The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2023-27

Workshop on Topics in Quantum Turbulence

16 - 20 March 2009

Numerical simulation of counterflow turbulence.

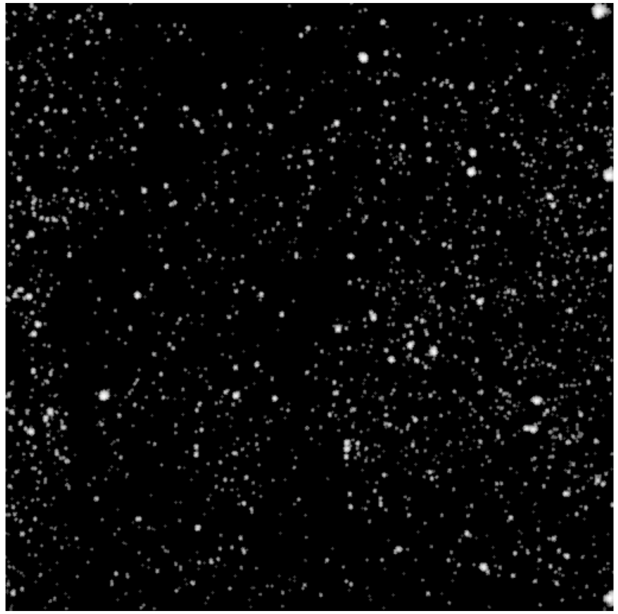
M. Tsubota
Osaka City University, Dept. of Physics
Osaka
Japan

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Numerical simulation of counterflow turbulence

Makoto Tsubota, Hiroyuki Adachi
Osaka City University, Japan

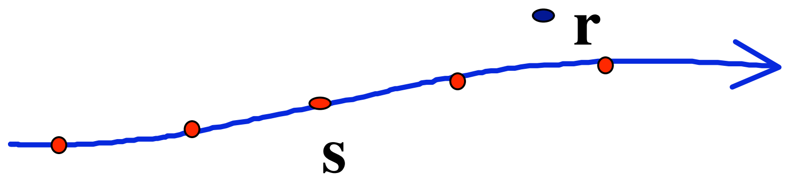
1. Making a steady state of counterflow turbulence by fully nonlocal Biot-Savart calculation
2. Obtaining the velocity distribution of vortices compared with the Maryland's observation



Visualization of counterflow by Paoletti et al.

Thanks to M. Paoletti, D. Lathrop

Vortex filament model (Schwarz)



A vortex makes the superflow of the Biot-Savart law, and moves with this local flow. At a finite temperature, the mutual friction should be considered.

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi}\mathbf{s}' \times \mathbf{s}'' + \frac{\kappa}{4\pi}\int_L' \frac{(\mathbf{s}_1 - \mathbf{r}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{r}|^3} + \mathbf{v}_{s,a}(\mathbf{s})$$

$$\dot{\mathbf{s}} = \dot{\mathbf{s}}_0 + \alpha\,\mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_0) - \alpha'\mathbf{s}' \times \left[\mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_0)\right]$$

The approximation neglecting the nonlocal term is called the LIA(Localized Induction Approximation).

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi}\mathbf{s}' \times \mathbf{s}'' + \mathbf{v}_{s,a}(\mathbf{s})$$

Schwarz's simulation(1) PRB38, 2398(1988)

