-16} $m^{-2/3}$ to ~ 10^{-19} $m^{-2/3}$

Inner scale: from millimetre to some ten millimetres depending on situation.

Outer scale: at ground level assumed of the order of height over ground. In high atmosphere many different values measured from half meter to hundred thousand meters.

Spectrum of Kolmogorov Model

Spectrum used by Tatarskii for the Kolmogorov model

$$\Phi_n(\kappa) = 0.033\, C_n^2\, \kappa^{-11/3} \exp(-\kappa^2/\kappa_m^2)$$

$$\kappa_m = 5.92/\, l_o$$

Inertial range

$$l_o < r < L_o$$

$$2\pi/L_o < \kappa < 2\pi/l_o$$

This approximate spectrum gives the correct dependence of the structure function in the inertial range and in the dissipation range and corresponds to an outer scale infinitely large.

Most theoretical results on propagation were obtained by this model

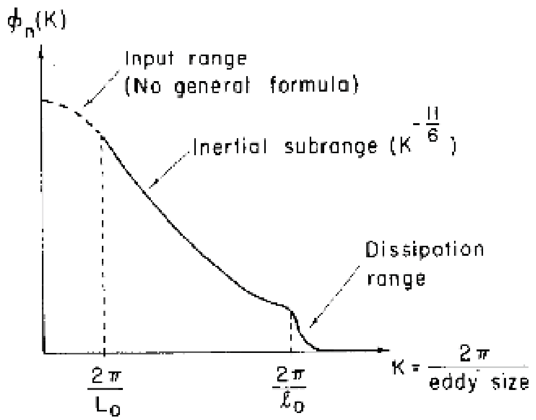


FIG. 16-8 Kolmogorov spectrum of the spectral density of the refractive index fluctuation.

Scheme of the Kolmogorov spectrum.

Small wavenumbers: input range

VON KARMAN MODIFIED MODEL

This spectrum also allows account of outer scale and is needed for propagation at near ground level:

$$\Phi_n(\kappa) = 0.033\, C_n^2\, \kappa^{-11/3} \exp(-\kappa^2/\kappa_m^2)\, (\kappa^2 + \kappa_o^2)^{-11/6}$$

$$\kappa_o = 1/L_o$$

at PRESENT THE MOST USED ONE in Theoretical Work.

A modification includes anisotropy

GAUSSIAN MODEL

Simple, useful when the inertial range is small, that is when inner and outer scale are not very different, so that they can be assumed to be 'equal'. It allows one to **obtain some basic results** in simple form.

Correlation function, structure function and spectrum are given in mathematical forms easy to hand:

$$B_n(r) = \langle \delta\mu^2 \rangle \exp(-r^2/r_o^2)$$

$$D_n(r) = 2 \langle \delta\mu^2 \rangle [1 - \exp(-r^2/r_o^2)]$$

$$\Phi_n(\kappa) = \langle \delta\mu^2 \rangle r_o^3 (8 \pi^{3/2})^{-1} \exp(- k^2 r_o^2 /4)$$

where

r_o **scale** (one scale model)

$\langle \delta\mu^2 \rangle$ **rms** of the refractive index

THEORY OF PROPAGATION IN RANDOM MEDIA

Wave equation. Scalar approximation: U component of the e.m. field

3) $$\nabla^2 U = \frac{(1+\delta\mu)^2}{c^2}\frac{\partial^2 U}{\partial t^2}$$

∇^2 Laplacian

- Equation 3 requires small refractive index fluctuations, almost always satisfied

- Average value of the refractive index is assumed = 1, good approximation for the atmosphere.

In the homogeneous, isotropic case $\langle\delta\mu\rangle = 0$

Solutions by use of complex exponentials

Scalar approximation and time dependence

$$U = v(P) \exp(-i\omega t)$$

$$v(P) = A(P) \exp[i\varphi(P)]$$

$v(P)$ complex amplitude, $A(P)$ amplitude, $\varphi(P)$ phase.

In the absence of fluctuations and homogeneous, isotropic medium let the complex amplitude be:

$$v_o(P) = A_o(P) \exp[i\varphi_o(P)]$$

In the case of **small fluctuations** there are a number of different ways to find approximate solutions of Eq. 3.

1 - **Born approximation**: consists in developing the field in a number of subsequent terms, of which the first is the unperturbed solution.

2 - **Ritov approximation**. Consists in developing the logarithm of the field in series. More efficient. However very limited for horizontal propagation.

Detailed developments of this method were given by V.I. Tatarskii and by Akira Ishimaru in, now classical, books.

By using the spectrum Tatarskii obtained the **approximate spectra and correlation or structure functions for phase and log-amplitude, $\chi = \ln(A/A_o)$, in a number of different limiting cases**. For instance in the case of plane waves, in the geometrical optics region, where the path L is short enough, $L \ll l_0^2/\lambda$, that diffraction by the smallest inhomogeneities can be neglected.

3 - Extended Huygens-Fresnel principle. This method is useful for both cases of small and strong amplitude fluctuations.

SOME BOOK TITLES

which can be of interest, without any sake of completeness.

V. I. Tatarskii, Wave propagation in a turbulent medium. McGraw-Hill, New York, 1961

V. I. Tatarskii, The effects of the turbulent atmosphere on wave propagation. Springfield, Va, 1971

A. Ishimaru, Wave propagation and scattering in random media. Volume 2, Academic Press, 1978.

L. C. Andrews, R. L. Phillips and C. Y. Hopen Laser beam scintillation with applications. SPIE Press, Bellingham, WA, 2001.

PHASE AND PHASE DEPENDENT QUANTITIES

-**PHASE FLUCTUATION** is the first effect of propagation through turbulence. And is the most important one for systems operating in the atmosphere. We already saw the diffraction example.

-**AMPLITUDE FLUCTUATION** is consequence of phase fluctuation.

Other CONSEQUENCES OF PHASE FLUCTUATIONS

- **DIFFERENTIAL ANGLE-OF-ARRIVAL FLUCTUATION** at each point of a wave front

-**WANDERING** of a beam as a whole (often called angle of arrival fluctuation)

Some results by Tatarskii (Rytov method)

PLANE WAVE: GEOMETRICAL OPTICS REGION

In geometrical optics region, $L \ll l_0^2/\lambda$, he found:

- phase structure function:

4) $D_\varphi(r) \approx 2.9\, k^2 C_n^2 L\, r^{5/3}$ $\quad r \gg l_o$

5) $D_\varphi(r) \approx 3.3\, k^2 C_n^2 l_o^{-1/3} L\, r^2$ $\quad r \ll l_o$

$k = 2\pi/\lambda$, λ wavelength

- and Log-amplitude variance

6) $B_\chi(0) = 3\, C_n^2\, l_o^{-7/3}\, L^3$

All valid when **outer scale is much larger than inner scale**.

