The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

**2023-1**

**Workshop on Topics in Quantum Turbulence**

***16 - 20 March 2009***

# CRITICAL VELOCITIES AND TRANSITION TO TURBULENCE IN OSCILLATING SUPERFLUID FLOWS

W. Schoepe

*Regensburg University*
*Germany*

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

# CRITICAL VELOCITIES AND TRANSITION TO TURBULENCE

# IN OSCILLATING SUPERFLUID FLOWS


**W. Schoepe**

**Regensburg University**


**Workshop on Topics in Quantum Turbulence**

**ICTP Trieste**

**March 2009**

## OUTLINE

1. Introduction: terminology of the forces on an oscillating sphere

2. Data analysis: how to analyze the velocity vs. driving force data, identification of $v_c$

3. Results: temperature dependence and frequency dependence of $v_c$

4. Theoretical models

5. Summary, conclusions and comments

Literature: Hänninen & Schoepe, arXiv:0801.2521 and J. Low Temp. Phys. 153, 189 (2008)

# DRAG FORCES ON AN OSCILLATING SPHERE

## Normal fluid drag:

### Hydrodynamic regime

Stokes regime $F_D = \lambda v$, with $\lambda = 6\pi\eta R(1+R/\delta)$ where $\eta \rightarrow \eta_n$ and $\rho \rightarrow \rho_n$

nonlinear behaviour expected when Strouhal number $\equiv$ amplitude/diameter $\geq 1$

### Ballistic regime (below 0.7 K)

$F_D = \lambda v$ with $\lambda = \rho_n \langle c \rangle \pi R^2$ where $\langle c \rangle$ is average quasiparticle velocity

## Superfluid turbulent drag:

$F_D(v,T) = \gamma(T)(v^2 - v_0^2)$ where $v \geq v_0$ and $\gamma(T) = ½\, C_D\, \rho_s(T)\, \pi R^2$ ($C_D \approx 0.36$ spheres)

Stable oscillation amplitude when average energy gain from driving force = average energy loss from drag force. ONLY for LINEAR drag simply: driving force = linear drag force.

$$\int \mathrm{F} \cdot \mathrm{v}\, \mathrm{dt} = \int \mathrm{F_D}(\mathrm{v}) \cdot \mathrm{v}\, \mathrm{dt}$$

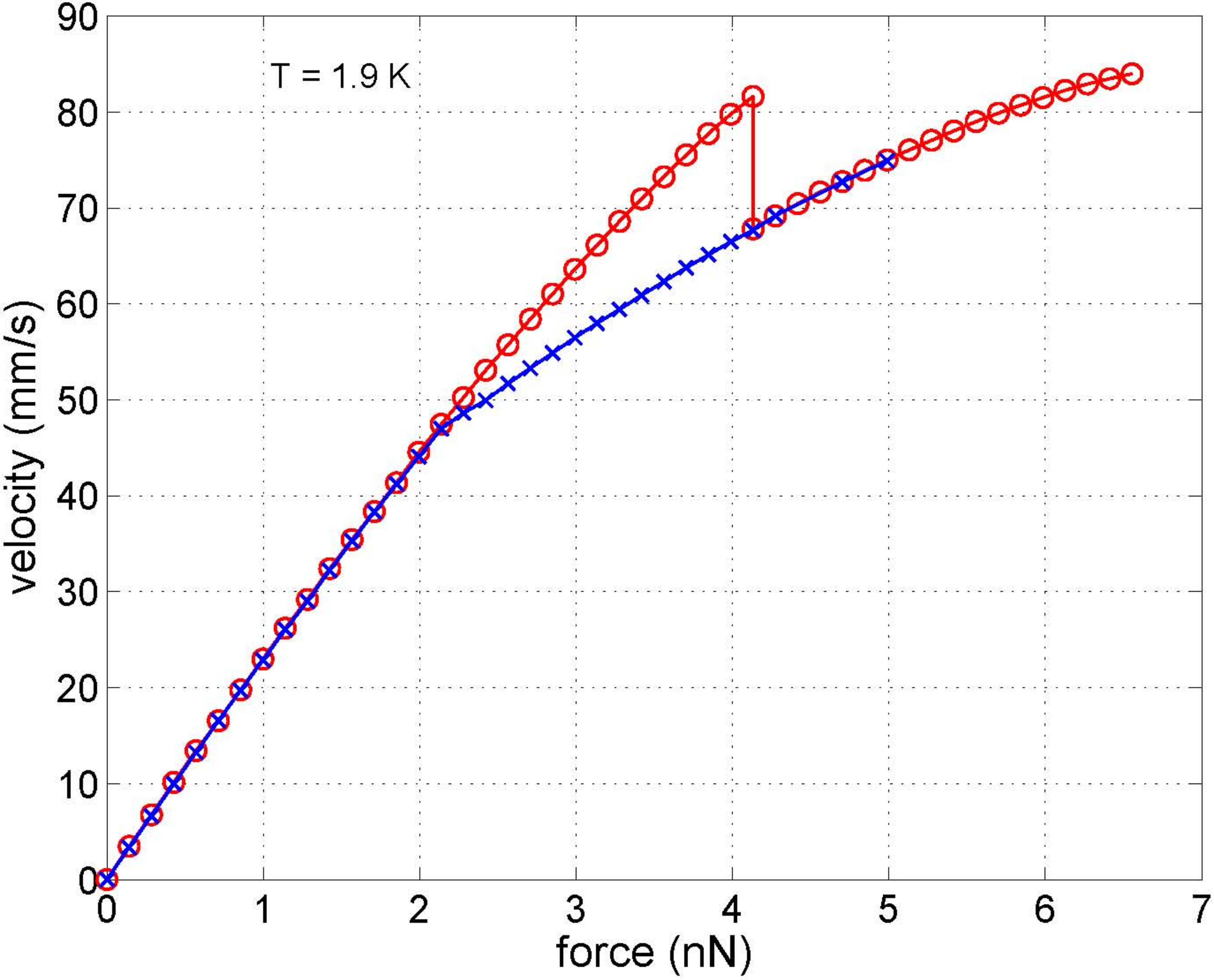
$$\mathrm{F_D}(\mathrm{v}) = \gamma(\mathrm{T})(\mathrm{v}^2 - \mathrm{v}_0^2), \;\; \mathrm{v} \geq \mathrm{v}_0$$

