The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
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2023-26

Workshop on Topics in Quantum Turbulence

16 - 20 March 2009

Computations on Vibrating Structures

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Computations on vibrating structures

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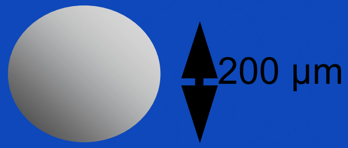
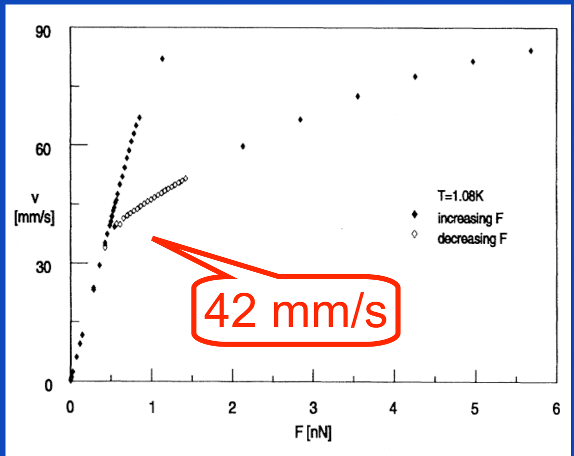
Thanks to R. Hänninen, W. F. Vinen

Ref. S. Fujiyama and MT, Phys. Rev. B79, 094513(2009); arXiv:0812.0268v2

0. Introduction -QT created by an vibrating object-

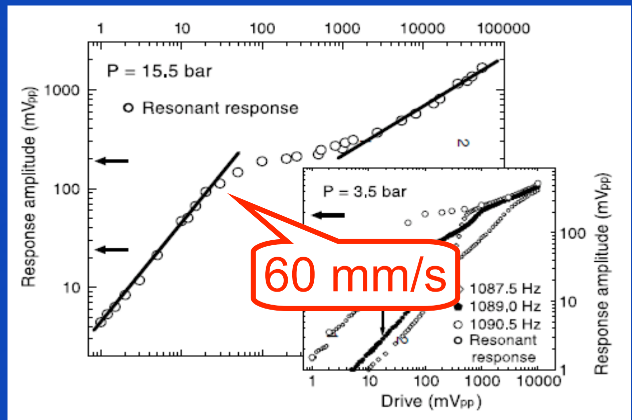
Microsphere





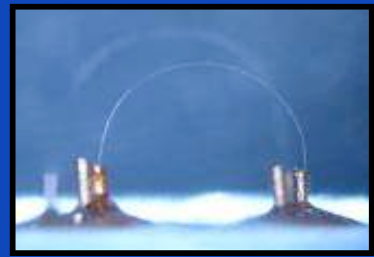
J. Jager, B. Shuderer, W. Schoepe, Phys. Rev. Lett. **74**, 566 (1995).

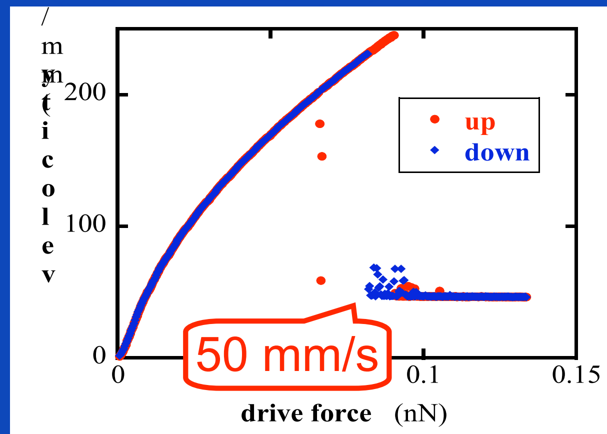
Grid



H.A. Nichol, L. Skrbek, P.C. Hendry, P.V.E. McClintock, Phys. Rev. Lett. **92**, 244501 (2004).

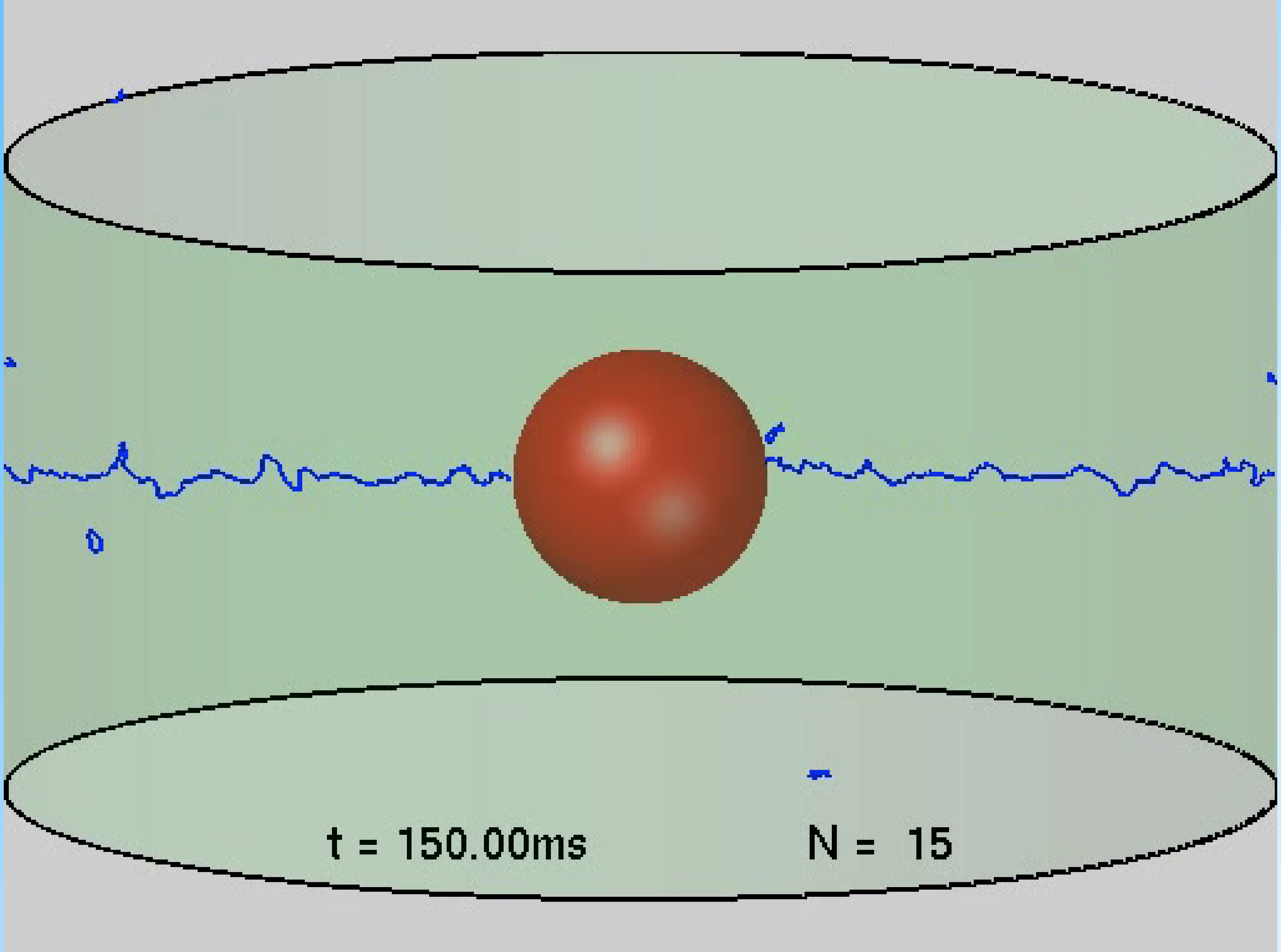
Wire





H.Yano, N. Hashimoto, *et al*, Phys. Rev. B **75**, 012502 (2007).