Dependence of phase structure function linear with path.

Some results for PHASE from Rytov method, outside the limit of geometrical optics, $L \gg l_0^2/\lambda$

A) $r \ll l_o$

$$D_\varphi(r) \approx 1.61 \; k^2 C_n^2 l_o^{-1/3} L \; r^2$$

Different coefficient with respect to geometrical optics result.

B) $r \gg l_o$

$$D_\varphi(r) \approx 2.9 \, k^2 C_n^2 L \, r^{5/3} \qquad r \gg (\lambda L)^{1/2}$$

$$D_\varphi(r) \approx 1.45 \, k^2 C_n^2 L \, r^{5/3} \qquad r \ll (\lambda L)^{1/2}$$

NOTE: LINEAR DEPENDENCE ON PATH LENGTH

Evaluation of phase correlation function

We now evaluate a **general formula, independent of the turbulence model**, valid for short paths, $L << l_0^2 /\lambda$. Use is made of the Geometrical optics approximation.

Coherent plane wave, of unit amplitude, propagating in x direction impinges on a layer of turbulence of thickness L. Complex amplitude

$$v(P) = e^{ikx}$$

Phase at path end without turbulence would be $\varphi_o = kL$.

In the presence of turbulence, geometrical optics gives

7) $$\varphi = k\int_0^L (1 + \delta\mu)\, dx$$

where integral is along a ray path. At point P at path-end, phase fluctuation

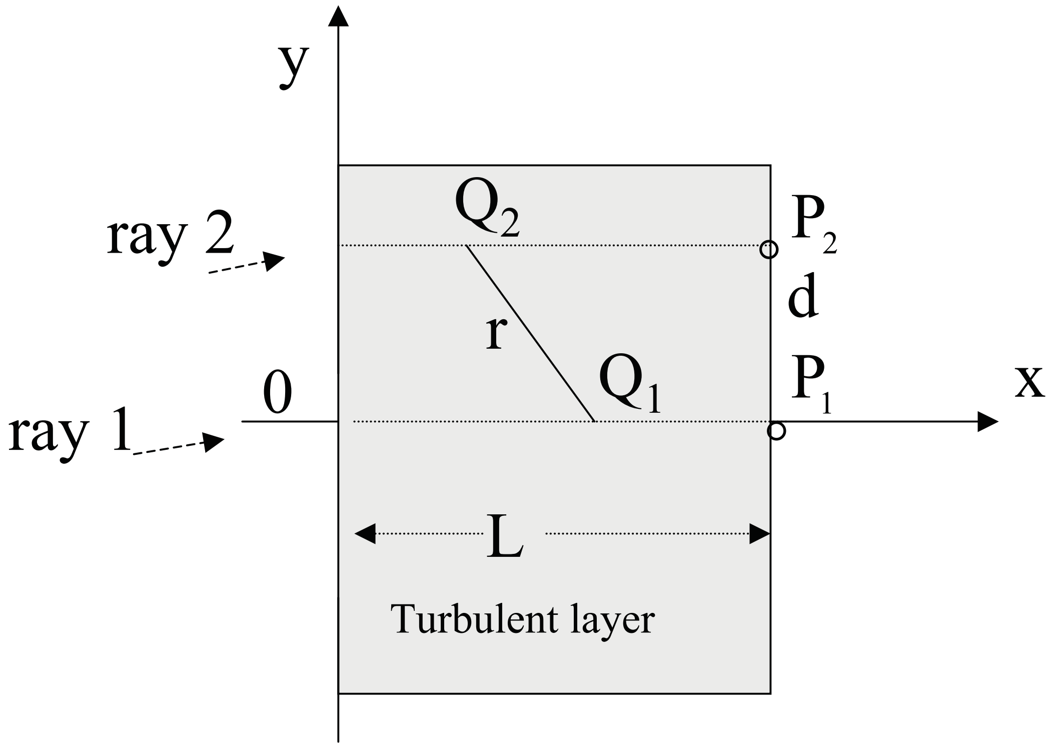
8) $$\delta\varphi = \varphi - \varphi_o = k\int_0^L \delta\mu\, dx$$

At point P_1 at the end of path 1:

$$\delta\varphi_1 = \delta\varphi(P_1) = k\int_o^L \delta\mu(Q_1)\,dx_1$$

At point P_2 on a parallel path spaced d

$$\delta\varphi_2 = k\int_o^L \delta\mu(Q_2)\,dx_2$$



Section in

xy plane

Correlation function between phase fluctuations at the end of the path, by definition, is

$$B_\varphi(r) = <\delta(\varphi_1)\delta(\varphi_2)>$$

After simple elaboration use of previous formulas gives:

$$B_\varphi(d) = k^2 \int_o^L \int_o^L B_n(r)\, dx_1\, dx_2$$

Meaning of this important result:

Correlation of phase fluctuations at the path end is related to correlation of the refractive index fluctuations at all couples of points along the two paths

APPLICATION OF THE GENERAL FORMULA TO A SIMPLE EXAMPLE: GAUSSIAN MODEL OF TURBULENCE

Introduction of the Gaussian correlation function of refractive index

$$B_n(r) = <\delta\mu^2> \exp(-r^2/r_o^2)$$

into the general formula gives rise to integrals which can be evaluated. In the case of a path length long enough $L > r_o$ (common case) the complete expression for the phase correlation function reduces to:

$$B_\varphi(d) = \sqrt{\pi}\ k^2 <\delta\mu^2> L\ r_o \exp(-d^2/r_o^2)$$

and

$$B_\varphi(0) = \sqrt{\pi}\ k^2 <\delta\mu^2> L\ r_o$$

Comment: Phase correlation and phase variance are linearly dependent on path. GENERAL RESULT, already noted

COMPARISON BETWEEN PHASE RESULTS

Tatarskii theory assumes outer scale very large, and gives dependence of phase structure function on points separation, d:

$$D_{\varphi} \propto d^2 \qquad \text{for } d < l_o$$

$$D_{\varphi} \propto d^{5/3} \qquad \text{for } d > l_o$$

Gaussian model gives

$$D_{\varphi} \propto 1 - \exp(-d^2 / r_o^2)$$

Both give the correct quadratic behaviour in the dissipation range. For the Gaussian model a simple series development around d=0 allows one to confirm the quadratic behaviour.

