The Abdus Salam International Centre for Theoretical Physics

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IAEA International Atomic Energy Agency

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Workshop on Topics in Quantum Turbulence

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Measurement Methods of Vibrating Mechanical Objects Used for Generation and Detection of Quantum Turbulence

P. Skyba

Slovak Academy of Sciences
Inst. of Experimental Physics
Kosica
Slovak Republic

Notes on measurement methods of vibrating objects in low temperature physics

Peter Skyba

Talk outline:

Introduction

Voltage measurement (vibrating wires technique)

Current measurement (tuning forks, torsional oscillators, grids technique)

Conclusion

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Vibrating objects

Traditional resonators used: vibrating wires, spheres, grids, oscillating discs,..

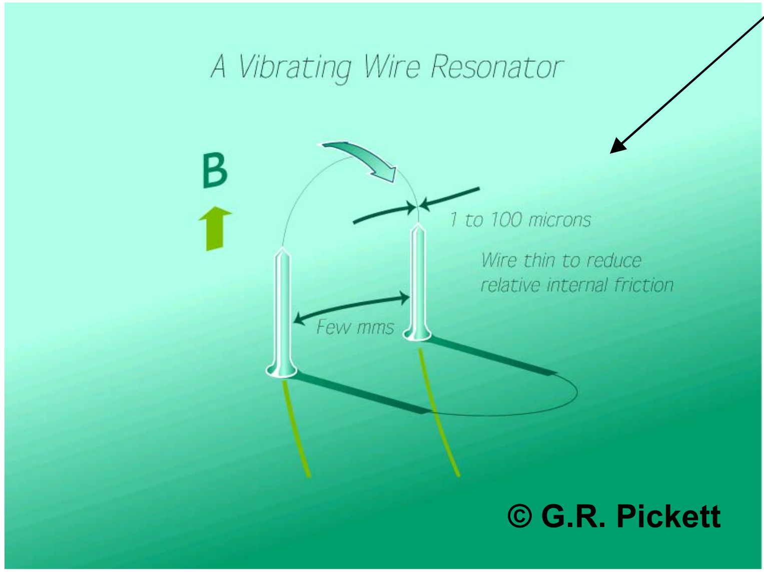
- resonators are immersed in liquid and bring into forced oscillations,
- properties of the liquids inferred from the resonance characteristics

Equation of motion:

$$\boxed{\ddot{x} + \gamma\dot{x} + \omega_0{}^2 x = f_0 \exp(i\omega t)}$$

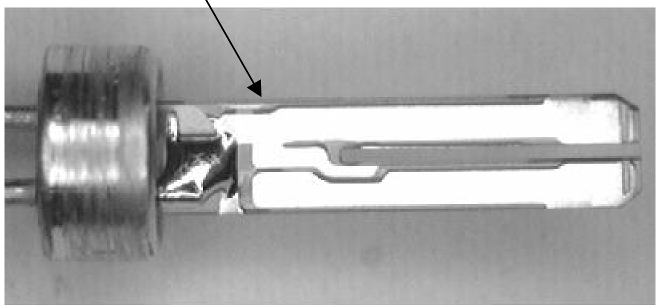
Physical quantity being measured - voltage or current.

Vibrating wire



S.N.Fisher et al., PRL 63 2566 (1989)

Tuning fork

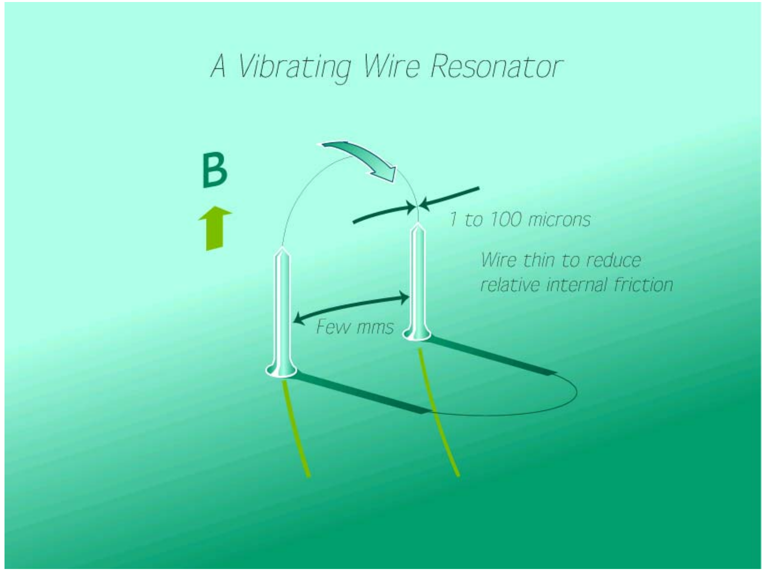


Dave Clubb et al., JLTP 136 Nos.1/2 (2004), 1
Rob Blaauwgeers et al., JLTP 146 Nos. 5/6 (2007)
plus many others

