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Workshop on Topics in Quantum Turbulence

16 - 20 March 2009

Dynamics of quantum turbulence in 4He in the T = 0 limit

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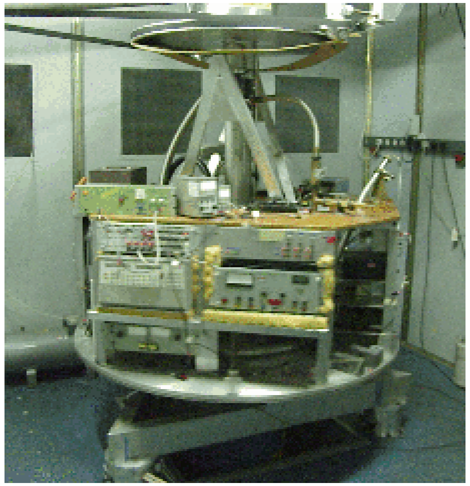
Topics on Quantum Turbulence, ICTP Trieste, 16 March 2009

Dynamics of quantum turbulence in ^{4}He in the $T = 0$ limit

Andrei Golov

Paul Walmsley, Joe Vinen, Henry Hall, Alexander Levchenko, Steve May, Peter Tompsett

1. Motivation to study dynamics of homogeneous turbulence in the $T = 0$ limit
2. Main achievements of past (1955-2000)
3. Manchester results (2006-2009)



The scope of this talk: evolution/dissipation of vortices and their tangles in as homogeneous and steady conditions as possible – in ^{4}He at $T < 1$ K

Part 1. QT - turbulence in SF. In superfluid component, it is a tangle of quantized vortices. Their discrete nature makes dynamics at quantum scales unusual.

We are interested in either $T = 0$ or $T < 1$ K (depleted, non-turbulent normal component)
One-fluid turbulence with a broad range of scales – from classical to quantum.

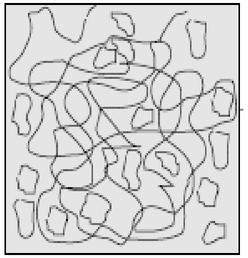
There are three mainstream types of turbulence:
classical (hydrodynamic) turbulence, magnetodynamic turbulence, wave turbulence.

QT is not a simplification of classical turbulence.
We believe it is a special type that can (if you prepare it so) look like classical turbulence
at a limited range of scales
but should be very different at quantum scales (where it is more like wave turbulence).

Hence, there are plenty of new physics, especially at short- and meso-scales.

Quantum Turbulence (QT) at $T = 0$: purely superfluid component, no normal component.
QT = *dynamical tangle of quantized vortices*: a huge range of lengthscales (dissipation at ~ 3 nm).

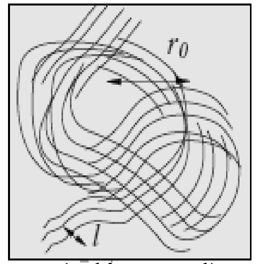
Take a tangle of average length L (typical intervortex distance $l = L^{-1/2}$ ~ 0.1 – 1 mm):



(random)

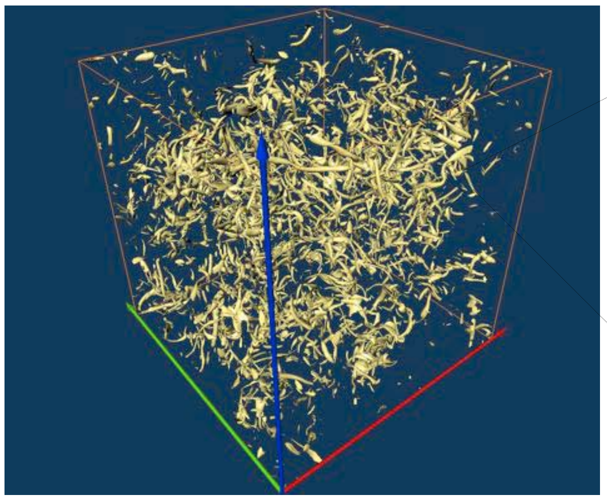
if vortices are random, then no flow on scales > l ,

if locally aligned, then there is flow on large scale.

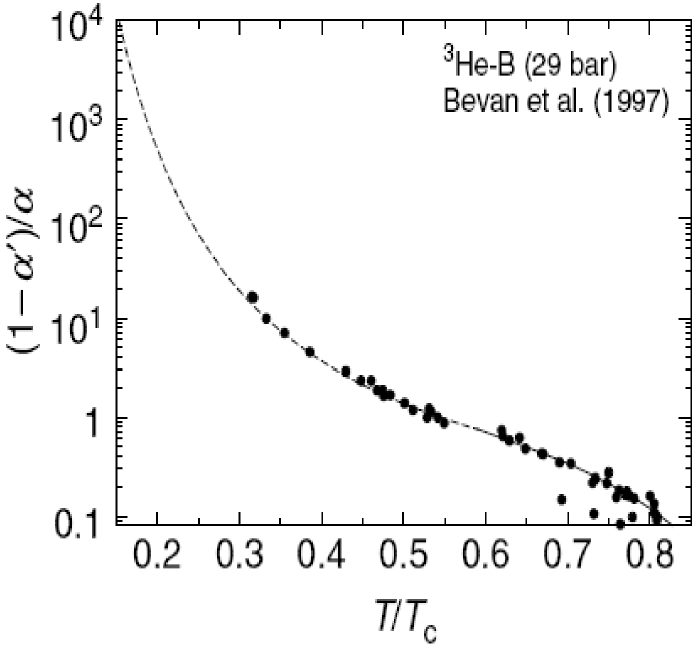


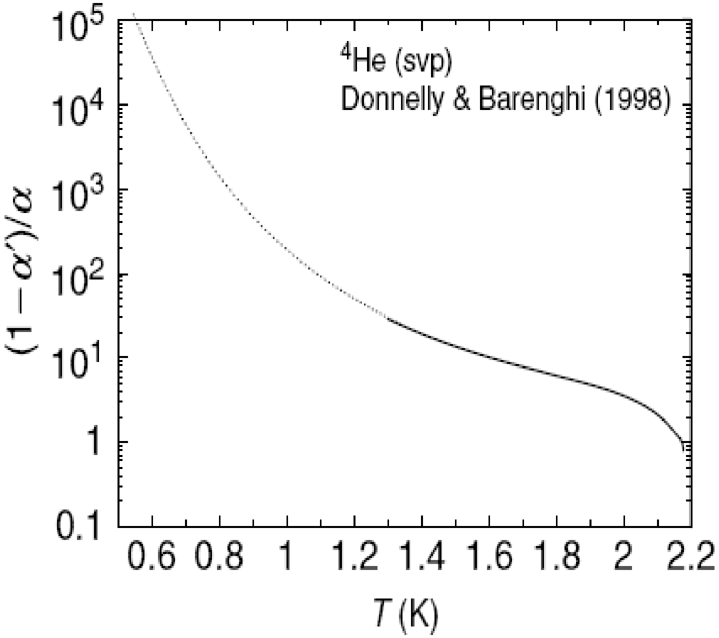
(aligned)

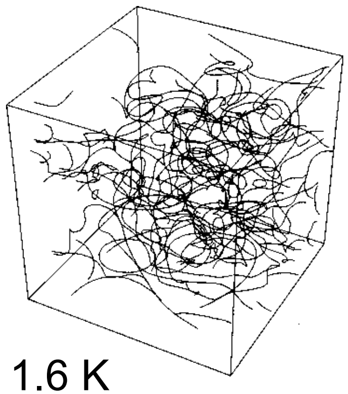
On scales >> l, coarse-grained description is that of classical fluids:



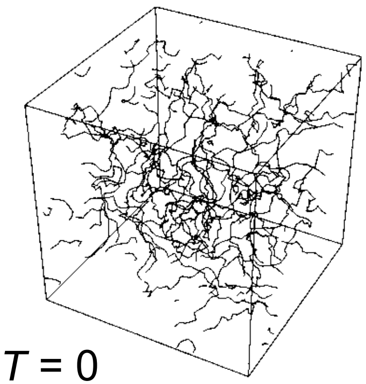
d	Quasiclassical	$l = L^{-1/2}$	Quantum	dissipation	k
4.5 cm	Kolmogorov	0.03 - 3 mm	Kelvin waves	λ ~ 3 nm	



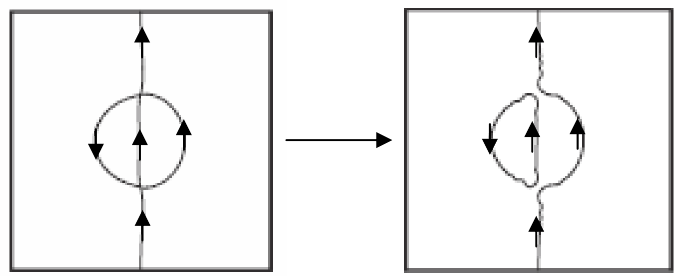




T = 1.6 K



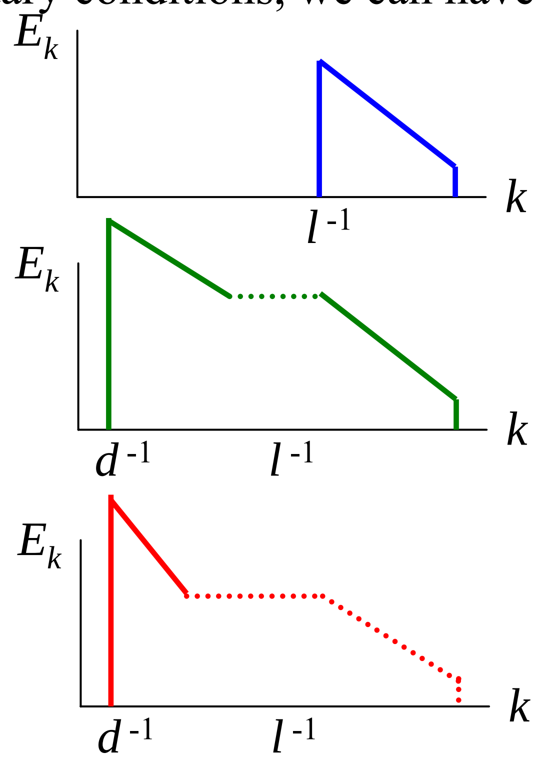
T = 0



Simulations by Tsubota, Araki, Nemirovskii (PRB 2000)

Energy spectra for different QT

Depending on pumping lengthscale, and on geometry and boundary conditions, we can have:



1. Ultra-quantum (a.k.a. "random", "Vinen") turbulence.
Free decay: $E(t) \sim L(t) \sim t^{-1}$

2. Quasi-classical ("Kolmogorov") homogeneous isotropic turbulence (HIT). Free decay: $E(t) \sim t^{-2}$, $L(t) \sim t^{-3/2}$

3. Quasi-classical rotating turbulence.
Free decay (approximately): $E(t) \sim t^{-1}$, $L(t) \sim t^{-1}$

General assumption: $\mathrm{d}E/\mathrm{d}t = -\nu(\kappa L)^2$

Some questions to be answered experimentally:

- Is it true QT can behave classically at $T = 0$ or at higher T?

- Is it true the dissipation is through phonon emission at short length scales, at least in ^{4}He?
  If yes, how will large-scale energy reach those short length scales?
  What type of inertial cascade is relevant: non-linear Kelvin waves, self-reconnections, etc.?

- Is the dissipation rate different for different spectra?

- Can one study quantum cascade on individual vortex lines (either straight lines or rings), perturbed either locally or by interaction with other lines?

At $T < 1$ K, it would be handy to be able to apply force to vortex lines and to detect their motion (injected ions seem to be the only available tool for ^{4}He).

Part 2. Let's quickly revise important relevant steps for QT in ^{4}He

Quantized vortices and QT predicted (Feynman 1955)
QT discovered in rotating helium and counterflow experiments ($T > 1$ K)
(Hall and Vinen 1956, Vinen 1957)

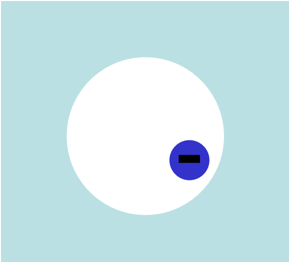
Injected ions were used successfully to investigate vortices at all temperatures $0 < T < 1.8$ K
(unlike another technique, second sound, only useful at $T > 1$ K)

Injected ions (mainly, electron bubbles) were used to detect the presence of vortices:

Careri et al., Nuovo Cimento **18**, 957 (1960) (turbulence detected by ions)
Careri et al., Phys. Lett. **1**, 61 (1962), Ions in Rotating Lilquid Helium II.

Recent review on injected ions: Borghesani, A. F. *Ions and Electrons in Liquid Helium*. Oxford University Press 2007.

