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Diffusion of an Inhomogeneous Vortex Tangle and Decay of the Superfluid Turbulence

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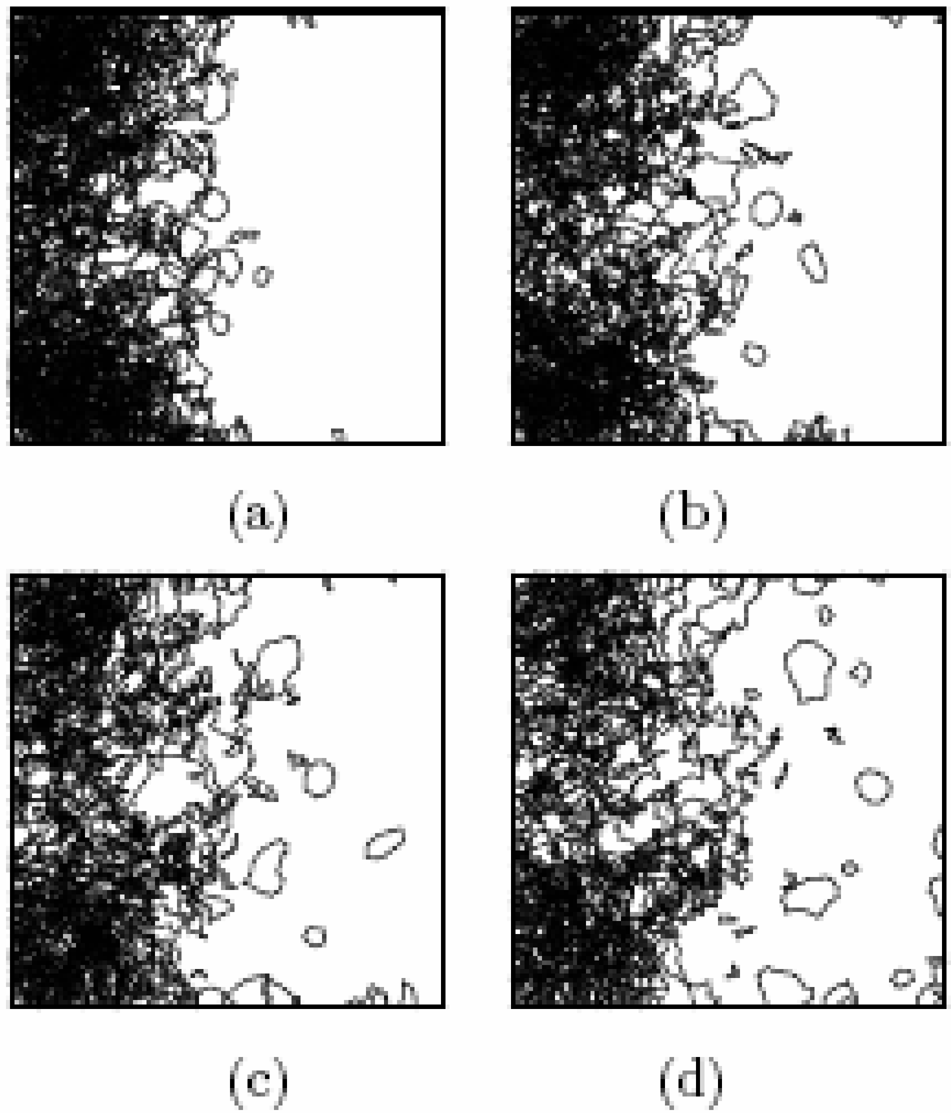
Introduction and scienific background

- **Idea that inhomogeneous vortex tangle evolves in a diffusive-like manner appeared pretty long ago. Thus, in 1988 van Beelen & Geurst, who observed the regions of high vortex line densities L(r,t) --- "plugs" in the channel with the counterflowing He II, proposed that this phenomenon appeared due to diffusion of quantity L(r,t).**

Numerical Illustration (M. Tsubota, T. Araki and W.F. Vinen, (2003)).

Authors determined the diffusion constant to be equal to 0.1κ



Applications to the decay problem?