How remnant vortices develop to turbulence under AC flow



T=0 K

R. Hänninen, M. Tsubota, W.F. Vinen, Phys. Rev. B **75**, 064502 (2007)

1. Problems

- How can we understand the critical velocity, including the dependence on the frequency?
- What is the observed hysteresis?
- How can we understand the drag force? This is related with the classical-quantum correspondence for QT by vibrating structures. etc.

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- How can we understand the critical velocity, including the dependence on the frequency?
- What is the observed hysteresis?
- How can we understand the drag force? This is related with the classical-quantum correspondence for QT by vibrating structures. etc.

Contents of this work

1. We developed a new method of how to calculate the drag force from the simulation by the vortex filament model.
2. The obtained drag coefficients are consistent with some observations.

The result strongly supports the classical-quantum similarity also for QT created by vibrating structures (AC QT).

Ref. S. Fujiyama and MT, Phys. Rev. B79, 094513(2009); arXiv:0812.0268v2

1. Drag force

$$F_D = \frac{1}{2} C_D \rho A U^2$$

C_D: drag coefficient

ϱ: fluid density

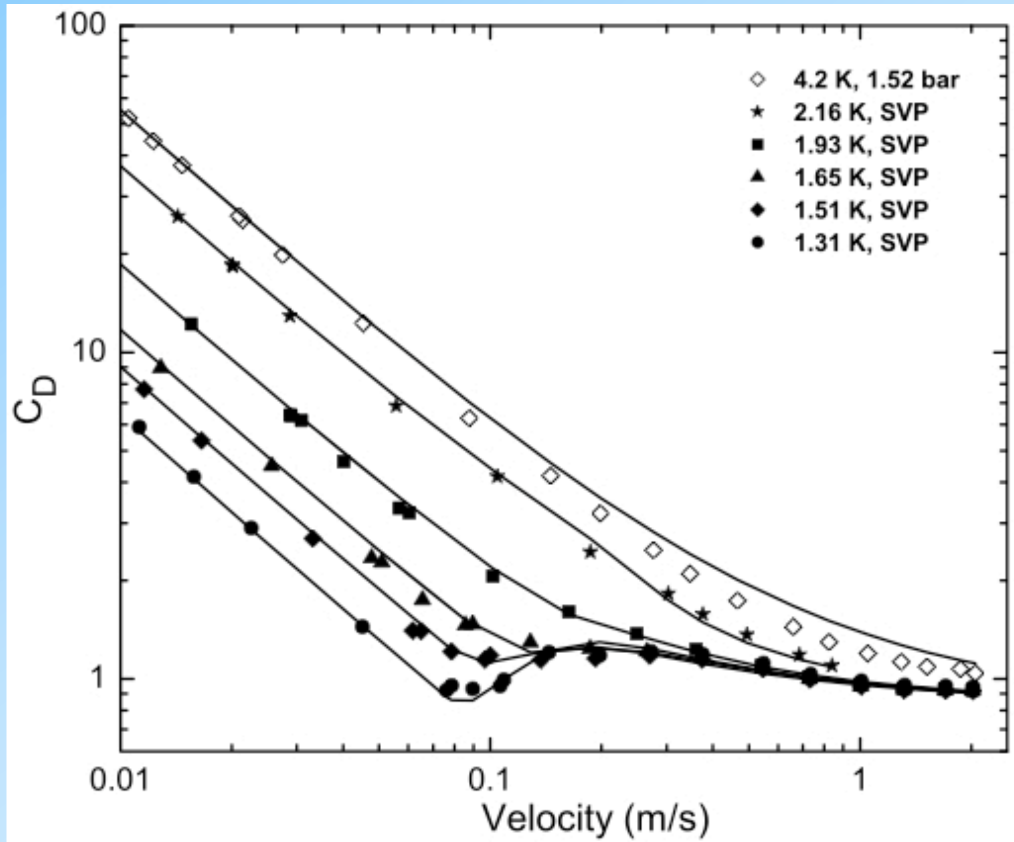
A: area of the object normal to the flow

U: flow velocity

Low velocity: $C_D \propto U^{-1}$

Stokes' drag force

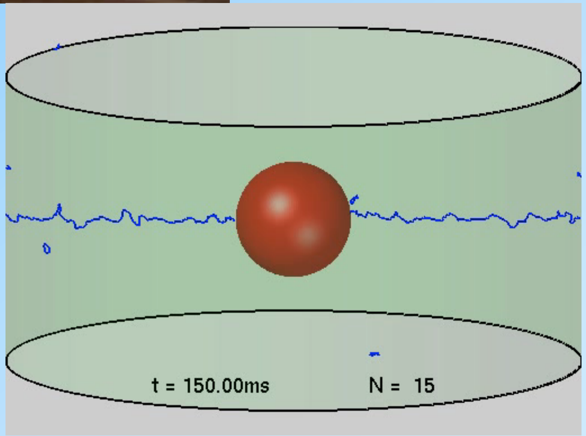
High velocity: $C_D \approx O(1)$



L. Skrbek and W. F. Vinen, in *Progress in Low Temperature Physics*, Vol. 16, p.193, case of a vibrating tuning fork

How to calculate the drag force?

A straightforward method would be to calculate the velocity due to vortices over the sphere surface and get the pressure.



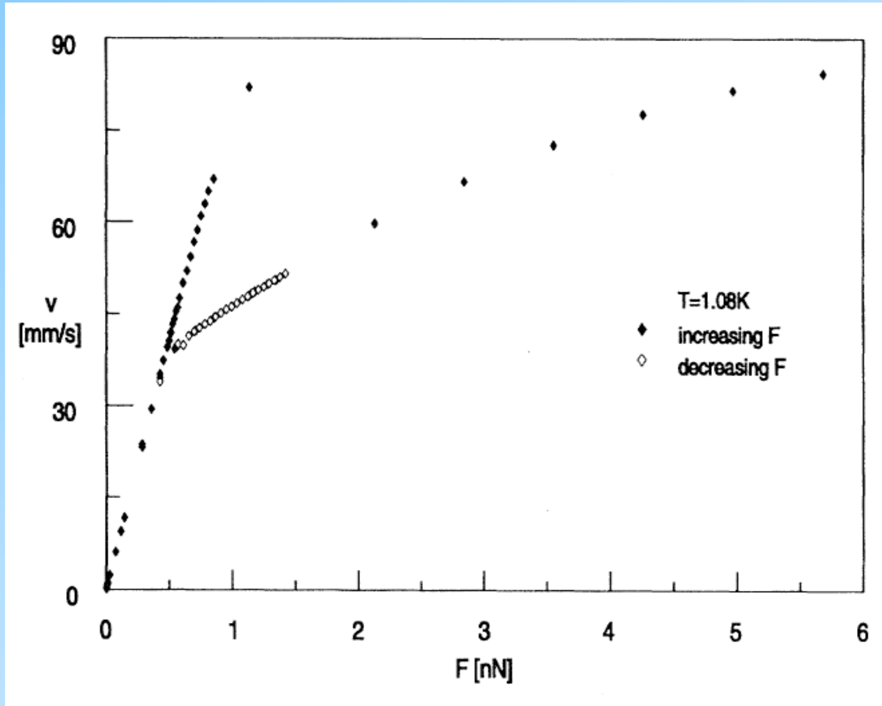
It is possible in the cases with a few vortices, which is actually done for the coupled dynamics of a sphere and vortices.

cf. D. Kivotides, C. F. Barenghi, Y. A. Sergeev, PRB75, 212502(2008): 77, 014527(2008)

However, it is quite difficult to do for a dense tangle!