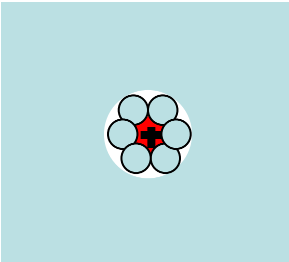
Injected ions: structure



Negative ion: bare electron in a bubble (Atkins 1959) :

p	0 bar	25 bar
R_-	17 Å	12 Å
m_-	243 m_{He}	87 m_{He}

(Ellis, McClintock 1982)



Positive ion: cluster ion ("snowball") (Ferrell 1957) :

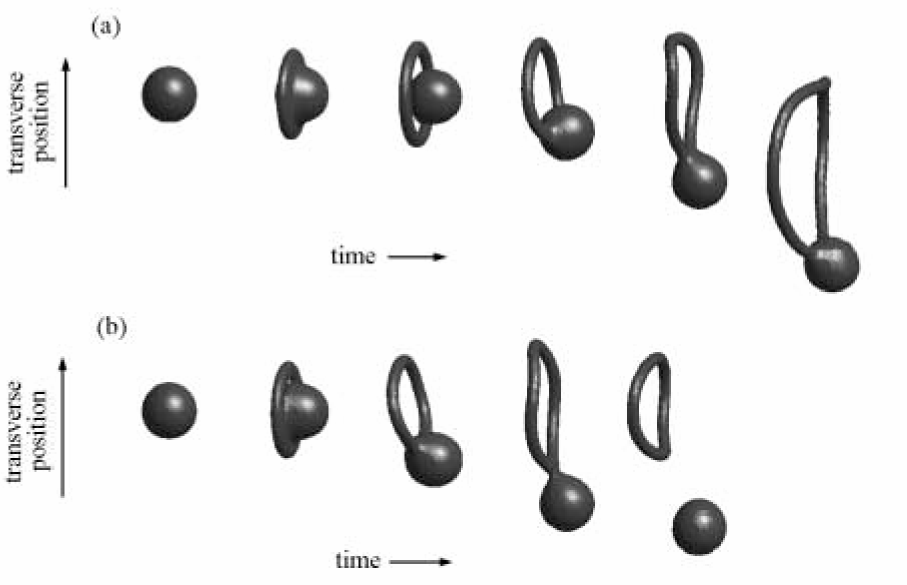
p	0 bar	25 bar
R_+	7 Å	9 Å
m_+	~30 m_{He}	~50 m_{He}

Ions - spherical probe particles that can be pulled by external force.

Proved extremely useful for studies of excitations in bulk He and vortices.

By changing pressure and species, one can cover R = 7–17 Å, m/m_{He}= 30-240.

Ion-ring complexes:



Proper E-v relations:
Roberts and Donnelly, Phys. Lett. **31A**, 137 (1970)

$$E = \tfrac{1}{2}\rho\kappa^2 R[\ln(8R/a) - \tfrac{3}{2}] ,$$

$$v = (\partial E/\partial p)_a = (\kappa/4\pi R)[\ln(8R/a) - \tfrac{1}{2}] ,$$

$$p = \rho\kappa\pi R^2 .$$

Experiment: Rayfield and Reif (1964), Rayfield (1968), Careri et al.(1065), Bruschi et al.(1966)
McClintock, Bowley, Nancolas, Stamp, Moss (1980, 1982, 1985)
Theory for V_c: C.M.Muirhead, W.F.Vinen, R.J.Donnelly, Phil. Trans. R. Soc. A**311**, 433 (1984).

Recent simulations:
T.Winiecki and C.S.Adams, Europhys. Lett. **52**, 257 (2000)
Berloff abd Roberts (2000)
Berloff and Youd, PRL 99, 145301 (2007)

Injected ions were used to create vortex rings, tag them and explore their dynamics at small fields and all $T < 1$ K. (Rayfield and Reif, 1964)

PHYSICAL REVIEW VOLUME 136, NUMBER 5A 30 NOVEMBER 1964

Quantized Vortex Rings in Superfluid Helium*

G. W. Rayfield† and F. Reif
Department of Physics, University of California, Berkeley, California
(Received 10 July 1964)

Evidence is presented to show that charged particles in superfluid helium at low temperatures can be accelerated to create freely moving charge-carrying vortex rings in the liquid. The circulation of these vortex rings can be determined by measuring their energy and velocity; it is found to be equal to one quantum h/m, where h is Planck's constant and m is the mass of a helium atom. The core radius of the vortex is approximately 1 Å. The dynamical properties of such a vortex ring moving under the influence of external forces can be described by a dispersion relation $E \propto p^{1/2}$ connecting its energy E and momentum p; it can also be understood in detail in terms of the hydrodynamic Magnus force. Experiments are described which verify the essential validity of this dynamical analysis. Vortex rings can interact with various quasiparticles in the liquid, i.e., with rotons, phonons, and He^3 impurities. The scattering of these quasiparticles by vortex rings can be investigated by experiments designed to study the temperature dependence of the rate of energy loss of such rings moving through the liquid. In this way it is possible to measure the effective momentum-transfer cross sections for scattering of the various quasiparticles by vortex lines. The cross section thus deduced is 9.5 Å for scattering of rotons and 18.3 Å for scattering of He^3 atoms. The experiments yield only scant information about scattering of phonons, but are not inconsistent with the magnitude of the phonon scattering cross section expected on theoretical grounds.

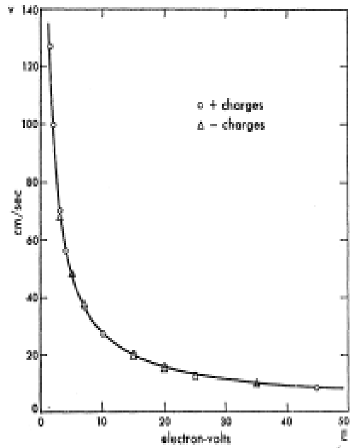


FIG. 2. Relation between the velocity v and energy E of a vortex ring. The points are experimental data for positive and negative charge carriers. The curve is the theoretical relation following from (5) and (6) with $\kappa = h/m$ and $a = 1.2$ Å.

$$E = \tfrac{1}{2}\rho\kappa^2 R[\eta - (7/4)],$$

$$v = (\kappa/4\pi R)(\eta - \tfrac{1}{4}),$$

$$\eta \equiv \ln(8R/a).$$

$$E = \tfrac{1}{2}\rho\kappa^2 R[\ln(8R/a) - \tfrac{3}{2}],$$

$$v = (\partial E/\partial p)_a = (\kappa/4\pi R)[\ln(8R/a) - \tfrac{1}{2}],$$

$$p = \rho\kappa\pi R^2.$$

Proper relations:
Roberts and Donnelly, Phys. Lett. **31A**, 137 (1970).

Drag force on charged vortex rings at T < 1.4 K. (Rayfield, Reif, Careri, Mazzoldi et al.)

$$\mathfrak{F} = -dE/dz = \alpha(T)\chi(E) \quad (\chi \equiv \eta - \tfrac{1}{4}).$$

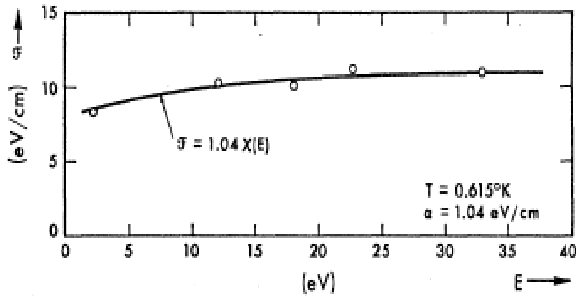
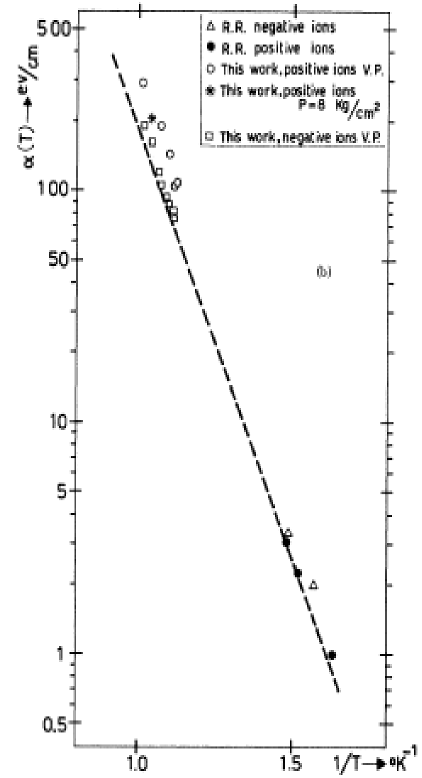


FIG. 7. The frictional force $\mathfrak{F}$ on a vortex ring as a function of its energy E at a given temperature T=0.615°K.



Careri, Cunsolo, Mazzoldi, Santini, PRL **15**, 392 (1965)
Bruschi, Maravigla, Mazzoldi, PR **143**, 143 (1966)
Rayfield, PR **168**, 222 (1968).

Ring's shape under a point force. Simulations for $E = 10^5$ V/cm
Samuels and Donnelly, PRL **67**, 2505 (1991).

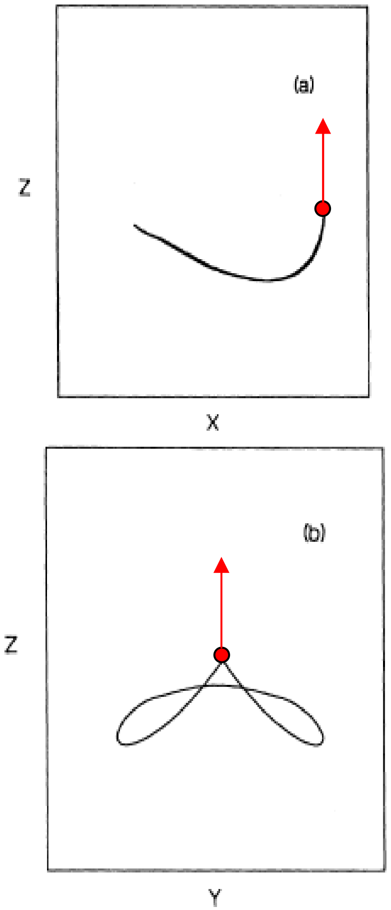


FIG. 1. The final shape of a vortex ring with a fixed ion moving in the z direction in an electric field. The ion is located at the cusp seen in (b). The z axis is exaggerated by a factor of 10.

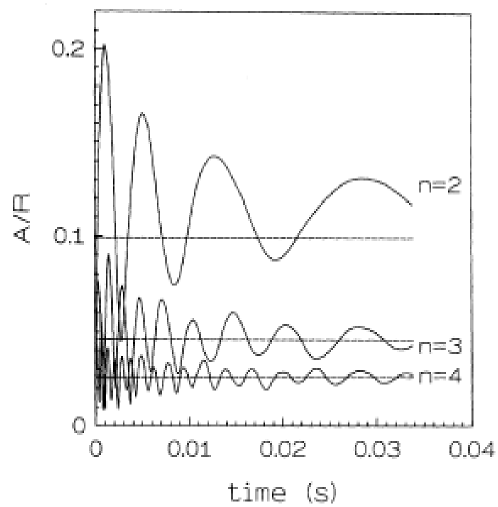


FIG. 2. Amplitudes of the lowest three harmonics vs time. The dashed lines are the amplitudes predicted from Eq. (12).

Charged vortex rings were used to detect presence of vortices

Schwarz, Donnelly, Phys. Rev. Lett. **17**, 1088 (1966) Quantized Vortex Rings in Rotating Helium II
Schwarz, Phys. Rev. **165**, 323 (1968) (T = 0.3 K)

The characteristics of the beam are obtained by scanning it across the narrow opening in front of the collector. Although general features are repeatable, the details vary from run to run. We ascribe this to varying accumulations of surface charge on the electrodes. The profile of the beam measured at the collector turned out to be surprisingly wide (>1 cm), although its outer edges were sharply defined. It seems that the shape of the beam is determined mainly by the collimating properties of the source and first grid, although some space-charge spreading may also be present. The apparent energy of the rings in the beam as determined by measuring the current as a function of back voltage is distributed about a value E_0 which is typically about 60% of eV_1. The cause of this apparent deficit in energy is not known, although similar effects have been noted by other investigators.[3] It may be connected with the angle at which rings enter the analyzing region.

[3]G. W. Rayfield and F. Reif, Phys. Rev. 136, A1194 (1964).

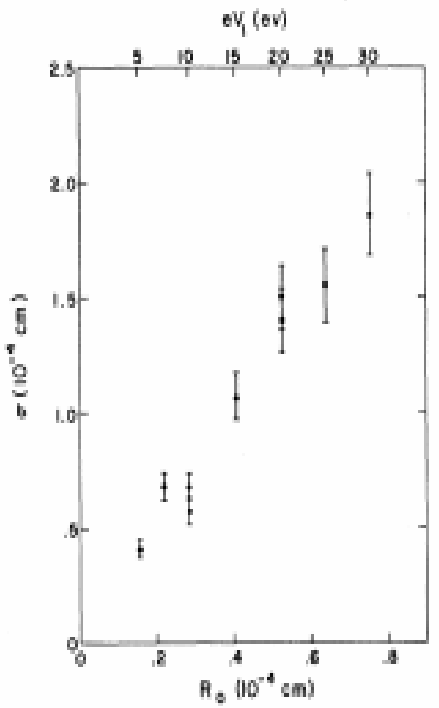


FIG. 2. Capture cross section for vortex rings incident on vortex lines as a function of the radius and energy of the rings.

PHYSICAL REVIEW VOLUME 165, NUMBER 1 5 JANUARY 1968

Interaction of Quantized Vortex Rings with Quantized Vortex Lines in Rotating He II†

K. W. Schwarz
Department of Physics and James Franck Institute, University of Chicago, Chicago, Illinois
(Received 29 May 1967)

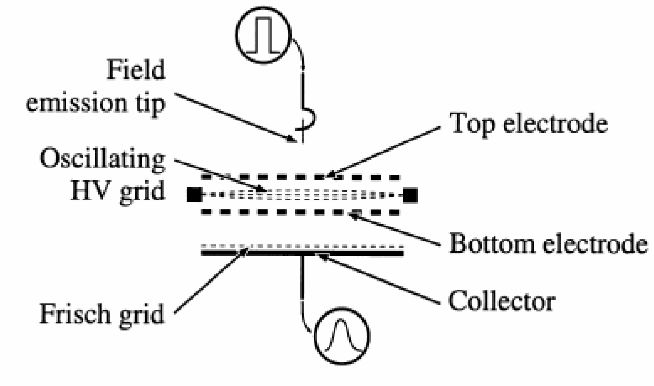
The effect of steady rotation on a beam of charged vortex rings in He II has been investigated. The rotation-dependent changes in the beam are interpreted in terms of strong ring-line interactions experienced by a fraction of the rings, plus collective effects which are the same for all rings. An approximate calculation based on inviscid hydrodynamics and a uniform distribution of vortex lines yields fair agreement with the observed behavior.

