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IAEA International Atomic Energy Agency

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Workshop on Topics in Quantum Turbulence

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Theory of Low Temperature Decay

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Theory of Low Temperature Decay

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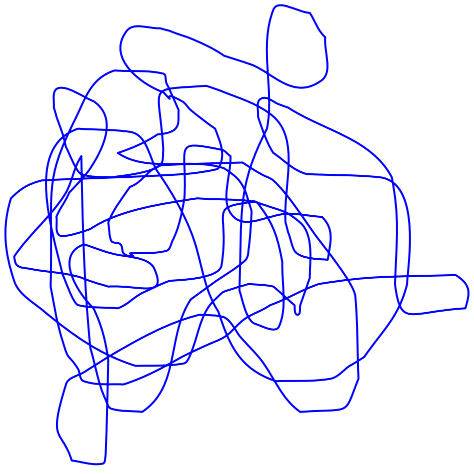


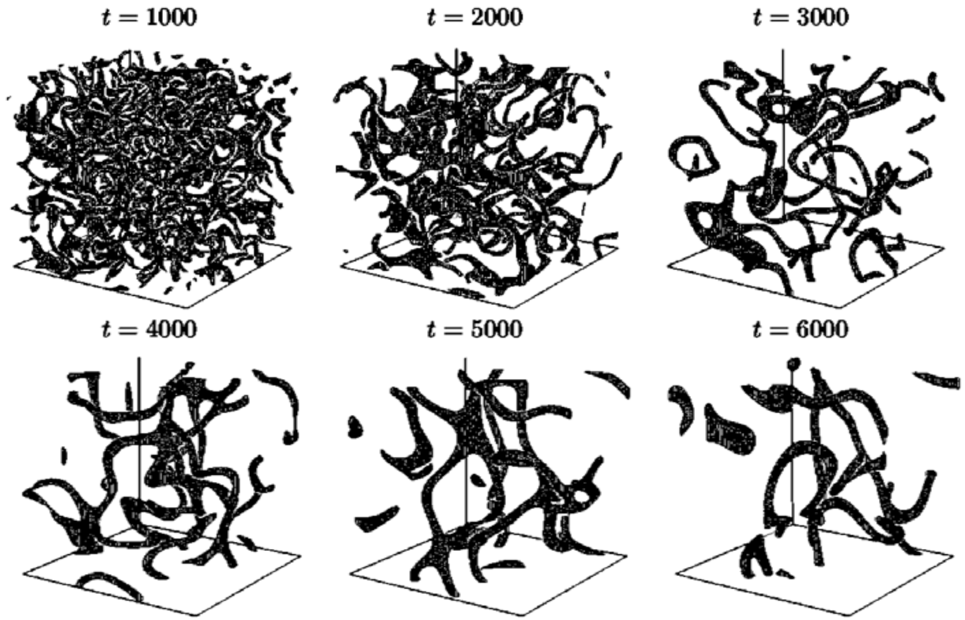
The tangle can be chaotic (non-structured),

or essentially polarized (Kolmogorov regime).

We will focus on the non-structured case.

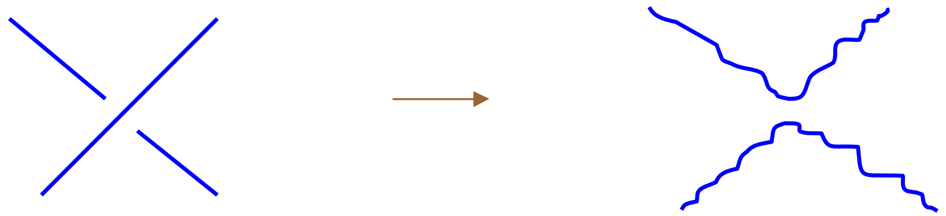
BEC kinetics and superfluid turbulence:

Kibble-Zurek-type picture with a distinct non-trivial mechanism



Simulation of Gross-Pitaevskii equation.   N. Berloff and B. Svistunov, 2002

See recent experiment by Brian Anderson and collaborators, *Nature* ***455,*** *948 (2008)*.



Vortex line reconnections generate Kelvin waves.

