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Workshop on Topics in Quantum Turbulence

16 - 20 March 2009

Vortex motion and dissipation in superfluid 3He-B at very low temperatures

F.M. Krusius

Helsinki University of Technology
Low Temperature Lab.
Finland

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

VORTEX MOTION AND DISSIPATION IN SUPERFLUID ^{3}He-B AT VERY LOW TEMPERATURES

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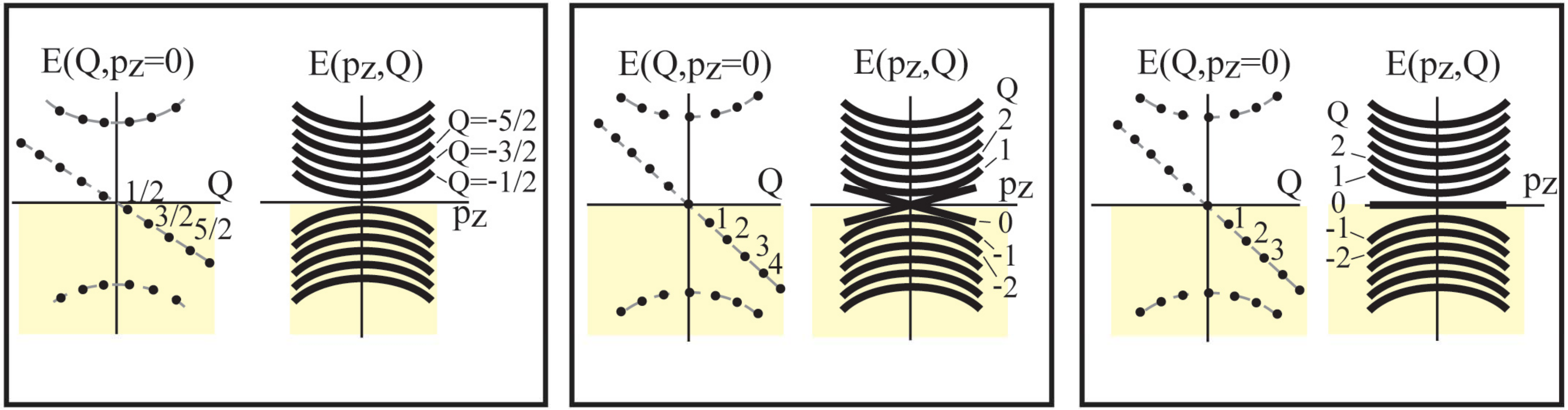
SPECTRUM OF FERMIONS

Caroli - Matricon - de Gennes bound states in potential well of vortex core



s-wave superconductor

^{3}He-B non-axisymmetric vortex n = 1

^{3}He-A singular-core vortex n = 1

Asymmetric branch

Quasiparticles can have zero energy

$E(Q) = - Q\, \omega_0 = 0$, when $Q = 0$

$\omega_0 = \Delta^2/E_F \ll \Delta$

Volovik: The Universe in a He droplet
(Cambridge Univ. Press, 2002)

VORTEX MOTION AND DISSIPATION IN SUPERFLUID ^{3}He-B

- Vortex front propagation in stationary state & spin up of superfluid component
  ⇒ large excess dissipation in addition to mutual friction below 0.35 T_c

- Sudden stop of rotation & spin down of superfluid component
  ⇒ no excess dissipation above 0.18 T_c

(first two items: circular cylinder with axis ∥ rotation axis)

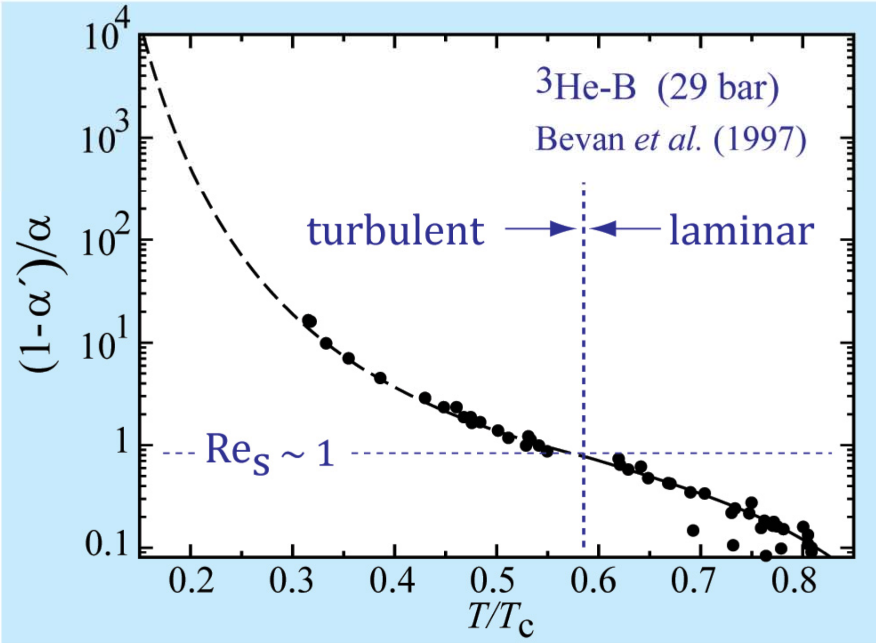
- Spin down of volume with broken axial symmetry
  ⇒ decay as in homogeneous turbulence

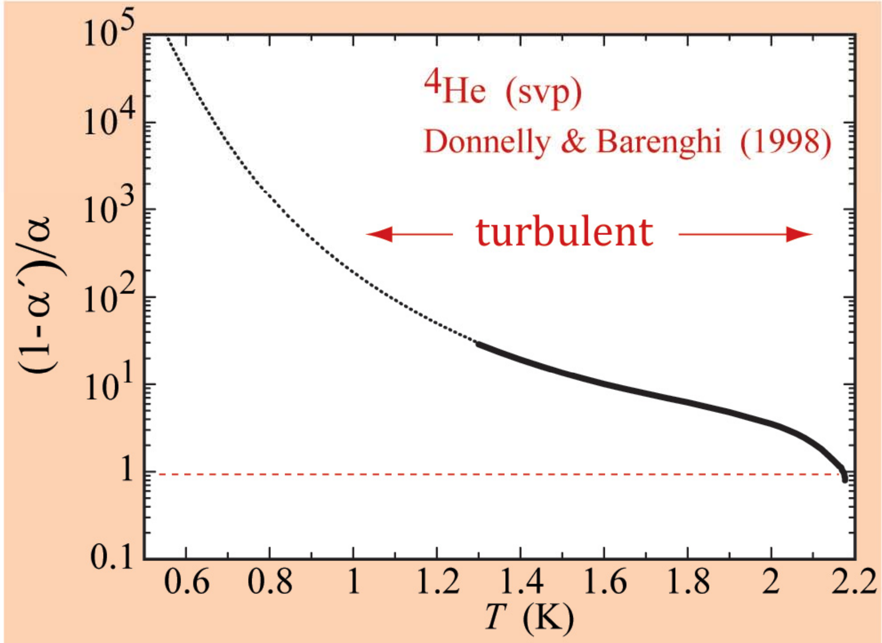
- Decay of vibrating-grid-generated turbulent tangle (Lancaster)
  ⇒ decay as in homogeneous turbulence

⇒ Polarization governs dissipation below 0.35 T_c
- no excess dissipation without reconnections?

He SUPERFLUIDS - ^{3}He-B and ^{4}He

- Superfluid Reynolds number $Re_s = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{1 - \alpha'}{\alpha}$





- Normal fluid kinematic viscosity $\nu = \eta/\rho$

$\nu \sim 1$ cm^2/s (1 mK) — locked to laminar coflow with reference frame

$\nu \sim 10^{-4}$ cm^2/s (1 K) — turbulent coflow with superfluid fraction

- Superfluid coherence length $\xi(T,P)$

$\xi(T=0) > 10$ nm — weak pinning

$\xi \sim 0.1$ nm — strong surface pinning

- Fermi system — T=0 dissipation mechanism unknown

Bose system — quantitative estimates of phonon emission

- Experimental situation: qualitatively similar for $T \rightarrow 0$ limit

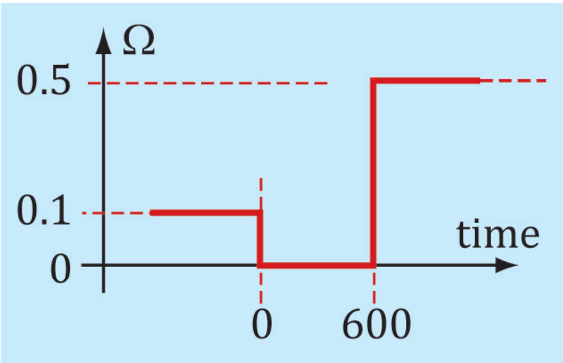
TRANSITION TO TURBULENCE

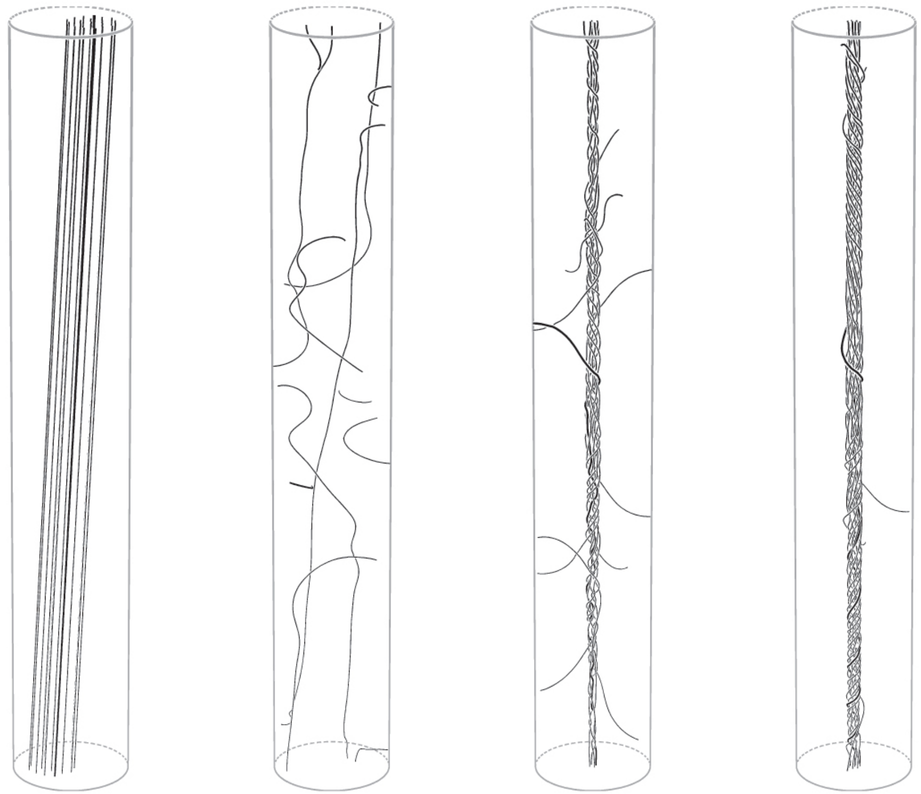
- Probed with rotating flow
- Different ways of starting turbulence by releasing in vortex-free superfluid counterflow:

  - vortex bundle → immediate **bulk turbulence** → highest transition temperature
  - one or a few seed vortices → **precursory vortex generation** followed by turbulence
    → lower and case-dependent transition temperature

- Example: **remanent vortices**

Rotation drive for remanent vortices





t = 0 | 600 s | 700 s | 800 s

N = 22 vortices | 12 | 12 | 12

- Situation in laminar temperature regime: no new vortices

- Numerical calculation at 0.4 T_c
  vortices tilted by 1.4^{o}
  smooth walls

R=3 mm
L=80 mm (scaled by 0.5)
P=29.0 bar
$\alpha = 0.18, \alpha' = 0.16$

TRANSITION TO TURBULENCE

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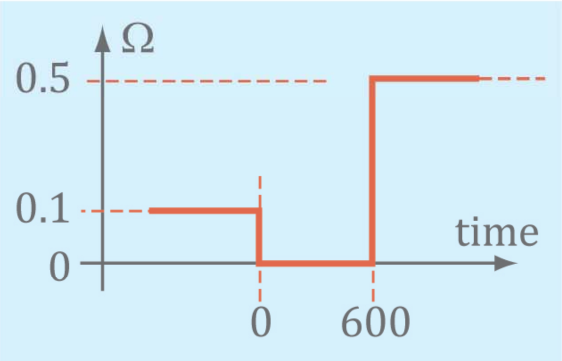

- Numerical calculation at $0.4\ T_c$
  vortices tilted by 1.4^o
  smooth walls

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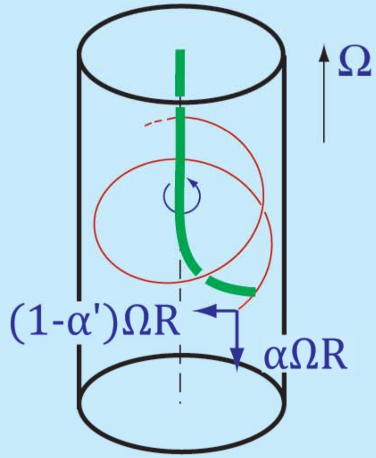
SPIRAL VORTEX MOTION