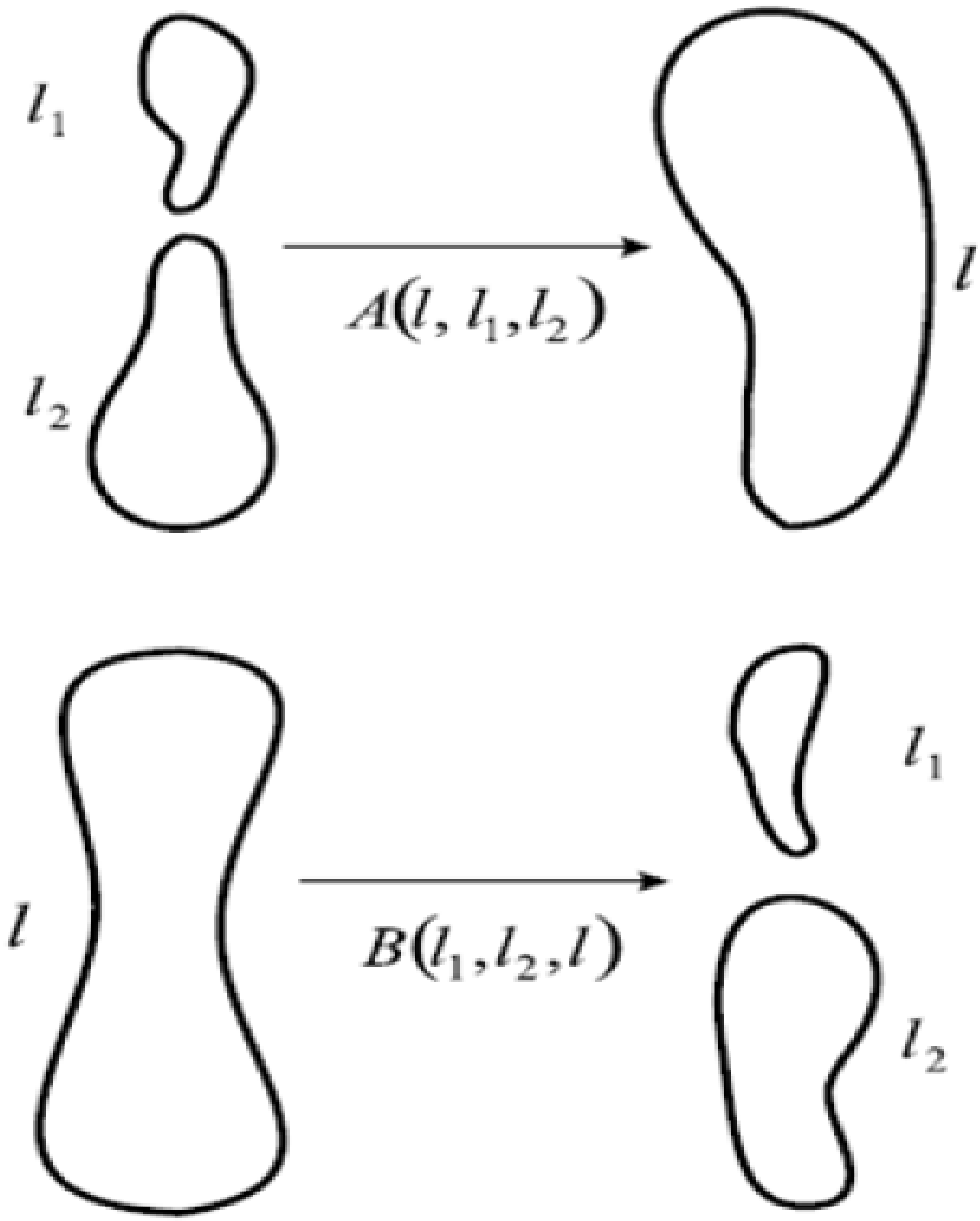
- Especial interest to the diffusion processes arises in context of the problem of decay of the vortex tangle at zero temperature.
- Mechanism of the diffusion-like spread of the vortex tangle with its subsequent degeneration usually is usually ignored, just grounding on the value 0.1κ for the diffusion constant. This small value of thediffusion coefficient does not lead to correct time of decay.

Purpose of work

- In the present paper we develop the theory describing the evolution of an inhomogeneous vortex tangle on the bases of kinetics of the merging and breaking down vortex loops. We showed that evolution of a weakly inhomogeneous vortex tangle obeys the diffusion equation with the coefficient equal to about 2.2 κ, which exceeds approximately twentyfold as large the value obtained in paper. We present arguments that the diffusion constant would be underestimated in \cite{Tsub-diff} due to especial procedure used by the authors. We used the diffusion equation to describe the decay of the vortex tangle at very low temperature. Comparison with the recent experiments on decay of the superfluid turbulence \cite{Pickett-2006},\cite{Golov07} is made.

Statement of problem: 1

Vortex loops composing the vortex tangle can move as a whole with some drift velocity V_{L} depending on their structure and their length. The flux of the line length, energy, momentum etc., realized by the moving vortex loops takes a place. Situation here is exactly the same as in usual kinetic theory with the difference that the "carriers" of length are not point particles but are are extended objects (vortex loops), which possess an infinite number of degrees of freedom with very involved dynamics. In addition, while collision (or self-intersection) of elements of filaments the reconnection of the lines occurs, and loops either merge or split, losing their individuality and turning into other loops.


