The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2024-15

Spring School on Superstring Theory and Related Topics

23 - 31 March 2009

Aspects of scattering amplitudes and colider physics in conformal field theories
Lecture 2

J.M. Maldacena

Institute for Advanced Study
Princeton
U.S.A.

L2

- DISCUSS SOME ~~[illegible]~~ FEATURES OF SCATTERING AMPLITUDES IN $\mathcal{N}=4$ SYM.

- PLANAR AMPLITUDES, $N \to \infty$ $g^2 N = \lambda = \text{fixed}$

- STRONG COUPLING, $\lambda \gg 1$

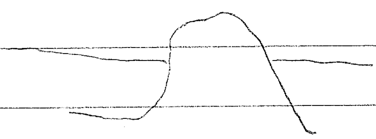
- APPEARANCE OF EXTRA SYMMETRY
  - "DUAL CONFORMAL SYMMETRY"
  - RELATED TO INTEGRABILITY
  - CONSTRAINS THE AMPLITUDES

- FIND A RELATION BETWEEN AMPLITUDES AND WILSON LOOPS_

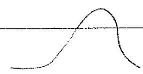
SCATTERING AMPLITUDES FROM CLASSICAL STRINGS.

1 FLAT SPACE

POTENTIAL SCATTERING

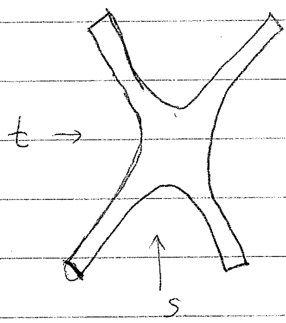


TUNNELING. OR



→ WKB APPROXIMATION , $S_{clas} \gg \hbar$

• STRING SCATTERING - OPEN STRINGS.



• $\int dz_i \langle \prod_i e^{ik_i\cdot X(z_i)} \rangle$

$|z_i - z_j|^{k_i\cdot k_j \alpha'}$

. $k_i\cdot k_j\alpha' \gg 1$ → LARGE ~~[illegible]~~ KINEMATIC INVARIANTS.

. SADDLE POINT

$$\sum_{\substack{j\neq i \\ j=1}}^{n} \frac{k_i\cdot k_j}{z_i-z_j} = 0 \qquad (A)$$

• CLASSICAL STRING SOLUTION IN SPACETIME

2 $X^\mu = \sum_{j=1}^{n} i k_j^\mu \log |z - z_j|^2$

- SOLVES THE EQUATIONS $\partial^2 X^\mu = 0$.
- THE VIRASORO CONSTRAINTS BOIL DOWN TO (A)
- SOLUTION IS PURELY IMAGINARY IF $k_i \cdot k_{j} > 0$ (SPACELIKE)
- 4-POINTS.

$t + s + u = 0$.

$t > 0,\ s > 0 \rightarrow$ u-channel. $\int dz\, |z|^{\alpha' s} |1-z|^{\alpha' t}$

$S_{class} \approx \alpha' s \log s + \alpha' t \log t \underbrace{- \alpha'(s+t) \log (s+t)}_{\alpha' u \log u}$ $\quad \frac{s}{z} = \frac{t}{1-z} \rightarrow z = \frac{s}{s+t}$

$\mathcal{A} = e^{-[\alpha' s \log s + \dots]}$

AS $s, t \rightarrow \infty$ WITH THEIR RATIO FIXED $\mathcal{A} \sim e^{-\alpha' s \underbrace{f(\theta)}_{\text{Angles}}}$

- LARGE ENERGY ~~FIXED ANGLE SCATTERING~~
- S_{cl}. LARGE $\rightarrow$ WKB ✓
- $e^{-(\text{REAL})} \rightarrow$ EXPONENTIALLY SUPPRESSED $\rightarrow$ TUNNELING.
- COLLISION TYPICALLY PRODUCES HIGHLY EXCITED STRING STATES.
- SMALL PROBABILITY FOR NOT EXCITING ANYTHING ELSE
- ALL TREE LEVEL $\rightarrow$ LOOP CORRECTIONS $\sim g^n e^{\frac{-\alpha' s f(\theta)}{n}}$