Vibrating structures were used to pump liquid helium through an orifice, charged vortex rings used to detect the produced vortices.
Emission of individual rings is detected as well as of their tangles.
Tangle's decay within seconds observed.

G. Gamota, *Phys. Rev. Lett.* **31**, 517 (1973).
Experiment with an array of vibrating orifices (Nuclepore membrane, orifices diameters 5 mkm, resonance frequency 1.17 kHz) at 0.3 - 0.5 K to generate a beam of monoenergetic vortex rings of diameter 1.7 - 3.2 mkm (decreasing as the flow drive was increased) and detect them by a transverse beam of charged vortex rings

Eventually, vibrating sphere was used by Schoepe et al. (90s),
and vibrating grid to generate and ions to detect QT were used (McClintock et al. 2000).

Observation of vortex tangles leaking from the ion emission region and decaying within a minute (McClintock et al. ~ 1974 - 1985)



Wind tunnels (flow channels) were used to pump liquid helium through an orifice, charged vortex rings used to detect the produced vortices.
Emission of individual rings is detected as well as of their tangles.
Tangle's decay within seconds observed.

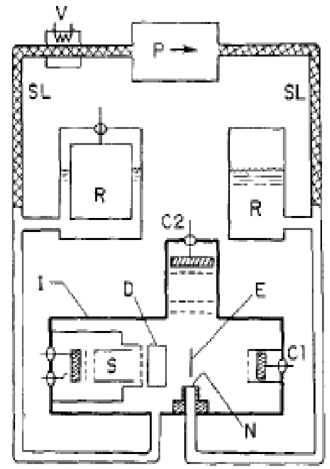
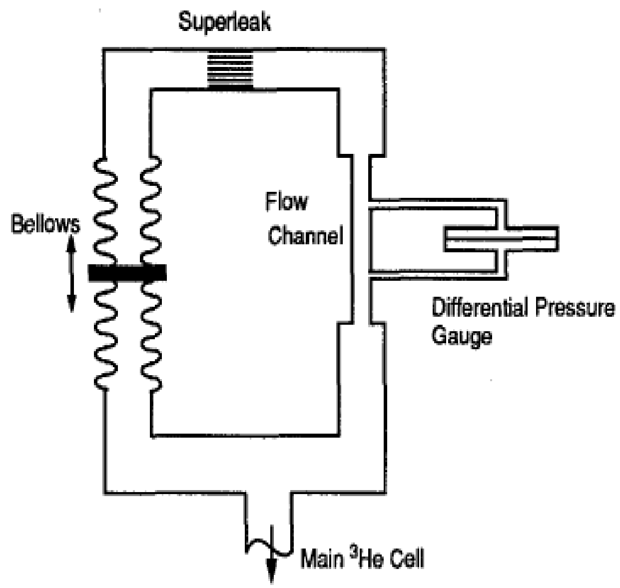


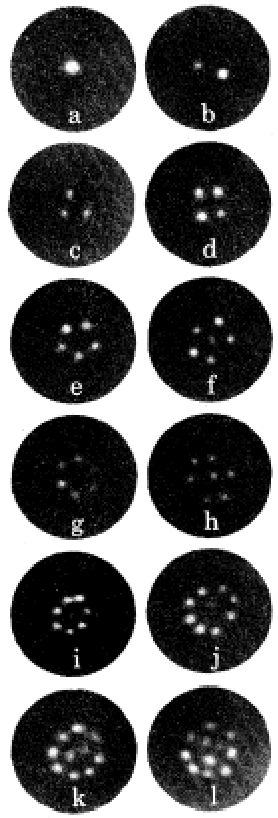
Fig. 1. Schematic diagram of the apparatus. The lettered components are identified in the text. Components of the interaction cell I are drawn approximately to scale.

Journal of Low Temperature Physics, 33, 243 (1978)
Observations of Quantized Vorticit Generated Superfluid ^{4}He Flow Through 2mkm-Diameter Orifices in Helium II
B. M. Guenin and G. B. Hess

Wind tunnels can be built using fountain-effect pumps (see further publication B. M. GUENIN and G. B. HESS Physica 1Ol B (1980) 285 and (two) bellows – like the one used by Saundry and Bozler with superfluid ^{3}He! "Breakdown of 3He-A Superflow in Low Magnetic Fields," P.D. Saundry, M.R. Thoman, L.J. Friedman, C.M. Gould, and H.M. Bozler, *J. Low Temp. Phys.* **86**, 401 (1992).



Ions used to detect/image presence of vortex arrays at rotation, and of their stability (Packard et al. 70-s)



Ion interaction with vortices, transport along them, emission of Kelvin waves
Role of rotons, phonons, ^{3}He impurities (Glaberson et al. mid 70-s)

Part 3. In what follows, I will talk about our experiments in Manchester in 2006-2009.

Plan:

1. Decay of turbulence created mechanically in a large container: $L(t) \sim t^{-3/2}$.

2. Use of ions to create charged individual rings or tangles.

3. Turbulence in rotating frame: decay *L(t),* etc.

Our contribution

Experimental challenges were:

1. How to generate **quasiclassical** turbulence at $T < 1$ K?
Impulsive spin-down of a cubic container is a way to stir the liquid.

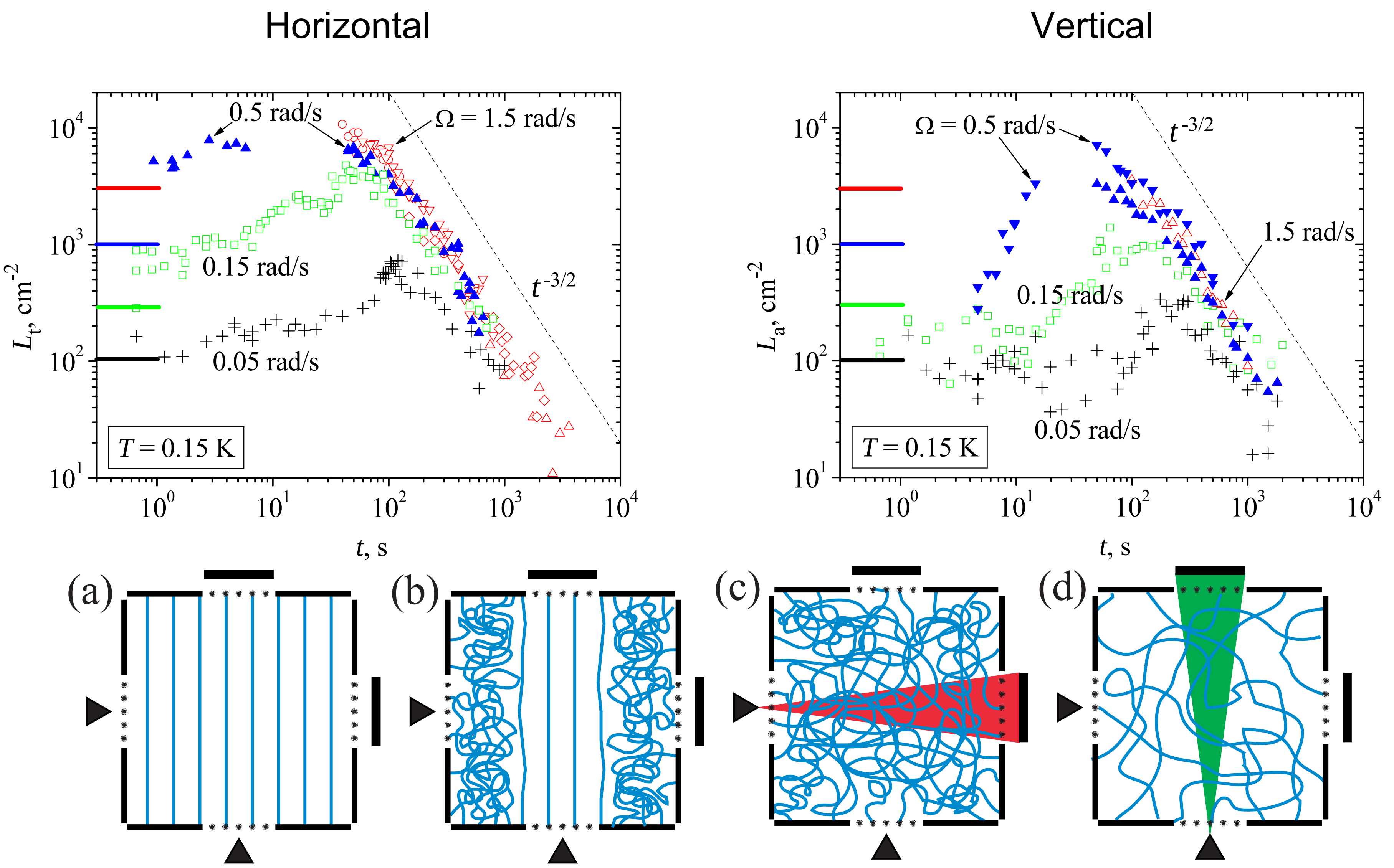
This technique has been widely used for classical turbulence (e.g. van Heijst, et al. 1989).
At $T > 1$ K, turbulence upon spin-down was observed by Hall&Vinen 1956, Lane&Reppy 1965.

2. How to ensure that turbulence is nearly homogeneous and isotropic?
Use large container ($d >> l$); sample turbulence in its centre far from walls.

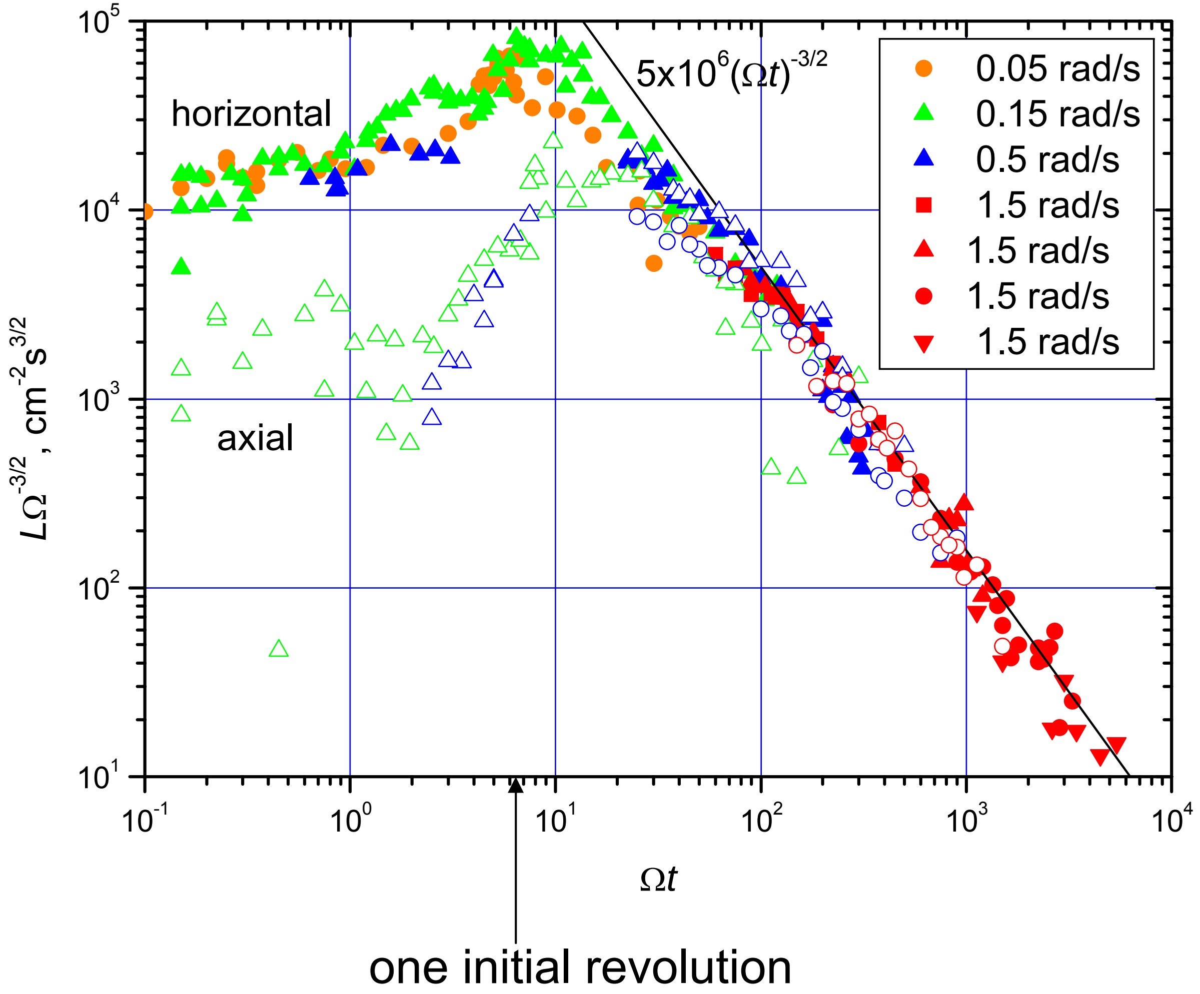
3. How to detect turbulence at $T < 1$ K?
Shoot a cloud of ions through and measure their loss proportional to L.
Charged vortex rings ($R \sim 1$ μm) are very sensitive probes at $T < 0.8$ K !

