The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2023-3

Workshop on Topics in Quantum Turbulence

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Between the Kolmogorov and Kelvin-wave cascades

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Between the Kolmogorov and Kelvin-wave cascades

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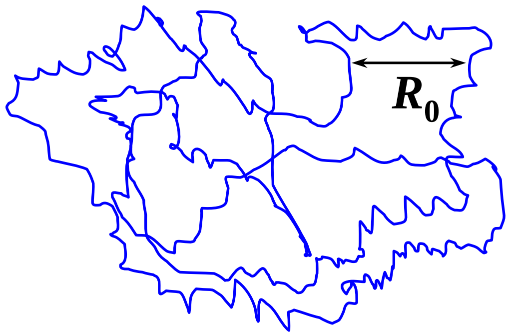
Outline

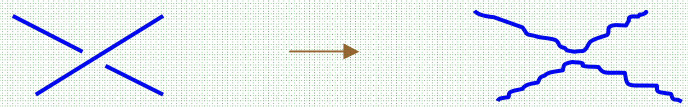
- What do we learn from the decay of non-structured tangles?
  bottleneck-like effect, Kelvin-wave cascades
- Quasi-classical tangles in detail
  Kolmogorov vs. Kelvin-wave cascade
  bottleneck proposal
  reconnection-driven crossover regime
- Finite T effects
  Scanning the cascade by mutual friction cutoff

Non-structured tangles at T=0: Decay Scenario overview



•Random reconnections at the largest scale $\lambda \sim R_0$

generate energy flux $\theta \sim \kappa^3 \ln^2(R_0 / a_0)\, R_0^{-2}$:



Generation of Kelvin Waves

Nonlinear KW interactions are too weak to support the flux θ !

$\lambda < R_0$: **“Bottleneck” accumulation of energy…**

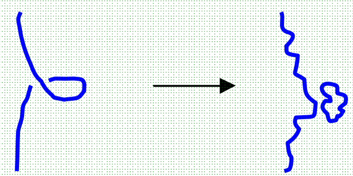
… until:

•Self-reconnections

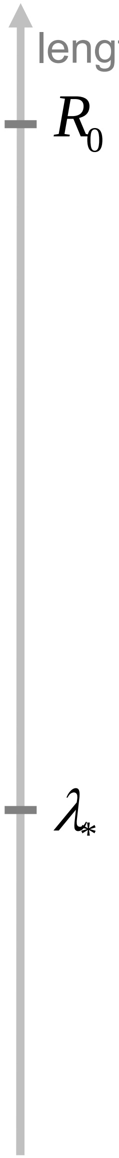
large amplitudes

$b_k k \sim 1$

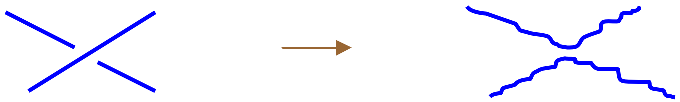
Svistunov (1995)



Non-structured tangles at T=0: overview



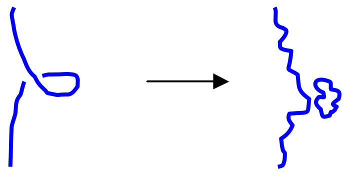
•Reconnections at $\lambda \sim R_0$:

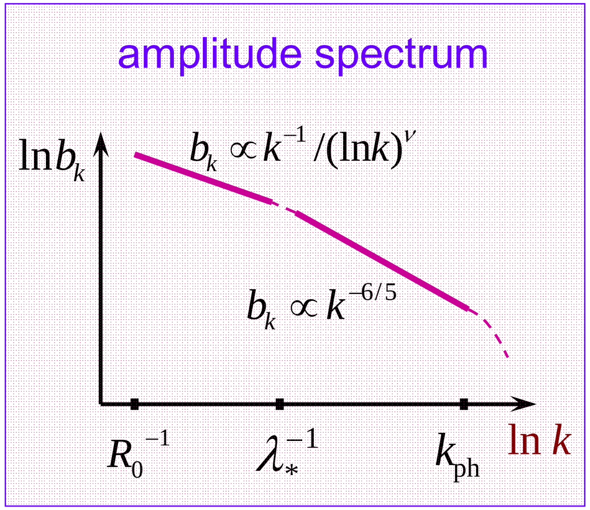


•Self-reconnections at $\lambda < R_0$, large amplitudes:

B.V. Svistunov (1995).

$b_k k \sim 1$

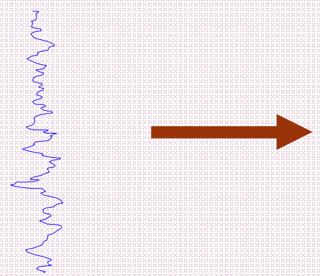


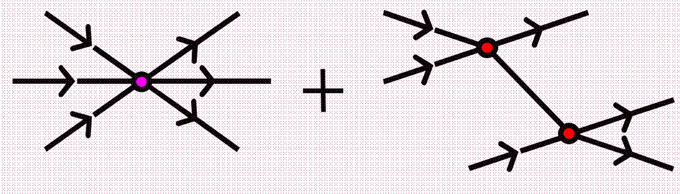


•Purely non-linear kinetics at $\lambda < \lambda_* \ll R_0$,small amplitudes:

E.V. Kozik and B.V. Svistunov, Phys. Rev. Lett. 92, 035301 (2004).

