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Spring School on Superstring Theory and Related Topics

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Black Holes as Effective Geometries

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Black Holes as Effective Geometries TRIESTE 2009

Based on: 0802.2257 JdB, Denef, El-Showk, Messamah, van den Bleeken

0807.4556 JdB, El-Showk, Messamah, van den Bleeken

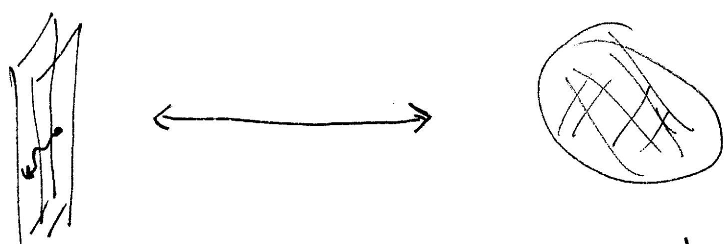
+ work in progress

also on:

⇒ Review with more background material and references:

0811.0263 V. Balasubramanian, JdB, El-Showk, Messamah

- Well known fact: black holes are thermal objects with entropy $S = A/4G$
- most microscopic countings of S in string theory use



bound state of branes
weak coupling
open strings

black hole
strong coupling
closed strings

transition occurs when size of bound state is of the order of the Schwarzschild radius

Counting is done at weak coupling (field theory) and then extrapolated to strong coupling

works if number of states does not jump ⟶ BPS black holes

⟶ use index rather than total # of states

- Most examples involve a 1+1 field theory & use Cardy's formula:

$$\log(\#\text{ states w/ values } L_0, \bar{L}_0) \sim 2\pi\sqrt{\frac{cL_0}{6}} + 2\pi\sqrt{\frac{c\bar{L}_0}{6}}$$

Well-known examples:

D1–D5–p black hole, dual involves a $N=(4,4)$ superconformal $d=2$ field theory $\rightarrow$ Cardy ✓

D0–D2–D4 dual involves a $N=(0,4)$ superconformal $d=2$ field theory $\rightarrow$ Cardy ✓.

D0–D2–D4–D6 (eg. BMPV black hole) no $d=2$ CFT description entropy related to growth of Gopakumar-Vafa invariants

$\frac{1}{16}$ BPS BH in AdS_5 no $d=2$ CFT description precise field theory explanation of the entropy has not been given

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– Would like to go beyond just counting S, ultimately ask dynamical questions. (what happens to an infalling observer, how does info come out, etc etc)

– For example, can we follow the individual degrees of freedom as we increase g_s? Is there a closed string description of the microstates? Would be very helpful.

– Knowing the microstates would also help understand how pert. gravity breaks down near the horizon. Naively, curvatures can be made arb. small, so effective field theory should be good?

– Various arguments suggest that the breakdown should be nonlocal (see eg. Giddings, 0705.2197) :
  - ∞ redshift near the horizon
  - AdS/CFT is nonlocal
  - cross section is bounded by $(\log E)^{D-2}$ in local unitary field theory. Gravity (black holes) has $\sigma \sim E^{(D-2)/(D-3)}$
  - pert. theory breaks down when black holes form (2 particles, resum graviton exchanges in the eikonal approximation)

Very difficult to make precise statements due to lack of obvious local observables in quantum gravity. AdS/CFT only computes an S-matrix

Would like to advocate the following picture:

① Grav. entropy arises from coarse graining microstates $\Psi \in \mathcal{H}$

② For almost all Ψ, "$\langle \Psi | g_{\mu\nu} | \Psi \rangle$" looks like a black hole geometry to great accuracy

③ small supersymmetric black holes: can realize all Ψ in terms of smooth supergravity solutions alone

④ large susy BH's: supergravity + string modes are needed

⑤ required nonlocality arises because the fluctuations in the metric are much larger than naively expected near the horizon: $\langle \Psi | g^2 | \Psi \rangle - \langle \Psi | g | \Psi \rangle^2$ is enhanced.

⑥ Low energy effective field theory breaks down due to large quantum effects (macroscopic quantum effects) → novel mechanism

⑦ Can see this explicitly in examples.

Crucial ingredient:
How does one obtain the Hilbert space of microstates from smooth supergravity solutions?