$$\gamma(\mathrm{T}) = \frac{1}{2} \mathrm{C_D}\, \rho_\mathrm{S}(\mathrm{T})\, \pi\, \mathrm{R}^2$$

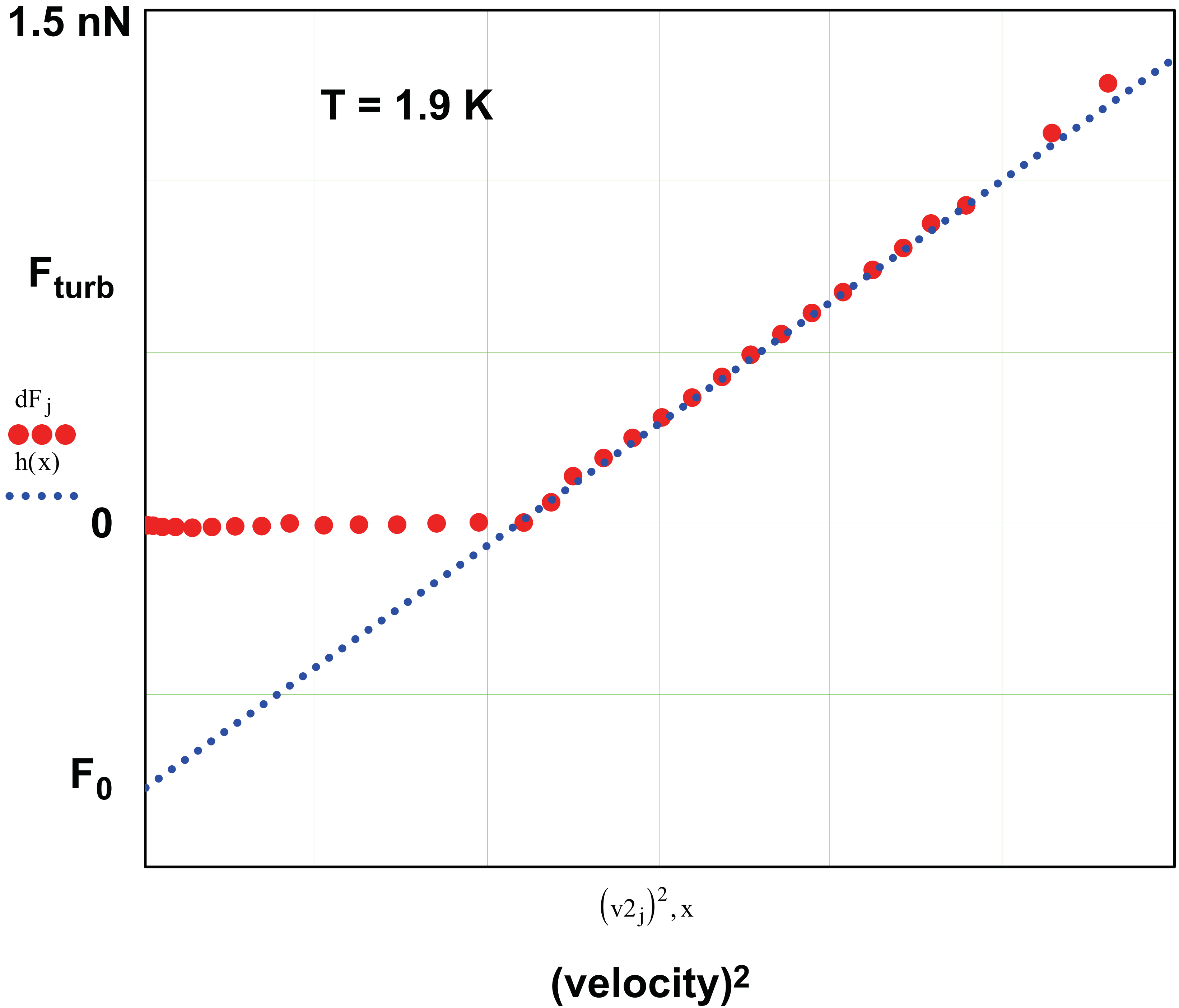
$$\mathrm{F}(\mathrm{v}) = \frac{8\gamma(\mathrm{T})}{3\pi}(\mathrm{v}^2 - \tfrac{3}{2}\mathrm{v}_0^2)$$