$b_k k \ll 1$



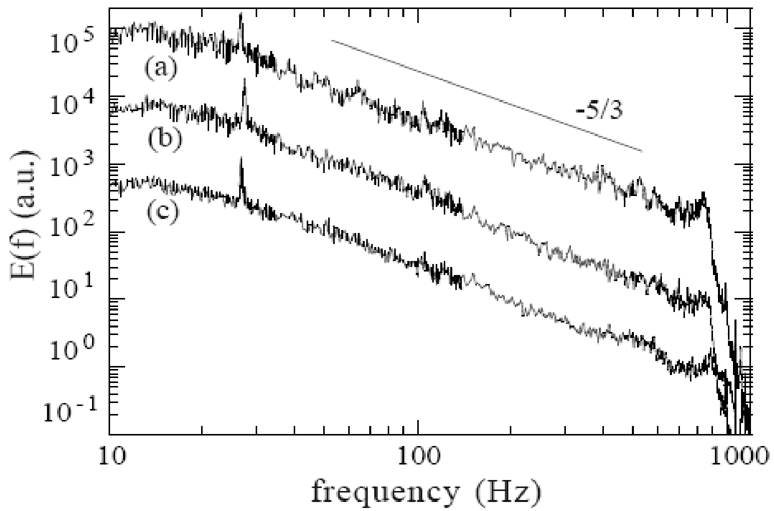


3-kelvon scattering

Superfluid Turbulence Behaving Classically

experiments:

J. Maurer and P. Tabeling, in superfluid He-4:



Classical Kolmogorov law at large scales

$$E(k) \sim \varepsilon^{2/3} k^{-5/3}$$

When turbulence is generated classically (by "stirring")

S. R. Stalp, L. Skrbek, and R. J. Donnelly, Phys. Rev. Lett. **82**, 4831 (1999).

In simulations

C. Nore, M. Abid, and M.E. Brachet, Phys. Rev. Lett. **78**, 3896 (1997).

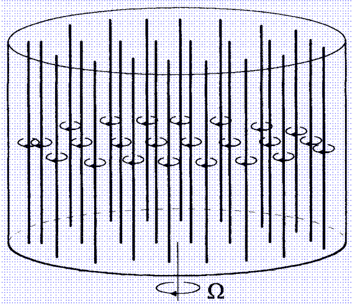
T=0 :

He-3: D. I. Bradley, D. O. Clubb, S. N. Fisher, A. M. Gue'nault, R. P. Haley, C. J. Matthews, G. R. Pickett, V. Tsepelin, and K. Zaki, Phys. Rev. Lett. **96**, 035301 (2006).

He-4: P. M. Walmsley, A. I. Golov, H. E. Hall, A. A. Levchenko, and W. F. Vinen, Phys. Rev. Lett. **99**, 265302 (2007).

Superfluid Turbulence Behaving Classically: general picture

- main parameter -- **Kolmogorov energy flux** ε
- ε is formed by the **largest** classical eddies
- classical eddy in a superfluid $\leftrightarrow$ **array of vortex lines** $\rightarrow$

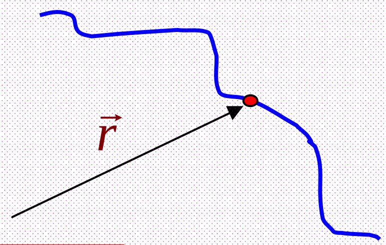


The existence of the Richardson cascade in superfluids is <u>not trivial</u> !!!

Formal proof:

Biot-Savart equation for quantized vortex lines:

$$\frac{d\vec{r}}{dt} = \frac{\kappa}{4\pi}\int (\vec{r}_0 - \vec{r}) \times d\vec{l}_0 \,/\, |\vec{r}_0 - \vec{r}\,|^3$$