Kelvin waves can, in principle, cascade down in the wavelength space, but…

Large and small parameters controlling kinetics of superfluid turbulence at T=0

Local induction: $\Lambda = \ln\left(R / a_0\right) \gg 1$

Pure Kelvin-wave cascade: $\frac{\text{Amplitude}}{\text{Wavelength}} \ll 1$

Becomes progressively smaller down the cascade

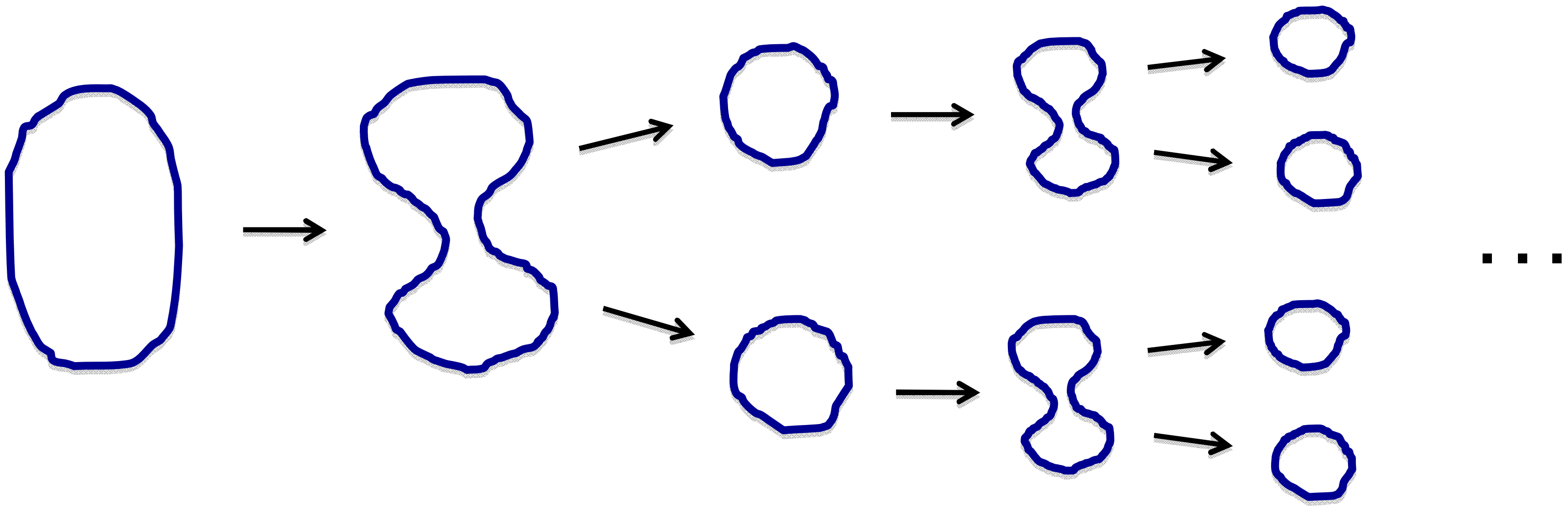
Kelvon-phonon interaction: $\frac{\kappa\lambda^{-1}}{c} \ll 1$

"Non-relativistic" parameter; guarantees weakness of coupling to phonons

Conservation laws controlling kinetics of superfluid turbulence at T=0

Energy conservation	To a good approximation, energy scales as the vortex line length.
Momentum conservation	For a vortex ring, momentum scales as the (algebraic) area.
Angular momentum conservation	= conservation of the number of kelvons: Kelvons cannot scatter inelastically
Integrability of the local-induction limit	Suppression of kelvon scattering processes **(!)**

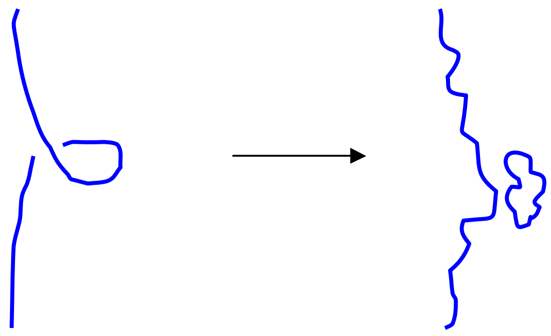
Absence of Feynman’s cascade



Feynman’s cascade is inconsistent with simultaneous conservation of energy and momentum.

