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Theory of Vibrating Structures

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Theory of vibrating structures

The onset of turbulence in the flow of superfluid ^4He past an obstacle

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Introduction I

- The title of this review is misleading, because we have no widely accepted theory of the generation of turbulence by a vibrating structure in a superfluid.
- Why do people spend time on experiments on vibrating structures? One answer is that they are fairly easy to do, in comparison with the study of steady flow past an obstacle. But they are harder to interpret, as is the case with the corresponding experiments with classical fluids. In classical fluids what is happening seems often to be very complicated, and detailed theories are difficult to develop. Virtually nothing would be known about the classical cases without the help of visualization. In the superfluid case we have as yet no visualization; all we have are experimental studies of the dependence of drag on velocity.
- So how can we go about trying to understand the experimental results in the quantum case? We must clarify what we are trying to explain. We must ask what features of the drag *versus* velocity relation do we wish to understand?
- Here we encounter a difficulty: we can identify what appears to be an important and significant feature in the drag curve for a particular structure, only to discover that that it is not reproducible in another structure, even when the two structures seem to be essentially identical.

Introduction II

- Here I shall made an admission. I shall allow myself to be guided by features of the experimental results that are not always present. You may object to this approach, saying that I am choosing to accept experimental results that fit in to my preconceived ideas, and to reject those that do not.

- However, I shall take the view that significant features can appear if the conditions are favourable, and that we should take advantage of these favourable conditions in guiding us towards a general understanding. These is some danger in doing this, but it seems to me that otherwise we can make no progress. Of course, I take the view also that we ought then to try to find out experimentally why conditions are favourable sometimes, but not always. Indeed one important message emerges from my approach: the need for more experiments, especially more systematic experiments covering wide ranges of variable with different well-characterized structures.

Classical dimensionless parameters

- Steady flow is characterized by a single dimensionless parameter: the Reynolds number $\mathrm{Re} = DU/\nu$ *U*=characteristic velocity; *D* characteristic dimension; ν= kinematic viscosity

- Oscillating flows require two dimensionless numbers: for example
  - the Keulegan-Carpenter number $K_C = \frac{2\pi a}{D}$ *a*=amplitude of oscillation
  - the Stokes number $\beta = \frac{\omega D^2}{2\pi\nu}$ ω=angular frequency

The dimensionless drag and inertia coefficients

- As in the case of the corresponding classical experiments the force F on the oscillating structure can be written in terms of the velocity U as

$$F = \frac{1}{2}\rho A C_D |U| U + \rho V C_M \frac{dU}{dt}$$

A is the projected area of the structure; V is its volume; ρ is the total fluid density;

C_D is the dimensionless “drag coefficient”;

C_M is the dimensionless “inertia coefficient”, which determines the added effective mass.

- Classical experiments are usually carried out with an imposed sinusoidal velocity $U = U\exp(-i\omega t)$; then we write

$$F = \frac{1}{2}\rho A C_D |U| U + i\omega\rho V C_M U$$

- In experiments on superfluids there is usually an imposed sinusoidal force, but we shall assume that the same formula can be used.
- Measurements of C_M in superfluids often produce strange results, which we shall ignore, pending more systematic experiments.

Observed drag coefficients in classical fluids

- In the simplest cases this seems to have the form

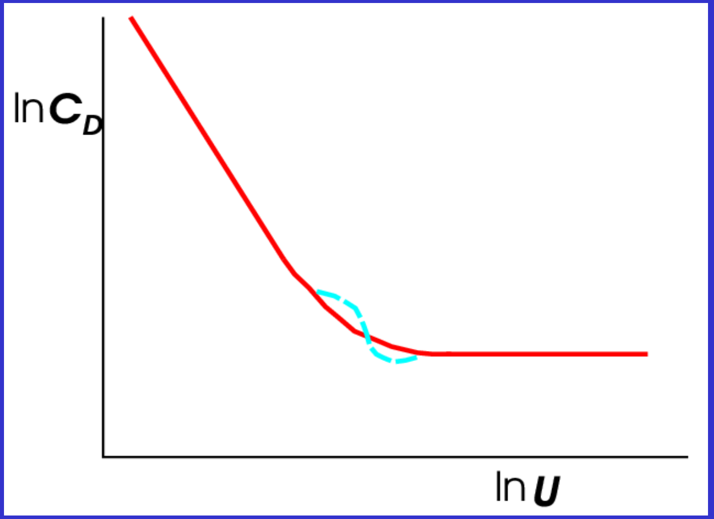
$$C_D = 2\alpha \frac{S}{A}(\omega\nu)^{1/2}\frac{1}{U} + \gamma$$

or

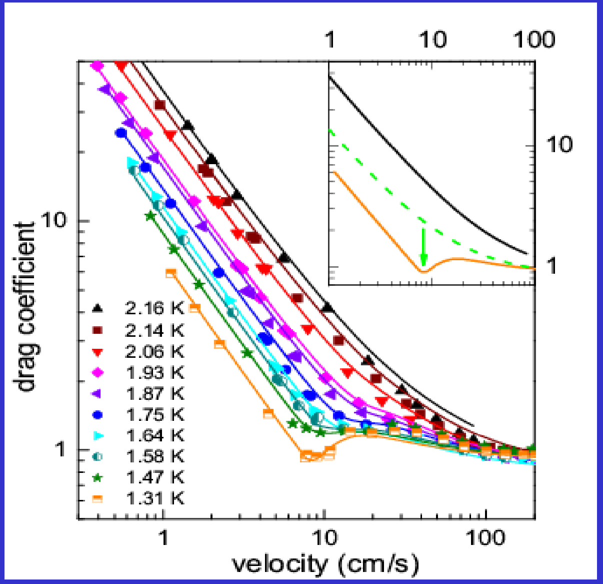
$$C_D = 2(2\pi)^{1/2}\alpha \frac{S}{A}\frac{1}{\beta^{1/2}}\frac{1}{K_C} + \gamma$$

where α is a dimensionless parameter depending on the shape of the structure;

