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Holography and strongly coupled model building

Lecture 3

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Trieste '09, Lecture #3

Kachru

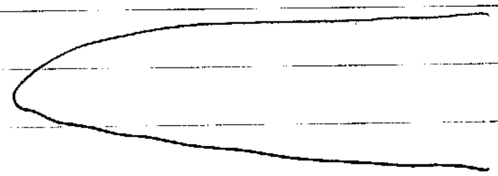
III. Solutions with exp small $\cancel{\text{SUSY}}$

At the end of the last lecture, we had constructed gravity solutions dual to the gauge theory ($\mathcal{N}=1$)

	$SU(M+N)$	$SU(N)$
$A_{1,2}$	$M+N$	$\bar{N}$
$B_{1,2}$	$\overline{M+N}$	N

$$W_{tree} \sim \epsilon\epsilon ABAB$$

For $N = kM \quad k \in \mathbb{Z}^+ \Rightarrow$

conifold pt smoothed $\Big\{$ 

$$\sum z_i^2 = 0 \rightarrow \sum z_i^2 = \varepsilon^2 \qquad \varepsilon \simeq e^{-\frac{2\pi K}{gM}}$$

Tip geometry $\sim$ a large S^3 pierced by M units of F_3^{RR} flux

$$F_3 \sim f\,\epsilon_{ijk}$$

$$a_0^2 = e^{-\frac{4\pi K}{3gM}} \qquad f \simeq \frac{2}{\sqrt{g_s^3 M}\; b_0} \qquad U(1)$$

$$ds^2 = a_0^2\, dx^\mu dx_\mu + g_s M b_0^2 \left(\tfrac{1}{2} dr^2 + r^2 d\tilde{\Omega}_2^2 + d\Omega_3^2\right)$$

a) SUSY solutions via D-brane probes (cf hep-th/0112197)

Natural idea: Given the tiny warped scale at the IR tip, can we locally do ~~SUSY~~ there to generate $F_{\cancel{SUSY}} \sim a_0^2 \sim e^{-4\pi k/3g M}$?

Sure.

Consider a $\overline{D3}$ probe in the KS solution with $N = kM$.

- No "extra D3"s in smooth SUGRA backgrnd $\Rightarrow$ $D3/\overline{D3}$ annihilation is impossible, despite the presence of N units of D3 charge.

- Dynamics? Determined by the DBI action on the probes.

$$S_{\overline{D3}} = -\frac{T_3}{g_s}\int d^4x \; \mathrm{Tr}\sqrt{\det(G_{11})\,\det(Q)}$$

$$- T_3 \int \mathrm{Tr}\left[2\pi i \, i_\Phi i_\Phi B_6 + C_4\right]$$

where:

$$Q^i_j = \delta^i_j + \frac{2\pi i}{g_s}[\Phi^i, \Phi^k](G_{kj} + g_s C_{kj})$$

and

$$i_\Phi i_\Phi B_6 = \Phi^n \Phi^m B_{mnpqrs} \frac{dy^p dy^q \cdots dy^s}{4!}$$

<u>Note</u>: $dB_6 = \frac{1}{g_s^2} *_{10} H_3 = \frac{-1}{g_s} dV_4 \wedge F_3$

with $dV_4 = a_0^4 d^4x$, & used ISD nature of the KS solution which relates H_3 & F_3 ($*_6 G_3 = i G_3$ w/ $G = F - \tau H$).

- First, imagine a single $\overline{D3}$ probe $\rightarrow$ drop all commutator terms in $S_{\overline{D3}}$:

$$S = \frac{T_3}{g_s} \int d^4x \sqrt{g_4}\, \mathrm{Tr}\, e^{4A} [2 + \frac{1}{2} e^{-2A} \partial^\mu \Phi^i \partial_\mu \Phi^j g_{ij}]$$

So $\exists$ a <u>potential</u> $\sim e^{4A} \longrightarrow$

$\overline{D3}$ [or COM of $p>1$ $\overline{D3}$s]
attracted to the TIP.

What is the dynamics AT the tip? Expanding out the matrices in $S_{\overline{D3}}$ $\rightarrow$

$$V_{eff} = \frac{T_3}{g_s}\left\{ p - i\frac{4\pi^2}{3} f\, \epsilon_{ijk} \mathrm{Tr}\,[\Phi^i, \Phi^j]\Phi^k - \frac{\pi^2}{g_s^2}\mathrm{Tr}\left([\Phi^i,\Phi^j]^2\right) + \cdots \right\}$$

Where are the extrema?