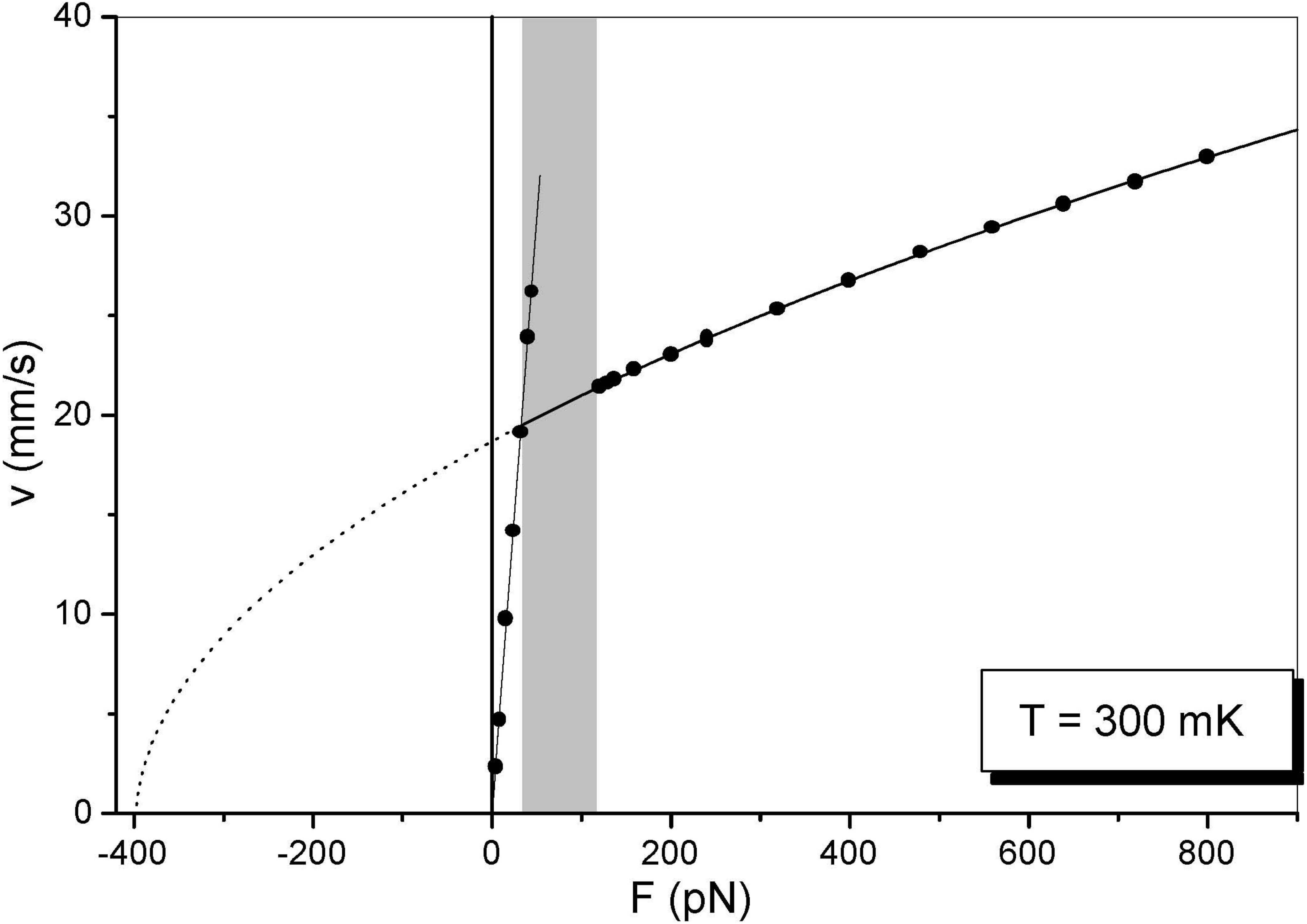
$$\mathrm{v}_\mathrm{C}^2 = \tfrac{3}{2}\mathrm{v}_0^2$$

$$\mathrm{F}(\mathrm{v}) = \lambda \mathrm{v} + \frac{8\gamma(\mathrm{T})}{3\pi}(\mathrm{v}^2 - \mathrm{v}_\mathrm{C}^2)$$