Suppose $M =$ space of smooth sugra solutions with the same quantum numbers as a given black hole

"smooth" means nonsingular, possibly after including higher order corrections.

intuition: these should correspond to pure states

mixed states with entropy will be described by solutions with a horizon.

Often M is a phase space. [set of all solutions of sugra is certainly a phase space]

symplectic form:

$$\omega = \int d\Sigma^\mu \, \delta\left(\frac{\delta \mathcal{L}}{\partial(\partial^\mu \phi)}\right) \wedge \delta\phi$$

$\downarrow$ momenta $\qquad \downarrow$ coords

ω follows from $\mathcal{L}_{sugra}$: tedious!

Given $(M, \omega) \xrightarrow{\text{quantize}} \mathcal{H}$

Quantization is complicated in general (and not unique)
Often can use geometric quantization, eg if $(M, \omega) =$ kähler.

- find line bundle $\mathcal{L}$ with $c_1(\mathcal{L}) = \omega$
- define $\mathcal{H} = H^0(M, \mathcal{L})$, space of holomorphic sections of $\mathcal{L}$
- local picture: holomorphic functions $\psi(z)$, $\langle \psi | \psi \rangle = \int_M e^{-K} \bar{\psi}(z) \psi(z)$

example: $\mathbb{R}^2$, $K = z\bar{z}$ (harmonic oscillator) because $\omega = \partial\bar{\partial} K \sim dz \wedge d\bar{z} \sim \delta p \wedge \delta q$

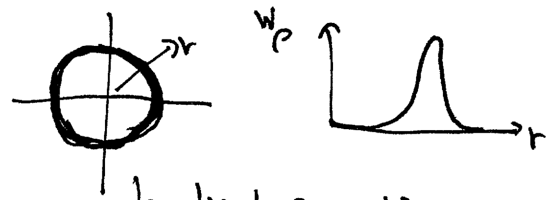
$\psi(z) = z^n \iff (a^\dagger)^n |0\rangle$

Another useful property : phase space density

$$\rho = \sum c_{n,m} |n\rangle\langle m| \implies W_\rho(z,\bar{z}) = \sum_{n,m} c_{n,m} \frac{\bar{z}^n}{\sqrt{n!}} \frac{z^m}{\sqrt{m!}} e^{-K}$$

W_ρ tells us where ρ is localized on phase space.

e.g. $\rho = |n\rangle\langle n| \implies$



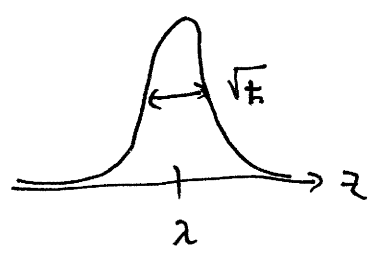
localized on a ring

important: coherent states $\quad e^{\bar{\lambda} a^+}|0\rangle = |\lambda\rangle \quad (\leftrightarrow \psi(z) = e^{\bar{\lambda} z}$

$$\rho = |\lambda\rangle\langle\lambda| \implies W_\rho(z,\bar{z}) \sim e^{-z\bar{z}} e^{\bar{\lambda} z} e^{\lambda \bar{z}}$$
$$\sim e^{-\lambda\bar{\lambda}} e^{-|z-\lambda|^2}$$

Gaussian localized at $z=\lambda$.

$\Rightarrow$ coherent states are the best possible approximation of a point in phase space ! W_ρ fills out a Planck-size cell around $z=\lambda$



Another example:

$$\rho = \sum_n e^{-\beta n} |n\rangle\langle n| \implies W_\rho(z,\bar{z}) = e^{-(1-e^{-\beta}) z\bar{z}}$$

thermal state.