l_1
l_2
$A(l, l_1, l_2)$
l
l
$B(l_1, l_2, l)$
l_1
l_2

Statement of problem. 2

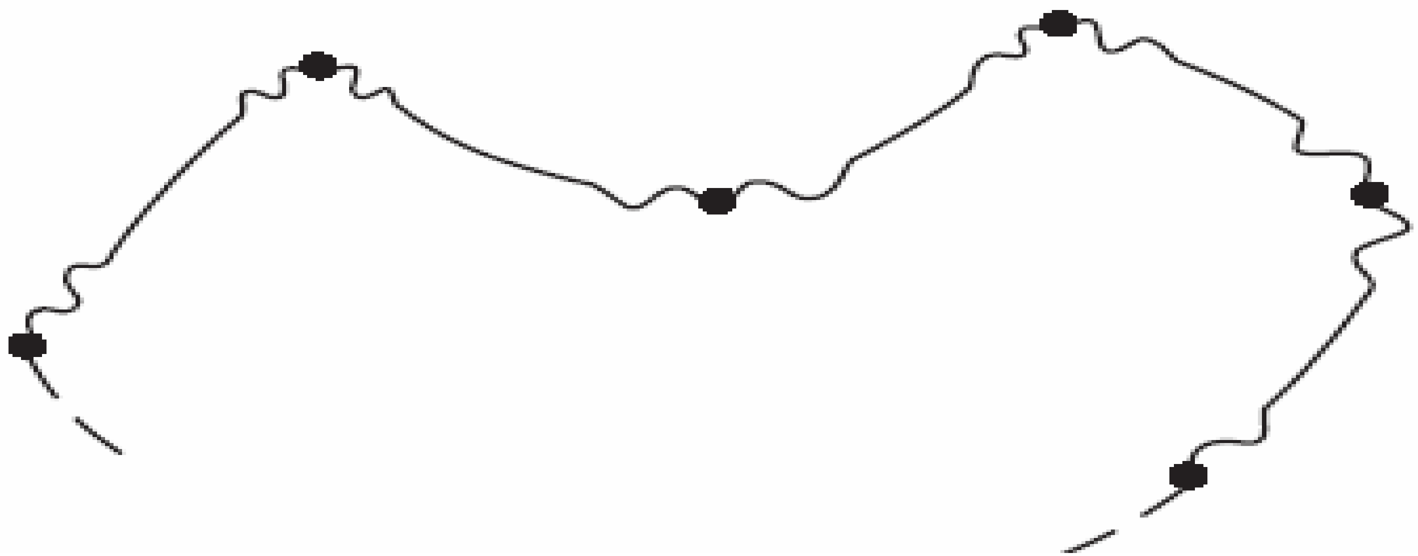
. Thus, the number of objects is not fixed and the full statement of this problem requires some analog of the secondary quantization method for extended object, or the string field theory, the problem of incredible complexity. Clearly, this problem can be hardly resolved in the nearest future. Some approach crucially reducing the number of degrees of freedom is required.

We offer to fulfill investigation basing on supposition that vortex loops have the Brownian structure (see Nemirovskii, PRB, (1998), PRL (2006) PRB, (2008)).

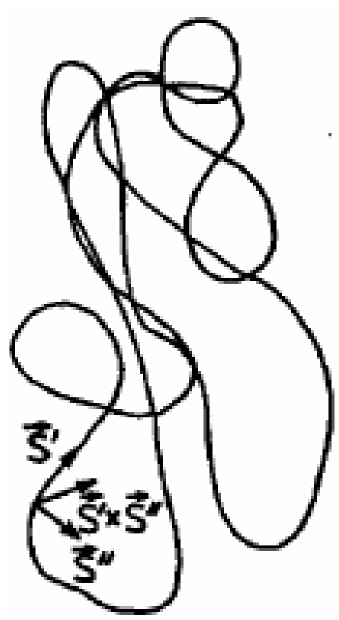
In fact this approach is not very new. For instance, in famous books by Kleinert there is described how the stochastic one-dimensional singularities (like linear dislocations, cosmic strings, polymer chains etc) can be considered as a set of Brownian loops.

Random walking (Brownian) structure

- The structure of any loop is determined by numerous previous reconnections. Therefore any loop consists of small parts which "remember" previous collision. These parts are uncorrelated since deterministic Kelvin wave signals do not have a time to propagate far enough. Therefore loop has a structure of random walk (like polymer chain).



Gaussian model of vortex loop



Main mathematical tool to descibe random walk is the Wiener distribution. We use it in form, which allows to take into account possible anisotropy and finite curvature.

$$P(\{\mathbf{s}(\xi,t)\}) \propto \exp\left(-\int_0^l\int_0^l \mathbf{s}'^{\alpha}(\xi,t)\Lambda_{\alpha\beta}(\xi,\xi')\mathbf{s}'^{\beta}(\xi',t)d\xi d\xi'\right).$$

Here $\Lambda_{\alpha\beta}(\xi,\xi')$ is Mexican-hat like function width ξ_0. The average loop can be imagine as consisting of many arches with mean radius of curvature equal ξ_0 randomly (but smoothly) connected to each other. Quantity ξ_0 is important parameter of the approach. It plays a role of the "elementary step" in the theory of polymer. It is low cut-of, theory does not describe scales smaller then ξ_0.Beeing a Gaussian function the Wiener distribution allows readily to calculate any average functional $\langle A(\{\mathbf{s}(\xi,t)\})\rangle$.

Statement of problem. 3

- ***Thus, besides of the structure parameters the Brownian loops have only degree of freedom, namely length l.***
- This conception allows to work with the distribution function n(l,t) of the density of a loop in the space of their lengths (number of loops of given length per unit volume). The distribution function n(l,t) obeys the Boltzmann type "kinetic" equation. Study of exact solution to this "kinetic" equation allowed to develop a theory of superfluid turbulence, which quantitatively describes main features of this phenomenon (see Nemirovskii, PRL (2006) PRB, (2008)).
- This approach turns to be useful in study the inhomogeneous vortex tangle. In this case we have to impose the coordinate dependence on the distribution function, that is to put n(l,r,t) and to modify "kinetic" equation with regard to inhomogeneous situation.
- In this work we restrict ourselves to the a bit more modest problem of evolution of the vortex line density. The corresponding theory can be develop in spirit of classical kinetic theory with the difference that the transport processes are executed with the extended objects - vortex loops. Accordingly the key questions is to evaluate the drift velocity V_{L} and the free path for the loop of size l.