In the inertial range the structure function from Tatarskii theory grows indefinitely, with a power 5/3. The Gaussian structure function increases then saturates for $d >> r_o$.

PHASE DEPENDENCE ON OUTER SCALE

As soon as measurements of phase were made it appeared that, at near ground levels (of interest for environmental research):

1 - the phase structure function saturates increasing the distance between the two points over the wavefront.

2 - saturation is due to outer scale and phase fluctuations strongly depend on outer scale

3 - measurements of outer scale and of structure constant, are possible by measuring phase fluctuations

The results of these measurements showed values of the outer scale much lower than expected. For example Consortini et al. found (Alta Frequenza, Special Issue URSI 38, p.149, 1969) values of the order of few centimeters, at a eight of about 1m over ground. Bertolotti and al. (Applied Optics, 13, p.1583, 1974) found values of the order of 1m at heights of 15-20m. Tendency to saturation is clear in the example of next page. It is now well established that the value of the **outer scale at near ground levels** strongly depends on the **local situation** and, when needed, is to be measured locally.

Bertolotti et al. Applied Optics 1974

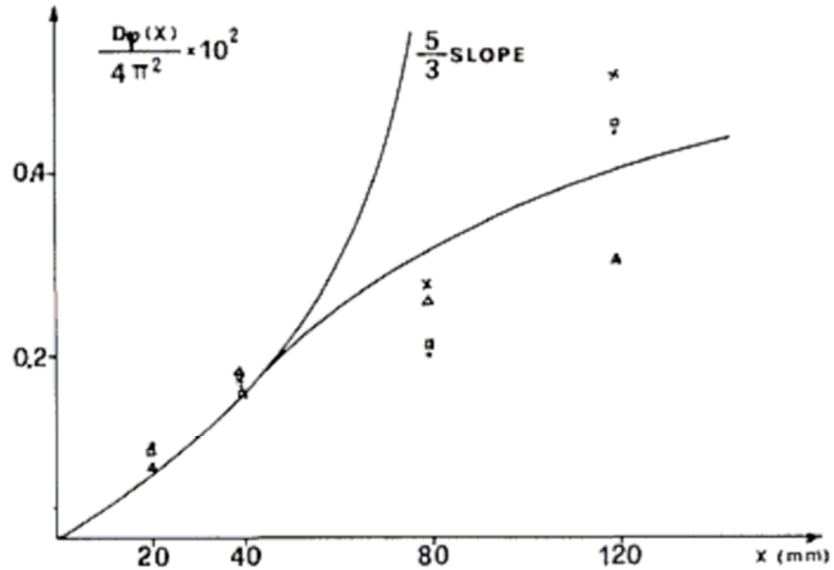


Fig. 3. Phase structure function vs x in the multibeam interferometer.

Extensive results on outer scale were recently summarized by V. Lukin in a SPIE Proc of Remote Sensing (2006).

His abstract and a table are here reported.

Outer scale of atmospheric turbulence

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ABSTRACT

In the early 70's, the scientists **in Italy (A.Consortini, M.Bertolotti, L.Ronchi), USA (R.Buser, Ochs, S.Clifford) and USSR (V.Pokasov, V.Lukin)** almost simultaneously discovered the phenomenon of deviation from the power law and the effect of saturation for the structure phase function. During a period of 35 years we have performed successively the investigations of the effect of low-frequency spectral range of atmospheric turbulence on the optical characteristics. The influence of the turbulence models as well as a outer scale of turbulence on the characteristics of telescopes and systems of laser beam formations has been determined too.

Keywords: turbulence, outer scale, model, vertical profile

Average outer scale measured by different authors

vertical propagation (courtesy of V.Lukin)

Year SU	Author	Outer scale size (m)	Instrument	Site
1984	Mariotti etal.	= 8	I2T	CERGA
1987	Colavita et al.	> 2000	Mark III	Mt. Wilson
1989	Tallon	5-8	Hartmann-Shack	Mauna Kea
1991	Rigaut et al.	= 50	COME ON	La Silla
1991	Nightingale	= 2	DIMM	La Palma
1993	Ziad et al.	5-100	Hartmann-Shack	OHP
1994	Agabi et al.	50-300	GSM1	OCA
1995	Busher et al.	10-100	Mark III	Mt.Wilson
1995	Fuchs	2,4-1.5	Ballons	Paranal

AMPLITUDE FLUCTUATIONS – SCINTILLATION

Amplitude fluctuations are consequence of phase fluctuations, they are due to curvature of wavefront which produces focusing and defocusing.

Scintillation = Intensity fluctuations

Quantity describing scintillation is the variance of intensity I(P).

I(P) is defined as the amplitude square

$$I(P) = v(P)\, v^*(P)$$

Scintillation is sensitive to the parameters of turbulence, fist to the **structure constant**, and mostly to the **inner scale** and is widely used to measure them over paths larger than some hundreds meters over horizontal paths. There are now a number of instruments, produced by specialized companies, which measure structure constant and inner scale by means of laser scintillation measurements.

Scintillation index, commonly denoted by σ_i^2 is the variance of the intensity at a point, (that is second centered moment of intensity) normalized to intensity square average. This involves higher (fourth) order correlations functions of the field and is outside our purposes. Here just some information.

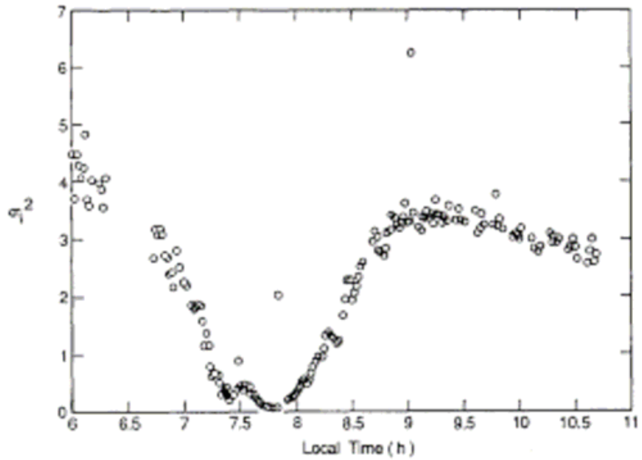
Measurements in cases of strong turbulence and/or long paths gave very high values of the scintillation index. For instance values of s_i^2 =4 were often found. It appeared that the range of validity of Rytov theory was too short and this stimulated a large effort to develop more general theoretical methods, and also numerical evaluation with powerful computers. The cover of a special issue of Applied Optics 1988, from Flatte's computations, gives an idea of the results.