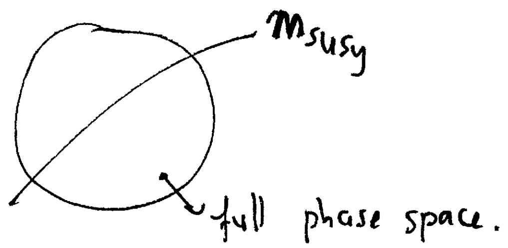
$\beta = 0$: ~~ground~~ $W=1$ everywhere

$\beta = \infty$: ground state

Back to supergravity

$\mathcal{M}$ = phase space of smooth soln's $\xrightarrow{\text{quantize}} \mathcal{H}$

- therefore smooth solutions are dual to coherent states (highly atypical states)
- in the large N limit ($\hbar \sim \frac{1}{N}$) coherent states become points in phase space, hence the word "microstate geometries"



- Quantizing subspaces of a phase space is meaningless
- Supersymmetry helps : first imposing susy & then quantizing = same as the other way around.
- Actual susy wavefunctions are localized near $\mathcal{M}_{susy}$
- not good for dynamical questions

<u>two examples</u> :

1. $\frac{1}{2}$ BPS states in $N=4$ SYM
   $\mathcal{M} = \{$ LLM droplets $\}$
   $\mathcal{H}$ = identical to that of N fermions in a harmonic oscillator potential

2. $\frac{1}{2}$ BPS states in the D1-D5 CFT (dual to $AdS_3 \times S^3 \times M_4$)
   $\mathcal{M} = \{$ LM geometries $\} = \{$ space of loops in $\mathbb{R}^4 \}$
   $\mathcal{H}$ = identical to that of 4 chiral bosons

($\mathcal{M}$ = phase space because solutions are <u>stationary</u>
i.e. carry momentum)

* In both cases have all $\frac{1}{2}$ BPS states realized in supergravity

* Can do AdS/CFT at the level of states!

coherent states $\longrightarrow$ classical geometry $\longrightarrow$ $\langle\lambda|O_k|\lambda\rangle$ one point functions

$$\rho = \sum \rho_\lambda |\lambda\rangle\langle\lambda| \longrightarrow \sum \rho_\lambda \langle\lambda|O_k|\lambda\rangle \text{ one point functions}$$

$\downarrow$

find bulk solution with these boundary conditions.

* This defines Smooth + Smooth + Smooth = black hole?

* Difficult in practise : easier is to use the fact that susy allows one to linearize the field eqns

→ can directly average geometries

→ result: smear solutions against the phase space density

$$\rho \Longleftrightarrow \int_M w_\rho(x) \text{ Smooth }(x)$$

→ explicit example later.

* features: – only get a reasonable geometry for "classical" ρ. (in general states are not classical)

– generic states map to a $\frac{1}{2}$ BPS black hole geometry, universal form, hard to distinguish from the dual of $\rho = \sum |\psi\rangle\langle\psi|$ sum over all states

- states are difficult to distinguish from each other and from the ensemble average (= BH)
- for fairly generic ensembles [eg occupy all modes of the 4 chiral bosons with different temperatures] get a geometry that depends on only a few quantum numbers: (no hair theorem) N, J, D

  N → charge, J → ang momentum, D → "dipole"

- puzzle: why does D appear and no other nonconserved charges?

All this was about small black holes, but would like to study large black holes.

Nice class of large supersymmetric black holes: D0-D2-D4

Allow for a decoupling limit so that the black hole sits in a spacetime that is asymptotic to $AdS_3 \times S^2 \times CY$

→ Dual CFT is $N=(0,4)$ SCFT, difficult to study, but Cardy works

(This class of black holes also includes black rings etc but that is not the focus here)

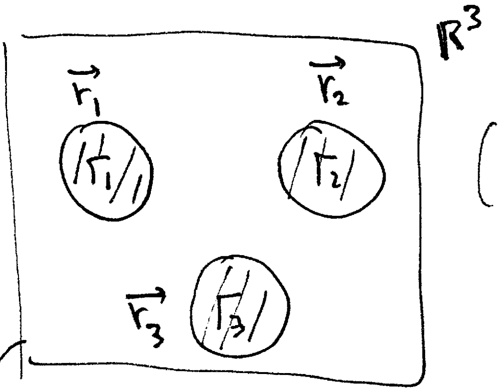
Would like to know space of smooth solutions of 5d SUGRA

→ don't know general answer, but do know a large class which also includes bound states of black holes

Start with type IIA on CY

Charge vector $\Gamma_i \in H^{even}(CY, \mathbb{Z}) = (p^0, p^A, q_A, q_0)$ $A = 1 .. \dim H^2(CY)$

$\#D6 \quad \#D4 \quad \#D2 \quad \#D0$



(4d understanding of p, q: magnetic and electric charges)