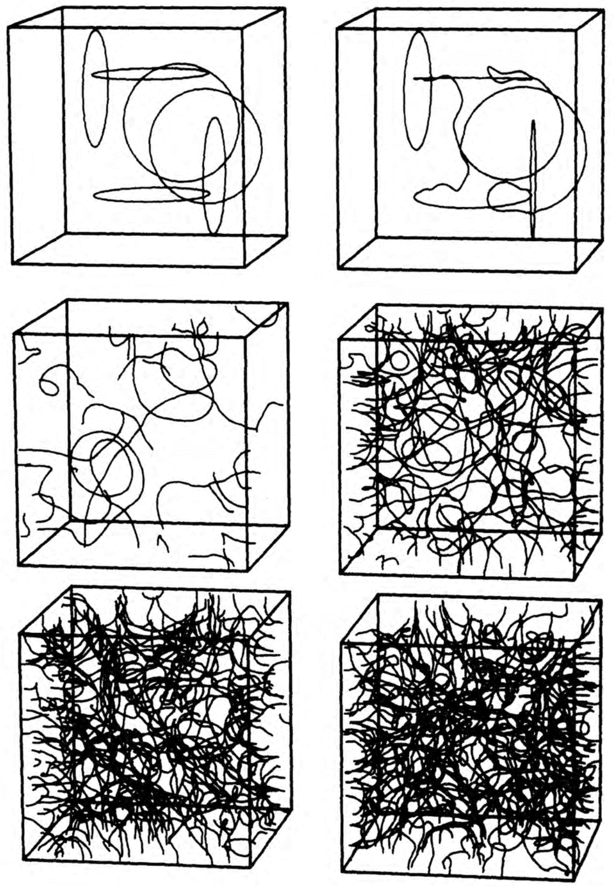


FIG. 4. Case study of the development of a vortex tangle in a real channel. Here, $\alpha = 0.10$, corresponding to a temperature of about 1.6 K, and $v_{s,0} = 75$ into the front face of the channel section shown. Upper left: $t_0 = 0$, no reconnections; upper right; $t_0 = 0.0028$, three reconnections; middle left: $t_0 = 0.05$, 18 reconnections; middle right: $t_0 = 0.20$, 844 reconnections; lower left: $t_0 = 0.55$, 12 128 reconnections; lower right: $t_0 = 2.75$, 124 781 reconnections.

Schwarz simulated the counterflow turbulence by the vortex filament model and obtained the steady state.

However, this simulation was unsatisfactory.

1. All calculations were performed by the LIA.

Schwarz's simulation(2) PRB38, 2398(1988)

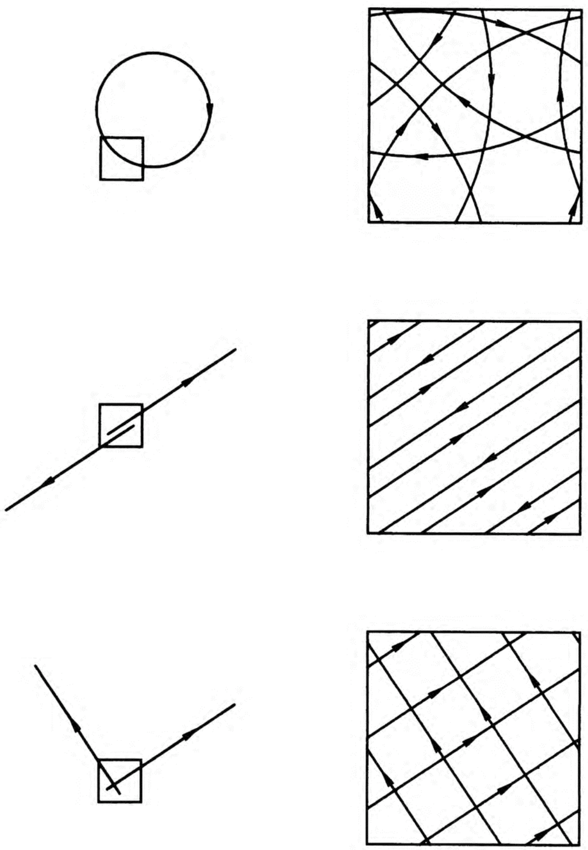


FIG. 8. Mapping of various vortex configurations into the computational volume, showing the appearance of the unit cell when all space is filled by the repetition of these objects. The end points of the lines represent equivalent points in the unit cell. Top row: closed loops; middle row: parallel infinite lines characteristic of a dead-end fluctuation; bottom row: infinite lines after randomizing procedure designed to reestablish three-dimensional behavior. The illustrations are intended to be purely schematic.

However, this simulation was unsatisfactory.

1. All calculation was performed by the LIA.
2. He used an artificial mixing procedure in order to obtain the steady state.