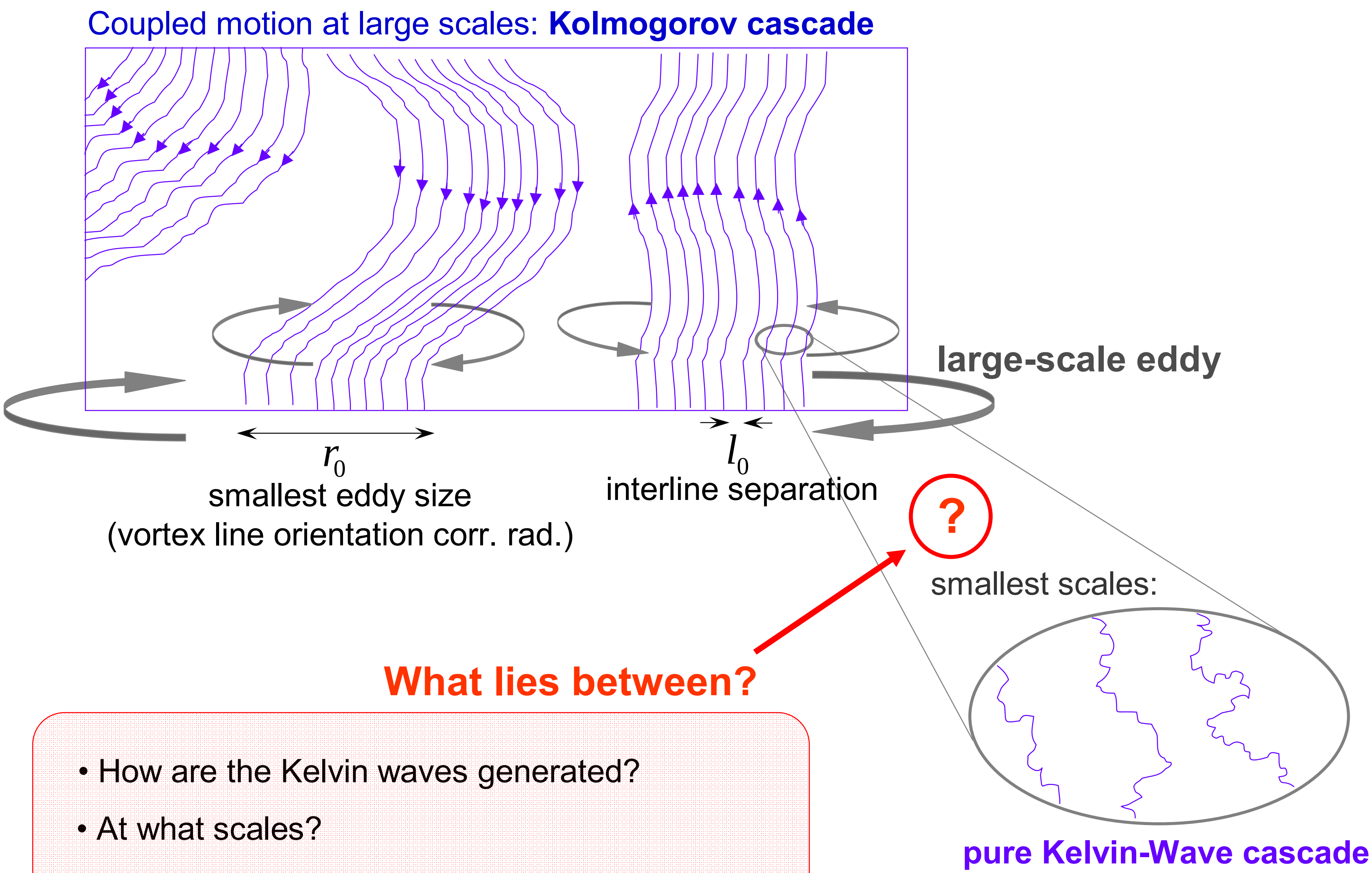


In terms of vorticity, $\vec{\omega}_{\vec{k}} = \kappa \int e^{-i\vec{k}\cdot\vec{r}}\, d\vec{l}$, becomes: (κ **drops out!!!)**

$$\frac{\partial \vec{\omega}_{\vec{k}}}{\partial t} = \vec{k} \times \int \frac{d^3 q}{(2\pi)^3}\, q^{-2} \left\{ \left(\vec{k}\cdot\vec{\omega}_{\vec{k}-\vec{q}}\right)\vec{\omega}_{\vec{q}} - \left(\vec{\omega}_{\vec{q}}\cdot\vec{\omega}_{\vec{k}-\vec{q}}\right)\vec{q} \right\}$$

Vorticity equation for an ideal incompressible classical fluid

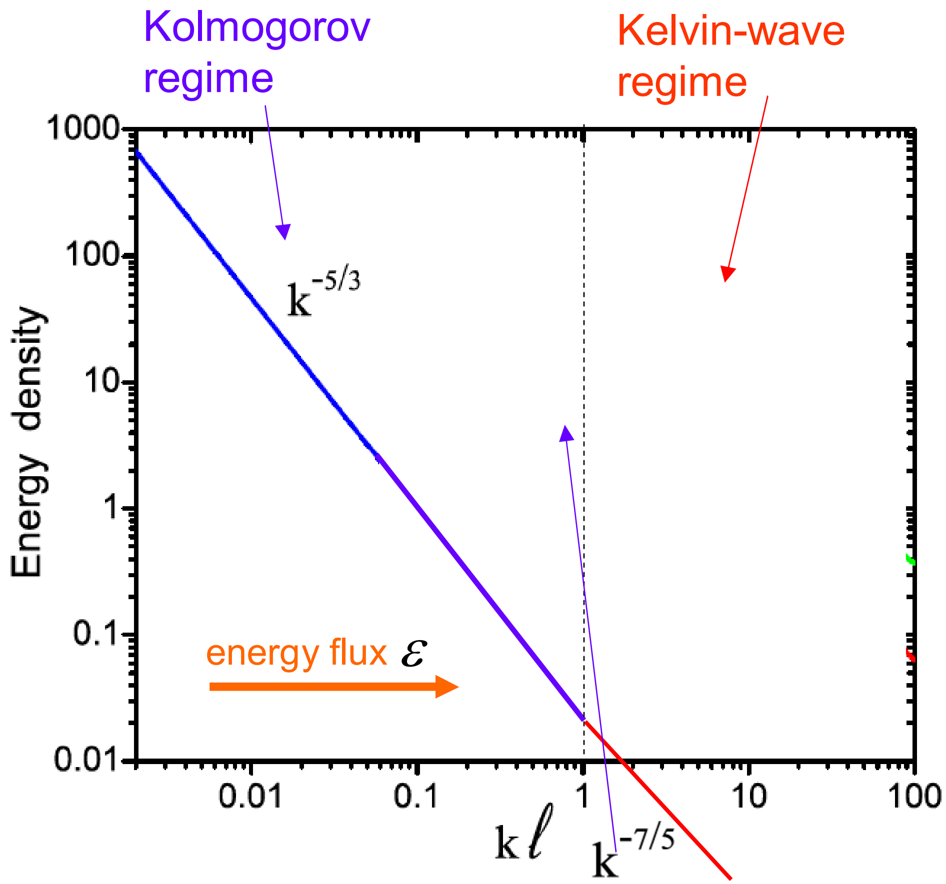
What is different at T=0?