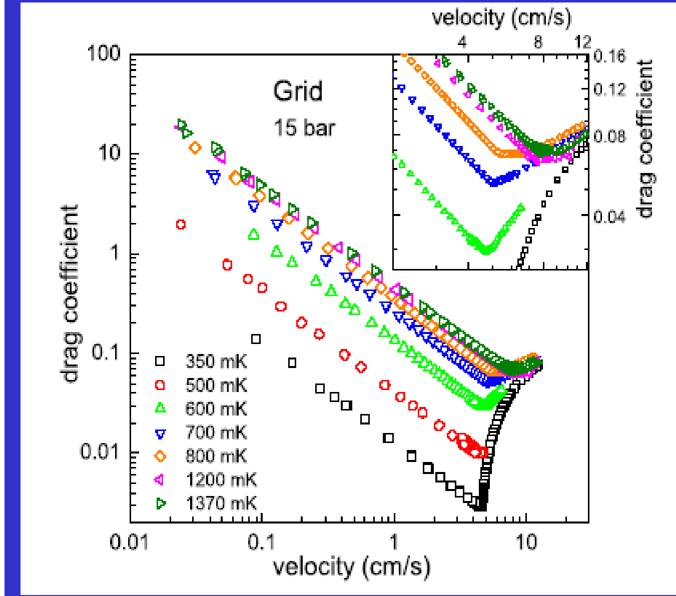
S is the surface area of the structure; and $\gamma \sim 1$.

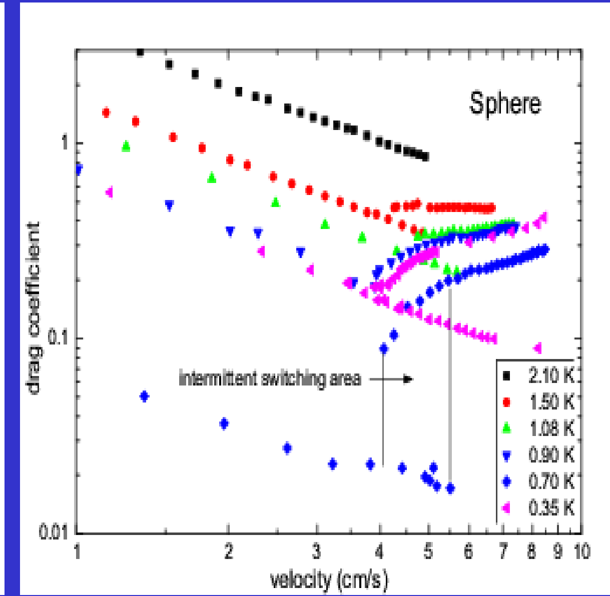


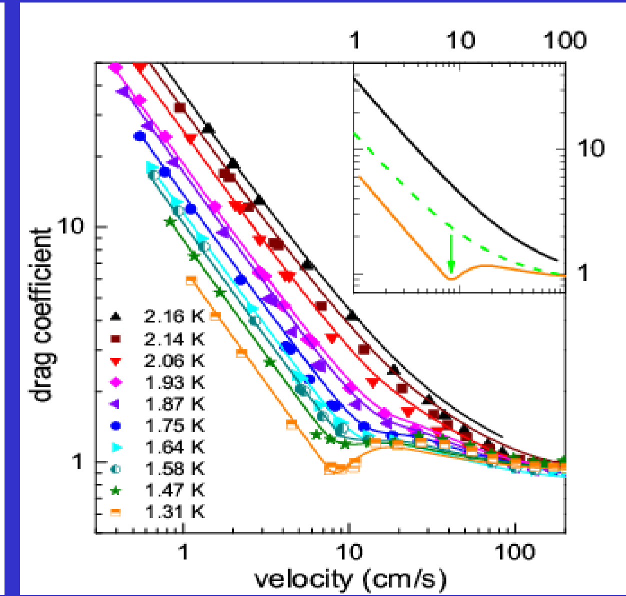
- Studies in normal ^{4}He with a vibrating fork support the simple formula.