- spiral motion of vortex end point
- when applied rotating flow $\mathbf{v} = \mathbf{v}_n - \mathbf{v}_s \sim \Omega R$ dominates
- Magnus force $\boldsymbol{f}_M = \rho_s \kappa\, \hat{\mathbf{s}} \times (\mathbf{v}_L - \mathbf{v}_s)$
- mutual friction force

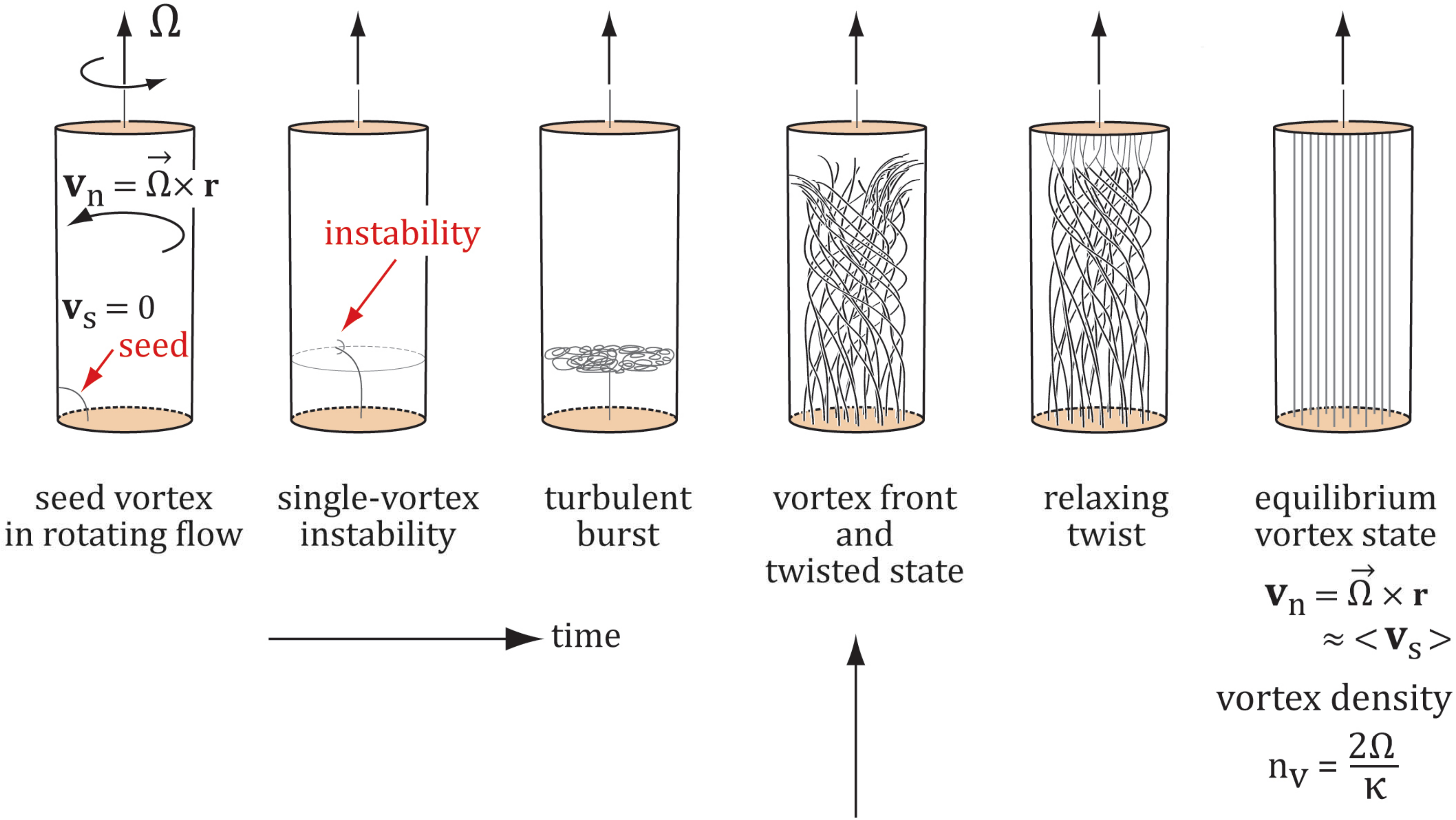
$$\boldsymbol{f}_{mf} = -\gamma_0\, \rho_s \kappa\, \hat{\mathbf{s}} \times [\hat{\mathbf{s}} \times (\mathbf{v}_n - \mathbf{v}_L)] + \gamma_0'\, \rho_s \kappa\, \hat{\mathbf{s}} \times (\mathbf{v}_n - \mathbf{v}_L)$$

- eq of motion for vortex element from balance of forces $\boldsymbol{f}_M + \boldsymbol{f}_{mf} = 0$

$$\mathbf{v}_L = \mathbf{v}_s + \alpha\, \hat{\mathbf{s}} \times (\mathbf{v}_n - \mathbf{v}_s) - \alpha'\, \hat{\mathbf{s}} \times [\hat{\mathbf{s}} \times (\mathbf{v}_n - \mathbf{v}_s)]$$

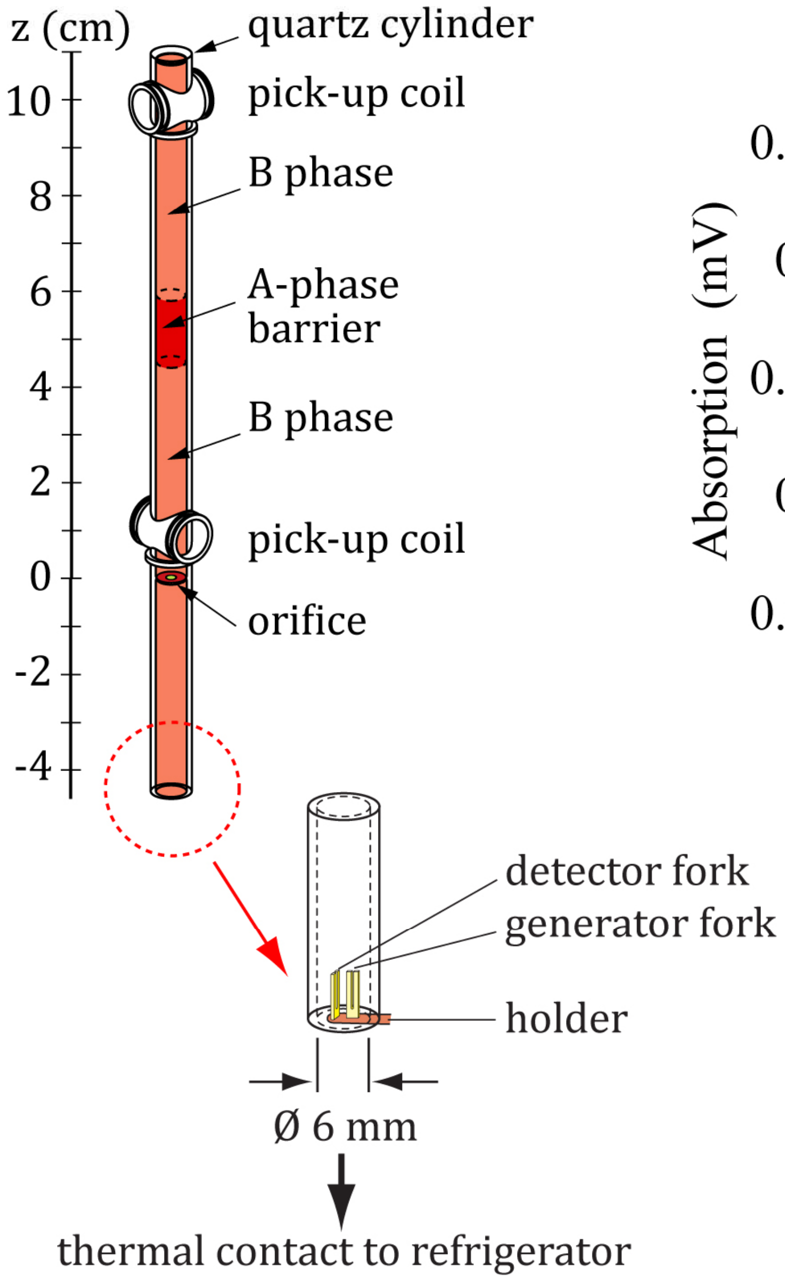
axial motion   azimuthal motion

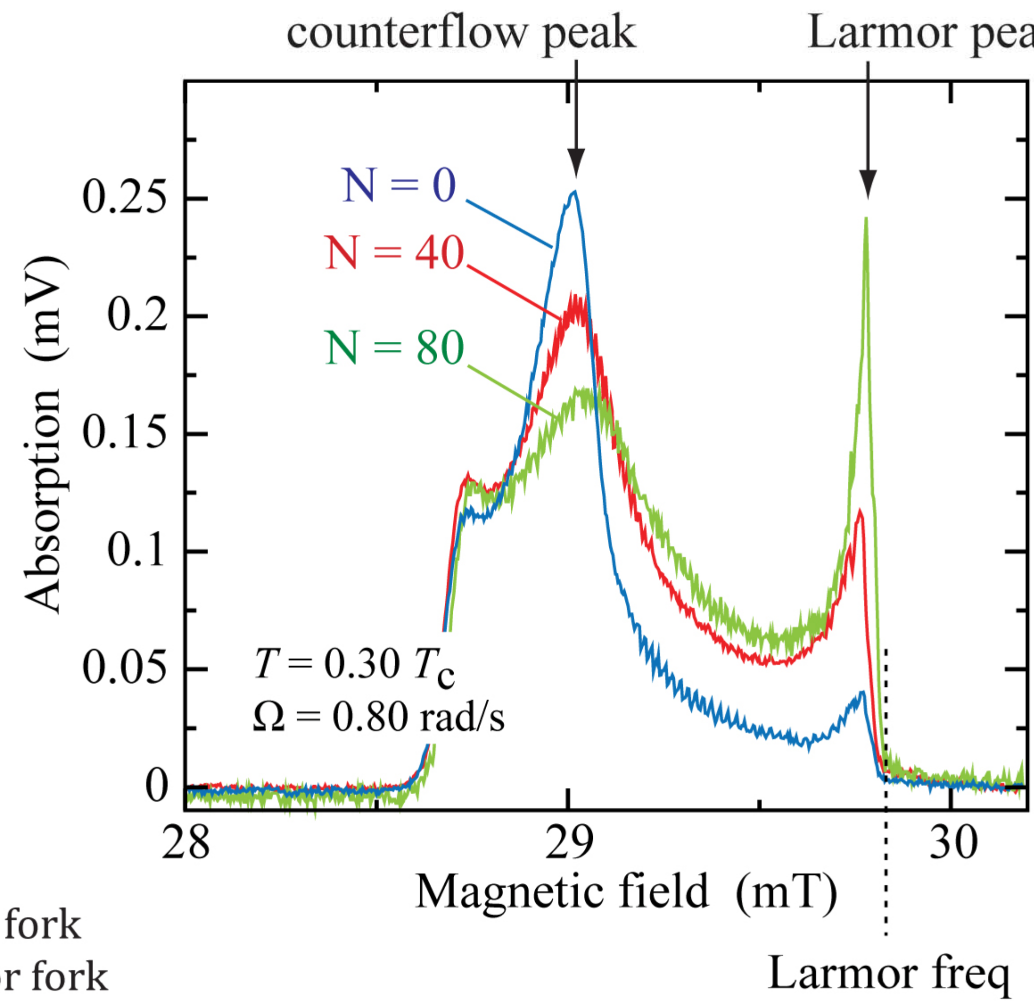
- **turbulent temperature regime**
  at $\Omega =$ const a sequence of transient states is traversed to final equilibrium
- the sequence represents **spin up of superfluid component**