2. Basic ideas for estimating a drag force

We confine ourselves to an equilibrium state of turbulence.

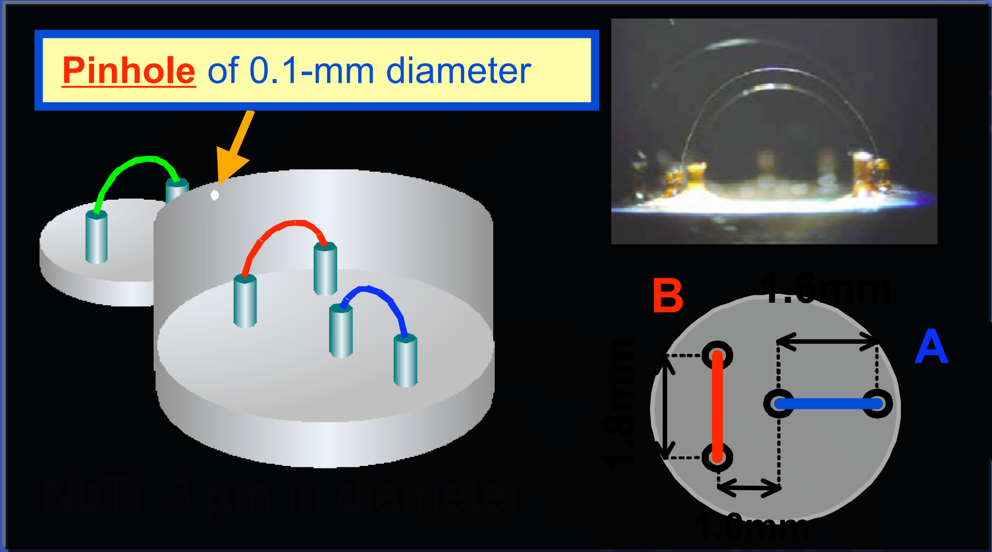


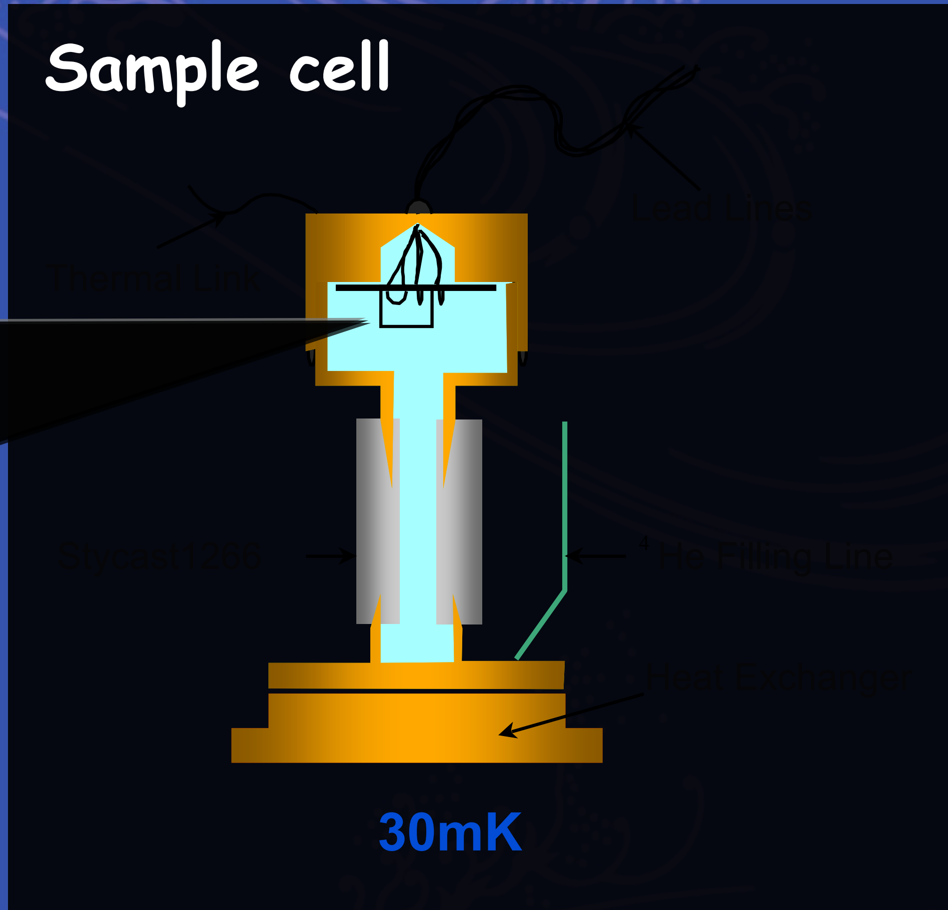
J. Jager, B. Shuderer, W. Schoepe, PRL **74**, 566 (1995)

We refer to the vibrating wire experiments in Osaka.

OCU Experimental setup

Configuration of vibrating wires



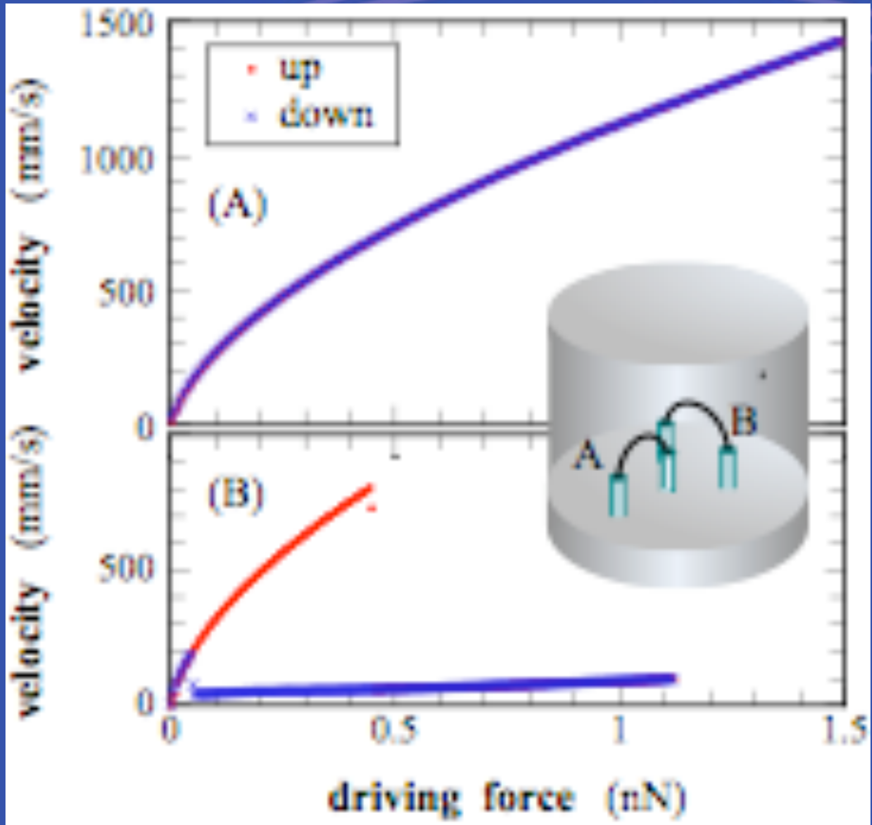


How to obtain a vortex-free vibrating wire

1. using a chamber with a pinhole
2. 20 hours filling of superfluid ^{4}He below 100 mK

The Osaka group got a vortex-free vibrating wire.