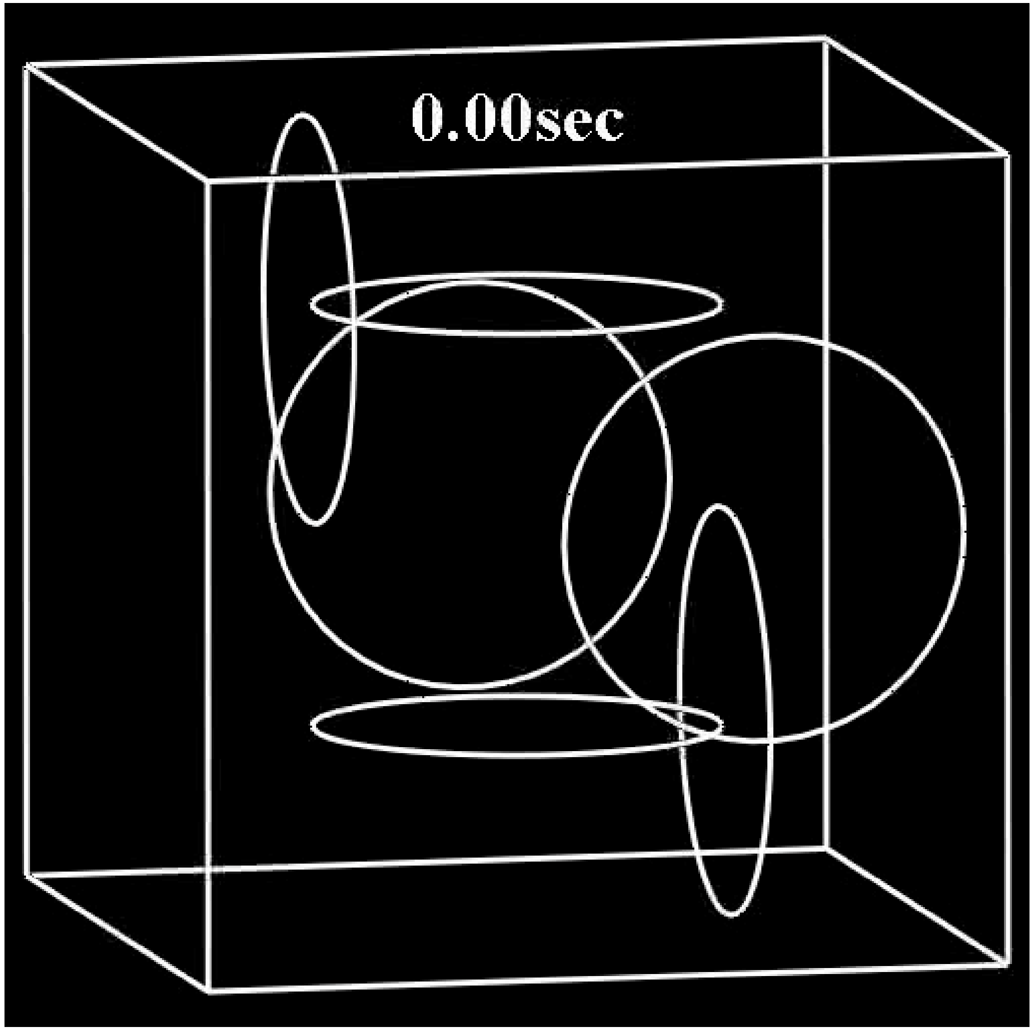
After Schwarz, there has been no progress on the counterflow simulation.

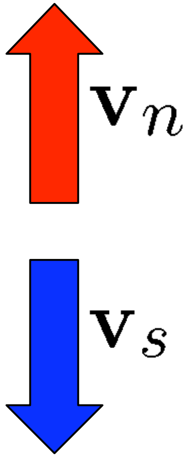
In this work we made the steady state of counterflow turbulence by fully nonlocal simulation to understand the Maryland's observation.

An important criterion of the steady state is to obtain

$$L = \gamma^2 |v_n - v_s|^2 \qquad \gamma = \frac{\pi B \rho_n \chi_1}{\kappa \rho \chi_2}$$