Will need the odd pairing $\langle \Gamma, \tilde{\Gamma} \rangle = p^0 \tilde{q}_0 - p^A \tilde{q}_A + q_A \tilde{p}^A - q_0 \tilde{p}^0$

(cf 4d electric-magnetic pairing $PQ' - P'Q$)

To describe corresponding gravity solution need only "Coulomb potential"

$$H = h + \sum_i \frac{\Gamma_i}{|\vec{r} - \vec{r}_i|} \in H^{even}(CY)$$

Full solution is expressed in terms of H only (see literature)

Necessary, <u>not sufficient</u> boundary condition for existence:

$$\forall j : \langle h, \Gamma_j \rangle + \sum_{i \neq j} \frac{\langle \Gamma_i, \Gamma_j \rangle}{|\vec{r}_i - \vec{r}_j|} = 0$$

(Also need to worry about reality of the metric)

- The constant h determines the asymptotics of the solution and satisfies $\langle h, \sum_j \Gamma_j \rangle = 0$ (ie it fixes the CY moduli at ∞)

- Existence of solutions will depend on h: walls of marginal stability

- Carry angular momentum: $\vec{J} = \frac{1}{2} \sum_{a<b} \frac{\langle \Gamma_a, \Gamma_b \rangle \vec{r}_{ab}}{r_{ab}}$

Illustrate this : eg $\langle \Gamma_1, \Gamma_2 \rangle = 1$
$\langle \Gamma_1, h \rangle = \alpha$
$\langle \Gamma_2, h \rangle = -\alpha$

Condition becomes $\alpha + \frac{1}{|\vec{r}_1 - \vec{r}_2|} = 0$. As $\alpha \uparrow 0$, $|\vec{r}_1 - \vec{r}_2| \to \infty$

and for $\alpha > 0$ solution no longer exists.

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* Many solutions exist , if $\sum_i \Gamma_i^{D6} = 0$ and $h = h^{(D0)}$ only

then the solution is asymptotic to $AdS_3 \times S^2$ ~~the~~

Which solutions are smooth ? The individual centers should not carry entropy → single bound state

(Again idea : $S=0$, pure states ↔ smooth
$S \neq 0$, mixed states ↔ horizon)

Obvious class of examples: single brane with only flux ,
(related by a large gauge transformation to single brane w/o flux
→ no entropy)

Recall WZ coupling $\int C \wedge e^F$ in DBI :

D6- brane with flux carries charges $(1, F, \frac{1}{2} F \wedge F, \frac{1}{6} F \wedge F \wedge F)$

Similarly for D0, D2, D4

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* example (D6 + flux) ($\overline{D6}$ + flux)

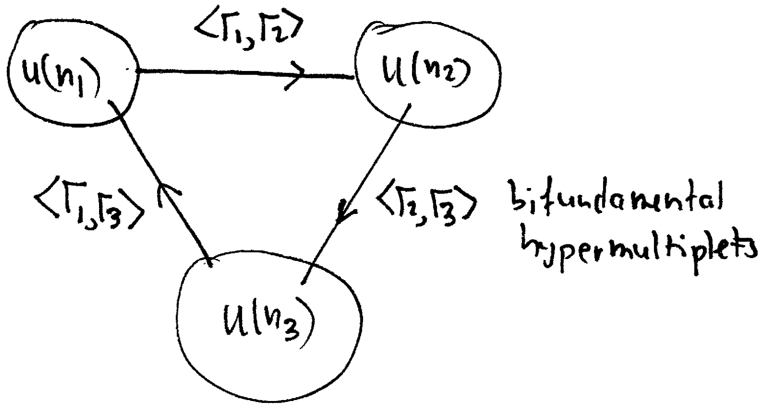
is identical to <u>global</u> AdS_3

In order to investigate $BH = (\sum \text{smooth solutions})$ we need to quantize these solution spaces.

Symplectic form = complicated

What is the open string picture? Quiver quantum mechanics (in IIA)

eg



Supergravity solution describes the Coulomb branch after integrating out the hypermultiplets.

