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Holographic Superconductors and Superfluids
Lecture 3

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Using AdS/CFT

- holographic dual for a 2+1 dim'l field theory
  - maximally supersymmetric $SU(N)$ Yang-Mills theory in 2+1 dim.s
  - coupling is dimensionful $[g^2] = $ mass
    flows to IR superconformal fixed point
  - 10 yr. old conjecture that the IR fixed pt. is dual to M-theory in an $AdS_4 \times S^7$ background (Maldacena, ...)
- we can describe a sector of this theory with the following 4d effective action (consistent truncation)

$$S = \frac{1}{2K^2} \int d^4x \sqrt{-g}\,(R - 2\Lambda) - \frac{1}{4g^2} \int d^4x \sqrt{-g}\, F_{\mu\nu} F^{\mu\nu}$$

$\Lambda = -3/L^2$ — negative cosm'al constant – space is asymp'ally AdS_4

$F_{\mu\nu}$ — gauge fields in bulk (gravity) dual to global symmetries in the bry (field theory)
- describes a $U(1)$ subgroup of the global $SO(8)$ R-symmetry

- classical grav'al description is valid at large N, $\frac{1}{K^2} \sim N^{3/2}$
- AdS is a hyperboloid w/ a time like direction, has a bry
- reminiscent of last time, we treat the on-shell classical action as a gen'ating fn'al for correlators in the field theory w/ the bry values of $g_{\mu\nu}$ and A_μ playing the role of external metric and gauge field $\Rightarrow$ way to compute $\langle JJ \rangle$, $\langle JT \rangle$, $\langle TT \rangle$!

- an important classical sol'n. - the dyonic black hole

$$\frac{ds^2}{L^2} = \frac{\alpha^2}{z^2}\left(-f(z)\,dt^2 + dx^2 + dy^2\right) + \frac{1}{z^2}\frac{dz^2}{f(z)}$$

$$A = h\alpha^2 x\,dy - q\alpha(1-z)\,dt \qquad \text{horizon} \quad z=1$$

$$f(z) = 1 + (h^2+q^2)z^4 - (1+h^2+q^2)z^3 \qquad \text{bry} \quad z=0$$

- calculating $\langle J^\mu \rangle$
  - near bry $A_\mu = a_\mu + b_\mu z + \ldots$
  - on-shell, Maxwell part of the action reduces to the bry term

$$W = \frac{\alpha}{g^2}\int d^3x\, \eta^{\mu\nu} A_\mu \partial_z A_\nu \qquad (A_z = 0 \text{ gauge})$$

  - $\langle J^\mu \rangle = \dfrac{\delta W}{\delta a_\mu} = \dfrac{\alpha}{g^2} b^\mu$

for our dyonic black-hole, $b_t = q\alpha$, $a_t = -q\alpha \equiv \mu$

$$\Rightarrow \rho = \langle J^t \rangle = -\frac{q\alpha^2}{g^2} = \frac{\mu\alpha}{g^2}$$

electric field of black hole is a charge density in FT!

- Ohm's Law $\quad \sigma_\pm = \dfrac{\pm i J_\pm}{E_\pm} \qquad J_\pm = J_x \pm i J_y, \quad E_\pm = E_x \pm i E_y$
  - we just saw $g^2 J_\pm = \lim_{z\to 0} \alpha\, \partial_z A_\pm$

we can think of $\pm i\alpha \partial_z A_\pm$ as a bulk magnetic field $\mathcal{B}_\pm$

$E_\pm$ as the bry limit of a bulk electric field $\mathcal{E}_\pm$

$$\Rightarrow \sigma_\pm = \lim_{z\to 0} \frac{\mathcal{B}_\pm}{g^2 \mathcal{E}_\pm}$$

[aside on (classical) electric magnetic duality

given $\frac{1}{4g^2}\int F_{\mu\nu}F^{\mu\nu}\sqrt{-g}\,d^4x$

we can think of $F_{\mu\nu}$ or $\sqrt{-g}\,F^{\mu\nu}$ as the fund'al field strength

$$\mathcal{B}_\pm \to -\mathcal{E}_\pm \;, \quad \mathcal{E}_\pm \to \mathcal{B}_\pm \;, \quad h \to -q, \; q \to h$$
]

$\Rightarrow$ we can calculate $\sigma_\pm$ very easily when $h = q = 0$

$$\sigma_\pm = \lim_{z\to 0} \frac{\mathcal{B}_\pm}{g^2 \mathcal{E}_\pm} = -\lim_{z\to 0} \frac{\mathcal{E}_\pm}{g^2 \mathcal{B}_\pm} = -\frac{1}{\sigma_\pm}\frac{1}{g^4}$$

$$\Rightarrow \quad \sigma_\pm = \pm\frac{i}{g^2} \quad \Rightarrow \quad \sigma_{xx} = \frac{1}{g^2}\,, \quad \sigma_{xy} = 0$$

freq. ind. conductivity when $\rho = B = 0$

- $\sigma_\pm(\rho, B)$ requires numerics in general but

$$\sigma_\pm(q,h) = -\frac{1}{g^4\,\sigma_\pm(h,-q)} \quad \text{from S-duality}$$

- hydro limit ($\omega \ll T$, $B/T^2 \ll 1$) is special

$$\left.\begin{aligned} \sigma_+ &= i\sigma_Q \frac{\omega + i\omega_c^2/\gamma + \omega_c}{\omega + i\gamma - \omega_c} \\ \omega_c &= \frac{B\rho}{\varepsilon + P} \qquad \gamma = \frac{\sigma_Q B^2}{\varepsilon + P} \end{aligned}\right] \text{forced by hydro}$$

P ← pressure

ε ← energy density

for this model, we can calculate

$$\sigma_Q = \frac{(sT)^2}{(\varepsilon+P)^2}\frac{1}{g^2} \quad \text{and we know} \quad \varepsilon = 2P$$

- notice the existence of a cyclotron pole
  - ω_c like $\omega_f = \frac{eB}{mc}$ free particle result

    but this ω_c is a collective fluid motion
  - Hartnoll et al. estimated that for $La_{2-x}Sr_xCuO_4$

$$\omega_c = 6.2 \text{ GHz} \frac{B}{1\,\text{Tesla}} \left(\frac{35K}{T}\right)^3$$

0.035 times the free e^- result $\left[\frac{\omega_c}{\omega_f} = 0.035\right]$

- unfortunately $\omega_c \sim \frac{1}{\tau}$ inverse scattering time

  may be unobservable (but graphene)

- recall σ lets us calculate θ via Ward identities

$$\overleftrightarrow{\theta} = -\overleftrightarrow{\sigma}^{-1} \cdot \overleftrightarrow{\alpha}$$

- in hydro limit $\theta_{xy} = -\frac{B}{T} \frac{i\omega}{((\omega + i\omega_c^2/\gamma)^2 - \omega_c^2)}$

- note that $\omega \to 0 \Rightarrow \theta_{xy} \to 0$

  vanishing is an artifact of (translation invariance $\Rightarrow \sigma \to \infty$)

- Hartnoll et al. suggested introducing an impurity scattering time

$$\omega \to \omega + i/\tau$$

$$\Rightarrow \lim_{\omega \to 0} \theta_{xy} = -\frac{B}{T} \frac{1/\tau}{((1/\tau + \omega_c^2/\gamma)^2 + \omega_c^2)}$$

a result which seems to reproduce some of the B and T dependence of high T_c superconductors

- Hartnoll and I made a few more improvements by calculating B and ρ dependence of τ