Response of a vibrating wire at 30 mK



VW(A) cannot make turbulence by itself even above 1 m/s !

↓

Effectively free of remanent vortices

While VW(B) is usual.

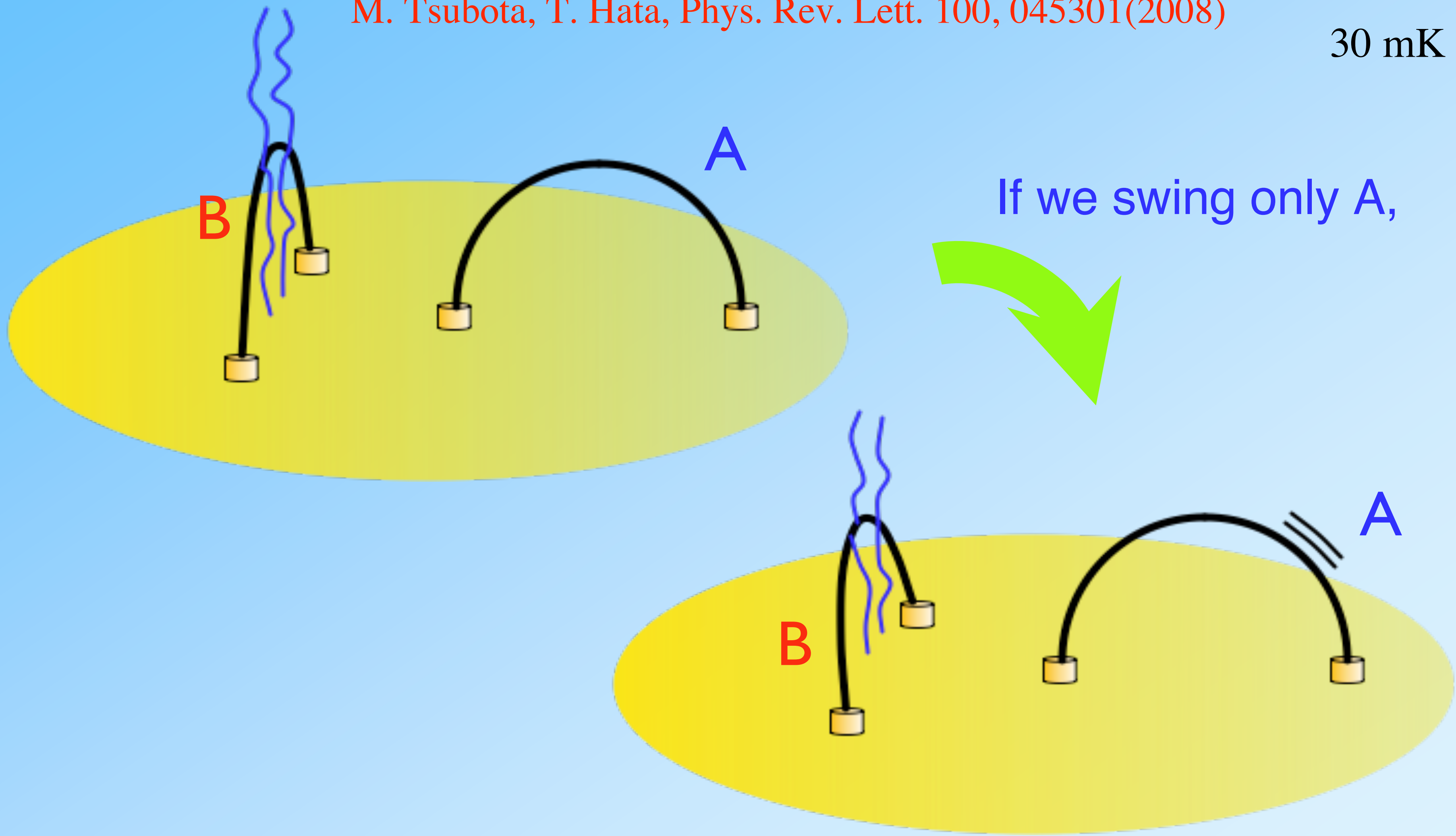
N. Hashimoto, R. Goto, H. Yano, K. Obara, O. Ishikawa, T. Hata, PRB76, 020504(2007)