Simulation by the full Biot-Savart law





BOX 2mm×2mm×2mm

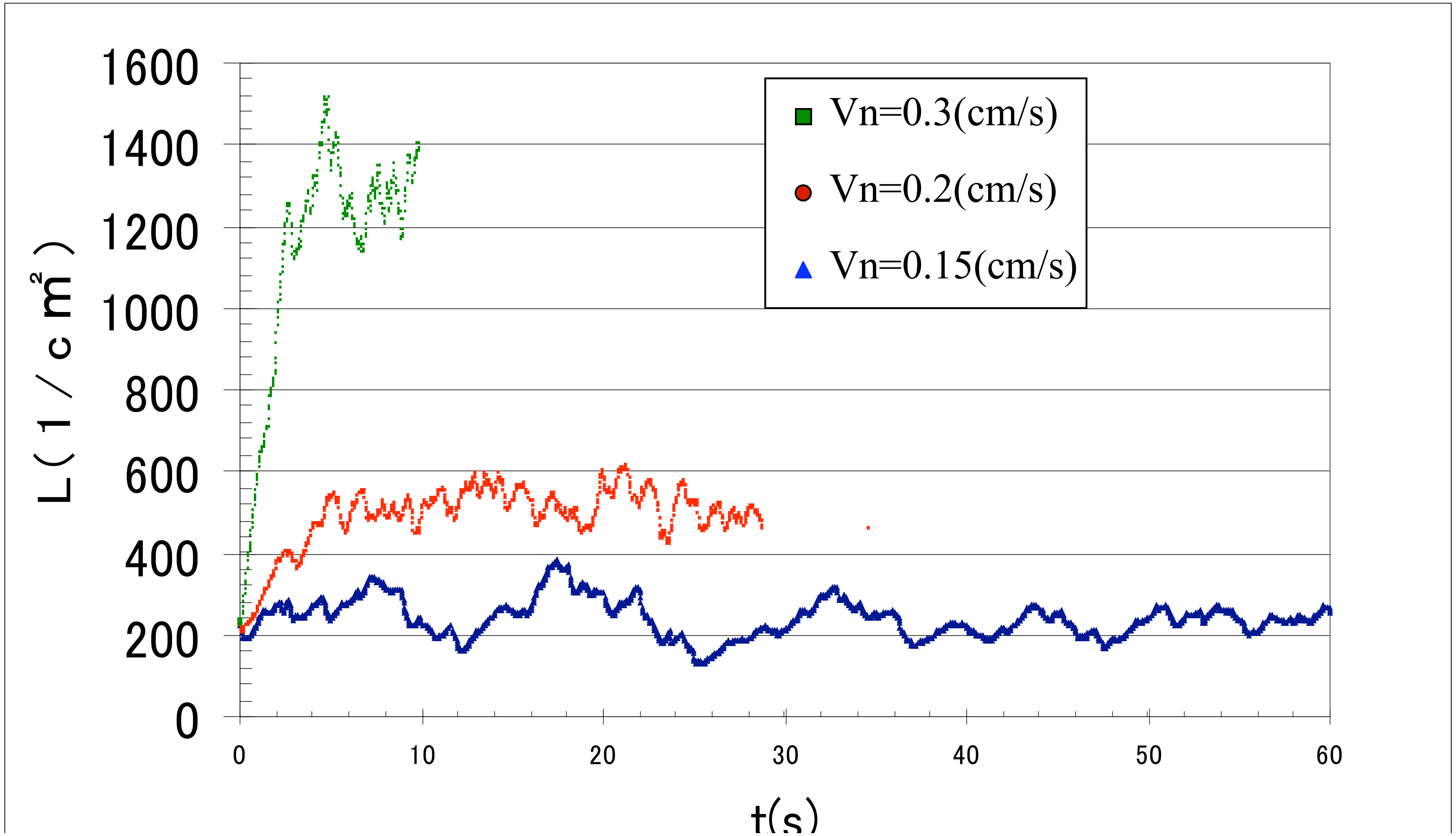
T=1.9(K)

Vn=3mm/s

Vs=2.71(mm/s)

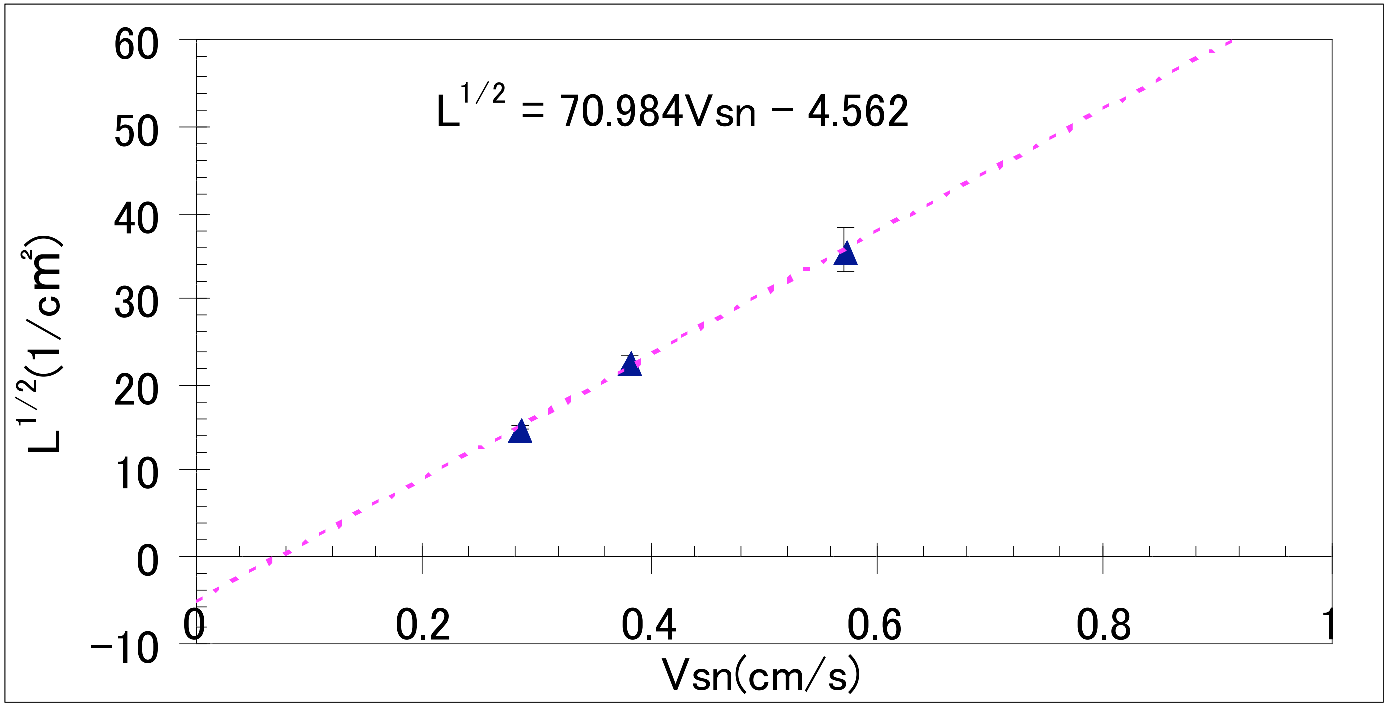
Periodic boundary conditions for all three directions