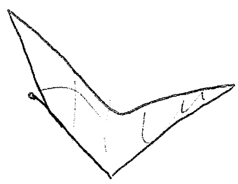
$\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{2}$ $\quad \frac{1}{3}\cdot\frac{2}{3}$ $\quad \frac{2}{3}$ $\quad \frac{2}{9}$

$\boxed{3}$

- T-duality. (ONLY ON THE DISK AMPLITUDE)

$$dY = i *_2 dX \qquad \partial Y = \partial X, \quad \bar\partial Y = -\bar\partial X$$

$$X^\mu \sim \alpha' i k^\mu \log|z|^2 \;\to\; Y = \alpha' i k^\mu \log\left(\frac{z}{\bar z}\right) \sim k^\mu \sigma$$

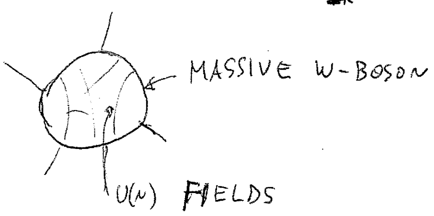


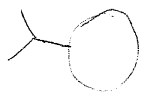
. LIKE A SOAP BUBBLE.

- YANG-MILLS THEORY, $\mathcal{N}=4$ SYM

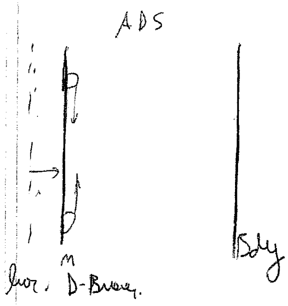
$U(N) \to U(N) \times U(m)$ or $U(1)^m$. $\quad m \ll N$ $\quad m;\ g^2 N = \text{FIXED}$, $N \to \infty$, $g \to 0$.

μ_{IR}





- IN ADS.



$$\frac{dz^2 + dx^2}{z^2} \qquad z_{IR} \sim 1/\mu_{IR}.$$

$k_{PROPER} \sim k \cdot z_{IR} \to \infty$ IF $\mu_{IR} \to 0$, $z_{IR} \to \infty$, $k = \text{FIXED}$.

$\boxed{4}$ - HIGH ENERGY SCATTERING IN AdS.

- CLASSICAL STRING SOLUTIONS.

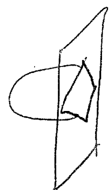
- T-DUALITY.

$$dY^\mu = i * \frac{dX^\mu}{z^2} \qquad r = \frac{1}{z}$$

. STRINGS ON

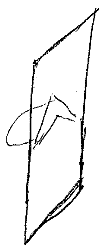
$$\frac{dy^2+dr^2}{r^2}$$

$$z = \infty \longrightarrow r = 0$$



$r=0$

. CONTOUR ON THE BOUNDARY WITH LIGHT LIKE PIECES.



. ORDER → PLANAR COLOR ORDERED AMPLITUDE

$$\mathcal{A} = \sum_{\text{PERMUT.}} Tr[T^{a_1} \dots T^{a_n}] \underbrace{A(p_1 \dots p_n)}_{\text{GETTING THIS.}}$$

$$\cdot\, \mathcal{A} = e^{-\overbrace{\frac{R^2_{AdS}}{\alpha'}}^{\sqrt{\lambda}} \times (AREA)} \times \text{FACTOR} \times \left(1 + \frac{1}{\sqrt{\lambda}}\right).$$

↑ DEPENDENCE ON POLARIZATIONS.

→ QUANTUM FIELDS ON THE CLASSICAL SURFACE

- SYMMETRY $\to \widetilde{SO(2,4)}$ DUAL CONFORMAL

$$k_i^\mu = y_i^\mu - y_{i+1}^\mu$$ ACTS ON y_i^μ AS CONFORMAL SYMMETRY

- DUALITY WITH WILSON LOOPS.

→ SAME COMPUTATION AS FOR WILSON LOOPS.

- @ WEAK COUPLING:

$n = 6$, 2-LOOPS

$$\mathcal{A}_{MHV} = \mathcal{A}_{TREE} \times \langle \text{WILSON LOOP} \rangle$$

. ORIGIN OF DUAL CONFORMAL SYMMETRY.