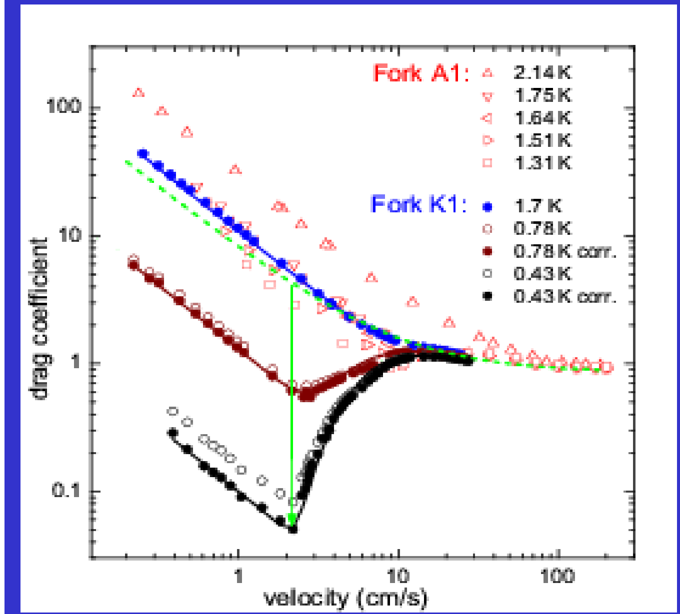


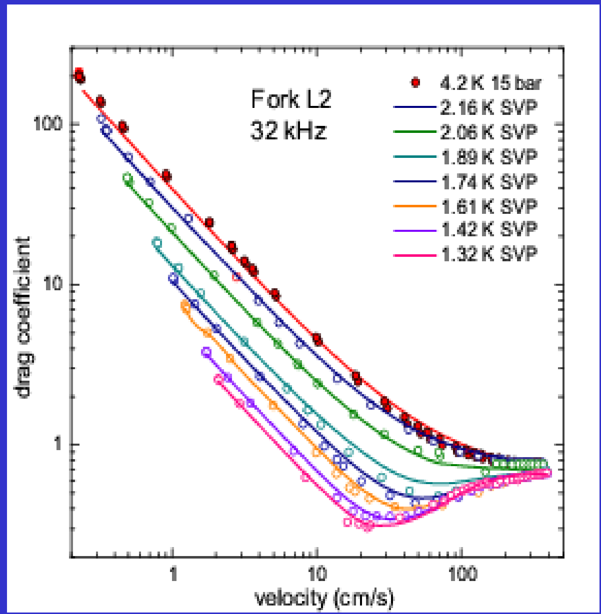
Observed drag coefficients in superfluid ^4He (Skrbek review)

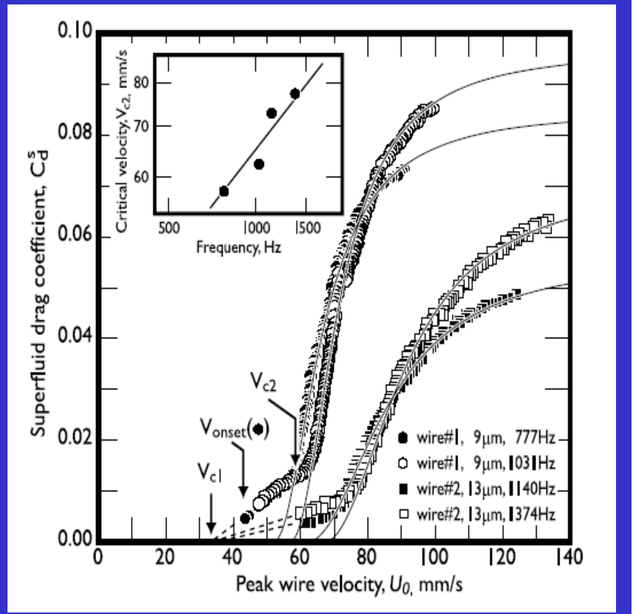












Comments on the observed drag coefficients I

- All have certain common features:
  - A region at low velocities where $C_D \sim 1/U$: due to laminar flow of normal fluid or ballistic scattering of normal-fluid excitations or nuisance damping; with potential flow of the superfluid component.
  - A sharp minimum, interpreted as a sharp superfluid critical velocity
  - A tendency for C_D to level off to a constant value ≤ 1 at high velocities: but often the data do not go to high enough velocities to be sure of the limiting behaviour at high velocity.
- A potentially important feature that may or may not be present
  - A region between the superfluid critical velocity and the limiting high-velocity behaviour where C_D goes through a maximum.
  - Some data show this maximum very clearly; others do not.
  - Whether the maximum does or does not exist seems not to be correlated with the type of structure.
- Hints from the vibrating wires and grids at very low temps that there can be slightly increased dissipation even below the superfluid critical velocity

Comments on the observed drag coefficients II

- ❑ Some evidence that the critical superfluid velocity is close to the velocity at which normal helium ($\nu \sim 10^{-8}$ m^2s^{-1}) starts to display a classical instability.
  - Examination of this classical instability for a variety of structures shows that it is associated with a critical Keulegan-Carpenter number that varies with the Stokes number as shown by the solid line, if $\nu \sim 10^{-8}$ m^2s^{-1}.
  - We see that observed critical superfluid velocities fit this curve rather well.
  - Perhaps this is an accident. Later we speculate that it is not an accident, and we shall discuss the possible significance of the value $\nu \sim 10^{-8}$ m^2s^{-1}.

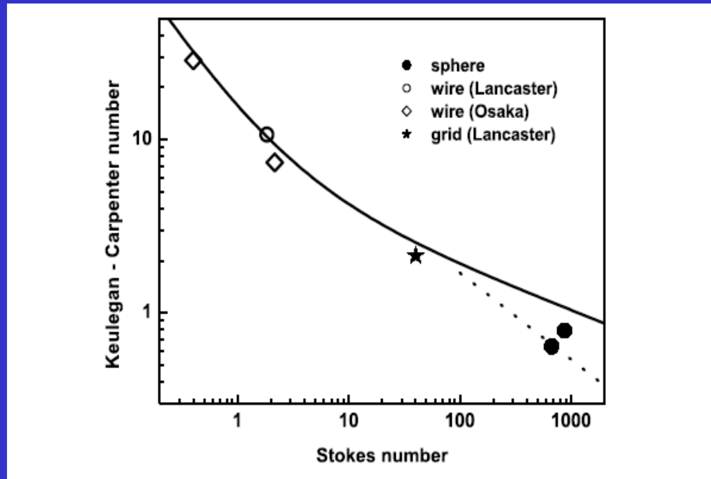
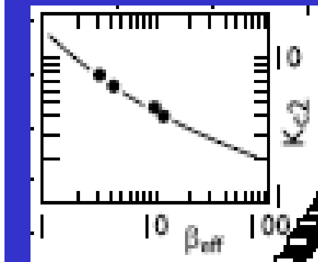


Fig. 23. A comparison of the observed critical values of K_C and β with those expected classically (eq. 11 and eq. 15), with an effective kinematic viscosity given by $\zeta = 0.11$.