Time-development of the line-length density L



T=1.9K

Relation between L and v_{sn}



T=1.9(K)

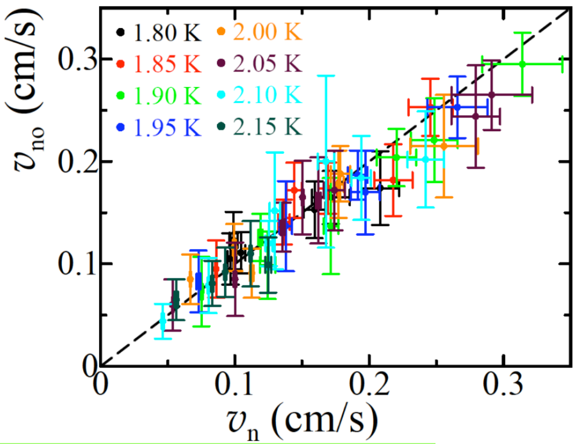
$$L = \gamma^2 |v_n - v_s|^2$$

The results are consistent with the observations by Childers and Tough, PRB13, 1040(1976).

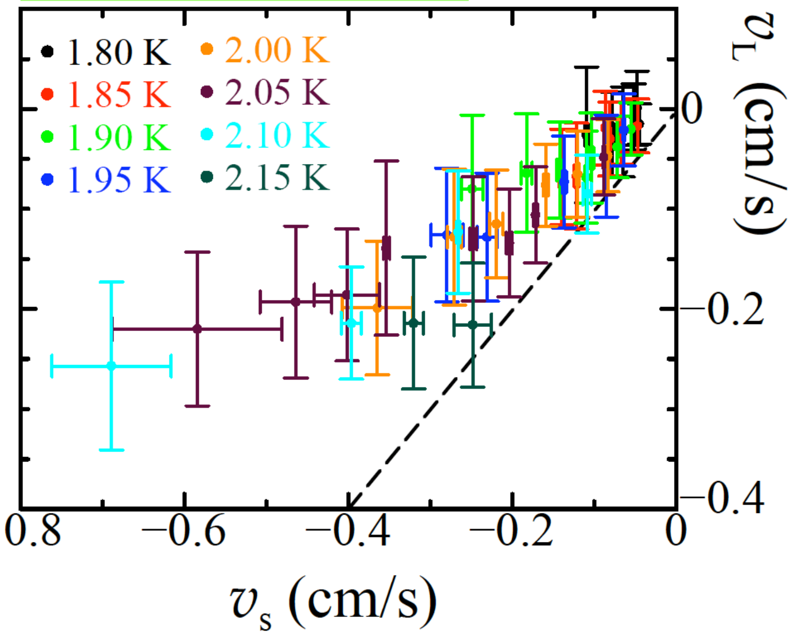
Observation of the velocity by the solid hydrogen particles

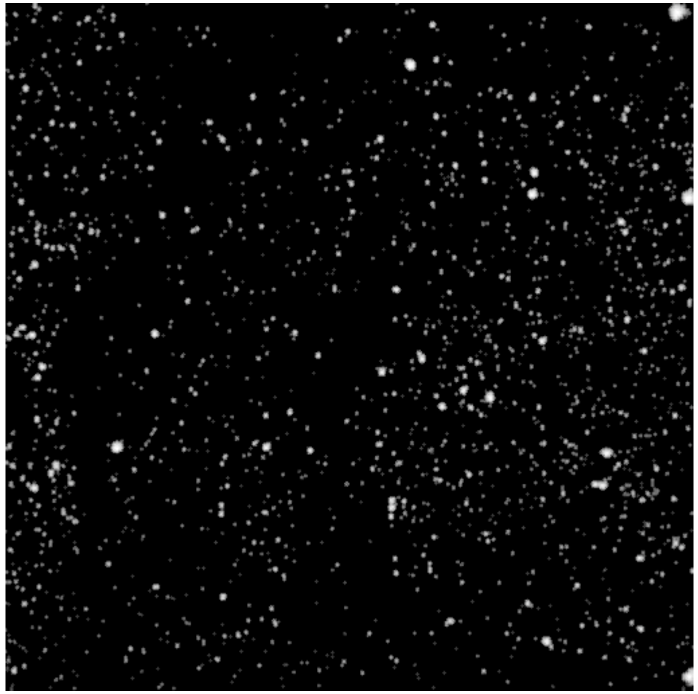
Paoletti,Fiorito,Sreenivasan,and Lathrop,J.Phys. Soc. Jpn. 77,111007(2008)