d	Quasiclassical	$l = L^{-1/2}$	Quantum	dissipation	k
4.5 cm	Kolmogorov	0.03 - 3 mm	Kelvin waves	$\lambda \sim 3$ nm	

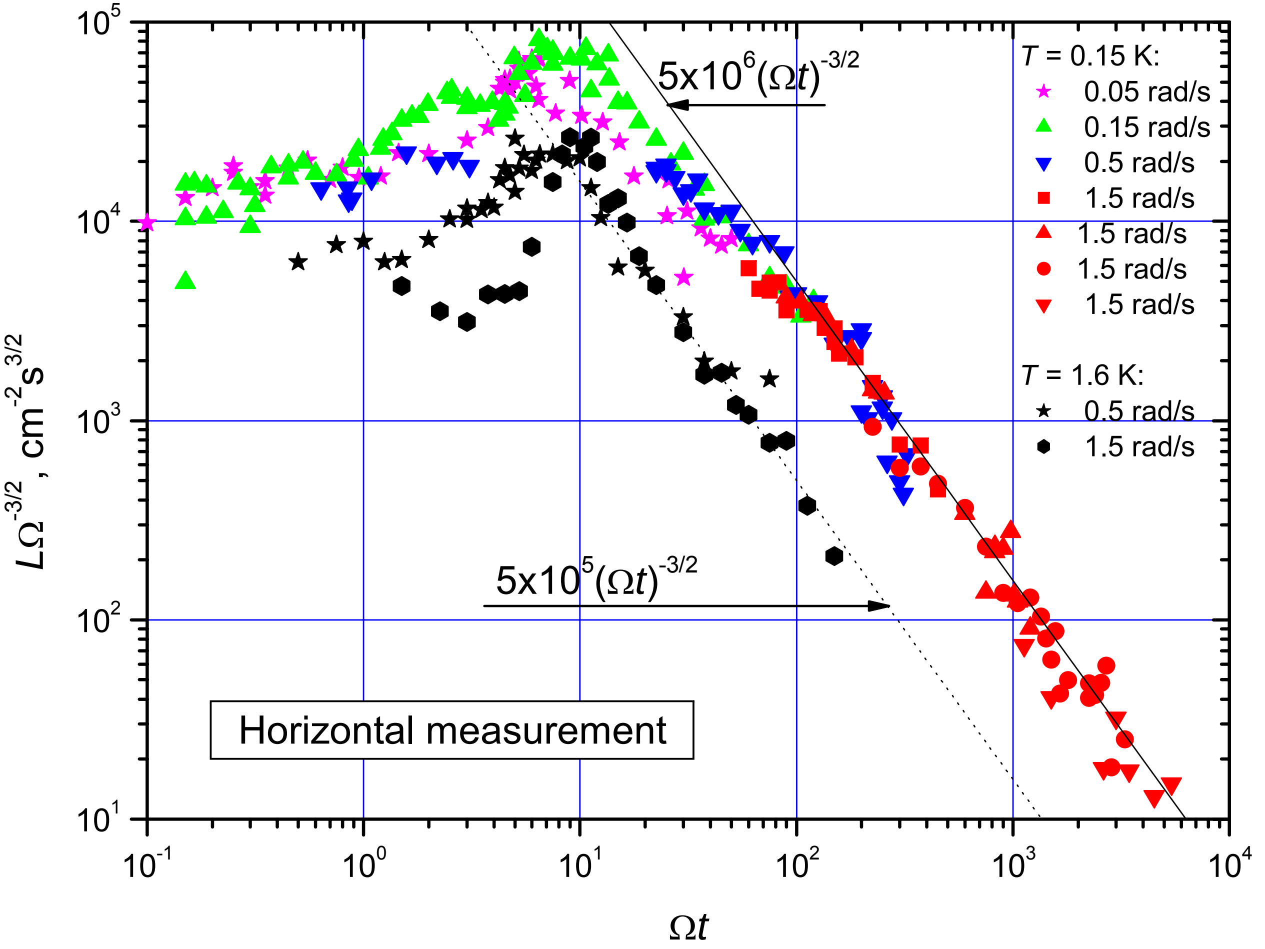
Horizontal vs. vertical direction



Scaling with Angular Velocity



Low vs. High Temperature: horizontal



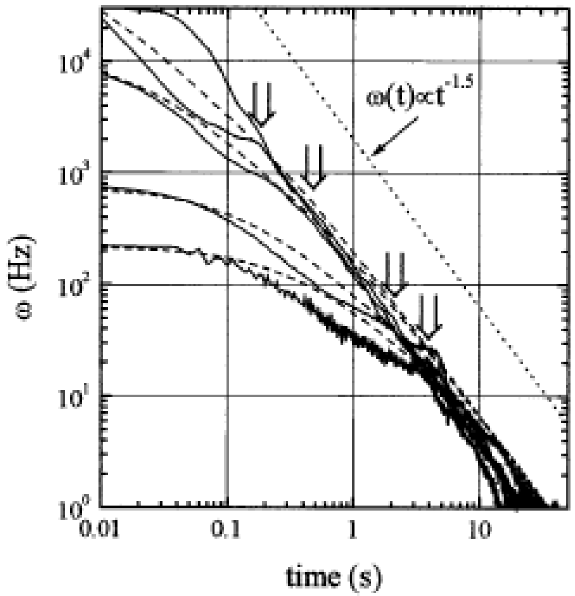
Walmsley, Golov, Hall, Levchenko, and Vinen, PRL **99**, 265302 (2007).

VOLUME 82, NUMBER 24 PHYSICAL REVIEW LETTERS 14 JUNE 1999

Decay of Grid Turbulence in a Finite Channel

Steven R. Stalp, L. Skrbek, and Russell J. Donnelly
Cryogenic Helium Turbulence Laboratory, Department of Physics, University of Oregon, Eugene, Oregon 97403
(Received 3 September 1998)

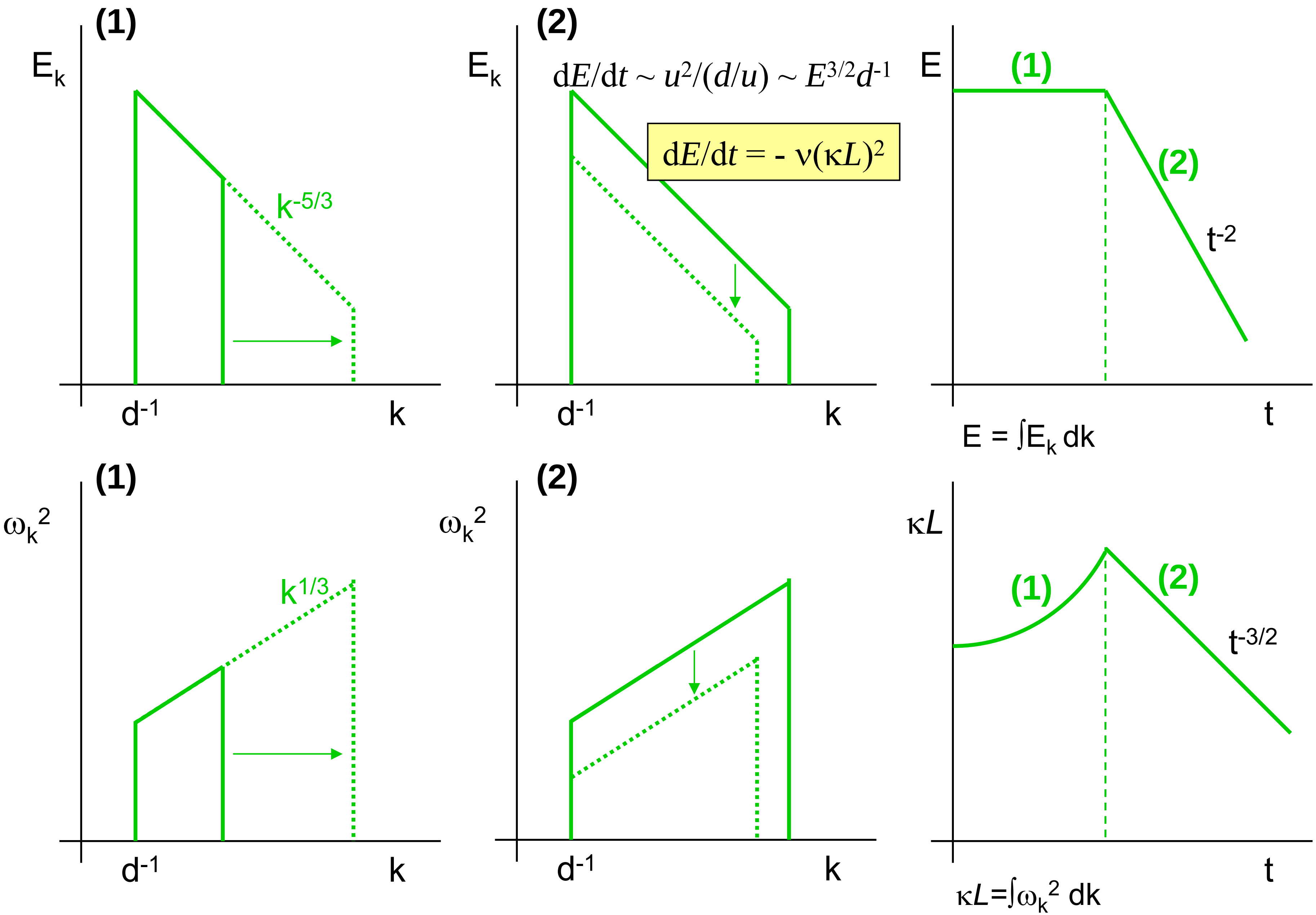
We develop a classical model for the decay of homogeneous and isotropic turbulence that takes into account the growth of the energy containing length scale and its saturation when it reaches the size of the containing vessel. The model describes our data on the decay of grid turbulence in helium II through 4 orders of magnitude in vorticity, as well as classical experiments on decaying turbulence.



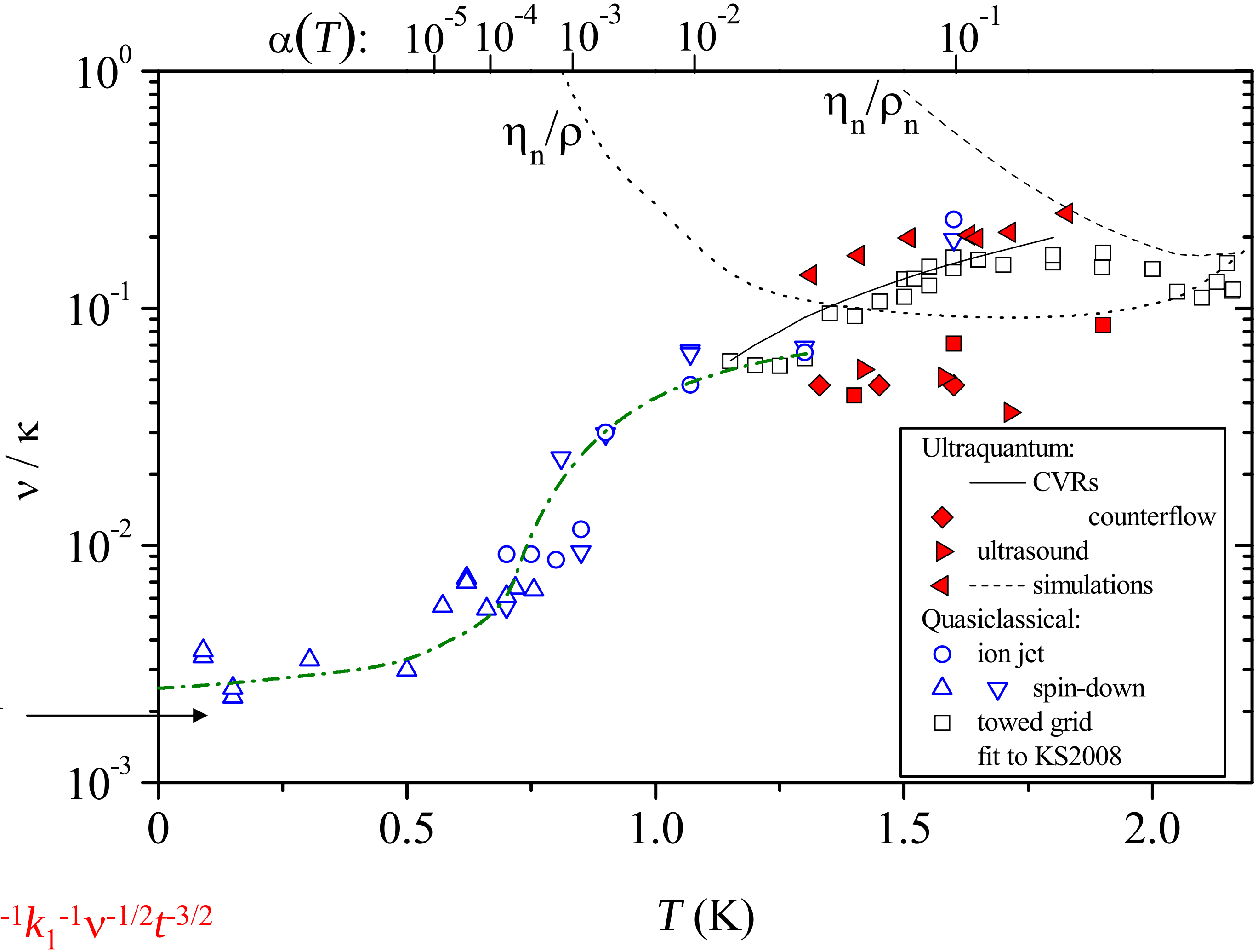
Followed by:
Skrbek, Niemela, Donnelly, PRL 2000;
Skrbek and Stalp, Phys. Fluids 2000;
Stalp, Niemela, Vinen, Donnelly, Phys Fluids 2002;
Niemela, Sreenivasan, and Donnelly, JLTP 2005.

