The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2023-22

Workshop on Topics in Quantum Turbulence

16 - 20 March 2009

Coupling between Different Kelvin Waves and Simulations on Rotating Superfluids

R.M. Hänninen

Helsinki University of Technology
Low Temperature Laboratory
Helsinki
Finland

Coupling between Different Kelvin Waves: Cascade and Inverse Cascade

R. Hänninen

Low Temperature Laboratory, Helsinki University of Technology

Workshop on Topics in Quantum Turbulence, Trieste, 2009

Outline

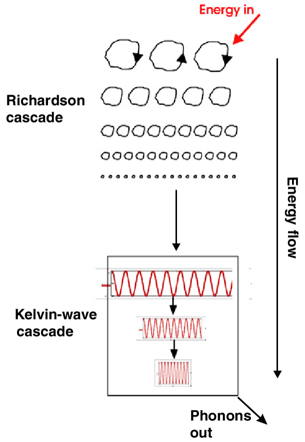
Introduction

Energy cascade in superfluids:

- Richardson cascade at large scales
- Kelvin-wave cascade at small scales

Vortex reconnections:

- provide a bridge between the two cascades
- important especially at low temperatures

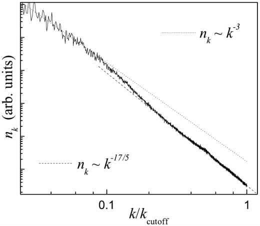


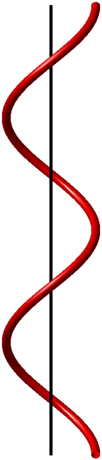
W.F. Vinen, J. Phys.: Condens. Matter **17** 3231 (2005).

Previous work on Kelvin-wave cascade

Kozik and Svistunov (PRL **94**, 025301 (2005)):

- steady state spectrum for K-waves
- $n_k \propto k^{-17/5}$





Vinen, Tsubota, Mitani (PRL. **91**, 135301 (2003)):

- drive K-waves with oscillating flow
- $n_k \propto k^{-3}$

Kelvin Waves

Simple helical wave: $x(z) = A_m \cos(2\pi mz/L)$, $y(z) = A_m \sin(2\pi mz/L)$

Left-handed ($m < 0$)

Combination

Right handed ($m > 0$)

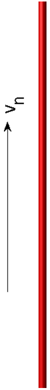
Kelvin Waves

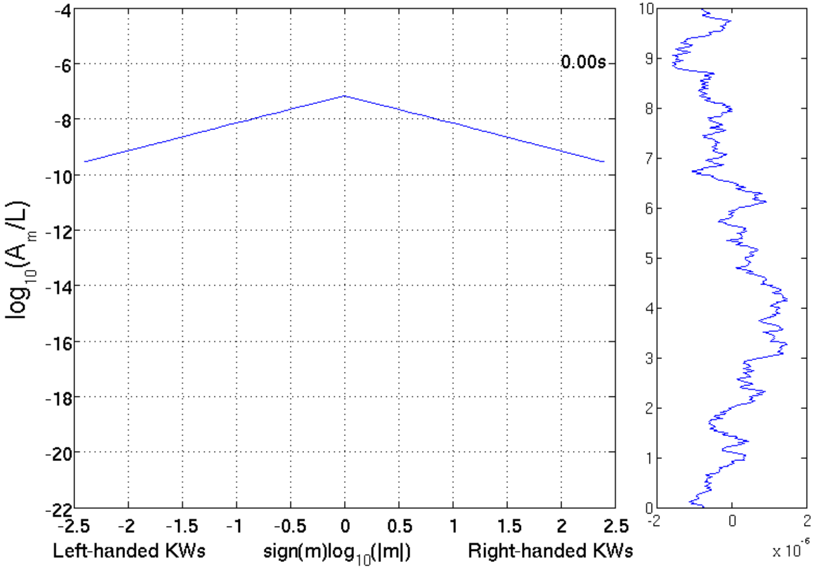
Glaberson instability for single vortex:

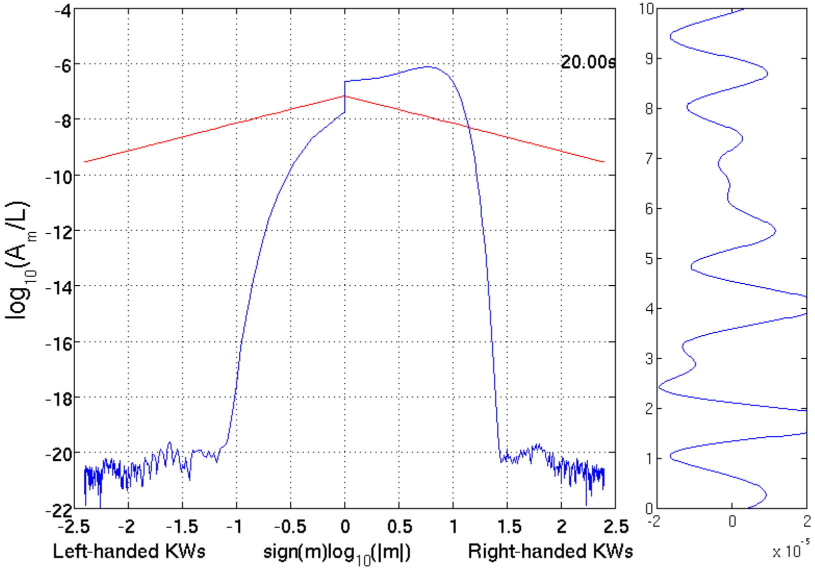
- Counterflow $v = v_n - v_s$ along vortex
- $v_c = (\Omega + \nu k^2)/k$, $\nu = (\kappa/4\pi)\ln(1/ka_0) \approx \kappa$
- $k_{opt} = \sqrt{\Omega/\nu} \Rightarrow v_c(k_{opt}) = 2\sqrt{\nu\Omega}$

In the following numerical example:

- periodic b.c., period $L \Rightarrow k = 2\pi m/L$, $m = \pm 1, \pm 2, \ldots$
- $\Omega = 0$, $L = 10$ mm, $v_n = 1$ mm/s, $v_n > v_c$ for modes m = 1,. . .,14
- ^{4}He-II with $\alpha = 0.1$ and $\alpha' = 0$
- initially $A_m \approx 10^{-6}\lambda_m$



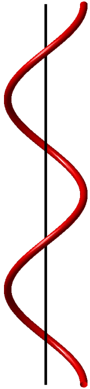

0.00s
log10(Am/L)
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5
Left-handed KWs
sign(m)log10(|m|)
Right-handed KWs
10
9
8
7
6
5
4
3
2
1
0
-2 0 2
x 10-6


20.00s
$\log_{10}(A_m/L)$
sign(m)$\log_{10}$(|m|)
Left-handed KWs
Right-handed KWs
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5
10 9 8 7 6 5 4 3 2 1 0
-2 0 2
x 10^{-5}

Coupling between different Kelvin waves

At low temperatures *Kelvin-wave cascade* should redistribute energy from small k-values to large k.
Consider a following simple case:

- no external velocities
- $T = 0$ $(\alpha = \alpha' = 0)$
- straight vortex (with KWs) with periodic b.c.
- assume that $A_m \ll \lambda_m$

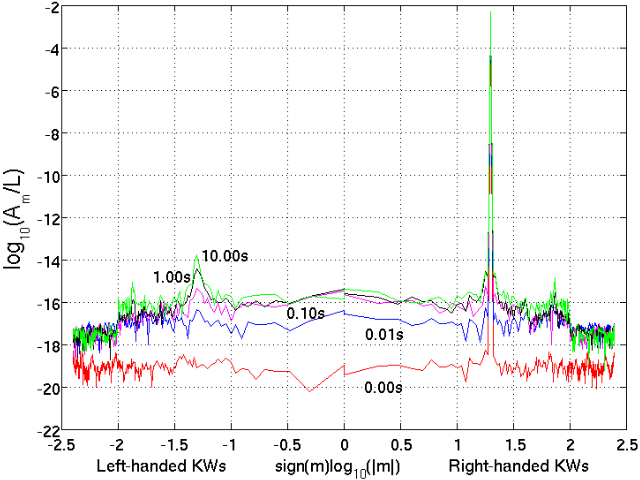


Parameters:

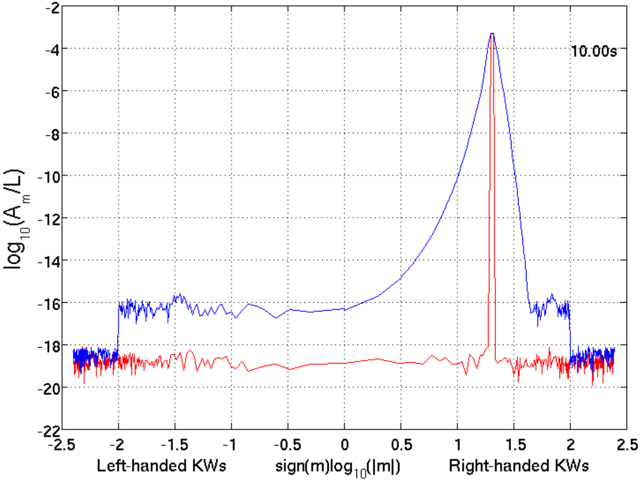
- $L = 10$ mm $(k = 2\pi m/L,\ m = \pm 1, \pm 2, \ldots)$
- ^{4}He-II
- Full Biot-Savart calculation

• E. Kozik and B. Svistunov, Phys. Rev. Lett. **94**, 025301 (2005).
• W.F. Vinen, M. Tsubota, and A. Mitani, Phys. Rev. Lett. **91**, 135301 (2003).