R. Goto, S. Fujiyama, H. Yano, Y. Nago, N. Hashimoto, K. Obara, O. Ishikawa, M. Tsubota, T. Hata, PRL100, 045301 (2008)

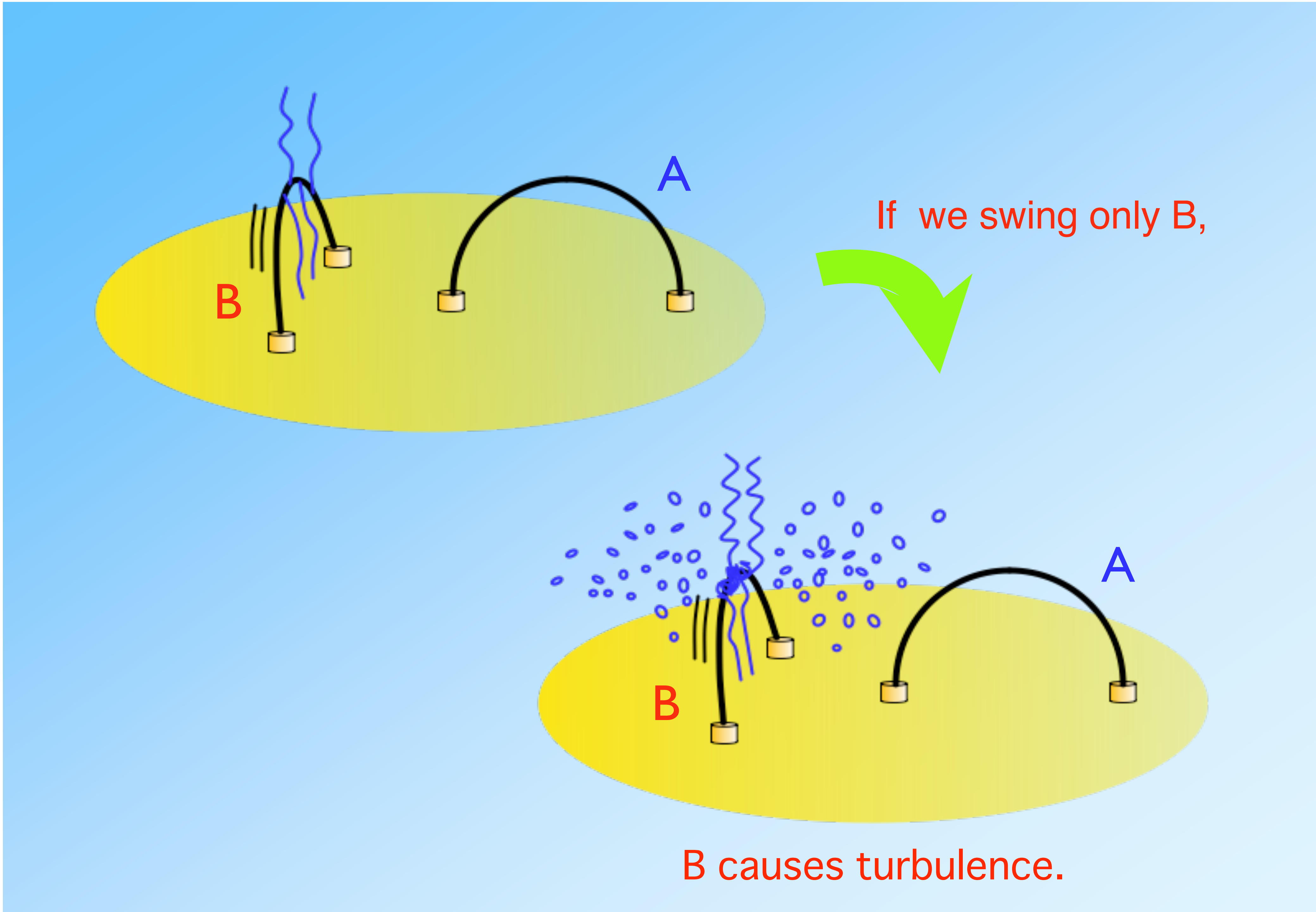
Turbulence due to free vortex loops

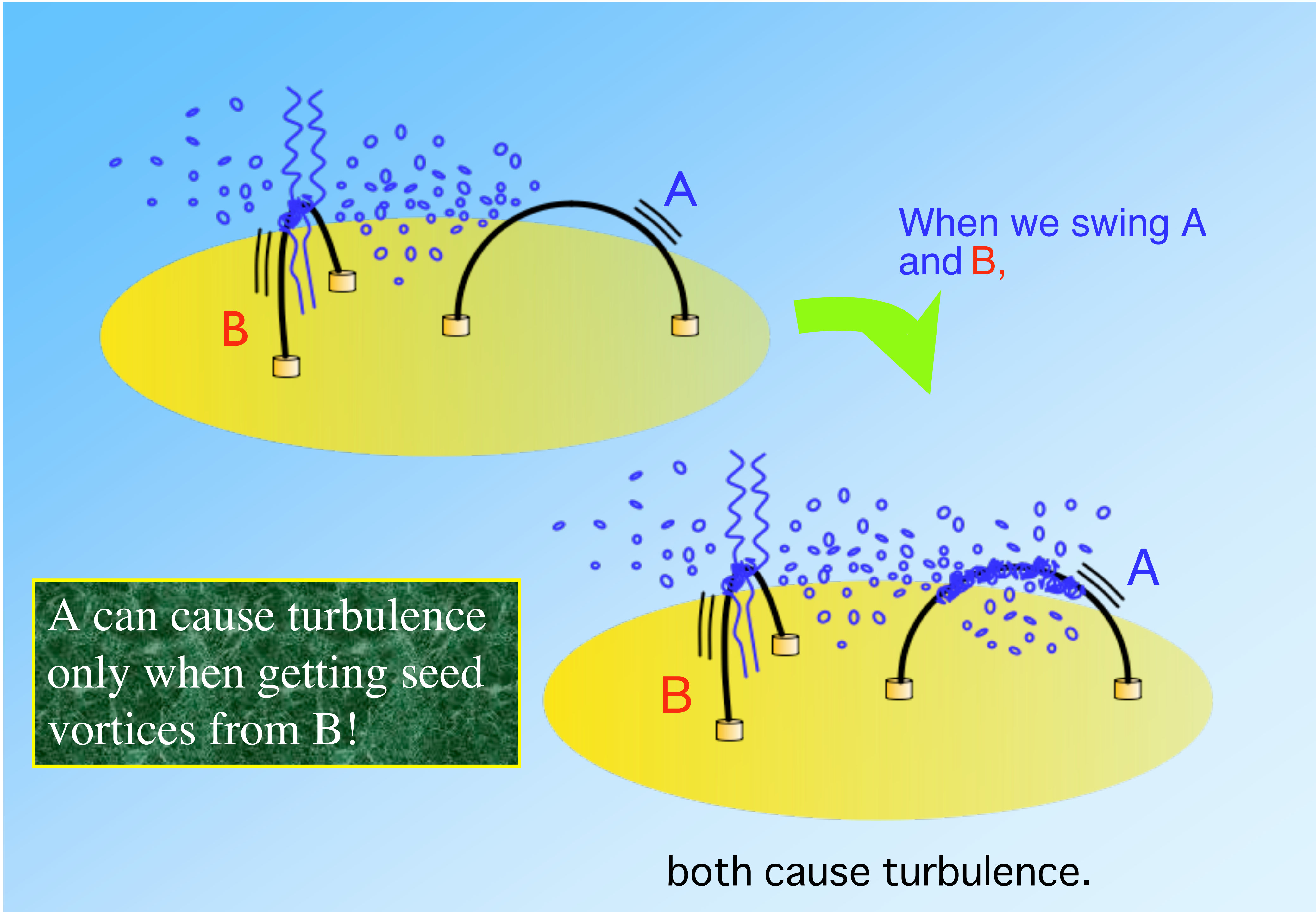
R. Goto, S. Fujiyama, H. Yano, Y. Nago, N. Hashimoto, K. Obara, O. Ishikawa, M. Tsubota, T. Hata, Phys. Rev. Lett. 100, 045301(2008)