Model for the free decay of quasi-classical homogeneous turbulence with saturated energy-containing length (scale of largest eddies limited by the container size d):
$L(t) \sim t^{-3/2}$

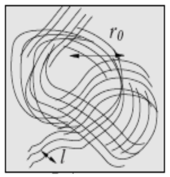
Spin-down to rest (purely classical model)



Effective viscosities for two types of HIT



Quasiclassical



$L(t) = (3C)^{3/2}\kappa^{-1}k_1^{-1}\nu^{-1/2}t^{-3/2}$

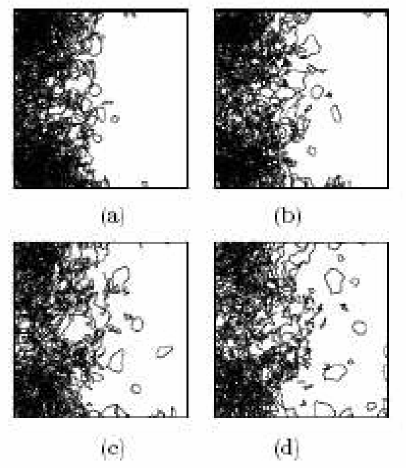
where $C \approx 1.5$ and $k_1 \approx 2\pi/d$.

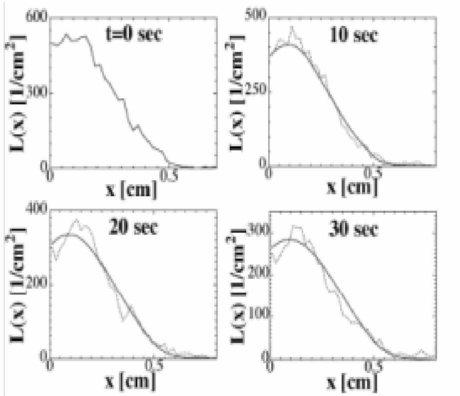
Some questions regarding interpretation still remain ($k_1 = 2\pi/d$ at all T ? Role of rotation? Non-Kolmogorov spectrum?). Further experiments (and numericals) should clarify the situation.

Diffusion model (no bulk dissipation)

Diffusion of Inhomogeneous Vortex Tangle and Decay of Superfluid Turbulence

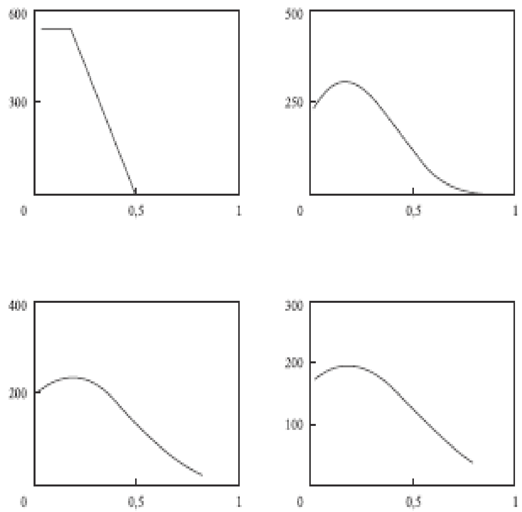
Sergey K. Nemirovskii arXiv:09.02.3720v1, 21 Feb. 2009

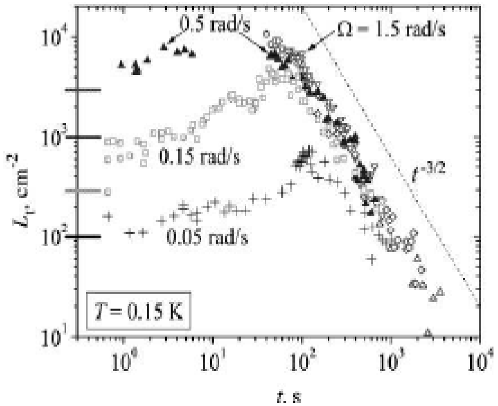


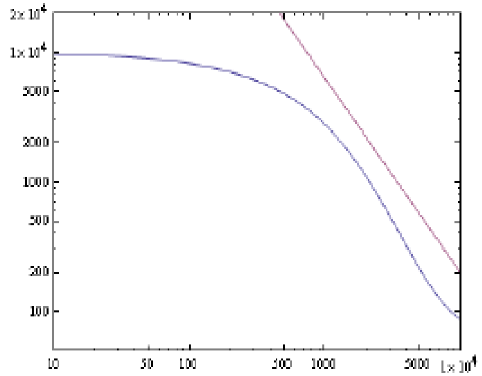


Tsubota, Araki, Vinen (2003)

$$\frac{\partial \mathcal{L}}{\partial t} = D_v \nabla^2 \mathcal{L}$$

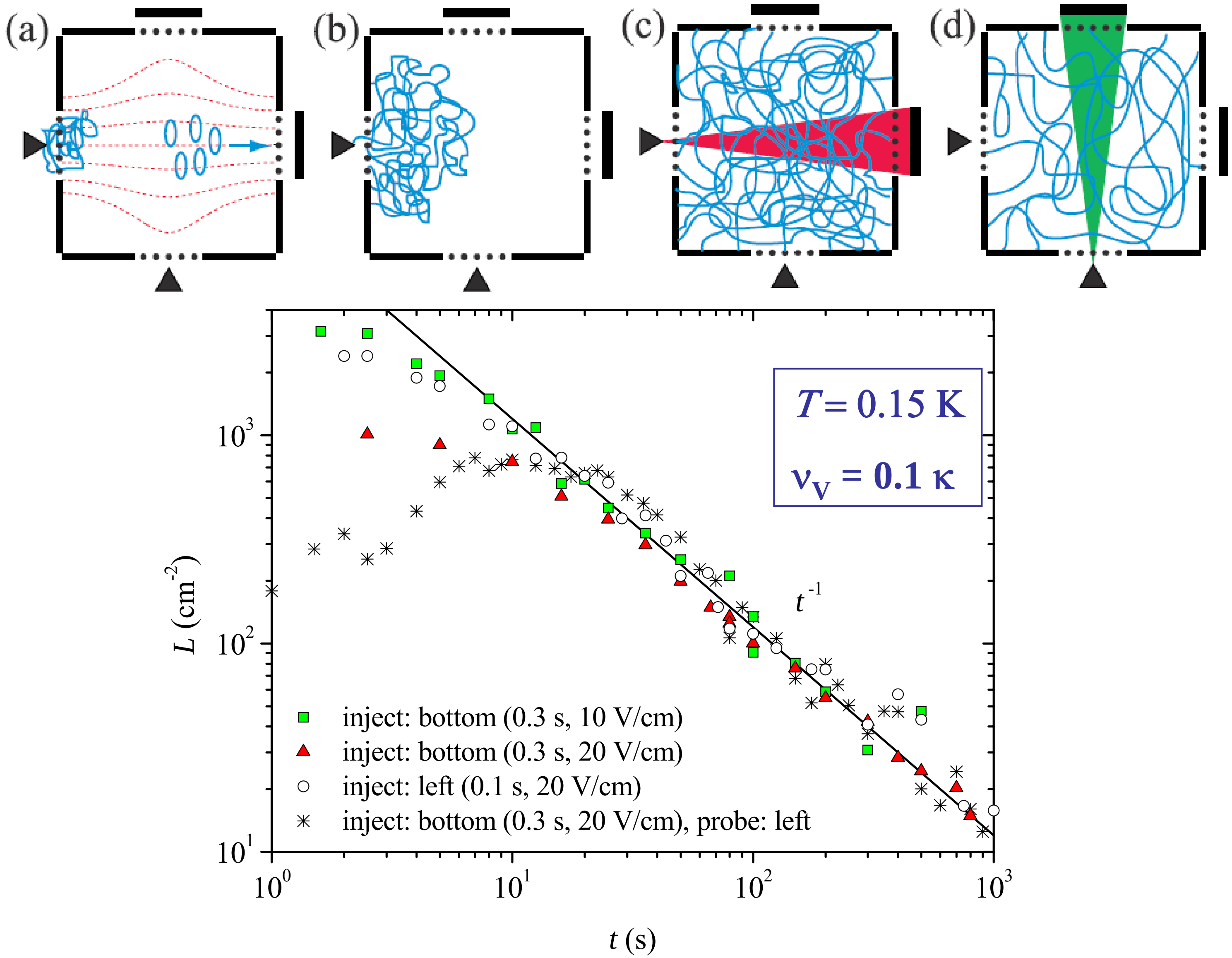






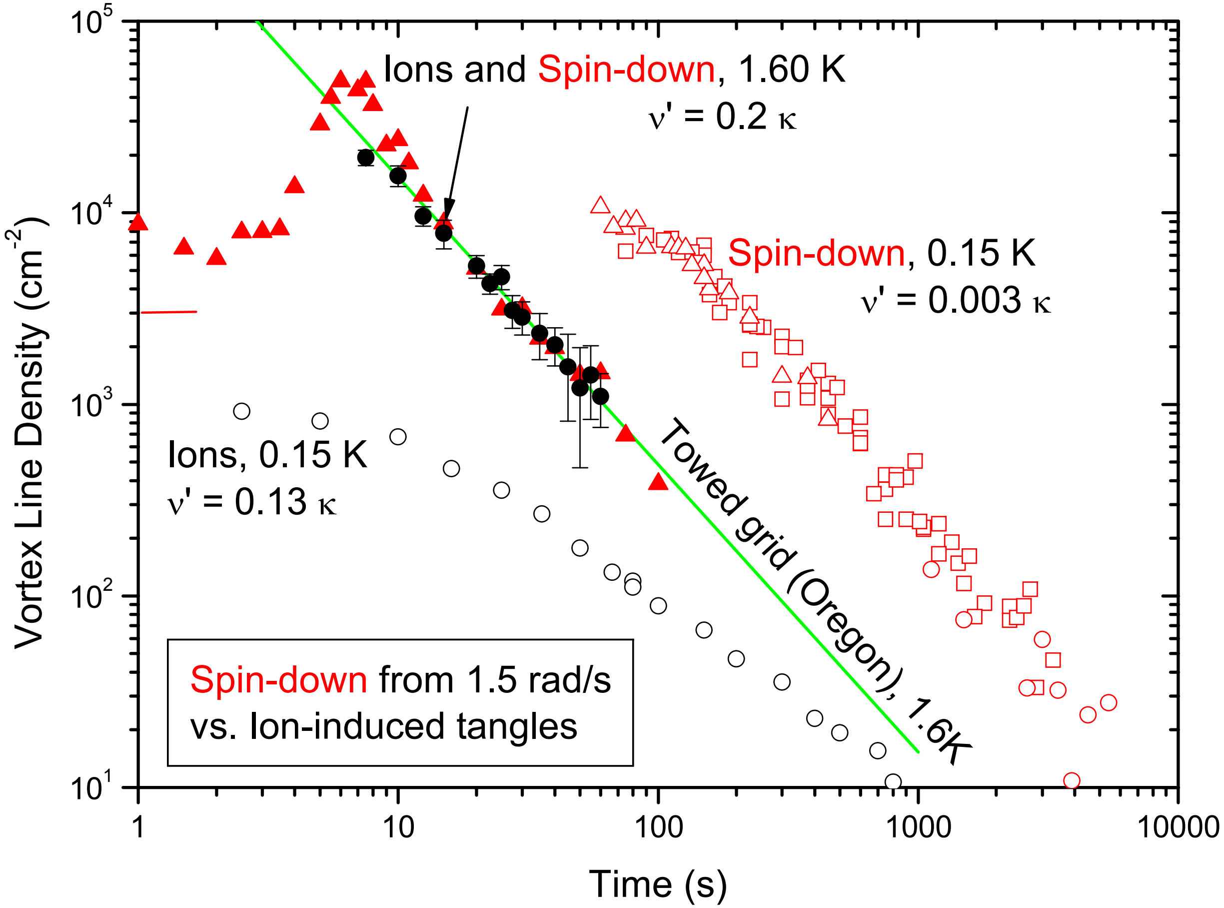
An alternative model, hardly applicable to our case
as it considers neither classical eddies nor bulk dissipation

Ultra-quantum turbulence (no large-scale flow)



Walmsley and Golov, PRL **100**, 245301 (2008)