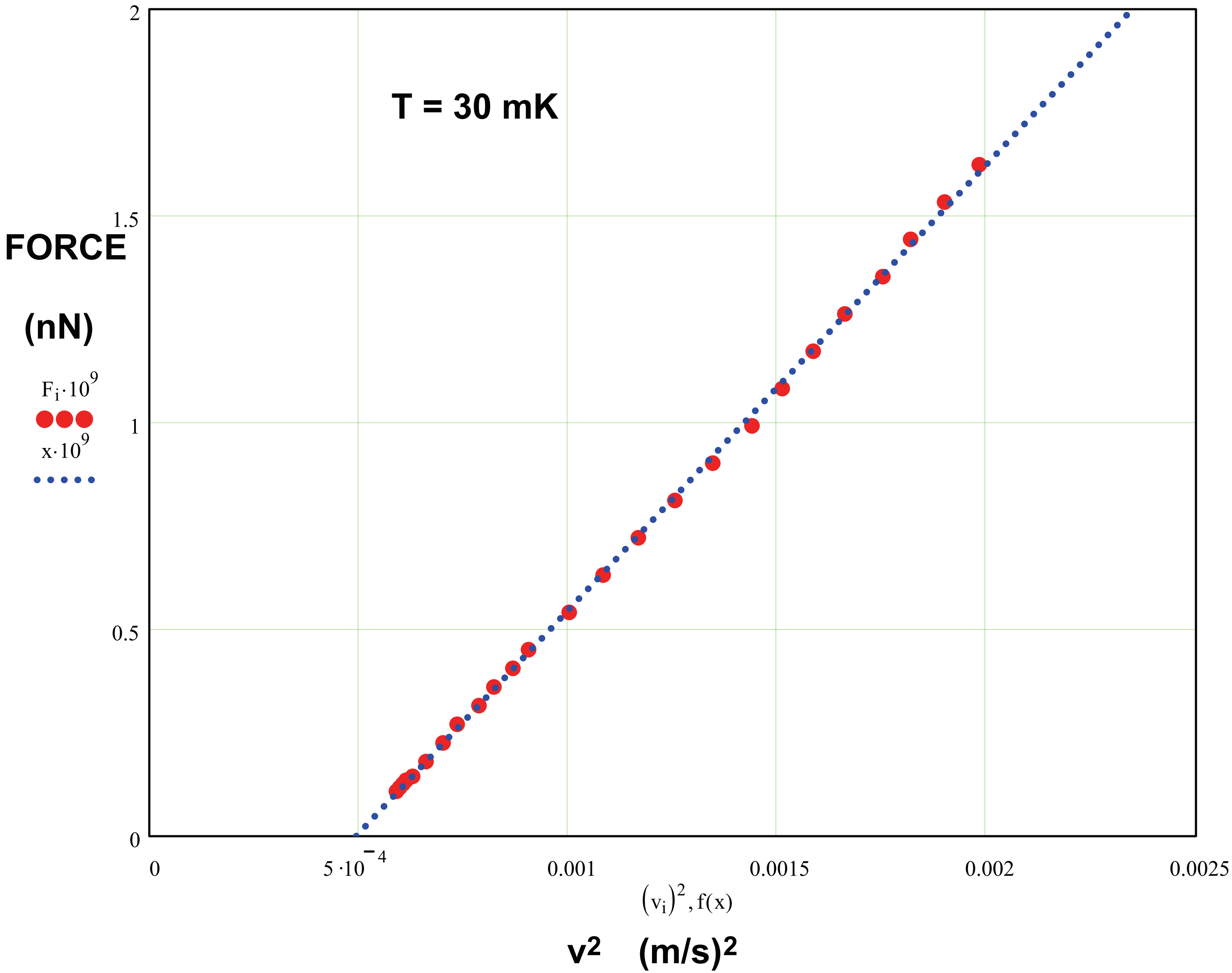


T = 1.9 K
velocity (mm/s)
force (nN)
0
10
20
30
40
50
60
70
80
90
0
1
2
3
4
5
6
7