One important theoretical problem was to explain the so called saturation of scintillation, that is saturation and subsequent decrease of the scintillation index, measured when the turbulence strength increased. An example of experimental results in the next page.

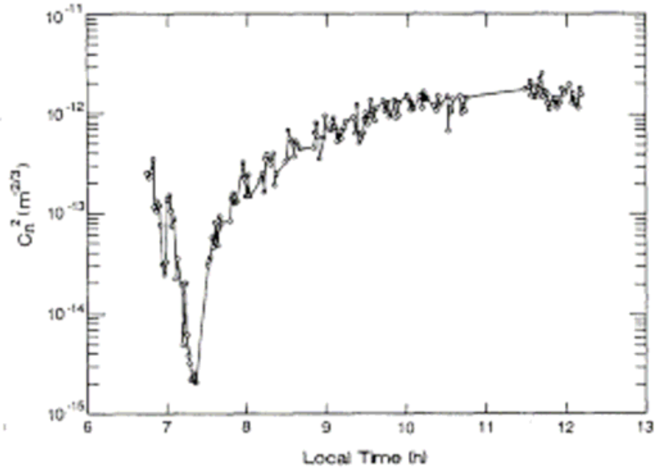
Another problem, which attracted much research, was to find the statistical distribution, probability density function, pdf, of the measured intensity in the case of strong scintillation and to investigate its evolution during changing turbulence conditions, e.g. during a day.

From: Vol. 10, No. 11/November 1993/J. Opt. Soc. Am. A

Consortini *et al.*



(a)



(b)

a) Example of scintillation index at a point of a laser beam, versus time, measured after a 1200 m horizontal path in the atmosphere.

b) Typical behaviour of the Structure Constant, versus time, measured over a 600 m parallel path.

The saturation at about 9h and subsequent decrease of the scintillation index is clear.
The Structure constant continuously increases.

EFFECT OF TURBULENCE ON SYSTEMS

It is well known that the behaviour of an imaging system can be described by its Optical Transfer Function (see lectures on images).

Could one also think of an Optical Transfer Function of the Atmosphere, OTF_A ? One should distinguish between long and short exposures.

Shortly, in the case of long exposure images and small fluctuations (Rytov approx), the normalized OTF_A of the atmosphere can be introduced and is given by*

$$OTF_A = \exp\left[-\tfrac{1}{2}\, WSF \right]$$

where the Wave Structure Function, WSF, is defined as

W S F = PHASE STRUCTURE FUNCTION +

+ LOG-AMPLITUDE STRUCTURE FUNCTION

*see e.g. J.W. Goodman, *Statistical Optics*, John Wiley and Sons, 1985, and also *Speckle Phenomena in Optics*, Roberts and Company, 2007.

The atmospheric turbulence can be therefore considered as a filter. The theoretical results showed also that the higher frequencies of the object spectrum are cut by turbulence, thus reducing the resolving power of systems, in a way analogous to aberrations.

The total optical transfer function of system and atmosphere OTF_T is the product of the optical transfer function of the system and the atmospheric one

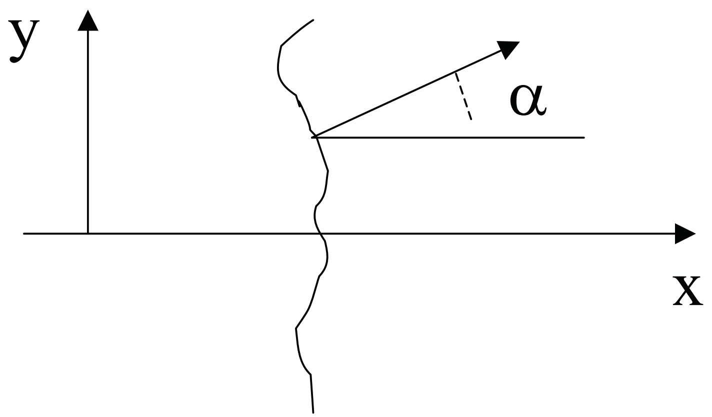
$$OTF_T = OTF_S \times OTF_A$$

It was also shown by Goodman that, for long path length and Rytov approximation, WSF quantity is equal to the phase structure function obtained in the case of the geometrical optics approximation (short path length).

Therefore our previous evaluations of the phase structure function can help us in evaluating the performance of system making long exposure images through the atmosphere

DIFFERENTIAL ANGLE OF ARRIVAL FLUCTUATIONS

Plane wave propagating in direction x. After path L , wavefront corrugates. At each point the normal to wavefront is no more parallel to x axis. Deviation angle is called (differential) angle of arrival. Two components α and β, in horizontal and vertical plane respectively

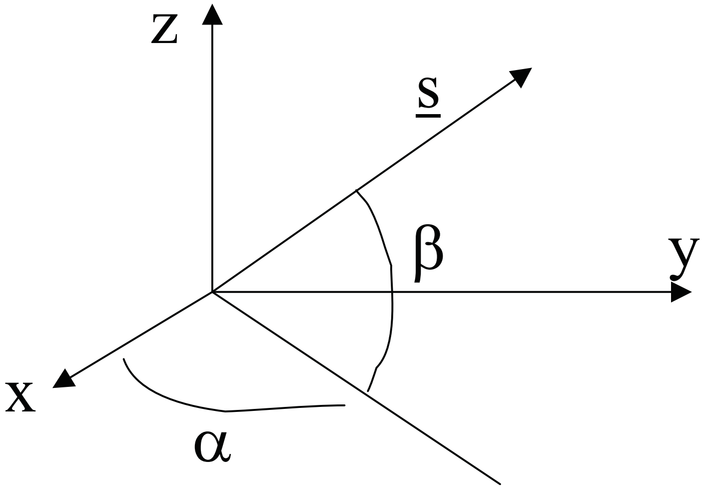


Section in horizontal plane xy

We now develop some simple formulas to evaluate α and β.
From geometrical optics equation:

$$\textbf{grad}\ \varphi = \mathbf{k}\ \ \mathbf{n}\ \ \underline{\mathbf{s}}$$

$$\text{grad } \varphi = k\, n\, (\cos\beta \cos\alpha\, \underline{i} + \cos\beta \sin\alpha\, \underline{j} + \sin\beta\, \underline{k})$$



Small fluctuations: quantities α and β are very small. Therefore:

$$\cos\alpha \sim \cos\beta \sim 1$$

$$\sin\alpha \sim \alpha \quad , \quad \sin\beta \sim \beta$$

Hence:

$$\partial\varphi/\partial y = k\,n\,\alpha \qquad \partial\varphi/\partial z = k\,n\,\beta$$

At each point of the wavefront, derivative of phase gives angle of arrival.

