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Interaction of Ballistic Quasiparticles with Quantized Vortices and Vortex Structure in 3He-B

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Interaction of ballistic quasiparticles with quantized vortices and vortex structures in ^{3}He-B

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Andreev scattering – measurement technique for studying quantized vortices and turbulence in ^{3}He-B in the limit $T/T_c \ll 1$.

Developed at the University of Lancaster, see e.g.

S.N. Fisher, A.J. Hale, A.M. Guénault, and G.R. Pickett, Phys. Rev. Lett. **86**, 244 (2001);

D.I. Bradley, S.N. Fisher, A.M. Guénault, M.R. Lowe, G.R. Pickett, A. Rahm, and R.C.V. Whitehead, Phys. Rev. Lett. **93**, 235302 (2004);

D.I. Bradley, D.O. Clubb, S.N. Fisher, A.M. Guénault, R.P. Haley, C.J. Matthews, G.R. Pickett, V. Tsepelin, and K. Zaki, Phys. Rev. Lett. **96**, 035301 (2006),

etc.

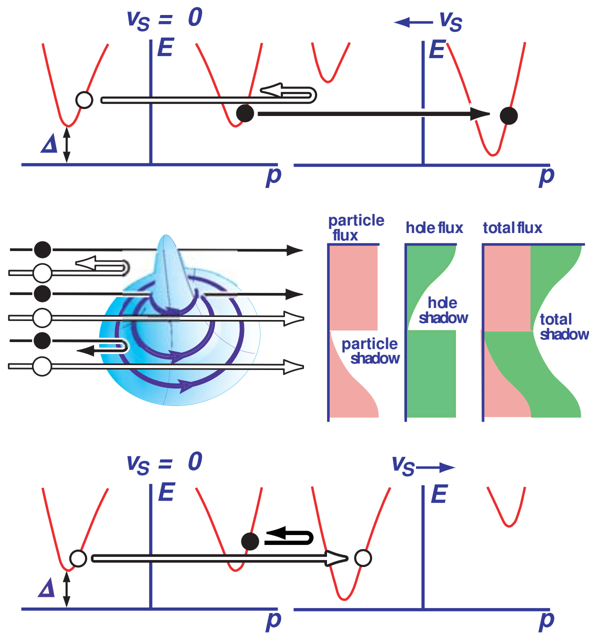
$$E = \sqrt{\epsilon_p^2 + \Delta_0^2} + \mathbf{p} \cdot \mathbf{v}_s$$

$$\epsilon_p = \frac{p^2}{2m^*} - \epsilon_F$$

Quasiparticles – excitations such that $\epsilon_p > 0$

Quasiholes – excitations such that $\epsilon_p < 0$

Andreev scattering near a single vortex



(S.N. Fisher, in *Vortices and Turbulence at Very Low Temperatures*, Springer, 2008, pp. 177-257)

On a spatial scale $\gg \xi_0$ an excitation can be considered a compact object of momentum $\mathbf{p}(t)$ and position $\mathbf{r}(t)$

$$E = \sqrt{\epsilon_p^2 + \Delta_0^2} + \mathbf{p} \cdot \mathbf{v}_s = \text{constant} \qquad \text{– effective Hamiltonian}$$

Equations of motion:

$$\frac{d\mathbf{r}}{dt} = \frac{\partial E(\mathbf{p},\mathbf{r})}{\partial \mathbf{p}} = \frac{\epsilon_p}{\sqrt{{\epsilon_p}^2 + \Delta_0^2}} \frac{\mathbf{p}}{m^*} + \mathbf{v}_s$$

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial E(\mathbf{p},\mathbf{r})}{\partial \mathbf{r}} = -\frac{\partial}{\partial \mathbf{r}}(\mathbf{p} \cdot \mathbf{v}_s)$$

Velocity field of superfluid vortex: $\mathbf{v}_s = \dfrac{\kappa}{2\pi r}\,\mathbf{e}_\varphi$

Integrals of motion:

$E(\mathbf{p}, \mathbf{r}) = E = \text{const}, \quad J = p_\varphi r = p_F \rho_0 = \text{const}$