The Drift Velocity

The drift velocity V_L is defined via an averaged quadratic velocity of the line elements (simple average velocity vanishes due to symmetry)

$$V_L = \sqrt{\left\langle \left(\frac{1}{l} \int \dot{\mathbf{s}}(\boldsymbol{\xi})^2 d\xi \right)^2 \right\rangle} = C_v \beta / \sqrt{l \xi_0}$$

$$\beta = (\kappa / 4\pi) \ln(l / a_0)$$

Here C_v is the constant of order of unity. Velocity V_l can be also estimated from the following qualitative consideration. Consequently considering the average loop as consisting of arches with the mean radius of curvature equal to ξ_0 randomly (but smoothly) connected to each other, we take its velocity as resulting velocity of all arches composing the loop. Since the arches randomly connected to each other, and have the velocity as for rings, $V_{arch} = \beta / \xi_0$ (directed along the normal), the resulting is averaged velocity is the "random walking" average, $V_l = \frac{1}{n} \sqrt{n} \beta / \xi_0$, where $n \sim l / \xi_0$ is a number of arches. Both ways lead to result that V_l is estimated as $V_l \sim \beta / \sqrt{l \xi_0}$.

Collision of loops

The drift motion is realized until the loop collides with other loops with subsequent formation of larger loop. Number of collision $P_{Col}(dt)$ per small interval dt can be estimated from the "kinetic equation" for the distribution function $n(l)$ of density of loops in space of their lengths l . The rate of change of density $n(l)$ due to collisions is

$$\begin{aligned}\frac{\partial n(l,t)}{\partial t} &= -\int\int A(l_1,l,l_2)\delta(l_2-l_1-l)n(l)n(l_1)dl_1dl_2 \\ &\quad -\int\int A(l_2,l,l_1,)\delta(l_1-l_2-l)n(l)n(l_2)dl_1dl_2\ ,\end{aligned}$$

The scattering cross-section $A(l_1,l,l_2)$ describes the rate (number of events per unit volume and unit time) of collision of two loops with lengths l and l_2 and forming the loop of length $l_1+l=l_2$,

$$A(l_1,l,l_2)=b_mV_l l_1 l.$$

Here b_m is numerical factor approximately equal to $b_m\approx 0.2$ and V_l is the characteristic velocity of the line elements.

Free path

The probability $P_{Col}(dt)$ for loop to collide with other loops (and reconnect) in a small interval dt is:

$$P_{Col}(dt) = \Lambda dt.$$

Calculation of the collision probability Λ we perform with use of the distribution function $n(l) = Cl^{-5/2}$

$$\Lambda(l) = 2\beta b_m \mathcal{L} \sqrt{\frac{l}{\xi_0}},$$

In usual way we conclude that the probability $P(x)$ for the loop of length l to fly the time t without collision is

$$P(x) = 2l\mathcal{L} \exp(-2lb_m \mathcal{L} x) \Rightarrow l_{free} \approx \frac{2.5}{l\mathcal{L}}$$

Flux through area element

$$\mathbf{J}_x = d\mathbf{j}_{+x} - d\mathbf{j}_{-x} = \frac{1}{4\pi}\int\int\int (d\mathbf{j}_{+x}(\theta,\varphi,l,R) - d\mathbf{j}_{-x}(\theta,\varphi,l,R)) \sin\theta d\theta d\varphi dl dR.$$

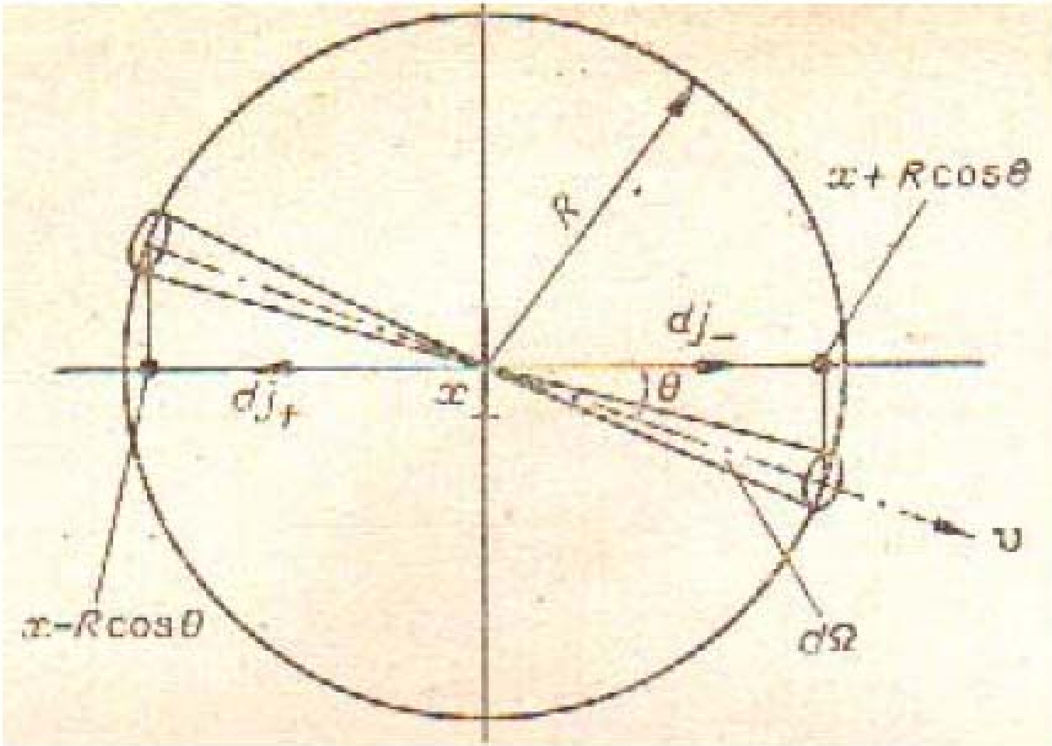


Figure 2: The net flux of length through the small area element placed in $x = 0$ and orientated perpendicularly to axis x