The role of self-reconnections

BVS, 1995



A self-reconnection produces a small ring, the ring gets re-absorbed by the tangle.

What's left behind are Kelvin waves!

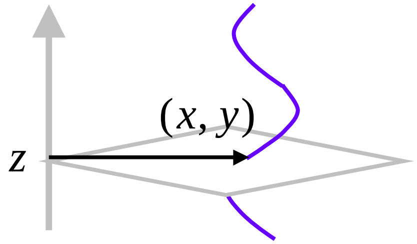
A smart model: Collapse supported cascade

BVS, 1995

canonical variables:

x – momentum

y – coordinate



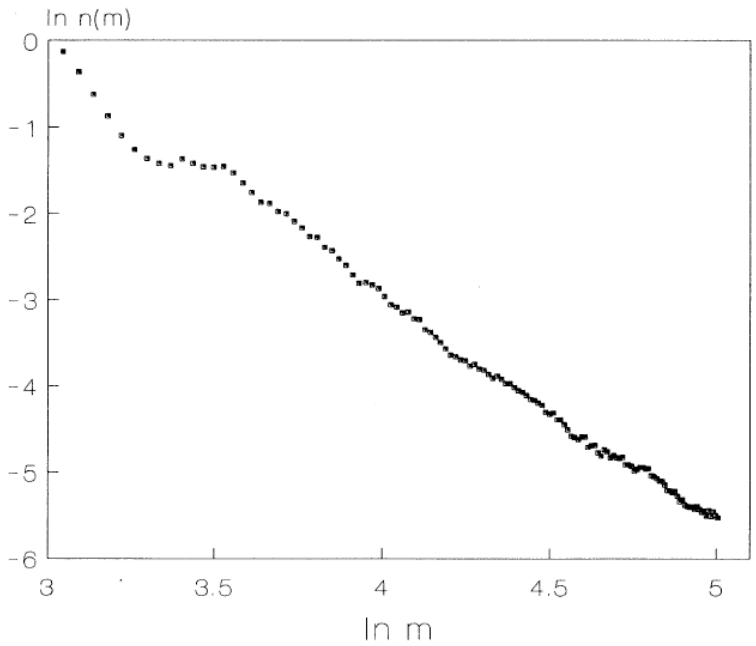
$$w(z) = x(z) + iy(z)$$

$$i\dot{w} = \frac{\delta H[w]}{\delta w^*}$$

$$H[w] = \int dz \sqrt{1 + |w'(z)|^2}$$

This model corresponds to the local induction approximation (and thus is integrable) as long as the function is smooth.

However, the time evolution naturally produces discontinuities, and these are qualitatively equivalent to self-crossings.



Pure Kelvin-wave cascade

Kozik and Svistunov, 2003

$$\text{Small parameter} = \frac{\text{Amplitude}}{\text{Wavelength}}$$

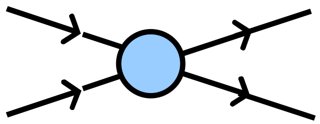
Becomes progressively smaller down the cascade.

1. Kelvon is a good elementary excitation

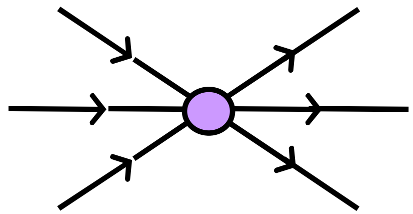
2. Number of kelvons is conserved (due to the rotational invariance)

