The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

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Workshop on Topics in Quantum Turbulence

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Experiments on Bose Condensates

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Atomic gas Bose-Einstein condensates

potential for quantum turbulence studies

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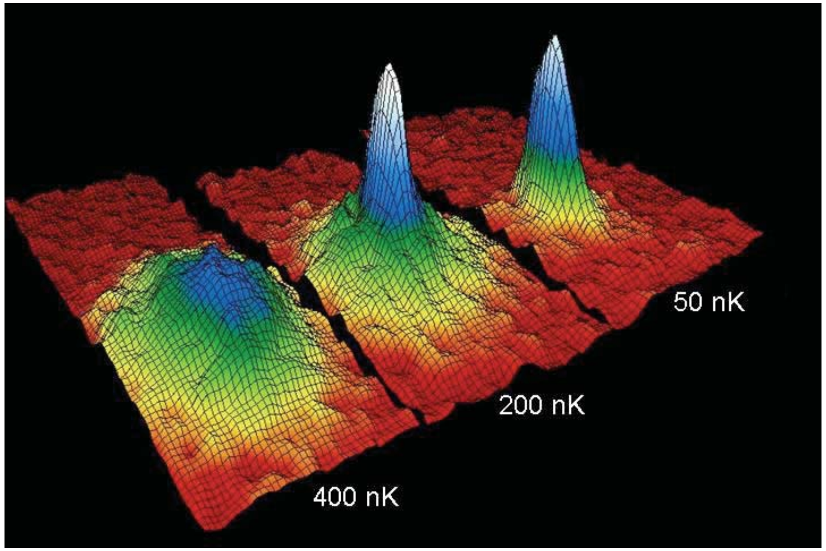
Outline

- Atomic gas BEC properties
- How to make an atomic gas BEC
- Techniques for probing and manipulating
- Superfluid/nonlinear wave phenomena
- Potential for quantum turbulence studies

BEC of a weakly interacting atomic gas

In 1995, a new "quantum liquid" was realized



1995 - BEC achieved in dilute gas of
Rb (JILA), Na (MIT) and Li (Rice).

This achievement leveraged decades of BEC research in the spin polarized hydrogen community

BEC of a weakly interacting atomic gas

The realization of BEC in a weakly interacting atomic gas marked the foray of atomic/optical physics into condensed matter physics… and there appears to be no turning back.

BEC research can be found in major laboratories throughout the world (>50 experimental groups).

Condensates have been produced in Rb, Na, Li, H, K, Cs, He*, Yb, Cr, and homonuclear diatomic molecules

BEC research impacts many areas of physics. Current research if primarily focused on atomic analogs of solid state systems and quantum information science.

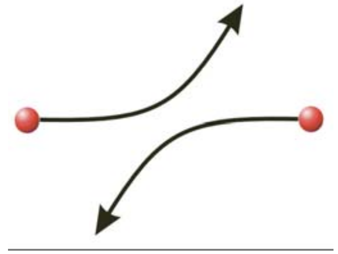
Typical atomic BEC numbers

Contrast with liquid helium

- number of atoms in the condensate, N_0: 10^3 to 10^8, typically 10^5
  much smaller
- densities n: 10^{13} to 10^{15} cm^{-3}
  much less
- characteristic size, R: 10 to 200 μm
  much smaller
- temperatures, T: 50 to 500 nK
  much colder
- nearly pure condensates $N_0/N > 95\%$ (hard to measure at this level)
  much larger
- weakly interacting: chemical potential, μ, 20 to 200 nK
  much weaker
- speed of sound, c_s, typically few mm/s
  much less
- coherence length, ξ, typically 0.3 μm
  much longer

Interactions

Primarily elastic 2-body scattering



Some inelastic 3-body at higher densities,
which leads to heating or loss

At T < ~1 mK, only s-wave for 2-body scattering,
characterized by s-wave scattering length, a_0

Interactions can be taken into account by a mean-field
strength only dependent on a_0

Theoretical description

The Bose condensate is described by a macroscopic wavefunction

$$\Psi(\vec{r}) = \sqrt{n(\vec{r})}e^{i\vartheta}$$

A lot of early research on atomic BECs involved the wave character – atom optics

In the weakly interacting (dilute) regime, $a_0 << n^{-1/3}$,
BEC obeys (quite well) the Gross-Pitaevskii equation (GPE)
a form of the non-linear Schrödinger equation

$$\left(\underbrace{-\frac{\hbar^2}{2m}\nabla^2}_{\text{kinetic energy}} + \underbrace{V(\vec{r})}_{\text{trap potential}} + \underbrace{\frac{4\pi a_0 \hbar^2}{m}|\Psi|^2}_{\text{interaction energy (density dependent)}}\right)\Psi = \underbrace{\mu}_{\text{chemical potential}}\Psi$$

Correlations accounted for by Bogoliubov equations

→ Quantum depletion $\sim\sqrt{na_0^3}$ is typically less than 1%

This description has been highly successful.

Thomas-Fermi distribution

We can use the GPE equation to predict the shape of the BEC

Typically, kinetic energy is much less than the interaction energy
→ neglect the kinetic energy term.

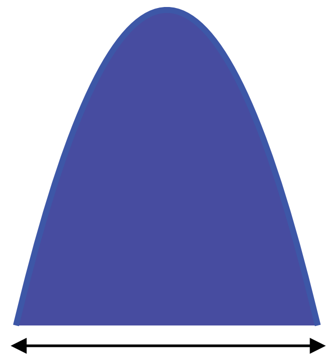
Condensates are usually made in a harmonic potential

→ $$V(r) = \frac{1}{2} m\omega^2 r^2$$

At T=0 (steady state), the GPE becomes $\left(V(\vec{r}) + U_0 |\Psi|^2\right)\Psi = \mu\Psi$

$$|\Psi|^2 = (\mu - m\omega^2 r^2)/U_0$$

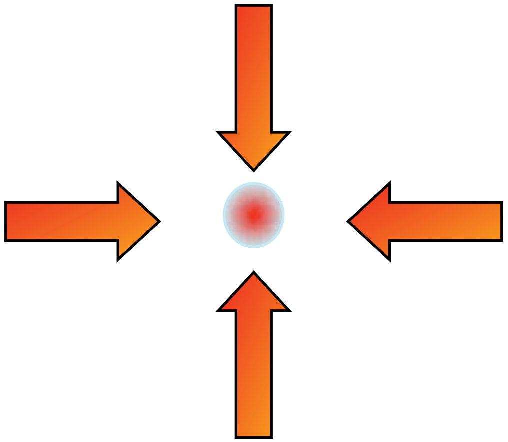
BEC density is an inverted parabola



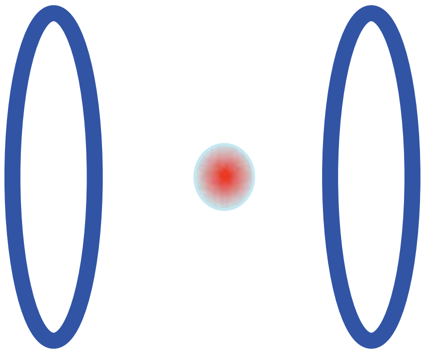
Characteristic size is given by R_{TF}
the Thomas-Fermi radius

How to make an atomic gas BEC

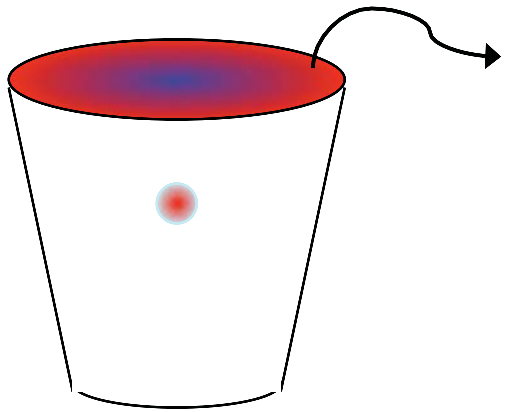
Basically 3 “easy” steps



laser cool
and collect atoms



transfer to a static trap
(conservative potential)



cool by
evaporation

cool 10 orders of magnitude

T~500 K   T~50 μK   T~50 nK

Types of containers (traps) for atoms

The low (nK) temperatures of the Bose gas prohibits confinement in a material container.

Typically, the atoms are held in either magnetic traps or optical traps, in an ultrahigh vacuum environment.

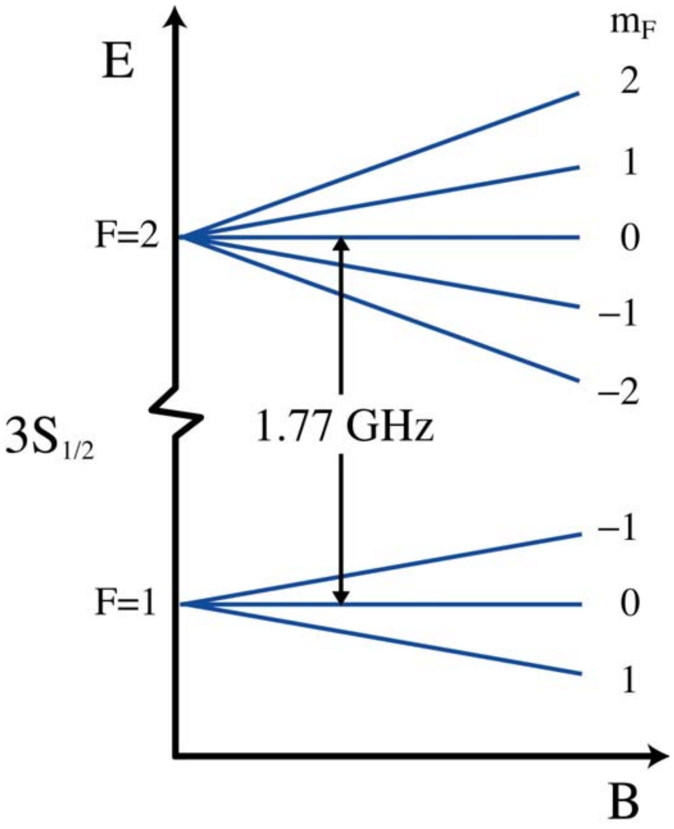
Magnetic Traps

Zeeman effect

Energy due to magnetic moment $\vec{\mu}$ interacting with a magnetic field $\vec{B}$

$$U = -\vec{\mu} \cdot \vec{B}$$

$$\vec{F} = \vec{\nabla}(\vec{\mu} \cdot \vec{B})$$



Can trap atoms in a suitable gradient magnetic field

Maxwell's equations only allow magnetic field minima in free space

⇨ Can only trap weak field seeking states (not the lowest energy state)

Advantages:

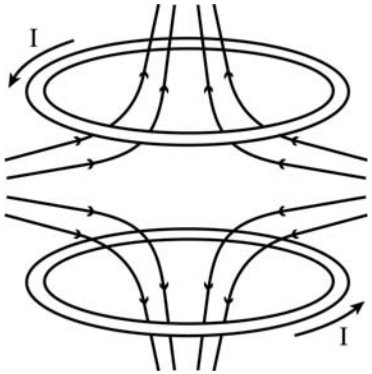
- conservative (static) potential
- large volume
- low cost

Disadvantages:

- can not trap the true ground state
- optical access is often diminished

Magnetic Traps (cont.) - Some examples

Quadrupole Trap

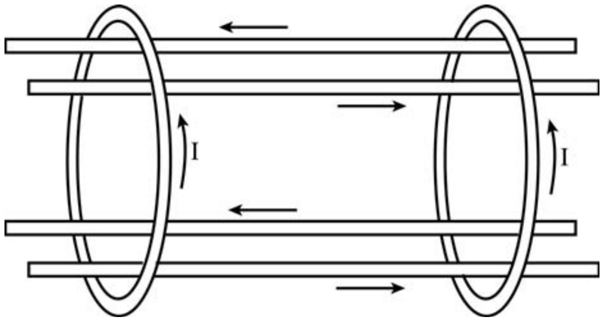


$$B \propto \sqrt{x^2 + y^2 + 4z^2}$$

Simplest design, but has a zero in the middle
can lose cold atoms due to Majorana transitions

1st neutral atom trap demonstrated

Ioffe Trap



$$B = B_0 + \alpha(x^2 + y^2) + \beta z^2$$

Not as strong as quadrupole
but has a bias field (no zero)

Magnetic traps typically made with field coils
Other possibilities: Permanent magnets
Microfabricated wires – “chip traps”

MPQ
Hänsch group

Optical Dipole Force Trap

Due to the AC Stark shift

Energy of dipole in electric field: $U_{dip} = -\vec{d} \cdot \vec{E}$

Oscillating electric field induces an oscillating dipole in the atom: $\vec{d} = \alpha\vec{E}$ $\quad \alpha$ polarizability