Upward particles



Downward particles



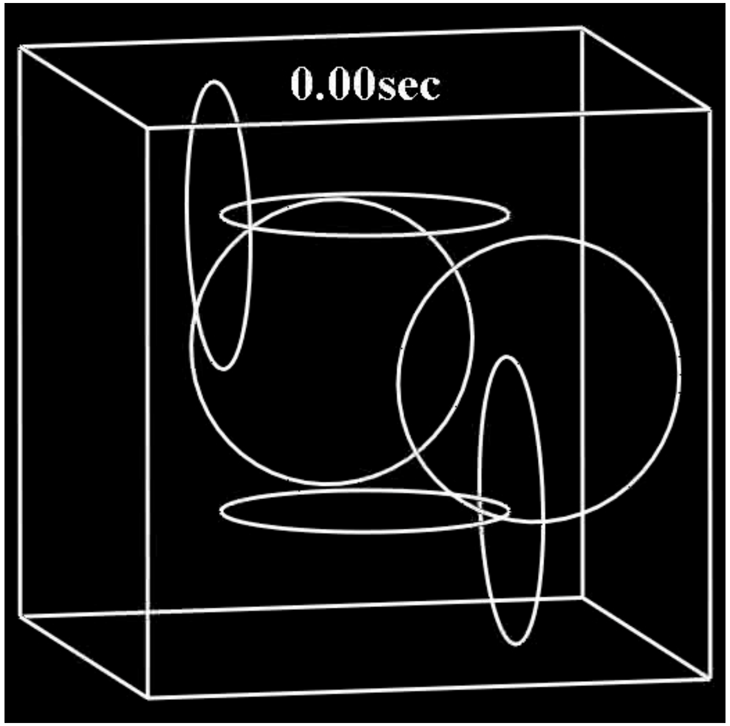


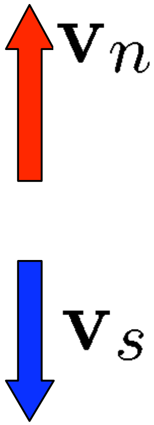
The broken line shows

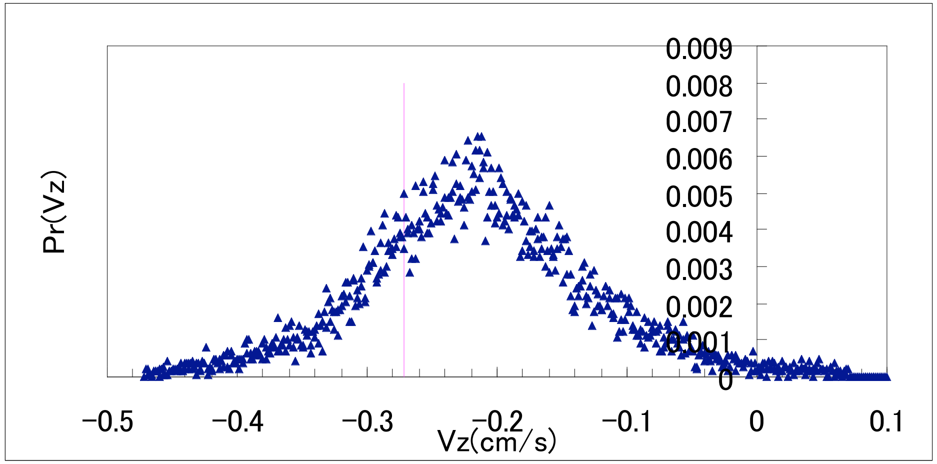
$$v_n = \frac{q}{\rho S T} \qquad v_s = -\frac{\rho_n}{\rho_s} v_n$$

The downward particles should be related with the velocity of vortices!