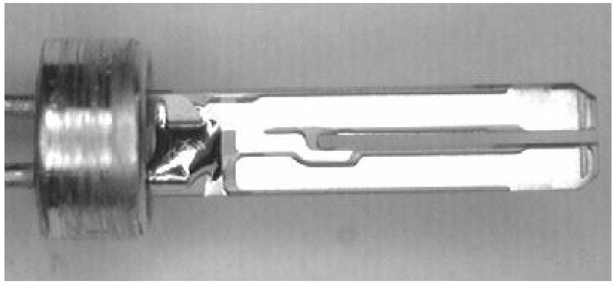
Mechanical resonators - basic properties

Equation of motion:

$$\boxed{\ddot{x} + \gamma\dot{x} + \omega_0^{\,2} x = f_0 \exp(i\omega t)}$$



tuning fork



$$\boxed{I(t) = \alpha \frac{dx(t)}{dt}}$$

α **–** is the constant

vibrating wire

$$\boxed{V_i \approx Bl \frac{dx(t)}{dt}}$$

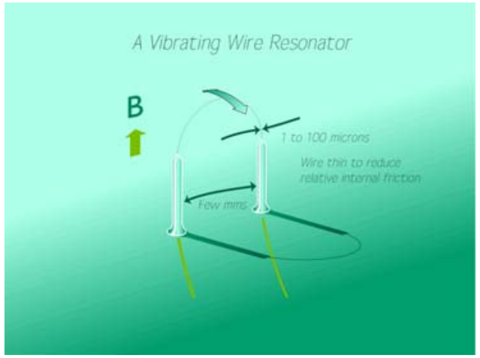
Velocity of the object:

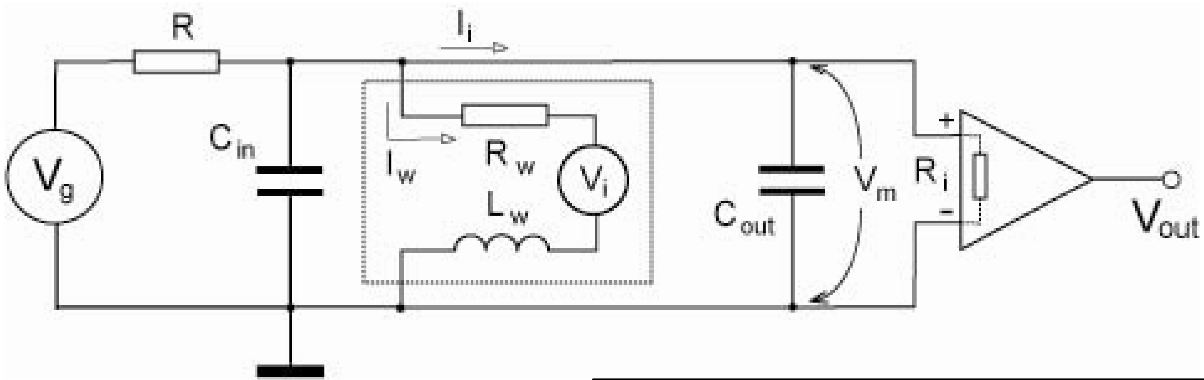
$$\boxed{v(\omega) = v(\omega_0)\frac{\gamma_2^2\omega^2 + i\omega(\omega_0^2 - \omega^2 - \gamma_1\omega)}{\left(\omega_0^2 - \omega^2 - \gamma_1\omega\right)^2 + \gamma_2^2\omega^2}}$$

Voltage measurements of vibrating wires - theory

Phase sensitive detection technique (lock in) in frequency range up to few kHz).

Typical experimental arrangement





$$V_{out} = K(\omega)V_i + V_{bgrd}$$

Analysis of the circuitry – superposition theorem

For the total voltage V_{out} being measured one can get (simplified form):

$$V_{out} = V_i\left(1 - \frac{i\omega L_w}{R'}\right) + V_g\left(\frac{R_w}{R} - \frac{i\omega L_w}{R}\right)$$

$$R' = \frac{R.R_i}{R + R_i}$$

$$V_i \approx Blv(\omega)$$

Transformers !

Voltage measurements of vibrating wires - experiment

Total measured voltage:

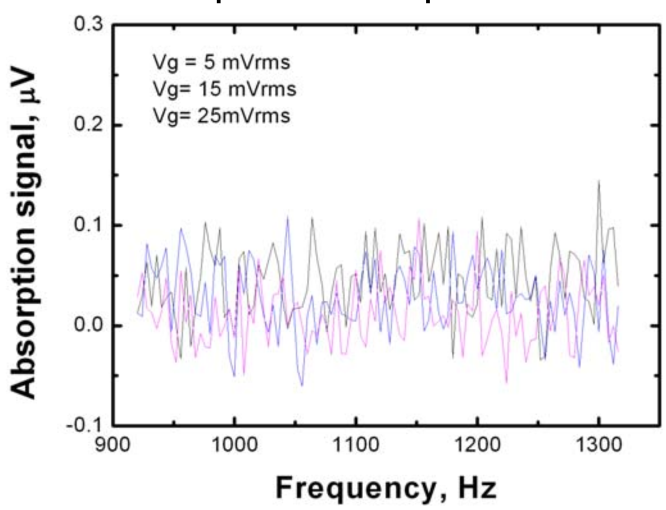
$$V_{out} = V_i\left(1 - \frac{i\omega L_w}{R'}\right) + V_g\left(\frac{R_w}{R} - \frac{i\omega L_w}{R}\right)$$