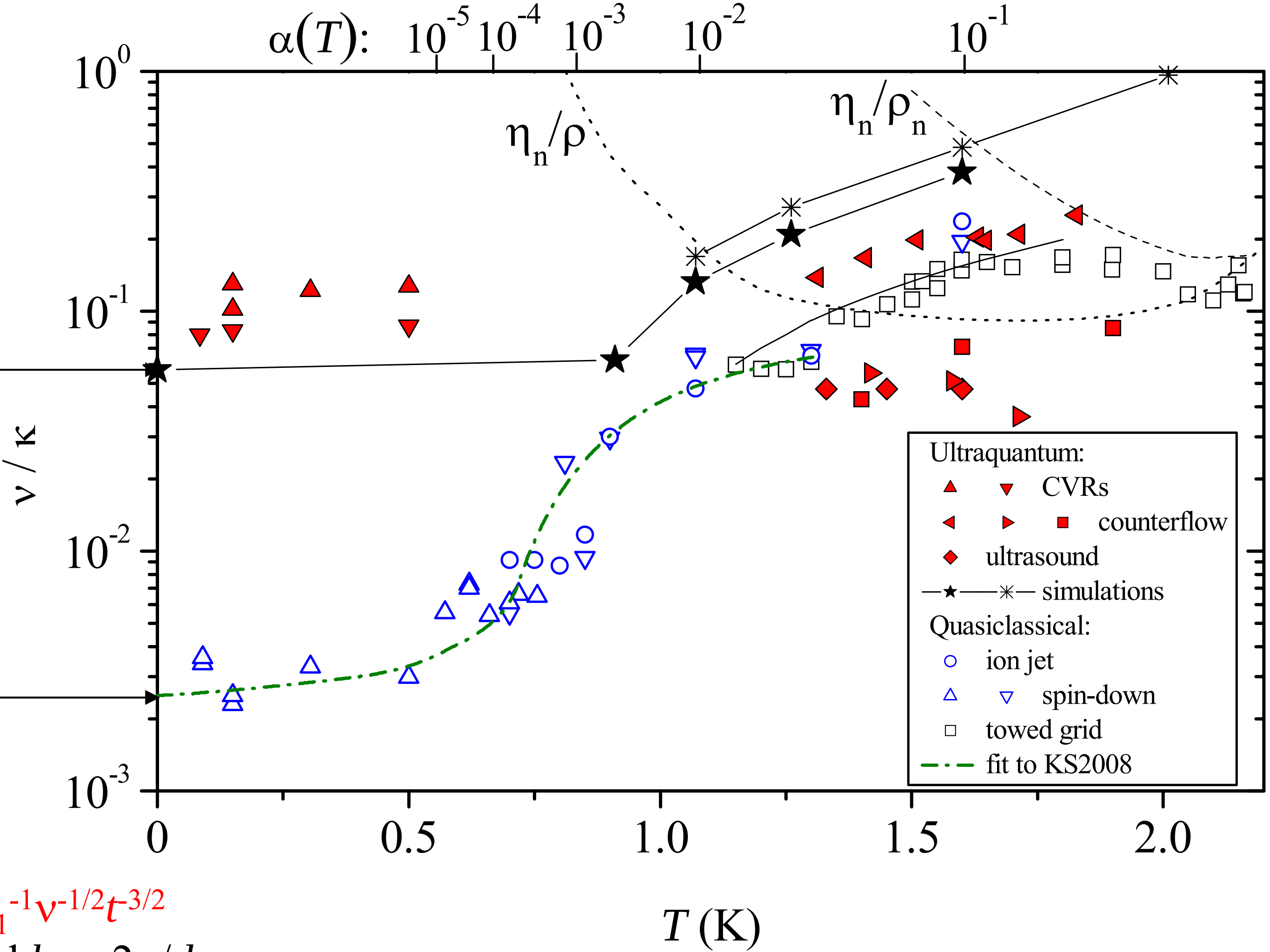
Low vs. High Temperatures



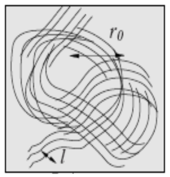
Effective viscosities for two types of HIT

Ultraquantum

$L(t) \approx 1.2\, \nu^{-1} t^{-1}$



Quasiclassical



$L(t) = (3C)^{3/2} \kappa^{-1} k_1^{-1} \nu^{-1/2} t^{-3/2}$

where $C \approx 1.5$ and $k_1 \approx 2\pi/d$.

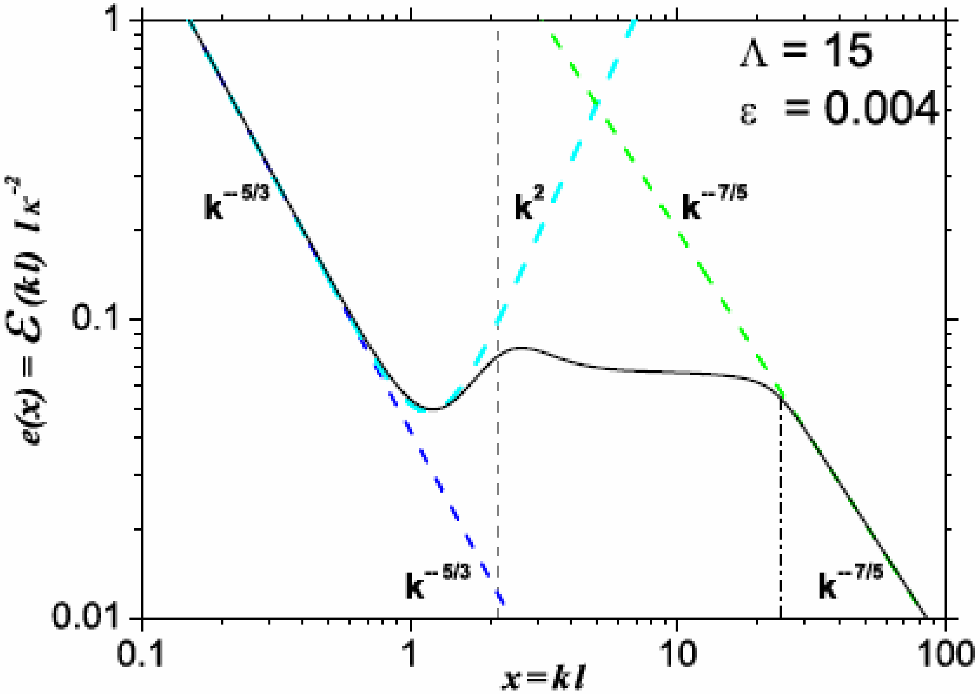
Simulations of non-structured tangles:

Tsubota, Araki, Nemirovskii (2000): $\nu \sim 0.06\,\kappa$ (frequent reconnections)

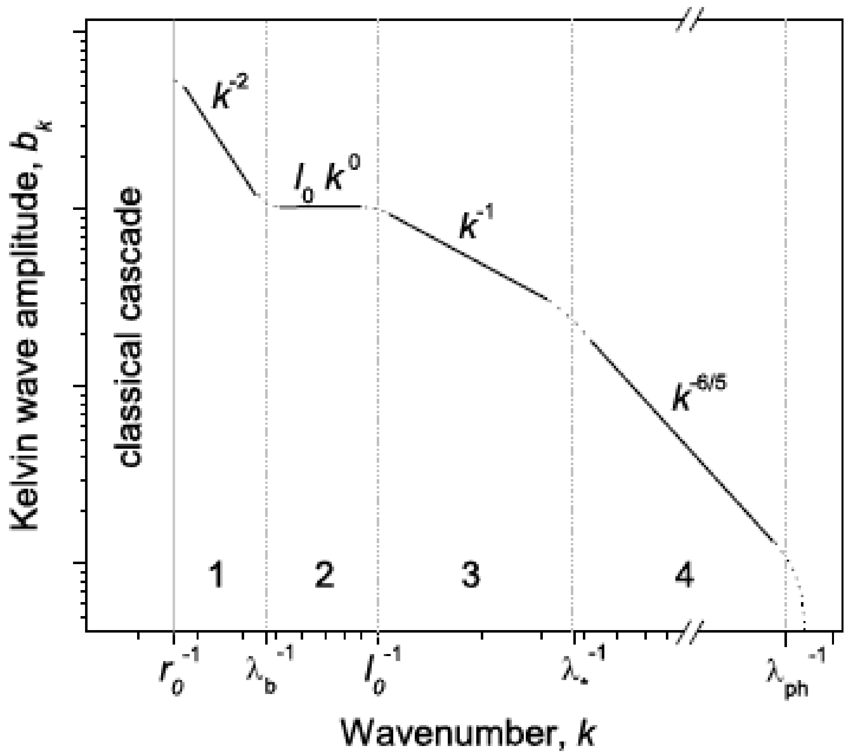
Leadbeater, Samuels, Barenghi, Adams (2003): $\nu \sim 0.001\,\kappa$ (no reconnections)

Energy spectra (Q-Cl homogeneous isotropic turbulence)

L'vov, Nazarenko, Rudenko, 2007-2008
(bottleneck, pile-up of vorticity at mesosclaes ~ l)



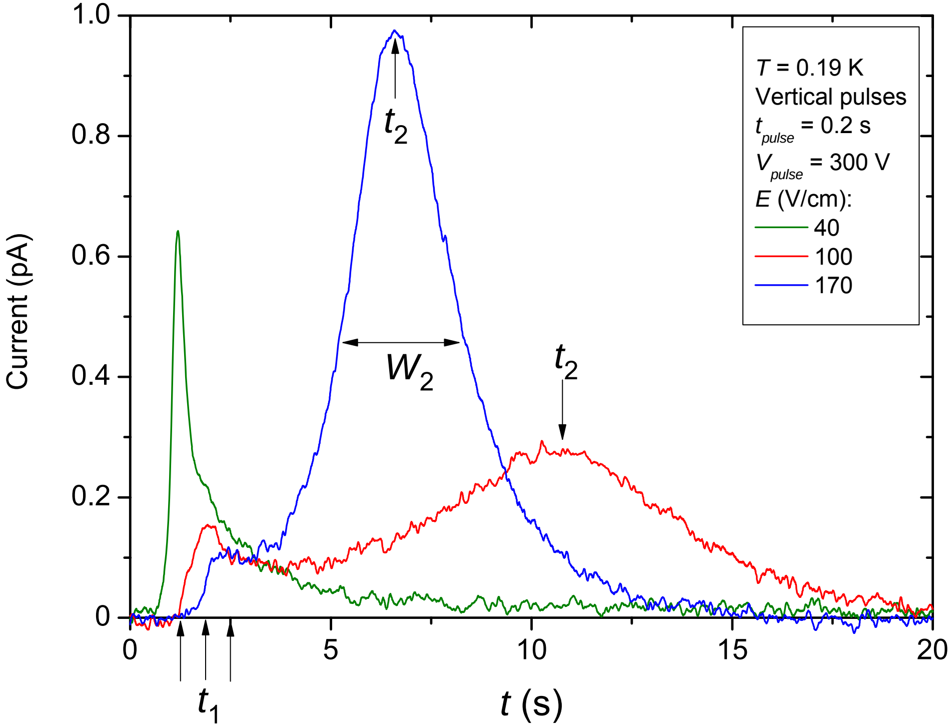
Kozik and Svistunov, 2007-2008
(reconnections, fractalization,
build-up of vorticity at mesoscales ~ l)

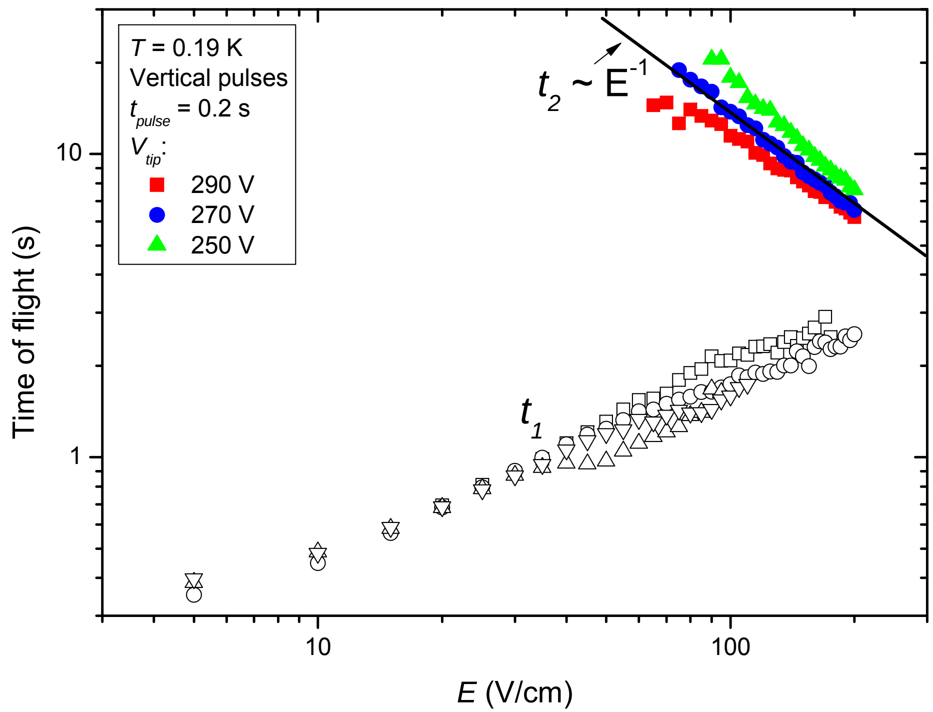


I.e. at $T = 0$, it is expected to have excess L at scales ~ l.
Unlike classical techniques (which are usually sensitive to velocity at large scales),
For QT, the convenient observable is L (vorticity $\langle\omega^2\rangle \sim (\kappa L)^2$).