Not hard to see that $\exists$ extrema where

$$[\Phi^i, \Phi^j] = -i g_s^2 f\, \epsilon_{ijk} \Phi^k$$

-- which up to a rescaling, is just the commutation relations satisfied by SU(2) generators!

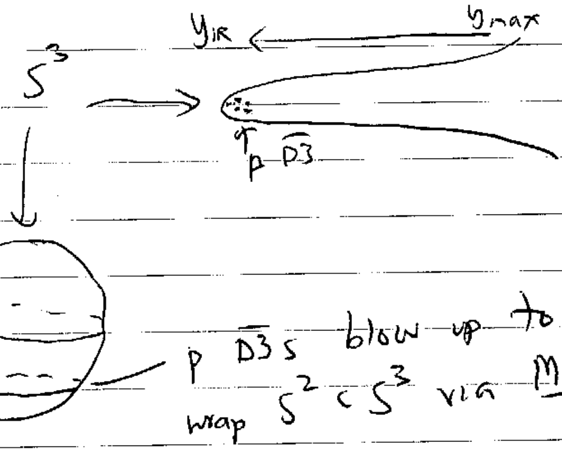
- p $\overline{D3}$s $\rightarrow$ need a p dim'l SU(2) representation

- $\exists$ critical pts for each one, ie any partition of p
- Minimal $V \to p$ dim'l irrep
- Can check that the radius of the blob of expanded $\overline{D3}$ is

$$R^2 \simeq \frac{4\pi^2}{M^2} p^2 \times \left(R_{S^3}\right)^2$$

$\to$ only reliable if $p \ll M$; otherwise, blob $\gtrsim$ size of space @ the tip!

So in the gravity regime, N D3s + M D5s + p $\overline{D3}$s has one branch of {vacua} where:



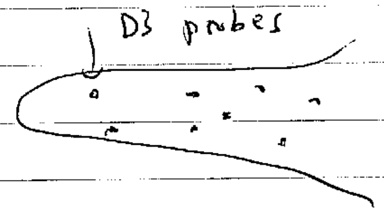
$N = KM$

p $\overline{D3}$s blow up to wrap $S^2 \subset S^3$ via Myers effect

SUSY energy $\sim p\, T_{\overline{D3}} \times e^{-\frac{8\pi K}{3gM}}$ } exp small.

Note that $\exists$ <u>another</u> state, visible in same gravity theory, w/ <u>same</u> <u>charges</u>:

- KS solution with $\tilde{N} = (K-1)M$, $\tilde{M} = M$, and $M-p$ D3s:

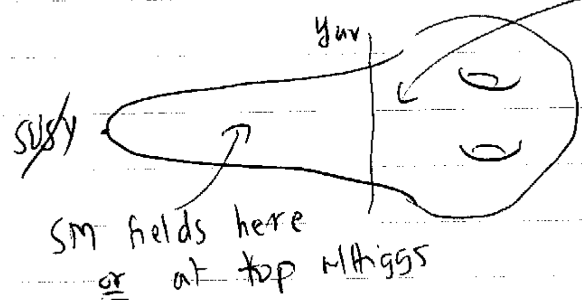


In fact, $\exists$ a brane/flux annihilation process that allows ~~SUSY~~ states to decay to these SUSY states. But, metastable if $p \ll M$.

b) <u>~~SUSY~~ solutions as normalizable perturbations of IIB SUGRA modes in conifold throat</u>

(cf arXiv: 0801.1520)

Our eventual goal is to use these ~~SUSY~~ states in a model where:



As long as no SM modes have support @ the tip, the ~~SUSY~~ effects on SM should be capturable if we know the ~~SUSY~~ gravity solution away from the tip at $r \gg \epsilon^{2/3}$.

In the SUSY theory with M D5s and N D3s, this sol'n was given by Klebanov & Tseytlin.

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} \, dX^\mu dX^\nu + e^{-2A(r)} \left[dr^2 + r^2 e_\psi^2 + r^2 \left(\sum_{i=1}^{2} e_{\theta_i}^2 + e_{\phi_i}^2 \right) \right]$$

where:

$$e_\psi = \frac{1}{3} \left(d\psi + \sum_{i=1}^{2} \cos\theta_i \, d\phi_i \right)$$

$$e_{\theta_i} = \frac{1}{\sqrt{6}} d\theta_i$$

$$e_{\phi_i} = \frac{1}{\sqrt{6}} \sin\theta_i \, d\phi_i$$

The warp factor is given by:

⑧

$$e^{-4A} = \frac{27\pi g_s}{4r^4}\left[N + \frac{3g_sM^2}{2\pi}\left(\log\left(\frac{r}{r_{uv}}\right) + \frac{1}{4}\right)\right]$$

$$\tau_{IIB} = \text{constant}$$

$$F_5 = (1+*)\cdot -27\pi\left[N + \frac{3g_sM^2}{2\pi}\log\left(\frac{r}{r_{uv}}\right)\right]$$

$$\times \ \mathrm{Vol}\,(T^{1,1}) \leftarrow \ = e_\psi \wedge e_{\theta_1} \wedge e_{\phi_1} \wedge e_{\theta_2} \wedge e_{\phi_2}$$

+ 3-Form fluxes:

$$F_3 = \frac{gM}{2}\, e_\psi \wedge (e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2})$$

$$B_2 = \frac{9g_sM}{2}\,[e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2}]\,\log\left[\frac{r}{r_{uv}}\right]$$

This KT solution preserves the full $SU(2) \times SU(2)$ global symmetry of the conifold.

The $\cancel{S}$USY state:

- Open string probe analysis → embiggened $\overline{D3}$ s @ the IR end of the geometry yield $\cancel{S}$USY.
- At long distance from tip, should be able to find a gravity sol'n encapsulating $\cancel{S}$USY

(a "KT" version of the ~~SUSY~~ state).

The smeared ~~SUSY~~ solution is:

$$ds^2 = r^2 e^{2a(r)} \eta_{\mu\nu} dx^\mu dx^\nu +$$

$$e^{-2a(r)} \left[\frac{dr^2}{r^2} + e^{2b(r)} e_\psi^2 + e^{2c(r)} \sum_i (e_{\theta_i}^2 + e_{\phi_i}^2) \right]$$

where - at leading order in M/N

- at leading order in $S = p e^{-\frac{8\pi k}{3gM}}$

$$e^{-2a} = \left(\frac{1}{2} + \frac{S}{32 r^4} \right) \sqrt{27\pi g N}$$

$$e^{2b} = 1 + \frac{S}{r^4}$$

Dilaton $\phi = \log g_s + \frac{1}{r^4} [-3 S \log(r)]$

$+ \cdots$ (Δ 3-form fluxes at $O(S)$)

The UPSHOT of all of the changes, is that:

- The $T^{1,1}$ is "squashed"
- Non-ISD (1,2) flux sourced
- ϕ now runs

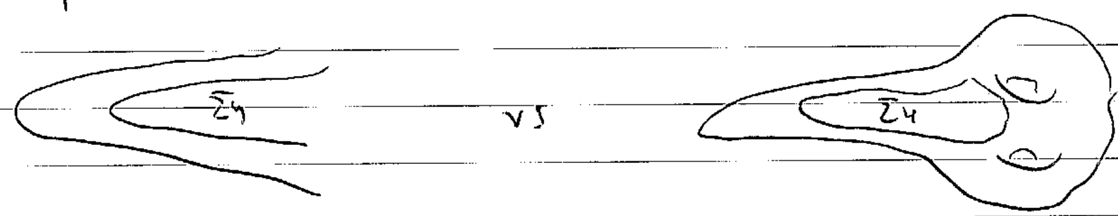
All are $\mathcal{O}(S')$, normalizable perturbations; this is a $\cancel{\text{SUSY}}$ state in KS field theory.

IV. Coupling $\cancel{\text{SUSY}}$ to SM

a) Adding a SM / GUT gauge group

Compared to the strongly coupled KS theory, the SM group is weakly gauged. So we should try to embed it in the throat by adding eg an SU(5) global symmetry.

Intuitively, we can get new gauge sector by adding D7s in throat. The D7 gauge fields are at $g_{YM}^{D7} \to 0$ in non-compact limit, but have finite g_{YM}^{D7} after compactification:



$$\frac{1}{(g_{YM}^{D7})^2} \sim \frac{\mathrm{Vol}(\Sigma_4)}{g_s} \rightarrow \begin{cases} \infty & \Sigma_4 \text{ non-compact} \\ \text{finite} & \Sigma_4 \text{ compact} \end{cases}$$

Want SUSY, MSSM or GUT, so choose a SUSY D7 embedding → specified by a holomorphic embedding (we'll avoid gauge bundle subtleties for now).

Easiest choice:

Conifold $\quad \sum z_i^2 = \varepsilon^2$

$\Sigma_4 \quad z_4 = \mu \quad$ "Kuperstein embedding"

K D7s on Σ_4 ⇒ get additional SU(k) "gauge" group! $k \geq 5$ → this is good enough.