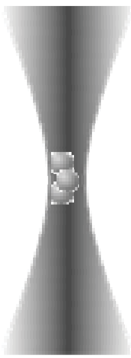
$U_{dip} = -\alpha E^2$ $\quad E^2 \propto I$ the laser intensity

In general, $\alpha = \alpha(\omega)$

$\alpha > 0$ for $\omega - \omega_{atom} < 0$ (d in phase with E)
$\alpha < 0$ for $\omega - \omega_{atom} > 0$ (d out of phase with E)

Red detuning:

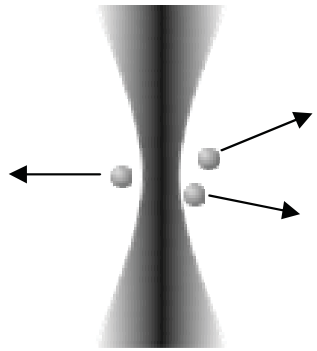
Atoms are trapped in the intensity maxima



$\omega_{laser} - \omega_{atom} < 0$

Blue detuning:

Atoms are repelled from the intensity maxima



$\omega_{laser} - \omega_{atom} > 0$

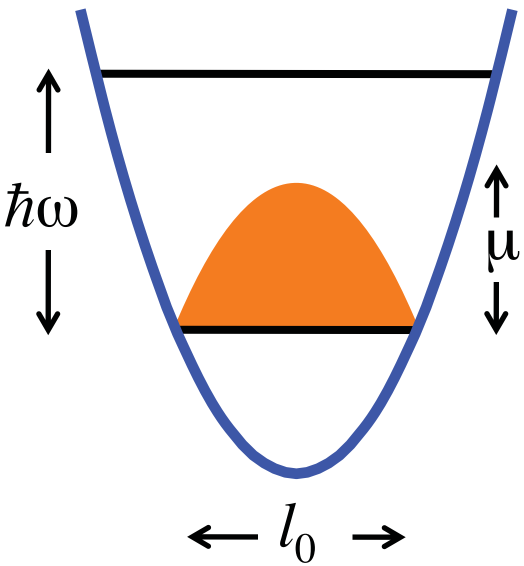
Wide variety of BEC clouds

Atomic BECs have been created in different shapes and sizes

spherical, pancake (oblate), cigar-shaped, ring-shaped

with different dimensionalities

3D, 2D and 1D



Reduced dimensionality $\longrightarrow$ confinement in (bare) ground state of trap

$\mu < \hbar\omega$

Collisions are still 3D ($a_0 < l_0$)

The dimensionality can often be continuously changed.

Probing and manipulating the BEC

Basically, two techniques:

Optical

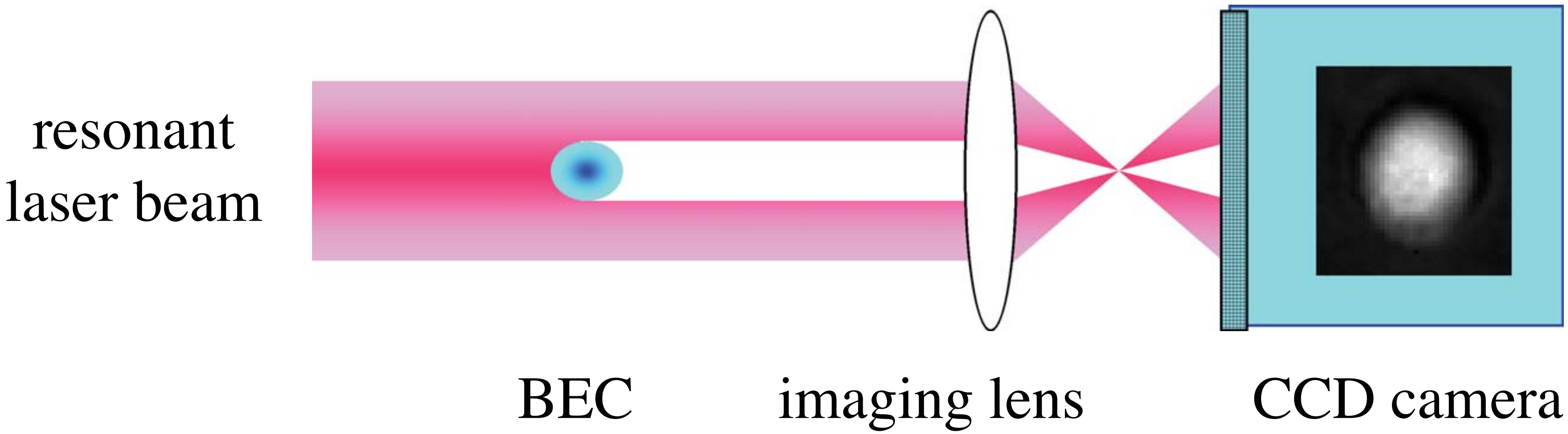
Shape deformation

Probing the atomic gas BEC

Imaging the BEC

By far, the most popular technique for observing the BEC is to take a picture of the cloud.

Typically use absorption imaging of a resonant laser beam



This provides a measurement of the spatial profile of the line density

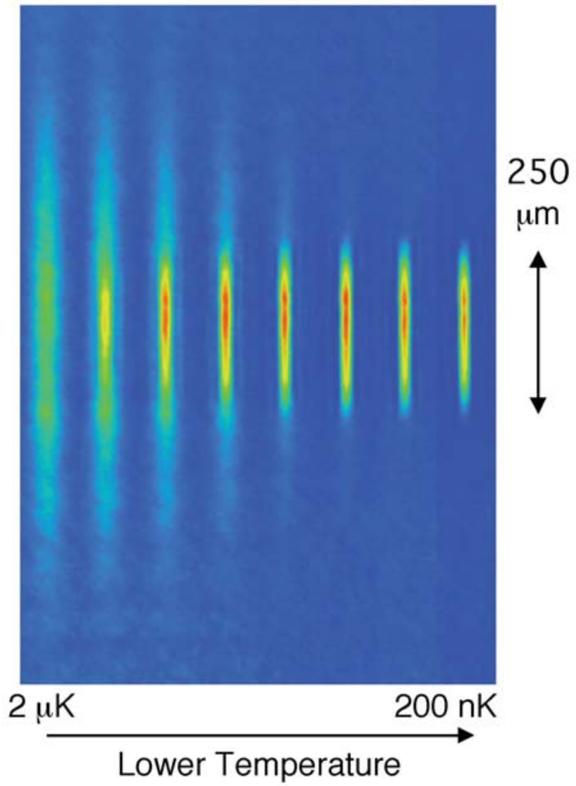
Lambert – Beers law $$I(x,y) = I_0(x,y)e^{-\sigma \int n(x,y,z)dz}$$

This technique is destructive, destroys the condensate.

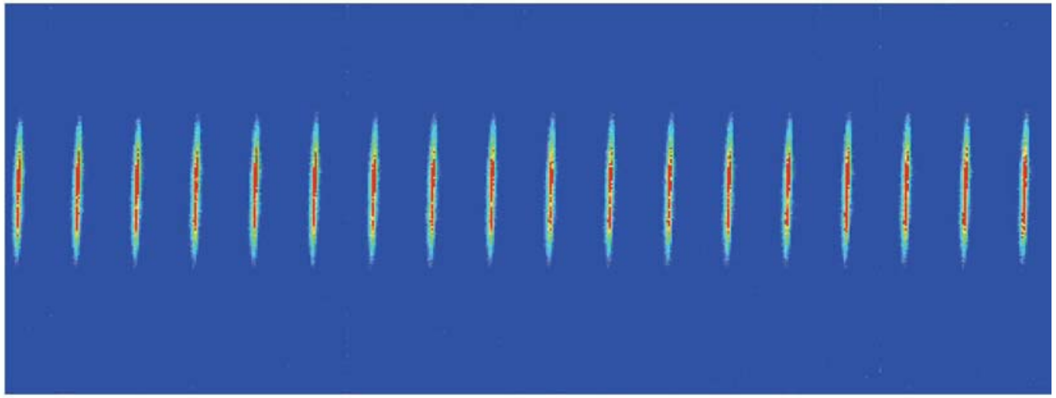
Probing the atomic gas BEC (cont.)

Phase contrast imaging enables taking multiple pictures, since each image only results in a small absorption of the probe beam.

MIT: M.R. Andrews, *et al.*, *Science* **273**, 84 (1996)



1 ms per frame

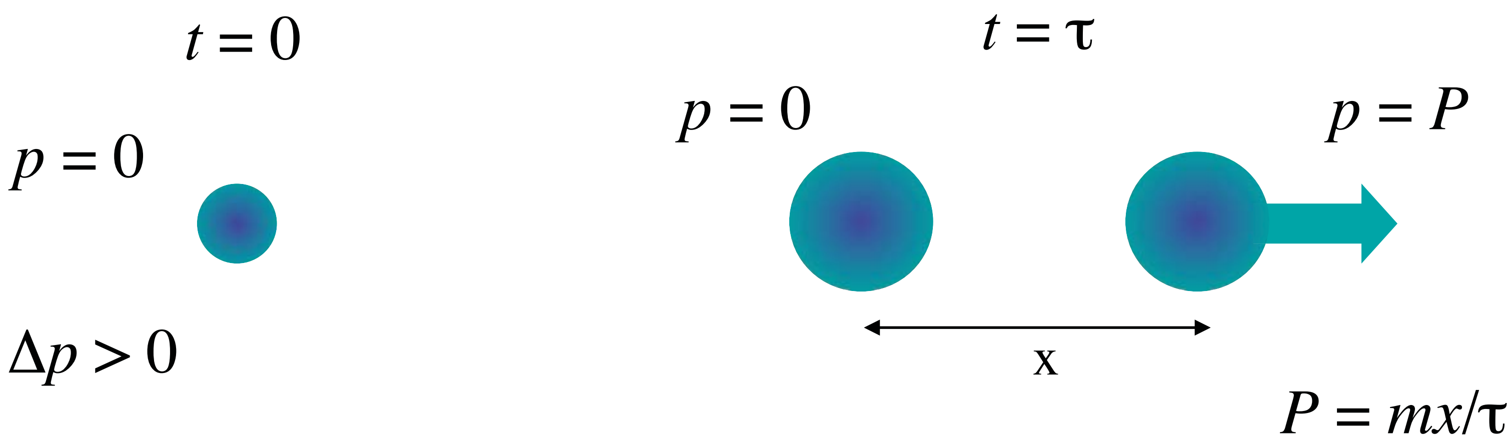


Minimal absorption per frame

Probing the atomic gas BEC (cont.)

Measuring the momentum distribution

Turn off the trap and let the atomic cloud evolve for some time-of-flight (TOF) period

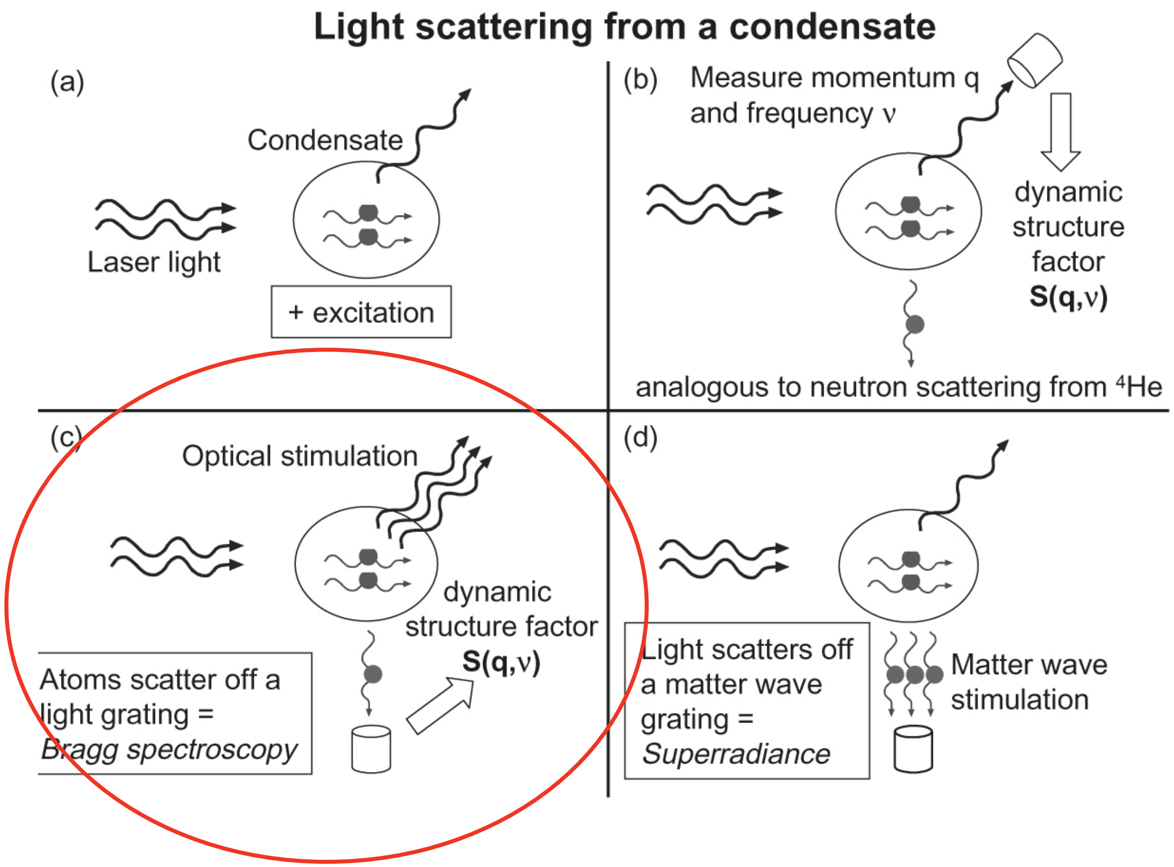


Measure both momentum spread and center of mass momentum of the atomic cloud

Scattering Properties

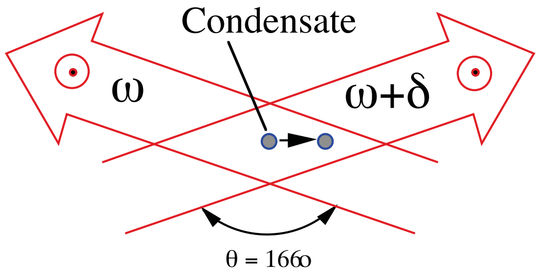
atom-atom scattering and atom-photon scattering