Wires ↑

Bradley *et al*

- ❑ The observed frequency-dependence of the superfluid critical velocity?
  - Is it $\omega^{1/2}$? Above graph is consistent with its being close to (but not exactly) $\omega^{1/2}$. But perhaps not a universal prefactor.
- ❑ Switching and hysteresis.

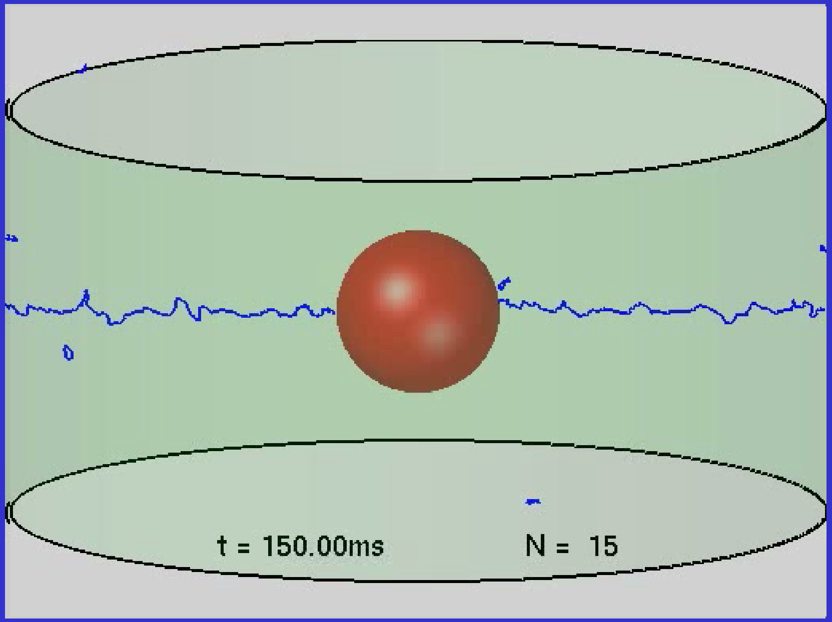
General comments on the theory of vibrating structures in ^{4}He

- What are trying to explain?
  - The existence, magnitude and frequency-dependence of the superfluid critical velocity.
  - Equally importantly, the form of the drag coefficient *versus* velocity at velocities exceeding this critical value.
  - Switching and hysteresis

- Possible approaches
  - Computer simulations: difficult to follow through to the form of the drag coefficient if vortex density grows to a large value. Good for very small structures (eg thin wires). But difficulty in imposing appropriate boundary conditions (rough surfaces).
  - Some sort of "statistical" approach, relying on vortex growth equations of the type originally developed in connection with thermal counterflow turbulence, or on concepts like quasi-classical flow and eddy viscosity. Good for large structures.

Computer simulations I

- Smooth oscillating sphere, $T = 0$, $R = 100\ \mu$m with remanent vortex (200Hz)
  (Hanninen, Tsubota, Vinen, PRB **75**, 064502, 2007)