ρ_0 – impact parameter

<u>Quasiparticle</u>

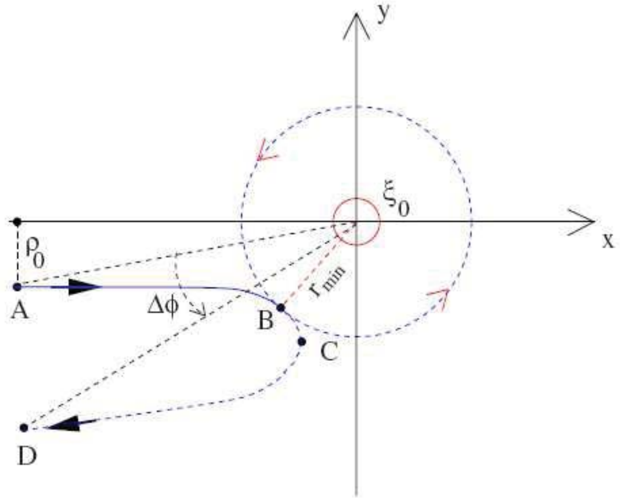
incident: $\epsilon_p > 0,\ p_r < 0$

moving away: $\epsilon_p > 0,\ p_r > 0$

<u>Quasihole</u>

incident: $\epsilon_p < 0,\ p_r > 0$

moving away: $\epsilon_p < 0,\ p_r < 0$



Unless $\rho_0 = 0$ ($J = 0$), the radial velocity will eventually vanish either because

$p_r = 0$ (classical turning point)

or

$\epsilon_p = 0$ (Andreev turning point)

$$T \ll T_c \qquad \Longrightarrow \qquad \frac{\epsilon_p}{\Delta_0} \ll 1$$

$$\left(\text{spectrum } E \approx \Delta_0 + \frac{(p - p_F)^2}{2\Delta_0 / {v_F}^2} \right)$$

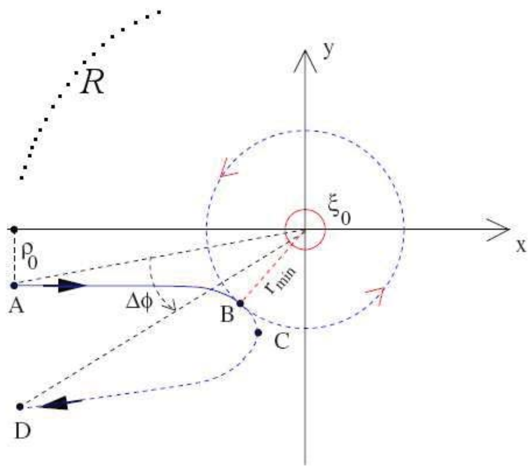
$\Longrightarrow$ approximate analytical solution of the equations of motion.

Return time of the excitation:

$$\tau \approx \frac{R}{v_F} \frac{2\Delta_0}{\epsilon_p}$$

Andreev reflection angle:

$$\Delta\varphi \approx \frac{\pi\hbar}{p_F (3\pi\xi_0\rho_0)^{1/2}} \ll 1$$



Locus of Andreev reflection
(X and Y – nondimensional coordinates scaled by ξ_0)

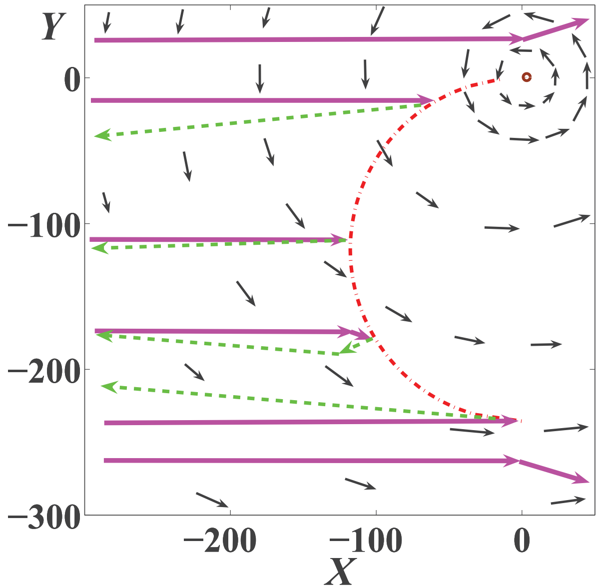
Maximum value of the impact parameter which causes Andreev reflection:

$$\rho_{0c} \approx 3\pi\xi_0 \left(\frac{\Delta_0}{\epsilon_p}\right)^2$$

Example: $k_B T/\Delta_0 \approx 0.1$

$\epsilon_p \approx k_B T \Longrightarrow \rho_{0c} \sim 10^3 \xi_0$

$\epsilon_p \approx (\Delta_0 k_B T)^{1/2}$
$\Longrightarrow \rho_{0c} \sim 10^2 \xi_0$