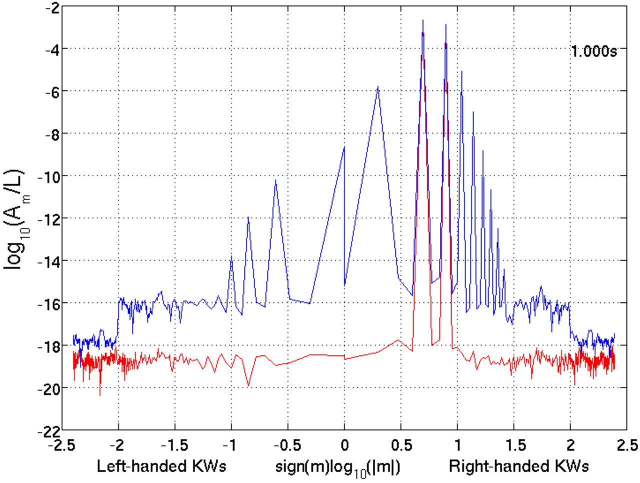
Single Kelvin mode ($m = 20$, $A_m/\lambda_m = 0.01$)



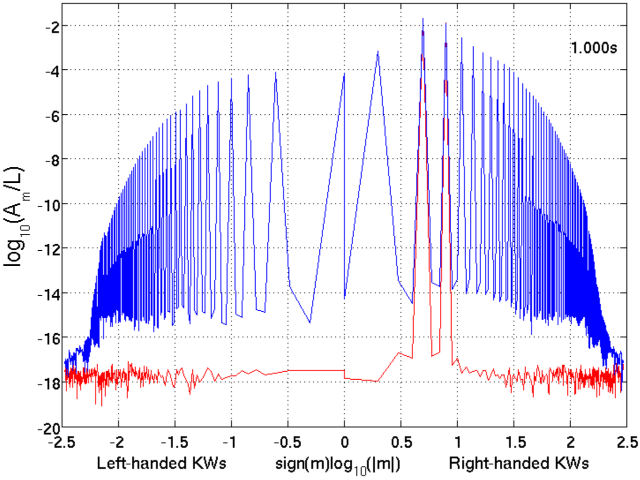
Two Kelvin modes ($m = 20{,}21$ $A_m/\lambda_m = 0.01$)



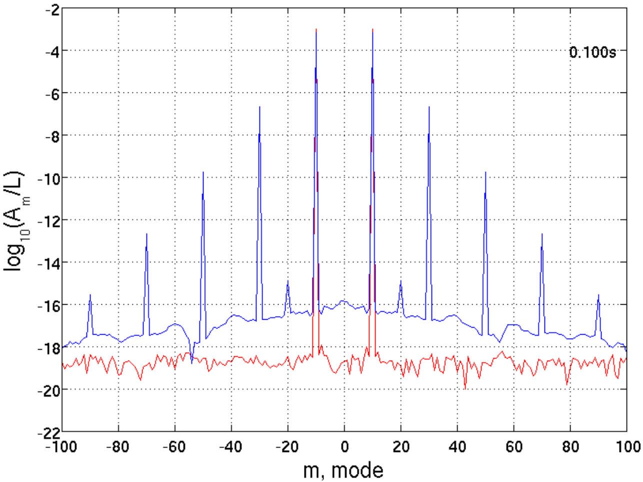
Two Kelvin modes ($m = 5,8$ $A_m/\lambda_m = 0.01$)



Two Kelvin modes ($m = 5,8$ $A_m/\lambda_m = 0.1$)



Finite temperature (m = -10,10 $A_m/\lambda_m = 0.01$, $\alpha = 1$)



What about realistic wall boundaries?

Vortex must terminate perpendicular to the wall:

- initial vortex must have at least two modes with different sign (i.e. m=1 and n=-1) and appropriate amplitudes/phases
- similar results

Coupling between Different Kelvin waves

Two Kelvin waves (modes m and n) on a straight vortex:

- produces cascade and inverse cascade
- only modes $m + p|m - n|$, $p = \pm 1, \pm 2, \ldots$ are excited
  - perhaps related to momentum and energy conservation laws
- strength of the cascade $\propto A_m A_n$ (?)
- cascade works also at finite temperatures

Numerical difficulties:

- difficult to verify a (quasi) steady state:
  - with small amplitudes it takes too long to reach
  - with larger amplitudes the fourier presentation fails (at later times x and y are not single valued functions of z any more)
- with more complicated configurations the accurate determination of K-waves is difficult:
  - in the energy spectrum $E(k)$ the contribution coming from straight vortex $\propto 1/k$ is too dominant

Thank You Very Much for Your Attention!