D-term eqns $\iff$ $\langle h, \Gamma_j \rangle + \sum_{i \neq j} \dfrac{\langle \Gamma_i, \Gamma_j \rangle}{|\vec{r}_i - \vec{r}_j|} = 0$ &

Symplectic form can also be obtained: exact at one loop $\rightsquigarrow$ safely extrapolate to large g_s.

Result: solution space M_{2N-2} is $2N-2$ dimensional

$$\Big(= \underbrace{3N-3}_{\text{center of mass}} - \underbrace{(N-1)}_{\text{number of independent constraints}} \Big)$$

$$\omega \sim \sum_{i \neq j} \langle \Gamma_i, \Gamma_j \rangle \frac{\varepsilon_{abc} (r_i - r_j)^a \, d(r_i - r_j)^b \wedge d(r_i - r_j)^c}{|r_i - r_j|^3}$$ like a monopole field

ω still needs to be restricted to M_{2N-2}

<u>example</u> Γ_1 Γ_2 $\langle \Gamma_1, \Gamma_2 \rangle = 1 \quad \langle \Gamma_1, h \rangle = -\langle \Gamma_2, h \rangle = \alpha$

$$\frac{1}{|\vec{r}_1 - \vec{r}_2|} + \alpha = 0$$

Clearly: $M_2 = S^2$

One finds $\omega = |\langle \Gamma_1, \Gamma_2 \rangle|$ (unit volume form)

geometric quantization: $$ds^2 = \frac{dz\, d\bar{z}}{(1+z\bar{z})^2} |\langle \Gamma_1, \Gamma_2 \rangle|$$

$$= + |\langle \Gamma_1, \Gamma_2 \rangle| \, \partial \bar{\partial} \log (1+z\bar{z})^2$$

$$\Rightarrow \quad K = |\langle \Gamma_1, \Gamma_2 \rangle| \log (1+z\bar{z})^2$$

$$\Rightarrow \quad \langle \Psi_1, \Psi_2 \rangle = \int d^2 z \, \frac{1}{(1+z\bar{z})^{2|\langle \Gamma_1, \Gamma_2 \rangle| + 2}} \, \bar{\Psi}_1(\bar{z}) \Psi_2(z)$$

$$\Rightarrow \quad \mathcal{H} = \{1, z, \ldots, z^{2|\langle \Gamma_1, \Gamma_2 \rangle|}\}$$

→ standard angular momentum multiplet on S^2

→ Landau levels on S^2

<u>aside</u>: easier is to use the fact that S^2 is also <u>toric</u>

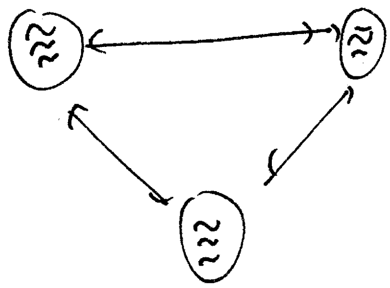
$$\omega = \underbrace{|\langle \Gamma_1, \Gamma_2 \rangle| \; d(\cos\theta)}_{} \wedge \underset{\substack{\uparrow \\ u(1) \text{ action}}}{d\varphi}$$

moment map $\mu : S^2 \to \mathbb{R}$ given by $\mu(\theta, \varphi) = |\langle \Gamma_1, \Gamma_2 \rangle| \cos\theta$.

number of states $= \#(\mathrm{Im}(\mu) \cap \mathbb{Z})$

→ can use this in more general cases too.

When are smooth solutions microstates of a <u>single</u> black hole?



three smooth bbbs very far apart look more like the microstates of a bound state of three black holes

→ restrict to "scaling" solutions: solutions where the centers can approach each other arbitrarily closely

→ need at least three centers

$$\left\{\begin{array}{l} \dfrac{\langle\Gamma_1,\Gamma_2\rangle}{r_{12}} + \dfrac{\langle\Gamma_1,\Gamma_3\rangle}{r_{13}} + a_1 = 0 \\[2ex] \dfrac{\langle\Gamma_2\Gamma_1\rangle}{r_{12}} + \dfrac{\langle\Gamma_2,\Gamma_3\rangle}{r_{23}} + a_2 = 0 \\[2ex] \dfrac{\langle\Gamma_3,\Gamma_1\rangle}{r_{13}} + \dfrac{\langle\Gamma_3,\Gamma_2\rangle}{r_{23}} + a_3 = 0 \end{array}\right.$$