Evolution of the vortex line density

$$\mathbf{J}_x = \frac{1}{6}\frac{\beta}{\sqrt{\xi_0}\mathcal{L}b_m}\frac{1}{4\xi_0^2}\frac{\partial}{\partial x}\left(\xi_0^{1/2}\mathcal{L}\right) \tag{3}$$

There are possible two variants. a).Quantity $\xi_0(x)$ is independent variable (e.g. due to the own vortex line dynamics, or we inject the loops of definite size). b).Mean curvature $\xi_0(x)$ and $\mathcal{L}(x)$ are not independent, there is correspondence between them (Schwarz, PRB, 1988, Nemirovskii, PRL, 2006)

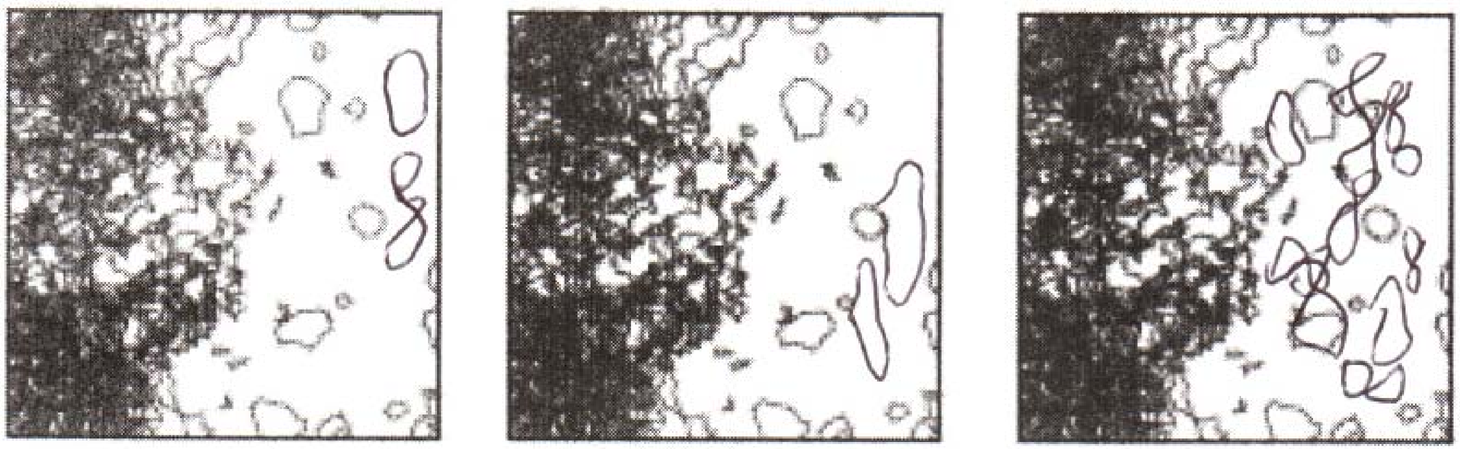
$$\xi_0 \approx 0.27\mathcal{L}^{-1/2}.$$

Tthe rate of change of quantity $\mathcal{L}$ obeys to the diffusion type equation

$$\frac{\partial \mathcal{L}}{\partial t} = D_v \nabla^2 \mathcal{L} \approx 2.2\kappa \nabla^2 \mathcal{L}$$

Boundary conditions. 1. Smearing of tangle.

1. Smearing of tangle. Let us consider a first the case when vortex tangle is placed in some restricted domain of superfluid helium. Let us consider also that vortex line density is not too high, this allows the relatively large loop to be radiated. They move slowly, the smaller loop run down larger loop, then collide and reconnect with them. So outside of initial domain the well developed tangle is formed. This, secondary, vortex tangle smoothly join with the initial tangle inside domain. This implies that in this case no boundary conditions are required at all, and evolution of the vortex line density obeys equation in infinite space with the initial distribution $\mathcal{L}(x, 0)$ inside domain.



Boundary conditions. 2. Radiation of loops.