Kelvon Hamiltonian:

$$H_{\mathrm{kw}} = \sum \varepsilon_k a_k^{+} a_k \quad +$$



$+$

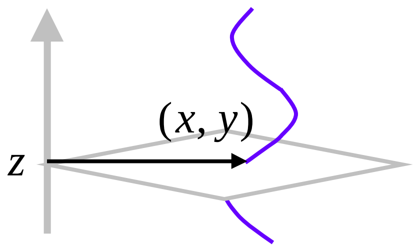


$+ \ldots$

canonical variables:

x – momentum

y – coordinate



$$w(z) = x(z) + iy(z)$$

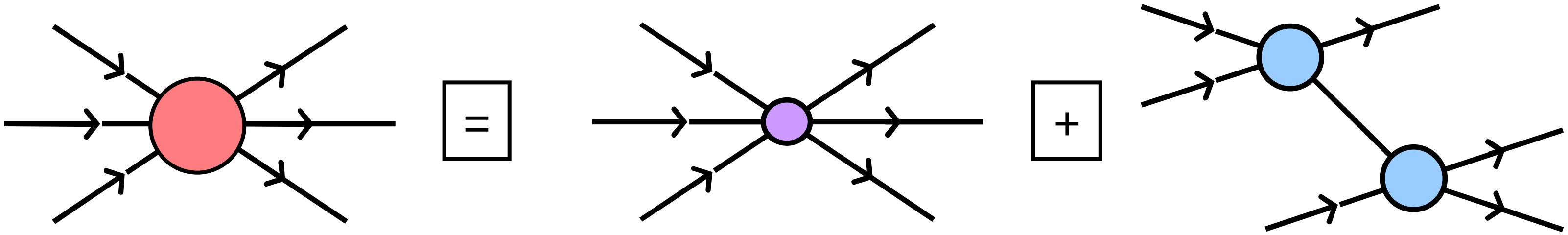
$$a_k \propto \int w(z) e^{ikz} dz$$

Kinetic equation

Non-trivial two-kelvon scattering is absent because of 1D

Kinetics are driven by

three-kelvon elastic scattering:



kelvon occupation number

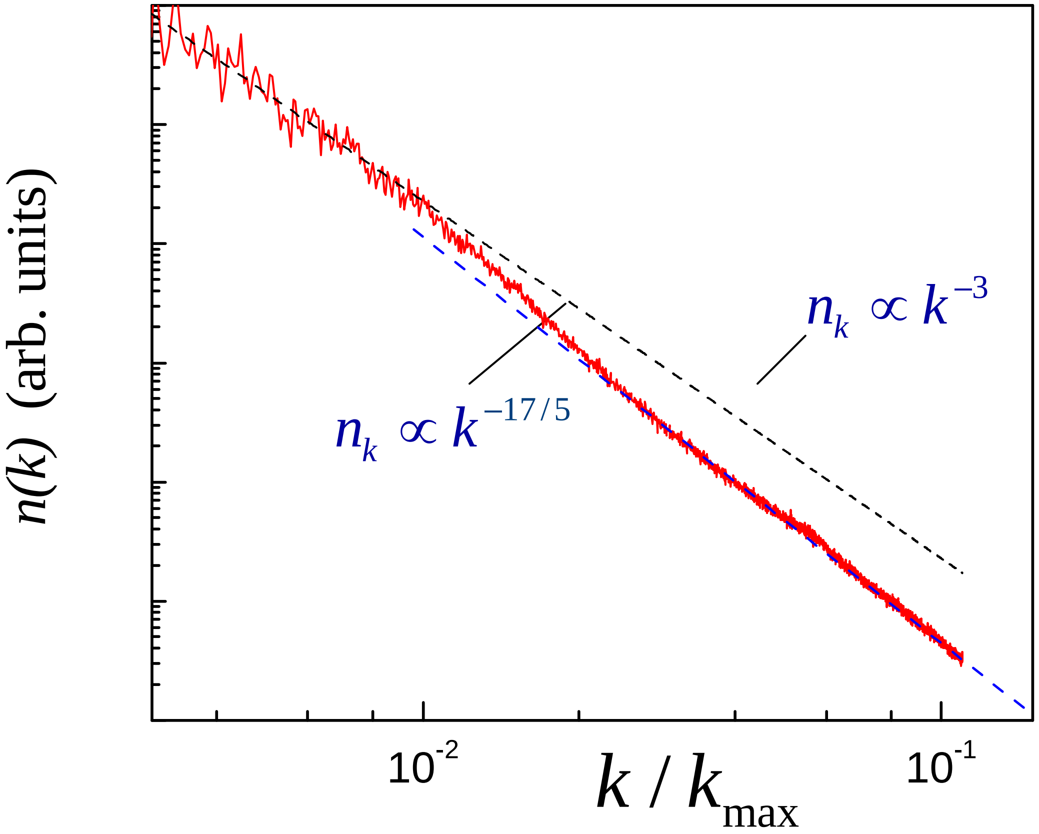
dimensional analysis of the collision term →