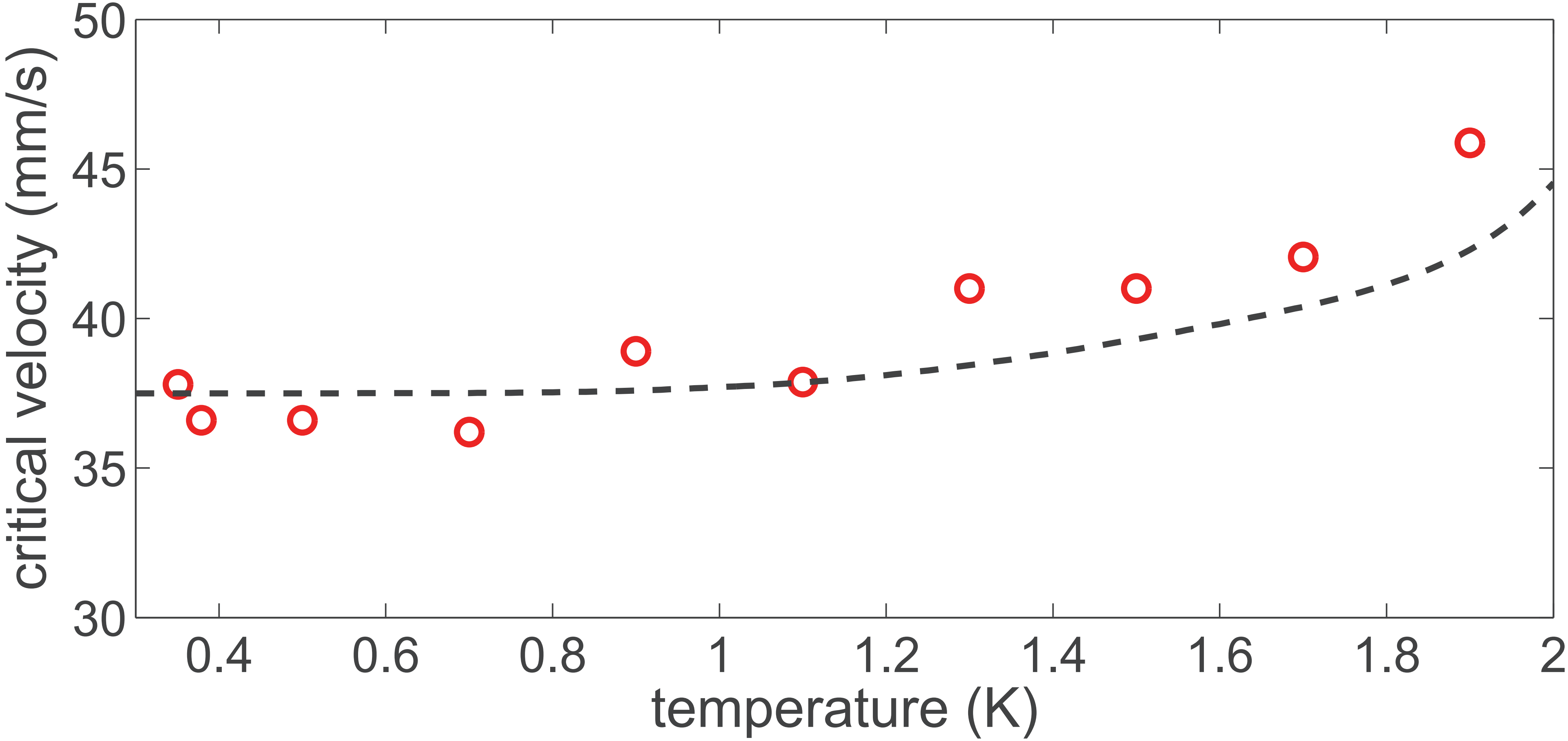

1.5 nN
T = 1.9 K
F turb
dF j
h(x)
0
F 0
(v2 j)2, x
(velocity)2



40
30
20
10
0
v (mm/s)
-400
-200
0
200
400
600
800
F (pN)
T = 300 mK

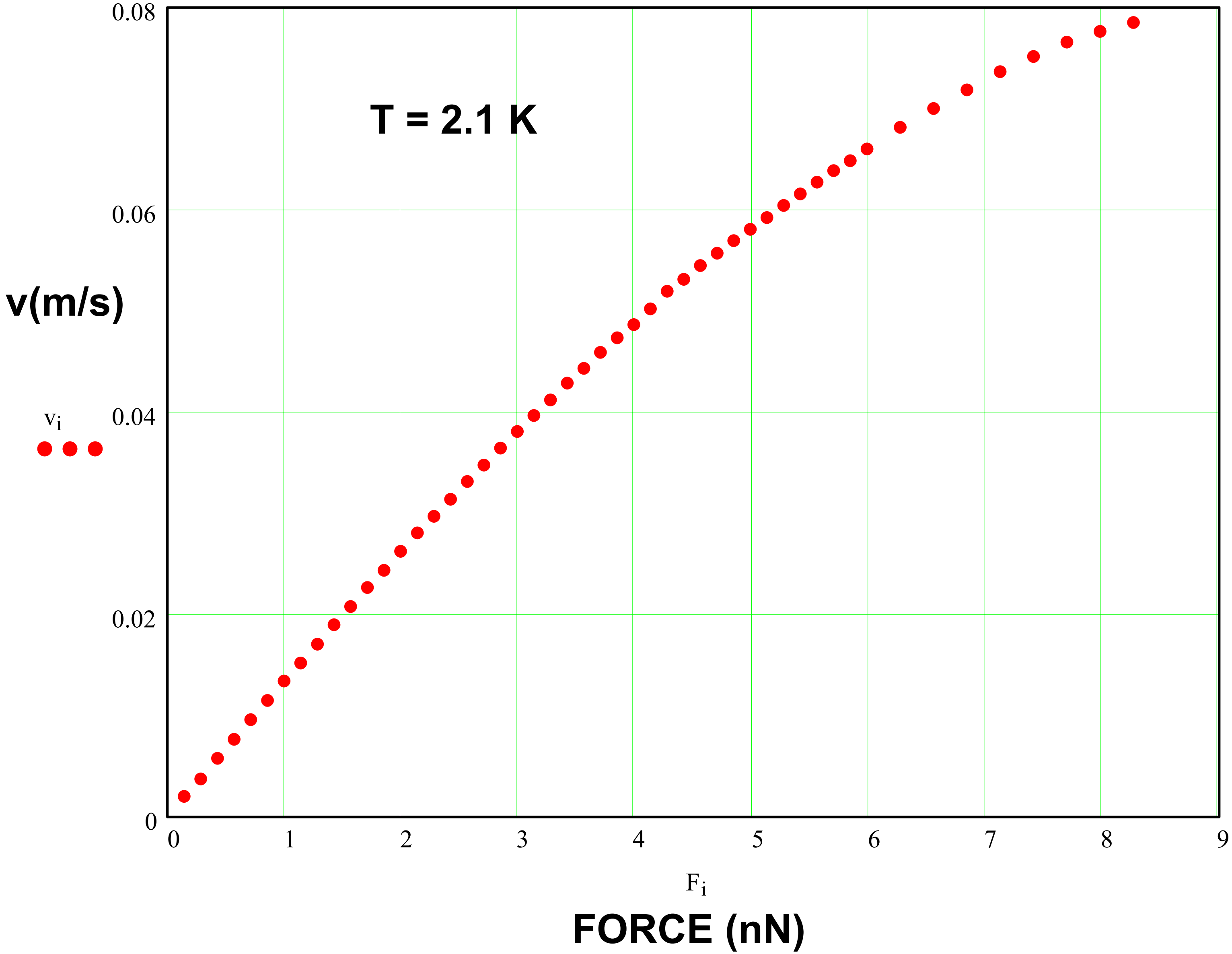


T = 30 mK
FORCE
(nN)
F_i·10^9
x·10^9
2
1.5
1
0.5
0
0
5·10^-4
0.001
0.0015
0.002
0.0025
(v_i)^2, f(x)
v^2 (m/s)^2