Heat transfer through the velocity field of a vortex

Heat carried by incident quasiparticles:

$$\delta Q_{inc} = \int_{\Delta_0}^{\infty} N(E) v_g(E) E \frac{\partial f(E)}{\partial T} \, \delta T \, dE$$

$$\Longrightarrow \qquad \frac{Q_{tr}}{Q_{inc}} \approx \frac{1}{2} \left\{ 1 + \exp\left(-\frac{\Delta_0}{k_B T} \frac{3\pi\xi_0}{2\rho_0} \right) \right\}$$

Experiments at ultralow temperatures: $\Delta_0/k_B T \sim 10$

$\Longrightarrow$ cross-section of the thermal flux $R_0 \sim 10\xi_0$

$\Longrightarrow$ 52% of the total heat is transferred through

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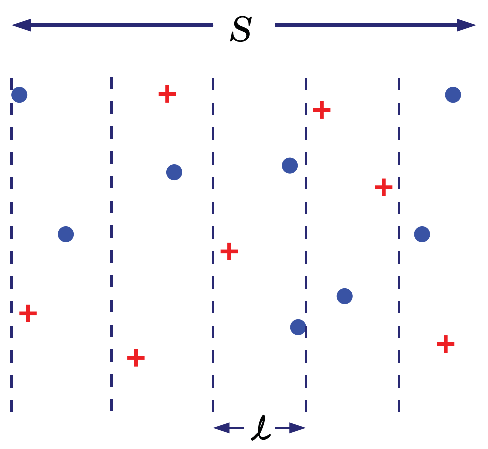
If the heat current has the cross-section $\sim 10^2\xi_0$

$\Longrightarrow$ 82% is transferred through

$\Longrightarrow$ the reflection takes place in the close vicinity of the core



Heat transfer in a vortex gas

Effective

radius of the vortex $R_0 = \ell/2$

Vortices are well separated:

$$\xi_0 \ll \ell, \qquad \frac{\Delta_0}{k_B T}\,\frac{3\pi\xi_0}{\ell} \ll 1$$

$$\Downarrow$$

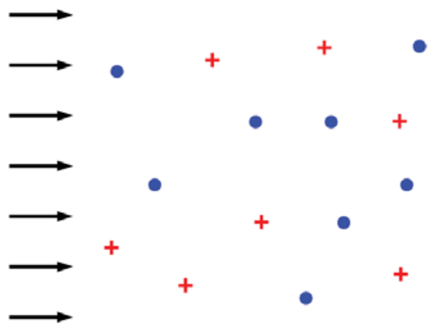
$$\ell = \left\{ -\frac{\Delta_0}{k_B T}\,\frac{3\pi\xi_0 S}{2\ln(Q_{tr}/Q_{inc})} \right\}^{1/2}$$

Example (experiment by Fisher *et al.*, PRL **86**, 244 (2001))

$Q_{tr}/Q_{inc} \approx 0.75$, $S \sim 2\times 10^{-1}$ cm, $\Delta_0/k_BT \sim 10$

$\Longrightarrow \ell \sim 1.62 \times 10^{-2}$ cm

Interaction of ballistic quasiparticles with vortex structures



Velocity field:

$$\mathbf{v}_s(\mathbf{r},\, t) = \sum_{i=1}^{i=N} \mathbf{v}_i(\mathbf{r},\, t)$$

where

$$\mathbf{v}_i(\mathbf{r},\, t) = \frac{\kappa_i}{2\pi|\mathbf{r} - \mathbf{r}_i(t)|^2}[-\mathbf{i}(y - y_i(t)) + \mathbf{j}(x - x_i(t))]$$