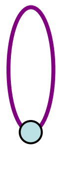
In strong field E, injected ions can be transported by two different means

(see talk by Paul Walmsley)

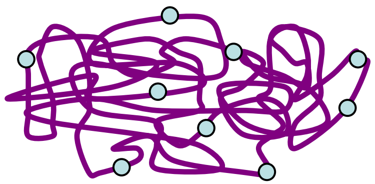




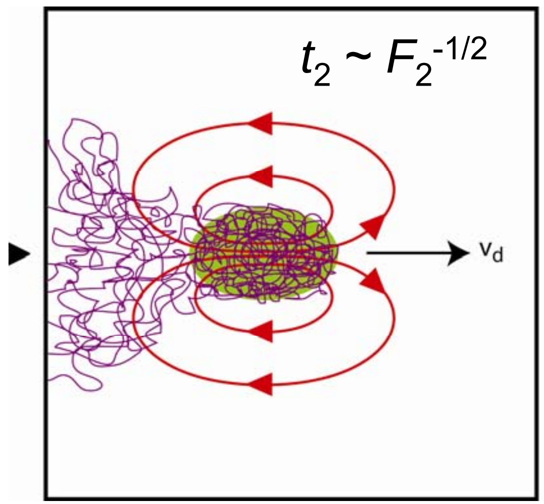
CVR's:



Charged tangle:

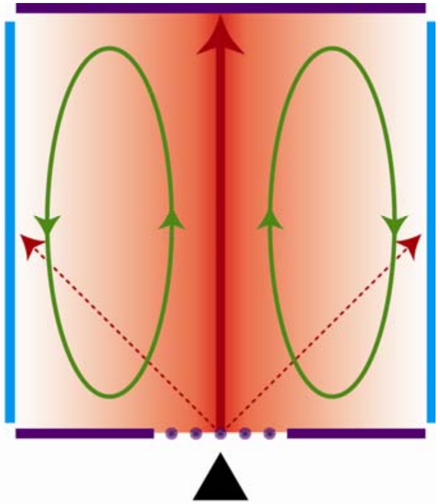


Total force:
$F_2 = Q_2 E$

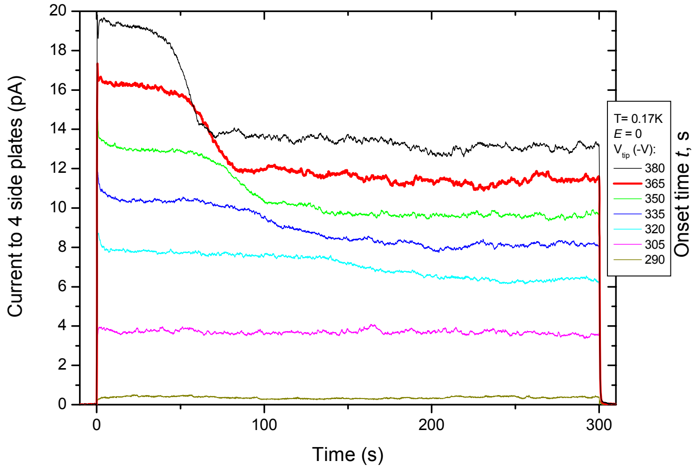


Model: electroconvection

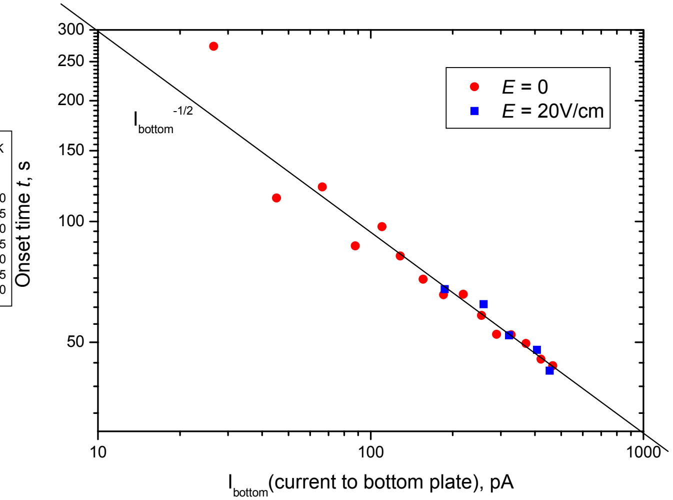
Large scale motion of fluid



Steady ion injection

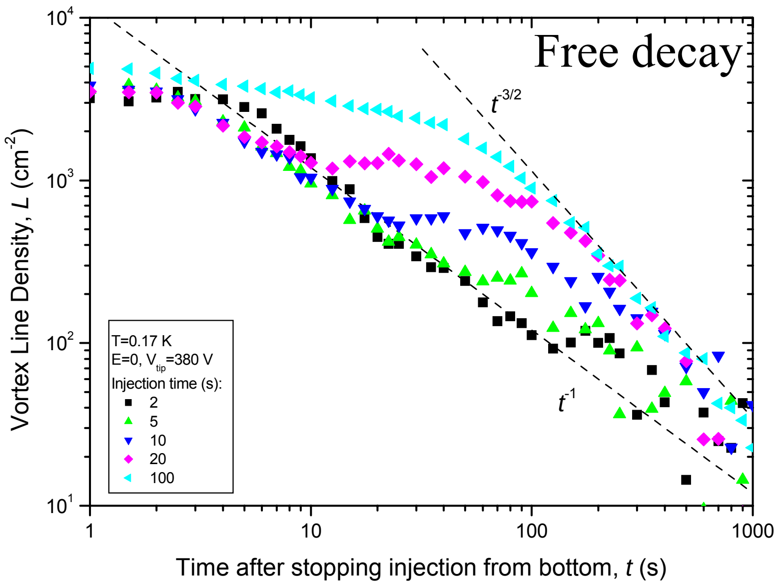


Switching time vs current



Inject from bottom for 2 – 500 s,

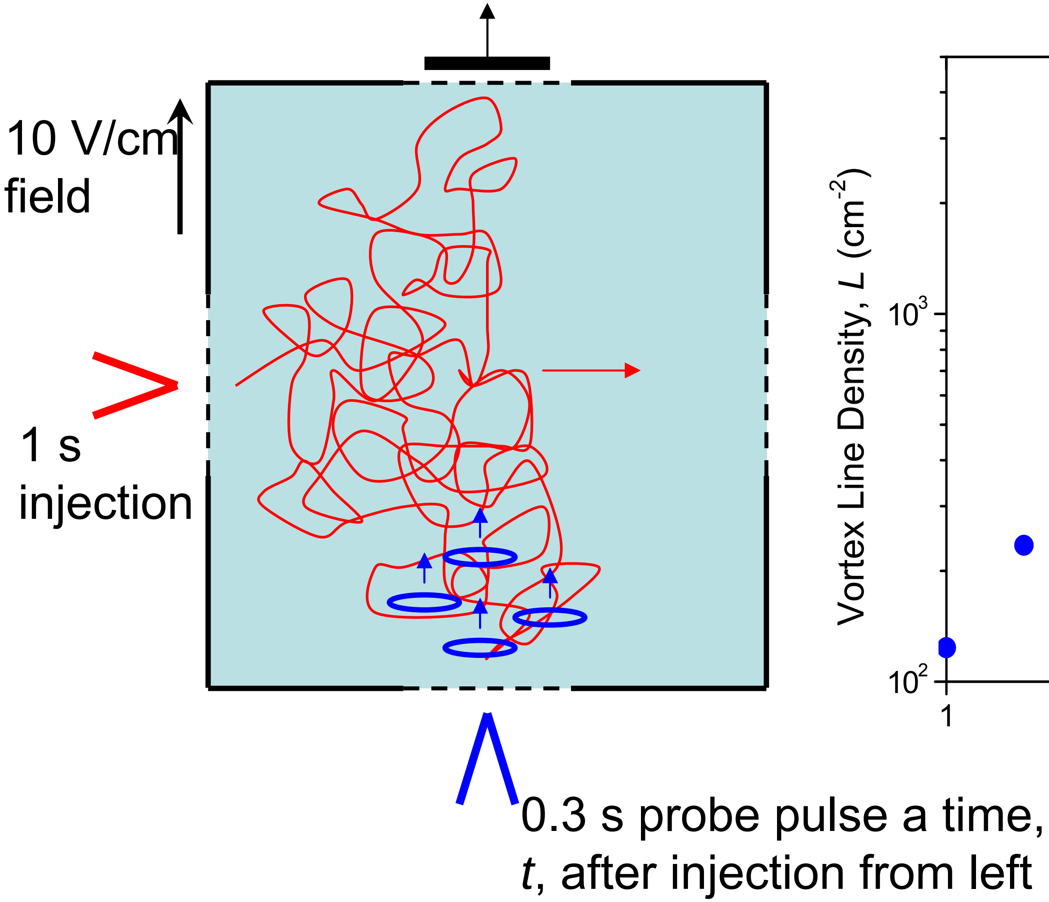
then wait time t and probe L horizontally.

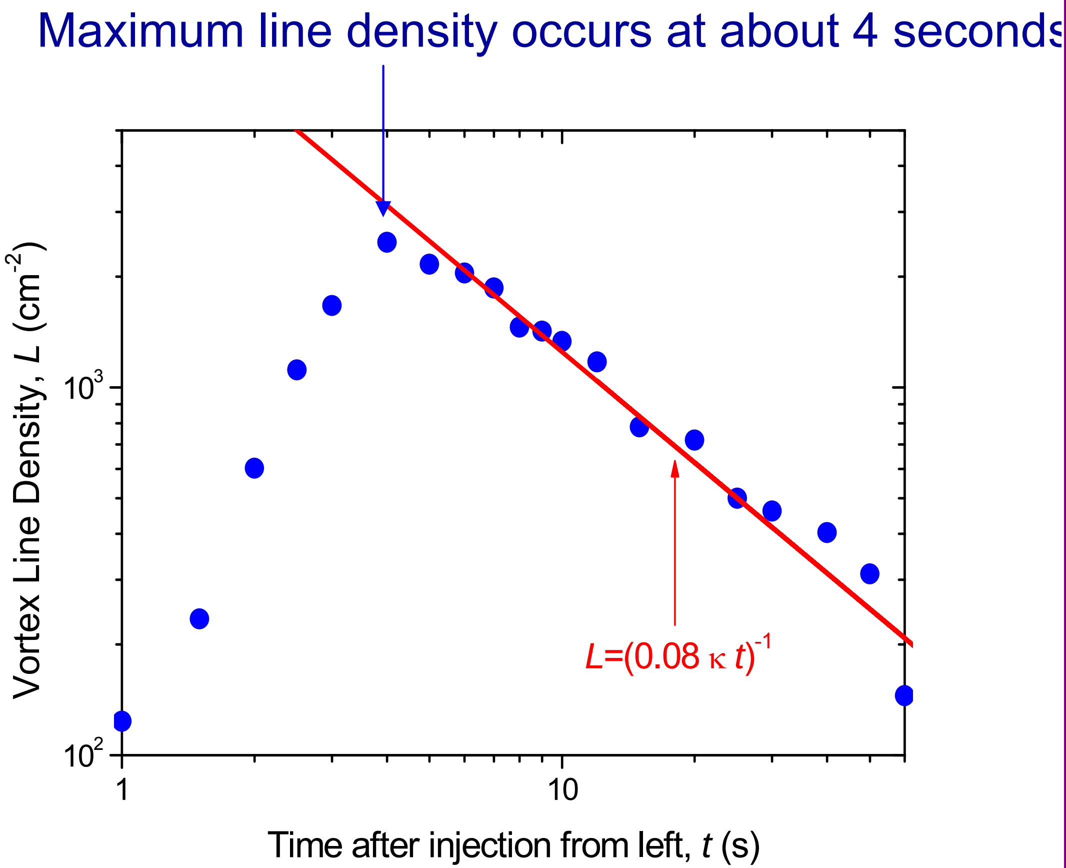


Tangle Growth & Decay in Centre of Cell

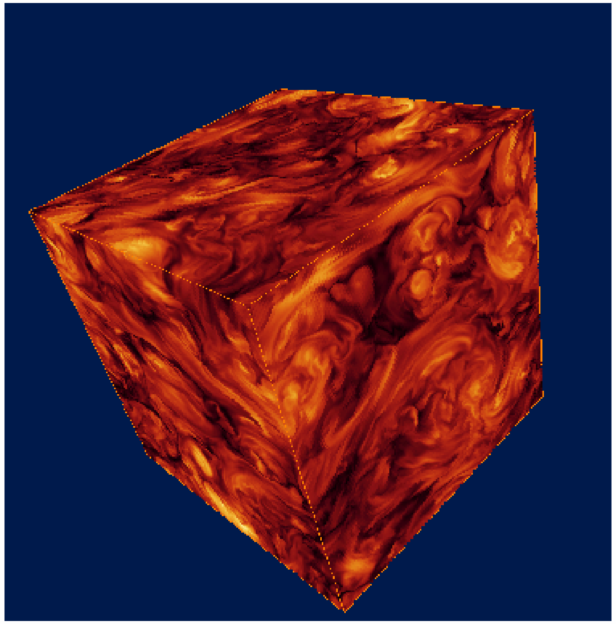
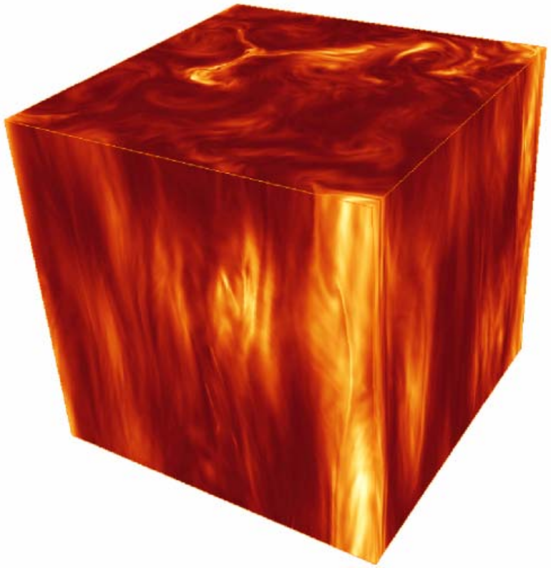
We can probe the growth of the tangle by first sending a pulse from the left tip and then use a pulse from the bottom tip to probe the vortex line density in the centre of the cell.

