The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

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Spring School on Superstring Theory and Related Topics

23 - 31 March 2009

Aspects of scattering amplitudes and colider physics in conformal field theories
Lecture I

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TOPIC OF THE LECTURES

- TALK ABOUT SOME LORENTZIAN OBSERVABLES RELATED TO THE ONES WE MEASURE AT COLLIDERS.

- L1 - CUSP ANOMALOUS DIMENSION -

- L2 - SCATTERING AMPLITUDES AT STRONG COUPLING VIA STRING THEORY FOR $\mathcal{N}=4$ SYM.
  - T- DUALITY
  - AMPLITUDE $\Leftrightarrow$ WILSON LOOPS -

- L3 - ENERGY CORRELATION FUNCTIONS IN CFT_S
  - SMALL ANGLE LIMIT

- L4 - ENERGY CORRELATION FUNCTIONS AT STRONG COUPLING -

C 1

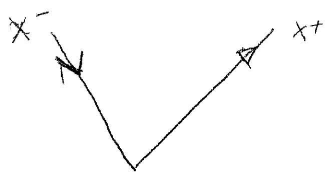
THE CUSP ANOMALOUS DIMENSION

- TALK ABOUT THE CUSP ANOMALOUS DIMENSION – $\Gamma_{cusp}(\alpha)$.
  - IMPORTANT FOR MANY (LORENTZIAN) EXCLUSIVE PROCESSES. (RESUMMATION FORMULAS).
    - AMPLITUDES $e^{-\Gamma_{cusp}(\alpha)\,(\log \mu_{IR})^2}$
    - WILSON LOOPS $e^{-\Gamma_{cusp}(\alpha)\,(\log \mu_{UV})^2}$ ✓
    - SUDAKOV FACTOR
    - HIGH SPIN OPERATORS $\Delta - S \sim 2\,\Gamma_{cusp} \log S$. KORCHEMSKY
    - ALTARELLI–PARISI KERNEL NEAR $x \sim 1$ (EVOLUTION OF PDF's). $P_{q \leftarrow q}(z) \sim \frac{2\Gamma_{cusp}}{(1-z)} +$ (see e.g. hep-ph 060722
  - IT IS MAINLY A LORENTZIAN OBJECT $\to$ NO SIMPLE EUCLIDEAN VERSION –

- WE WILL DISCUSS IT IN DETAIL – CONSIDER A CONFORMAL FIELD THEORY

$\boxed{C2}$

1 WILSON LOOP POINT OF VIEW.



U(1) THEORY.

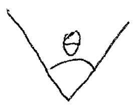
$$W \sim e^{-\int dx^+ dx^- \langle A_+ A_- \rangle} \sim e^{-\frac{g^2}{4\pi^2}\int_0^\infty dx^+ dx^- \frac{1}{x^+ x^-}}$$

$$W \sim e^{-\frac{g^2}{4\pi^2}\left(\log\left(\frac{\mu_{UV}}{\mu_{IR}}\right)\right)^2}$$

- BOTH A UV AND AN IR DIVERGENCE

SIDE COMMENT:

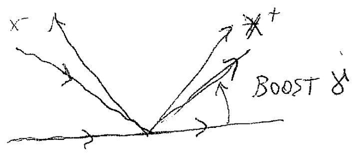
- EUCLIDEAN WILSON LOOP



$$W \sim e^{-\tilde{\Gamma}_c(\theta)\left(\log \frac{\mu_{UV}}{\mu_{IR}}\right)}$$

$\downarrow$

- SPACELIKE IN LORENTZIAN SIGNATURE

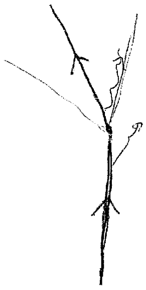


$$W \sim e^{-\tilde{\Gamma}_c(\gamma)\, \log \frac{\mu_{UV}}{\mu_{IR}}} \longrightarrow e^{-\Gamma_{cusp} \cdot \gamma\, \log\left(\frac{\mu_{UV}}{\mu_{IR}}\right)}$$

$$\tilde{\Gamma}_c(\gamma) \sim \gamma\, \Gamma_{cusp}.$$

$\boxed{3}$

- CONNECTION TO ACCELERATION RADIATION (OR BREMSSTRAHLUNG)



SUDDEN CHANGE IN VELOCITY.

→ CREATION OF THE NEW COULOMB FIELD.

→ PULSE OF RADIATION → IS A COHERENT STATE.