Velocity of the ith vortex point:

$$\frac{d\mathbf{r}_i(t)}{dt} = \sum_{j=1, j\neq i}^{i=N} \mathbf{v}_j(\mathbf{r}_i)$$

Effective Hamiltonian:

$$E = \sqrt{\epsilon_p^2 + \Delta_0^2} + \mathbf{p} \cdot \mathbf{v}_s(\mathbf{r},\, t) \qquad (\neq \text{constant})$$

Equations of motion:

$$\frac{d\mathbf{r}}{dt} = \frac{\partial E(\mathbf{p}, \mathbf{r})}{\partial \mathbf{p}} = \frac{\epsilon_p}{\sqrt{{\epsilon_p}^2 + \Delta_0^2}} \frac{\mathbf{p}}{m^*} + \mathbf{v}_s$$

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial E(\mathbf{p}, \mathbf{r})}{\partial \mathbf{r}} = -\frac{\partial}{\partial \mathbf{r}}(\mathbf{p} \cdot \mathbf{v}_s)$$

Non-dimensional variables. Scales:

distance ξ_0, velocity $\dfrac{\xi_0}{\kappa}$, time $\dfrac{\xi_0 p_F}{\Delta_0}$, momentum p_F, energy Δ_0

Numerical solution

Non-dimensional Andreev shadow of a single vortex:

$$S_0 = 3\pi(\Delta_0/\epsilon_p)^2$$

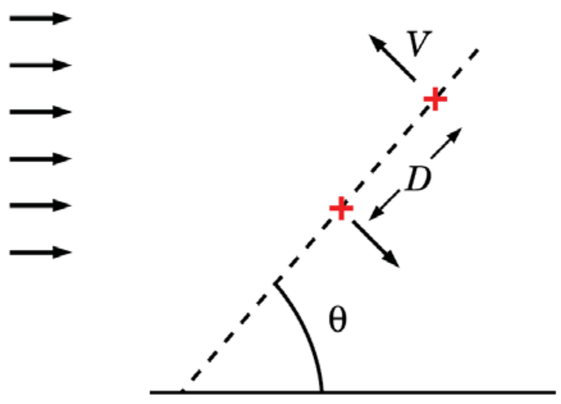
General result:

In the presence of a vortex structure (e.g. vortex-vortex or vortex-antivortex pair, or a vortex cluster) the partial screening takes place so that the total shadow, $S = S_1 + S_2 + ...$ is not necessarily the sum of shadows of individual (isolated) vortices, i.e.

$$S = \sum_{i=1}^{N} S_i \neq N S_0$$

In all following examples $\mathbf{\Pi}_0 = (1.0001, 0) \Longrightarrow S_0 \approx 269$

1) Vortex-vortex pair



Non-dimensional
distance between vortices $D = d/\xi_0$

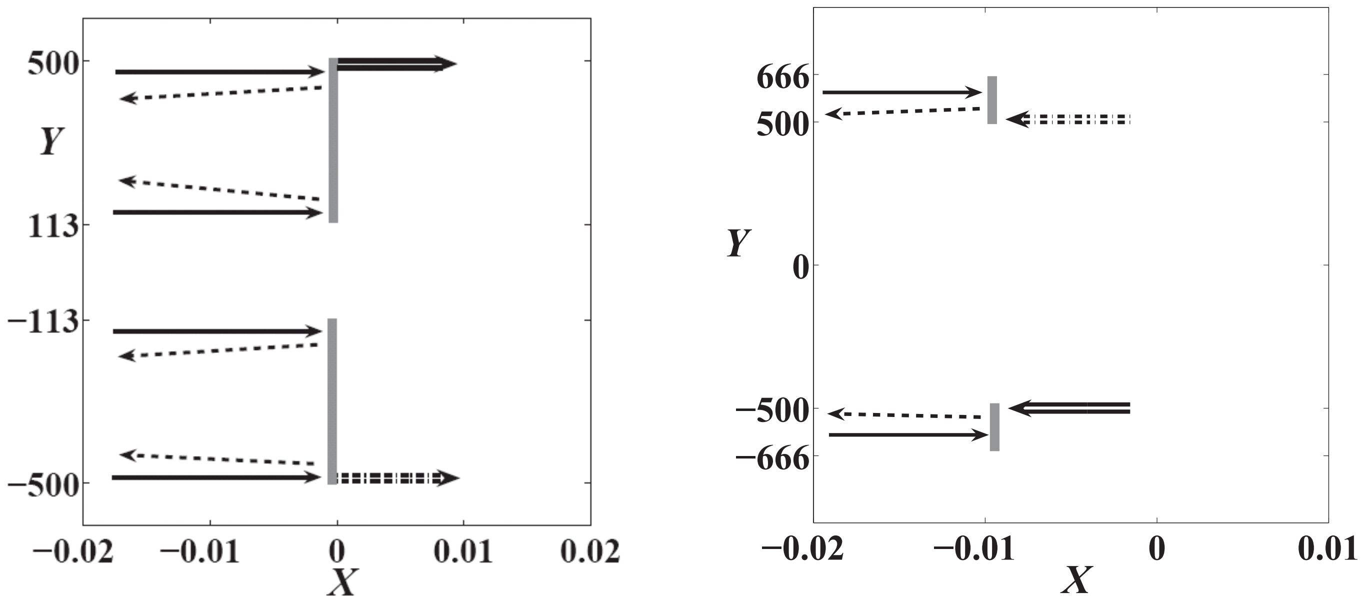
Non-dimensional
velocity of rotation $V_{rot} = 1/(2\pi D)$