At a given point $Q(x_1)$ along path 1, (see fig next page) angle α is obtained by introducing φ from Eq. 7

9)

$$\alpha(x_1) = \frac{\partial}{\partial y}\left(\int_0^{x_1}[1+\delta\mu_1]ds_1\right) =$$

$$= \int_0^{x_1}\frac{\partial}{\partial y}\delta\mu_1\,ds_1$$

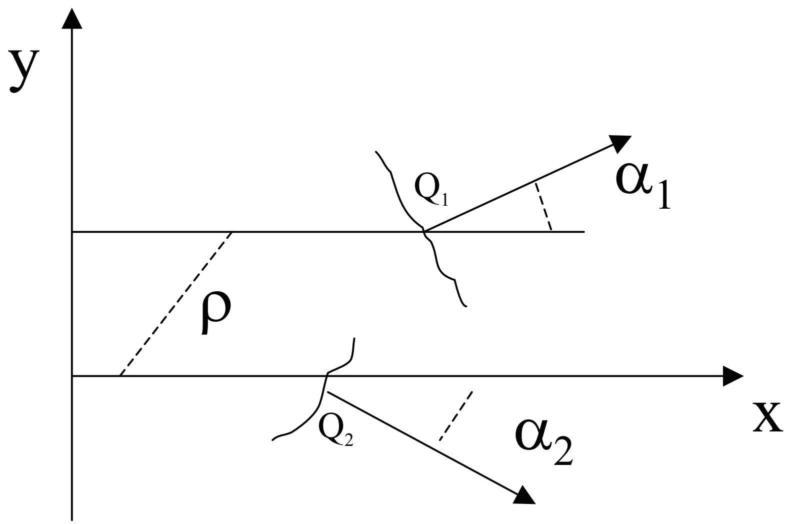
In first term n was replaced by ~ 1. Analogous expression for β.

Correlation function of angle of arrival at points $Q_1=Q_1(x_1)$ and $Q_2=Q_2(x_2)$ along two parallel paths:

$$B_\alpha(Q_1,Q_2) = <\alpha_1,\alpha_2> =$$

$$= \int_0^{x_1}\int_0^{x_2} \frac{\partial^2}{\partial^2 y_1 y_2} B_n(\rho)\, dx_2\, dx_1$$

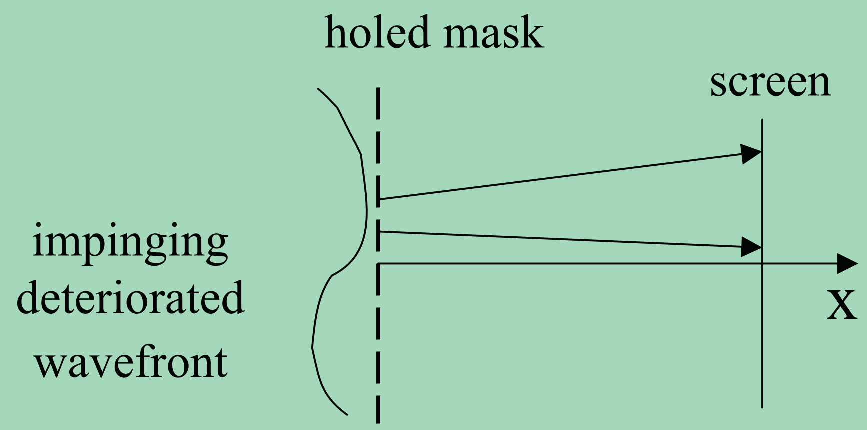
Important: **Correlation function of angle of arrival** requires derivatives of refractive index correlation function at **any two points of the paths** before x_1 and x_2



Angle of arrival correlation functions are sensitive to the turbulence **inner scale** and methods of measuring the inner scale, based on them, have been developed in the literature.

Differential angle of arrival is used in **Adaptive optics** : Hartman test (now also called Shack-Hartman test) is used for measuring the wavefront shape. Adaptive optics systems, initially for astronomy purposes, now are of great interest in atmospheric propagation, either horizontally or with slant paths.

Hartman test: holed mask on which the deteriorated wavefront impinges and a number of beams are produced. On a screen moving spots are obtained. Measurement of "frozen" spot positions on the screen give the instantaneous angles of arrival and, after elaboration, wavefront. "Frozen" means that all procedure, including wavefront correction is to be made in a time not larger than 1 ms.



Hartman test scheme

THIN BEAM PROPAGATION

Thin beams are an important tool to investigate the atmosphere locally.

Thin beam is beam whose lateral dimension is not larger than the dimension of the smallest inhomogeneity, inner scale.

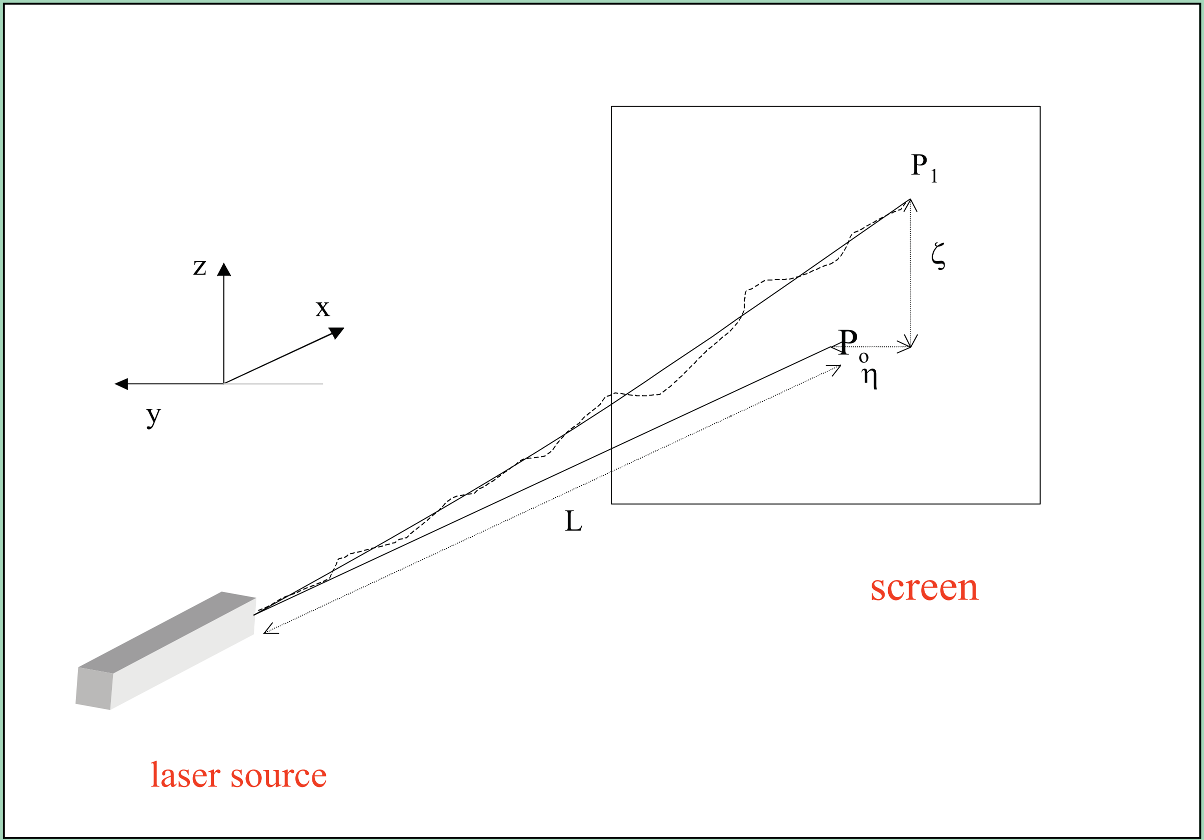
Generally a laser beam propagating horizontally in the atmosphere remains thin if the propagation path is short enough, e.g. a few meters if not collimated or some tens meters if collimated. Typical values of the inner scale are several mm or larger.