Number of ways for light to scatter off a BEC



Bragg Scattering

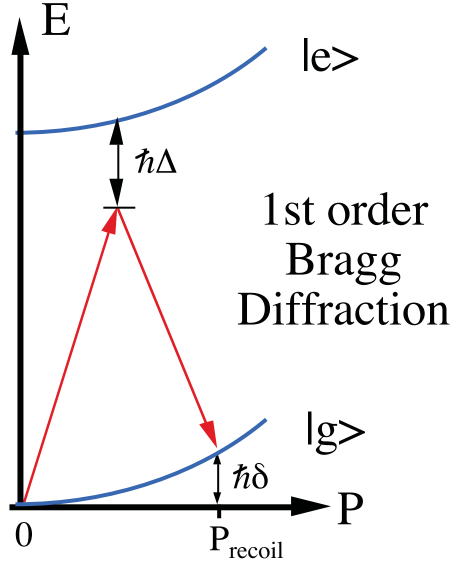
$(\Delta m_F = 0)$



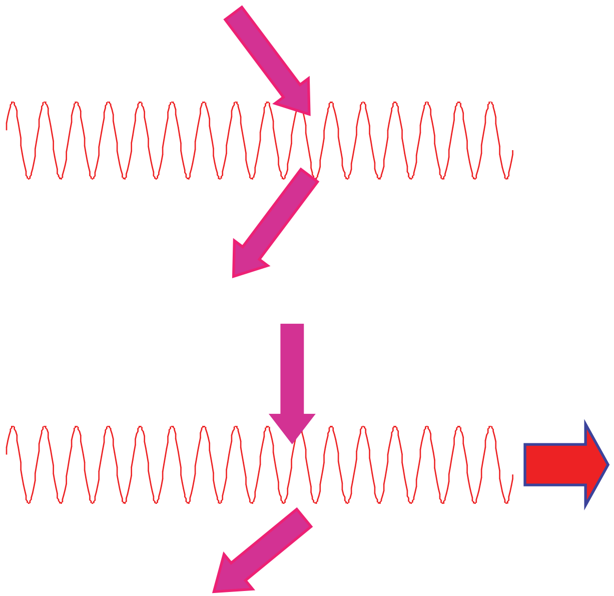
$P_{recoil} = 2\hbar k \sin(\theta/2) \approx 2\hbar k$

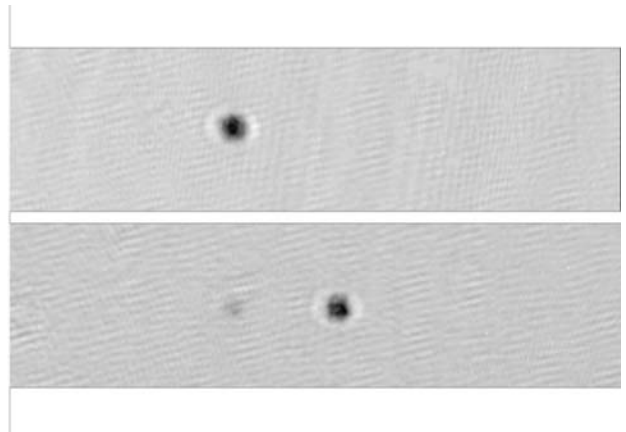
$\hbar\delta = P^2/2M$

$\hbar\Delta \sim 1.8\text{GHz}$



Atoms scatter off light grating
(analogous to Bragg scattering of x-rays)





BEC before
$p = 0$

BEC after
$p = 2\hbar k$

NIST: M. Kozuma *et al*., PRL **82**, 871 (1999)

Bragg spectroscopy

Use 2-photon Bragg scattering to measure the velocity distribution

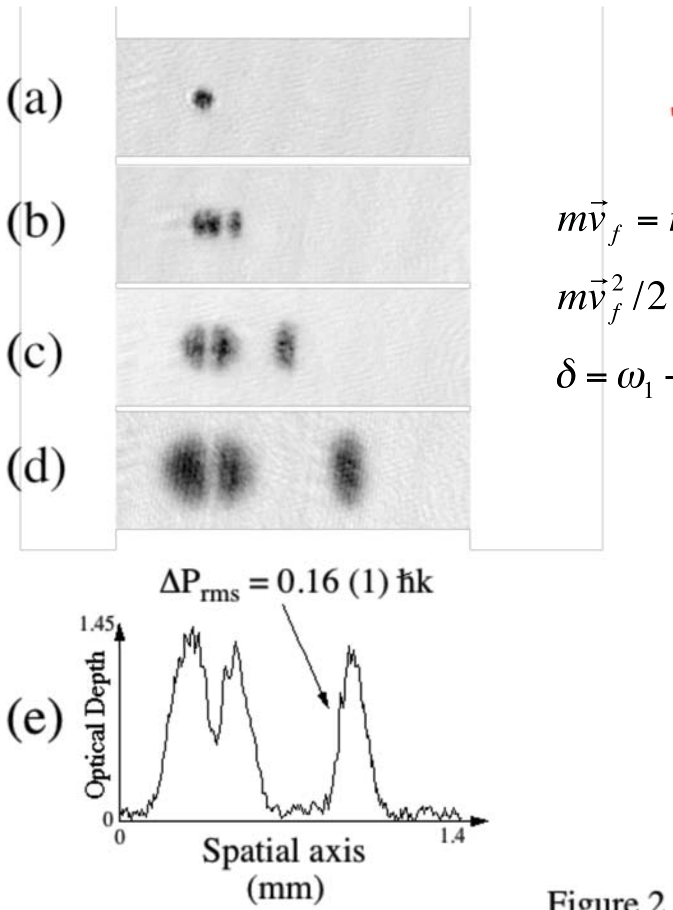
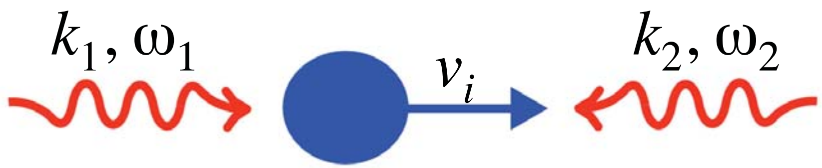


Figure 2

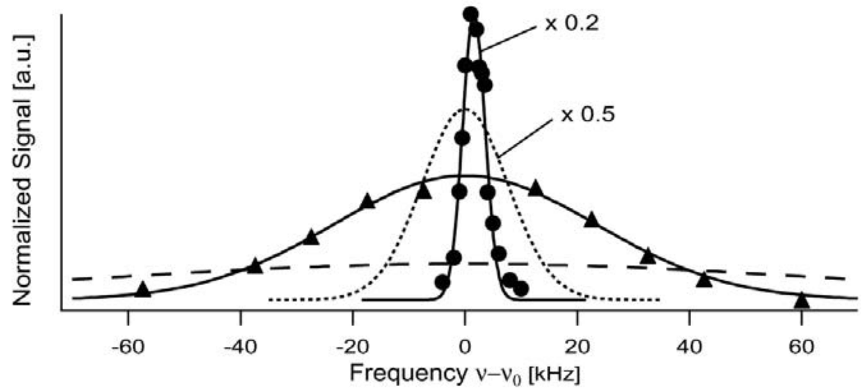


$$m\vec{v}_f = m\vec{v}_i + \hbar(\vec{k}_1 - \vec{k}_2)$$

$$m\vec{v}_f^{\,2}/2 = m\vec{v}_i^{\,2}/2 + \hbar\vec{v}_i(\vec{k}_1 - \vec{k}_2) + \hbar^2(\vec{k}_1 - \vec{k}_2)^2/2m$$

$$\delta = \omega_1 - \omega_2 = m(\vec{v}_f^{\,2} - \vec{v}_i^{\,2})/2\hbar = \vec{v}_i(\vec{k}_1 - \vec{k}_2) + \hbar(\vec{k}_1 - \vec{k}_2)^2/2m \approx 2kv_i + 4\omega_R$$

Doppler shift

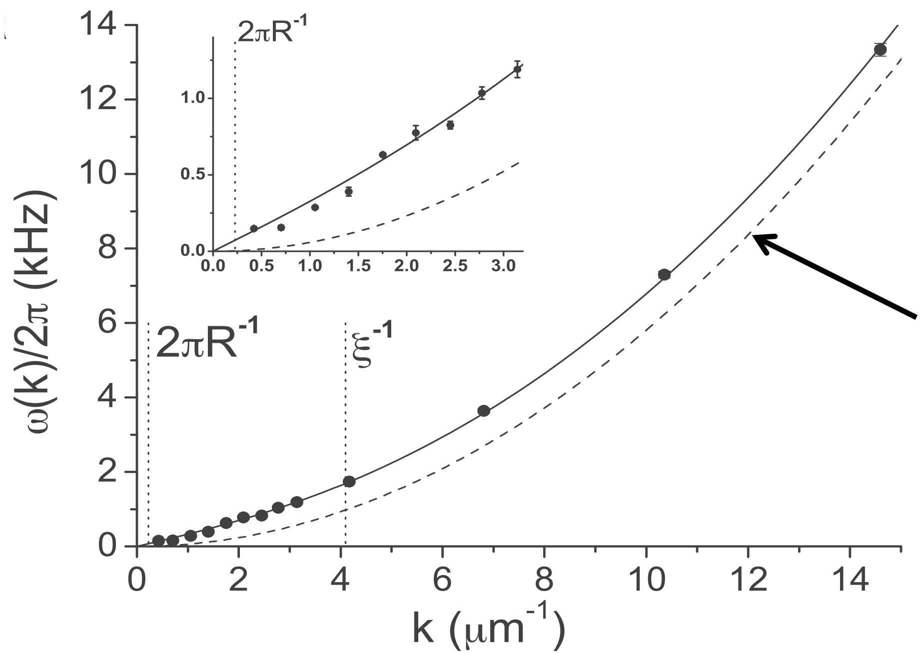


NIST: M. Kozuma *et al.*, PRL **82**, 871 (1999)

MIT: J. Stenger *et al.*, PRL **82**, 4569 (1999)

