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Simulations on Vibrating Structures and a Simple Model for the Critical Velocity

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Simulations on Vibrating Sphere and Tilted Rotating Cylinder

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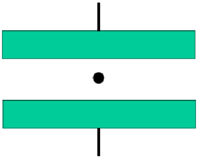
Outline

1. Vibrating Sphere
   - Introduction
   - Numerical simulations with vortex filament model
   - What is the steady state like?
2. Tilted rotating cylinder: steady state turbulence?
   - Background and motivation
   - Motion of ideal fluid
   - Vortex configurations and steady states
   - Summary

Vibrating structures in helium superfluids

Many different structures:

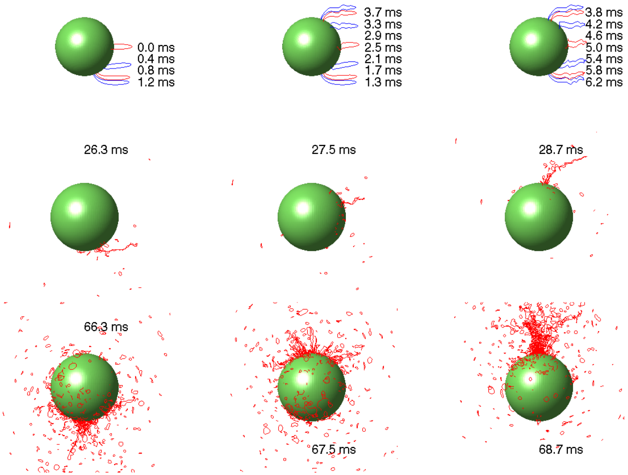
- grids
- wires
- forks
- spheres
  - most simple case numerically



- M. Niemetz and W. Schoepe, J. Low Temp. Phys. **135**, 447 (2004).
- J. Jäger, B. Schuderer, and W. Schoepe, Physica B **210**, 201 (1995).
- J. Jäger, B. Schuderer, and W. Schoepe, Phys. Rev. Lett. **74**, 566 (1995).
- W. Schoepe, Phys. Rev. Lett. **92**, 095301 (2004).

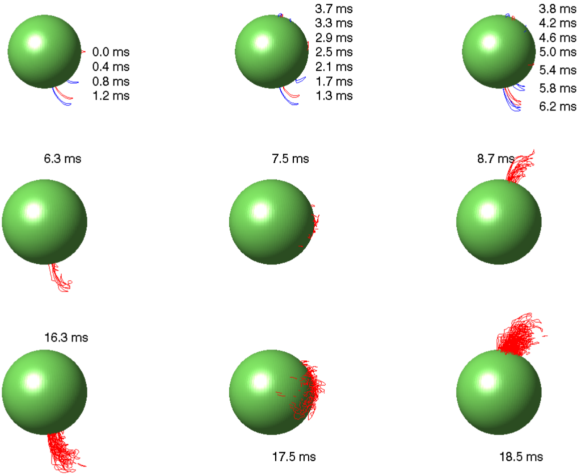
Oscillating flow around sphere, $v > v_c$

^{4}He-II, $R = 0.1$ mm, $v = 150$ mm/s, $f = 200$Hz, T=0:



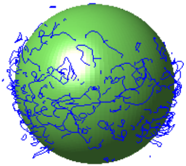
Oscillating flow around sphere, $v > v_c$

^{4}He-II, $R = 0.1$ mm, $v = 200$ mm/s, $f = 200$Hz, T=1.5K (Stokes flow for v_n):



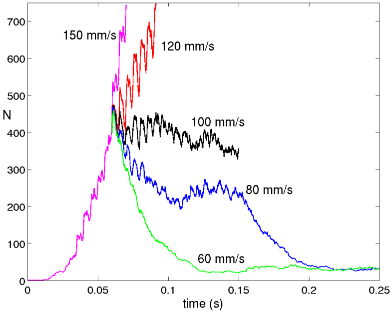
Oscillating flow around sphere, $v > v_c$