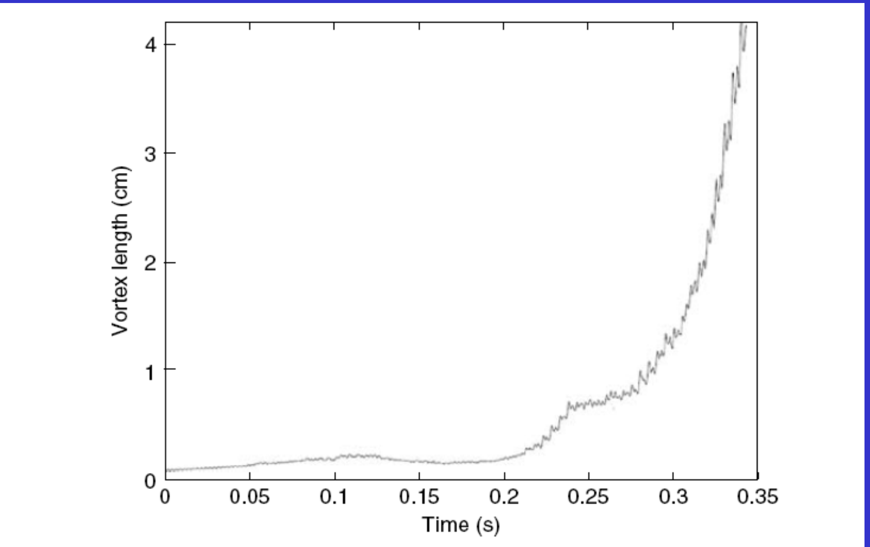


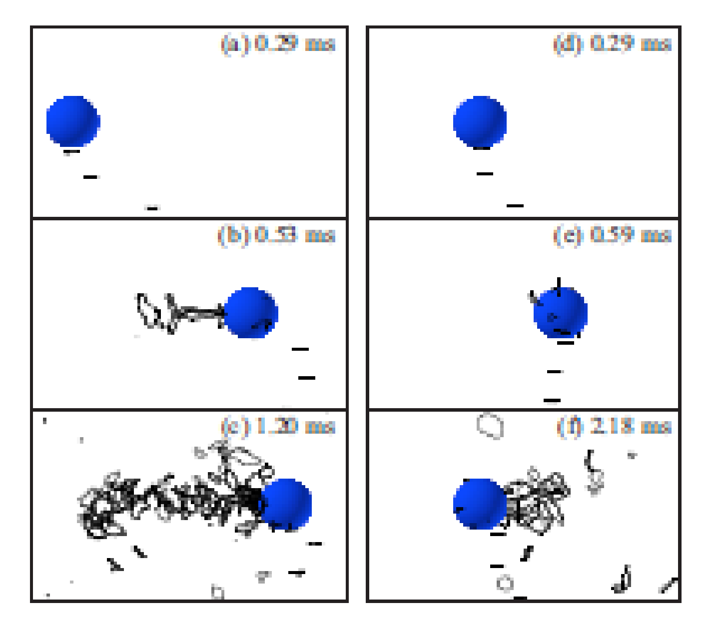
FIGURE 22 Growth with time of the total length of vortex line near the sphere in the simulations of Hänninen et al. (2007).

movie

- Suggests rapid increase in vortex density when velocity exceeds critical value of about 120 mm s^{-1}; critical velocity significantly larger than observed by Schoepe with a rough sphere. No hysteresis.
- But computer limitations did not allow us to follow the increase in vortex density to a steady state.
- Need for nucleating vortex. Does the form of the nucleating vortex matter? Focus on behaviour as velocity is reduced from a large value.

Computer simulations II

- Smooth oscillating sphere, $T = 0$, $R = 3\mu m$, triggered by injected vortex rings (1.59 kHz). Goto et al, PRL 100, 045301 (2008); Fujiyama & Tsubota, PRB 79, 094513, 2009



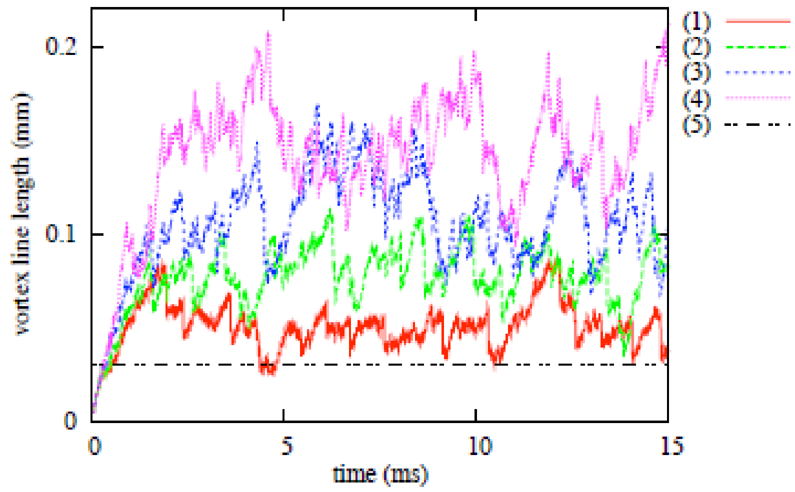
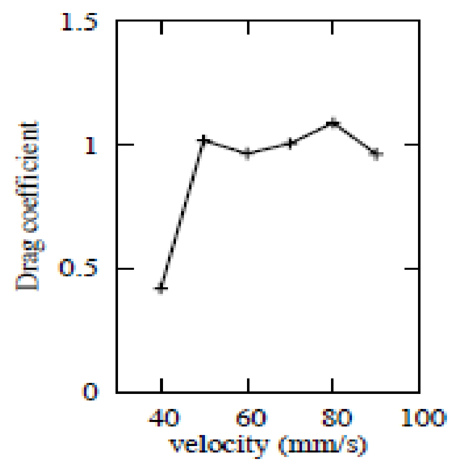


FIG. 3: (color online). Vortex line length at different velocities of the oscillation. (1): 30 mm/s, (2): 50 mm/s, (3): 70 mm/s, (4): 90 mm/s, (5): vortex line length in the absence of the sphere.

Note that a steady state (albeit with large fluctuations) can be achieved with this very small sphere at velocities that are not too high

Computation of drag coefficient →



Why does $C_D \to 1$?

An avalanche described analytically

- The simulations with the large sphere suggest that when the velocity of the sphere exceeds a critical value the vortex density suddenly grows in a kind of avalanche.
- Hanninen & Schoepe (JLTP **153**, 189, 2008) have suggested that such an avalanche might be described by a generalization by Kopnin of an equation that was originally suggested in connection with thermal counterflow

$$\frac{dL}{dt} = \beta\left[v_s L^{3/2} - \kappa L^2\right] \qquad \beta = A(1-\alpha') - B\alpha$$

  This equation describes the rate of change of line density when the superfluid moves at a uniform, time independent, velocity v_s relative to a fixed normal fluid.