Measurement of excitation spectrum

Bogoliubov spectrum measured by Bragg spectroscopy



$$E(p) = \sqrt{\varepsilon_p\left(\varepsilon_p + 2mc_s^2\right)}$$

$$\varepsilon_p = p^2/2m$$

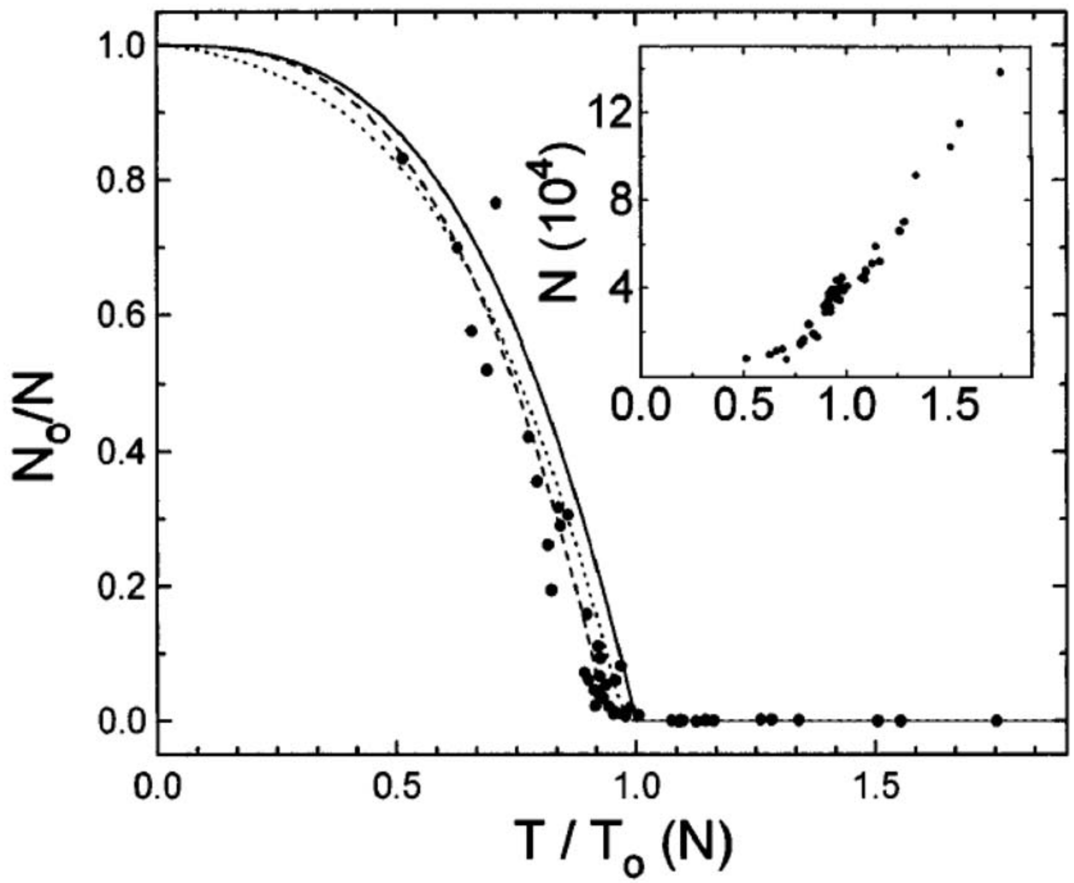
Free particle spectrum

$$E(p) = \varepsilon_p$$

Weizmann: J. Steinhauer *et al*., PRL **88**, 120407 (2002)

BEC Transition

First measurements on a Bose-Einstein condensate of dilute atomic gas



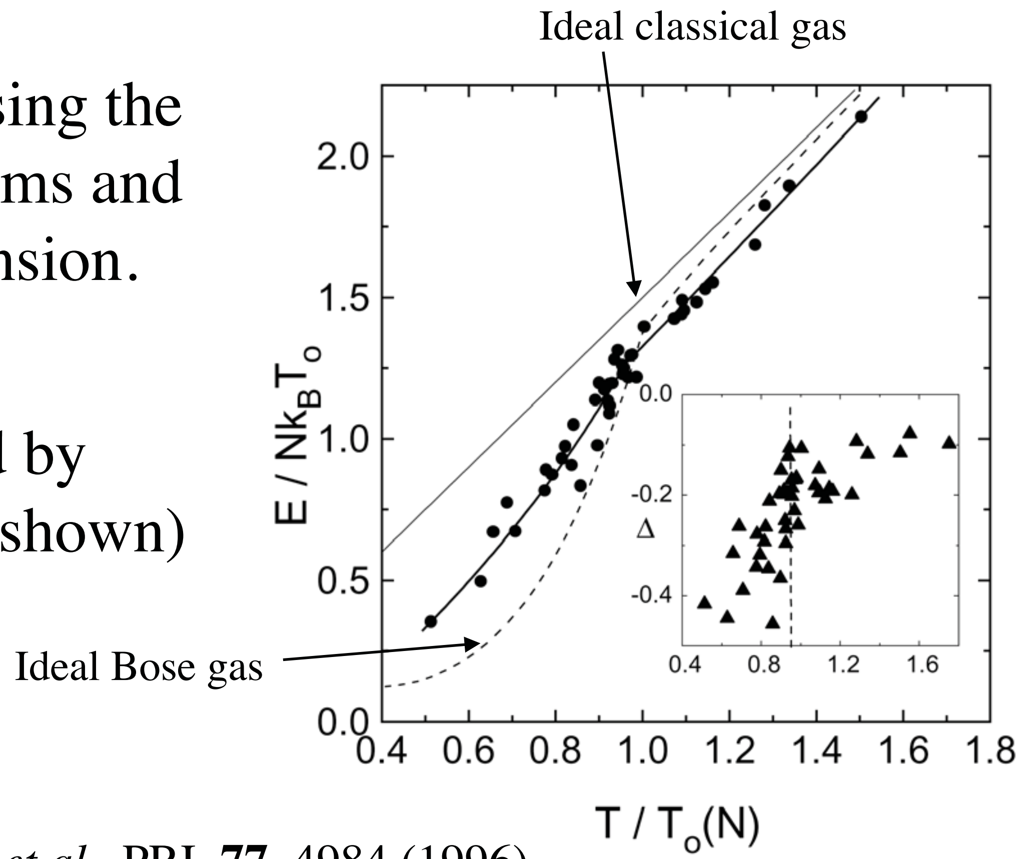
JILA: M.H. Anderson *et al*., Science **269**, 198 (1995)

Specific Heat

Measurement of the energy content (T integral of specific heat)

Data taken by releasing the trapped cloud of atoms and measuring the expansion.

Data well described by finite T theory (not shown)



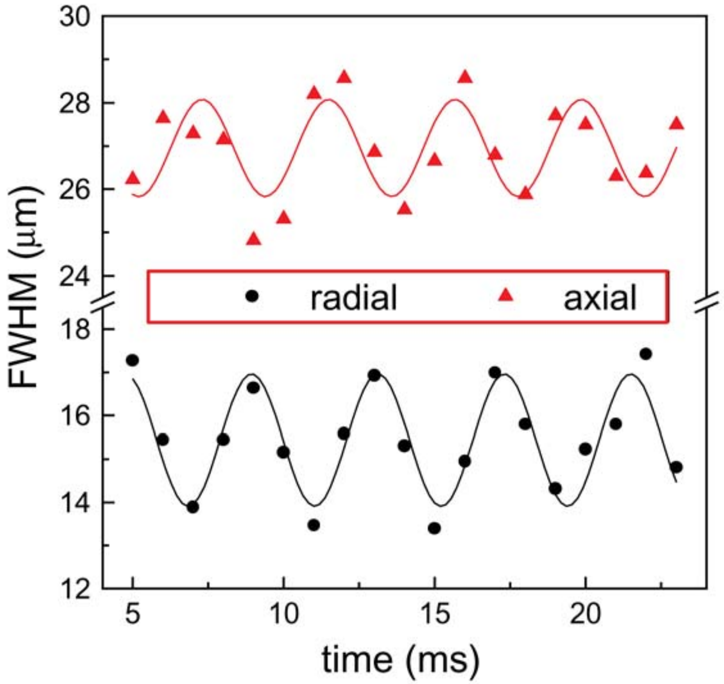
JILA: J.R. Ensher *et al*., PRL **77**, 4984 (1996)

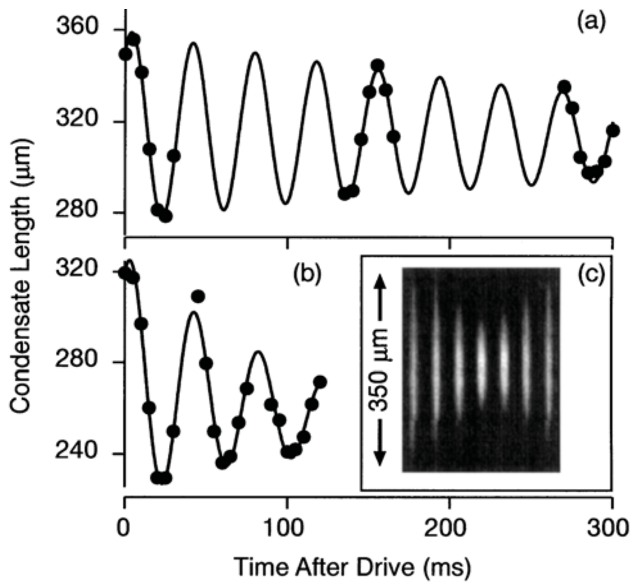
Thermodynamic properties of BEC are very difficult to measure

Excitations of collective modes

Modulate the trapping potential and then release the condensate

Oscillation frequencies shifted from trap frequencies due to interactions





JILA: D.S. Jin *et al.*, PRL **77**, 420 (1996)
D.S. Jin *et al.*, PRL **78**, 764 (1997)

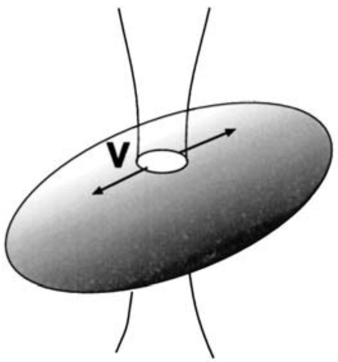
MIT: D.M. Stamper-Kurn *et al.*, PRL **81**, 500 (1998)

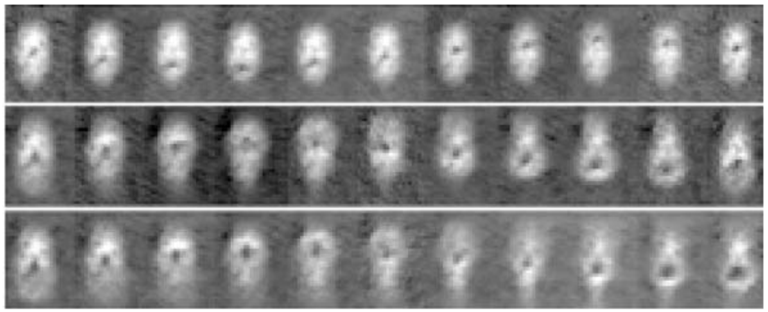
Critical velocity measurements

MIT: R. Onofrio *et al*., PRL **85**, 2228 (2000)

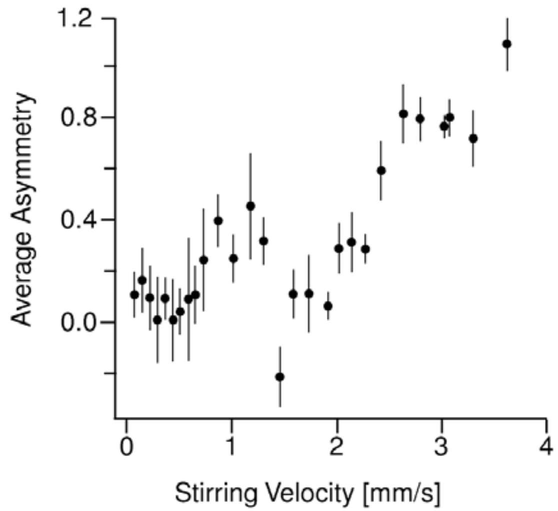
Scan focused, blue-detuned (repulsive potential) laser through BEC

Look for drag of BEC as function of scan speed





in situ images of same cloud

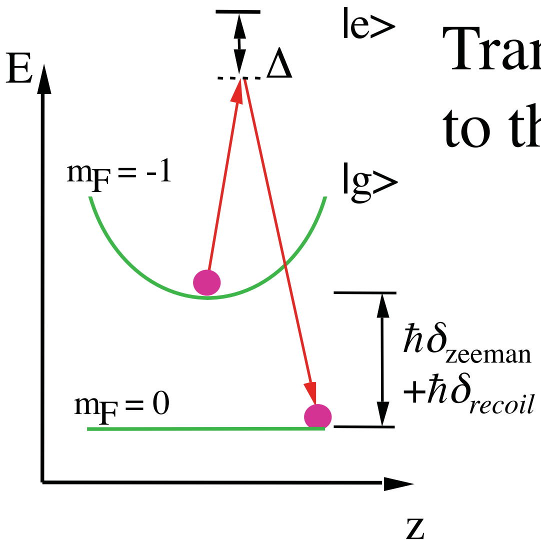


Found critical velocity 5 times smaller than sound velocity $v_c = 5c_s$

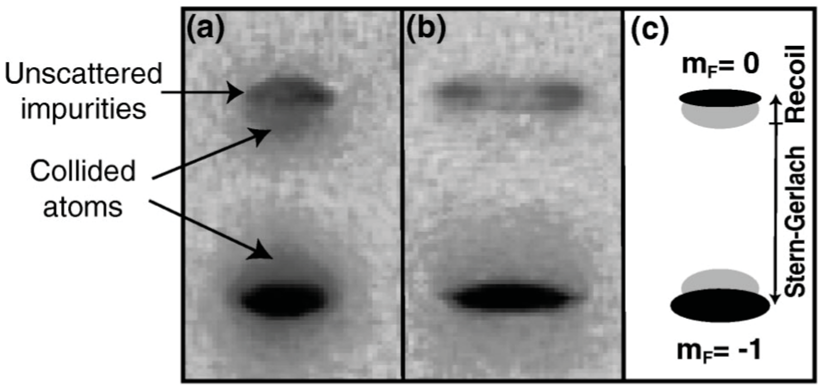
Most likely to due vortex nucleation induced by the scanning laser