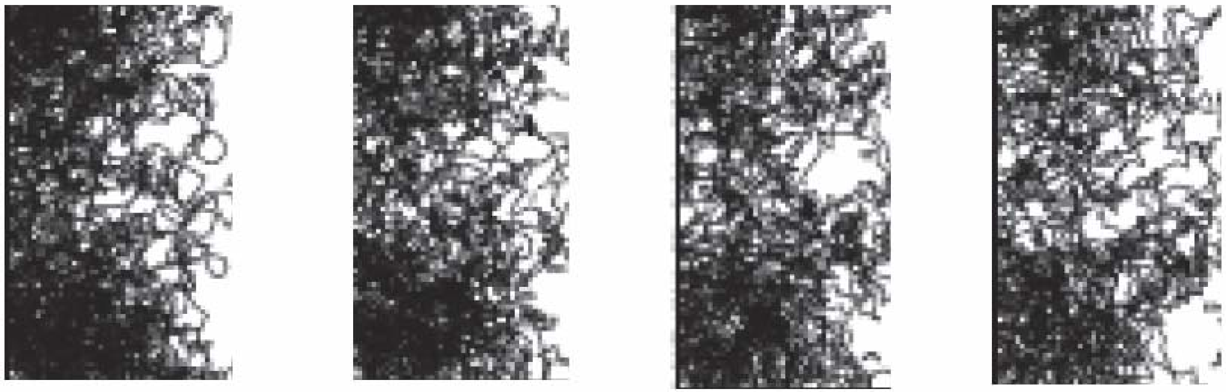
2. Radiation of loops. The second situation can be realized when the radiated vortex loops run away and does not influence the initial vortex tangle. It can happen, for instance if the initial tangle is very dense, so it can radiate only very small loop, which rapidly propagate. These loops run away without interaction with each other and with initial tangle where they are radiated from. Other hypothetical variant is if there is some trap on the boundary absorbing vortex loops. Thus vortex loops escape from the initial domain do not back influencing original vortex tangle. In both case the boundary conditions can be found assuming that diffusive like flux of length near boundary $\mathbf{J}_x = -D_v \nabla \mathcal{L}(x_b, t)$ coincides with the flux executed by vortex loops radiated through the (right) boundary $\mathbf{J}(x_b, t)$.

$$C_{rad}\mathcal{L}^{\frac{3}{2}} + \frac{1}{2} D_v \nabla \mathcal{L}(x_b, t) = 0.$$

Boundary conditions. 3. Solid walls.

3. Solid walls. Vortices can annihilate on the solid wall, they can undergo pinning and depinning radiating vortices back to the bulk of helium. Surely this requires a special treatment which goes beyond the scope of the work. The one possible way is to consider the solid wall as a "partial" trap, which catches the loops and re-emit part of them back into the volume. Formally it can be written as previous condition with additional term describing back flux. Without detailed analysis it can be supposed that this flux is proportional to the vortex line density on the boundary $\mathcal{L}(x_b, t)$ with the coefficient C_{back} depending on dynamics of line on the wall (jumps between pinning sites, Kelvin waves dynamics near the wall etc.). Thus the boundary condition in this case can be written in form.

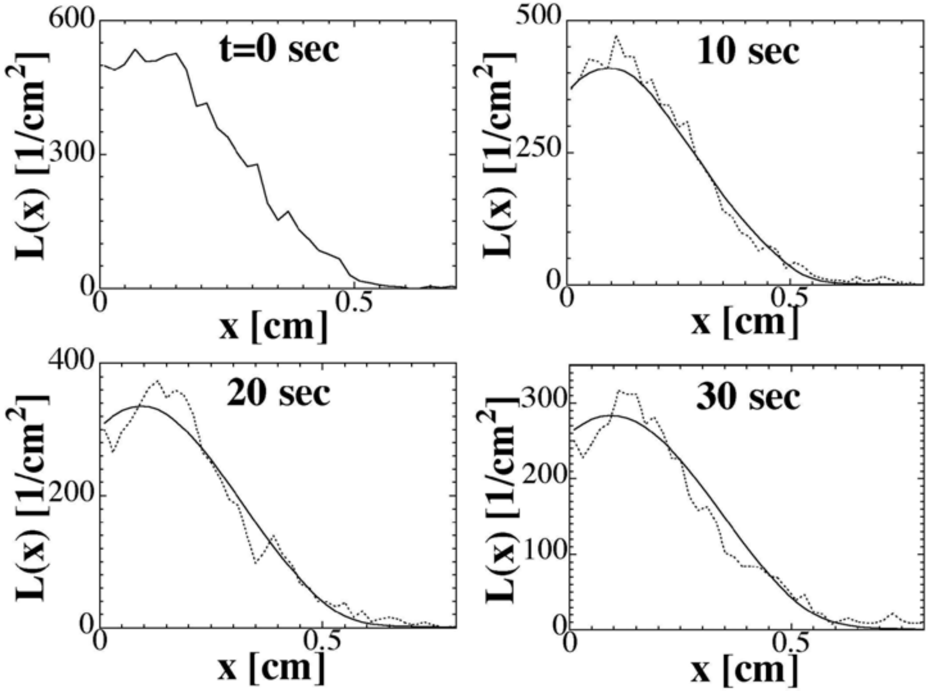
$$C_{rad}\mathcal{L}^{\frac{3}{2}} + \frac{1}{2}D_v \nabla \mathcal{L}(x_b, t) - C_{back}\mathcal{L} = 0.$$