30 mK

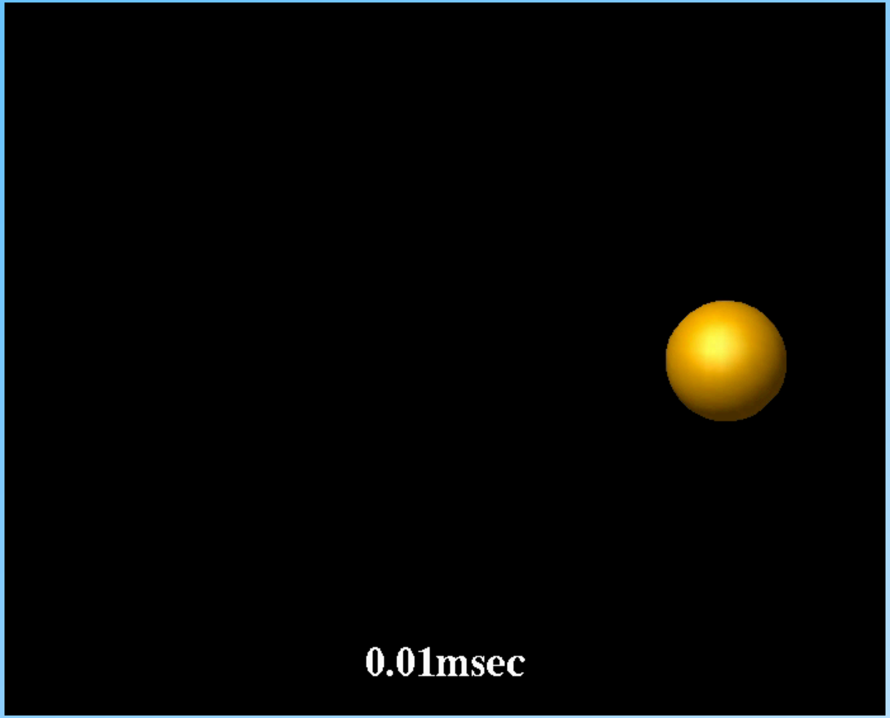


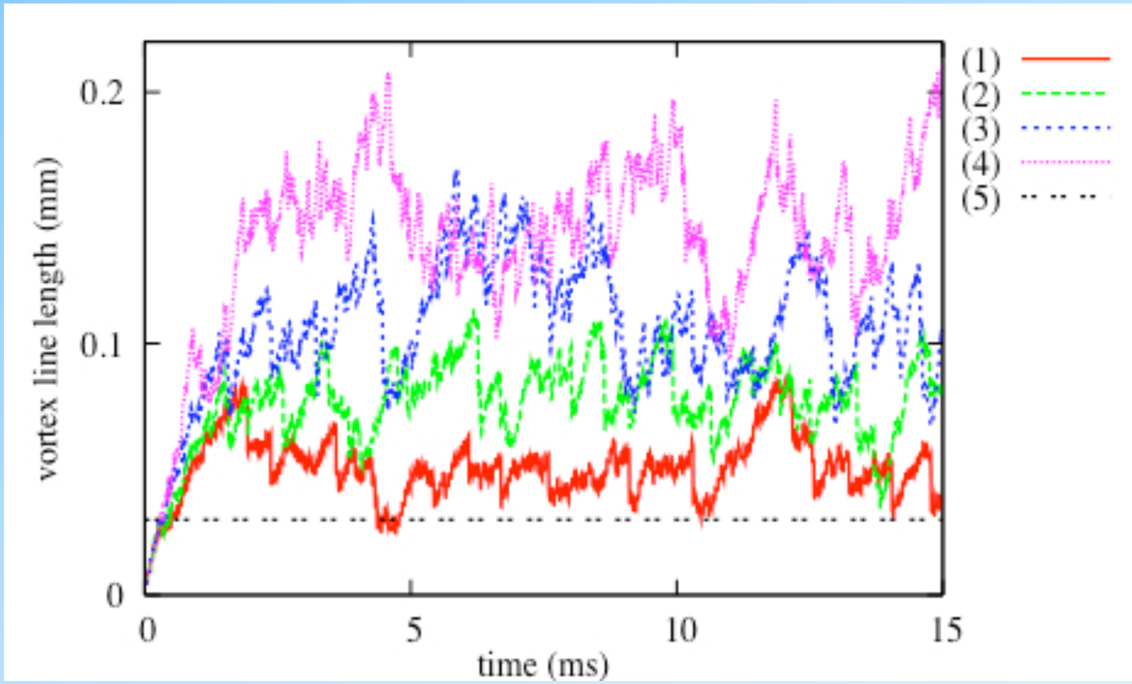
A never causes turbulence.


A
B
If we swing only B,
A
B
B causes turbulence.


A
B
When we swing A and B,
A
B
A can cause turbulence only when getting seed vortices from B!
both cause turbulence.

QT by an oscillating sphere getting vortex rings





Parameters for the sphere

Radius 3μm

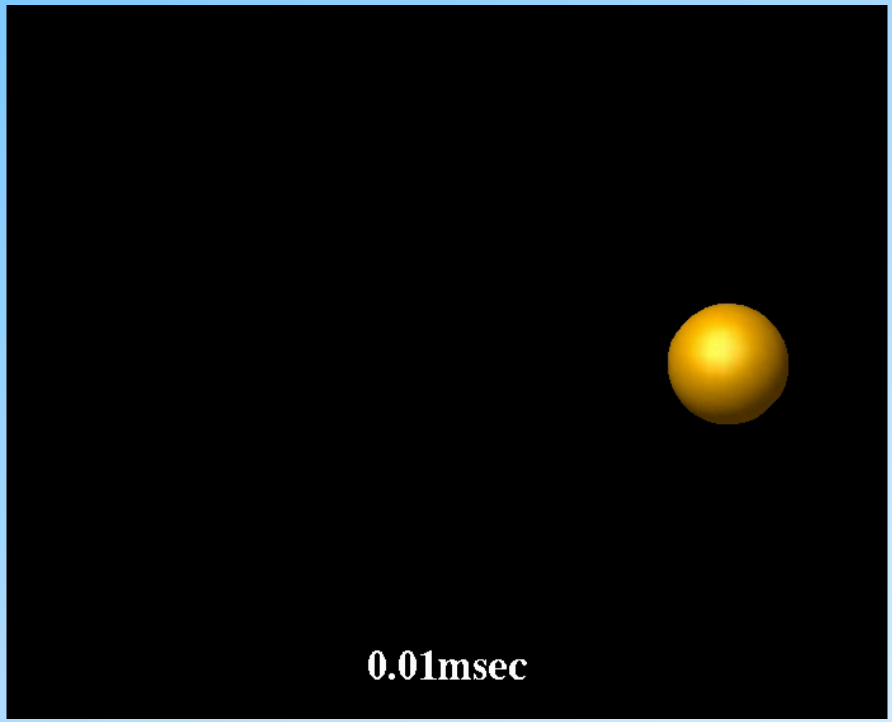
Frequency 1590 Hz

Velocity 137 mm/s

Vortex line length for different velocities (1)30mm/s, (2)50mm/s, (3)70mm/s, (4)90mm/s, (5) without a sphere

Basic idea for calculating the drag force

Output energy



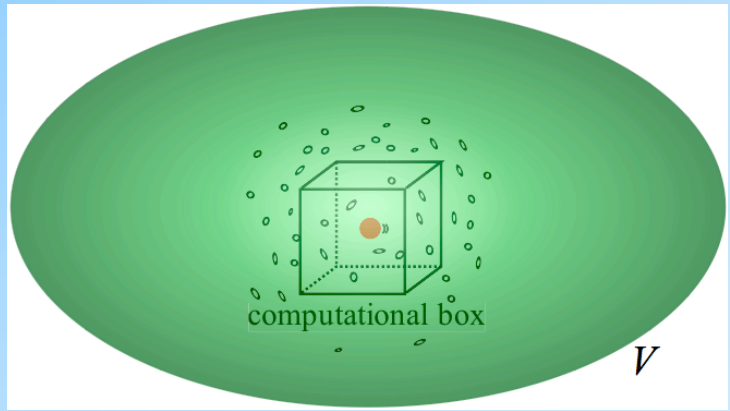
Input energy

(Output energy) - (Input energy) = (Work by the drag force)

The energy is estimated by the length of the vortex lines.

3. Formulation (1/4)

- Vortex rings are continuously supplied to the oscillating sphere.
- Vortices amplified by the sphere escape from the computational box.
- We have a statistically equilibrium state in the computational box.

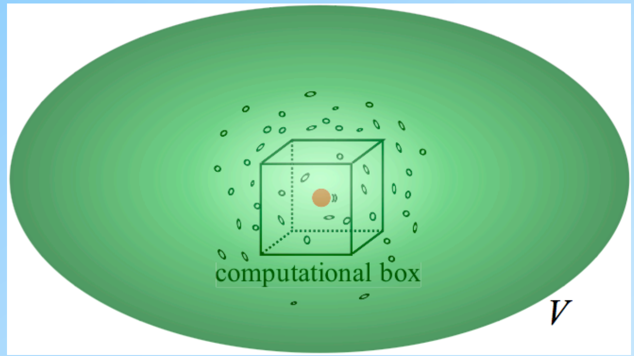


The superfluid kinetic energy is $K = \int_V \frac{1}{2}\rho_s v_s^2 dV$.