$$n_k \propto k^{-17/5}$$

or for Kelvin wave amplitude ($b_k \propto \sqrt{n_k k}$): $b_k \propto k^{-6/5}$

17/5

Kozik and Svistunov, 2003



Initial condition:

$n_k \propto k^{-3}$

External forces:

drain, *no* source

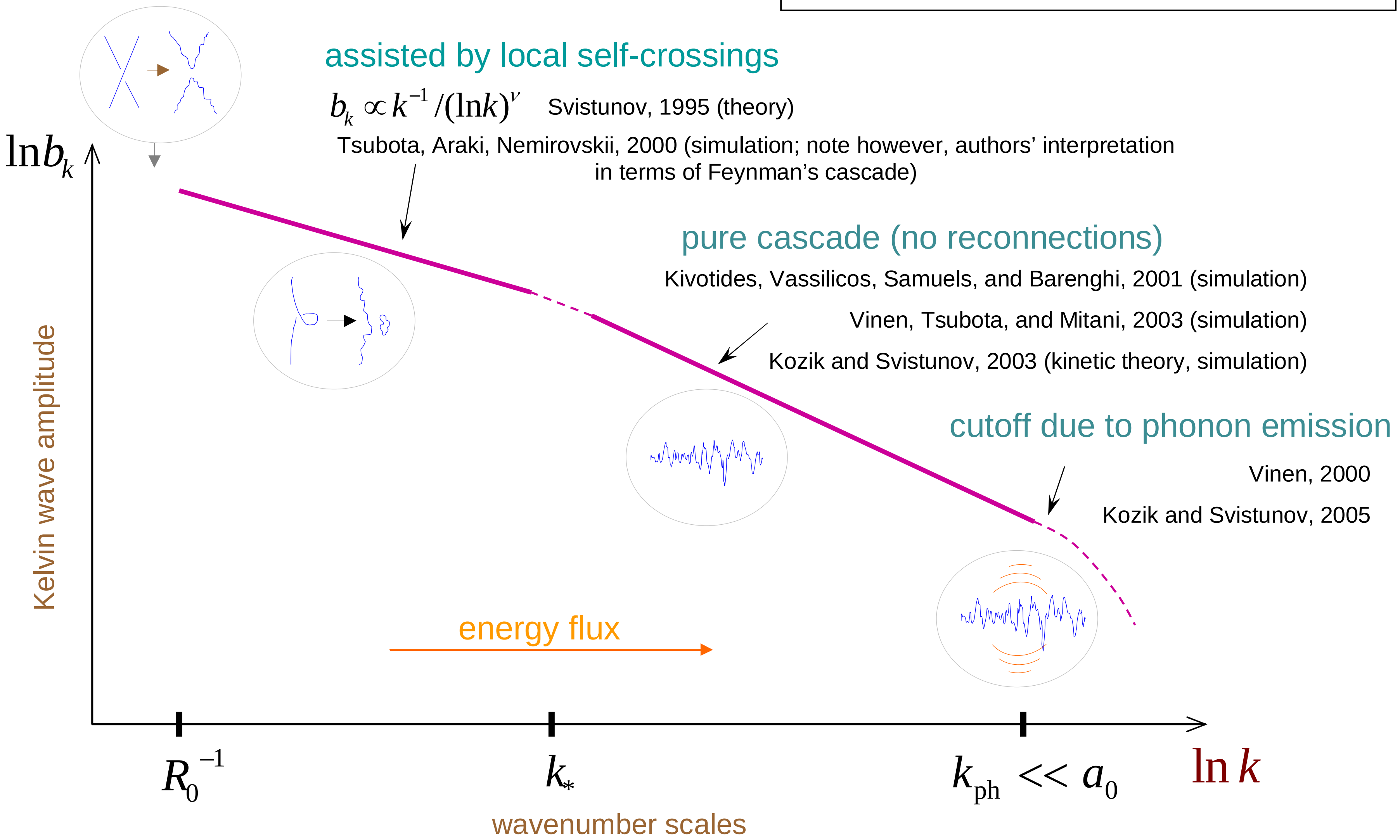
Evolution picture:

a back-wave propagating from large-wavenumber region towards smaller wavenumbers transforms $n_k \propto k^{-3}$ into $n_k \propto k^{-17/5}$.

Kelvin-wave cascade in the chaotic tangle

(Circumstantial) experimental evidence:

Davis, Hendry, and McClintock, 2000



Emission of sound, and more…

What are phonons in the presence of vortices?

Phase field: $\Phi(\vec{r}) = \Phi_0(\vec{r}) + \varphi(\vec{r})$

$\oint_{\text{around vortex}} \nabla\Phi_0 \cdot d\vec{l} = \pm 2\pi$ singular

non-singular $\oint \nabla\varphi \cdot d\vec{l} = 0$

$\Delta\Phi_0(\vec{r}) = 0$ (away from a vortex)

But how about the causality?

To find the canonical variables, use the Lagrangian.

Vortex – Phonon Lagrangian

$$L = \int d\mathbf{r}\left[-n\dot{\Phi}_0 - \eta\dot{\varphi} + \eta\dot{\Phi}_0\right] - H_{\mathrm{kw}} - H_{\mathrm{ph}} - H_{\mathrm{int}}$$

vortex

$$\oint_{\text{around vortex}} \nabla\Phi_0 d\mathbf{r} = 2\pi$$

$$\nabla^2\Phi_0 = 0$$

$$H_{\mathrm{kw}} \approx \sum_k \varepsilon_k a_k^+ a_k$$

$$H_{\mathrm{ph}} \approx \sum_{\mathbf{q}} \omega_{\mathbf{q}} c_{\mathbf{q}}^+ c_{\mathbf{q}}$$

phonons

$$\oint_{\text{anywhere}} \varphi\, d\mathbf{r} = 0$$

$\{a_k, a_k^+\}$, $\{c_{\mathbf{q}}, c_{\mathbf{q}}^+\}$ are not canonical !

variable transformation:

$\{a_k, a_k^+\}$, $\{c_{\mathbf{q}}, c_{\mathbf{q}}^+\}$ (eliminating $\eta\dot{\Phi}_0$) → $\{\tilde{a}_k, \tilde{a}_k^+\}$, $\{\tilde{c}_{\mathbf{q}}, \tilde{c}_{\mathbf{q}}^+\}$

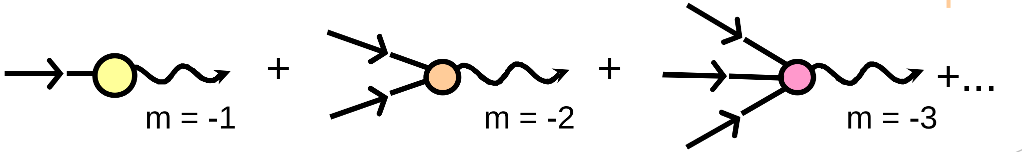
canonical

Vortex – phonon Hamiltonian

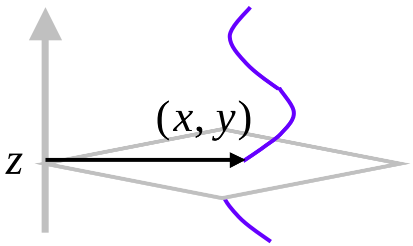
$$\text{Energy} = \sum_k \varepsilon_k a_k^+ a_k + \sum_{\mathbf{q}} \omega_{\mathbf{q}} c_{\mathbf{q}}^+ c_{\mathbf{q}} + H_{\text{int}}$$