measure propagation velocity of vortex front

NMR MEASUREMENT OF FRONT PROPAGATION

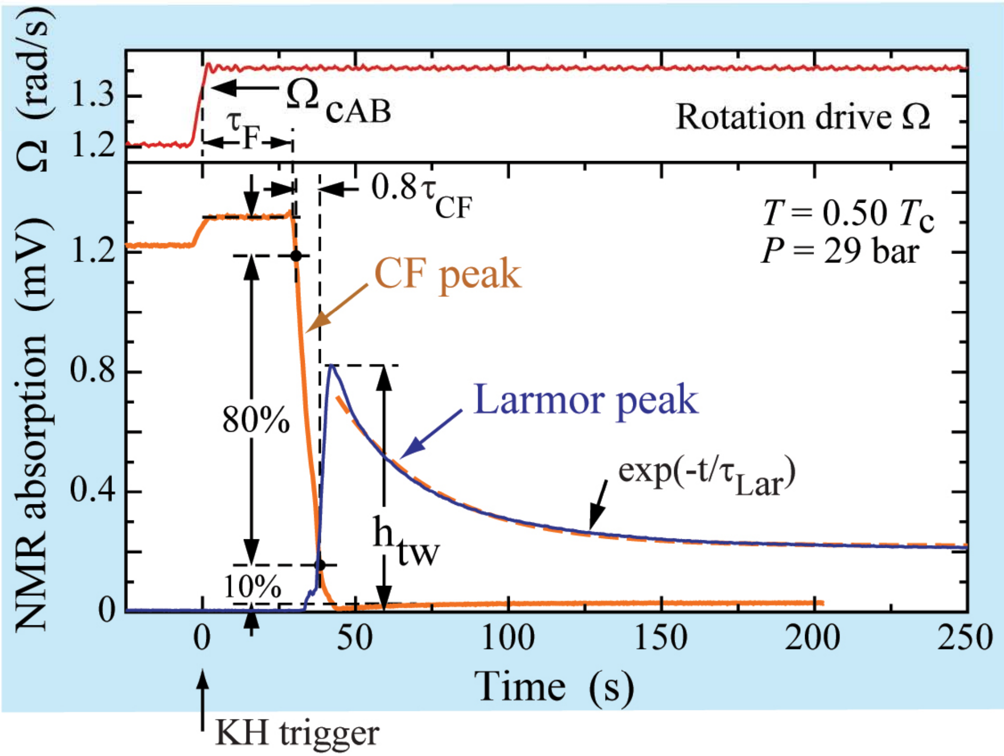




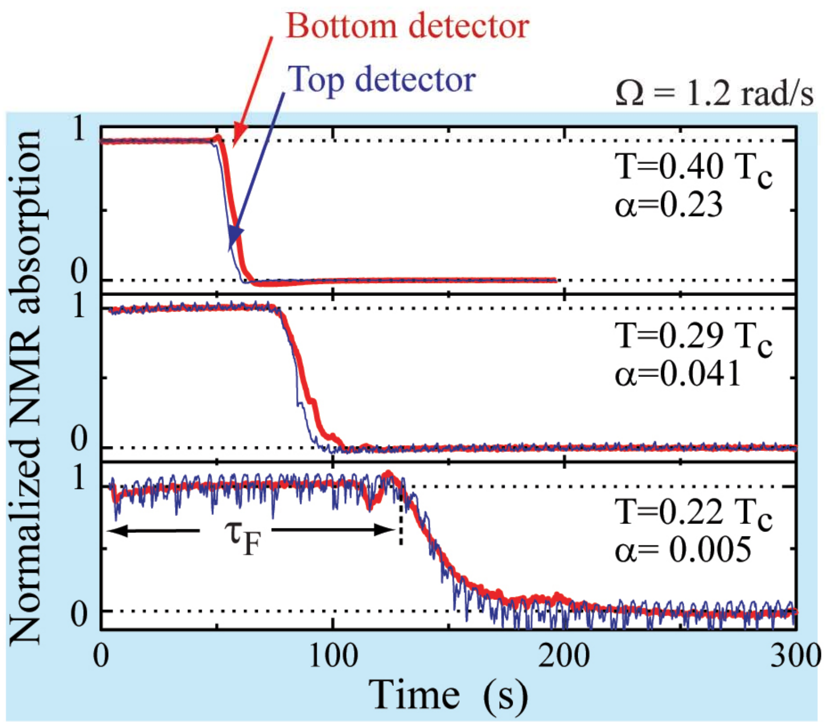
- **Origin of NMR spectrum:**
  - the line shape is a mapping of the orientational distri bution of the order parameter axis over the cross section of the cylinder
  - the azimuthally flowing counterflow deflects the axis from the magnetic field orientation

NMR MEASUREMENT OF FRONT PROPAGATION

Measurement procedure



Velocity signals



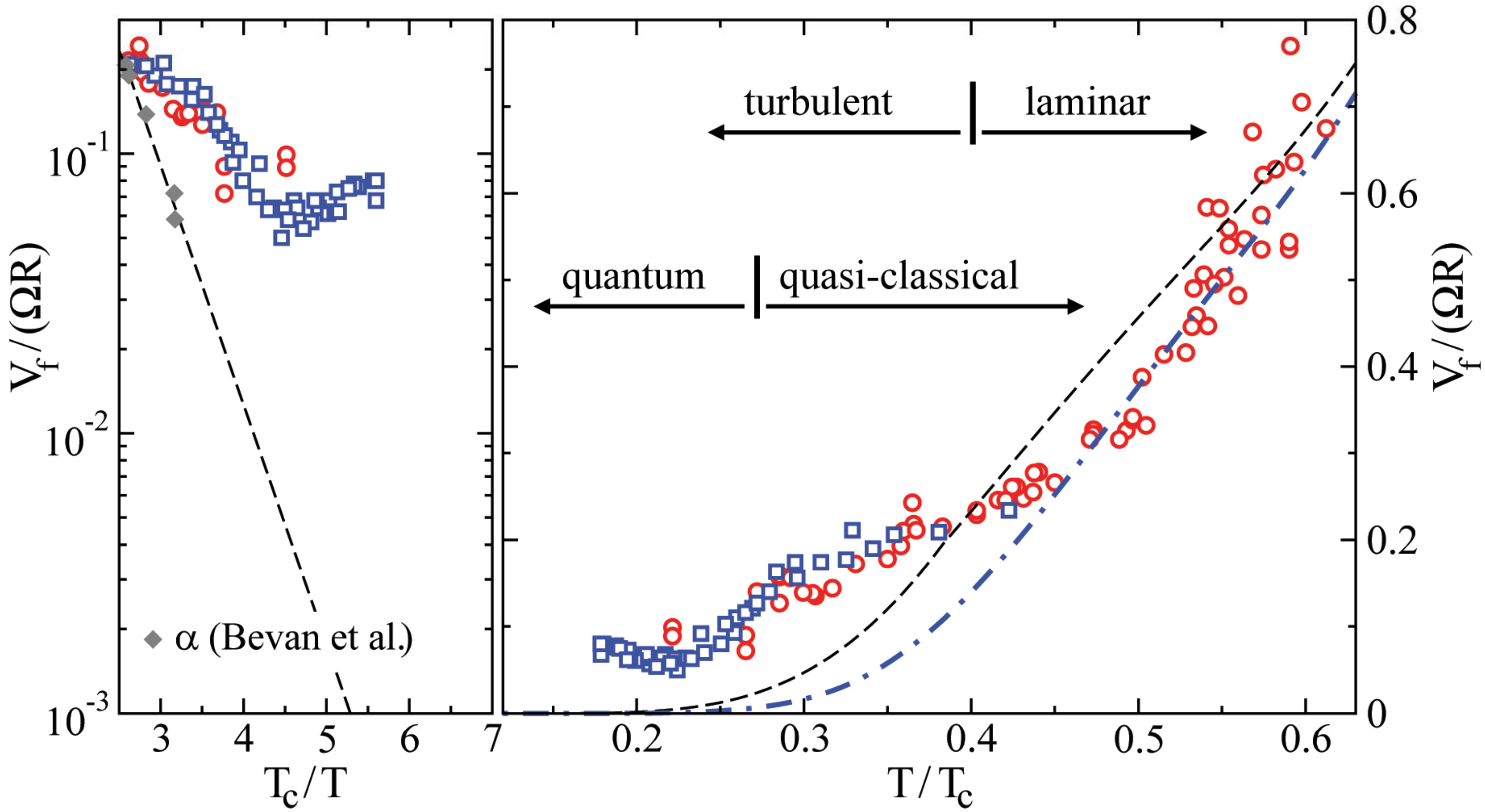
NMR peak heights vs time

- τ_F → velocity V_f of front from AB interface to detector
- τ_{CF} → max thickness of front in axial direction
- τ_{Lar} → unwinding of twisted vortex cluster
- h_{tw} → axial counterflow from the twist

CF peak height vs time

- Vortex bundle injection from AB interface: Equal flight lengths for top and bottom
- Remanent vortices from orifice: flight path is the whole length of cylinder
- Results agree for different lengths of flight path

VORTEX FRONT VELOCITY



- ○ Vortex injection from AB interface
- □ Remnant vortices around orifice start vortex front expansion
- – – Mutual friction $\alpha(T)$ (from Bevan et al. JLTP 1995)
- ·–· Kinetic energy of twisted cluster subtracted

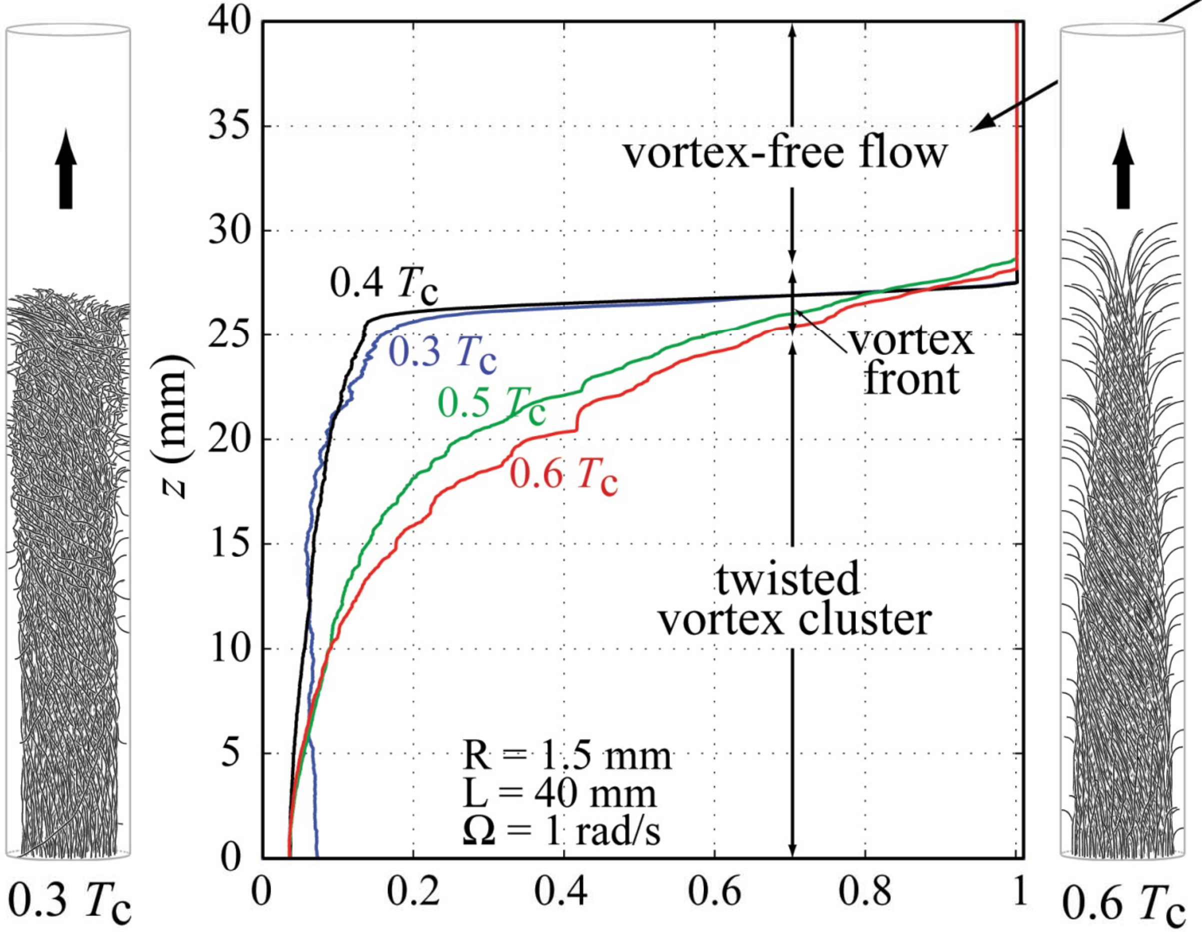
Large excess dissipation localized in vortex front!