^{4}He-II, $R = 0.1$ mm, $v = 120$ mm/s, $f = 200$Hz, T=1.5K (Stokes flow for v_n):

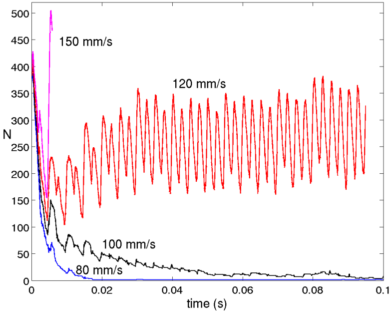


Oscillating flow around sphere

^{4}He-II, $R = 0.1$ mm, $f = 200$Hz:



Vortex number, T=0.



Vortex number T=1.5K. (Stokes for v_n.)

Critical velocity $v_c \approx 100$ mm/s.

Numerical problems/chalenges

- difficult to reach a steady state
- limited range of length scales
- hard to estimate the numerical dissipation at low temperatures
- rouch surfaces may alter results
- what is the normal component doing at finite temperatures?
- calculation of the damping force is a complicated task
- no clear evidence for $v_c \propto \sqrt{\kappa\omega}$

What is the steady state like?

"Large" structures (spheres, grids, forks, thick wires):

- large number of vortices, vortex avalanche?
- mimicing normal flow? (no such indication in simulations with sphere of R=100μm)

"Small" structures (thin wires)

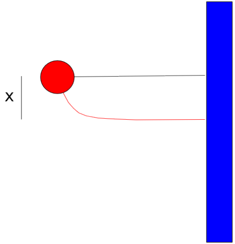
- simulations do not indicate a vortex avalanche (sphere of R=10μm, numerical problem?)
- can a single (or few) vortex cause the observed damping for small structures?

Consider Osaka measurements for thin wires at low temperatures: H. Yano *et al.* PRB **75**, 012502 (2007).

Effect of single vortex

Extra return force due to vortex tension:

$$T_v = \rho_s \kappa^2 \ln(\ell/a)/2\pi$$

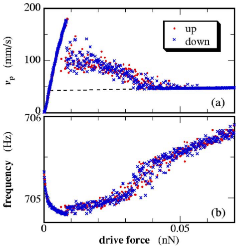


- direction depends on the vortex orientation.
- simulations $\rightarrow \eta \approx 90^o$ when oscillation amplitude, x, is close to sphere radius, R, therefore $\sin\eta \approx x/\sqrt{x^2+R^2}$.

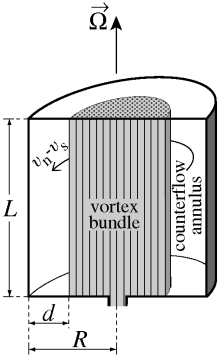
All this results that $\omega^2 = \omega_0^2 + T_v/(RM)$, where M is the mass of the wire.

Osaka parameters: $R = 1.25\mu$m, R_{loop}=1mm $\rightarrow$ $M \approx 10^{-10}$kg.

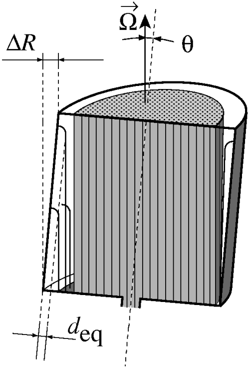
$$\Delta\omega \approx 0.25\mathrm{Hz}$$



Tilt of the cylinder axis



Perfect alignment

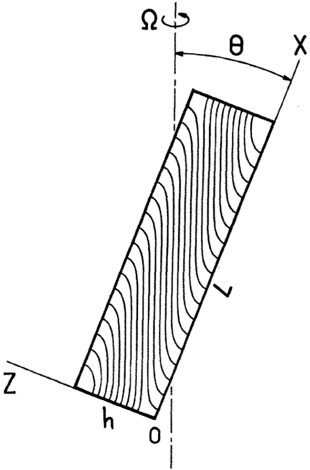


Small misalignment

Previous results

Result by Mathieu *et al.*, PRB 1984:

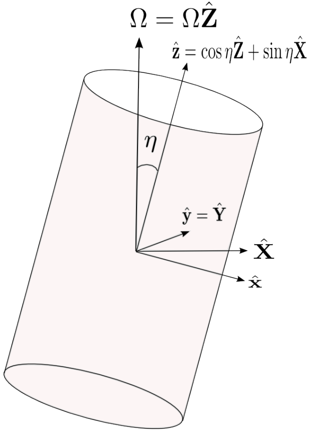
- Using continuous model for vorticity
- Rectangular cavity with infinite lenght along y-direction
- Assumed that $\omega \parallel \mathbf{\Omega}$ deep in the cavity



Tilted rotating cylinder

Motion of ideal fluid:
Consider a situation (at some particular time) where $\mathbf{\Omega} = \Omega\hat{\mathbf{Z}}$ and the cylinder axis is along $\hat{\mathbf{z}} = \cos\eta\hat{\mathbf{Z}} + \sin\eta\hat{\mathbf{X}}$. Cartesian$(X, Y, Z)$ and cylindrical (ρ, ϕ, z) coordinates are related by

$$
\begin{aligned}
X &= \rho\cos\phi\cos\eta + z\sin\eta \\
Y &= \rho\sin\phi \\
Z &= -\rho\cos\phi\sin\eta + z\cos\eta.
\end{aligned}
$$



Tilted rotating cylinder

In cylindrical coordinates $\boldsymbol{\Omega}$ and $\mathbf{r}$ are given by

$$\begin{aligned}\boldsymbol{\Omega} &= \Omega\left[-\sin\eta\cos\phi\hat{\boldsymbol{\rho}}+\sin\eta\sin\phi\hat{\boldsymbol{\phi}}+\cos\eta\hat{\mathbf{z}}\right]\\ \mathbf{r} &= \rho\hat{\boldsymbol{\rho}}+z\hat{\mathbf{z}}\end{aligned}$$

Without vortices $\mathbf{v}=\nabla\Phi \Rightarrow \nabla^2\Phi=0$.

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

Boundary condition: $\frac{\partial\Phi}{\partial n}=\hat{\mathbf{n}}\cdot(\boldsymbol{\Omega}\times\mathbf{r})$ implies:

$$\frac{\partial\Phi}{\partial\rho}_{|\rho=R}=\Omega z\sin\eta\sin\phi,\qquad \frac{\partial\Phi}{\partial z}_{|z=\pm L/2}=-\Omega\rho\sin\eta\sin\phi$$

These are in lab. frame. In rotating frame:

$$\mathbf{v}=\nabla\Phi-\boldsymbol{\Omega}\times\mathbf{r}$$

Tilted rotating cylinder

Trial $\Phi(\rho,\phi,z) = \Omega(\rho z + g(\rho,z))\sin\phi\sin\eta$:

$$\begin{aligned}
\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial g}{\partial\rho}\right) + \frac{\partial^2 g}{\partial z^2} - \frac{1}{\rho^2}g &= 0 \\
\frac{\partial g}{\partial\rho}_{|_{\rho=R}} = 0 \qquad \frac{\partial g}{\partial z}_{|_{z=\pm L/2}} &= -2\rho.
\end{aligned}$$

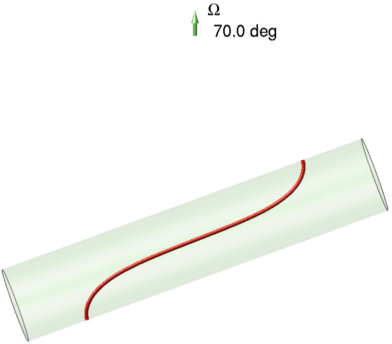
Velocity components in the rotating frame are then given by

$$\begin{aligned}
v_\rho &= \Omega\sin\phi\sin\eta\frac{\partial g}{\partial\rho} \\
v_\phi &= \Omega\cos\phi\sin\eta\frac{g}{\rho} - \Omega\rho\cos\eta \\
v_z &= \Omega\sin\phi\sin\eta(2\rho + \frac{\partial g}{\partial z})
\end{aligned}$$