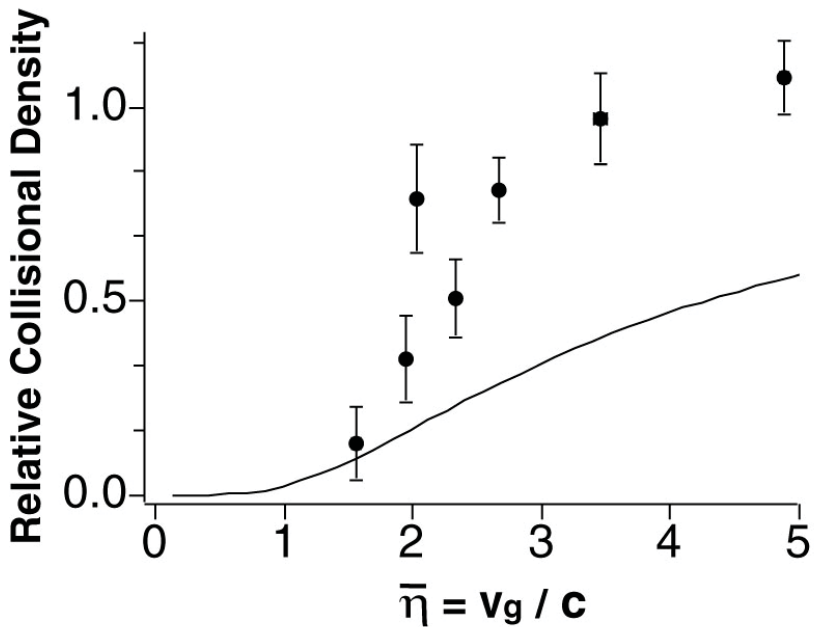
Reduction in scattering below c_s

Analogous to experiments looking at mobility of ions in liquid He.



Transfer a small number of atoms
to the untrapped (m=0) state

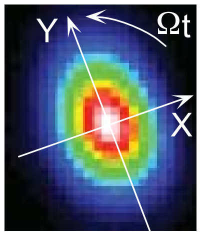


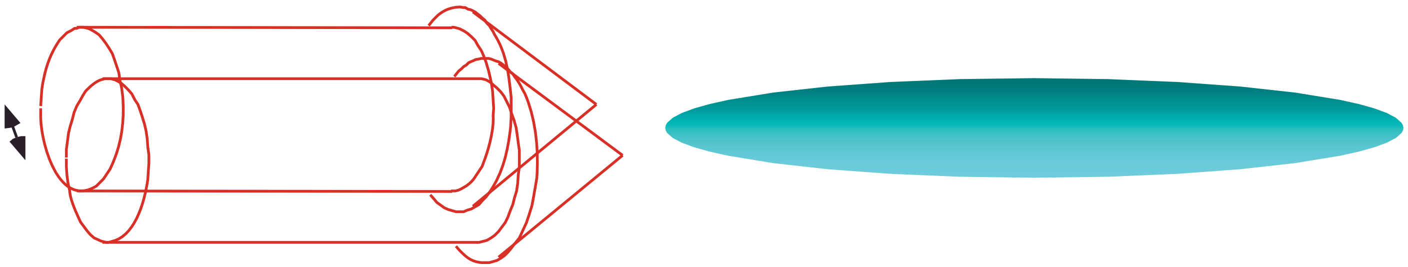


MIT: A. P. Chikkatur *et al*. PRL **85**, 483 (2000).

Generating vortices

Typically generated by "stirring" up the condensate.
Equivalent to rotating an asymmetric trap.





ENS: K.W. Madison *et al*., PRL **84**, 806 (2000)

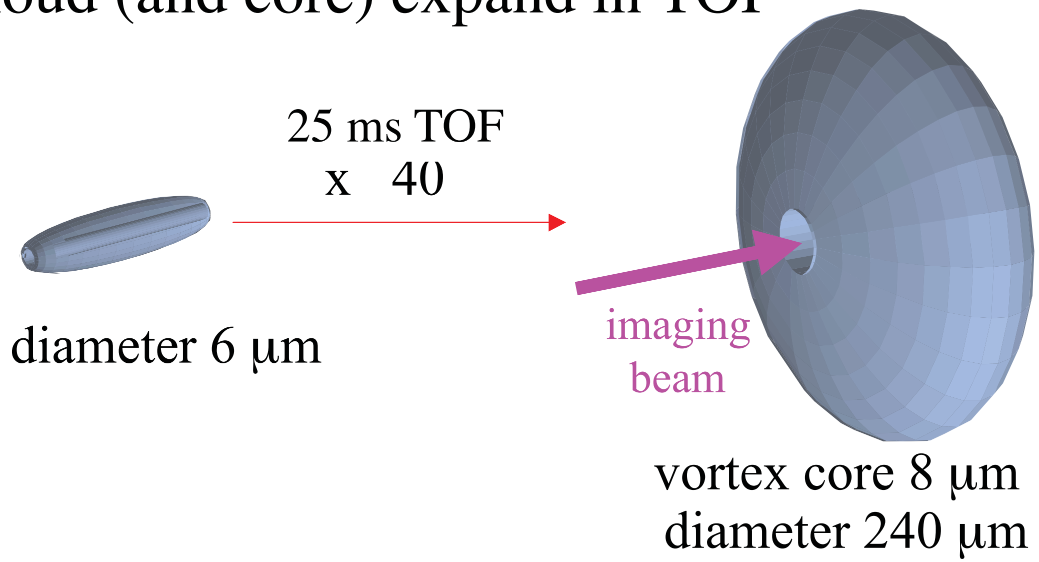
Imaging Vortices

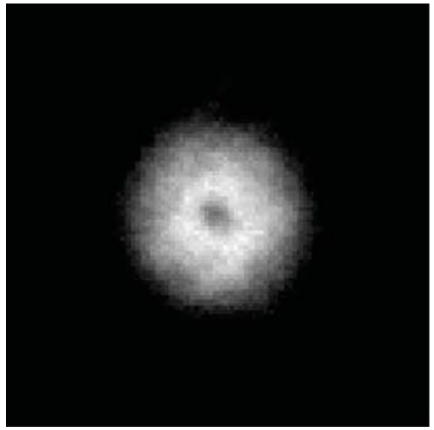
A single vortex is not visible *in situ* (below optical resolution limit)

$$\xi = \frac{1}{\sqrt{8\pi\rho\, a}} = 0.2\,\mu\mathrm{m}$$

$\rho = 2\ \ 10^{14}\ \mathrm{cm}^{-3}$
$a = 5.5\ \mathrm{nm}$

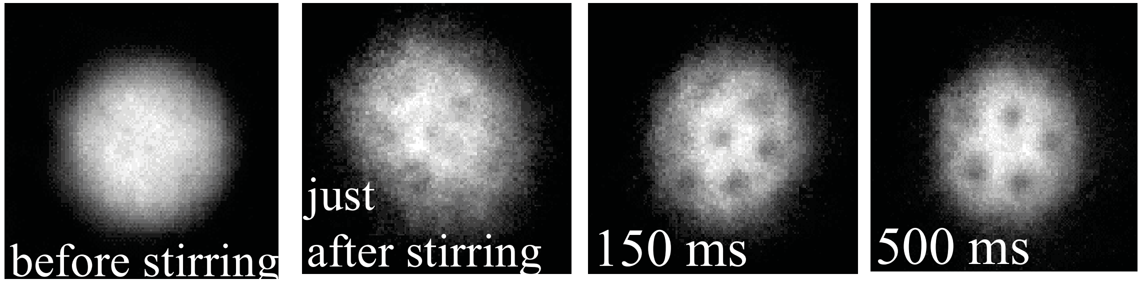
Let cloud (and core) expand in TOF





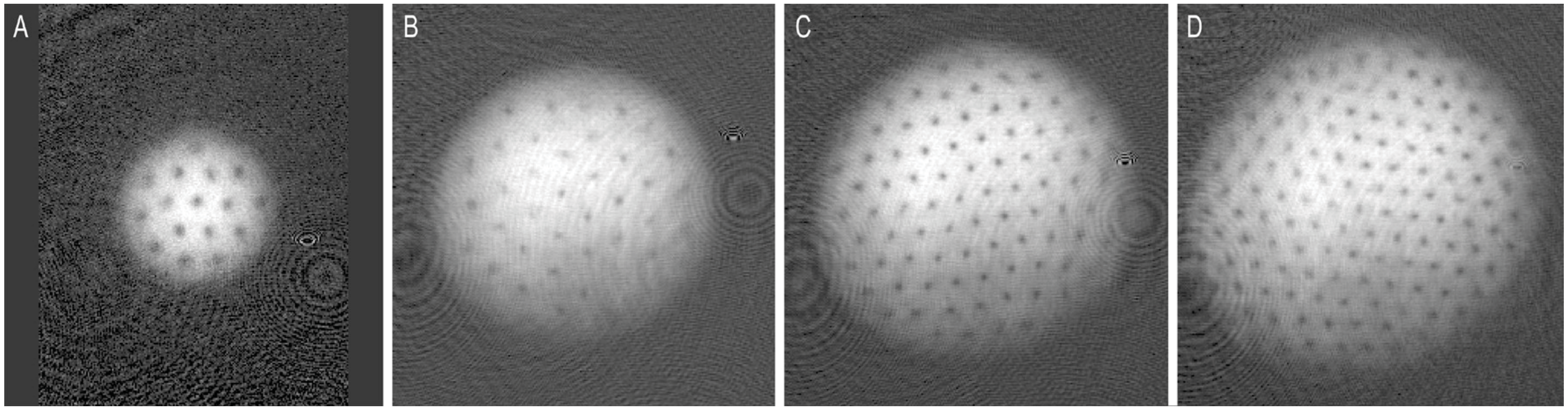
Vortex arrays

Vortices arranged in triangular (Abrikosov) lattice



ENS: K.W. Madison *et al*., PRL **84**, 806 (2000)

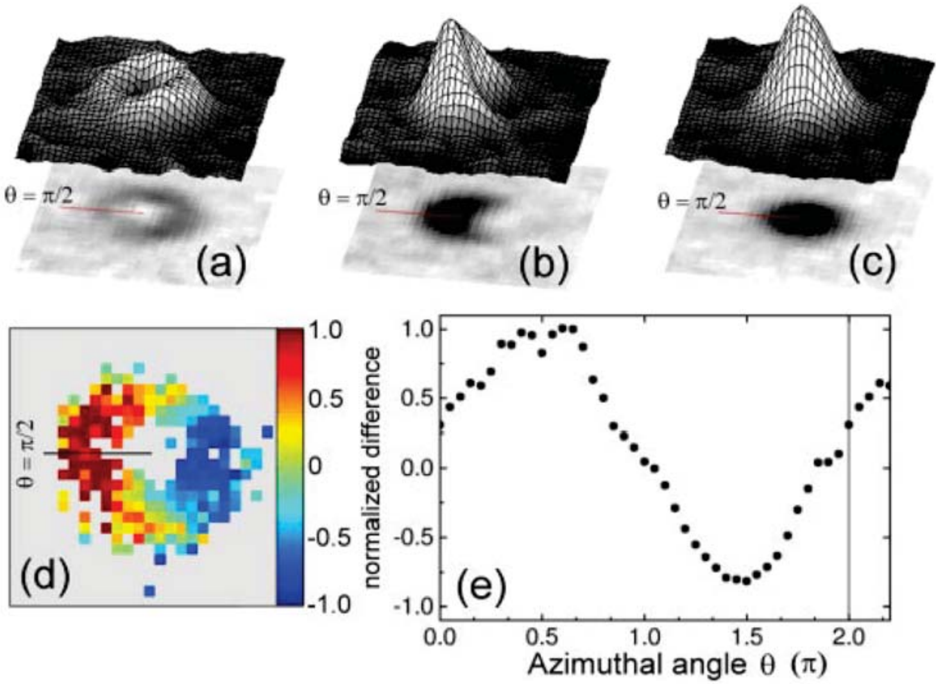
Spinning up large condensates



MIT: J.R. Abo-Shaeer *et al*., Science **292**, 496 (2001)