At each point of the path the refractive index fluctuations produce lateral deviations of the beam from its initial direction (wandering). On a screen at the end of the path, the beam impinges in a place different from that in the absence of turbulence (unperturbed). Its position changes randomly in time, around an average position. Wandering is measured by the horizontal and vertical distances from the average position.

EXAMPLE: Thin beam propagation

General case: Both gradient and random fluctuations of n are present

$P_o(x_o, y_o)$ unperturbed position. From now on gradient is assumed zero.



WANDERING

Lateral fluctuations

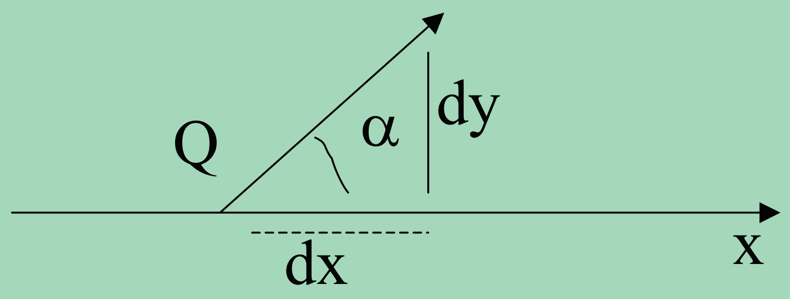
Horizontal fluctuation

$\eta = y_0 - y_1$

Vertical fluctuation

$\xi = z_0 - z_1$

Wandering can be evaluated from **differential angle of arrival**



If at a point Q of the path the horizontal component of the differential angle of arrival is α, the displacement after a path dx is (α is small):

$$dy = \alpha\, dx$$

After path L, the horizontal fluctuation (in direction y) is:

10) $$\eta = \int_0^L \alpha\ dx = \int_0^L \int_0^x \frac{\partial}{\partial y} \delta\mu\ ds\ dx$$

Where the Equation 9 for α was used.

Thin beam wandering is a phase related effect and therefore is very sensitive.

It is possible to show that the variance of the displacements of the beam increases with the path length L, as L^3. This **third power dependence** makes wandering very important for measurements over short paths.

Correlation of the lateral fluctuations of two parallel rays, see further, also increases with the third power of the path:

$$B_y(d) = \langle \eta_1 \eta_2 \rangle \propto L^3$$

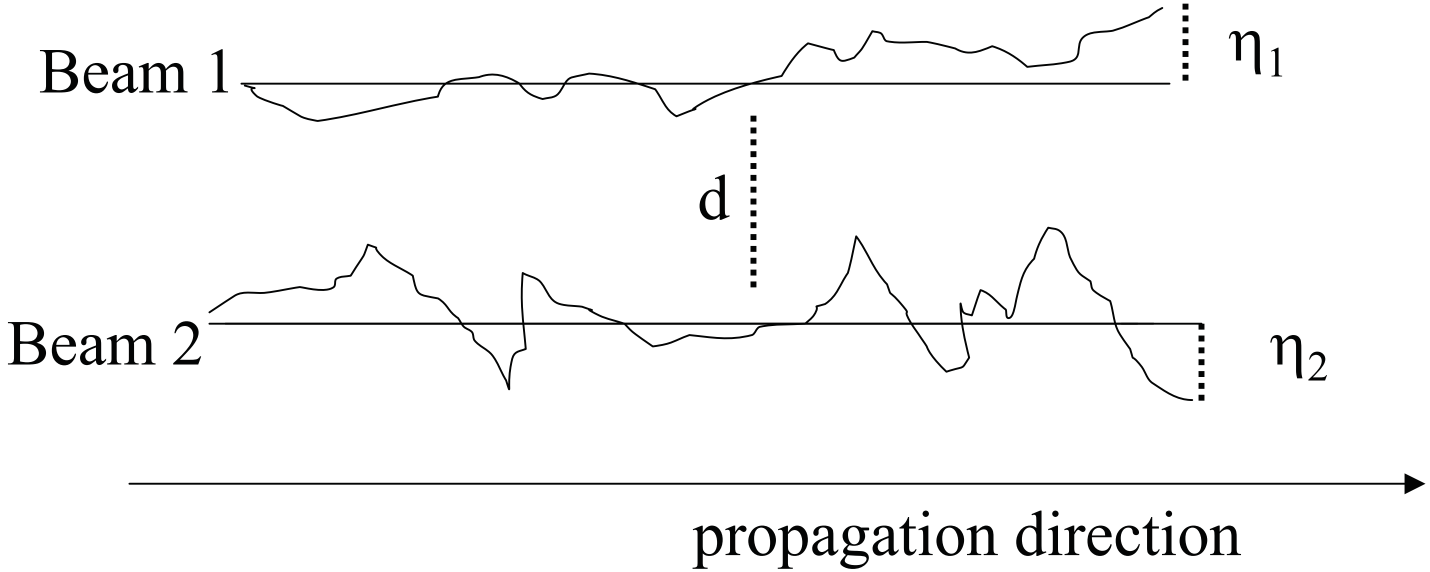
$$B_z(d) = \langle \xi_1 \xi_2 \rangle \propto L^3$$

Wandering and correlation of wandering are utilized in a number of applications to measure the **parameters of turbulence** and also **the gradient of the refractive index.**

Example: Thin beam wandering theory based on von Karman spectrum.