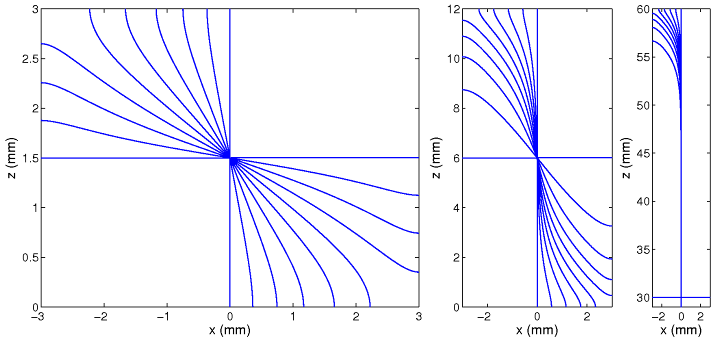
Now one can do vortex filament calculations in rotating frame with $\mathbf{v}_n = 0$ and $\mathbf{v}_s = \mathbf{v} + \mathbf{v}_\omega + \mathbf{v}_b$, where $\mathbf{v}$ is due to rotation.

Single vortex inside rotating tilted cylinder

Minimum energy configuration when:
$\Omega = 50$ mrad/s
$R = 3$ mm
$L = 30$ mm
$(T = 0.5T_c)$



Configuration for Singe Vortex, $\Omega = 0.5\text{rad}/\text{s}$



Here $\eta = 0, 10, 20, \ldots,$ and 90 degrees. Vortex is practically on the xz-plane (defined by rotation and cylinder axis). (*Left*) $L = R = 3$mm, (*Middle*) $L = 4R = 12$mm, and (*Right*) $L = 20R = 60$mm (only upper half is plotted)

Vortex array in infinitely long cylinder (g=0)

$$v_\rho = 0 \qquad v_\phi = -\Omega\rho\cos\eta \qquad v_z = 2\Omega\rho\sin\phi\sin\eta$$

Vortex density due to reduced azimuthal counterflow is

$$n_v = 2\Omega\cos\eta/\kappa \tag{1}$$

Axial velocity affects the vortex configuration only when it is above the Glaberson critical value. Since $v_{\mathrm{z,max}} = 2\rho\Omega\sin\eta$, the vortices suffer the instability if ($\nu = \kappa\ln(1/ka_0)/(4\pi) \approx \kappa$)

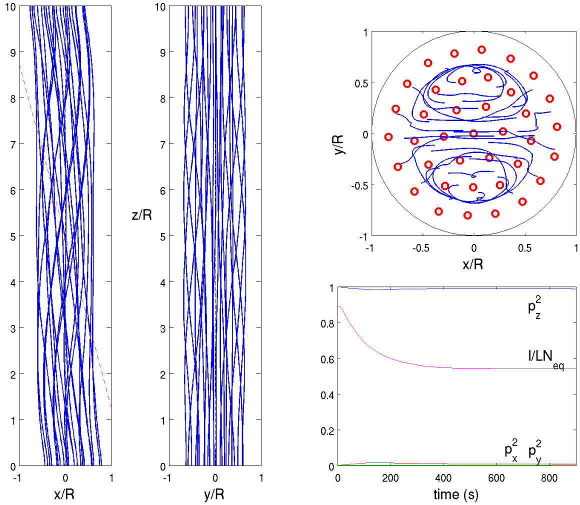
$$N > N_{\mathrm{c}} = 2\pi\frac{\nu}{\kappa}\cot^2\eta \approx 2\pi\cot^2\eta. \tag{2}$$

Numerically(* with $\eta = 30^\circ$ ($N_c \approx 19$) a vortex cluster with N=22 is stable and N=25 Kelvin waves appear.