Eltsov et al. PRL **99**, 265301 (2007)

NUMERICAL CALCULATION OF VORTEX FRONT PROPAGATION

Kinetic energy

$$\mathcal{E} = \frac{1}{4}\pi \rho_s \Omega^2 R^4$$



$$\mathcal{E}(z) = \int (\mathbf{v}_n - \mathbf{v}_s)^2 \, d^2r \, / \int (\Omega r)^2 \, d^2r$$

- Steady-state propagation with axial velocity V_f:

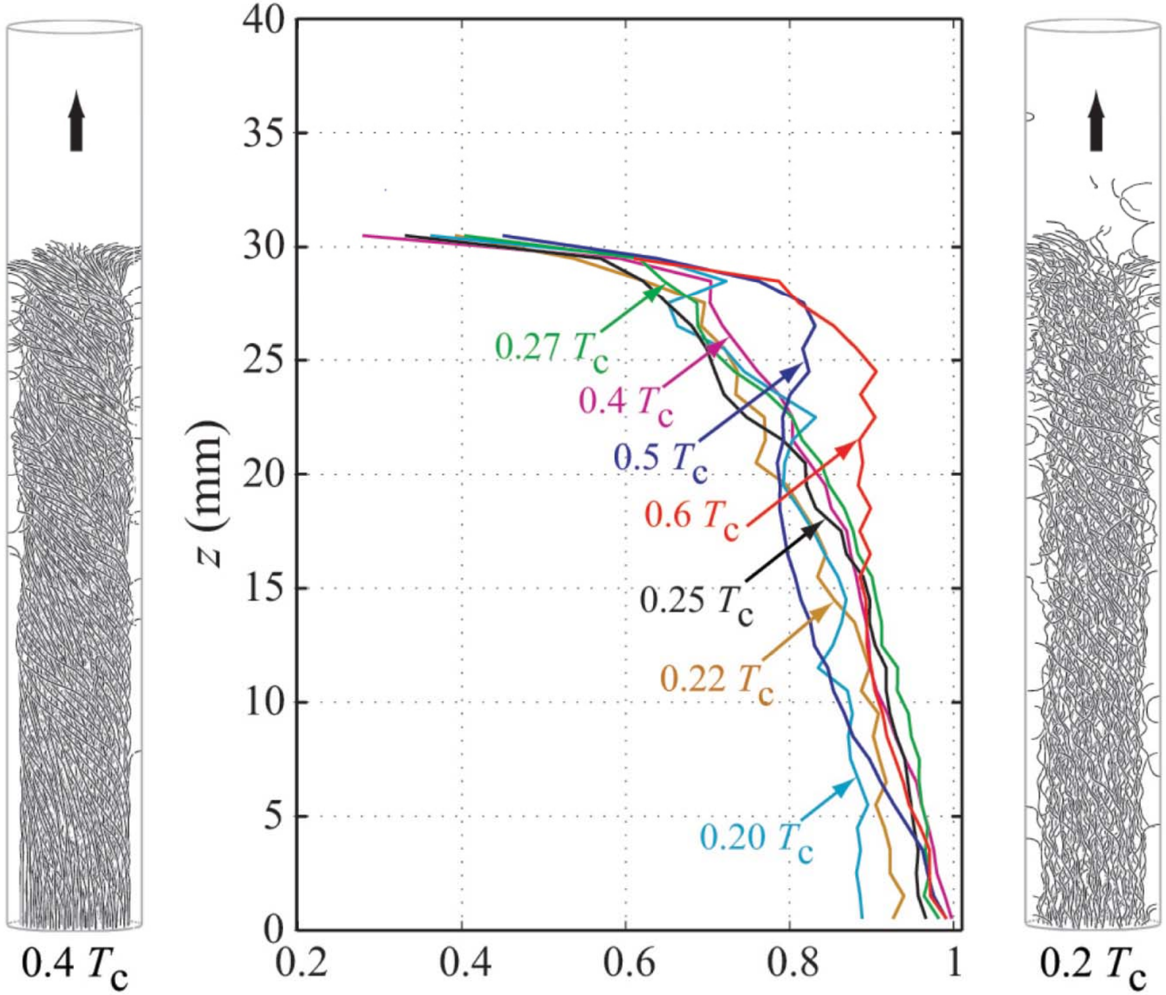
$$\frac{d\mathcal{E}}{dt} = \frac{1}{4}\pi \rho_s \Omega^2 R^4 V_f$$

- Sharp fronts below 0.4 T_c
  ⇒ dissipation concentrated in the front

- Analysis of front structures: increasingly reconnections & Kelvin waves with decreasing temperature

NUMERICAL CALCULATION OF VORTEX FRONT PROPAGATION

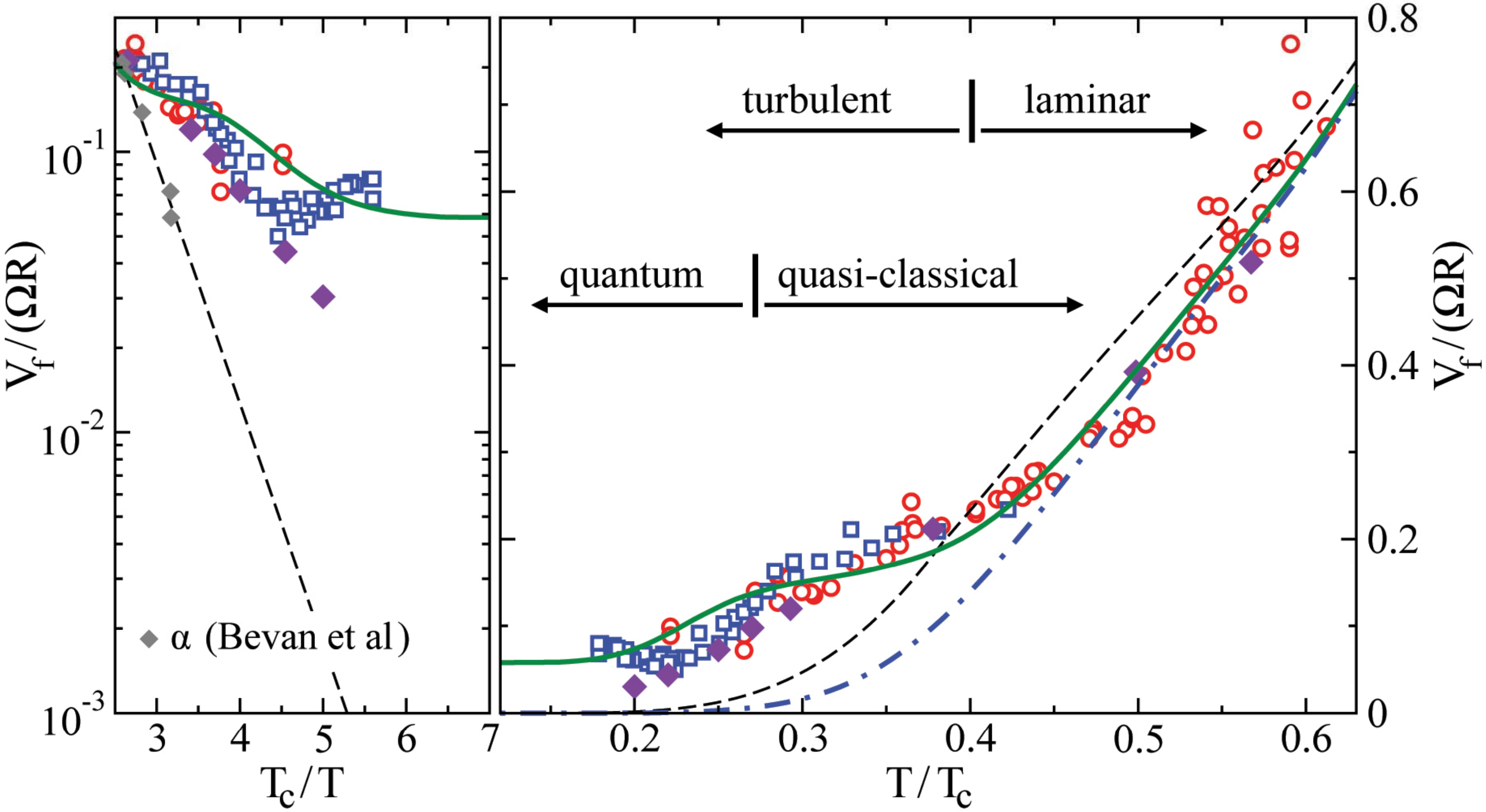
Polarization along rotation axis



- High polarization, but declining towards the front
- Characteristic for cylinder aligned along rotation axis

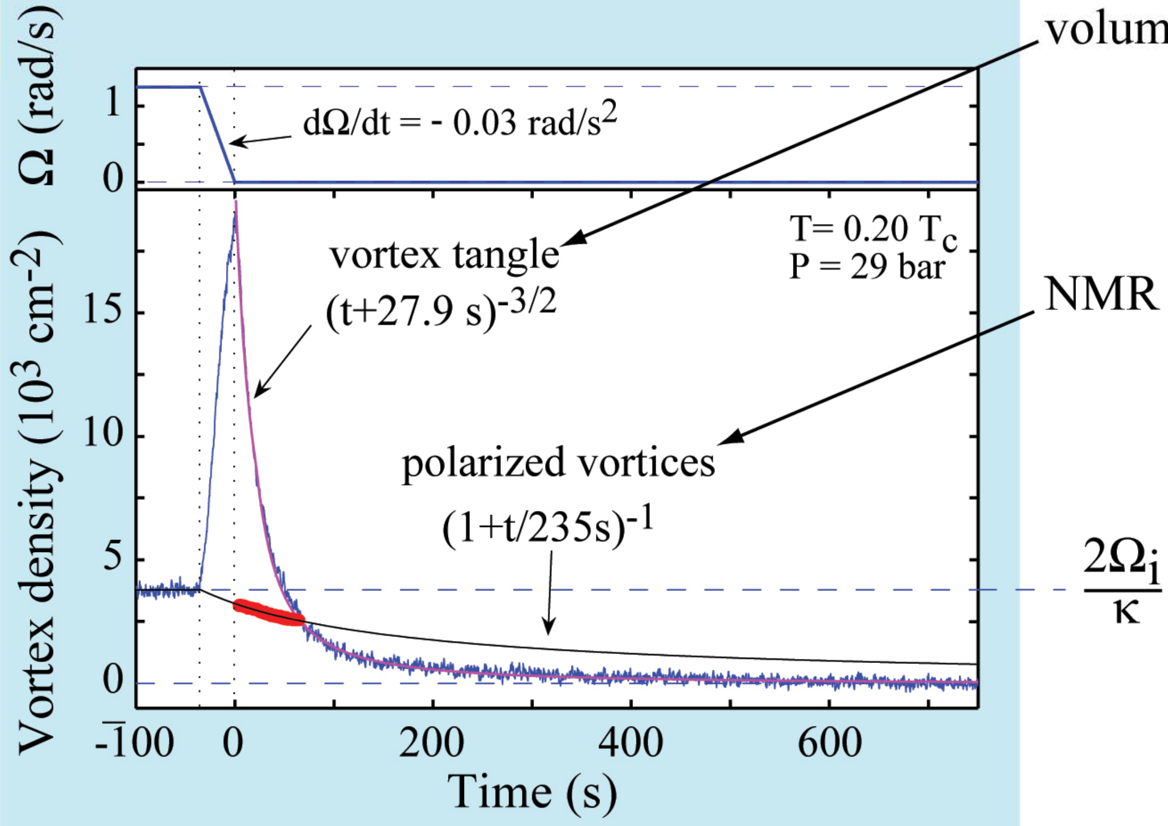
$$p_Z = \int (\hat{\mathbf{s}} \cdot \hat{\mathbf{z}})^2 \, d\xi \, / \int d\xi$$

VORTEX FRONT VELOCITY



- ○ Vortex injection from AB interface
- □ Remnant vortices around orifice start vortex front expansion
- – – Mutual friction $\alpha(T)$ (from Bevan et al. JLTP 1995)
- · – · Kinetic energy of twisted cluster subtracted
- ◆ Numerical calculations (vortex filament model)
- —— Analytic model: corrections from twisted cluster, turbulent energy transfer, and quantum bottleneck

SPIN DOWN OF SUPERFLUID FRACTION



- two different responses depending on symmetry of environment

Laminar response for superfluid vorticity $\vec{\omega}_s = \vec{\Omega}(t) + \frac{1}{2} \nabla \times \mathbf{v}_s$ when $\mathbf{v}_n = \vec{\Omega} \times \mathbf{r}$

Euler eq.

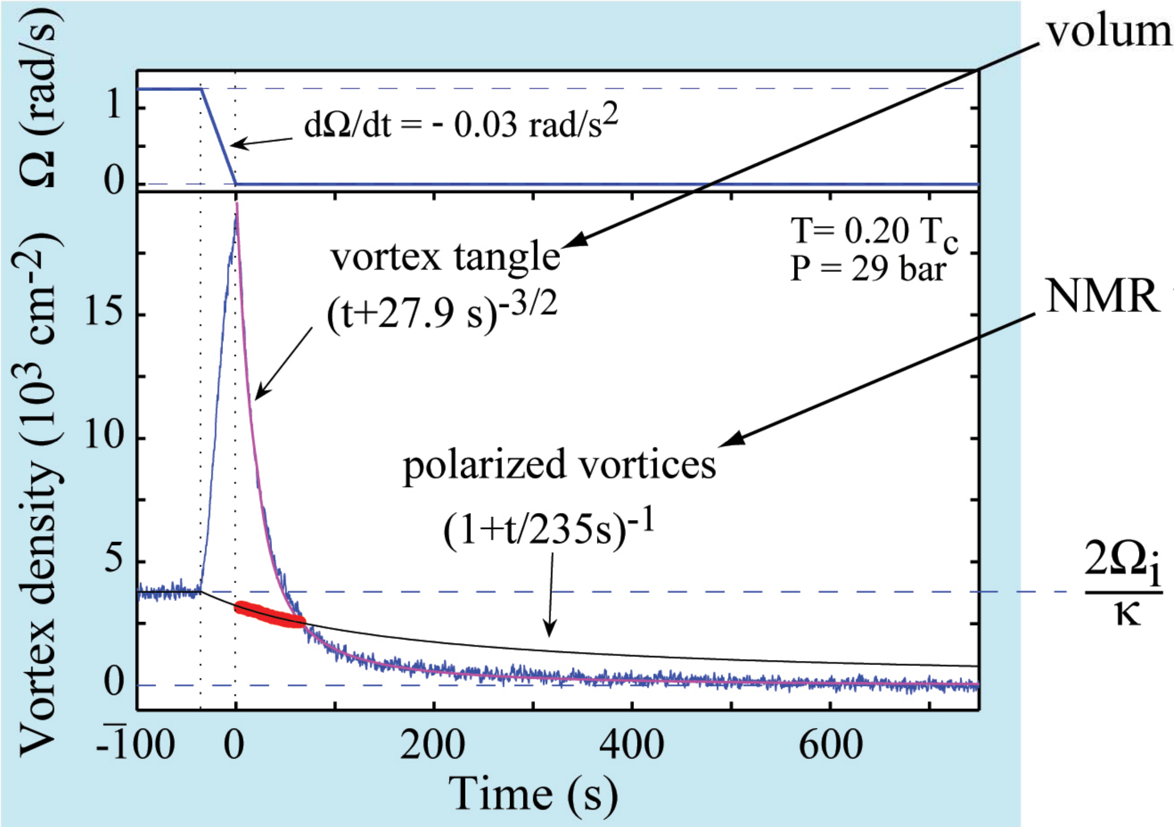
$$\frac{d\omega_s}{dt} + 2\,\alpha\,\omega_s\,[\omega_s - \boldsymbol{\Omega}(t)] = 0$$