$r_{12} \to 0$, $r_{13} \to 0$, $r_{23} \to 0$ requires $r_{12} \sim \varepsilon\langle\Gamma_1,\Gamma_2\rangle$, $r_{13} \sim -\varepsilon\langle\Gamma_1,\Gamma_3\rangle$, $r_{23} \sim \varepsilon\langle\Gamma_3,\Gamma_3\rangle$

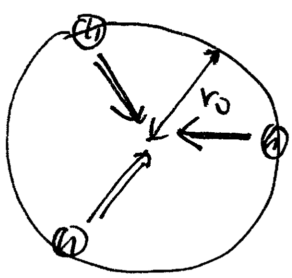
with $\varepsilon \to 0$

(or similar with different signs)

$\Rightarrow$ $\{\langle\Gamma_1,\Gamma_2\rangle, \langle\Gamma_3,\Gamma_1\rangle, \langle\Gamma_2,\Gamma_3\rangle\}$ have to obey the triangle inequalities

Can quantize this system: get a finite number of states

→ $\exists\, r_0$ such that



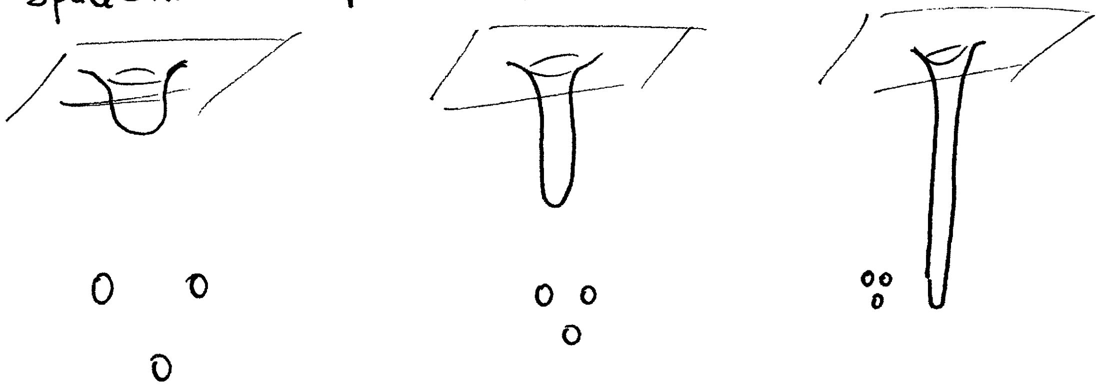
covers one Planck cell

~~below r_0:~~

Geometries where the centers are much closer are not good classical geometries: they are not dual to a coherent state which fills a large region of phase space

Roughly, r_0 is given by $\vec{J}(r_0) \sim 1$.

Space-time interpretation?



An ∞ deep throat develops as the centers approach each other

$\Rightarrow$ very deep throats are <u>not good</u> classical geometries!
despite the fact that curvatures remain small

$\Rightarrow$ Effective field theory breaks down

$\Rightarrow$ Phenomenon is non-local

$\Rightarrow$ Phase space is highly squeezed down the throat
(& not related to proper length in spacetime)

This is good for another reason:
An arbitrarily deep throat has approx scale invariance $\Rightarrow$
Spectrum of theory becomes continuous. But CFT's
should not have a continuous spectrum (on the cylinder).