Example: $D = 100$, $\theta = 90^o$

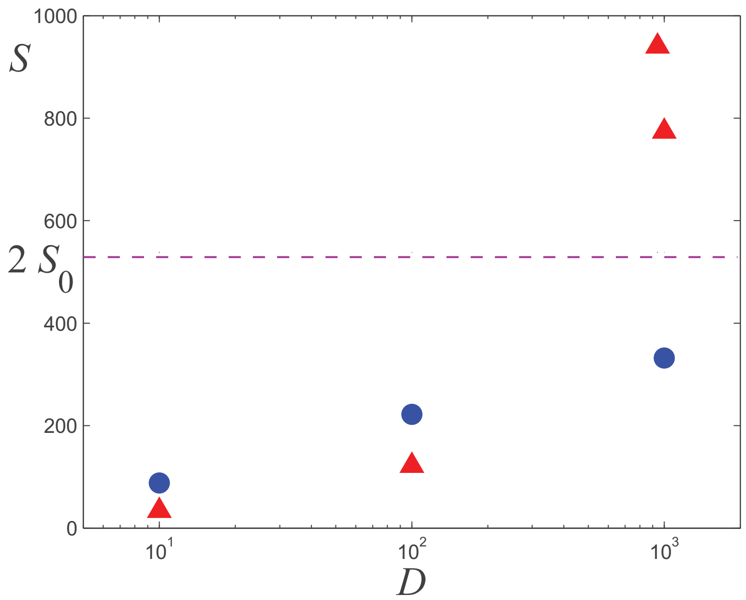
$$S_1 = 44.7 < S_0, \quad S_2 = 443.3 > S_0$$

$$S = 488 < 2S_0 = 538$$

2) Vortex-antivortex pair (2D model of a vortex ring)



$\underline{\theta = 90^o}$

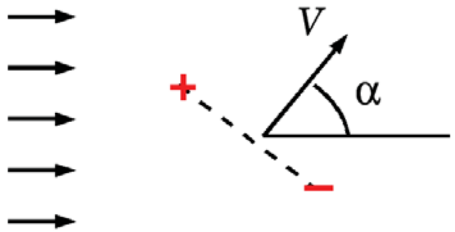


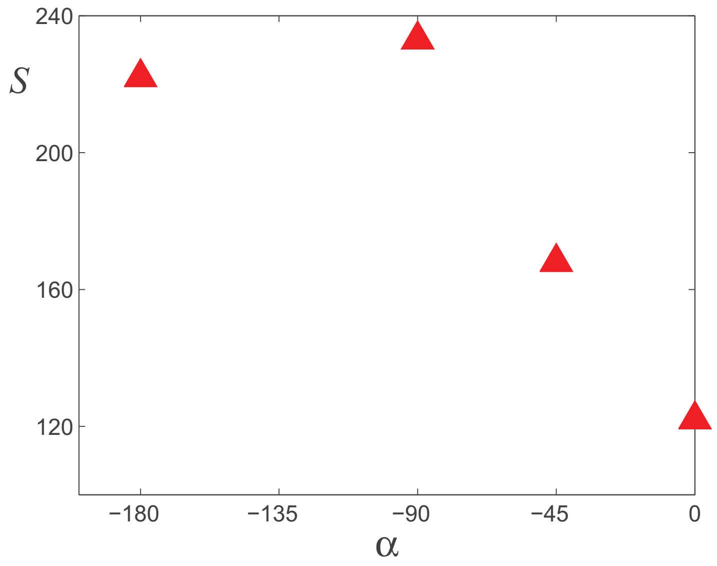
▲ – quasiparticles and the vortex-antivortex pair moving in the same direction