- How are the Kelvin waves generated?
- At what scales?

Energy Bottleneck idea

L'vov, Nazaranko, Rudenko, PRB **76**, 024520 (2007).



Bottleneck accumulation of energy

Pure Kelvin-wave cascade is unable to sustain the Kolmogorov energy flux!

key parameter:

$$\Lambda = \ln(l_0 / a_0) >> 1$$

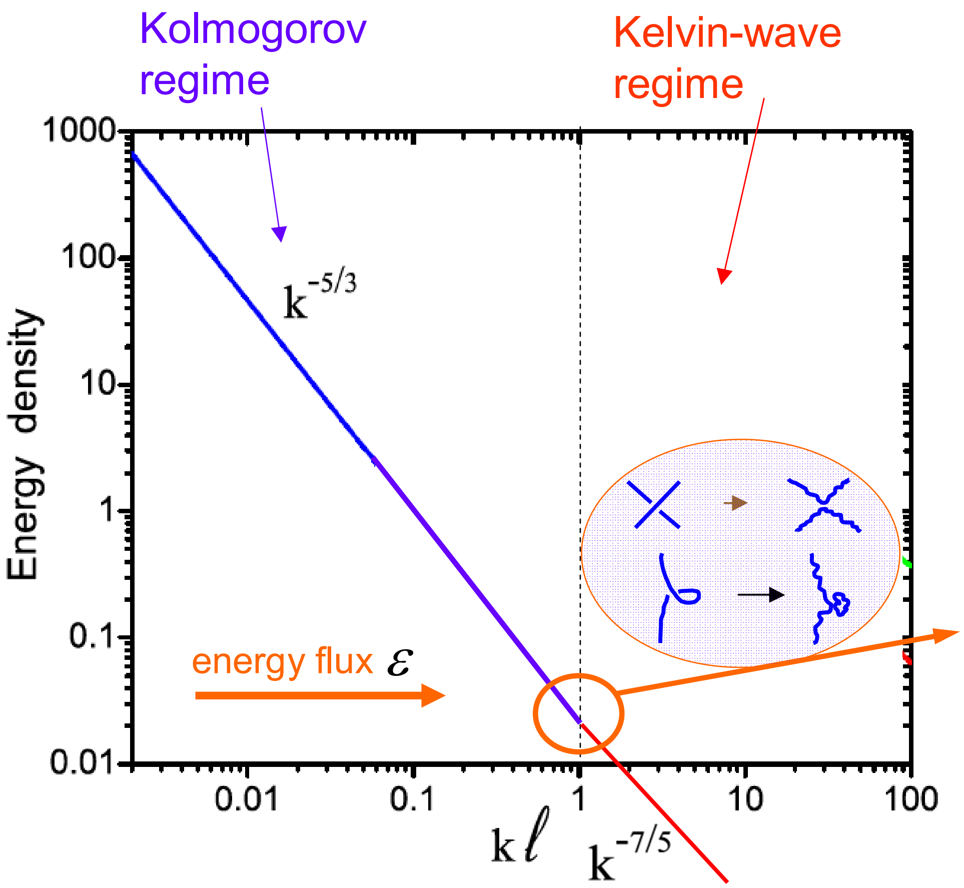
~ 15 in He-4

One step back…

key parameter:

$$\Lambda = \ln(l_0 / a_0) >> 1$$

~ 15 in He-4



Pure Kelvin-wave cascade is unable to sustain the Kolmogorov energy flux!

The same as in non-structured tangles!

What about reconnections here as well?

Interline separation = l

vortex-line curvature radius = l

energy flux due to reconnections:

$$\frac{\varepsilon_{rec}}{\varepsilon} \sim \Lambda^2 >> 1 \;\;\textbf{!!!}$$

Anti-bottleneck???

Something must be happening before we reach the scale l !!!

Superfluid Turbulence Behaving Classically: Classical to Quantized crossover

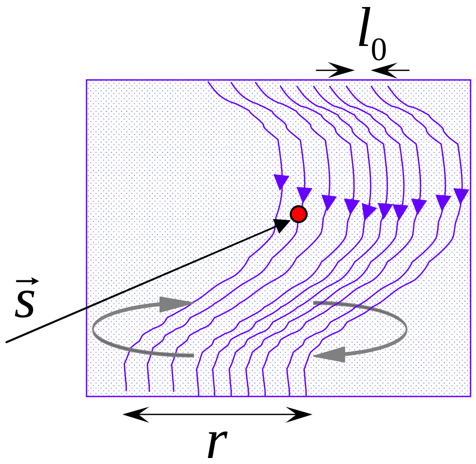
Kozik and Svistunov, 2007

T=0, only degrees of freedom --- vortex lines:

$$\dot{\vec{s}} = \vec{v}(\vec{s}), \qquad \vec{v}(\vec{r}) = \frac{\kappa}{4\pi}\int \frac{(\vec{s}_0 - \vec{r}) \times d\vec{s}_0}{|\vec{s}_0 - \vec{r}|^3}$$

$$\vec{v}(\vec{s}) = \vec{v}^{\,\mathrm{SI}}(\vec{s}) + \vec{v}^{\,\mathrm{I}}(\vec{s})$$

$\int_{\text{containg } \vec{s}} \ldots$ (self-induced) $\int_{\text{the rest}} \ldots$ (induced by other lines)



controlling parameter

$$\Lambda = \ln(l_0 / a_0) >> 1$$

~ 15 in He-4

Kolmogorov vs. quantized regime:

Competition between v^{SI} and v^{I} !