* with $L = 30$ mm (periodic b.c.), $R = 6$ mm, $\Omega = 100$ mrad/s and $T = 0.5T_c$

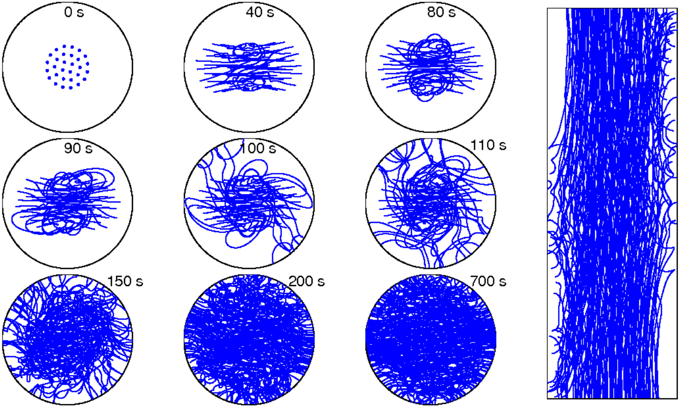
Finite cylinder, small tilt or Ω

$L = 10R = 30\text{mm}$, $\Omega = 50\text{mrad/s}$, $\eta = 15^o$, **static** steady state:



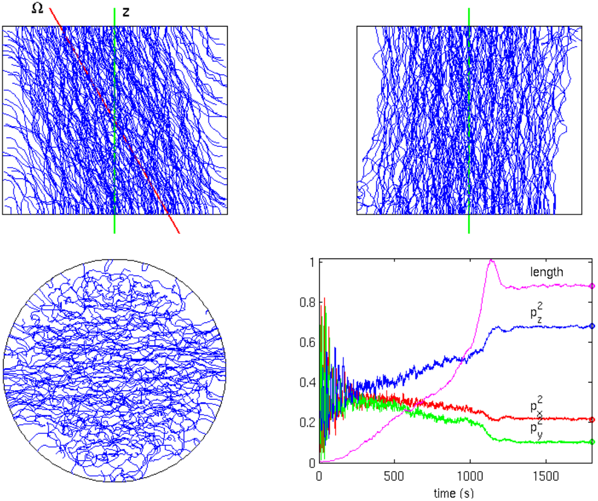
Finite cylinder, large tilt or Ω

$L = 6R = 18\text{mm}$, $\Omega = 0.25\text{rad/s}$, $T = 0.4T_c$, $\eta = 30^o$, **dynamic** steady state:



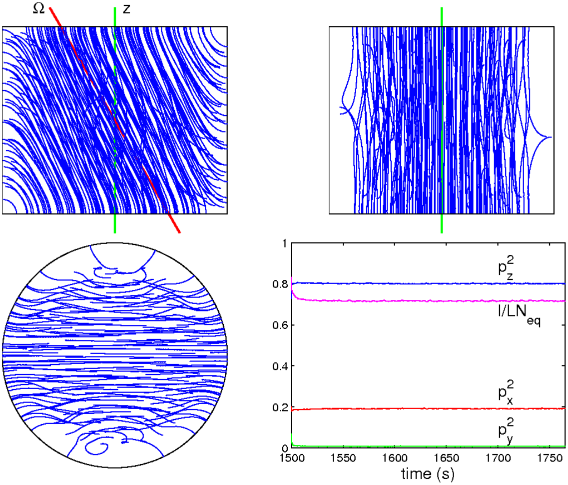
Finite cylinder, large tilt or Ω

$L = 5\text{mm},\ R = 3\text{mm},\ \Omega = 0.25\text{rad/s},\ T = 0,\ \eta = 30^o$:



Finite cylinder, large tilt or Ω

$L = 5\text{mm}$, $R = 3\text{mm}$, $\Omega = 0.25\text{rad/s}$, high T $(\alpha = 10, \alpha' = 0.1)$, $\eta = 30^o$:



Tilted rotating cylinder:

- with large enough tilt vortices become unstable
- unexpected configuration for single vortex
- possible to obtain dynamic steady state and polarized turbulence

Thank You Very Much for Your Attention!