IS INTEGRABILITY.

$$K_y^\mu = \int d\sigma \left(\frac{y^2 \partial_0 y}{r^2} + \cdots \right).$$

$$= \int d\sigma \left(y^2 \partial_1 X + \cdots \right)$$

$$= \int d\sigma \; y \underbrace{(\partial_1 y)}_{\frac{\partial_0 X}{z^2}} \partial X \quad = \int d\sigma \underbrace{(y)}_{\int^\sigma d\sigma' \underbrace{\partial_1 y}_{\frac{\partial_0 X}{z^2}}} \frac{\partial_0 X \, X}{z^2}$$

$$= \int d\sigma \left(\int^\sigma d\sigma' \, j_0^P(\sigma') \right) j_0^{DIL}(\sigma)$$

$$= Q_2^P$$

→ 2nd NONTRIVIAL INTEGRABILITY CHARGE IN THE ORIGINAL VARIABLES.

$\boxed{6}$

- DUAL CONFORMAL + CONFORMAL = ALL INFINITE NUMBER OF INTEGRABLE CHARGES.

- IMPOSING DUAL CONFORMAL SYMMETRY WE ARE IMPOSING SOME OF THE CONSTRAINTS OF INTEGRABILITY.

  - PLANAR ONLY
  - FULL QUANTUM THEORY → ONLY $\mathcal{N}=4$ SYM.

- POWER OF DUAL CONFORMAL SYMMETRY.

- NAIVE:

  A = FUNCTION OF CROSS RATIOS.

  4- POINT → NO CROSS RATIOS FOR LIGHT-LIKE SEPARATED POINTS.

  5- POINTS ↗

  6- POINTS → 3 CROSS RATIOS. (V.S. 8 MANDELSTAM INVARIANTS)

- ~~HERE~~ PRECISE → BROKEN BY IR REGULATOR IN A CONTROLLED WAY

  IR DIV $e^{-\Gamma_{cusp}\left(\log(\mu^2/s_{i,i+1})\right)^2}+\dots$

$\Rightarrow$

$$\mathcal{A} = \underbrace{\mathcal{A}_{IR\text{-}DIV} \times \mathcal{A}_{BDS}}_{e^{-\Gamma_{cusp} \times \left(\frac{\mathcal{A}_{1\text{-}loop}}{A_{tree}}\right)}} \times F_{INITE}\,(\text{CROSS-RATIOS})$$

CANCELS VARIATION...

$\cancel{e^{\Gamma_{cusp} \times (1\text{-}loop)}}$

(. COMPUTING WILSON LOOPS IS SIMPLER THAN COMPUTING AMPLITUDES)

- FULL STRING WORLD SHEET

- $AdS_5 \times S_5$ + WORLDSHEET FERMIONS.

32 $\theta_{\alpha i}$ ($\uparrow SU(4)$), $\theta_{\dot\alpha}^{\ i}$, $\vartheta_{\dot\alpha i}$, $\vartheta_{\alpha}^{\ i}$,

- CHOOSE A K-SYMMETRY GAUGE WHICH SETS $\vartheta_\alpha^{\ i} = 0$.

- & WRITE ACTION SO THAT ONLY $\partial\theta_{\alpha i}$ APPEARS.

- POSSIBLE BECAUSE $\{Q^{\alpha i}, Q^{\beta i}\} = 0$. (ABELIAN).

- T- DUALIZE X^μ & ALSO $\theta_{\alpha i}$ (8-FERMIONIC VARIABLES).

8

- GET BACK SAME SIGMA MODEL

  (IN $\varphi_\alpha^{\,i} = 0$ K-GAUGE).

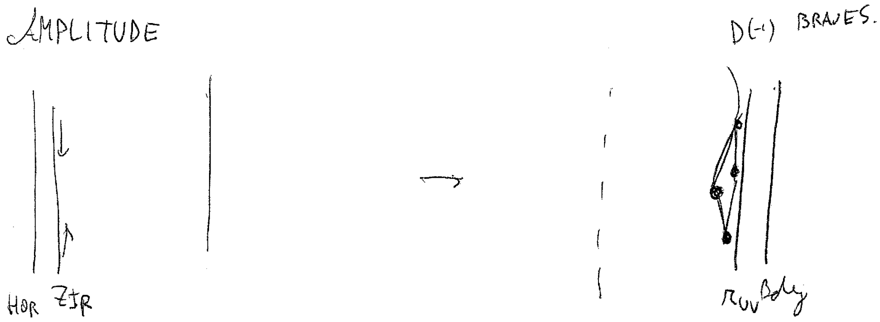
- NO DILATON SHIFT (ONE LOOP DETERMINANT FOR CHANGE OF VARIABLES VANISHES)
- HIGHER LOOPS $\rightarrow$ NO OBVIOUS WAY TO COMPACTIFY $\theta_{\alpha i}$