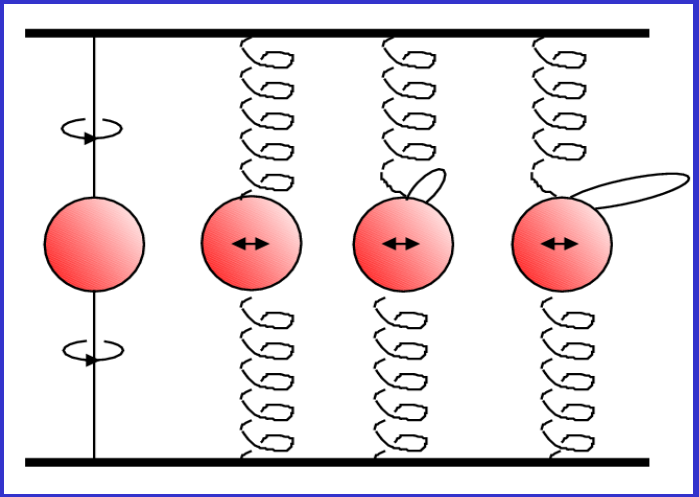
- Some difficulties
  - Kopnin's derivation (PRL **92**, 135301, 2004). Based on the HVBK equations. Take some sort of course-grained average; then put vorticity $\omega = \kappa L$. But in this context $\omega \neq \kappa L$; κL is related to the enstrophy, not the vorticity.
  - A generalization to the case of an oscillating flow seems questionable.
- Nevertheless the basic idea of an avalanche may be right, at least in some cases. It may lead to a critical superfluid velocity $\sim (\kappa\omega)^{1/2}$ but does not account for the detailed dependence of C_D on velocity.

Another argument for vortex growth when $v_s > (\kappa\omega)^{1/2}$

❑ A critical velocity ~ $(\kappa\omega)^{1/2}$ comes from a simple dimensional if it is assumed to be independent of the size of the structure.

❑ Another argument is simply this:

- Oscillation at frequency ω generates Kelvin waves with half wavelength ~$(\kappa/\omega)^{1/2}$.
- A reconnection at the surface of the sphere produces a vortex loop of radius $R \sim (\kappa/\omega)^{1/2}$.
- This loop will expand under the influence of the velocity U of the sphere if $U > \sim \kappa/R \sim (\kappa\omega)^{1/2}$.



Reconsideration of the experimental results I

- Especially from larger structures, where high densities of vortex lines are probably generated.

- Let us remember that when the superfluid contains a high density of vortex lines it seems to behave like a classical fluid, on length scales greater than the line spacing ℓ. If both fluids are present and are not forced to move with different velocities, the turbulent motions in both fluids are strongly coupled, and we have in effect, on scales $>\ell$, a single classical fluid (quasi-classical behaviour).

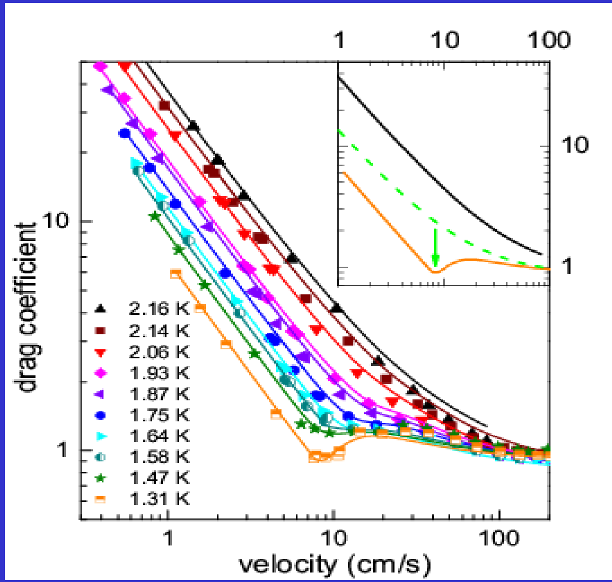
- What is the effective kinematic viscosity of this quasi-classical fluid?

  - If there is ***no normal fluid***, it is presumably something like an eddy viscosity, due to turbulent motion on a scale ℓ. In that case the effective kinematic viscosity is of order κ.

  - If ***both fluids are present***, the effective kinematic viscosity arises from a combination of this eddy viscosity and the normal-fluid viscosity. It is still of order κ (for ^{4}He).

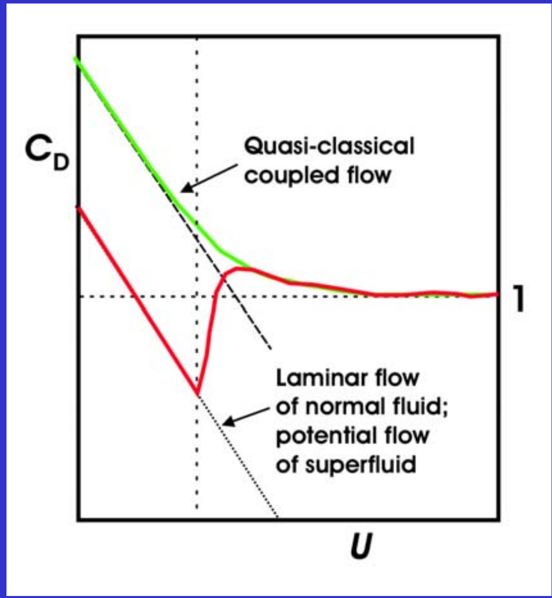
Reconsideration of the experimental results II

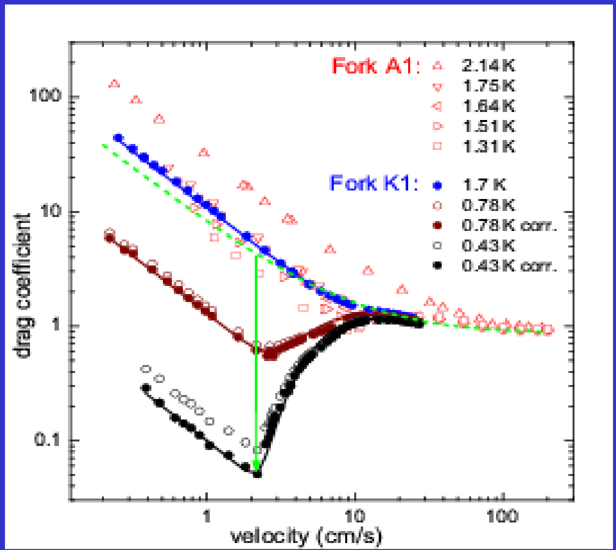
❑ Look now at the results that are often observed with vibrating forks.