If B = 0 T, then V_i = 0 Volts, and V_{out} is:

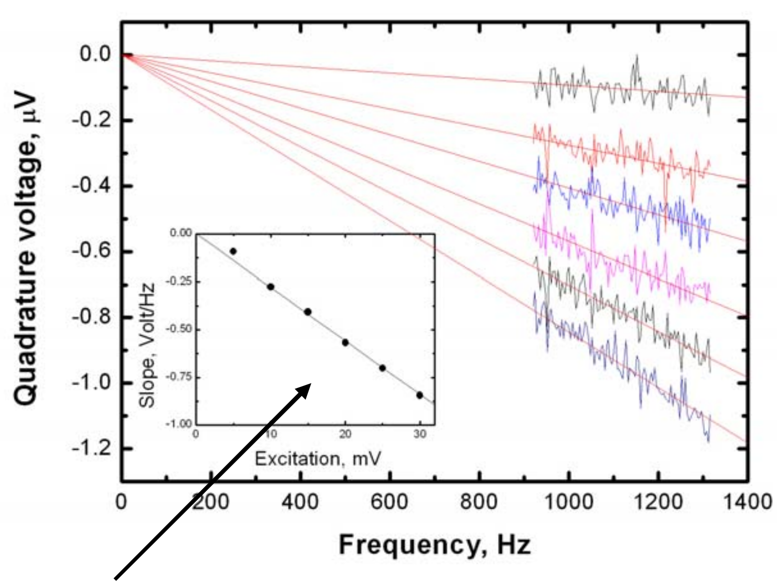
$$V_{out} = \frac{R_w}{R}V_g - \frac{i\omega L_w}{R}V_g$$

If R_w=0, then in phase component is zero

In phase component



Quadrature component



R_w and L_w of the wire can be measured.

Slope is L_w/R

Comment on L_w!

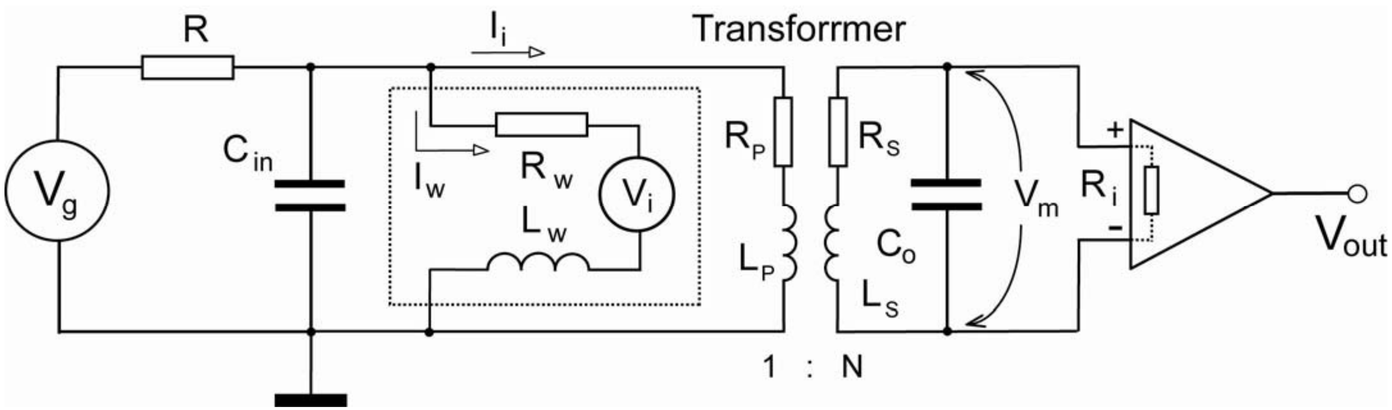
Vibrating wire with transformer – theory

As T→ 0K, in superfluid 3He-B dominant scattering process is Andreev reflection. Therefore, the wire (and any other mechanical resonator) is driven at low velocities.

$$\boxed{V_i \approx Blv}$$

This induced voltage becomes comparable with noise voltage of preamplifier as it operates at room temperature. Low temperature transformers are used to improve the signal to noise ratio.