$v^{\mathrm{SI}} \sim \Lambda \kappa / r$

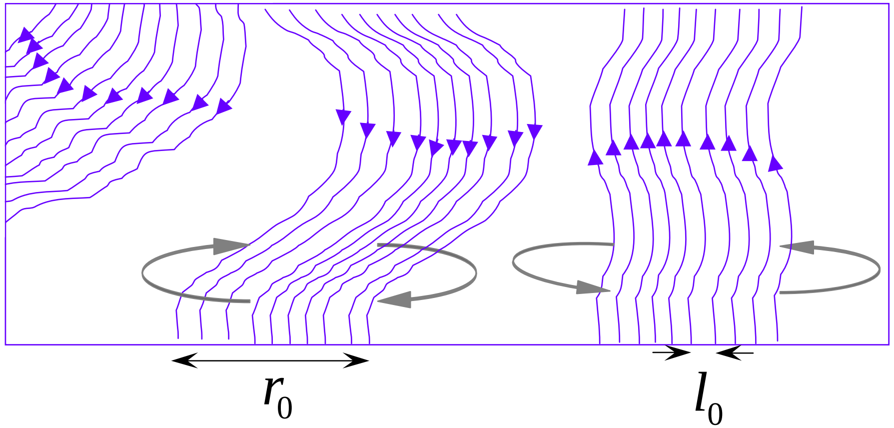
$v^{\mathrm{I}} \sim N_r \kappa / r$

N_r – number of lines in eddy of size r

Crossover scale:

$$r_0 \sim \Lambda^{1/2} l_0$$

The crossover scale r_0



Kolmogorov regime ($v_r \sim (\varepsilon r)^{1/3}$ **):**

number of lines in an eddy of size r

$$N_r \sim \varepsilon^{1/3} r^{4/3} / \kappa$$

competes with the **L.I.A. parameter**

$$\Lambda_r = \ln(r / a_0) \sim \Lambda = \ln(l_0 / a_0)$$

$N_r >> \Lambda$ - quasi-classical regime

$N_r << \Lambda$ - quantized regime

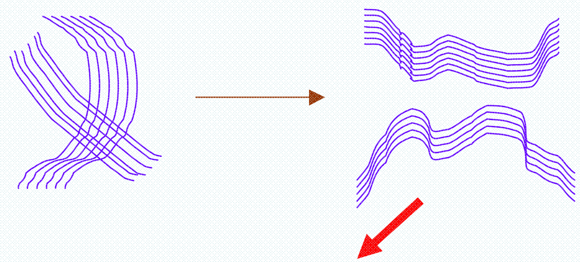
crossover scale $r_0 \sim \Lambda^{1/2} l_0$

interline separation $l_0 \sim (\Lambda \kappa^3 / \varepsilon)^{1/4}$

At the scale r_0 :

- vortex lines are moving independently
- bundle decoherence time > vortex-line turnover time

reconnections of bundles



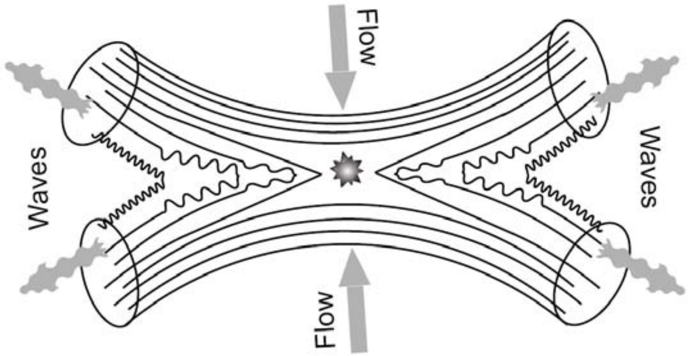
generation of Kelvin waves!!!

The controversy!