Schematically (low *T*) →

Fitting formula:
see talk by Skrbek





- Consider what happens as we reduce U *(hysteresis)*
- At high U we expect quasi-classical flow: $C_D \sim 1$
- If this persists to lower U we get the green line.
- But as U approaches the transition to laminar flow C_D drops, rather suddenly , leaving no contribution from the superfluid. Significant?

Reservations?

- The maximum in C_D is not always seen. Perhaps the transition from turbulent superfluid to potential-flow superfluid can in some cases be more gradual. But, if so, we do not know why.
- The fact that the superfluid critical is close to the laminar-turbulent transition in the quasi-classical fluid may be an accident. But if the Hanninen-Schoepe scenario were right, one would expect no connection between these two transitions.
- Values of the effective kinematic viscosity deduced by fitting a formula based on our model seem to vary from one fork to another is a strange way (although the order of magnitude is always right).
- These are serious reservations, but more experiments will throw light on the situation.
- But suppose there really is a connection between the superfluid critical velocity and the quasi-classical transition. Why might it exist? (Blazkova, Schmoranzer, Skrbek, Vinen: PRB in press)
- This connection would provide another explanation for the $\omega^{1/2}$ frequency dependence.

A difference between classical and quantum turbulence

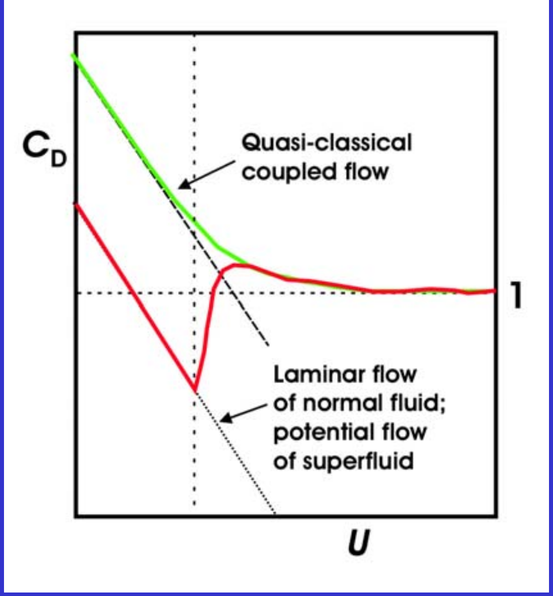
- The connection may be related to an interesting difference between quantum and classical turbulence.

- In ***classical turbulence*** we can generate an initial turbulent state on a single large length scale (>> Kolmogorov dissipation length): a scale corresponding to, for example, the size of an obstacle.

- In ***quantum turbulence*** this impossible, in the sense that large scale turbulence is possible only in the presence of a suitably large density of vortex lines, suitably polarized, which allows rotation on a large length scale; i.e. the large scale turbulence requires the presence of small scale turbulence on a scale of order the vortex-line spacing.

A possible connection with the quasi-classical transition?

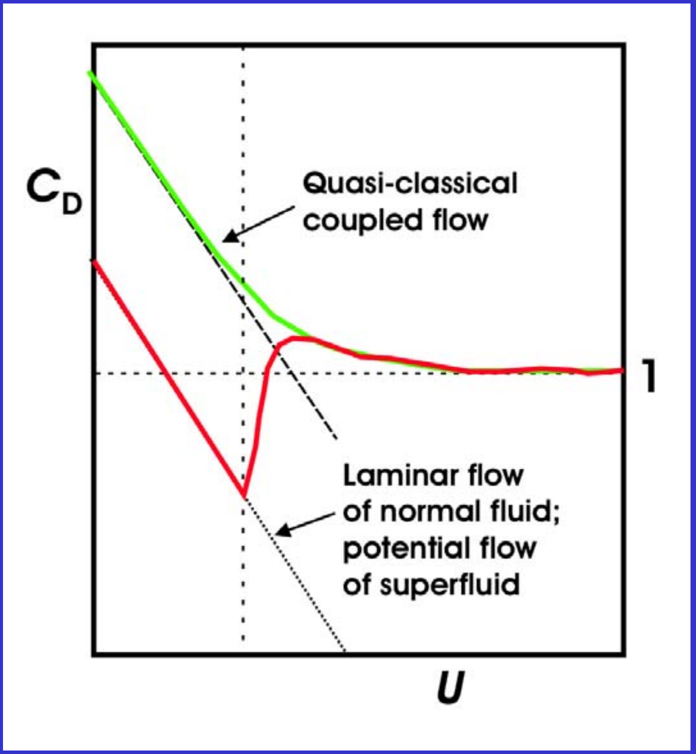
- Thus quasi-classical coupled flow requires a minimum density of vortex lines. That minimum density may be such that the line spacing is much less than the viscous penetration depth for the coupled fluids.
- But, as we say, this high density of vortex line means strongly developed turbulence at short length scales. An efficient way of generating turbulence on a short length scale is by decay of turbulence on a large length scale through the action of the non-linear term [(v.∇)v] in the Navier-Stokes (or Euler) equation.