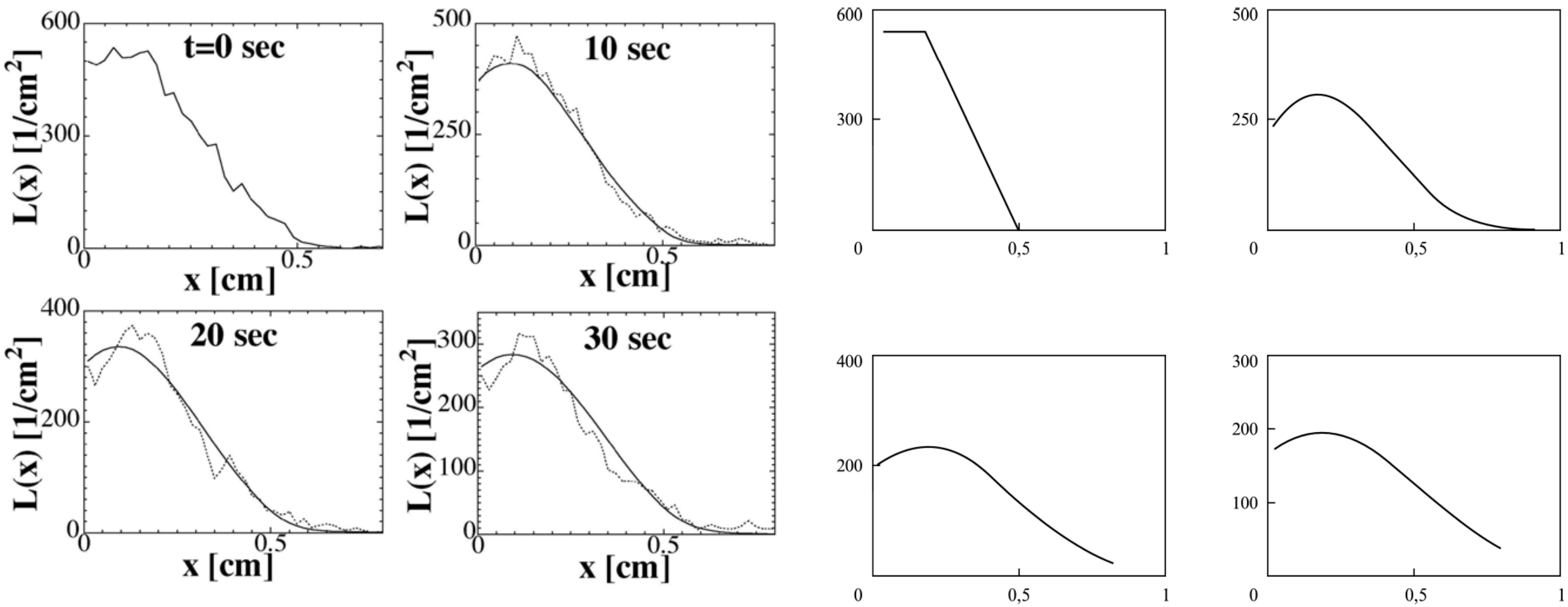
Tsubota’s group numerical experiment. I.

- As it was discussed in the Introduction, the contribution of the diffusion (or radiation of loops) is ignored, mainly due to smallness of the diffusion constant offered in paper Tsubota et al. Let us expect this work more thoroughly. In paper there was studied one-dimensional evolution (spacial spreading) of the vortex tangle concentrated initially in some domain of space and having there nonuniform distribution. On the rest pictures the distribution of the VLD at different moments of time is shown.To describe this evolution of the VLD it had been supposed the quantity L(x,t) to obey the diffusion equation with the additional term –
- $-\chi\_2(\kappa/2\pi)L^2$
- in the right hand side. In turn this term was introduced to describe the decay of the vortex tangle in early numerical simulation made Tsubota et al. Because of this additional term contribution of diffusion to the whole decay would be significantly underestimated. We would like to note that there was possible to choose another reasoning, namely, to consider that decay of the vortex tangle in both cited papers occurs mainly by the diffusion process (with the diffusion coefficient calculated in our work). We calculated spacial -temporal evolution of vortex tangle (under condition of numerical experiment by Tsubota et al.) with use of our diffusion coefficientIt can be concluded that approach developed satisfactory describes evolution of vortex tangle without any additional supposition.

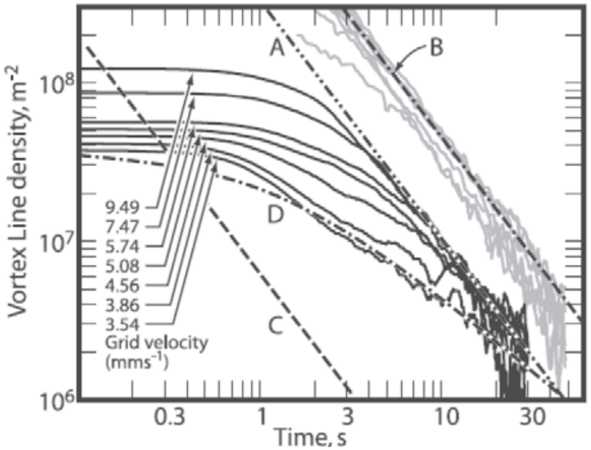
Tsubota’s group numerical experiment. II.



Tsubota's group numerical experiment. II.