$\mathcal{N} e^{g a^\dagger} |0\rangle$ $\qquad$ $\mathcal{N}$ = NORMALIZATION FACTOR

- PROBABILITY AMPLITUDE FOR NOT EMITING ANY PHOTON.

$$\langle 0 | \mathcal{N} e^{g a^\dagger} | 0 \rangle \sim \mathcal{N} \qquad \mathcal{N}^{-2} = e^{\sum_{\text{oscill}} g^2}$$

- THE WILSON LOOPS WE DISCUSS WERE COMPUTED USING FEYNMAN PROPAGATORS (ANALYTIC CONTINUATION FROM EUCLIDEAN SIGNATURE).

→ VACUUM FINAL CONDITION → AMPLITUDE FOR <u>NOT</u> EMITTING EXTRA PHOTONS.

- WHY DO WE CARE ...

C 4

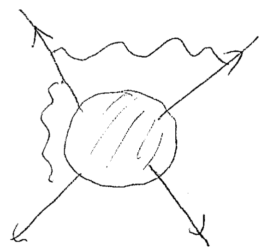
. AMPLITUDES.

. SCATTERING AMPLITUDES HAVE DOUBLE LOG DIVERGENCES.

. RESUM $\mathcal{A} = e^{\underbrace{-\Gamma_{cusp}\left(\log\left(\mu_{IR}/\sqrt{s}\right)\right)^2}_{\text{SUDAKOV FACTOR}}} + \text{TERM w/ SINGLE LOG.}$

(ASHOK SEN)

. WHY?



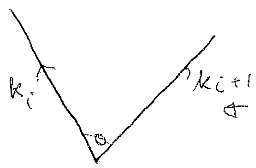
LOW ENERGY GLUON SEES OUTGOING PARTICLE AS A HARD LINE

⇒ APPROXIMATE IT AS A WILSON LOOP.

→ ONLY IR DIV (UV CUT OFF BY Q^2).

. PLANAR → CAN TREAT EACH INDEPENDENTLY

(ALSO WORKS FOR NON-PLANAR).



← SUCH WILSON LOOP $e^{-\Gamma_{cusp}\left(\log\left(\mu_{IR}/\mu_{UV}\right)\right)^2}$

WITH $\mu_{UV}^2 \sim S_{i,i+1} = k_i \cdot k_{i+1}$

↓

SCALE AT WHICH WE CAN'T TREAT THE EXCHANGED GLUON AS SOFT.

DOUBLE LOG IR DIVERGENCES OF AMPLITUDES GIVEN BY WILSON LOOP.

$\boxed{C5}$

- FIXED ORDER → IR div.

~~ALL OR~~
RESUMMED → SUPPRESION

→ BECAUSE WE ARE FORBIDDING SOFT & COLINEAR RADIATION!

→ NO DIVERGENCE IF WE CONSIDER INCLUSIVE OBSERVABLES.

→ WE CONSIDER THE FULL COHERENT STATE WHICH INCLUDES THE RADIATIO[N]

- IN SOME OBSERVABLES WE SOMETIMES PUT EXPERIMENTAL CUTS THAT PRECISELY FORBID THIS RADIATION.

→ THE CUT (LIKE μ_{SR}) DEPENDENCE OF THE OBSERVABLE IS DOMINATED ~~THE STORY~~ BY Γ_{cusp}.

ALL LOOPS

- WHY DOUBLE LOG?

SYMMETRIES: BOOSTS & TRANSLATIONS.

1 LOG PER SYMMETRY

$$ds^2_{3+1} = dx^+dx^- + dr^2 + r^2 d\varphi^2 = r^2 \left[\underbrace{\frac{dx^+dx^- + dr^2}{r^2}}_{AdS_3} + d\varphi^2 \right]$$

$AdS_3 \to \cos^2\beta\, d\gamma^2 + \sin^2\beta\, d\chi^2 - d\beta^2$

FLUX $F_{\gamma\chi}$ → LOCALIZED NEAR $\beta \sim \pi/4$

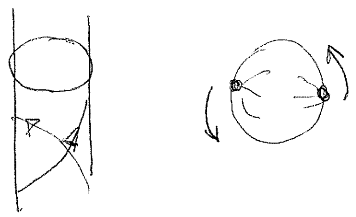
C6

– HIGH SPIN OPERATORS.

– CONSIDER IN SOME DETAIL TO OBTAIN A HAMILTONIAN PICTURE OF Γ_{cusp}. WE WANT TO EXPRESS Γ_{cusp} AS THE ENERGY (DENSITY) OF SOME STATE.

(AS IN: ANOMALOUS DIMENSIONS → ENERGIES OF STATES IN $\mathbb{R} \times S^3$)

• HIGH SPIN OPERATORS.



$S \gg 1$ → MOVING VERY FAST

$q (\overleftrightarrow{\partial}_+)^S \bar{q}$

NOT COLOR NEUTRAL.