Hamiltonian =

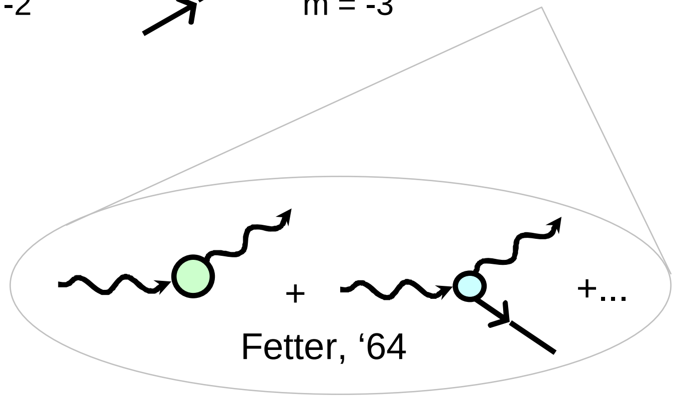
$$\sum_k \varepsilon_k \tilde{a}_k^+ \tilde{a}_k + \sum_{\mathbf{q}} \omega_{\mathbf{q}} \tilde{c}_{\mathbf{q}}^+ \tilde{c}_{\mathbf{q}} +$$



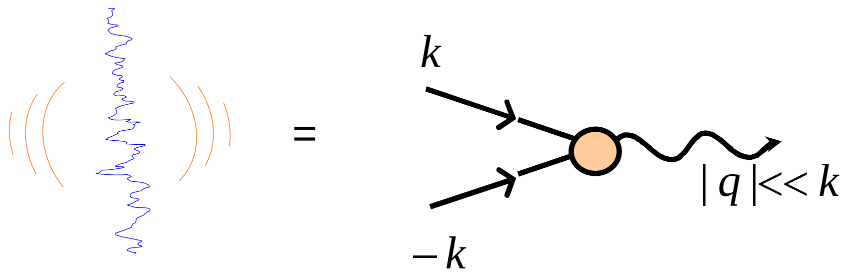
$$+ \; H_{\text{int}}$$



conserves
1. momentum along z
2. angular momentum along z

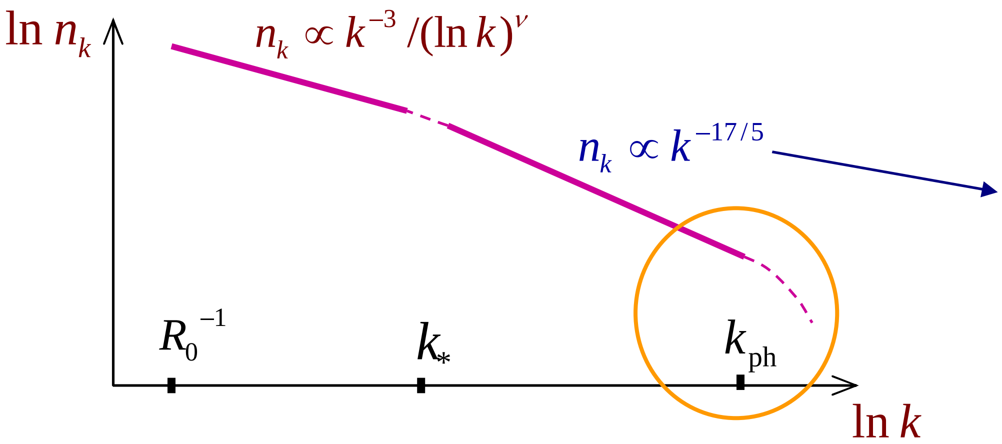


Sound radiation by superfluid turbulence



radiated power per unit length:

$$\Pi_k = \text{Const.}\,\frac{\varepsilon_k^6 k}{c^5 \rho}\left(n_k n_{-k}\right), \qquad \varepsilon_k = \frac{\kappa}{4\pi}\ln(1/ka_0)\,k^2$$