[Sonin, RMP 1987]

solution for polarized vortices

$$\omega_s(t) = \frac{\boldsymbol{\Omega}(t{=}0)}{1 + t/\tau_F} \quad \text{with } \tau_F = 2\,\alpha\,\boldsymbol{\Omega}(0) \approx \frac{\alpha\,\kappa\,N(0)}{\pi R^2}$$

SPIN DOWN OF SUPERFLUID FRACTION



- two different responses depending on symmetry of environment

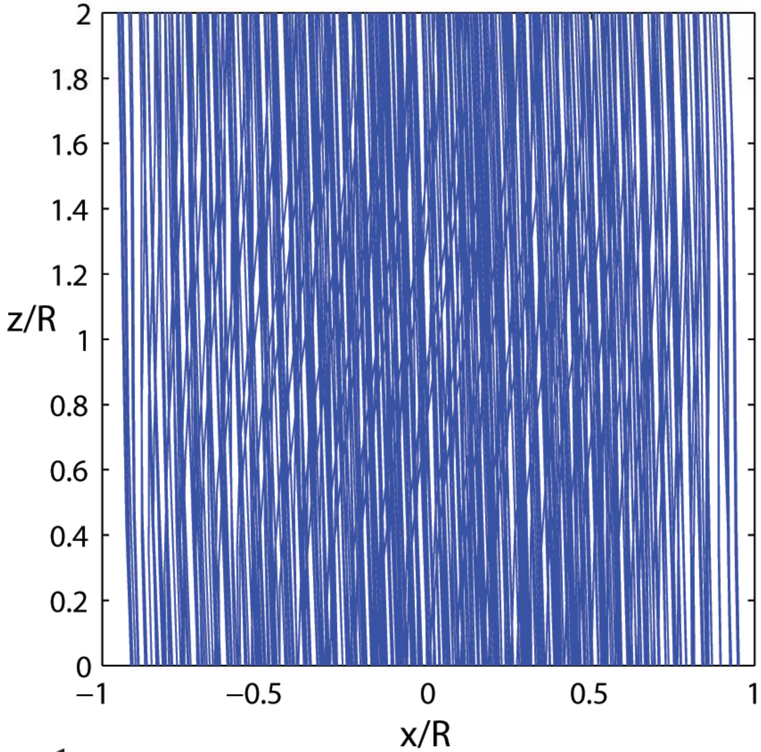
Decay of homogeneous turbulence [Stalp et al. PRL 1999]

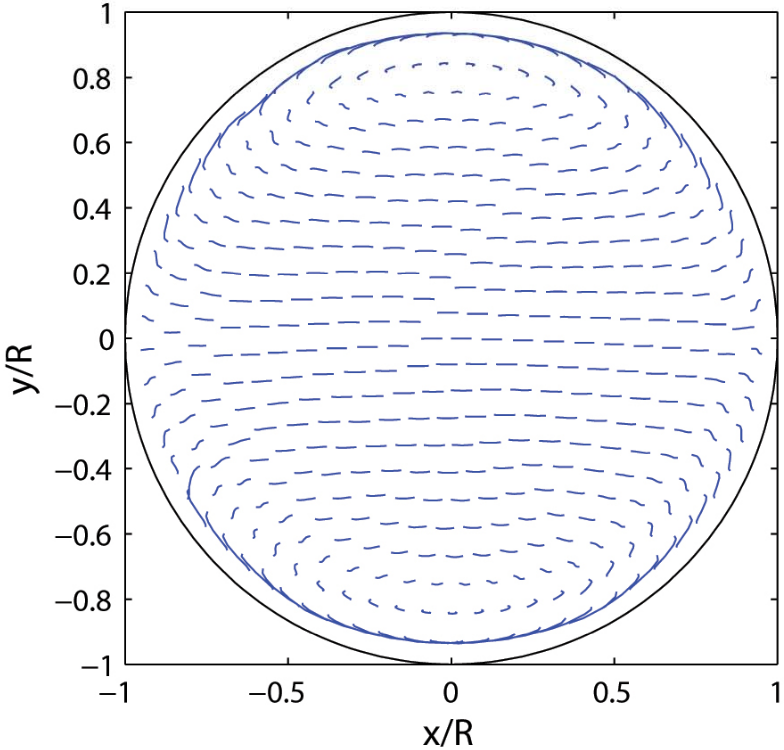
$$\frac{d\mathcal{E}}{dt} = - \nu\, \omega_s^2 = - \nu\, (\kappa\, L)^2$$

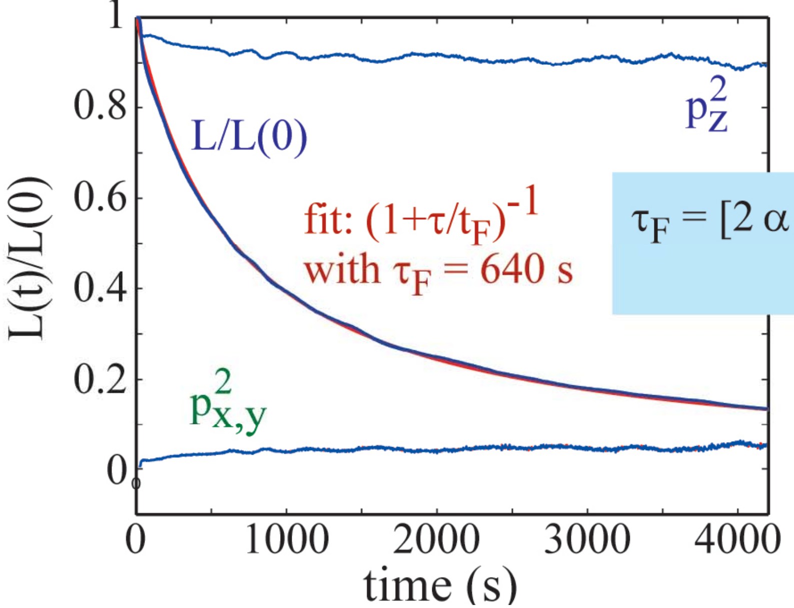
$$L(t) \propto \frac{d}{2\pi\kappa\,\sqrt{\nu}\;\; t^{3/2}} \propto [w - w_0]/w_0$$

change of width w in resonator response curve

Spin down from 0.5 rad/s - numerical example





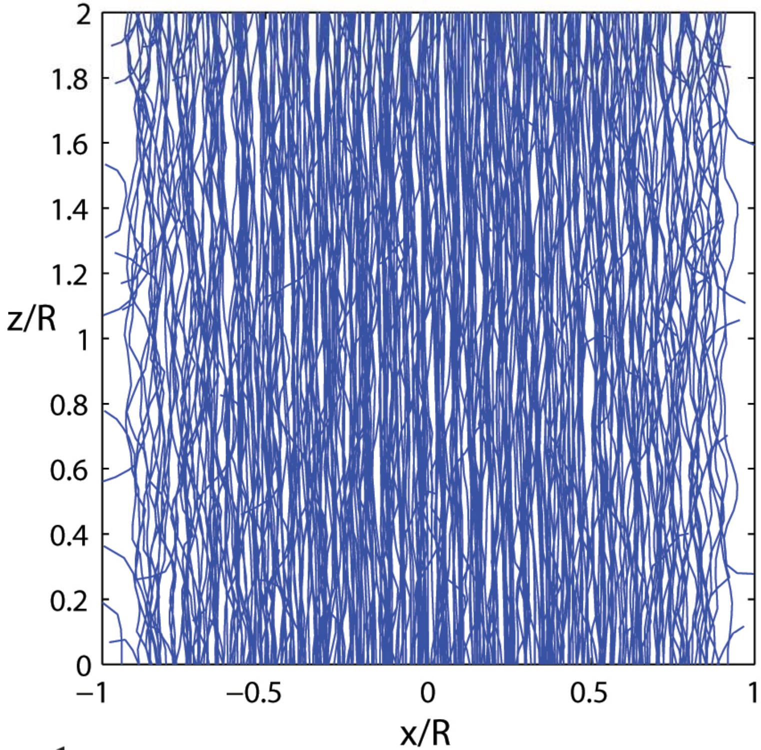


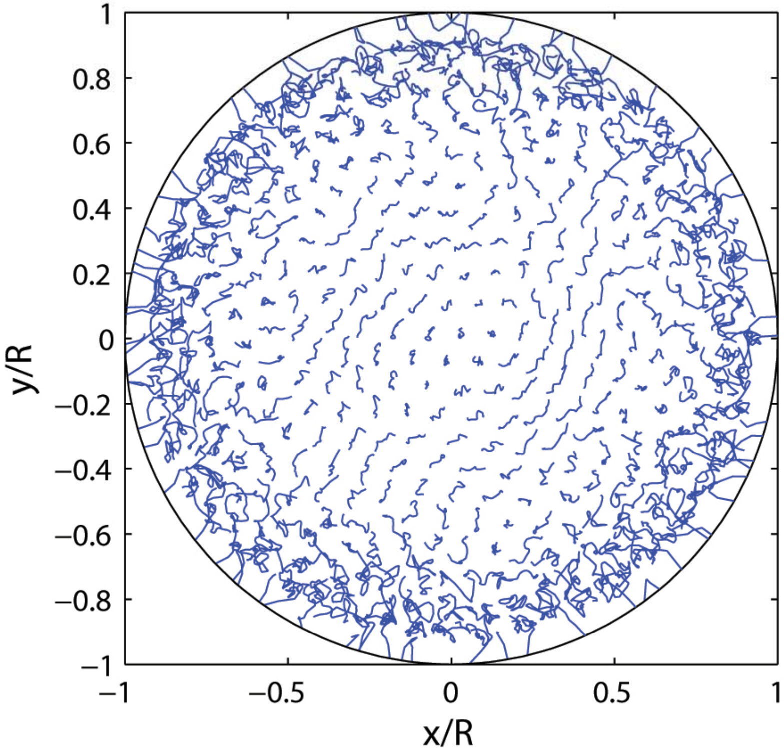
$t = 0$

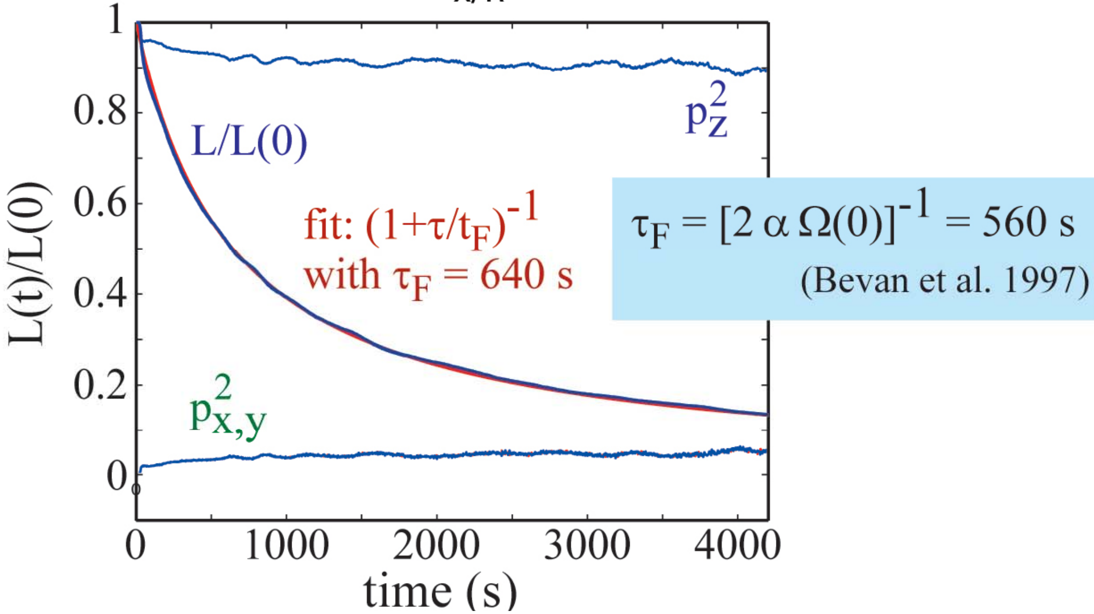
$\tau_F = [2\,\alpha\,\Omega(0)]^{-1} = 560$ s
(Bevan et al. 1997)

R = 3 mm
height = 6 mm
tilt 2^o cylinder axis from $\vec{\Omega}$
T = 0.20 T_c
$\Omega(0)$ = 0.5 rad/s
P = 29 bar