Correlation functions of horizontal and vertical fluctuations of parallel beams, at the end of a path, were theoretically investigated by using a von Karman model of turbulence. Expressions are available in terms of series developments. Without entering the details here main results:

- Correlations functions of lateral fluctuations of two paralled beams, spaced by d In plane correlation $B_y(d)$, out of plane $B_z(d)$.



- $B_y(d)$ and $B_z(d)$ are sensitive to all parameters of turbulence: Inner scale, outer scale, C_n^2.

- From measurements of these correlations one can obtain all parameters.

Example: Measurements in the atmosphere: 15m horizontal path at height h ~10 m. $B_y(d)$ and $B_z(d)$, normalized, plotted versus separation between the two beams. Note the different behaviour.

from *Consortini A. and O'Donnell K. "Beam wandering of thin parallel beams through atmospheric turbulence"*. Waves in Random Media, *3: S11-0S18, 1991*

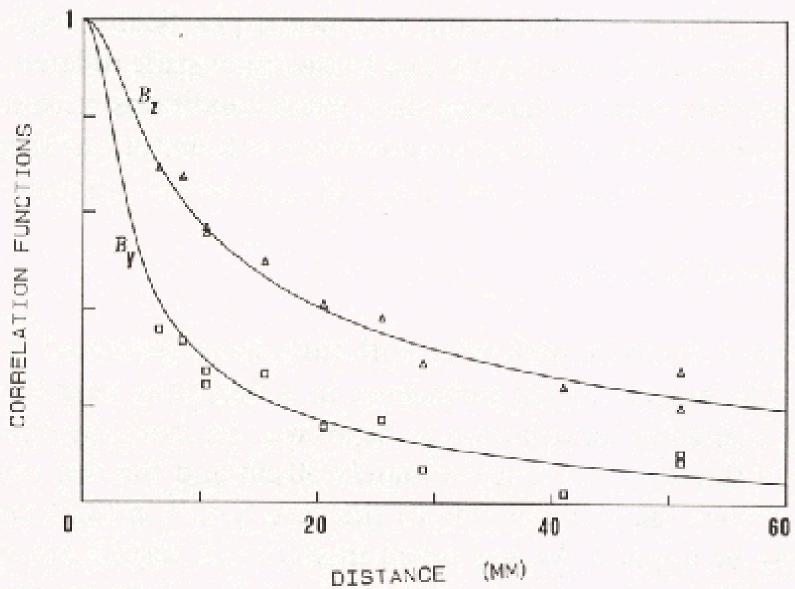
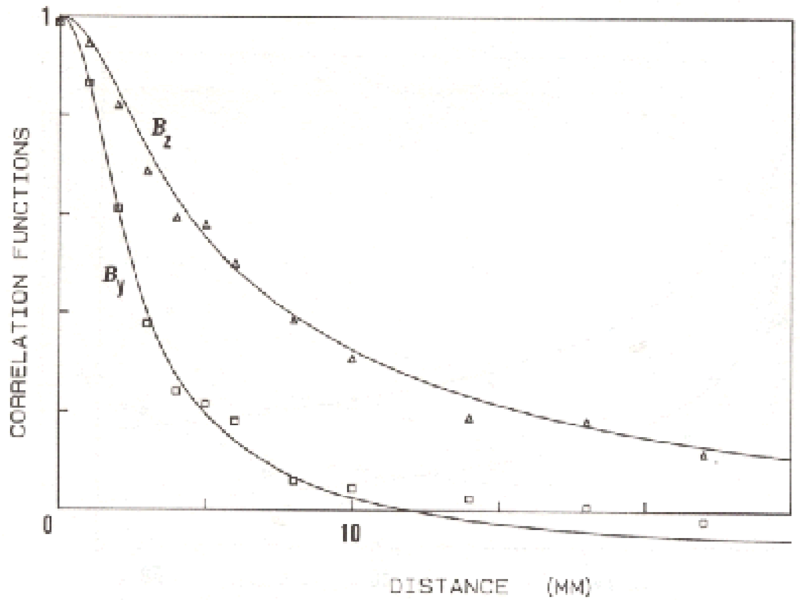


Figure 9. Same experimental data as figure 8 compared with modified von Karman model. l_0 = 7.7 mm, L_0 = 24.8 cm.

Comment: Variance depends on all parameters. Initial decay depends on inner scale, subsequent part on outer scale. Fitting allows one to obtain all of them.

Dependence on outer scale is clear in laboratory experiments where outer scale can be small and $B_y(d)$ becomes negative, anticorrelation when d ~ outer scale.



From: Waves in Random Media *as previously*

Figure 11. Laboratory measurements: in-plane (squares) and out-of-plane (triangles) measured correlation functions normalized and plotted against beam distance. Curves: fitting with modified von Karman model. $l_0 = 5$ mm, $L_0 = 28$ mm.

Important property on which a method of inner scale measurement is based.   Difference function Δ(d), defined as:

$$\Delta(d) = B_z(d) - B_y(d)$$

has a maximum when the ray spacing reaches the inner scale value.

A Consortini and K A O'Donnell

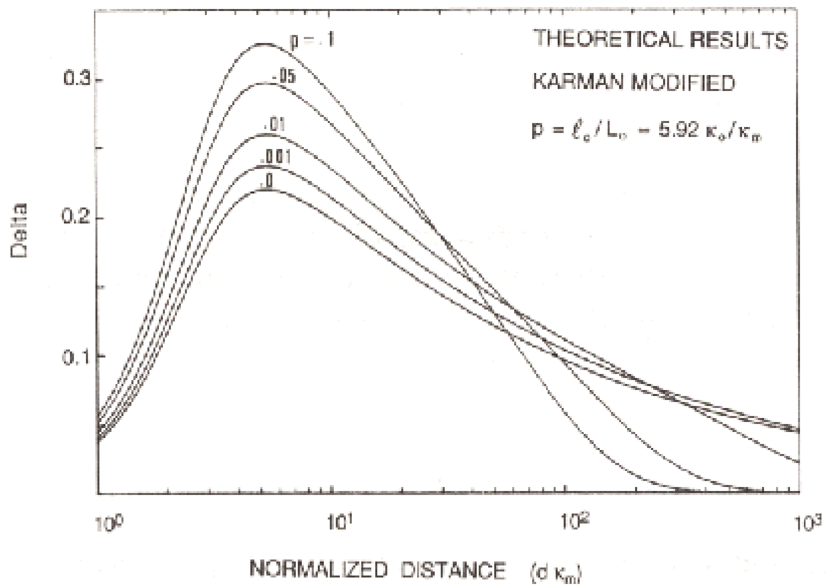


Figure 2. The normalized function Delta $= \Delta(d)/\sqrt{B_y(0)B_z(0)}$ plotted versus $\kappa_m d$ for a number of values of the scale ratio $p = l_0/L_0$ in the long-path case.