We finally want to derive

$$\frac{(\text{Output energy}) - (\text{Input energy})}{(\text{Time})} \approx \frac{dK}{dt} \approx u_p F_D$$

3. Formulation (2/4)



The difficulty

When we calculate $\dfrac{dK}{dt} = \dfrac{d}{dt}\int_V \frac{1}{2}\rho_s v_s^2 dV$,

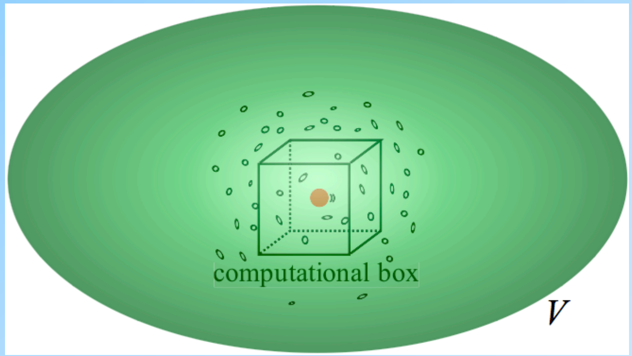
the time-derivative and the integration are not interchangeable because the sphere oscillates and the domain of the integration changes everytime.

To handle the difficulty, we make the integration domain include the inside of the sphere, and define the fluid density as

$$\rho(\mathbf{r},t) = \rho_s \theta\left(\left|\mathbf{r} - \mathbf{x}(t)\right| - a\right),$$

where $\mathbf{x}(t)$ is the center of the sphere and a is the radius.

3. Formulation (3/4)



$$V_{all} = V + (\text{sphere})$$

$$\frac{dK}{dt} = \frac{d}{dt}\int_{V_{all}} \frac{1}{2}\rho v_s^2 dV$$

$$= \int_{V_{all}} \frac{1}{2}\frac{\partial \rho}{\partial t} v_s^2 dV + \int_{V_{all}} \rho \mathbf{v}_s \cdot \frac{\partial \mathbf{v}_s}{\partial t} dV$$

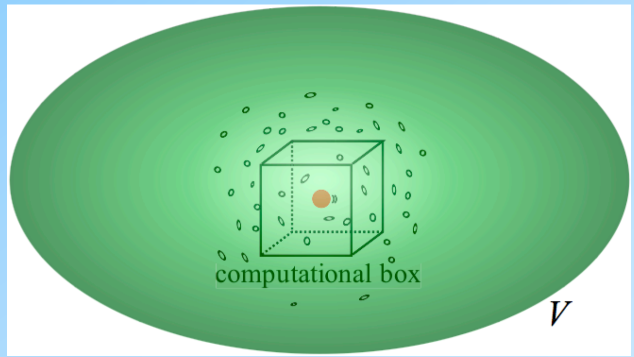
$$= \int_{V_{all}} \frac{1}{2}\frac{\partial \rho}{\partial t} v_s^2 dV + \int_{V} \mathbf{v}_s \cdot \left\{ -\nabla p - \rho_s (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s \right\} dV$$

$$= \int_{V_{all}} \frac{1}{2}\frac{\partial \rho}{\partial t} v_s^2 dV - \int_{V} \mathbf{v}_s \cdot \rho_s (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s dV - \int_{V} \mathbf{v}_s \cdot \nabla p dV$$

We can show the first and the second terms are canceled out.

As a result,

3. Formulation (4/4)



By using the incompressibility, we obtain

$$\frac{dK}{dt} = -\int_V \mathbf{v}_s \cdot \nabla p dV = -\int_S p\mathbf{v}_s \cdot d\mathbf{S}$$

Since $\mathbf{v}_s \cdot d\mathbf{S} = \mathbf{u}_p \cdot d\mathbf{S}$ over the sphere surface, the equation becomes

$$\frac{dK}{dt} = -\mathbf{u}_p \cdot \left(\int_S p d\mathbf{S}\right) = -\mathbf{u}_p \cdot \mathbf{F}_{f \to s}$$

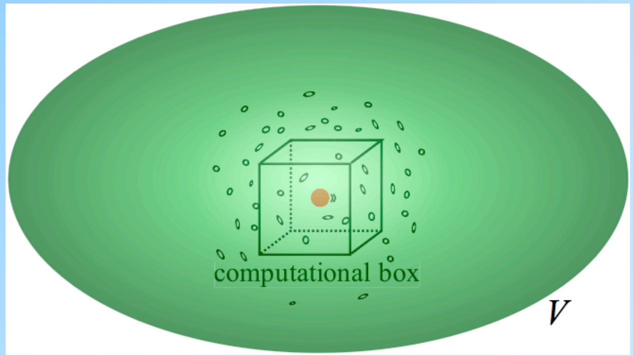
The force $\mathbf{F}_{f \to s}$ is resolved into the drag force $\mathbf{F}_D$ parallel to $\mathbf{u}_p$ and the lift force $\mathbf{F}_{lift}$ normal to $\mathbf{u}_p$. Then we obtain

$$\frac{dK}{dt} = -\mathbf{u}_p \cdot \mathbf{F}_D$$

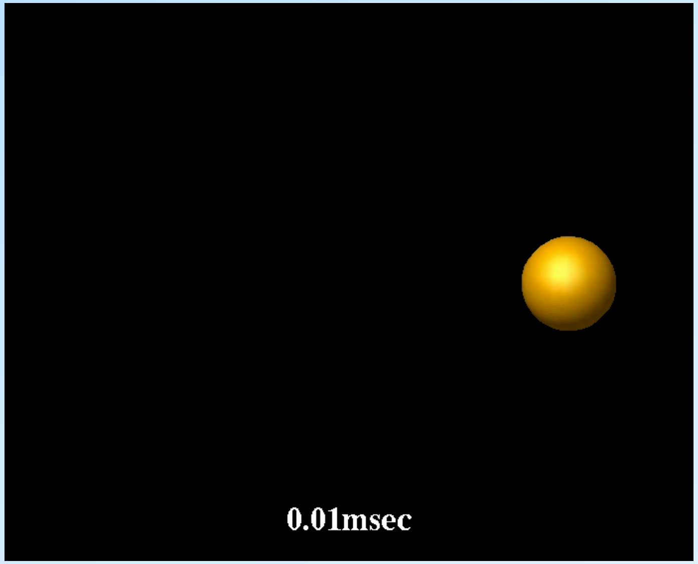
Assuming $\mathbf{u}_p$ is antiparallel to $\mathbf{F}_D$, we obtain $\dfrac{dK}{dt} \approx u_p F_D$.

4. the drag force from the simulation

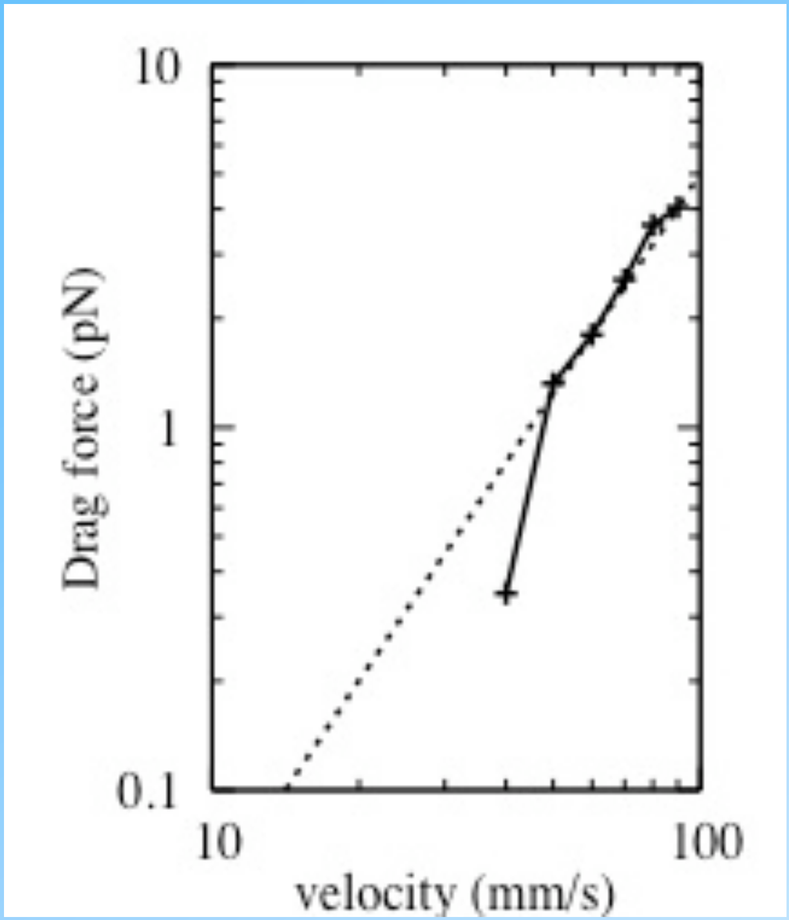
The power $\langle dK/dt \rangle$ is estimated from the energy of vortices escaping from the computational box.