Up to 300 vortices have been observed observed

Coherent generation of vortices



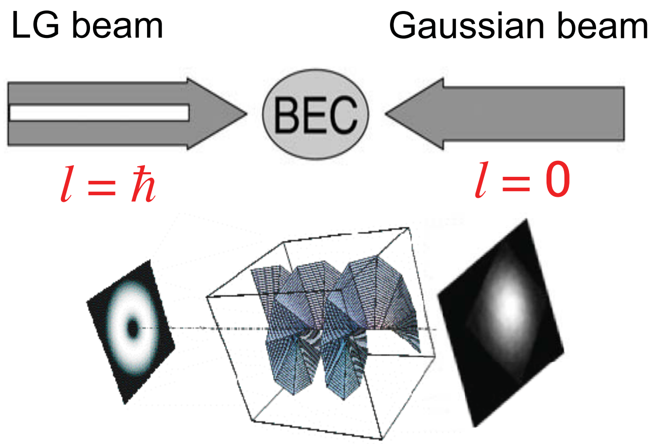
Coupling of 2-component BEC by rf and microwave fields

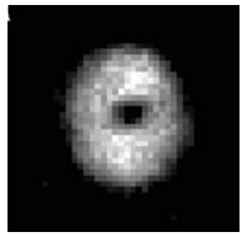
JILA: M.R. Matthews *et al*., PRL **83**, 2498 (1999)

First vortex in atomic BEC

Transfer orbital angular momentum of light to atoms

NIST: M.F. Anderson *et al*., PRL **97**, 170406 (2006)



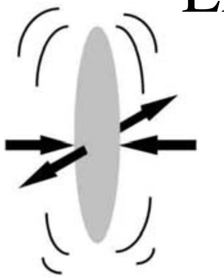


BEC vortex

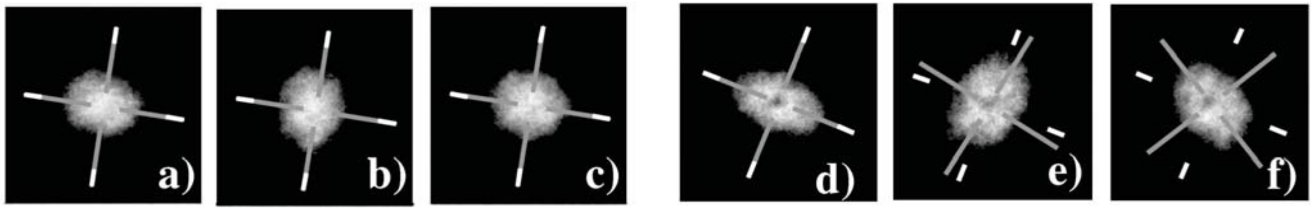
Other ways to detect vortices

Excite oscillations of the cloud – look for changes influenced by vortices

Quadrupolar oscillations ENS: F. Chevy *et al.*, PRL **85**, 2223 (2000)



Excite $m = +2$ and $m = -2$ modes



no vortex vortex

Angular momentum of cloud (due to vortex) lifts degeneracy of modes

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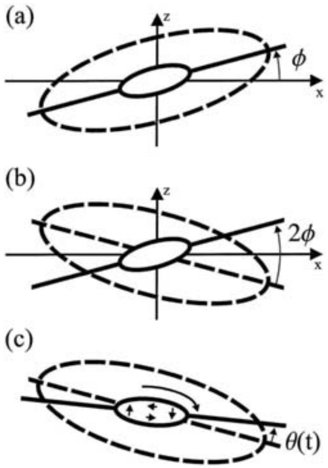
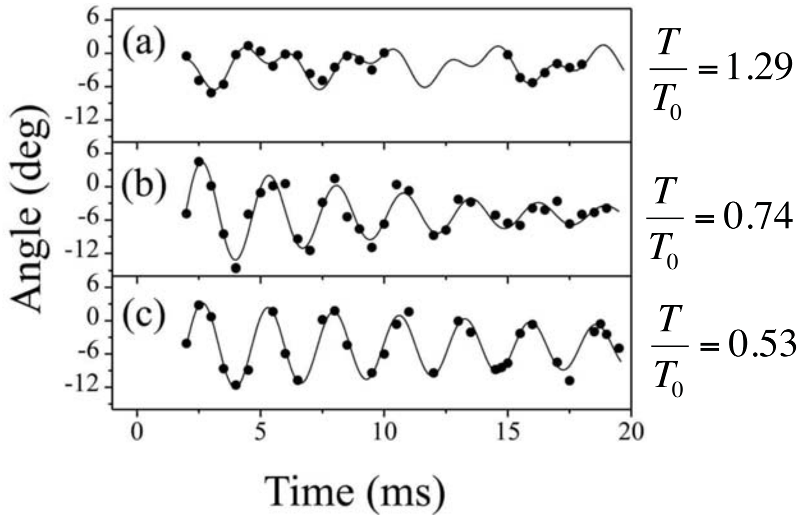
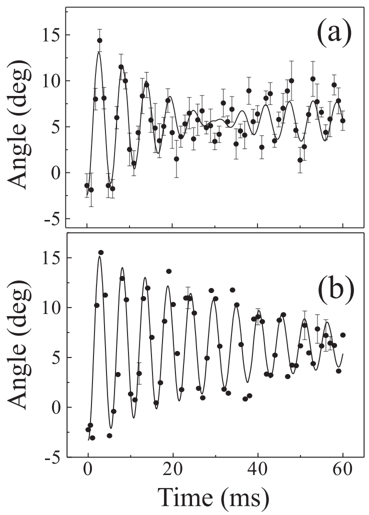
Scissors mode oscillations Analog of vibrating wire detector

Oxford: O. Marago *et al.*, PRL **84**, 2056 (2000), E. Hodby *et al.*, PRL **91**, 090403 (2003)







vortex

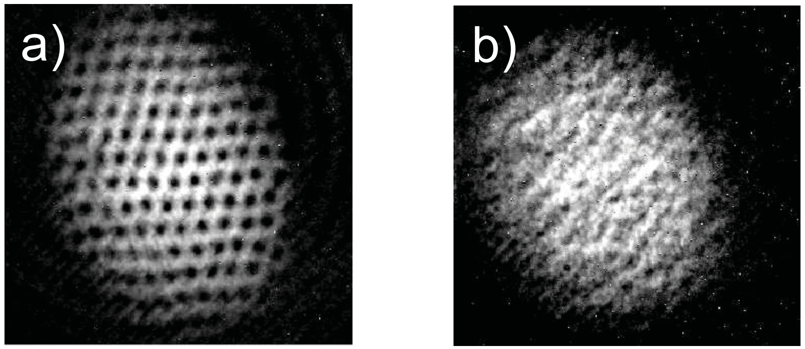
no vortex

Decay of vortex lattice

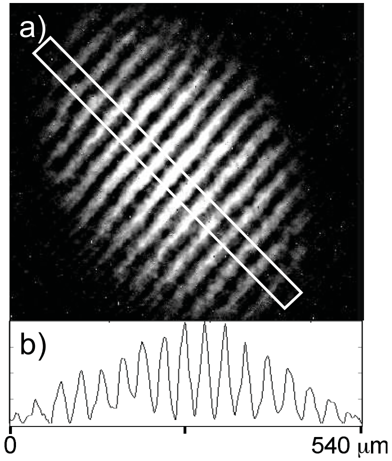
JILA: P. Engels *et al.*, PRL **89**, 100403 (2002)

Excite quadrupolar mode - look at influence on vortex lattice

Excite $m = -2$ mode (statically deform trap)

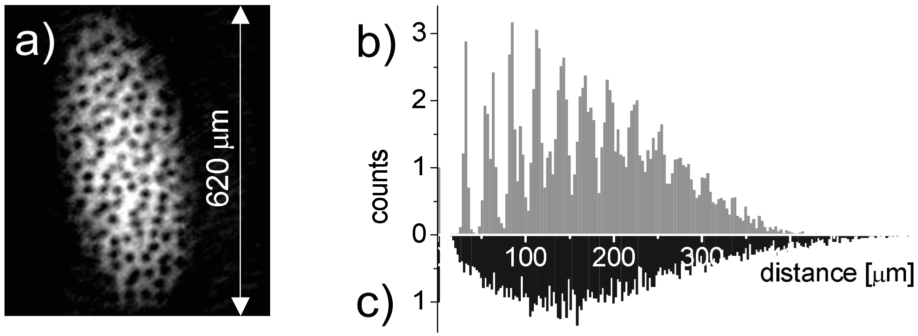


Observe disorder after 400 ms



t < 400 ms, transiently observe strong tilting of lattice

Excite $m = +2$ mode (rotate trap deformation)

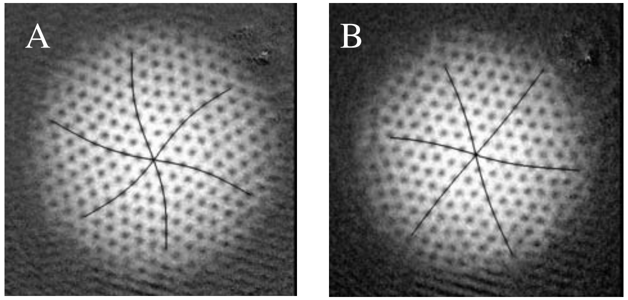


Observe melting of lattice

Other vortex lattice studies

More examples of the power of laser manipulation

Tkachenko modes JILA: I. Coddington *et al*., PRL **91**, 100402 (2003)



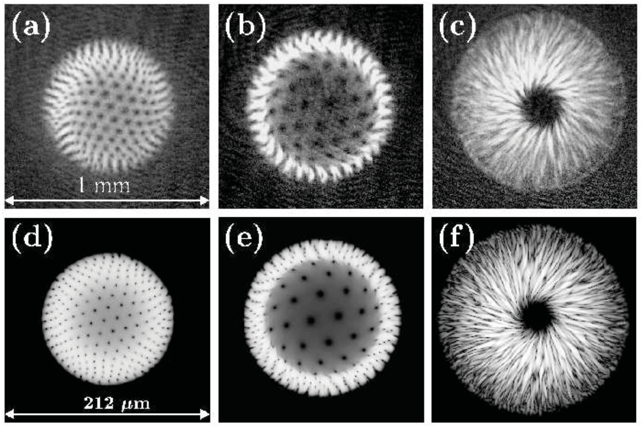
500 ms 1650 ms

Use resonant laser beam to remove some atoms from the central region

Sound propagation JILA: T. P. Simula *et al*., PRL **94**, 080404 (2005)

exp.

theory



pulse duration 5 ms 10 ms 30 ms

Repulsive laser pulse applied to the central region to excite radial sound wave

Kelvin modes

ENS: V. Bretin *et al.*, PRL **90**, 100403 (2003)

Create BEC with single, centered vortex
Excite either $m = +2$ or $m = -2$ quadrupole mode

Observed that $m = -2$ decays faster than $m = +2$

Possibly due to nonlinear decay (Beliaev) to Kelvin mode

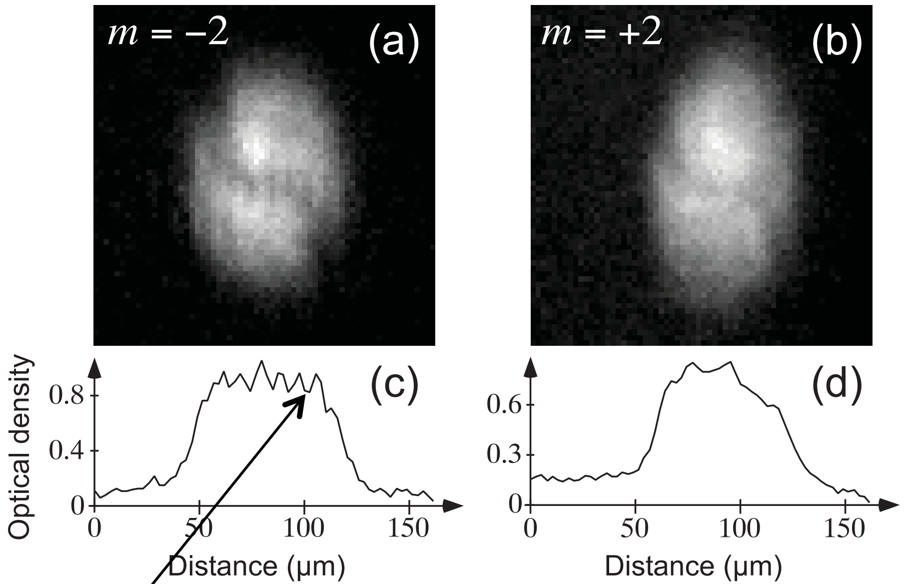
$$\omega_{-2} \longrightarrow k, \frac{\omega_{-2}}{2} \quad \text{and} \quad k, \frac{\omega_{-2}}{2}$$