- Perhaps the high density of vortex line required if the superfluid flow is to mimic classical turbulence can be generated only if the superfluid is already turbulent on a large length scale. So to get large scale turbulence you need to have small scale turbulence first; but to get sufficiently intense small-scale turbulence you must first have large scale turbulence.

Another view of this connection

- Suppose we start at a high velocity and gradually reduce the velocity.
- As long as the flow is fully turbulent, a sufficient density of vortex lines is maintained by the decay of the quasi-classical turbulence.
- But, as the velocity at which this quasi-classical turbulence would give way to quasi-laminar flow, this process for generating vortex lines starts to fail. The density of vortex lines falls to a low value, and we are left with laminar viscous flow of the normal fluid and (practically) potential flow of the superfluid.



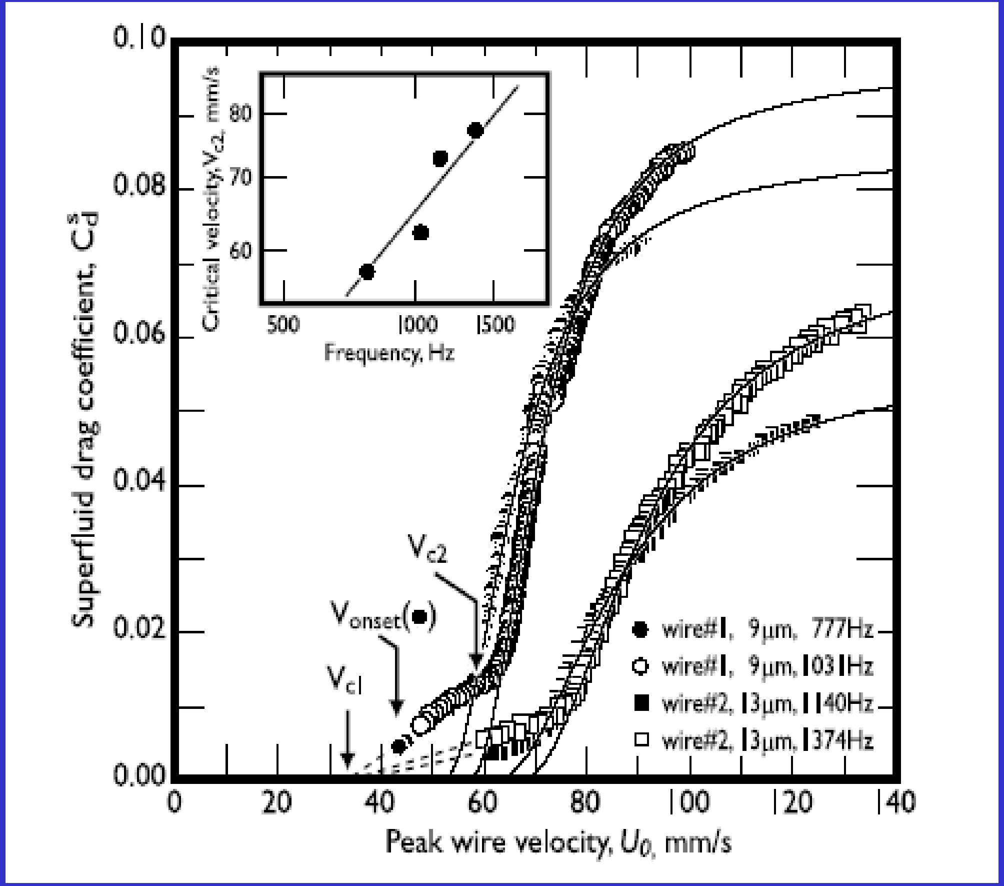
- What happens if we start at a low velocity and increase it. At first we have laminar flow of the normal fluid and potential flow of the superfluid. When the velocity exceeds the quasi-classical critical velocity this situation becomes unstable, or perhaps metastable. A transition to quasi-classical coupled turbulence can occur, perhaps with hysteresis.

Vortex lines and drag at subcritical velocities

- According to our discussion the vortex density and an associated drag ought not to disappear at velocities below the quasi-classical critical velocity; it ought simply to be very small (true for both the avalanche process and my alternative process)..

- A small, subcritical, dissipation would be most easily seen at very low temperatures.

- Very recent results from Lancaster may have seen it
  - Bradley *et al*., JLTP **154**, 97, 2009 - vibrating wire.
  - Garg *et al*, this meeting - vibrating grid)



The Hanninen-Schoepe critical velocity?

- Perhaps the mechanism underlying this critical velocity does exist, but only at velocities higher than those we are now considering.

- If the Hanninen-Tsubota-Vinen simulations are to be believed, then it probably is higher.

Closing comments.

- Practically the whole of this presentation has been speculation. But it does emphasize the need for more experimental evidence on a range of large oscillating structures, with a careful study of the way in which the drag coefficient varies with velocity over a wide range of velocity. There is also a need to visualize the flow.

- We are still a long way from an understanding of the way vibrating structures work in superfluid ^{4}He. But the possibilities we have been exploring do suggest that interesting and potentially important effects may well be relevant. We should not simply give up!

- We have not touched on superfluid ^{3}He – probably very different!

Thank you