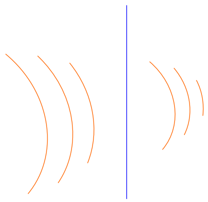


$$k_{\text{ph}} \sim \frac{\left[a_0 / R_0\right]^{6/31}}{\left[\ln(R_0 / a_0)\right]^{24/31}}\, a_0^{-1}$$

$$\lambda_{\text{ph}} \sim 1 \div 0.1 \mu m$$

Scattering

Elastic scattering



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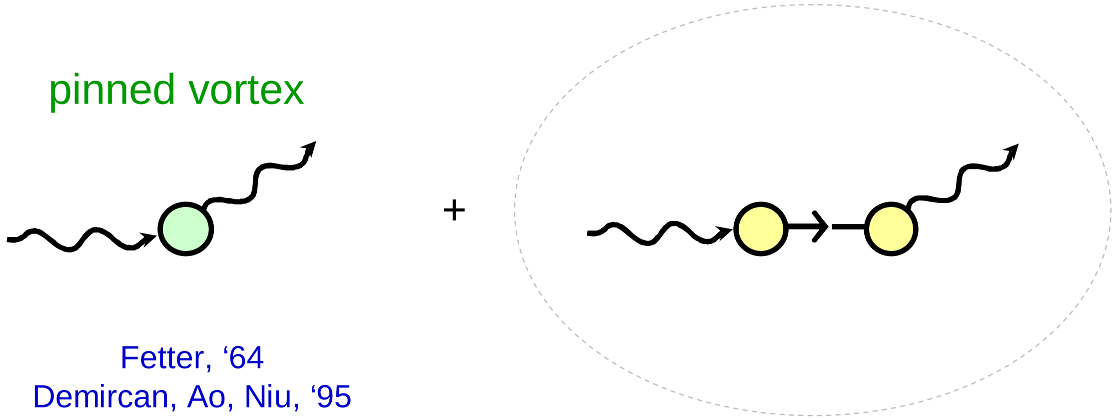
pinned vortex

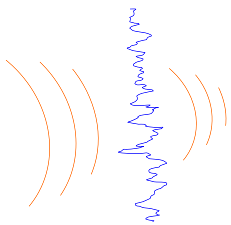


correct result: Pitaevskii, '58
Sonin, '76
(see also
Iordanskii)

Fetter, '64
Demircan, Ao, Niu, '95

Kozik and Svistunov, 2005

Inelastic scattering



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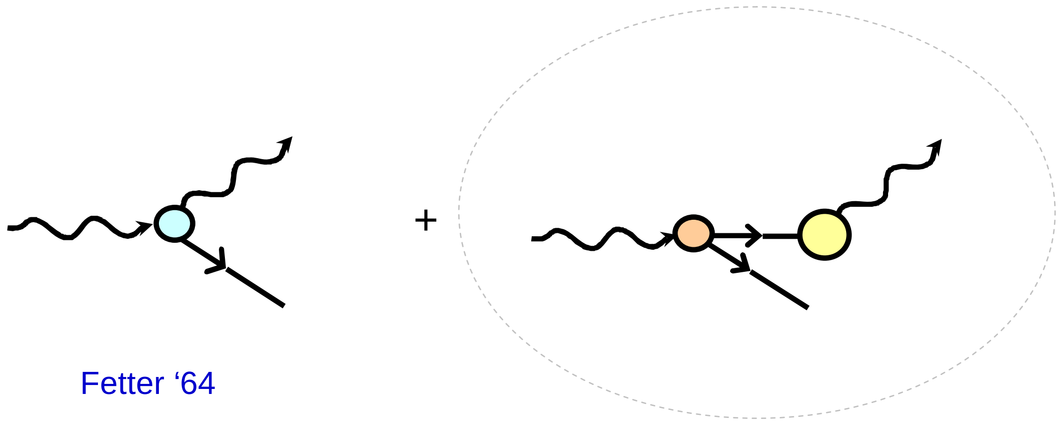

correct result

Fetter '64

Kozik and Svistunov, 2005

Conclusions

- Reasons for a physically rich Theory: Conserving quantities and small parameters

- Absence of Feynman’s cascade

- Self-reconnection supported range is inevitable.

- Theory of pure Kelvin wave cascade

- The Hamiltonian of vortex-phonon interaction: the answers for sound radiation by Kelvin wave cascade, and other vortex-phonon processes.