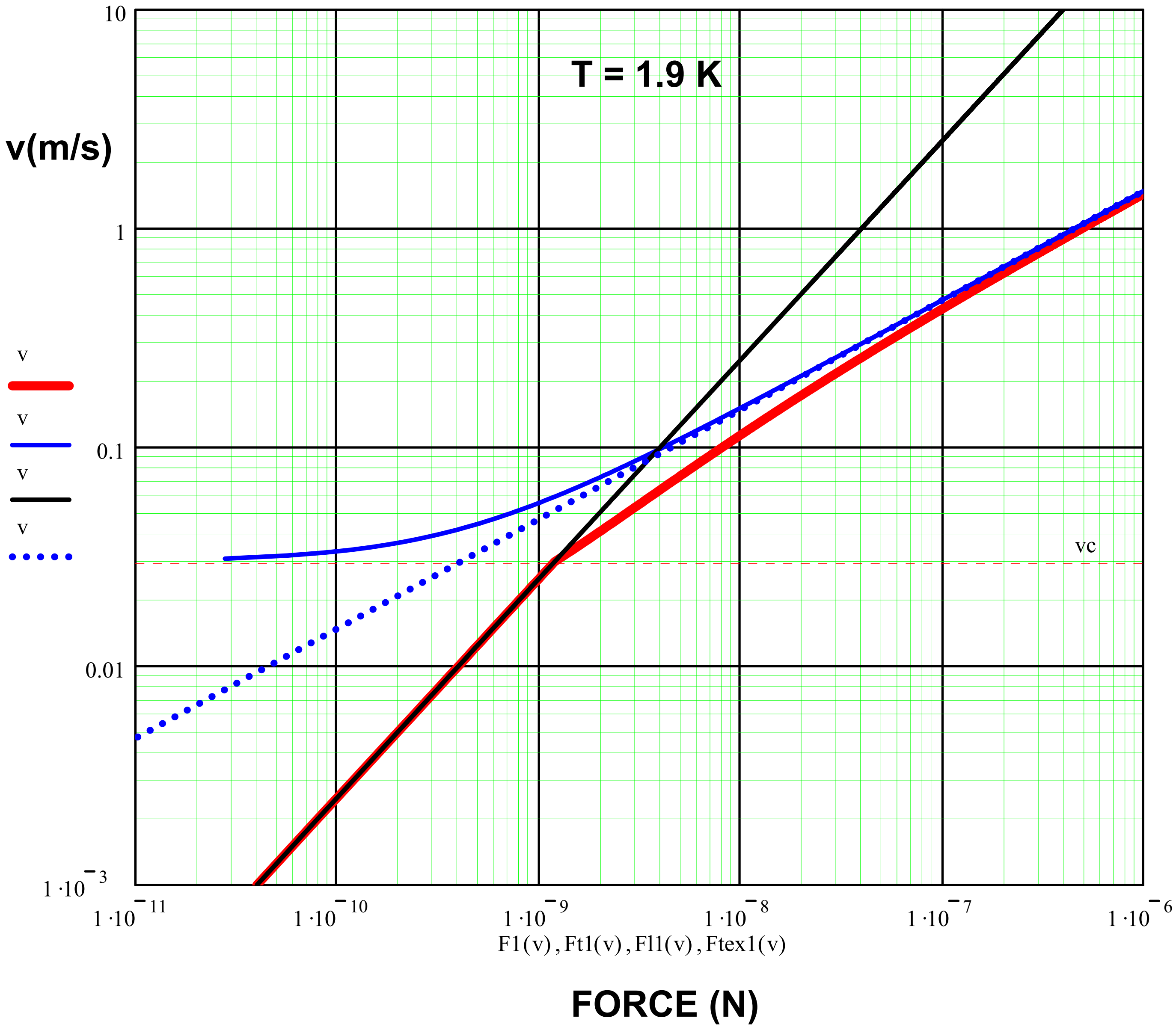


critical velocity (mm/s)
50
45
40
35
30
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
temperature (K)

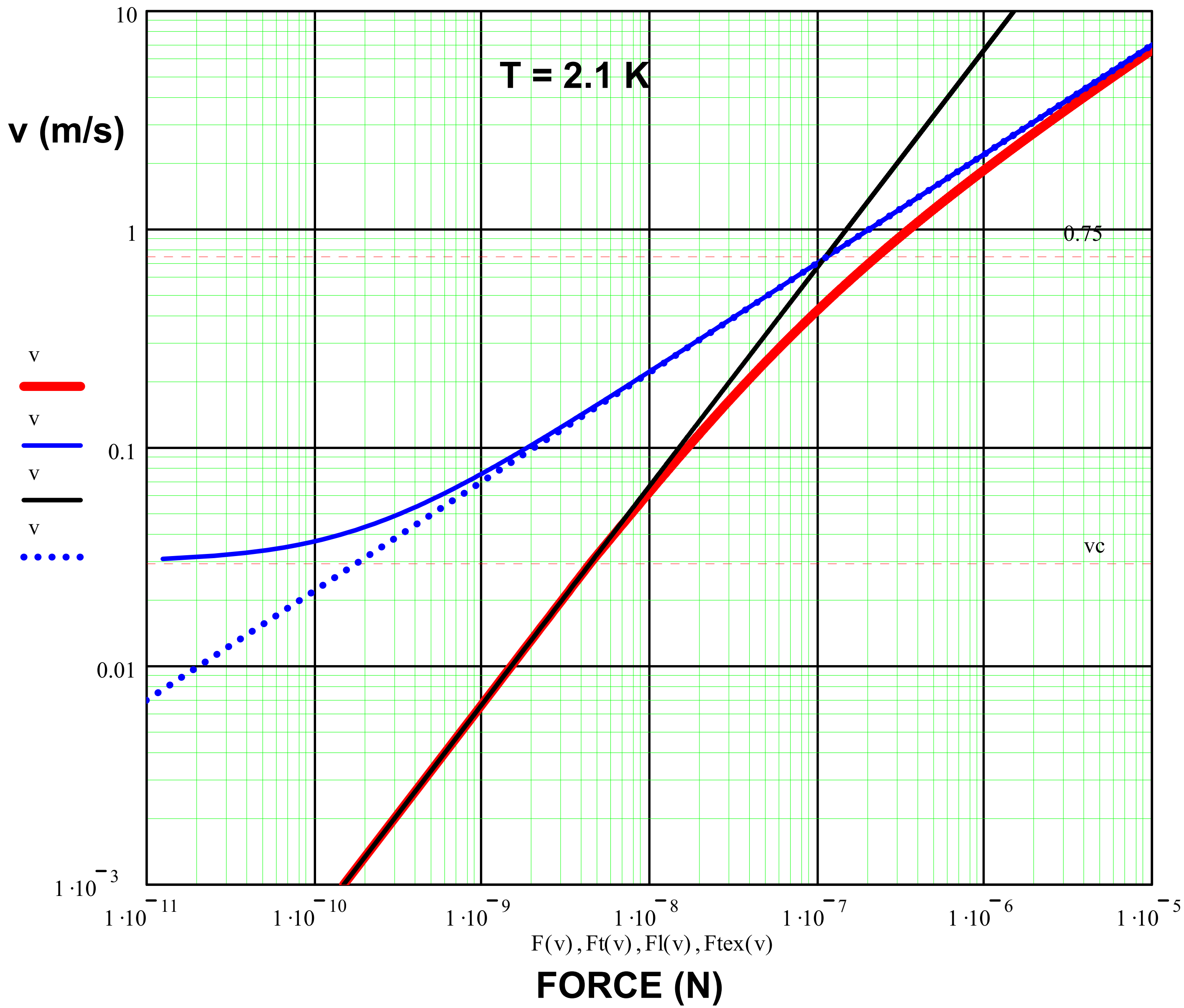
# Where is the critical velocity?