Pickett's group experiment (2006)



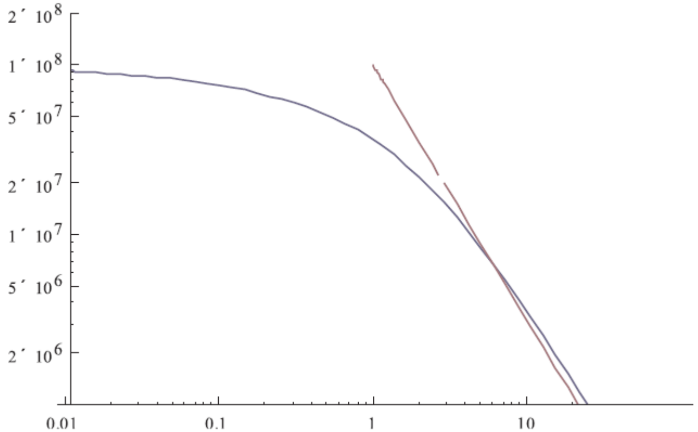
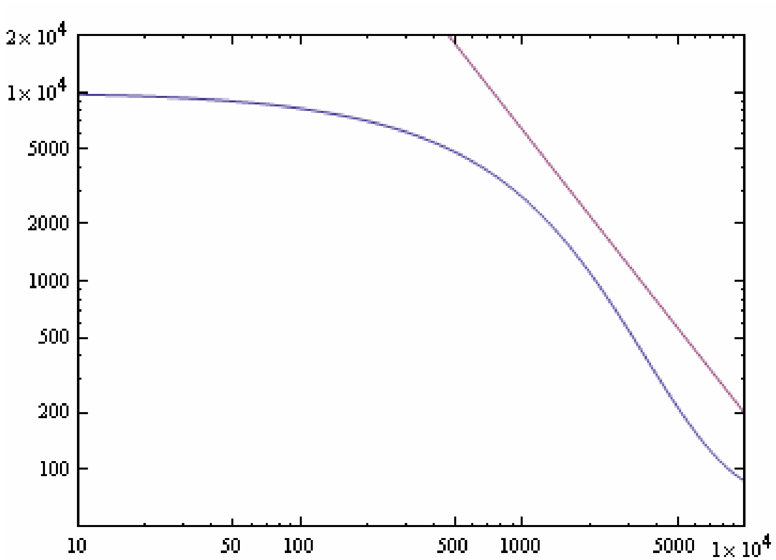


FIG. 5: Comparison of experimental data on the decay of superfluid turbulence obtained in [8] (upper figure) and our theoretical result (lower). We calculated temporal evolution of the averaged vortex line density (for initial condition $\mathcal{L} = 10^8\ 1/cm^2$) due to diffusion process described in the present paper. The initial domain of high vortex line density was created in the volume ^{3}He-B, so its diffusion like behavior should satisfy the first type boundary condition. The straight line in the lower figure exactly corresponds to line A in the upper figure 5 (which was named by authors of [8] as "limiting behavior").

Golov's group experiment (2007)



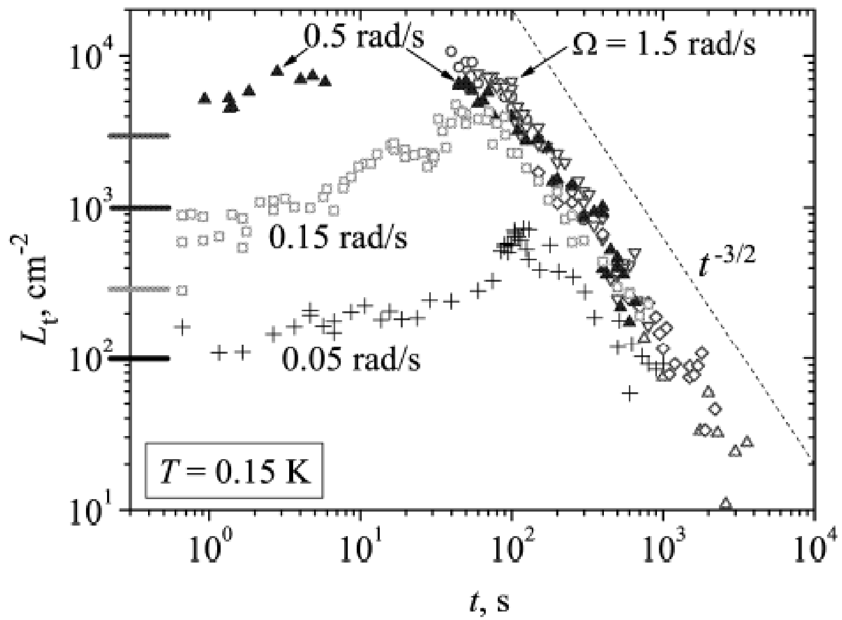


FIG. 6: Comparison of experimental data on the decay of superfluid turbulence obtained in [9] (upper figure) and our theoretical result (lower). We calculated temporal evolution of the averaged vortex line density (for initial condition $\mathcal{L} = 10^4\ 1/cm^2$) due to diffusion process described in the present paper. The superfluid turbulence in [9] was created in the cubic container with solid boundaries. Therefore we had chosen the third type boundary condition with the fitting parameter $C_{back} \approx 0.9$.

Conclusion

- In summary, the theory describing the evolution of inhomogeneous vortex tangle at zero temperature is developed on the bases of kinetics of merging and splitting vortex loops. By use of Gaussian model we calculated the flux of the vortex line density $L(x,t)$ in inhomogeneous vortex tangle and demonstrate that under certain circumstances it satisfies to the diffusion like equation with the coefficient equal approximately to 2.2κ. We use this equation to describe the decay of the vortex tangle at very low temperature.
- *Good agreement with the recent experiments on decay of the superfluid turbulence convinced us that the "diffusion contribution" into solution of the problem on decay of the vortex tangle is significant.*

Thank You !