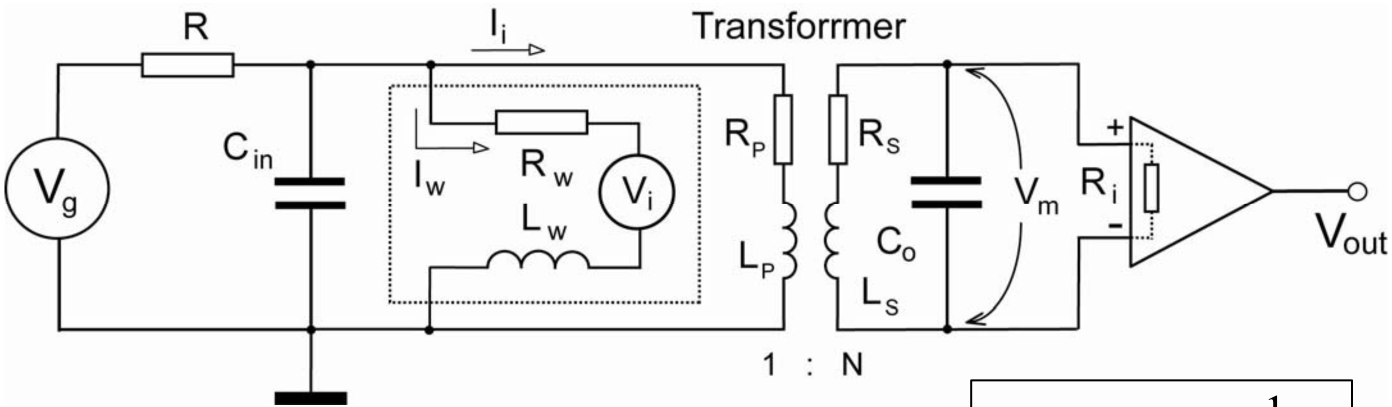
Typical experimental arrangement:



For transformer the “T” model has been used.

Vibrating wire with transformer – theory

Typical experimental arrangement:



Let assume that $R_i >> R_s$, R_i >>abs(iωM) and

$$\omega_0 << \omega_{ro} = \frac{1}{\sqrt{L_s C_o}}$$

Superposition theorem:

$$V_{out} = V_{bgrd} + k(\omega) V_i$$

$$V_{bgrd} = N \frac{V_g}{RR_p}\left[R_w R_p + \omega^2 L_w (L_p + M) - i\omega L_w R_p\right]$$

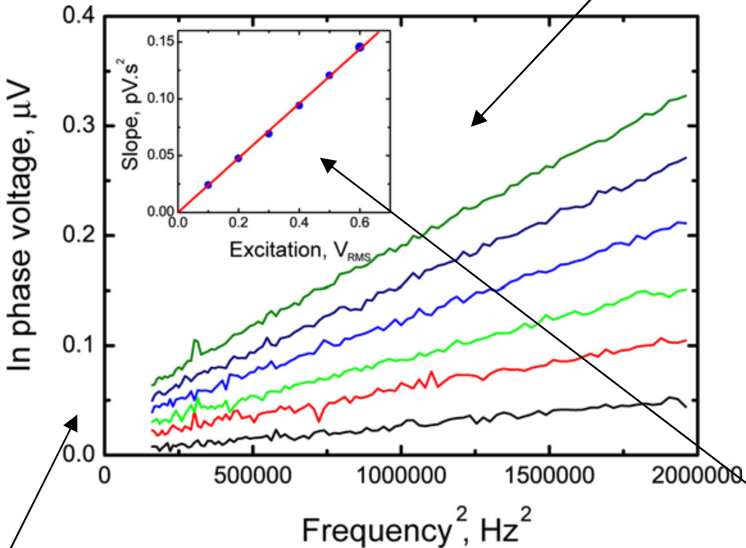
$$V_{sig} = N V_i \left[1 - \frac{i\omega^3 R (L_p + M)^2 L_w}{(RR_p)^2 + \omega^2 R^2 (L_p + M)^2}\right]$$

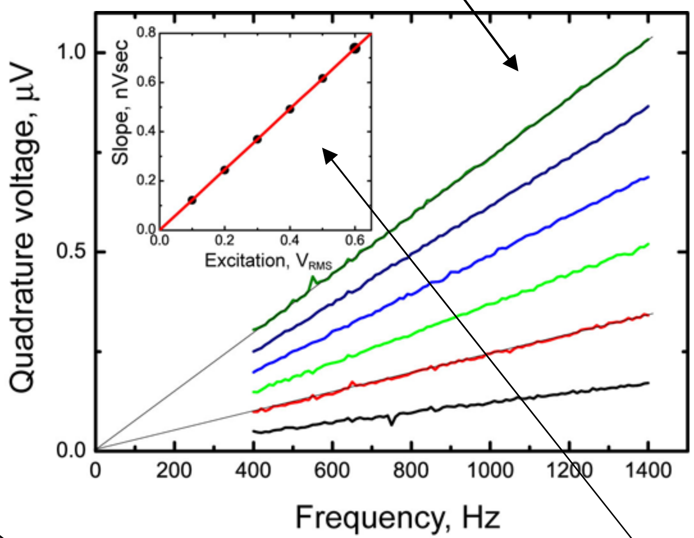
Higher transformer gain leads to a nontrivial amplitude-frequency response.