A SIMILAR DUALITY IN FLAT SPACE:

FLAT SPACE w/ ZERO RR FIELDS $\longrightarrow$ FLAT SPACE WITH SELF DUAL RR FIELDS

DIFFER AT HIGHER LOOPS.

- <u>FINALLY</u>:



DISK DIAGRAM w/ OPEN STRINGS BETWEEN D(-1) BRANES

$\downarrow$

LOWEST STATE = 16-COMPONENTS $\sim$ 8 BOSONS + 8 FERMION. OF SYM MULTIPLET.

- WILSON LOOP $\rightarrow$ SIMILAR BUT D(-1) $\rightarrow$ BDY.

9

• DIFFER IN THEIR BOUNDARY CONDITIONS.

→ WHAT IS THE FULL EXTENT OF THIS DIFFERENCE?
(ONLY A TREE LEVEL FACTOR)

⇝ PROPAGATION OF MASSIVE W-BOSON.

SIDE COMMENT:

WILSON LOOPS IN MOMENTUM SPACE:

$$W[\xi(x(s))] \longrightarrow W(\xi(p(s))) = \int \underset{\substack{\uparrow \\ \text{MEASURE}}}{\mathcal{D}X(s)}\, e^{i\int ds\, x(s)p(s)}\, W[\xi(s)]$$

$$\mathcal{A} = W[p(s)]$$

WITH THE SAME CONTOUR WE HAD ABOVE.

→ (REALLY → SUM OF TERMS.)

POLYAKOV – MCGREEVY
SEVER

Q: WHY IS THE WILSON LOOP IN MOMENTUM SPACE THE SAME AS THE WILSON LOOP IN POSITION SPACE?

10

TAKE HOME

① . HIGH ENERGY STRING AMPLITUDES → CLASSICAL WORLDSHEETS.

② . LARGE $\lambda = g^2 N$ AMPLITUDES IN $\mathcal{N}=4$ SYM → SURFACES IN AdS .

③ . STRINGS → EASY TO SEE INTEGRABILITY

- & - DUAL CONFORMAL SYMMETRY

- FULL SYMMETRY OF THE QUANTUM σ-MODEL.

④ . STRING AMPLITUDES SEEM TO BE DOMINATED BY IR DIVERGENCES AT STRONG COUPLING .

→ ARE HIGHLY SUPPRESSED

- REFERENCES FOR LECTURE II

- HIGH ENERGY SCATTERING IN STRING THEORY

  - GROSS & MENDE PLB 197, 129 (87)
    NPB 303, 407 (88)

- AMPLITUDES @ STRONG COUPLING & STRINGS IN ADS.

  ALDAY & J.M. 0705.0303.

- CONFORMAL WARD IDENTITIES FOR WILSON LOOPS.

  0712.1223
  - DRUMMOND HENN KORCHEMSKY SOKATCHEV

- WILSON LOOPS = AMPLITUDES IN PERTURBATION THEORY

  ↓ 0803.1466
  BERN - DIXON - KOSOWER - ROIBAN
  SPRADLIN, VERGU, VOLOVICH

  ↓ 0803.1465
  DRUMMOND HENN KORCHEMSKY SOKATCHEV

  2-loop
  6-gluon.

- DUAL CONFORMAL SYMMETRY IN PERTURBATION THEORY

  0807.1095 DRUMMOND - HENN KORCHEMSKY . SOKATCHEV

- FERMIONIC T-DUALITY
  &
  DUAL CONFORMAL SYMM. = INTEGRABILITY

  - BERKOVITS & J.M. 0807.3196
  - BEISERT, RICCI, TSEYTLIN 0807.3228

- FOR A GENERAL MODERN TREATMENT OF IR EFFECTS

  SEE : "SOFT COLLINEAR EFECTIVE FIELD THEORY"

  & GOOGLE FOR LECTURES BY I. STEWART