Spin down from 0.5 rad/s - numerical example



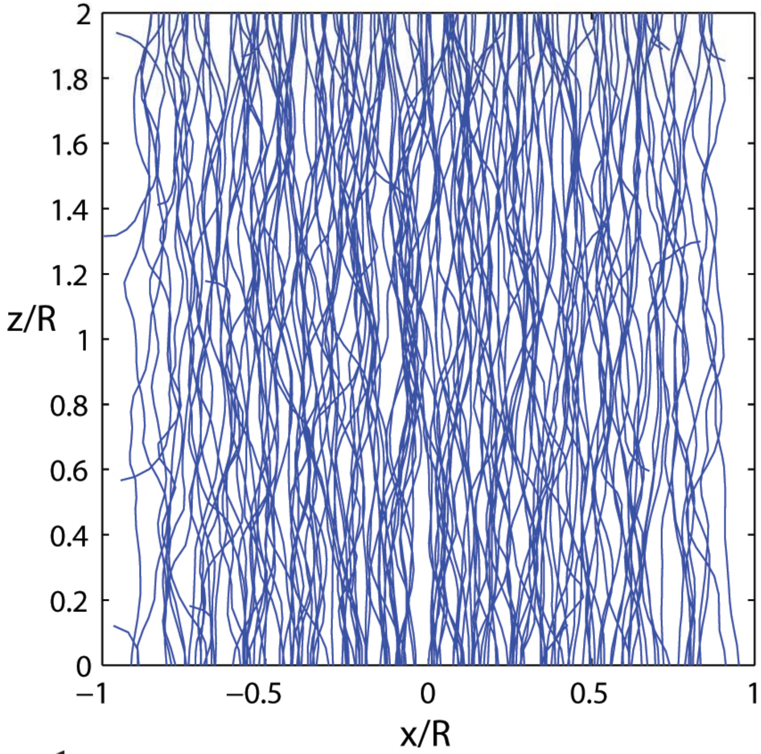


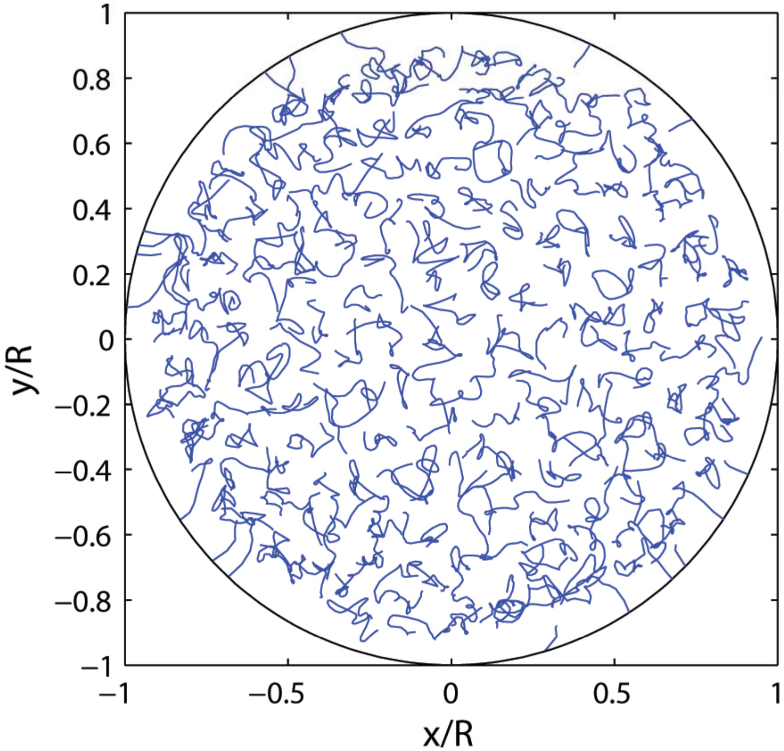


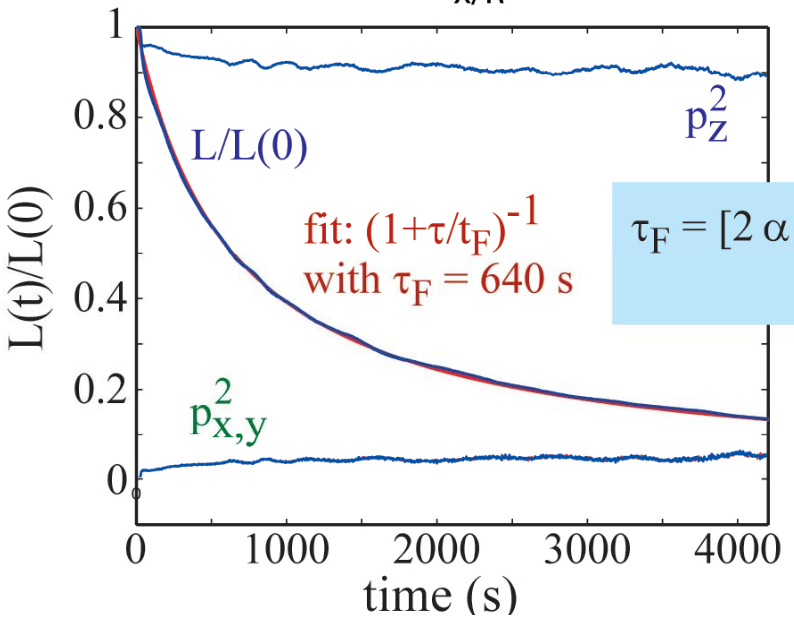
$t = 100$ s

$R = 3$ mm
height = 6 mm
tilt 2° cylinder axis from $\vec{\Omega}$
$T = 0.20\ T_c$
$\Omega(0) = 0.5$ rad/s
$P = 29$ bar

Spin down from 0.5 rad/s - numerical example





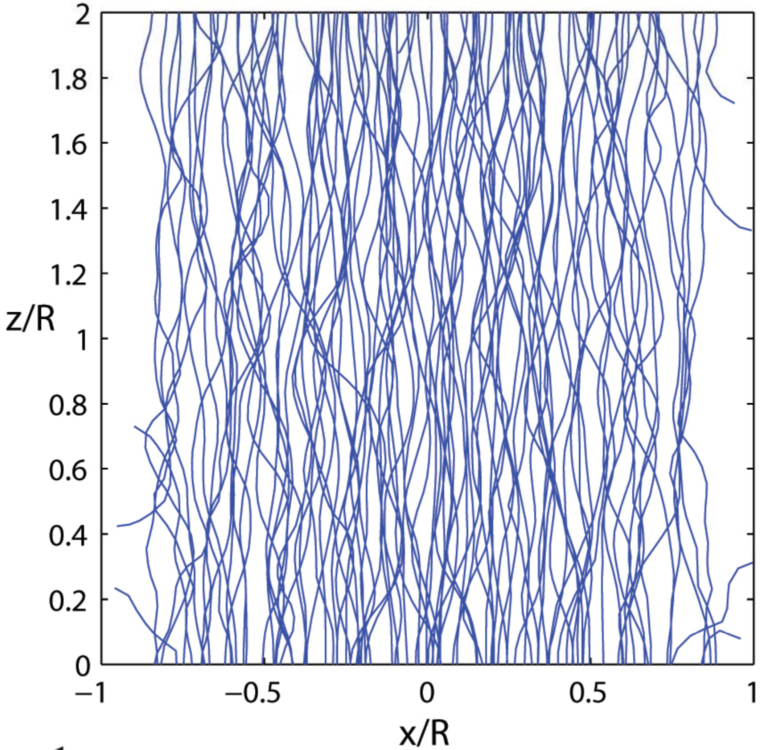


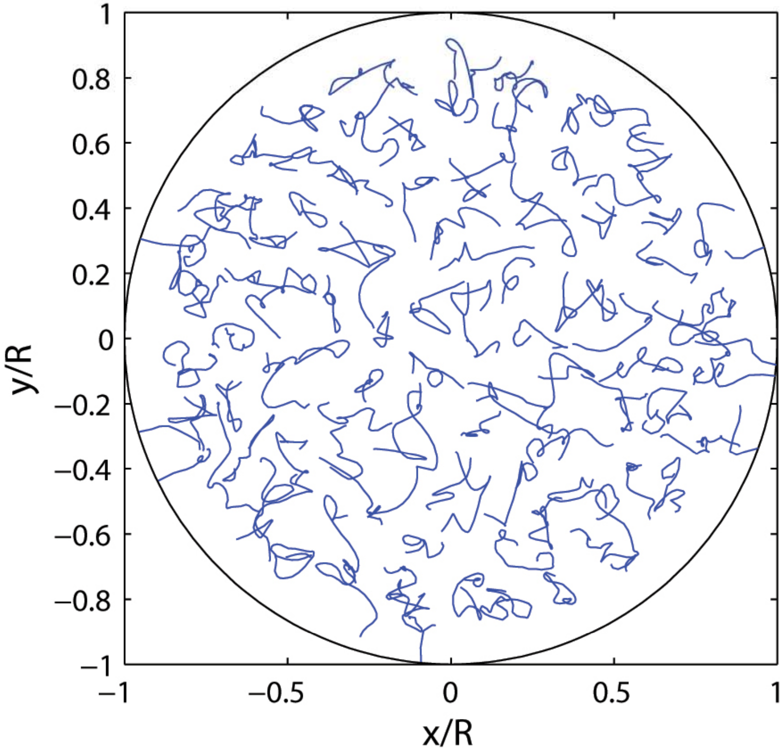
$\tau_F = [2\,\alpha\,\Omega(0)]^{-1}$ = 560 s
(Bevan et al. 1997)

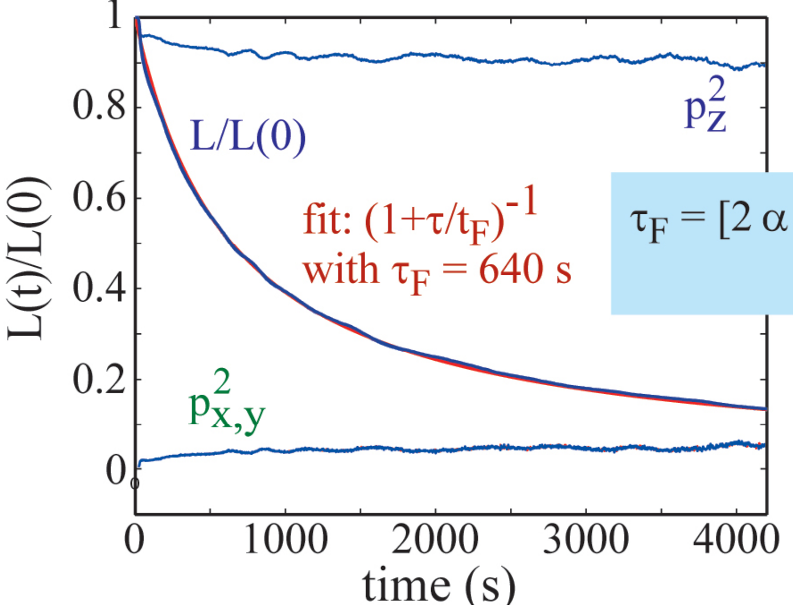
t = 1000 s

R = 3 mm
height = 6 mm
tilt 2^o cylinder axis from $\vec{\Omega}$
T = 0.20 T_c
$\Omega(0)$ = 0.5 rad/s
P = 29 bar

Spin down from 0.5 rad/s - numerical example







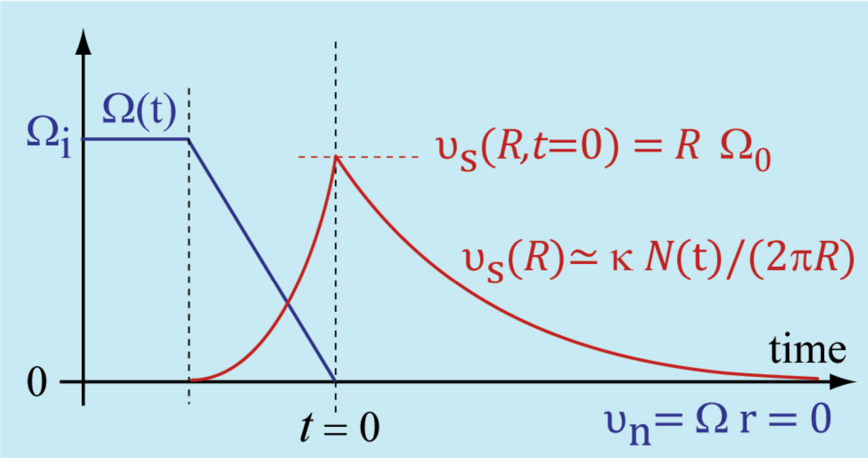
$\tau_F = [2\,\alpha\,\Omega(0)]^{-1}$ = 560 s
(Bevan et al. 1997)