Vibrating wire with transformer – experiment

Measurement performed on wire of diameter 4.5 μm having f_0 = 700 Hz.
Transformer with gain N=30 and B=0Tesla.

$$\boxed{V_{bgrd} = N\frac{V_g}{RR_p}\left[R_w R_p + \omega^2 L_w (L_p + M) - i\omega L_w R_p\right]}$$

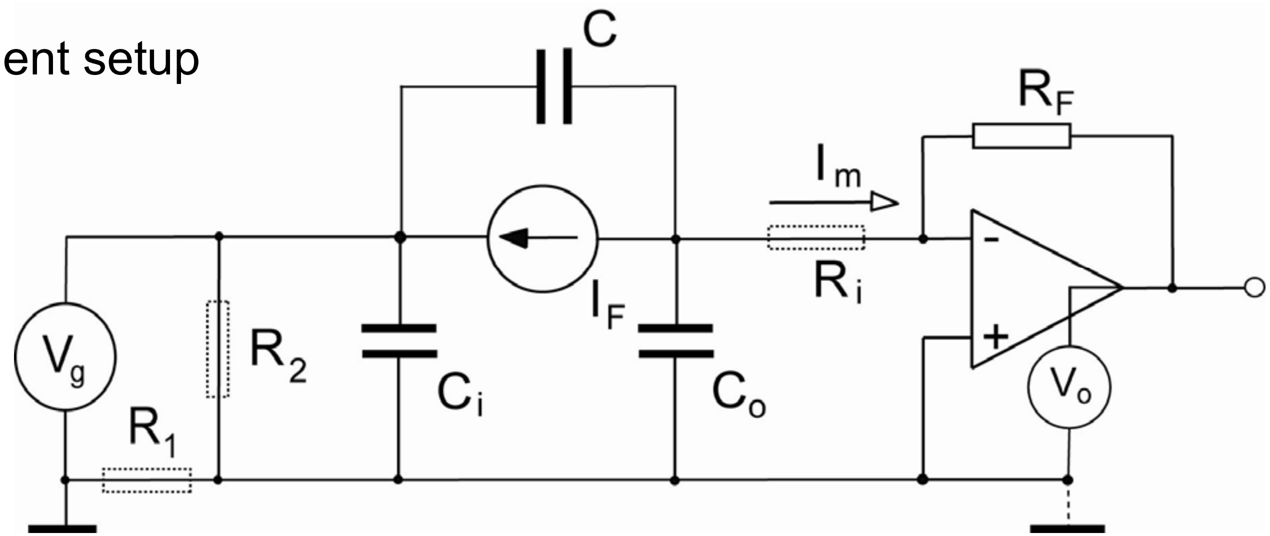




Allows determine R_w and slope determines $L_w(L_p+M)/RR_p$ Allows to determine L_w

Current measurements – tuning forks

Typical measurement setup



$$I_F(t) = \alpha \frac{dx(t)}{dt}$$

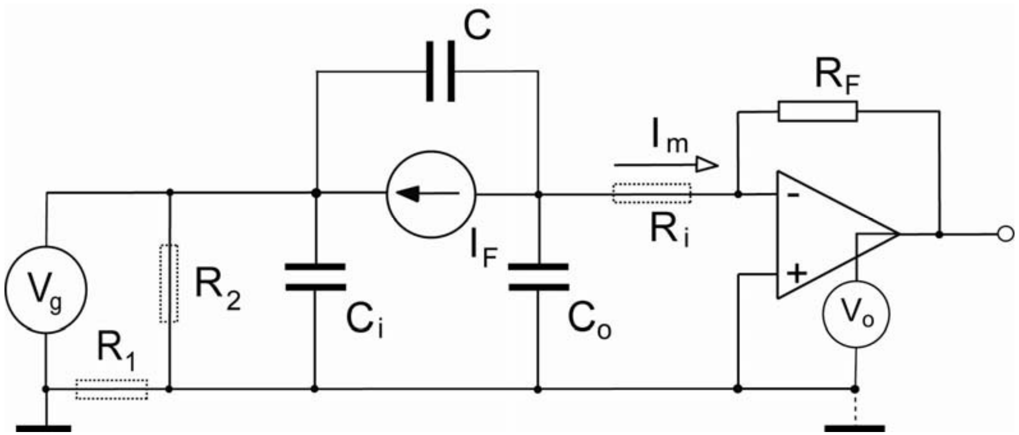
$\longleftarrow$

$$v(\omega) = v(\omega_0) \frac{\gamma_2^2\omega^2 + i\omega(\omega_0^2 - \omega^2 - \gamma_1\omega)}{\left(\omega_0^2 - \omega^2 - \gamma_1\omega\right)^2 + \gamma_2^2\omega^2}$$

Superposition theorem:

$$V_0 = V_{bgrd}(V_g) + Z(\omega) I_F$$

Current measurements – tuning forks



Bgrd voltage:

$$V_{bgrd} = -\frac{Z_2}{R_1 + Z_2}\frac{1}{1+(\omega C_0 R_i)^2}\left[\omega^2 R_F R_i C C_0 - \frac{R_1}{Z_2} + ik\omega R_F C\right].V_g$$

$$Z_2 = \frac{R_2}{1+i\omega R_2 C_i}$$

$$k = 1 - \frac{R1 C_0}{Z_2 C}$$

Signal voltage:

$$V_{sig} = -\frac{1-i\omega C_0 R_i}{1+(\omega R_i C_0)^2} R_F I_F(\omega)$$

Current measurements – tuning forks - experiment

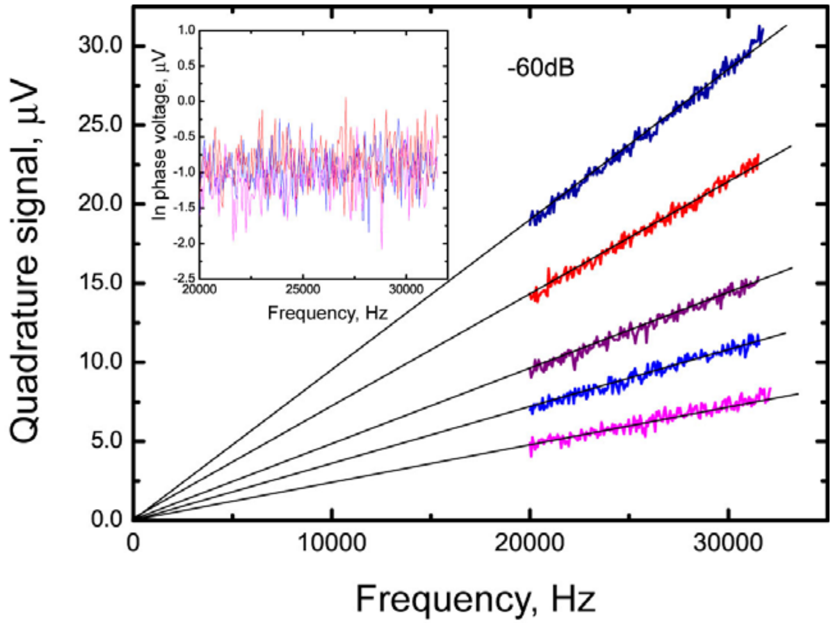
Let discuss first influence of the background voltage

$$V_{bgrd} = -\frac{Z_2}{R_1 + Z_2} \frac{1}{1 + (\omega C_0 R_i)^2} \left[\omega^2 R_F R_i C C_0 - \frac{R_1}{Z_2} + ik\omega R_F C \right] . V_g$$

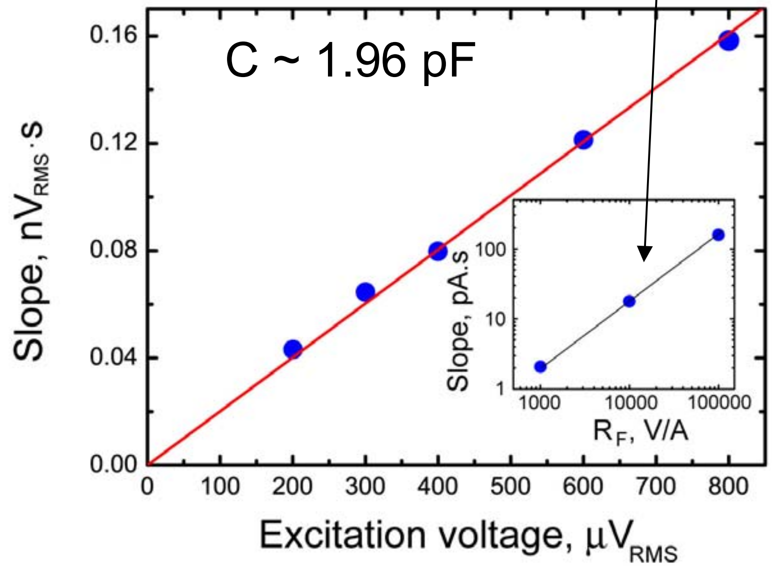
Proper I/V converter $R_i \rightarrow 0\Omega$ and $R_1 \rightarrow 0\Omega$, then