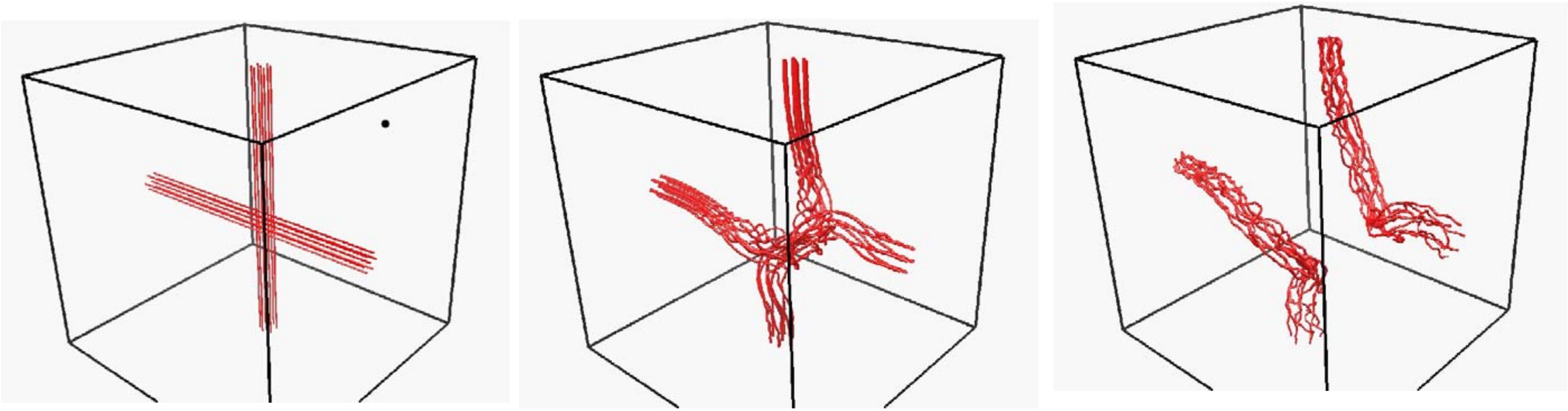
An objection by L'vov, Nazaranko, and Rudenko:

PRB **76**, 024520 (2007)

Bundle reconnections as seen by L’vov, *et al.:*



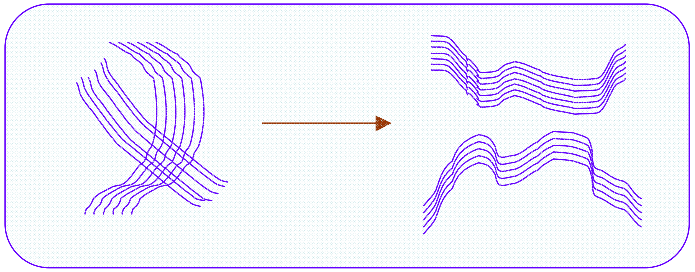
However, in simulations:



Alamri, Youd, and Barenghi, 2008

Scales below r_0

Reconnections of vortex-line bundles:



continues self-similarly with the spectrum

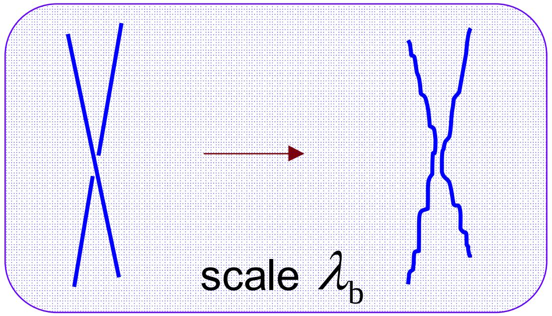
$$b_k \sim r_0^{-1} k^{-2}$$

in the range of scales

$$\lambda_b \sim \Lambda^{1/4} l_0 \ll \lambda \ll r_0$$

At the scale $\lambda_b \sim \Lambda^{1/4} l_0$, $b_k \sim l_0$ → the notion of bundles looses meaning

Reconnections of nearest-neighbor lines at small angles:



- Bottleneck-like regime!
- Not a true cascade!

Reconnections at λ_b lead to **energy build up** with the spectrum

$$b_k \sim l_0 (\lambda_b k)^{-1/2} \qquad b_k k \propto k^{1/2}$$

within the range $\lambda_c \sim l_0 / \Lambda^{1/4} \ll \lambda \ll \lambda_b$

At the scale λ_c , $b_k \sim \lambda_c$ → self-reconnections take over the energy flux

From Kolmogorov to Kelvin-wave cascade (large parameter $\Lambda = \ln(l_0 / a_0) >> 1$)

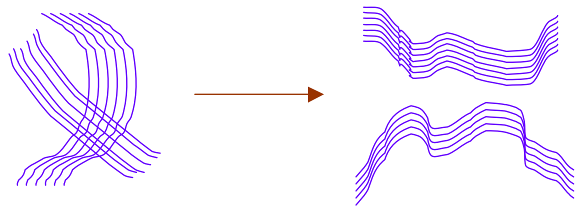
PRB 77, 060502, (2008).

length scale

$v^{I} \sim v^{SI}$ → crossover to QT

$r_0 \sim \Lambda^{1/2} l_0$

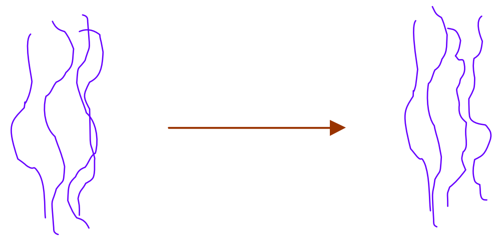
reconnections of vortex bundles



$b_k \sim r_0^{-1} k^{-2}$

$\lambda_b = \Lambda^{1/4} l_0$

reconnections between neighbors in the bundle

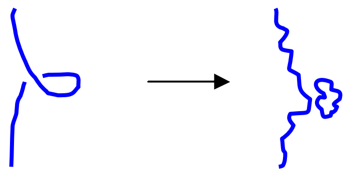


$b_k \sim l_0 (\lambda_b k)^{-1/2}$

$\lambda_c \sim l_0 / \Lambda^{1/4}$

self – reconnections

(vortex ring generation)