As the first trial, we obtained the velocity distribution on the vortices.







$$v_s = -\frac{\rho_n}{\rho_s} v_n = -0.271\text{cm/s}$$

T=1.9(K)

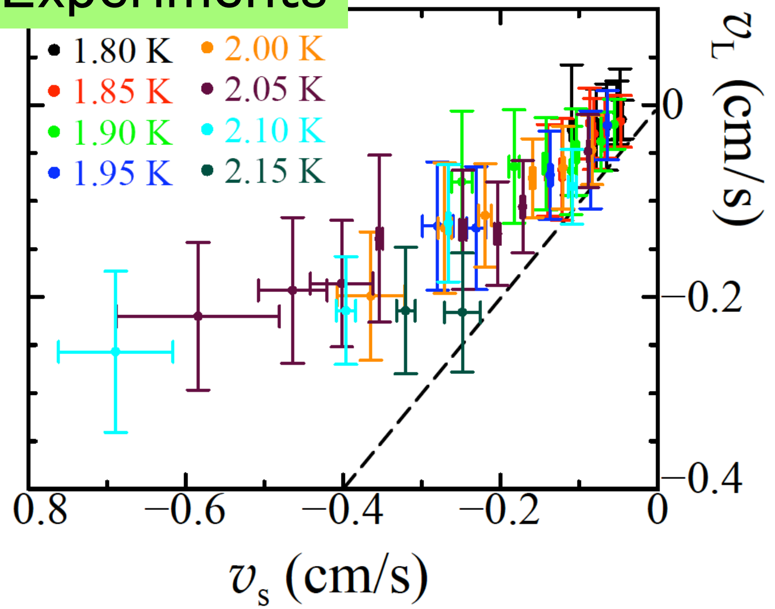
BOX 2mm × 2mm × 2mm

Vn=3(mm/s)

The peak is just shifted from the mean superflow velocity.

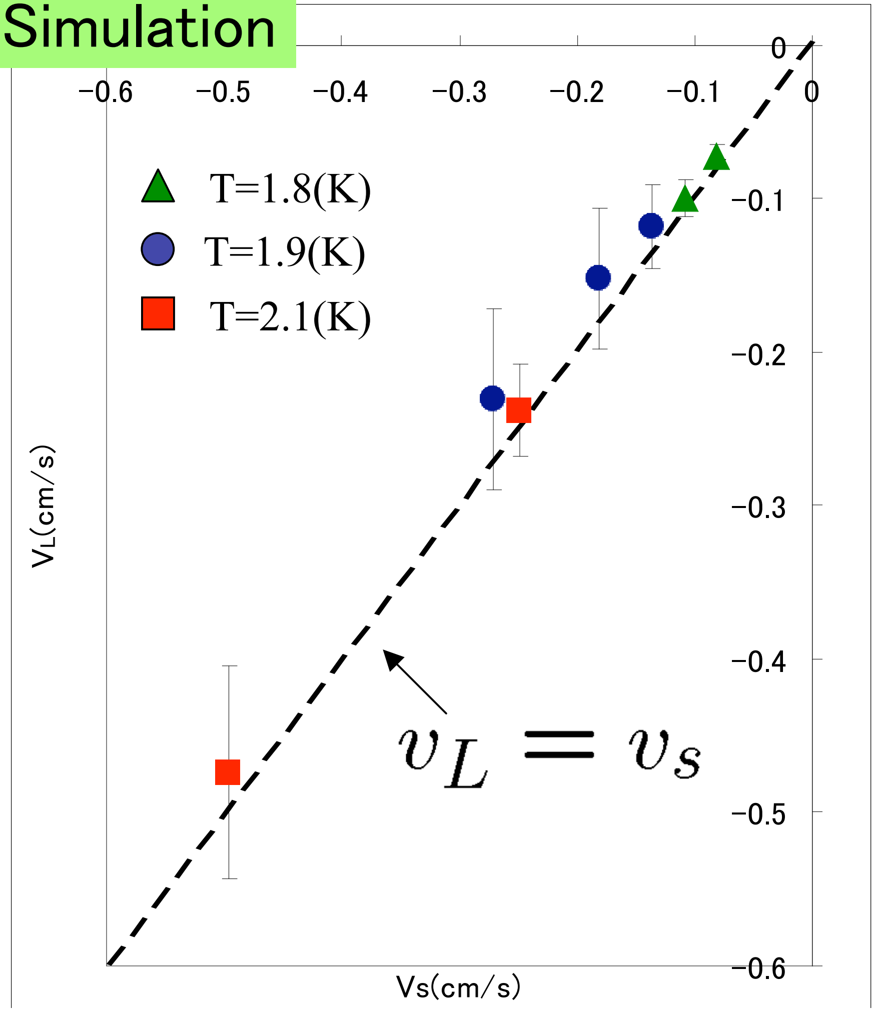
Velocity distribution(1)

Experiments



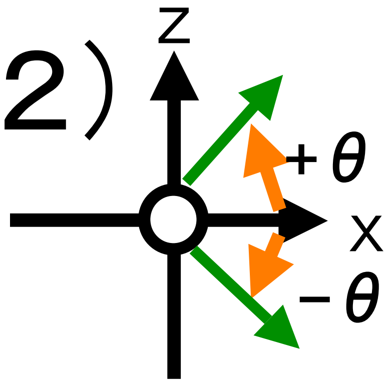
The simulation shows some qualitative behavior, but not quantitative agreement with the experiments.