−2 quad modes

kelvon modes

$$\omega(k) = \frac{\hbar k^2}{2m} \ln\left(\frac{1}{k\xi}\right)$$

$\xi = 0.3 \mu\text{m}$ gives $k = 0.8\ \mu\text{m}^{-1}$

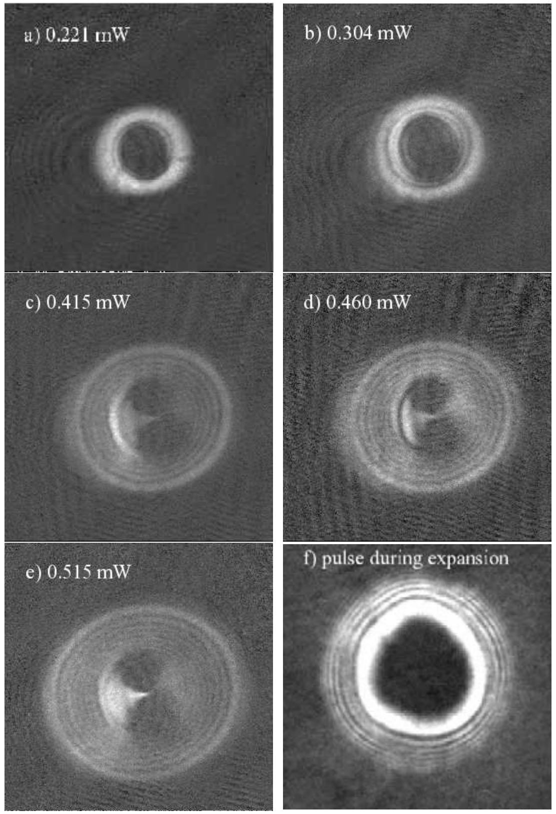


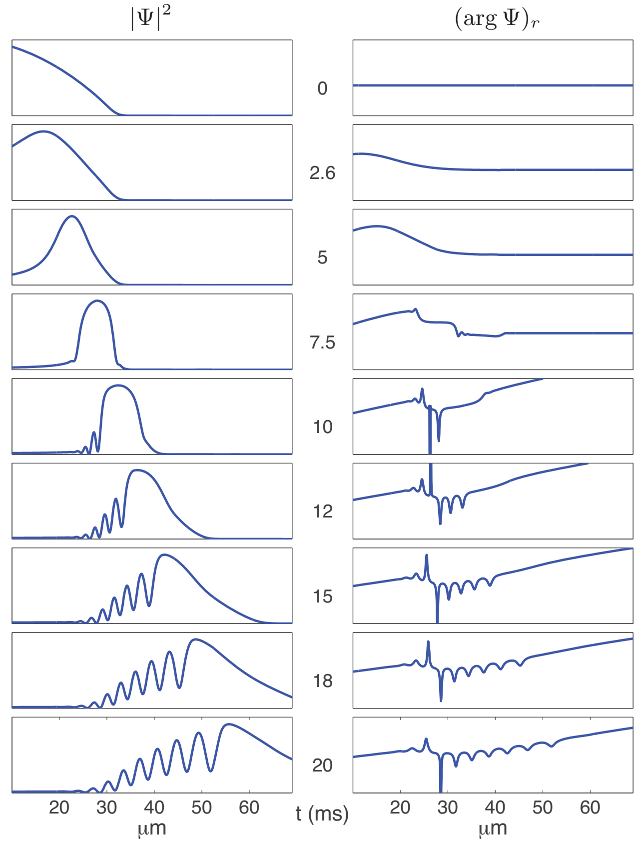
wiggles at $k = 0.7\ \mu\text{m}^{-1}$

Shock Waves

JILA: M.A. Hoefer *et al.*, PRA **74**, 023623 (2006)

Apply repulsive laser beam to center of condensate





Observe dispersive shock wave

Other interesting aspects

- tunable interactions

  Feshbach resonances
- other topological excitations

  solitons
- spatial coherence

  matterwave interferometry
- non-equilibrium phenomena

  spontaneous magnetization

Tunable interactions

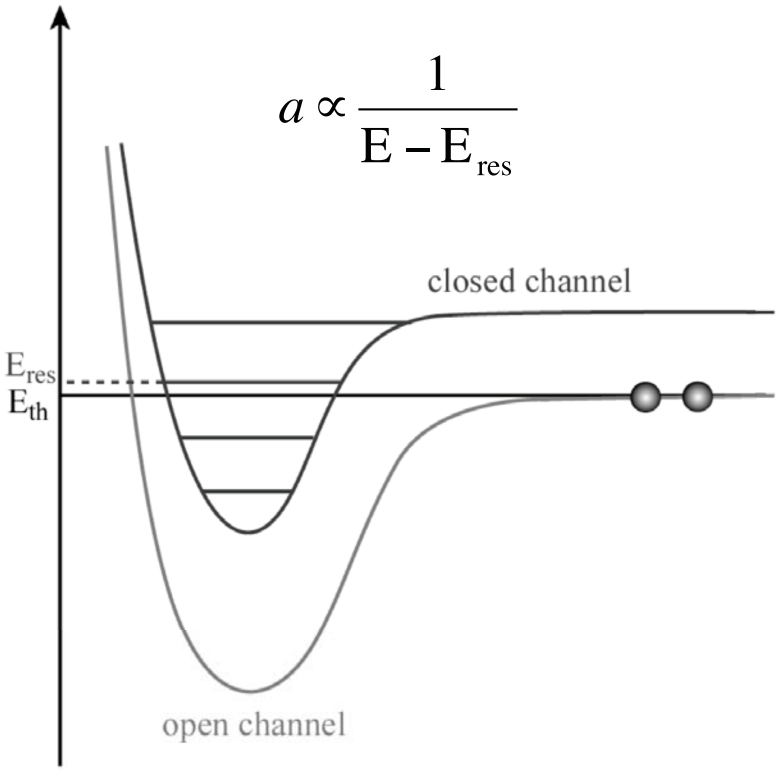
Feshbach resonance

Eindhoven: E. Tiesinga *et al*., PRA **47**, 4144 (1993)

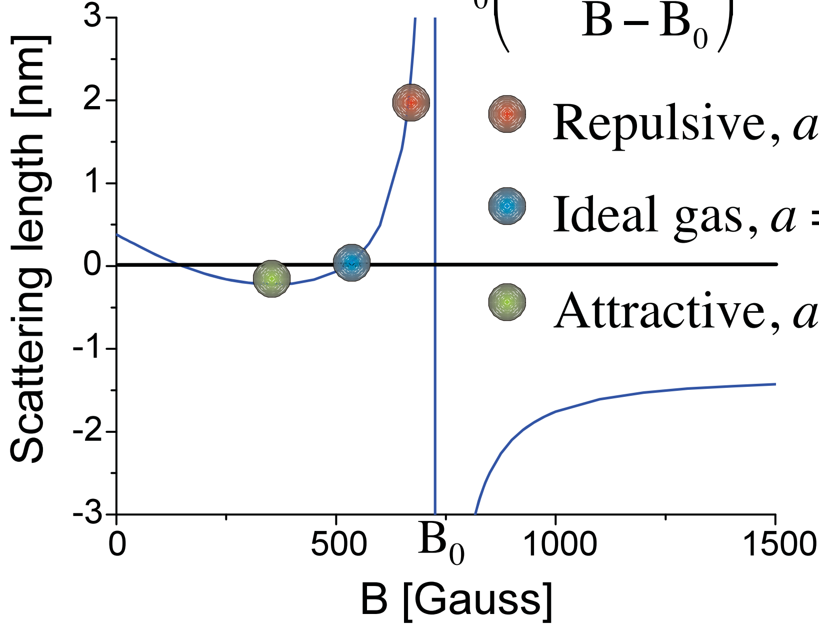
Scattering resonance between an open channel and a closed channel.

For two states with different magnetic moments, an external magnetic field can tune the bound state of the closed channel into resonance with the input (open) channel.

$$a \propto \frac{1}{E - E_{res}}$$



$$a = a_0\left(1 + \frac{\Delta B}{B - B_0}\right)$$



MIT: S. Inouye, *et al*., *Nature* ***392***, 151 (1998)

3-body losses prevent making *a* very large for bosons, not a problem for fermions

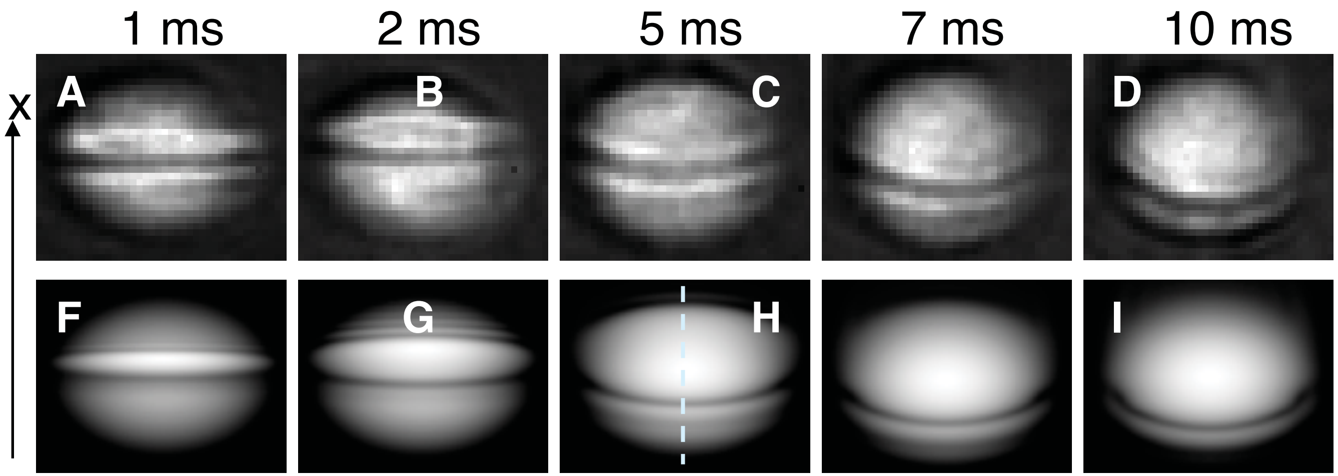
Other interesting aspects

- tunable interactions
  Feshbach resonances
- other topological excitations
  solitons
- spatial coherence
  matterwave interferometry
- non-equilibrium phenomena
  spontaneous magnetization

Soliton propagation in BEC

NIST: J. Denschlag *et al*., *Science* ***287***, 97 (2000)
Hannover: S. Burger *et al*., PRL **83**, 5198 (1999)

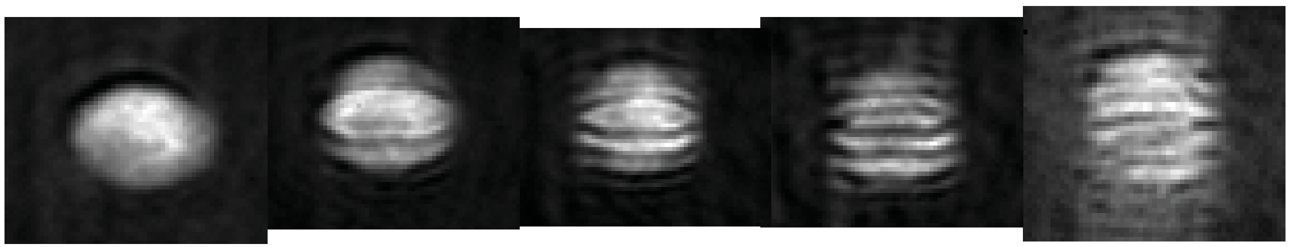
Imprint phase step on condensate to generate dark soliton



Experiment

Theory

Imprint larger phase step



100ns 600ns 900ns 1.3μs 2.5μs

Dark soliton decay to vortex ring JILA: B.P. Anderson *et al*., PRL **86**, 2926 (2001)

Other interesting aspects

- tunable interactions
  Feshbach resonances
- other topological excitations
  solitons
- spatial coherence length
  matterwave interferometry, 1^{st} and 2^{nd} order coherence
- non-equilibrium phenomena
  spin textures via rapid quench of ferromagnetic BEC
    UC Berkley: L.E. Sadler *et al*., *Nature* ***433***, 312 (2006)
  spontaneous vortex generation via Kibble-Zurek mechanism
    Arizona: C.N. Weiler *et al*., *Nature* ***455***, 948 (2008)

Other interesting aspects (cont.)

Tremendous variety of atomic properties and configurations

- Multi-component systems
  spinors, mixtures (Rb-K, Na-K, etc.)