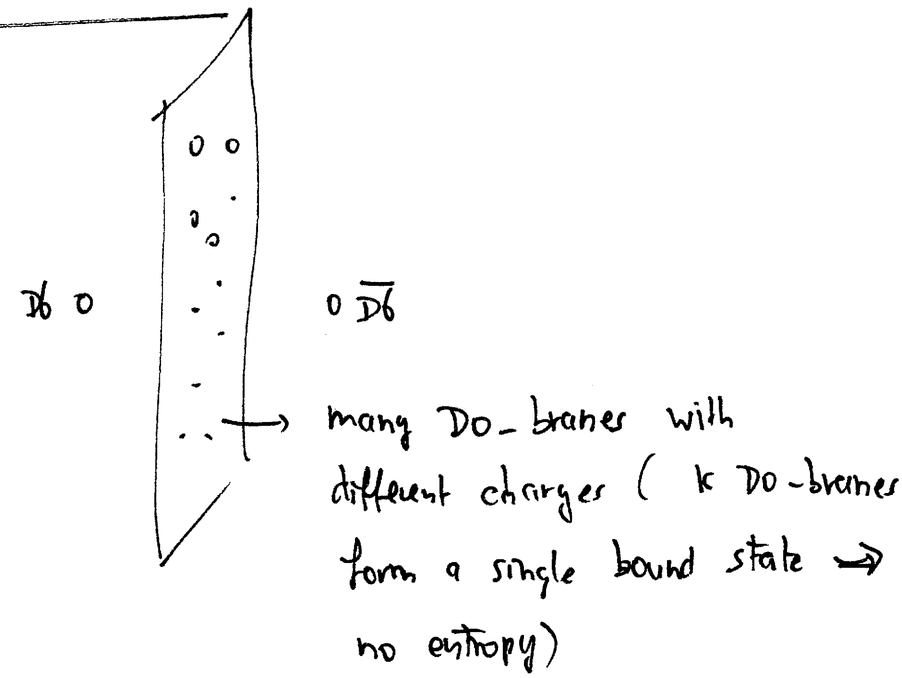
$\rightarrow$ This also resolves this apparent paradox.

Compute energy scale related to r_0 & express in boundary
data: predicts a gap of $\Delta(L_0) = \frac{1}{c}$ for eg a scalar
fluctuations in the throat

$\rightarrow$ looks reasonable - cf long strings would predict
such a gap

How many smooth solutions does supergravity have? Enough to explain the entropy of the black hole?

Best we could do

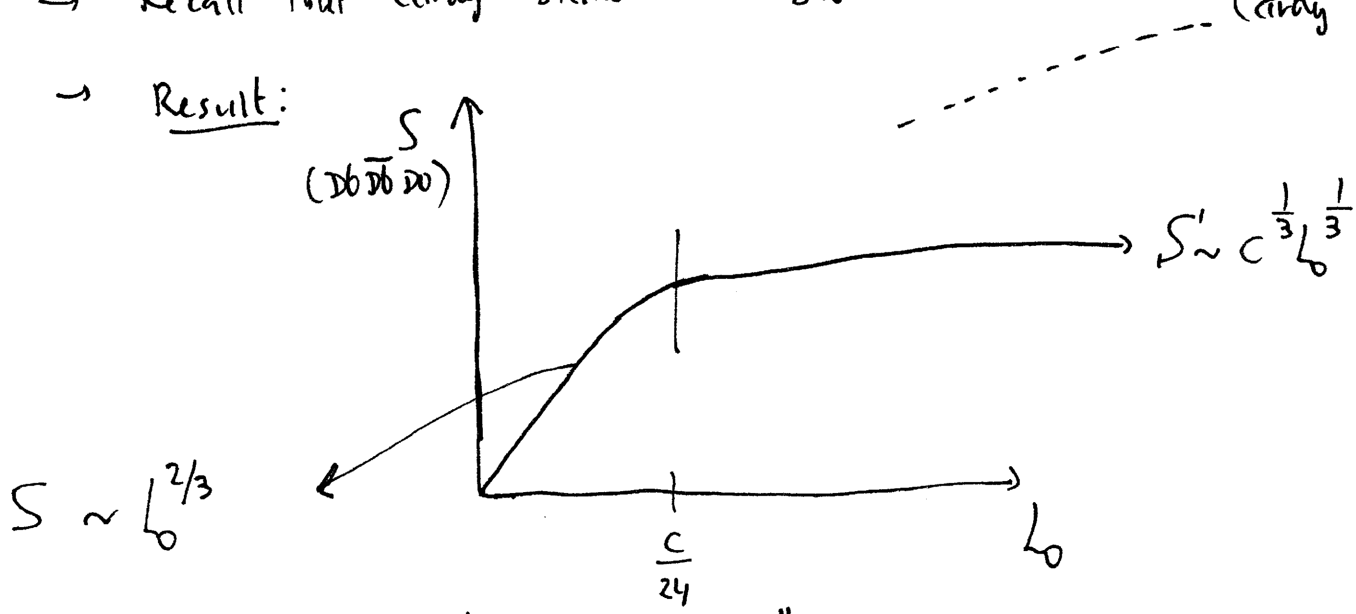


Quantize this space of solutions (use toric techniques)

→ translate this in terms of the quantum numbers of the dual CFT: this was an $N=(0,4)$ CFT with supersymmetric states that have $\begin{cases} \bar{L}_0 = \text{ground state} \\ L_0 = \text{anything} \end{cases}$

→ Recall that Cardy states that $S \sim 2\pi\sqrt{cL_0/6}$ ⟶ Cardy

→ Result:



Unfortunately, get $L_0^{1/3}$ instead of $L_0^{1/2}$ ....