From:
Waves in Random Media 3,
n°2, p.85-92, 1993

Theoretical evaluation for different models showed that, practically, the **maximum is independent of the model and of the outer scale.** Important property to “directly” measure inner scale in unknown situations in the environment.

Example: Evaluation of inner scale from experimental difference function. Inner scale resulted

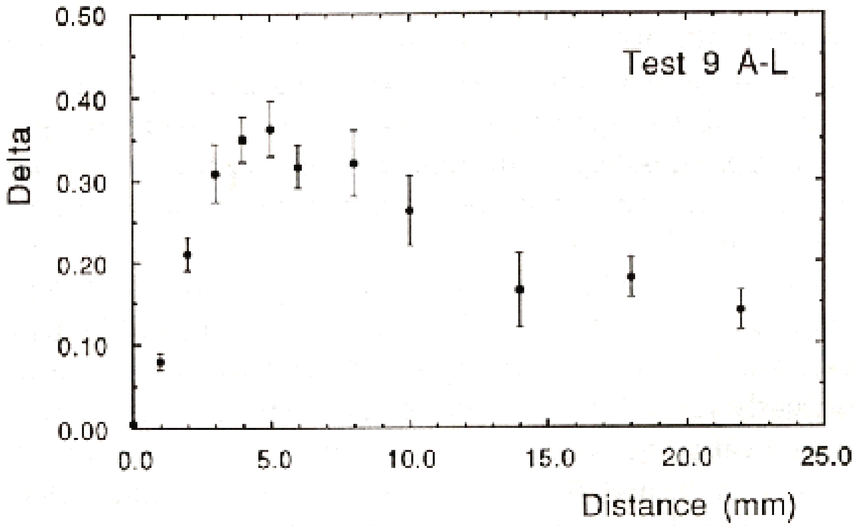


Figure 4. Laboratory measurements of the normalized correlation difference Delta = $\Delta(d)/\sqrt{B_y(0)B_z(0)}$: points represent average results of seven data sets; bars represent their standard deviations.

From:
Waves in Random Media 3, n°2, p.85-92, 1993

USE OF THIN BEAMS TO MEASURE INSTANTANEOUS GRADIENT OF REFRACTIVE INDEX

From Eq 10, one can show that the lateral, horizontal $\eta(t)$ and vertical $\zeta(t)$, **instantaneous fluctuations** of a ray on a screen, at the end of a path of length L, are given by:

$$\eta(t) = (L^2/2)\ \langle \partial\mu(\underline{r}, t)/\partial y \rangle$$

$$\zeta(t) = (L^2/2)\ \langle \partial\mu(\underline{r}, t)/\partial z \rangle$$

Here brackets < > denote path averages of the derivatives

$$\partial\mu(\underline{r}, t)/\partial y \quad \text{and} \quad \partial\mu(\underline{r}, t)/\partial z$$

of the refractive index fluctuations at a point, $\underline{r}$, with respect to the horizontal and vertical coordinates, respectively.

Equations are:

-valid in the limits of the geometrical optics approximation, and

-require that the beams are “thin”.

Concurrent conditions: allow measurements over several meters’ paths, that is **short paths**

METHOD to measure the refractive index gradient

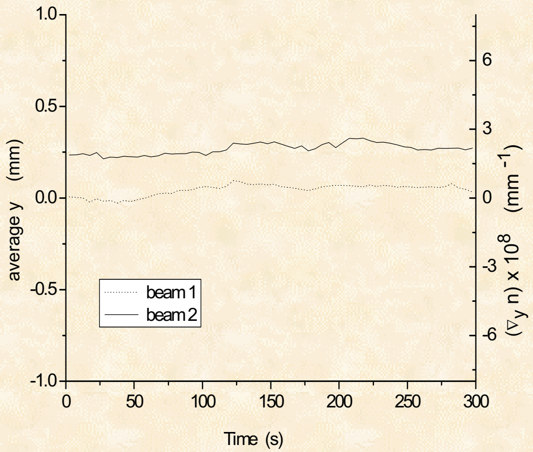
measurements of instantaneous values of $\eta(t)$ and $\zeta(t)$ give the instantaneous value of the components of the gradient transverse to the path, averaged over the path.

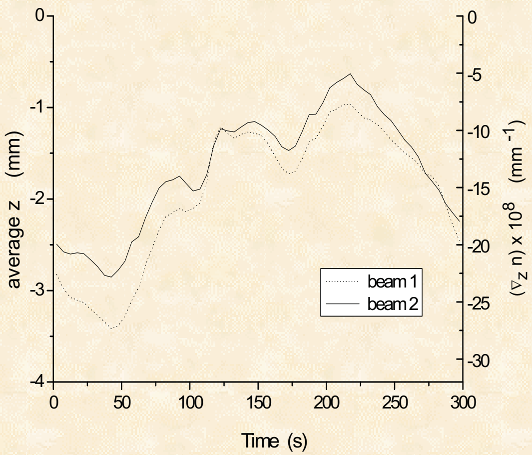
One can follow time evolution. In the next page an example of evolution of the gradient components of two parallel beams in the atmosphere is shown. Here also sensitivity to anisotropy is clear.

Left figure: horizontal coordinate time evolution of two parallel laser beams. The **gradient scale** is on the right side axis. Right figure: same for the vertical coordinate and gradient.

Note: sensitivity to anisotropy

Example of strong vertical excursion June 2004: open air, paved ground, two beams, d = 11.5cm. Time evolution of horizontal and vertical coordinates. Each point 10 s average.





Horizontal coordinate excursion < 0.2 mm

Vertical   coordinate excursion   ~ 2.5 mm

CONCLUSION

In these lectures some basics concepts of propagation of optical radiation through atmospheric turbulence and effects on image forming systems were given.

Some main models of turbulence were considered and some formulas derived, in simple cases, mainly for phase fluctuations.

Differential angle of arrival fluctuations and wandering, which are consequences of phase fluctuations, were also considered.

The first one is of importance for investigating turbulence and, mostly, for use in adaptive optics systems.

Wandering, having a 3^{rd} power dependence on path length, is the most sensitive quantity for environmental testing over short paths.