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Spring School on Superstring Theory and Related Topics

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Holographic Superconductors and Superfluids
Lecture 4

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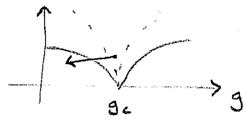
Holographic Superfluids and Superconductors

- try to build in the physics of the super-conducting phase transition into the holographic model



- two proposals for modifying the action - need something that will serve as an order parameter
  - add a $\mathbb{C}$ scalar

$$\delta S = -\int d^4x \sqrt{-g} \left(|D\Psi|^2 + V(|\Psi|) \right)$$

$$D = \partial - ieA$$

  - promote $F_{\mu\nu}$ to a $SU(2)$ gauge field $F^a_{\mu\nu}$

comments

- not totally clear how to embed either of these choices in string theory - phenomenological
- $V(|\Psi|)$ arbitrary - we will choose $V(|\Psi|) = -\frac{2}{L^2}\psi^*\psi$
  (tachyonic but above BF bound $m^2 = -\frac{2}{L^2} > -\frac{9}{4L^2}$)
- $SU(2)$ action more rigid but physics is messy
  - order parameter is a current
  - phase transition breaks rot'al invariance
- one nice thing about $SU(2)$ case in $d = 4$ dim.s is that the behavior of the system is analytic near T_c

- the scalar case

Gubser observed an instability for the scalar to condense when ρ gets too large

$$m_{eff}^2 = m^2 + g^{tt} A_t^2 = m^2 - \frac{z^2}{f} \frac{\varepsilon^2}{L^2} (1-z)^2$$

$$m_{eff}^2 \leq m^2 \quad \text{and} \quad \to m^2 \text{ when } z=0 \text{ or } 1$$

no need for ψ^4 term typical of LG!

- probe limit – let's assume $\kappa^2 \ll g^2$ and decouple gravity
  – Abelian-Higgs model in a fixed curved b.ground black hole in AdS_4

$$f = 1 - z^3$$

(work in units $\alpha = L = e = 1$)

$$z^2 \left(\frac{f}{z^2} \psi' \right)' = \left(\frac{m^2}{z^2} - \frac{A_t^2}{f} \right) \psi$$

$$A_t'' = \frac{2g^2}{z^2 f} \psi^2 A_t$$

soln. w/ $\psi \neq 0$ describes end pt. of instability

<u>bry conditions</u>

grand can. or can. ensemble

$z = 0$

$$A_t = \mu - g^2 \rho z + \ldots$$

$$\psi = a z + b z^2 + \ldots$$

horizon

$z = 1$

$A_t = 0$ (so $g^{tt} A_t^2 < \infty$)

$\psi < \infty$

<u>aside on scalars in AdS/CFT</u>

$$m^2 L^2 = \Delta(\Delta - 3)$$

Δ conf. scaling dim. of scalar dual to ψ

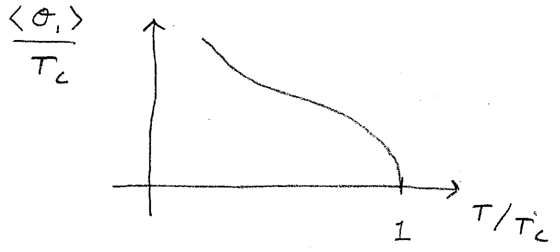
$$m^2 L^2 = -2 \Rightarrow \Delta = 1 \text{ or } 2$$

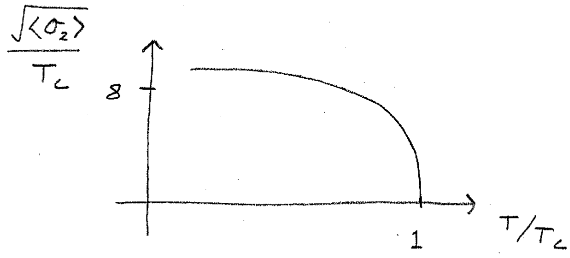
if we set $a = 0 \Rightarrow b \sim \langle O_2 \rangle$ for scalar w/ $\Delta = 2$

$b = 0 \Rightarrow a \sim \langle O_1 \rangle$ for scalar w/ $\Delta = 1$

③

- we have a well defined set of diff. eq.s which can be solved numerically





$$T_c \sim \rho^{1/2} \sim \mu$$

- 2nd order phase transition

$$\langle \mathcal{O}_1 \rangle \sim \langle \mathcal{O}_2 \rangle \sim (T_c - T)^{1/2} \quad \text{near } T_c$$