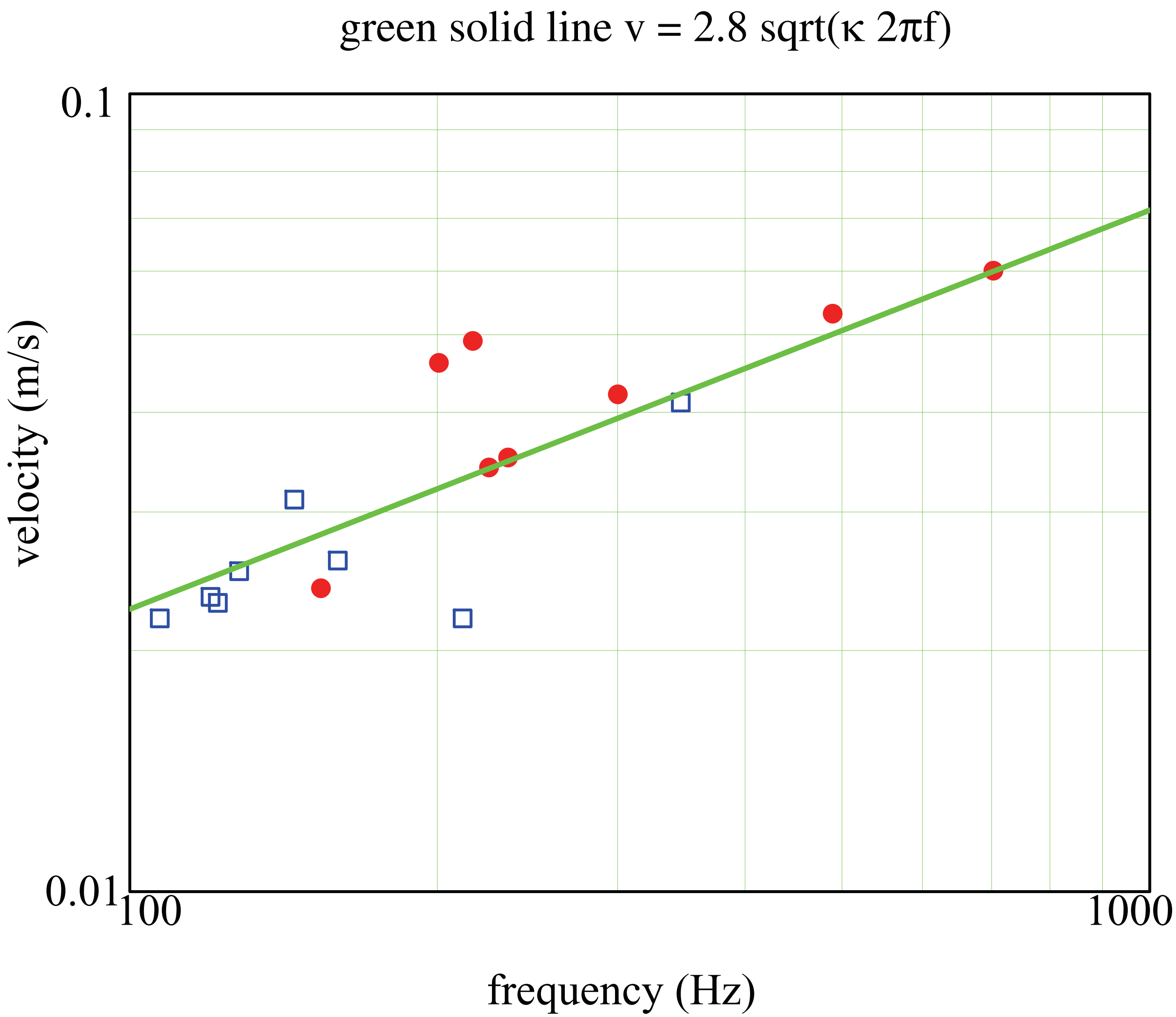


Discontinuity of the slope at the critical velocity: "kink"!



No kink! The critical velocity cannot be determined!





green solid line v = 2.8 sqrt(κ 2πf)
0.1
0.01
velocity (m/s)
100
1000
frequency (Hz)