$$\boxed{V_{bgrd} = -i\omega C R_F V_g}$$

Measurements in vacuum @ T ~1K



Various values of R_F

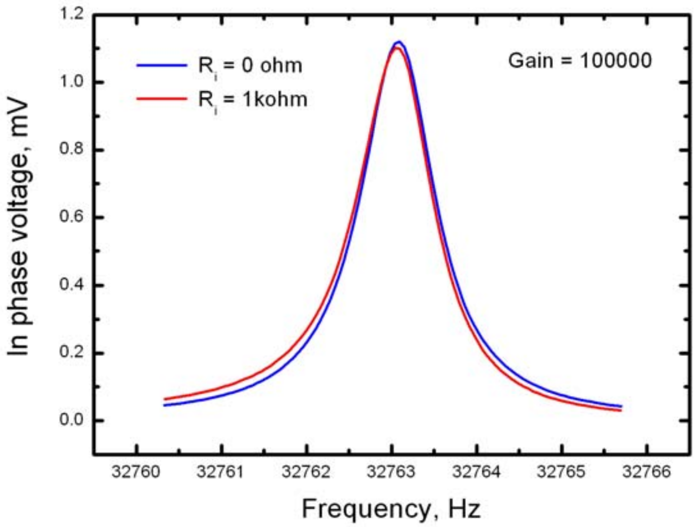


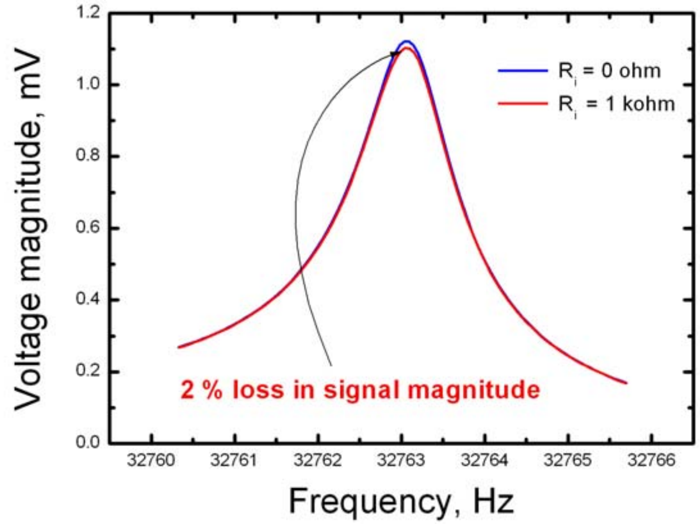
Current measurements – tuning forks - experiment

Effect of non zero input resistance ($R_i \neq 0\Omega$) of an I/V converter on measurements

$$V_{sig} = -\frac{1 - i\omega C_0 R_i}{1 + (\omega R_i C_0)^2} R_F I_F(\omega)$$

Amplitude and phase change due to non-zero R_i

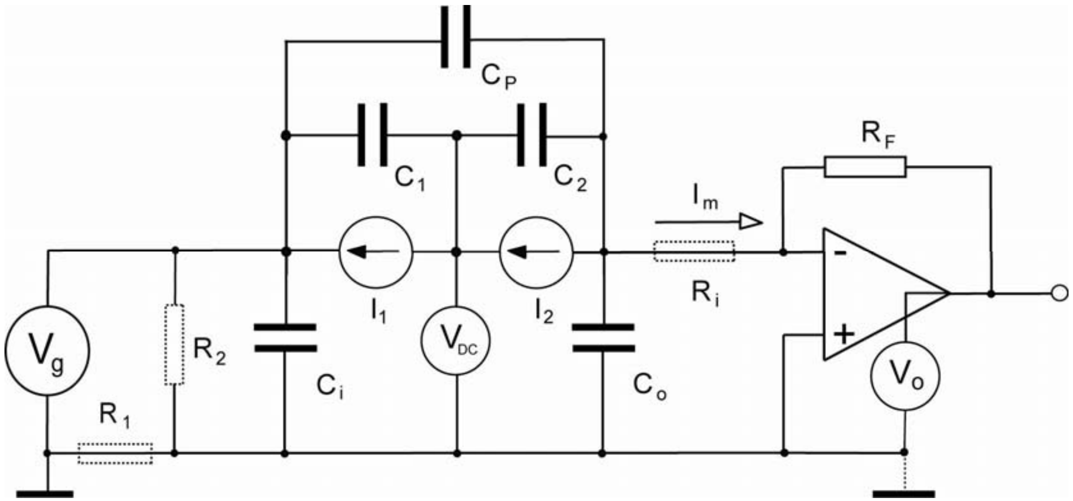




Nearly all commercial available lock-in amplifiers having I/V converters have Ri ≠ 0Ω

Current measurements – grids and torsional oscillators

Typical measurement setup



$$I_i(t) = V_{DC}\frac{dC_i}{dt} = -V_{DC}\frac{C_i}{d_i}v_i$$

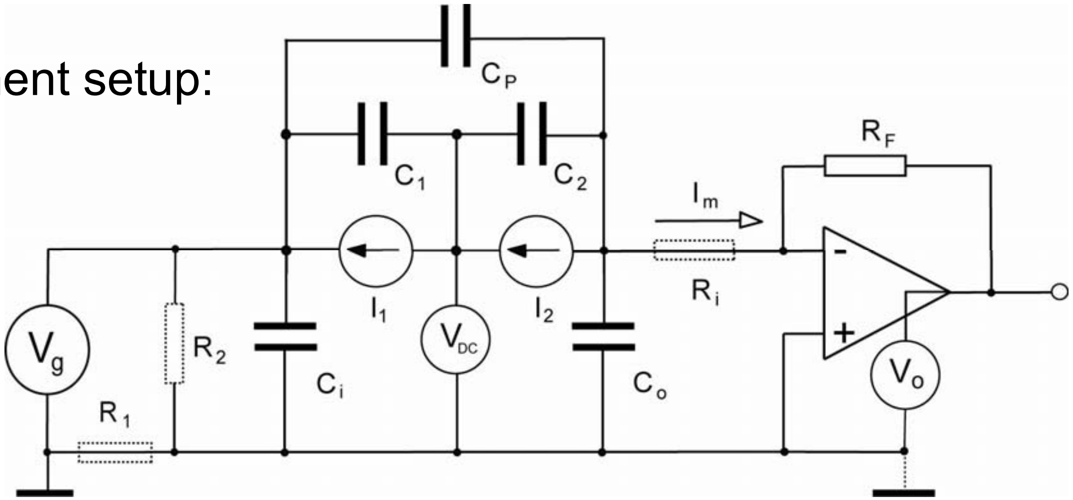
$$v_i(\omega) = v_i(\omega_0)\frac{\gamma_2^2\omega^2 + i\omega(\omega_0^2 - \omega^2 - \gamma_1\omega)}{\left(\omega_0^2 - \omega^2 - \gamma_1\omega\right)^2 + \gamma_2^2\omega^2}$$