(FOR COLOR NEUTRAL → DESCENDENT $\partial_+^S (q\bar{q})$)

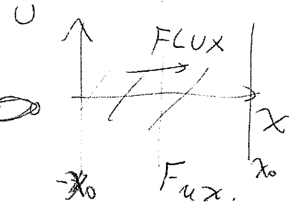
→ REPLACE BY LIGHT-LIKE WILSON LOOPS.

→ CHOOSE SPECIAL COORDINATES. $\mathbb{R}^4$ IS CONFORMAL TO $AdS_3 \times S^1$

$$\underbrace{-du^2 + dx^2 + d\sigma^2 + 2\sinh 2\sigma \, dx \, du}_{AdS_3} \quad \underbrace{+ d\varphi^2}_{S^1}$$

W.L. AT $x = \pm x_0$ → TAKE x_0 TO ∞.

• $\Delta - S \sim i\frac{\partial}{\partial u}$ • $S \sim \underbrace{e^{2x}}_{\cosh(2x)} \times$ DERIVATIVES



• GRAVITATIONAL WARP FACTOR $\sinh 2\sigma \, dx \, du$.
CONFINES THE FLUX AROUND $\sigma \sim 0$.

~~PROBL~~

C7

- PROBLEM BECOMES 1+1 DIMENSIONAL
- FLUX IN 1+1 DIMENSIONS

→ ENERGY DENSITY

$$\mathcal{E} \approx \frac{\text{ENERGY}}{\underbrace{\Delta x_0}_{2x_0}} \approx \frac{\Delta - S}{\log S} \simeq 2\Gamma_{cusp}(\lambda)$$

JUST A FUNCTION OF $\lambda = g^2 N$. (AT LARGE N).

- THIS IS THE ~~[illegible]~~ STATE WHOSE ENERGY WE COMPUTE.
- 2 SYMMETRIES → u & x TRANSLATIONS.

$$Z = e^{i 2\Gamma_{cusp} \Delta u . \Delta X}$$

- ANALYTIC CONTINUATION GIVES THE CUSP WILSON LOOP & REMOVES THE i → REAL ANSWER. FOR THE WILSON LOOP.

- RESUMMATION → USE SYMMETRY GENERATORS → EXPONENTIATING SOME HAMILTONIAN ...

- THIS ARGUMENT IS VALID IN OTHER DIMENSIONS TOO. → 2+1 DIMENSIONS
- NEEDS CONFORMAL SYMMETRY + CONSERVED FLUX.
- NOT VALID FOR ϕ^3 IN D=6 → SINGLE LOG DIVERGENCES IN AMPLITUDES.

$\boxed{C8}$ IN $\mathcal{N}=4$ SYM.

$\Gamma_{cusp}(\lambda)$

CAN BE COMPUTED USING INTEGRABILITY.

$\rightarrow$ INTEGRAL EQN $\rightarrow$ SOLUTION $\rightarrow$ GE $\Gamma_{cusp}(\lambda)$.

$\rightarrow$ EXPANSION AROUND WEAK & STRONG COUPLING IS KNOWN.

(MATHEMATICAL CURIOSITY:

$g^2 = \frac{\lambda}{(4\pi)^2}$ LOOP COUNTING PARAMETER.

$\Gamma_{cusp} = 4 g^2 - \frac{4}{3}\pi^2 g^4 \ldots$

$\uparrow$ $16\,\zeta(2)$

$g^{2l+2}\ (\ \zeta(2l) \ldots \zeta(l+k)\,\zeta(l-k))$

$\uparrow$ SUM OF ARGUMENTS OF ζ $\leq 2l$)

STRONG COUPLING EXPANSION

$f_{BES} = 2\,\Gamma_{cusp}^{AdS}$

$$\Gamma_{cusp} = 2g + \text{cnst} + \frac{1}{g} + \ldots$$
$$= 2\sqrt{\frac{\lambda}{4\pi}}$$

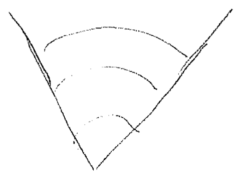
- AT STRONG COUPLING:

$$ds^2 = d^2\rho\ AdS_3 + d\varphi_1^2\ d^2\rho + d\rho^2.$$

$$\mathcal{E} \approx R_{AdS}^2\, T_{STRING} \sim \frac{\sqrt{\lambda}}{2\pi}$$

$\uparrow$ STRING AT $\rho = 0$

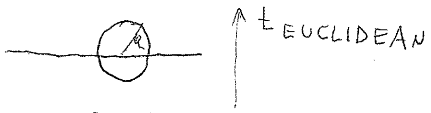
- WILSON LOOP: $\frac{dx^+dx^- + dz^2 + \ldots}{z^2}$ $\quad z^2 = 2x^+x^-$



SPACELIKE SURFACE.