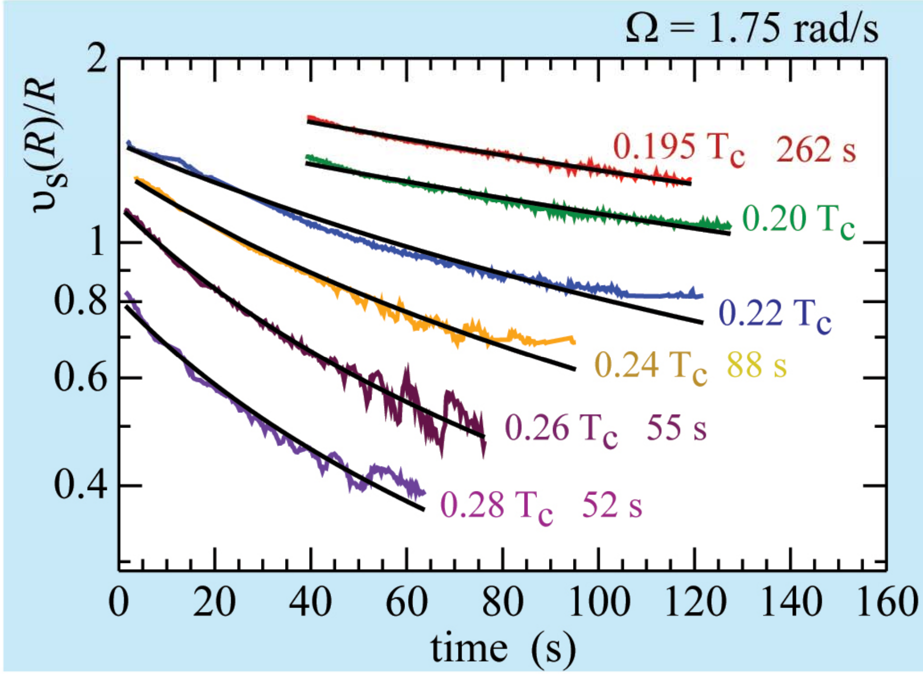
t = 2000 s

R = 3 mm
height = 6 mm
tilt 2^o cylinder axis from $\vec{\Omega}$
T = 0.20 T_c
Ω(0) = 0.5 rad/s
P = 29 bar

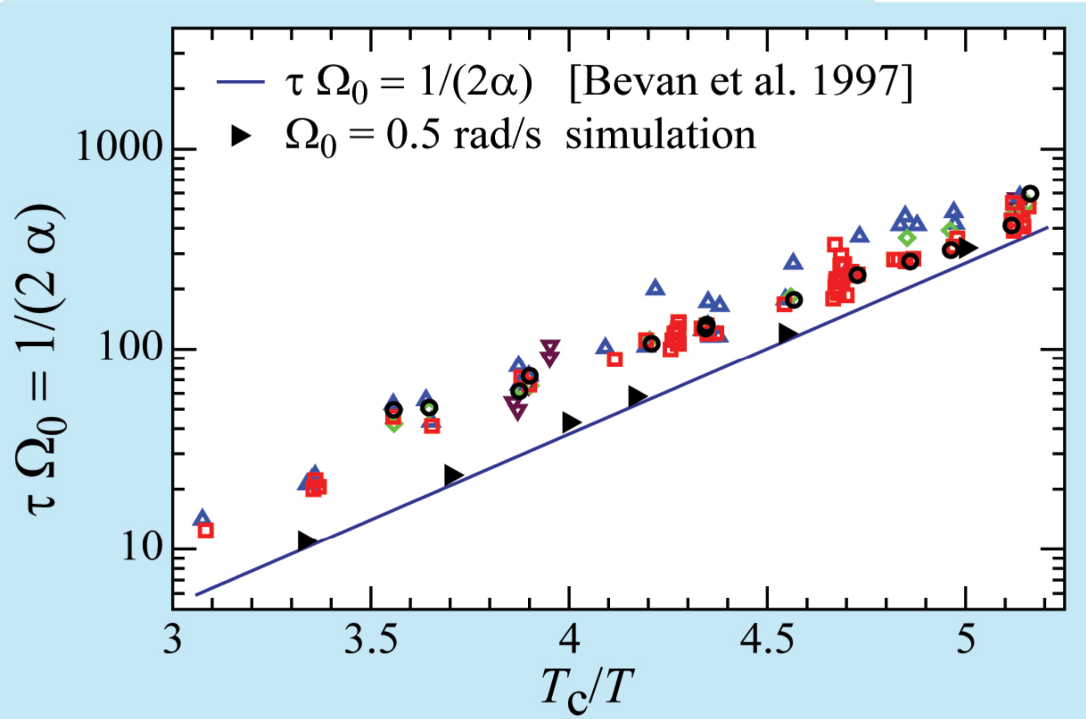
SPIN DOWN IN A CYLINDER

- no signal from turbulent tangle is detected
- large transient signal from the flow field created by remanent polarized vortices



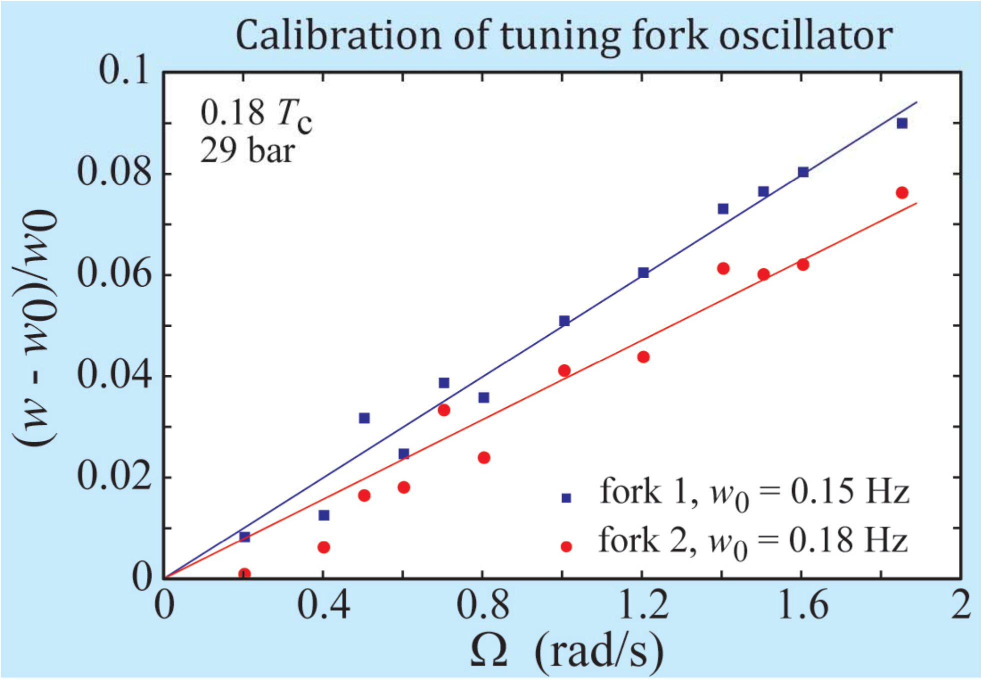


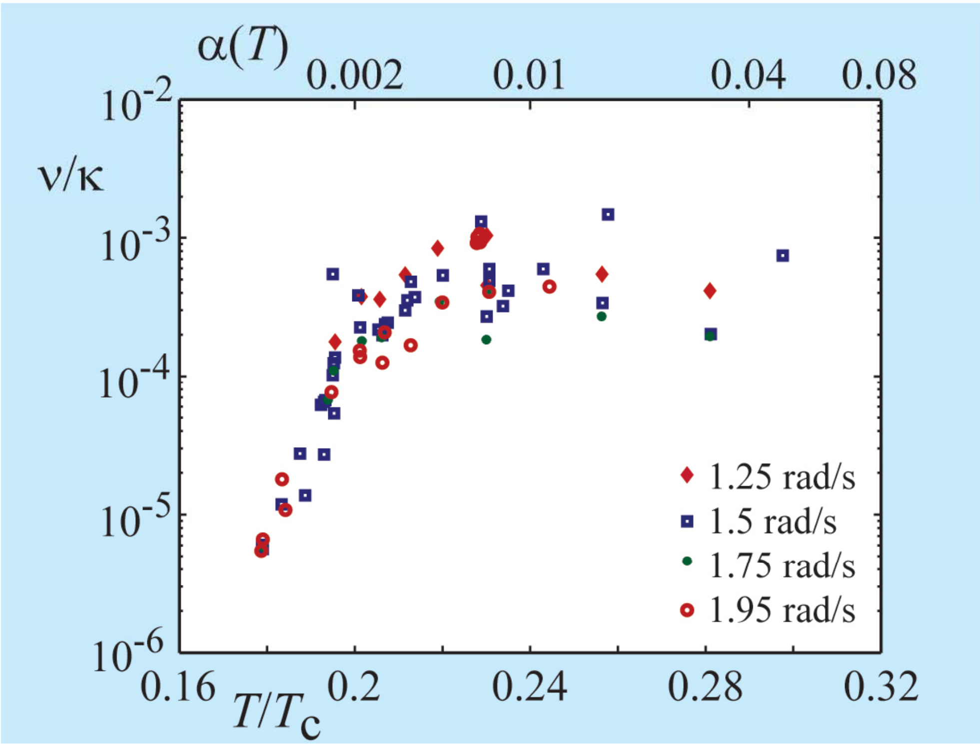
Fit: $\upsilon_s(t) = \dfrac{R\,\Omega_0}{1 + t/\tau_F}$



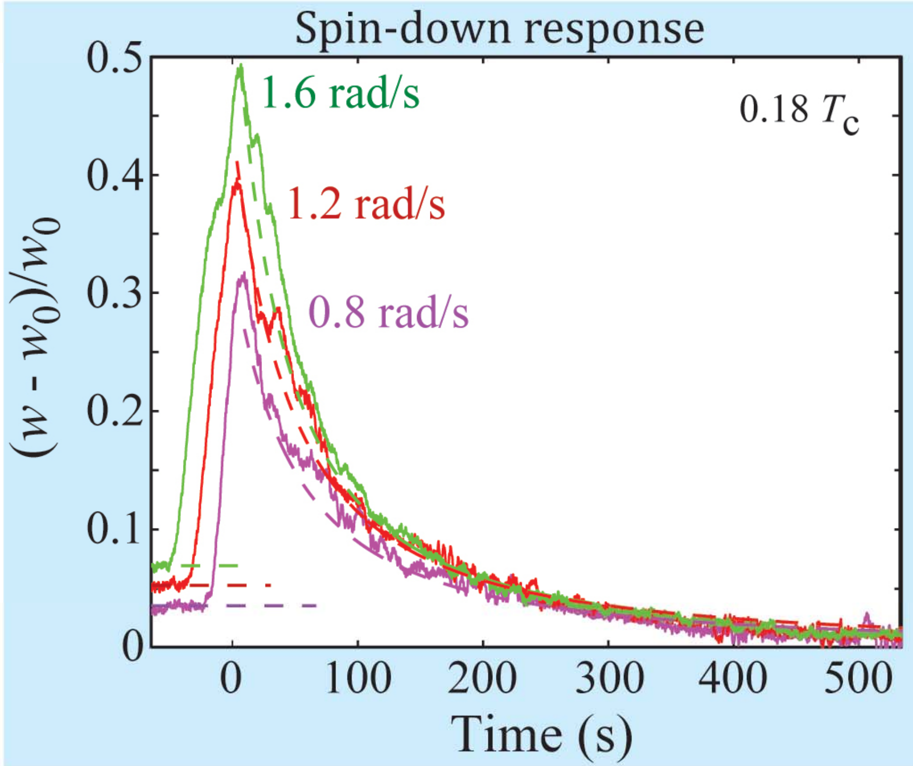
NMR measurement

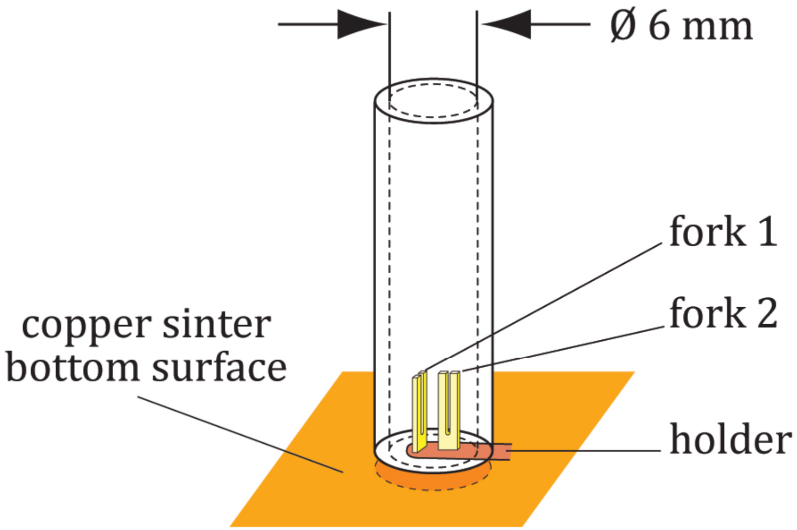
- ▽ $\Omega_i = 2$ rad/s
- △ $\Omega_i = 1.95$ rad/s
- ◇ $\Omega_i = 1.75$ rad/s
- □ $\Omega_i = 1.5$ rad/s
- ○ $\Omega_i = 1.25$ rad/s