Simulation



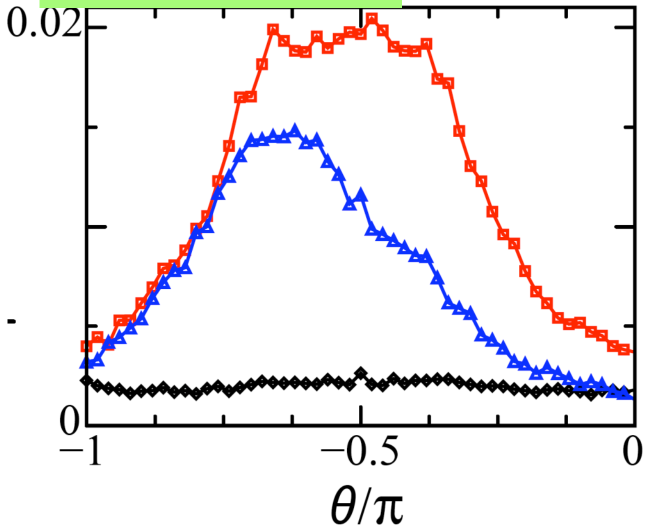
$$v_s = -\frac{\rho_n}{\rho_s} v_n$$

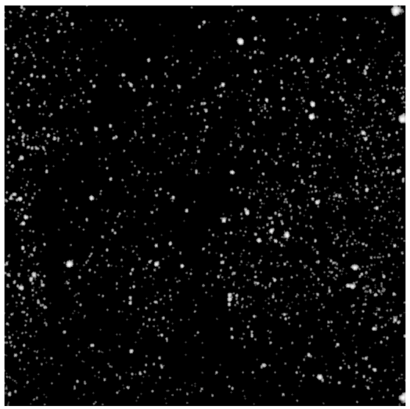
Velocity distribution（2）



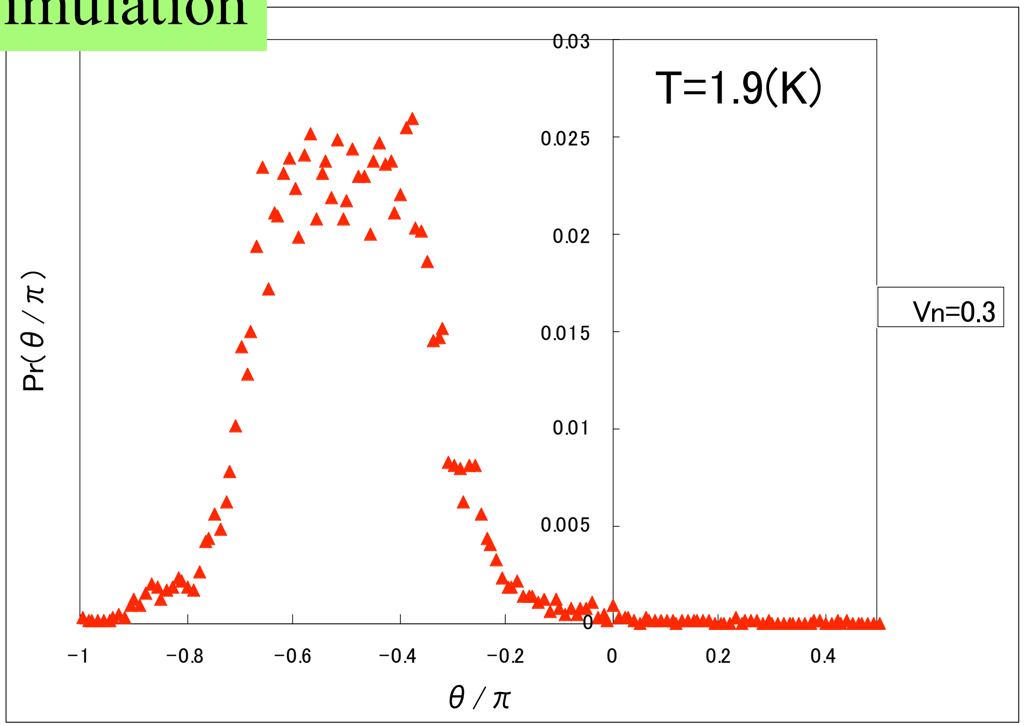
Direction of the downward particles

Experiments





Simulation



We obtained the similar distribution for other temperatures and velocities.

Summary

We performed the numerical simulation for counterflow turbulence by the full nonlocal Biot-Savart law.

1. We obtained statistically steady states.
2. We calculated the superfluid velocity distribution on the vortices and compared it with the Maryland's observation. They were qualitatively consistent.

Future developments

By considering the force acting on the particles, we will develop the analysis.