The tangle grows and fills the whole cell. $L \sim t^{-1}$, agrees with other measurements.





Turbulence in rotation

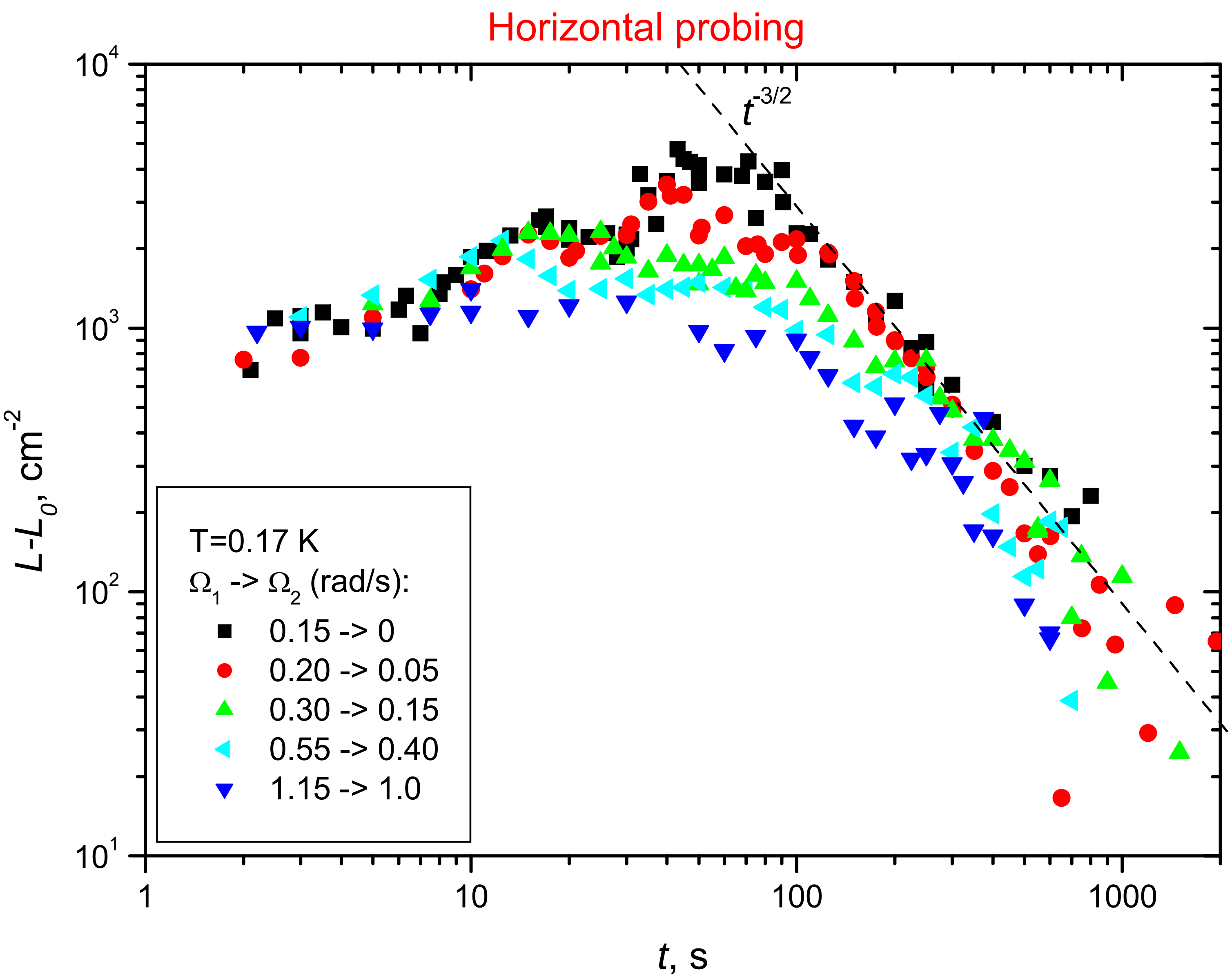
(illustrations of flow field in a stationary (left) and rotating (right) frame copied from http://www.ipp.mpg.de/~wcm/projects/rota/rot_hd.html)

We currently study turbulence decay at continuous rotation. This can help quantify the importance of tangle polarization, as well as to attempt a realization of quasi-2d turbulence and an inverse cascade in it.

E.g. rotating classical turbulence: Morize et al. Phys. Fluids **18**, 065107 (2006).

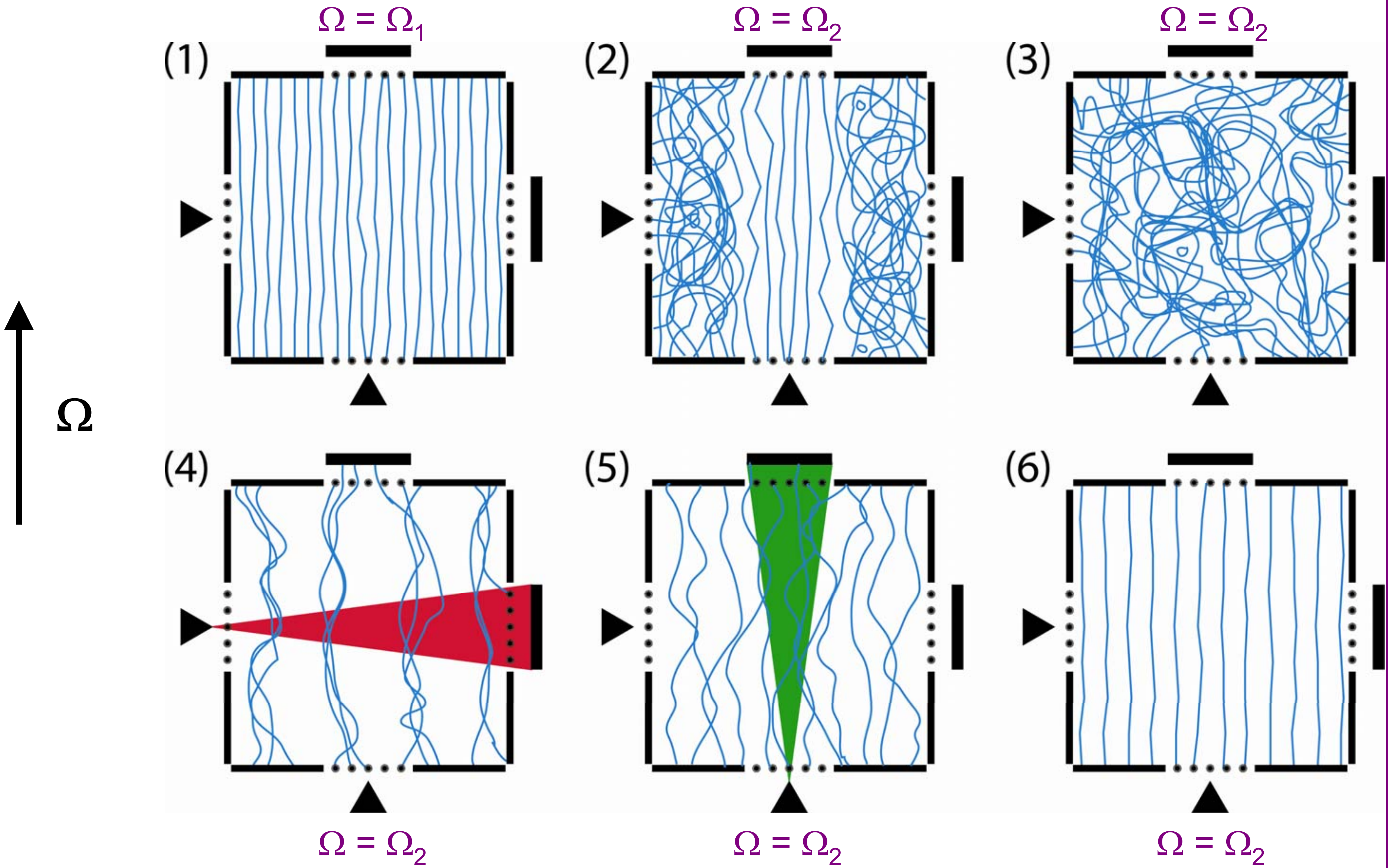
Results: Spin-downs, all with $\Delta\Omega$ = -0.15 rad/s

Vortex line density L minus the equilibrium density of the final state $L_0 = 2\,\Omega_2 / \kappa$



Spin-down: $\Omega_1 \rightarrow \Omega_2$

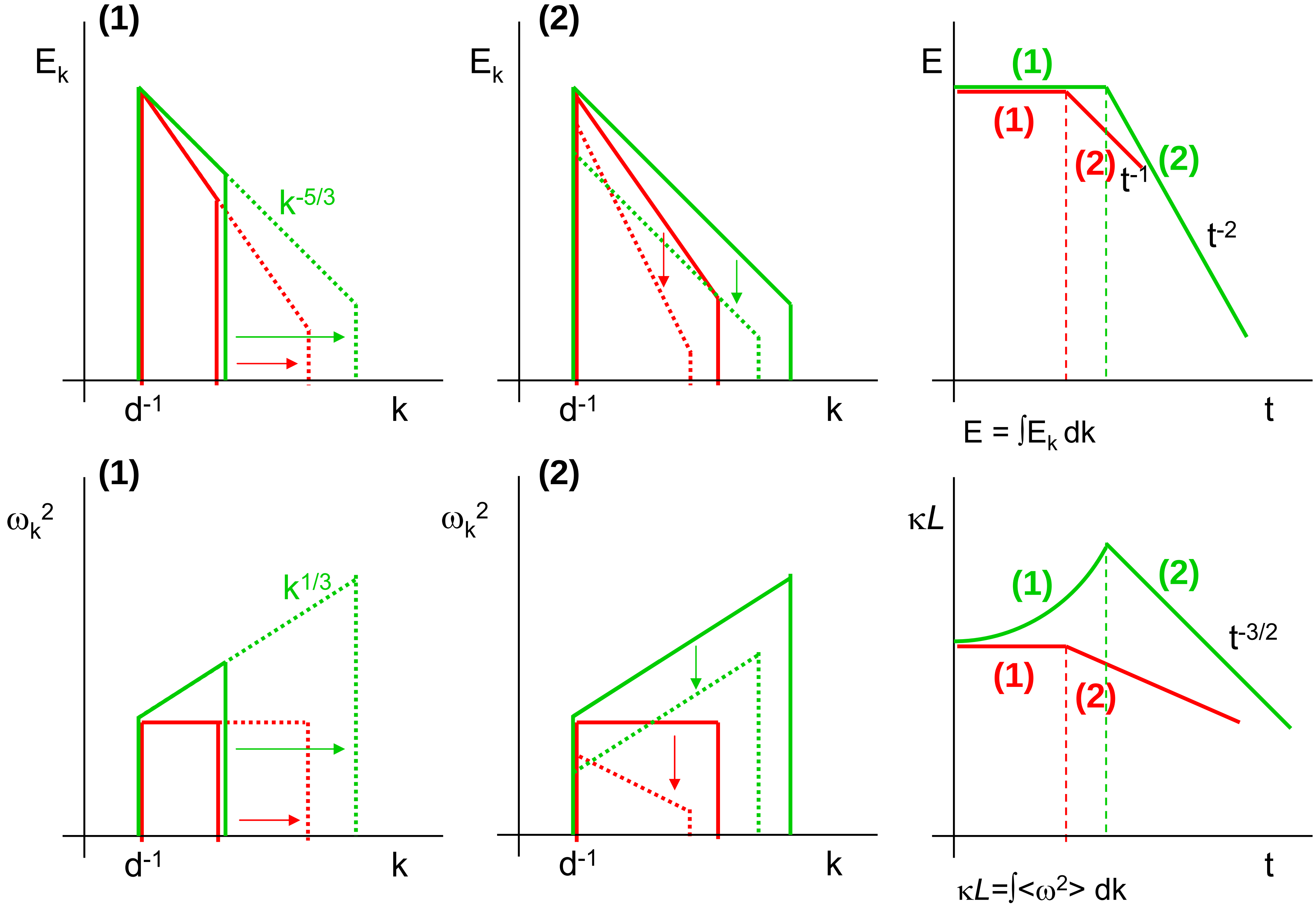
Rotate at Ω_1 for 10 minutes then at $t = 0$, spin-down to Ω_2.



QT in rotation: purely classical model

$\Omega=0$ (or $\Omega<<\Delta\Omega$)

$\Omega>>\Delta\Omega$



Conclusions

We used charged vortex rings to probe turbulence in superfluid ^{4}He in the $T = 0$ limit.

The decay of different types of turbulence, generated by injected ions or spin-down, studied.

Ultraquantum tangles decay as $L \sim t^{-1}$. This is consistent with Vinen's equation, $E \sim t^{-1}$, and the effective kinematic viscosity $\nu = 0.1\kappa$.

Quasiclassical tangles decay as $L \sim t^{-3/2}$. This is consistent with a developed Kolmogorov cascade truncated at cell size, $E \sim t^{-2}$. The effective kinematic viscosity $\nu = 0.003\kappa$.

In $T = 0$ limit, ν(ultraquantum) / ν(quasiclassical) ~ 30. This implies that, at the same overall L, structured tangles feed into the quantum cascade 30 times less energy than random ones.

Excitation of Kelvin waves on charged vortex rings at high fields observed.

A charged tangle in external field keeps accelerating the surrounding superfluid forward.

At continuous ion injection, switching between two steady regimes observed.

In rotating frame, turbulence possesses less vorticity, in agreement with classical expectations.