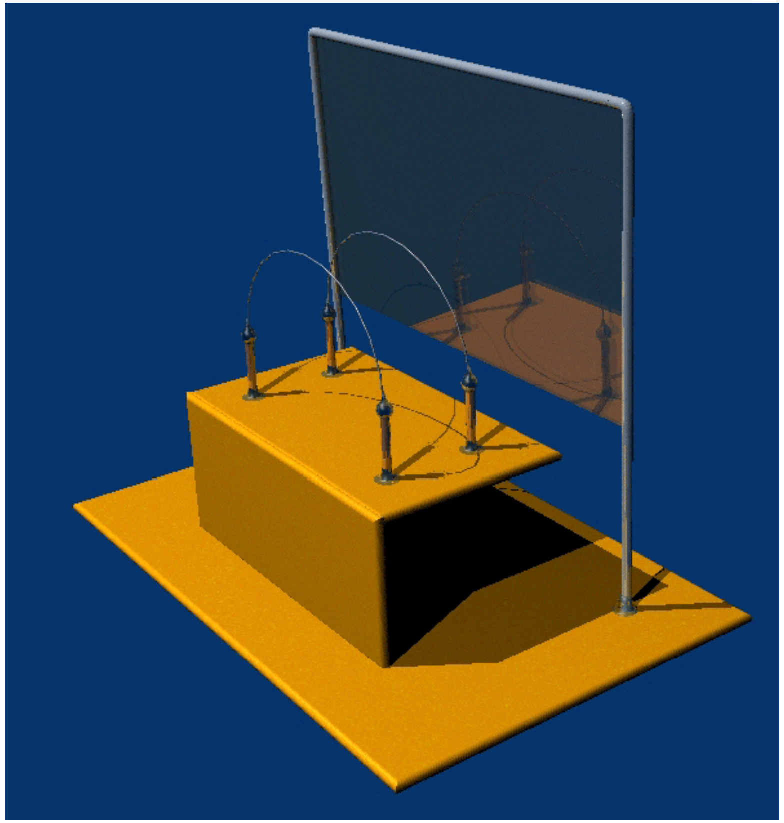


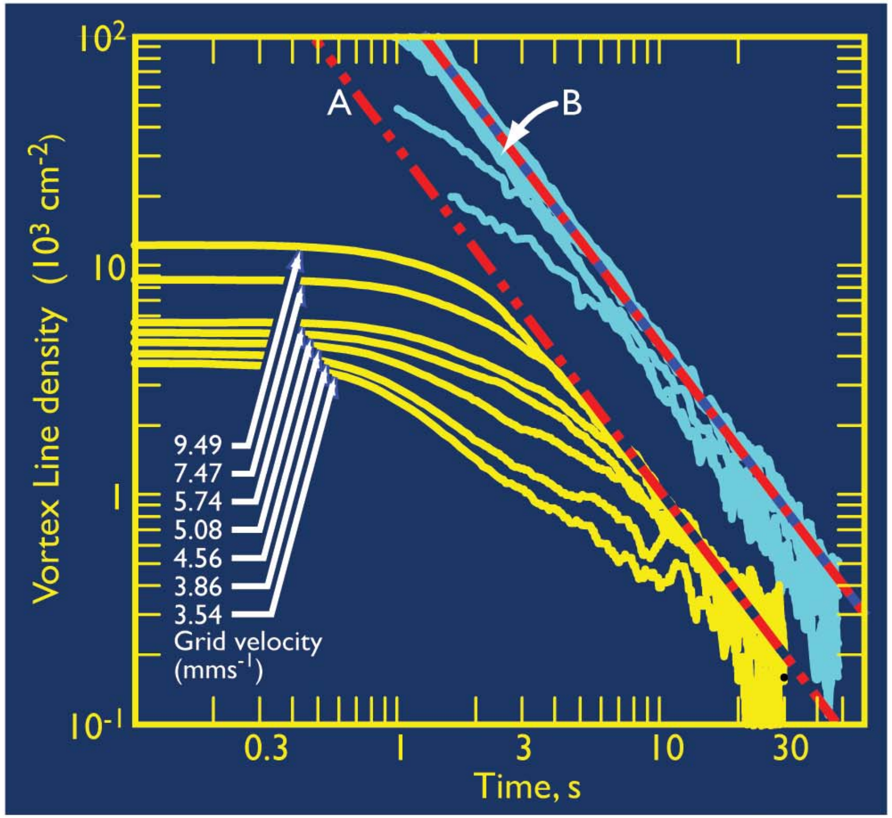
SPIN DOWN - BROKEN CYLINDRICAL SYMMETRY





DECAY OF VIBRATING-GRID-GENERATED TURBULENCE

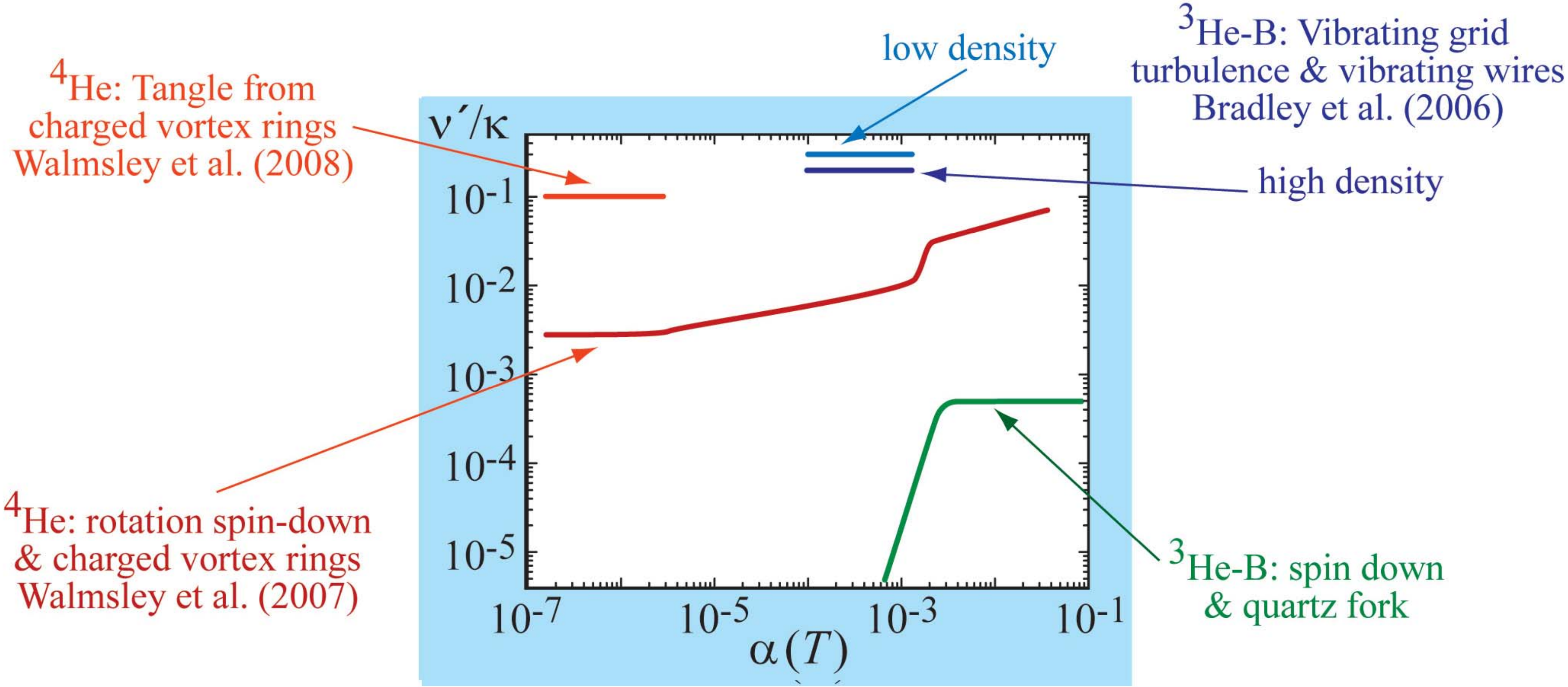




Bradley et al. (2006)

- tangle with $t^{-3/2}$ decay at high grid velocities
- tangled polarized vortex rings at low grid velocities with t^{-1} decay
- comparing spatial decay of Andreev reflected fraction on detectors #1 and #2 gives the size of the tangle as $d \approx 1.5$ mm
- decay results are for the closer detector
- ambient temperature depends on grid power; correction applied according to a thermometer reading
- calibration of vortex density: $f \approx L\, p_F\, \hbar\, d/(2 m_3 k_B T)$ Andreev reflected fraction
- $T < 0.2\ T_C$: no temperature dependence

DISSIPATION FROM TANGLE DECAY MEASUREMENTS



Decay analyzed as

- turbulence with motions at large scale d $\qquad L(t) = (3C)^{3/2}\, \dfrac{d}{2\pi\kappa}\, \dfrac{1}{\sqrt{\nu}}\, t^{-3/2}$
- flow restricted to scales < intervortex distance ℓ $\qquad L(t) = \dfrac{1}{4\pi}\, \ln(\ell/a_0)\, \dfrac{1}{\nu}\, t^{-1}$

- large differences in measured ν'/κ, although fit to $t^{-3/2}$ or t^{-1} is OK!
- reconnections are important: is it homogeneous isotropic turbulence?
- has the T = 0 temperature limit been reached in 3He-B?

VORTEX MOTION AND DISSIPATION IN SUPERFLUID ^{3}He-B

- Dissipation below $0.35\ T_C$ depends strongly on vortex polarization
  ⇒ no excess dissipation without reconnections?

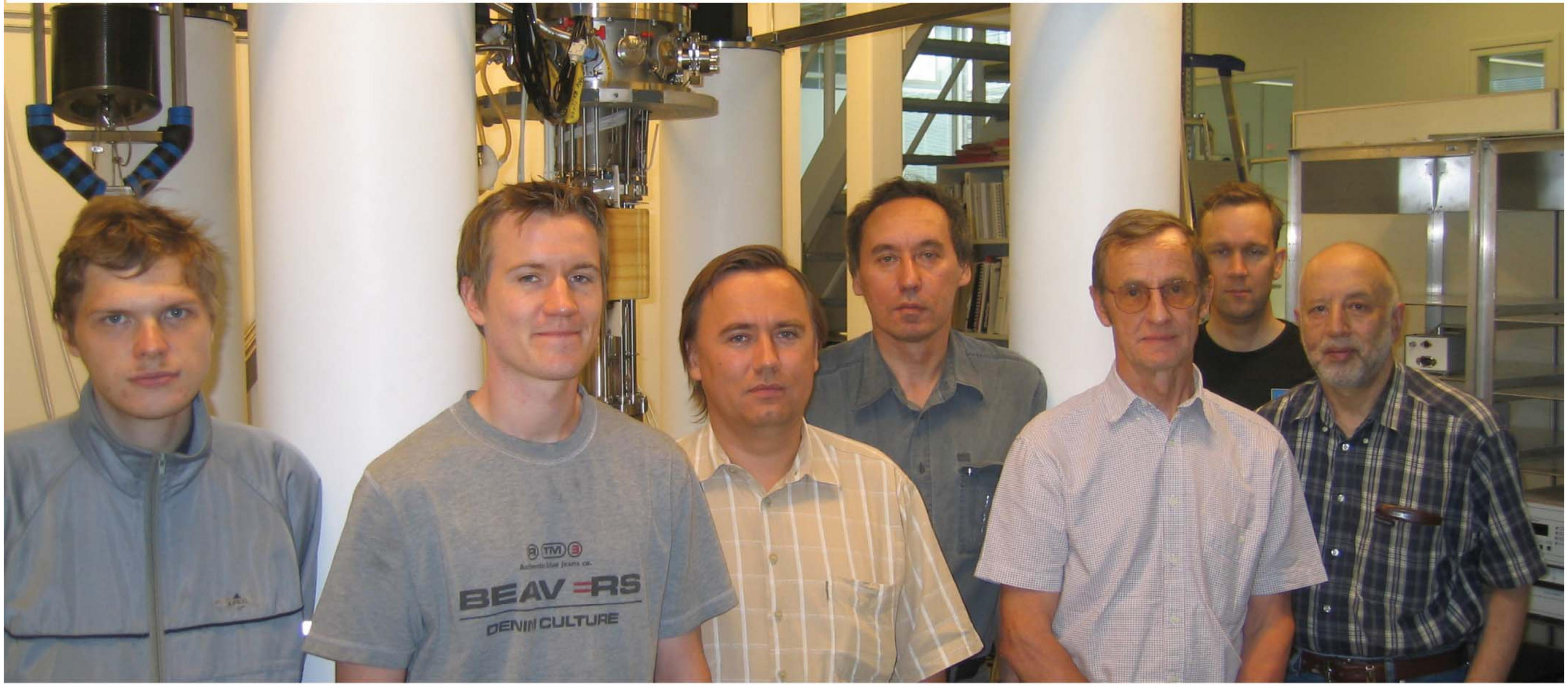
- Unresolved differences remain between decay measurements of "homogeneous & isotropic turbulence"

- What is the mechanism of $T = 0$ temperature dissipation in a p-wave Fermi superfluid

- More measurements are needed

- Measurements planned in rotating ^{3}He-B
  - thermal signal from propagating vortex front
  - NMR measurements of vortex tangle decay:
    - spin down of a volume with no axial symmetry
    - grid turbulence in rotation

VORTEX MOTION & TURBULENCE IN ROTATING ^{3}He-B BELOW 0.3 T_c



Experiment
V.B. Eltsov, R. de Graaf, P. Heikkinen,
J. Hosio, M. Krusius, R.E. Solntsev

Low Temperature Laboratory
Helsinki University of Technology

Theory
G.E. Volovik

Numerical calculations
R. Hänninen