## Theoretical Models

**1. Qualitative but very general:**

**the superfluid "Reynolds number" $Re_s = v\, l / \kappa$ is $>$1 for turbulence to exist,**

**assume that for amplitudes $a = v/\omega < 2R$ the characteristic length $l \sim a$, hence**

$$v \geq v_c \approx \sqrt{\kappa\omega}$$

**2. Compare time scale $1/\omega$ with relaxation time $\tau$ of the vorticity L:**

$$\dot{L} = \beta\left[v_s L^{3/2} - \kappa L^2\right]$$

**2 stable solutions:**

$$L = 0, \qquad L_{sat} = (v_s / \kappa)^2$$

**For $L_0 = L(0) << L_{sat}$ there is a simple approximation:**

$$L/L_{sat} = \left[ \left(L_{sat}/L_0\right)^{1/2} - (t/\tau) \right]^{-2}$$

**and finally for $L \rightarrow L_{sat}$ :**

$$L_{sat} - L \sim \exp(-t/\tau).$$

**with**

$$\tau = 2\kappa/\beta v_s^2 , \quad \beta = A(1-\alpha') - B\alpha, \quad A, B \sim 1.$$

$$(\tau = 2/\beta\kappa L_{sat})$$

**For turbulence to be fully developed in the oscillating flow field we need approximately**

$$\omega\tau \leq 1/4$$

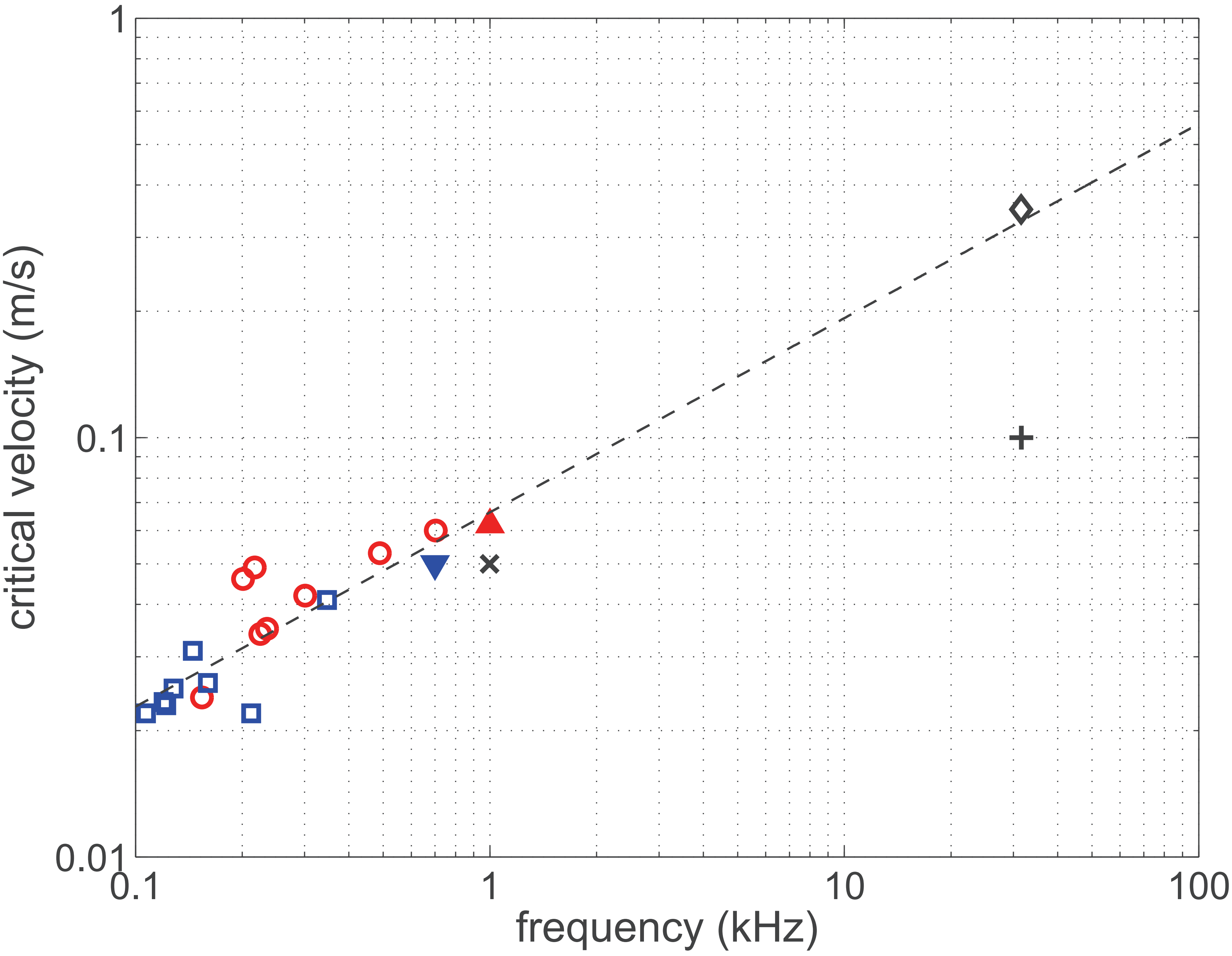
$$v_s > v_c \approx \sqrt{8\kappa\omega/\beta} = 2.8\sqrt{\kappa\omega/\beta}$$

$$v_c \sim \sqrt{\kappa\omega} \Rightarrow A_c = v_c/\omega \sim \sqrt{\kappa/\omega}$$

$$L_c = (v_c/\kappa)^2 = \omega/\kappa \Rightarrow \ell_c = 1/\sqrt{L_c} = \sqrt{\kappa/\omega}$$

$$\boxed{A_c \sim \ell_c}$$

**at the critical velocity: oscillation amplitude ~ vortex spacing !**



1
0.1
0.01
0.1
1
10
100
critical velocity (m/s)
frequency (kHz)

## Summary, conclusions and comments

1. The critical velocity is universal and depends only on the frequency and not on size or geometry of the oscillating object: $v_c \approx 2.6\ (\kappa\omega/\beta)^{1/2}$.

2. The superfluid turbulent drag force is $F_D = ½\ C_D\ \rho_s(T)\ \pi R^2\ (v^2 – v_0^2)$.

3. The critical velocity cannot be determined when there is no discontinuity of the slope of the v(F) curve at $v_c$ .

4. There are interesting analogies to both classical oscillatory and steady flow.