● – quasiparticles and the vortex-antivortex pair moving in opposite directions

$\underline{D = 100}$

Dependence on the angle α

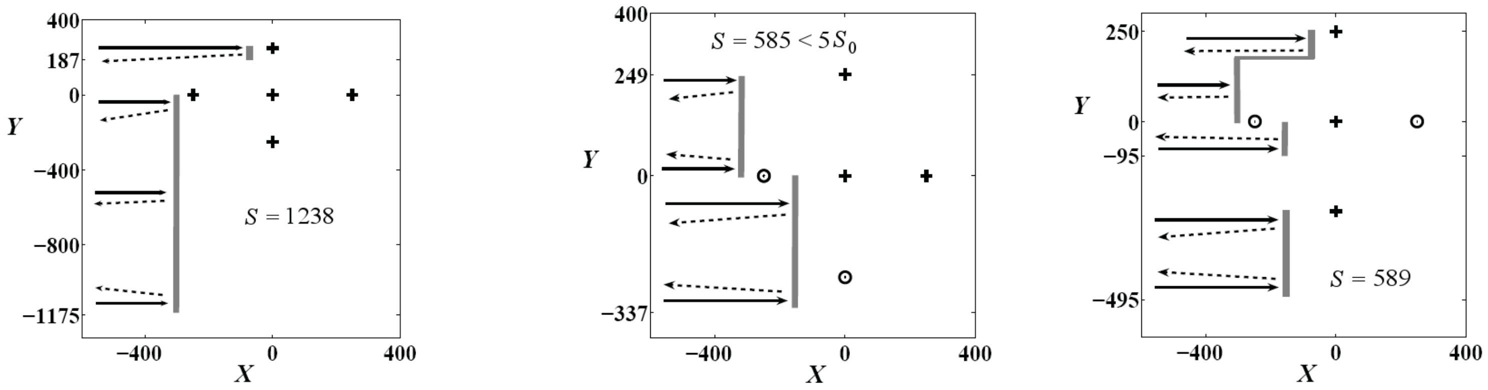




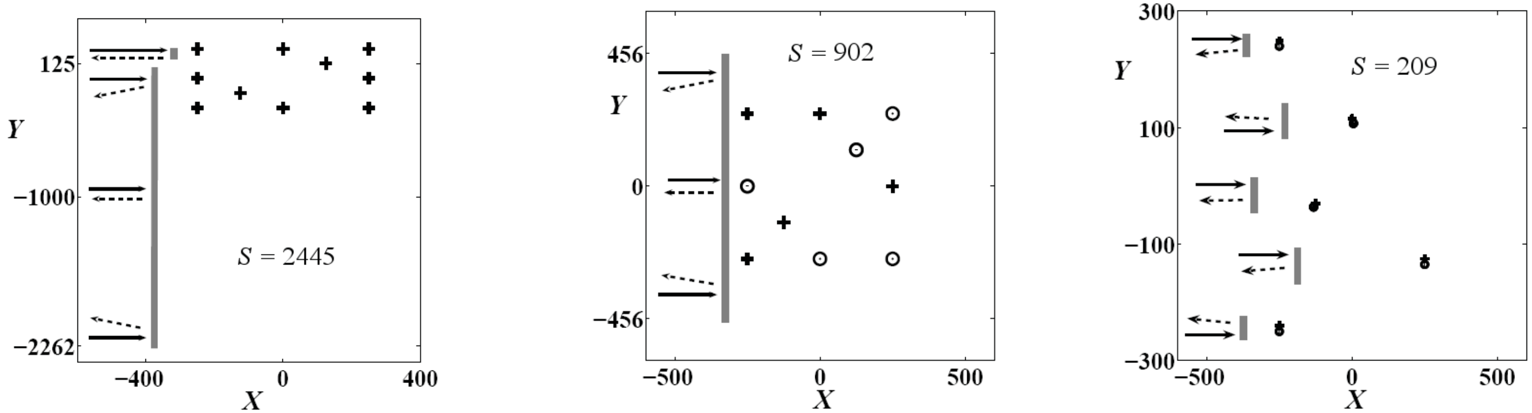
Vortex clusters

$+$ – vortices, $\circ$ – antivortices

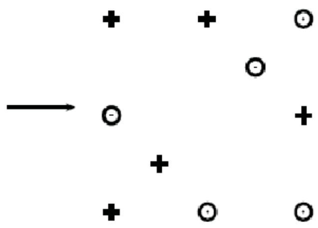
<u>5 vortices</u> $\quad 5S_0 = 1345$

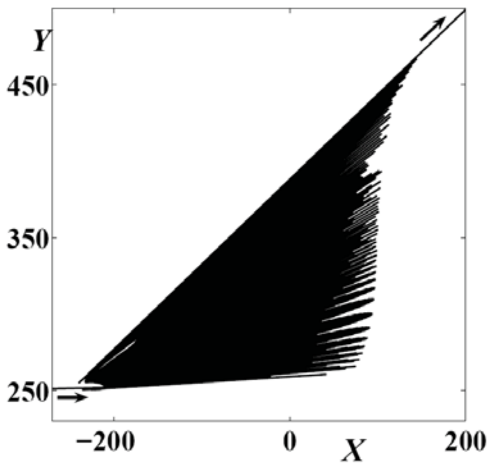


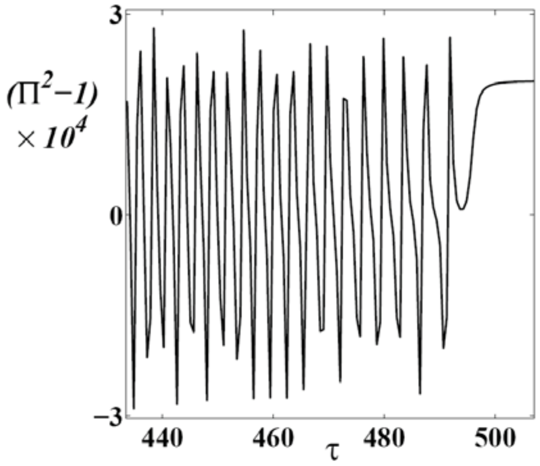
10 vortices $\qquad 10S_0 = 2690$



Multiple Andreev reflection







The quasiparticle turns into a quasihole many times before escaping as a quasiparticle

Conclusions and questions

1. The total Andreev shadow of a vortex structure is not necessarily the sum of shadows of individual (isolated) vortices. This does not mean that the interpretation given to recent experiments is incorrect; it is possible that, for a large random vortex system, the partial screening effects average out. If this is the case, screening effects can be accounted for by introducing a prefactor ($\sim O(1)$?).

2. Homogeneous isotropic turbulence contains coherent vortex structures and is organized in scales. However, provided we are interested only in large-scale properties, the partial screening effects can be accounted for as suggested in **1**.

3. In the case where there is large scale anisotropy, such as in rotating or inhomogeneous turbulence, Andreev technique should be used with more care. Perhaps, in order to gain information about geometry and anisotropy, Andreev reflection measurements should be performed in different directions and combined with numerical calculations.

4. To address the issues discussed in **1** − **3**, numerical investigations are needed in three dimensions with realistic vortex line density.

4. Upon impinging on complex vortex configurations, quasiparticle may experince multiple Andreev reflections, and the resulting reflection angle may not be small.

a) Is this a typical situation, in which case an interpretation of experimental results might be seriously affected, or just a rare event which can be ignored?

b) What are general conditions for the multiple reflection?

c) How does the resulting angle of multiple Andreev reflection depend on geometric and other properties of the complex vortex configuration?

5. Finally, these results are also relevant to the problem of interaction between rotons and quantized vortices in ^{4}He. In particular, the calculation of the mutual friction should be reconsidered in view of possible screening effects.