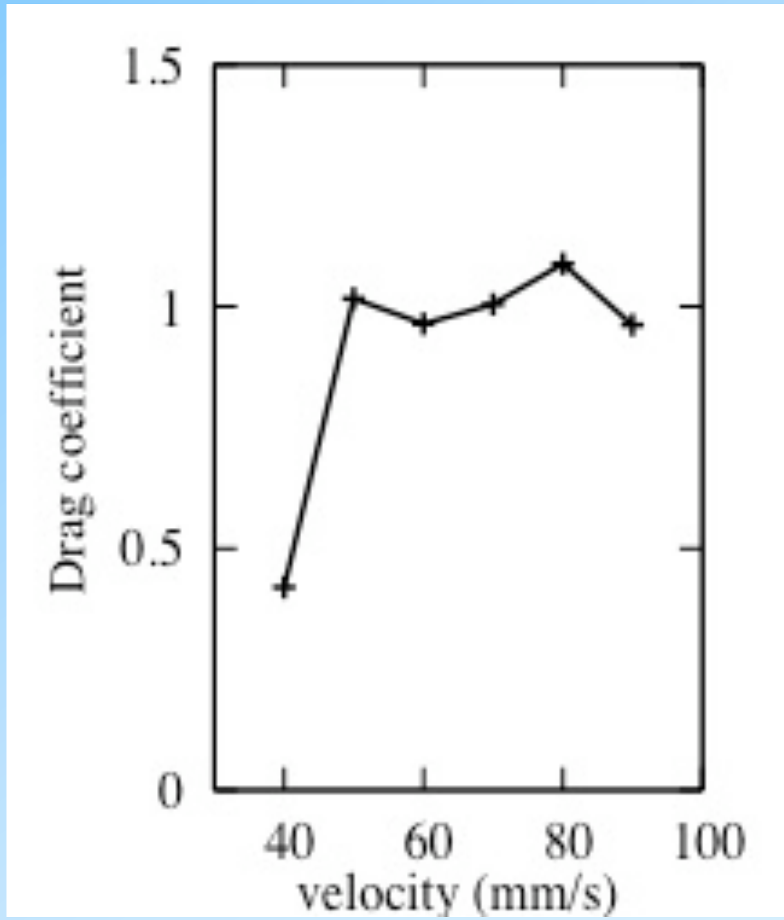


$$\frac{(\text{Output energy}) - (\text{Input energy})}{(\text{Time})} \approx \frac{dK}{dt} \approx u_p F_D$$



The obtained drag force and the drag coefficient





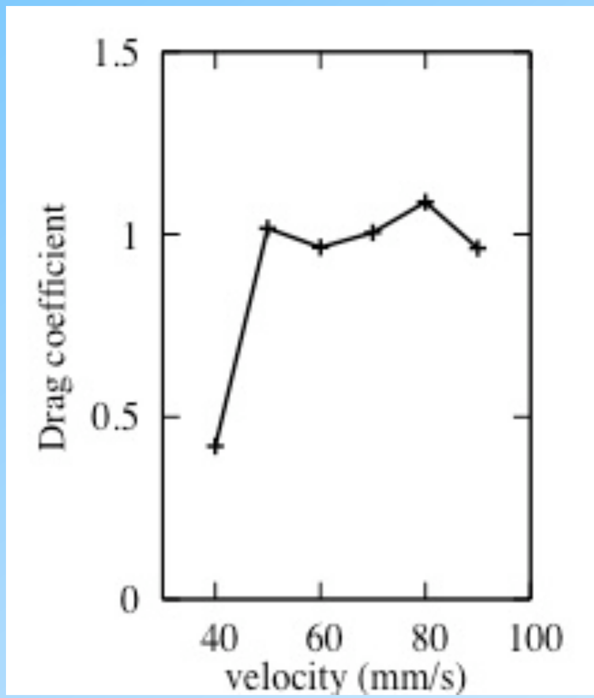
The dotted line $\propto U^2$

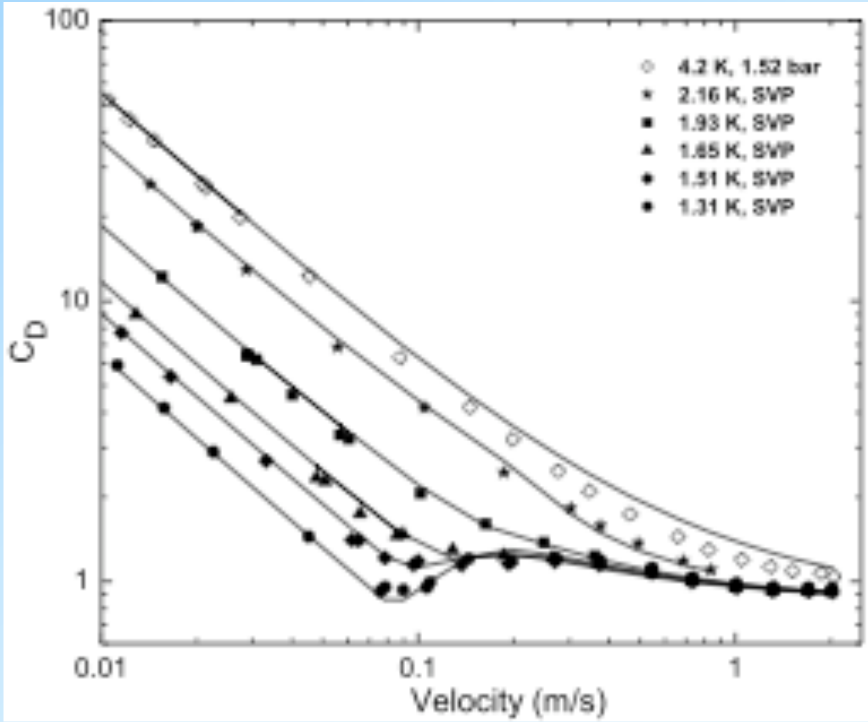
$$F_D = \frac{1}{2} C_D \rho A U^2$$

The turbulent state gives $C_D \approx O(1)$!

Summary

We obtained the drag force of turbulence created by an oscillating sphere. The results support the similarity between QT and CT even in this case.





Ref. S. Fujiyama and MT, Phys. Rev. B79, 094513(2009); arXiv:0812.0268v2