Superposition theorem (I_1 is not contributed to V_0):

$$V_0 = V_{bgrd}(V_g) + Z(\omega)I_2$$

Current measurements – grids and torsional oscillators

Grid measurement setup:



$$V_0 = V_{bgrd}(V_g) + Z(\omega) I_2$$

$$V_{bgrd} = -\frac{Z_2}{R_1 + Z_2} \frac{1}{1 + (\omega C_0 R_i)^2} \left[\omega^2 R_F R_i C_P C_0 - \frac{R_1}{Z_2} + ik\omega R_F C_P \right] . V_g$$

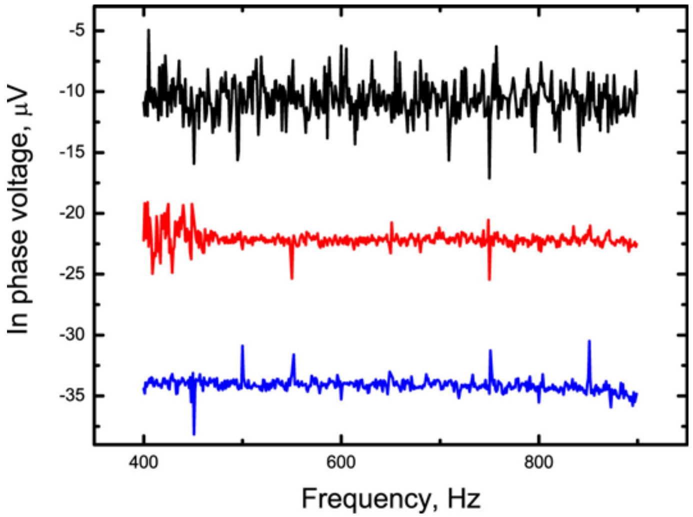
$$V_{sig} = -\frac{1 - i\omega C_0 R_i}{1 + (\omega R_i C_0)^2} R_F I_2(\omega) \longleftarrow I_2(t) = V_{DC} \frac{dC_2}{dt}$$

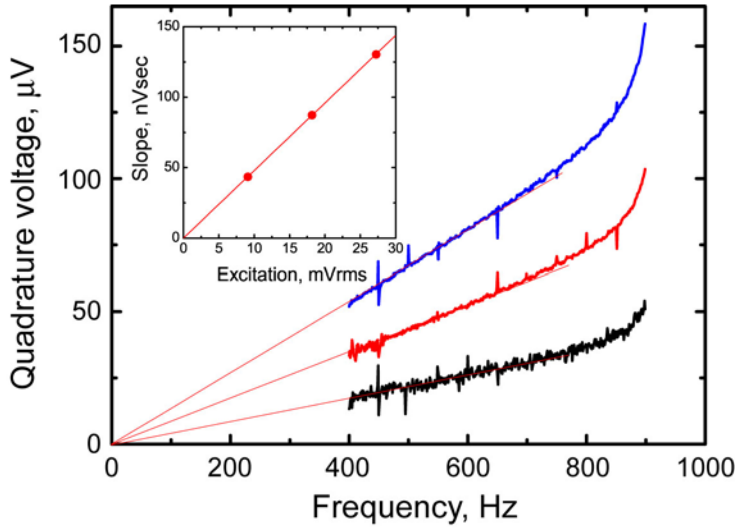
Current measurements – grids - experimental

Measurements performed using a Lancaster grid at f_0 ~ 1kHz and Q ~ 10^5

Proper I/V converter having $R_i \rightarrow 0\Omega$, then V_{bgrd} can be expressed as:

$$V_{bgrd} = -\frac{Z_2}{R_1 + Z_2}\left[-\frac{R_1}{Z_2} + i\omega R_F C_P\right].V_g$$





Allows ratio $R_1/(R_1+Z_2)$ to be determined experimentally ~ 0.0015

Allows C_P to be measured C_P 7.7 pF

Problem with ground!

Conclusion

Voltage measurements on vibrating wires:
- easy to measure and determine the background when B=0T ;
- direct wire measurements – an agreement between the model and experiment;
- transformers improve signal to noise ratio, but they change the frequency dependence of background (can be measured) and reduce input impedance of the preamplifier (R_i/N^2);
- higher N results in nontrivial (resonance) response of the background voltage;
- Instead of using R to determine current – use proper V/I converter.

Current measurement tuning forks, grids and torsional oscillators:
- impossibility to measure pure background like for vw's;
- non-zero input impedance of the I/V converter affects precision of measurements;
- ground problems – resistive attenuators dividers followed by a voltage follower providing impedance insulation and proper voltage source;

Acknowledgement

Acknowledgement to lads who shared their experience of using these devices with me:

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Thank’s for your attention.