Let's try to estimate the number of smooth BPS supergravity solutions:

$AdS_3 \times S^2$ has 5d supergravitons $\rightarrow$ tower of KK states.

Let's take a gas of supergravitons which are all BPS.
Before grav. backreaction, the BPS bound is saturated.
Gravity will try to lower the energy but this cannot happen.
$\rightarrow$ After backreaction presumably still BPS.
$\rightarrow$ Many will become singular, so the entropy of the supergravitongas gives an upper bound

From the KK reduction we get one. supergraviton with

$$L_0 = \ell + k, \quad \bar{J}_0 = \ell, \quad k, \ell = 0, 1, 2 \ldots$$

$$\Rightarrow \quad \sum N_{gas}(L_0)\, q^{L_0} = \prod_{\substack{k,\ell \geq 0 \\ (k,\ell) \neq (0,0)}} \frac{1}{(1-q^{\ell+k})} \quad \Rightarrow \quad N_{gas}(L_0) \sim L_0^{2/3}$$

So there is hope....

However, let's try to include some backreaction. One simple fact is the "stringy exclusion principle" which states that $\bar{J}_0^{tot}$ is bounded by $\frac{c}{6}$, follows from unitarity of the $N=4$ SCFT.

So we should compute

$$\sum N_{gas}(L_0, \bar{J}_0)\, q^{L_0}\, y^{\bar{J}_0} = \prod_{k\ell} \frac{1}{(1-q^{\ell+k}\, y^{\ell})}$$

and then estimate $\quad N_{gas}(L_0) = \sum_{\bar{J}_0=0}^{c/6} N(L_0, \bar{J}_0)$

Let's compute the asymptotics of $N(L_0, \bar{J}_0)$

this can obtained via a Legendre transformation

of the free energy:

$$N(L_0, \bar{J}_0) = \log Z - L_0 \cdot \log q - \bar{J}_0 \cdot \log y \Big|_{\text{legendre.}}$$

estimate $\log Z \simeq \sum_{\substack{lk \\ m=1}}^{\infty} \frac{q^{(l+k)m} y^{lm}}{m} \sim \sum_m \frac{1}{m} \frac{1}{1-(qy)^m} \frac{1}{1-q^m}$

Asymptotic region is when $\log Z \to \infty$ so $q \sim 1$ and $qy \sim 1$

$\to \log Z \sim \sum \frac{1}{m^3} \frac{1}{\log(qy) \log q}$

Now do the legendre transform. Result:

$$N(L_0, \bar{J}_0) \simeq \bar{J}_0^{1/3} (L_0 - \bar{J}_0)^{1/3}$$

From this we find that $\begin{cases} N_{gas}(L_0) \sim c^{\frac{1}{3}} L_0^{\frac{1}{3}} & \text{for } L_0 \gtrsim \frac{c}{6} \\ N_{gas}(L_0) \sim L_0^{2/3} & \text{for } L_0 \lesssim \frac{c}{6} \end{cases}$

Amazingly 1) this precisely matches the $D6\,\overline{D6}\,D0$ – counting

2) only taking the exclusion principle into account is sufficient to reduce the number significantly

$\Longrightarrow$ This strongly suggests smooth supergravity solutions will <u>not</u> be enough and stringy degrees of freedom need to be included

Outlook:

* do non-BPS black holes? The argument that the CFT should have a gap might still work here

* explore the quiver quantum mechanics picture in more detail: find further evidence for large quantum effects (superpotential might be crucial) not just in the ground state (in progress ...)

* can this be relevant for e.g. small black holes at LHC ??

* can a similar picture be developed for cosmological solutions?

* finally, really address those complicated dynamical BH questions