(LG mean field exponent)

- large gap $\sim \langle \mathcal{O}_1 \rangle \sim \sqrt{\langle \mathcal{O}_2 \rangle} \Rightarrow$ strong coupling

BCS weak coupling	$2 \cdot \text{gap}$	$= 3{,}54\, T_c$	
AdS/CFT		$= \infty$	for $\Delta = 1$
		$\approx 8 T_c$	for $\Delta = 2$
high T_c		$= (4 \text{ to } 7)\, T_c$	

<u>the conductivity</u>

$$\left[\begin{aligned} &(f A_x')' - \frac{\omega^2}{f} A_x = \frac{2g^2}{z^2} \psi^2 A_x \\ &A_x = \frac{E_x}{i\omega} + \frac{g^2}{\alpha} J^x z + \dots \qquad E_x \sim e^{-i\omega t} \\ &\sigma_{xx} = \frac{J^x}{E_x} \end{aligned}\right.$$

- in general we need numerics, but ...

for

- $T > T_c$, $\psi = 0$ and $\sigma_{xx} = \frac{1}{g^2}$ as before

- $T \approx 0$ and $b = 0$ case $(\Delta = 1)$

  then $\psi \approx \frac{\langle \mathcal{O}_1 \rangle}{\sqrt{2}} \frac{z}{g^2}$ is approx'ly linear (numerical observation)

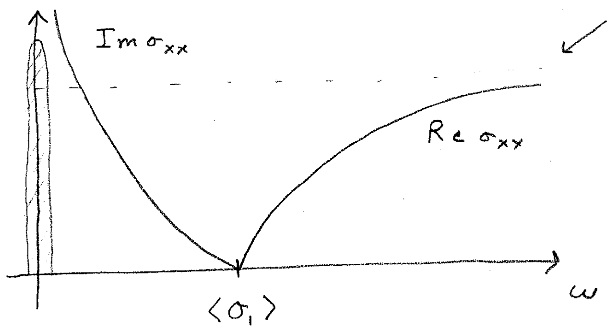
  and we get the KG eqn.

  $$A_x'' - \omega^2 A_x = \langle \mathcal{O}_1 \rangle^2 A_x$$

two cases:

$$A_x = \begin{cases} A_x^0 \, e^{-\sqrt{\langle \mathcal{O}_1 \rangle^2 - \omega^2}\, z} & \omega < \langle \mathcal{O}_1 \rangle \quad \left(\begin{matrix}\text{regular} \\ \text{as} \\ z \to \infty\end{matrix}\right) \\ A_x^0 \, e^{i\sqrt{\omega^2 - \langle \mathcal{O}_1 \rangle^2}\, z} & \omega > \langle \mathcal{O}_1 \rangle \quad \left(\begin{matrix}\text{outgoing plane} \\ \text{wave as } z \to \infty\end{matrix}\right) \end{cases}$$

$$\sigma_{xx} = \frac{i}{g^2} \frac{\sqrt{\langle \mathcal{O}_1 \rangle^2 - \omega^2}}{\omega} \operatorname{sgn}\left(\langle \mathcal{O}_1 \rangle^2 - \omega^2\right)$$



large ω like high T

indeed $\langle \mathcal{O}_1 \rangle = 2\,\text{gap}$

- there must be a δ-fn. peak at $\omega = 0$

  Kramers-Kronig relations $\quad \operatorname{Im} \sigma(\omega) = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{\operatorname{Re} \sigma(\omega') d\omega'}{\omega' - \omega}$

  $\Rightarrow \quad \operatorname{Re} \sigma(\omega) \sim \pi \langle \mathcal{O}_1 \rangle \delta(\omega) / g^2$ near $\omega = 0$

- classic textbook plot for a superconductor comments about superconductors vs. superfluids:

- normally, superfluidity is associated w/ spon. symm. breaking
  supercond. Higgs mechanism
- in the field theory, this broken $U(1)$ is global $\Rightarrow$ technically a superfluid
- for certain kinds of questions, e.g. σ_{xx}, the distinction is immaterial; for others, e.g. Meissner effect, it's very important.
- we have London equation $\vec{J} = -\frac{\langle O_1 \rangle}{g} \vec{A}$ (at small ω, $\vec{k} \ll \langle O_1 \rangle$) w/out Maxwell's equations
  - so the external fields produce currents, but the currents then do not produce countering fields