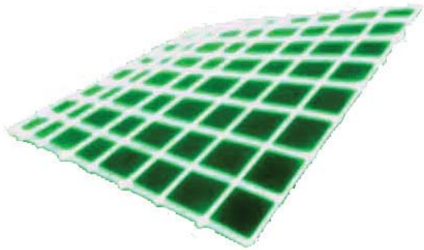
- Fermions – BEC to BCS superfluidity
  by changing a_0, it is possible to go from a
  molecular BEC to a BCS superfluid of paired fermions

- Dipolar gases (Cr, Li) – anisotropic interactions (p-wave)
  magnetic moment dominants the interactions
  recently, ground-state dipolar molecules realized (big dipoles)

Prospects for quantum turbulence studies

Nearly arbitrary shaped perturbations can be created by optical fields with a minimum length scale of $\lambda > \xi_0$ (i.e. smooth)

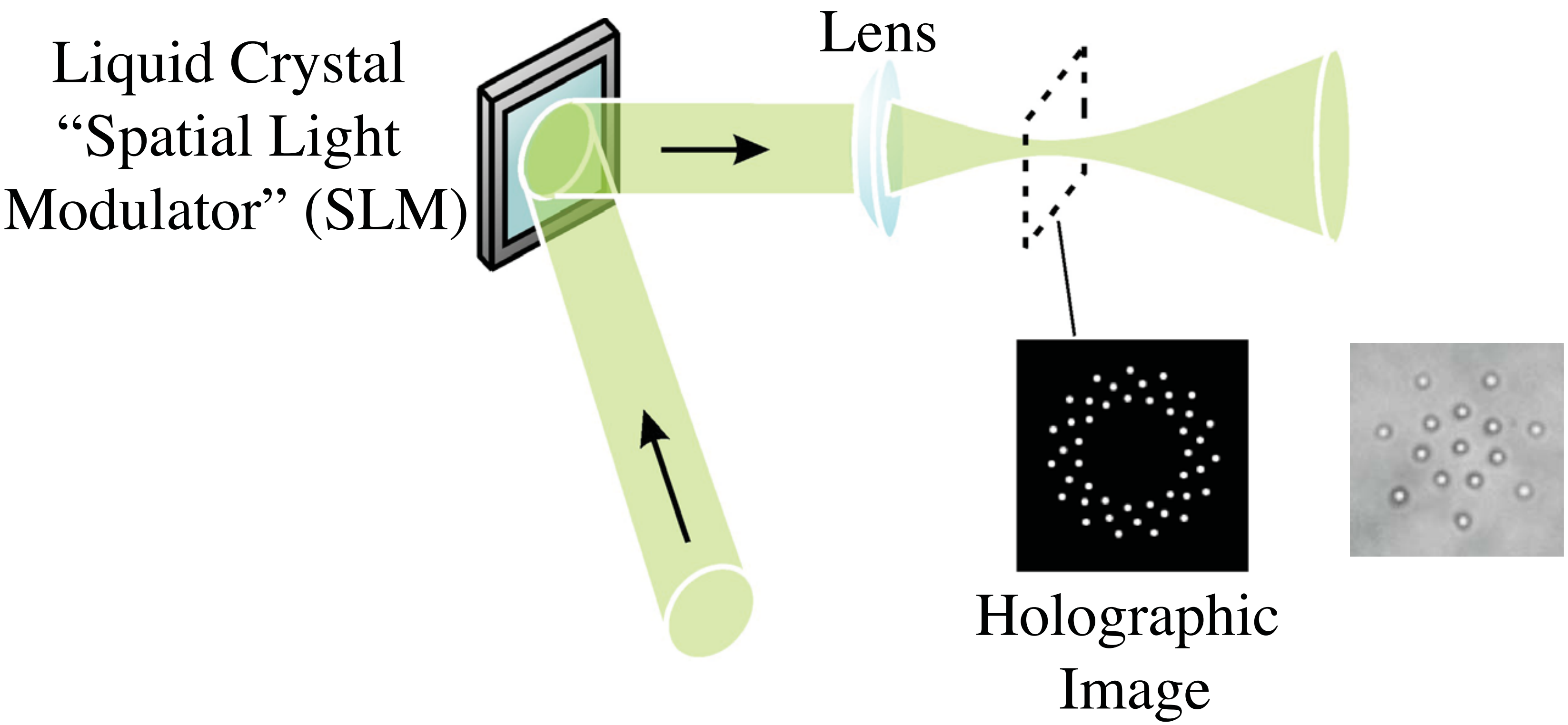
cylinders, grids?



Laser beams can be scanned easily at kHz rates (up to MHz possible). They can be turned on and off almost instantaneously.

Liquid Crystal Display Technology

Create arbitrary and dynamic light patterns



Prospects for quantum turbulence studies

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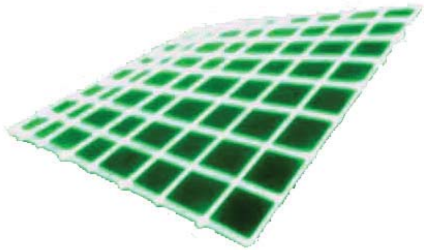
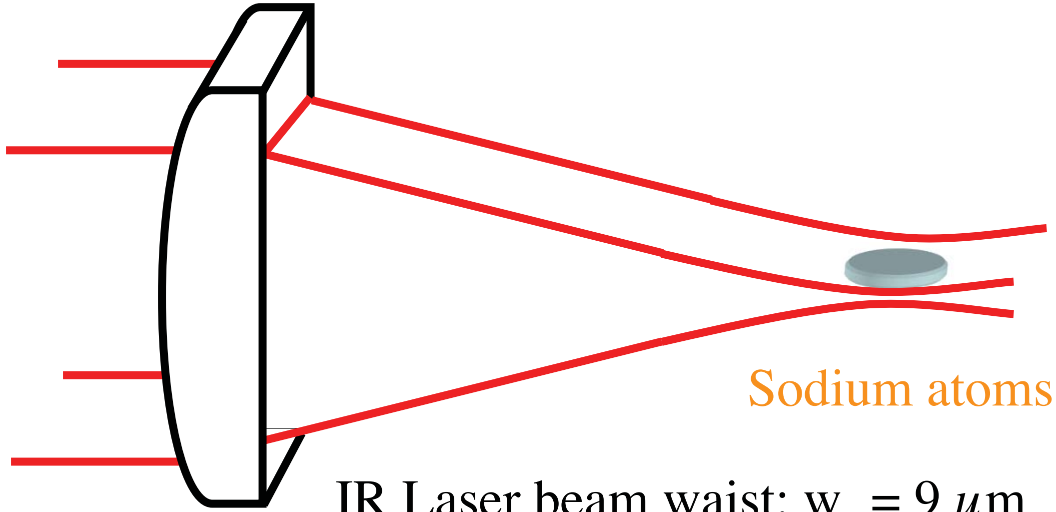


Laser beams can be scanned easily at kHz rates (up to MHz possible). They can be turned on and off almost instantaneously.

Dimensionality of system can be well controlled.

2D turbulence?

2D BEC



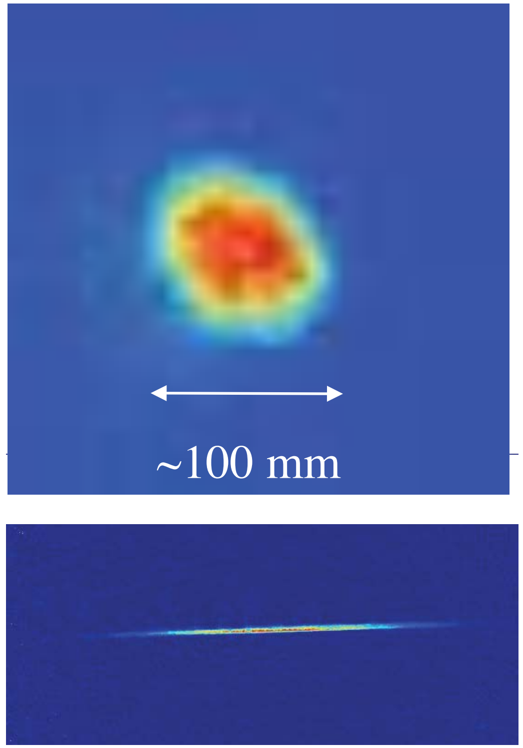
IR Laser beam waist: $w_z = 9\ \mu m$

$w = 800\ \mu m$

Rayleigh range: $z_R = 400\ \mu m$

400 mW $\longrightarrow$ ~1 kHz & ~20 Hz aspect ratio of 50

in situ images



from above

from side

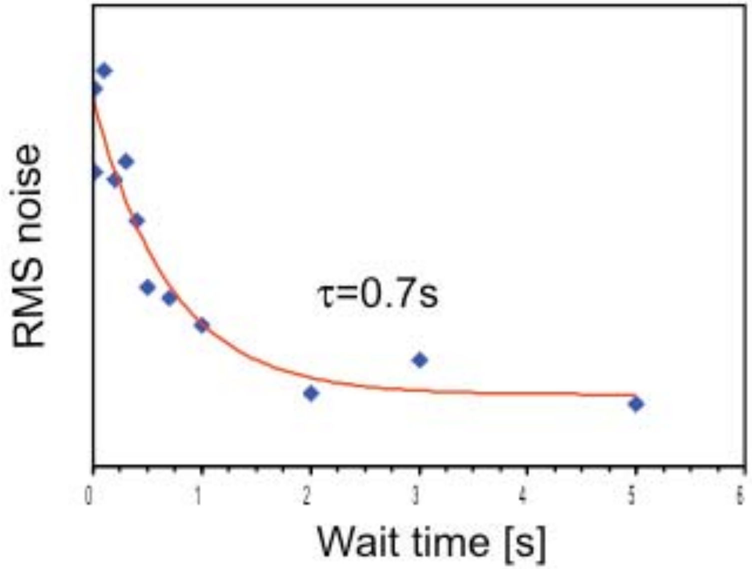
Recently studied 2D superfluid (BKT) transition

arXiv:0805.3519 to appear in PRL

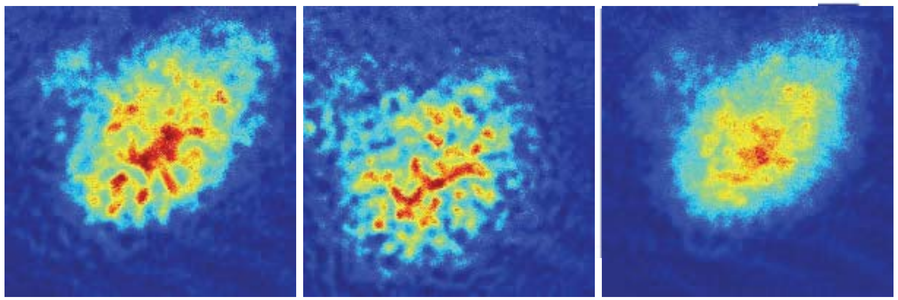
Excitations of a 2D BEC

The 2D BEC is displaced by an external force
Center of mass motion damps in less than 100 ms
→ Excitations (ripples, density fluctuations)

Vortices disappear in less than 1 s

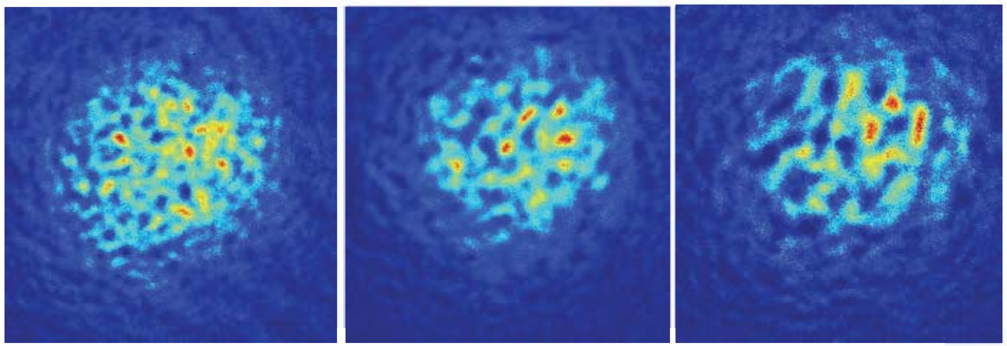


BEC at different times in the trap



200 ms 500 ms 5 s

BEC for different TOFs



5 ms 8 ms 10 ms

Excitations are vortex/anti-vortex
(they merge after TOF)
Lifetime of excitations ~1 s

Prospects for quantum turbulence studies

The physical size of (experimentally realizable) systems will probably not be big enough to see turbulence at large length scales.

$$\log\left(R/r_0\right) \sim 1$$ Small inertial range

On the other hand, it may be possible to study the detailed processes of quantum turbulence under controlled conditions.

For example: connection of vortex lines (not directly observed yet)
decay mechanism of Kelvin modes (cascades?)

Recent proposal: pinning vortex line to observe Kelvin mode decay.

T.P. Simula, T. Mizushima and K. Machida, PRA **78**, 053604 (2008).