$b_k \sim k^{-1}$

$\lambda_* = l_0 / \Lambda^{1/2}$

purely non-linear cascade of Kelvin waves

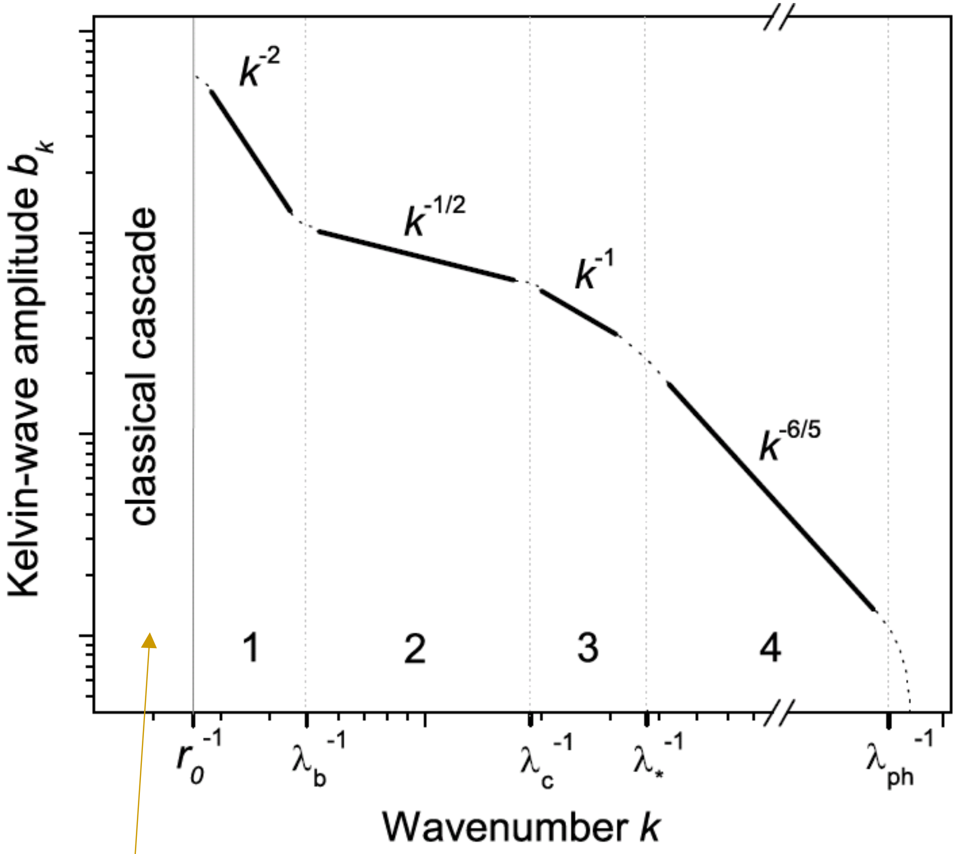
(no reconnections)

$b_k = (\Theta / \kappa^3 \rho)^{1/10} k^{-6/5}$

$\lambda_{ph} = [\, \Lambda^{27} \, (\kappa / c)^{25} \, l_0^6 \,]^{1/31}$ → phonon radiation

From Kolmogorov to Kelvin-wave cascade at T=0

Spectrum of Kelvin waves in the quantized regime:



$$E(k) \sim \varepsilon^{2/3} k^{-5/3}$$

1. reconnections of bundles
2. neighbor reconnections in the bundles
3. self-reconnections on single lines with vortex ring emission
4. non-linear Kelvin-wave cascade on single lines without reconnections

What happens if T is finite?

(EK and B. Svistunov, 2008)

Key parameter – mutual friction coefficient $\alpha = \alpha(T) \propto T^5, \; T \to 0$

Kelvin waves decay due to the mutual friction:

$$\dot{b}_k \sim -\alpha \, \omega_k \, b_k \qquad \omega_k = (\kappa/4\pi)\Lambda k^2$$

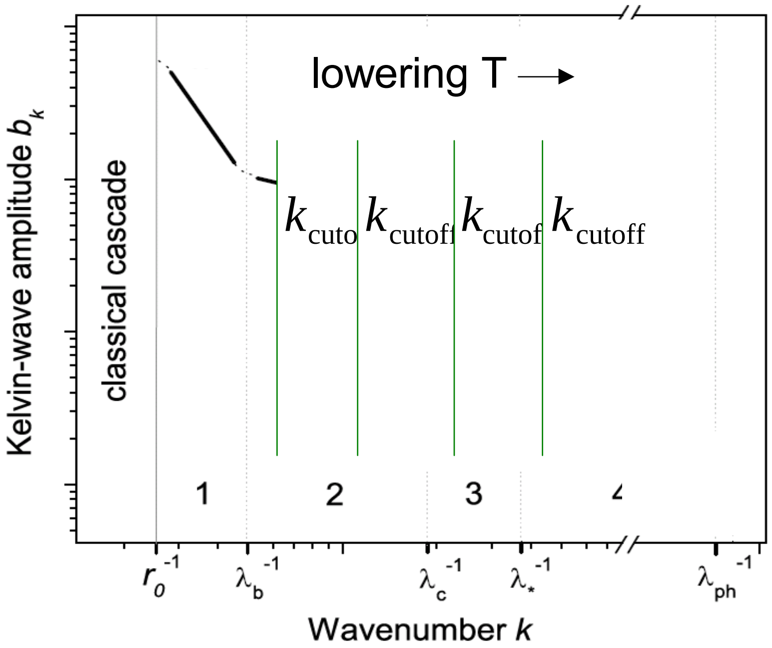
Dissipative energy flux (per unit line length):

$$\Pi(k) \sim \alpha \, \kappa \, \rho \, \omega_k^2 \, b_k^2 \longrightarrow k_{\mathrm{cutoff}}[\alpha(T)]$$