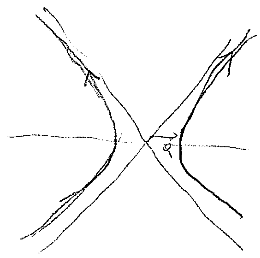
$\boxed{C9}$

- ALTERNATIVE:



CIRCLE

$\longrightarrow$ $t_{LORENTZIAN}$



ACCELERATING CHARGES FROM $t=0$

SU~~RFACE~~:

$$Z^2 + t_E^2 + X^2 = a^2$$

$$\longrightarrow \quad Z^2 = a^2 + X^+ X^-$$

→ TIMELIKE SURFACE

NO EXPONENTIAL SUPPESSION.

→ GET ENERGY FLUX IN THE WHOLE UPPER QUADRANT.

THE $a \to 0$ LIMIT IS <u>NOT</u> THE OTHER SURFACE.

→ DIFFERENT FUTURE BOUNDARY CONDITIONS_

$\boxed{C\ 10}$

- ~~REMARK ABOUT NON-CONFORMAL CASE.~~

~~FIRST~~ A WORD ABOUT DIMENSIONAL REGULARIZATION:

WILSON LOOPS.

$4-2\varepsilon$ DIMENSIONS & $\varepsilon < 0$ FOR IR

$\varepsilon > 0$ FOR UV.

FOCUS ON UV

$$\int_0^1 \frac{dx^+ dx^-}{x^+ x^-} g^2 \xrightarrow[\text{DIM REG.}]{} \int_0^{x_{IR}} \frac{dx^+ dx^-}{(x^+ x^-)^{1-\varepsilon}} g^2 \mu^{2\varepsilon}$$

$$\sim \frac{1}{\varepsilon^2} \cdot x \, g^2 \mu^{2\varepsilon} (x_{IR})^{2\varepsilon}$$

HIGHER ORDERS.

$$W \to e^{-\frac{1}{\varepsilon^2} \Gamma^{(-2)}_{cusp} (\lambda \mu^{2\varepsilon} x_{IR}^{2\varepsilon})}$$

$$\left(\lambda \frac{d}{d\lambda}\right)^2 \Gamma^{(-2)}_{cusp} = \Gamma_{cusp}(\lambda)$$

AMPLITUDES:

$$e^{-\frac{1}{\varepsilon^2} \sum_i \Gamma^{(-2)}_{cusp} \left(\lambda \mu_{IR}^{2\varepsilon} / s_{i,i+1}^{\varepsilon}\right) + \frac{1}{\varepsilon} g^{(-1)}}$$

↑ ANOTHER FUNCTION.

- IN THE NON CONFORMAL CASE → WE HAVE TO INTEGRATE OVER SCALES $\sim e^{\int_{\log s}^{\log \mu_{IR}} \frac{d\tau}{\tau} \left\{ \Gamma_{cusp}(\tau) (\log \tau/s) + G(g^2(\tau)) \right\}}$

& → SIMILAR FOR DIMENSIONALLY REGULATED CASE.

C10

SUMMARY & CONCLUSIONS

1. Γ_{cusp} IS A VERY IMPORTANT ANOMALOUS DIMENSION FOR LORENTZIAN PROCESSES.

2. LEADING IR DIVERGENCES OF AMPLITUDES
   → GIVEN BY Γ_{cusp}.
   ↓
   - SUPPRESSES EXCLUSSIVE PROCESSES
   - INCLUSSIVE ONES ARE FINITE → NO LIMIT ON SOFT OR COLLINEAR GLUONS.

3. CAN BE VIEWED AS THE ENERGY OF A PARTICULAR FLUX CONFIGURATION ON $AdS_3 \times S^1$ (OR $AdS_3 \times Z_2$)

4. GIVES THE TUNNELING AMPLITUDE FOR PRODUCING THE LONG RANGE COULOMB FIELDS

5. CONNECTED TO ACCELERATION RADIATION

SOME REFERENCES.

- THE IR STRUCTURE OF GAUGE THEORY AMPLITUDES:

  - STERMAN & TEJEDA-YEOMANS HEP-PH/0210130.

- $\Delta - S = \Gamma_{cusp} \log S$: KORCHEMSKY · Mod Phys Lett. A 4 (1989) 1257

- Γ_{cusp} IN $\mathcal{N}=4$ SYM:

  BEISERT - EDEN - STAUDACHER - HEP-TH/0610251

- STRUCTURE OF IR DIVERGENCES IN PLANAR AMPLITUDES IN $\mathcal{N}=4$ SYM:

  - BERN - DIXON - SMIRNOV HEP-TH/0505205

- WHAT I DISCUSSED ~~TOO~~ HERE ON THE $AdS_3 \times S^1$ & THE CUSP...

  ALDAY & JM . 0708.0672.