Scanning the cascade:

$$\ln [L(\alpha)/L_0] = \int_{\tilde{k}}^{k_{\mathrm{cutoff}}(\alpha)} (b_k k)^2 \, dk/k$$

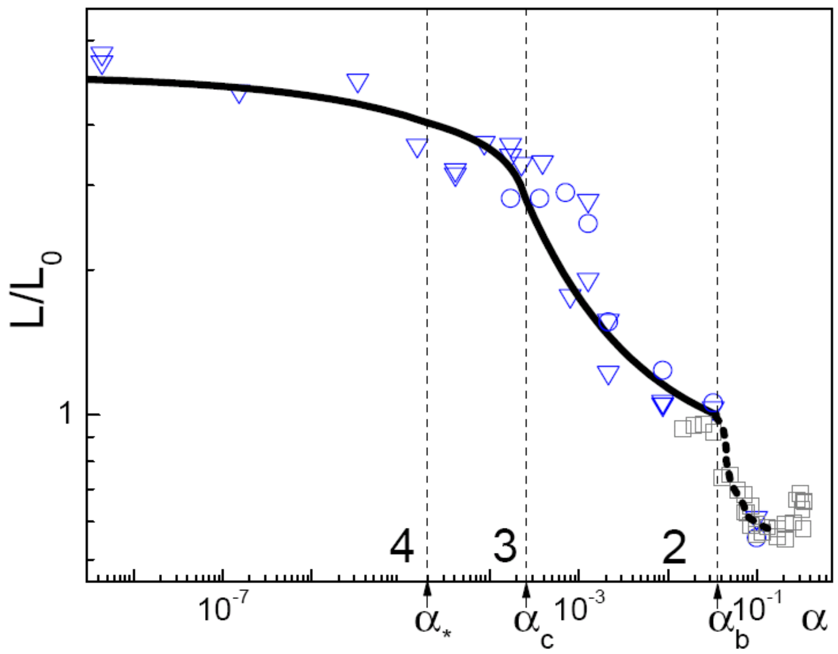
Vortex line length increase

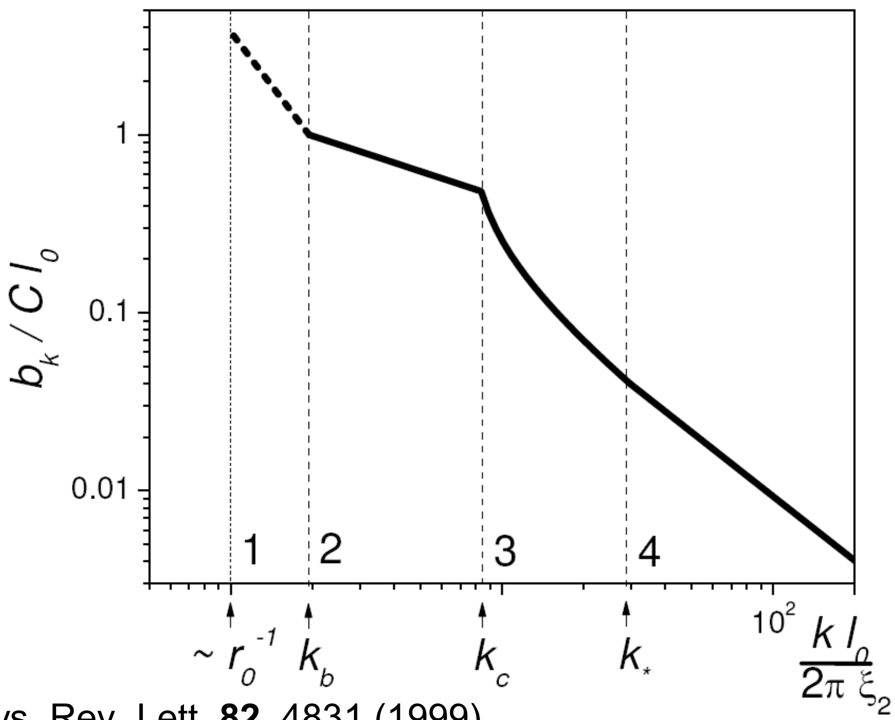


Fitting experimental data, extracting the Kelvin-Wave spectrum

Vortex line length increase: $\ln\left[L(\alpha)/L_0\right] = \int_{\tilde{k}}^{k_{\mathrm{cutoff}}(\alpha)} (b_k k)^2 \, dk/k$

vortex line length → Kelvin-wave spectrum





Stalp, Skrbek, and Donnelly, Phys. Rev. Lett. **82**, 4831 (1999).

Experiment: Walmsley, Golov, Hall, Levchenko, Vinen, PRL **99**, 265302 (2007)

Walmsley and Golov, Phys. Rev. Lett. **100**, 245301 (2008)

Conclusions

• A theoretical picture of the decay of Quasi-classical turbulence seems to emerge.

• The qualitative agreement between the theory and experiments is very promising

• Next steps: numerics (!), dissipation in 3He, rotating turbulence, calorimetry, (plausibly) Kelvin-wave spectroscopy, etc.