The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2024-16

# Spring School on Superstring Theory and Related Topics

*23 - 31 March 2009*

**Aspects of scattering amplitudes and colider physics in conformal field theories**
**Lecture 3**


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E1

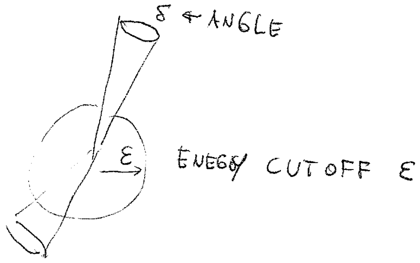
# ENERGY CORRELATION FUNCTIONS.

Q: WHY ARE WE COMPUTING AMPLITUDES?

- WOULDN'T IT BE NICE TO COMPUTE DIRECTLY FINITE QUANTITIES?
- PHYSICAL QUANTITIES SHOULD BE NICER....

- VARIOUS IR FINITE OBSERVABLES.
  - JET AMPLITUDES.
  - EVENT SHAPE VARIABLES. "THRUST DISTRIBUTIONS" $\quad T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \to \frac{d\sigma(T)}{dT}$
  - ENERGY CORRELATIONS.

- JET AMPLITUDES.

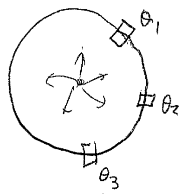



$$\sigma \sim \quad \sim e^{-\Gamma_{cusp} \log \delta \log Q/\varepsilon} + \dots$$

$T \sim 1$ ALSO SUPPRESSED BY $\Gamma_{cusp}$.

E 2.

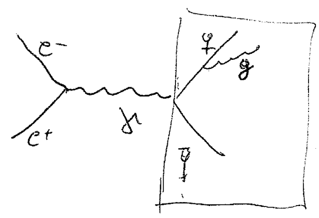
## – ENERGY CORRELATORS




- DETECTORS ON A SPHERE A $\infty$.
- MEASURE ENERGY PER UNIT ANGLE AT $\infty$. TOTAL ENERGY FLOW FOR ALL TIME.
- EACH EVENT $\rightarrow$ SERIES OF TICKS. $\rightarrow$ ENERGY DISTRIBUTION $\mathcal{E}(\theta)$.

$\langle \mathcal{E}(\theta_1) \cdots \mathcal{E}(\theta_n) \rangle$ = AVERAGE OVER ALL EVENTS.

- SIMPLEST CASE $\rightarrow$ WELL DEFINED INITIAL STATE $e^+e^- \rightarrow \gamma \rightarrow q\bar{q}$
$\rightarrow$ CFT




CFT

- $$\mathcal{E}(\theta_1) = \lim_{r \to \infty} r^2 \int_{-\infty}^{\infty} dt \; T_0{}^i \, \hat{n}^i$$

UNIT VECTOR IN DIRECTION $\hat{\theta}$ ON $S^2$

- DEFINED IN TERMS OF STRESS TENSOR CORRELATORS

$$\langle \Psi | \, \mathcal{E}(\theta_1) \cdots \mathcal{E}(\theta_n) | \Psi \rangle = \langle 0 | \mathcal{O}^\dagger \underbrace{\mathcal{E}(\theta_1) \cdots \mathcal{E}(\theta_n)}_{\text{PROBE THE STATE}} \mathcal{O} | 0 \rangle$$

NOT TIME ORDERED

CREATES THE STATE

E3

• TO UNDERSTAND THE NATURE OF THE OPERATORS IT IS CONVENIENT TO DO A CONFORMAL TRANSFORMATION.

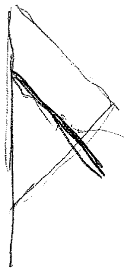
MINKOSWSKI SPACE $\mathbb{R}^{1,3}$

DETECTORS AT LARGE $r$.

$\longrightarrow$

DETECTORS ALONG THE NULL FUTURE BOUNDARY OF MINKOWSKI SPACE.

EXTEND



BRING TO A FINITE REGION

• MORE EXPLICITLY:

$$y^+ = -\frac{1}{x^+} \qquad y^- = x^- - \frac{x_1^2 + x_2^2}{x^+} \qquad y^{1,2} = \frac{x^{1,2}}{x^+}$$

$$dx^\mu dx_\mu = \frac{dy^\mu dy_\mu}{(y^+)^2}$$

$y^+ = 0$ IS $x^+ = \infty$

$\underset{\substack{\uparrow \\ \text{NULL DIRECTION}}}{y^-}, \ \underbrace{\vec{y}_T}_{\text{PLANE}}$

$\mathcal{E}(\theta) \xrightarrow[\text{TO}]{\text{RELATED}}$

$$\mathcal{E}(\vec{y}_\perp) = \int_{-\infty}^{\infty} dy^- T_{--}(y^-, y^+ = 0, y_1, y_2)$$

$\uparrow$ DEPENDS ON $y_1, y_2$ $\downarrow$ ON A PLANE.

ALL COMPONENTS OF $P_\mu$ ARE DETERMINED IN TERMS OF $\mathcal{E}(\vec{y})$.

$\boxed{E4}$

CONFORMAL MINK. SPACE

$$-(Z^{-1})^2-(Z^0)^2+(Z^1)^2+\dots+(Z_4)^2=0.$$

IN $\mathbb{R}^{2,4}$

WITH METRIC
$ds^2=-(dZ^{(-1)})^2-(dZ^0)^2+(dZ^1)^2+\dots+(dZ_4)^2$

IDENTIFICATION: $Z^\mu \sim \lambda Z^\mu$

GENERATORS:

$$\Theta_{MN} = Z_M \frac{\partial}{\partial Z^N} - Z_N \frac{\partial}{\partial Z^M}.$$

$Z^\pm = Z^{(-1)} \pm Z^4$

$\mu=0,1,\dots,3$ $Z^\mu$.

$x^\mu = \frac{Z^\mu}{Z^+}$

$$P_\mu \sim Z_\mu \frac{\partial}{\partial Z^-} - Z_- \frac{\partial}{\partial Z^\mu}$$

THE BOUNDARY OF MINK. SPACE IS AT $Z^+=0$.

(Metric → set $f(Z)=0$ eg $Z^+=1$)

$$P_\mu \Big|_{\substack{Z^+=0 \\ = Z_-}} \approx \underbrace{Z_\mu \frac{\partial}{\partial Z^-}}$$

ALL PROPORTIONAL TO THIS GENERATOR

- LORENTZ SYMMETRY = $SO(1,3)$ = $SL(2,\mathbb{C})$ = GROUP OF CONFORMAL TRANSFORMATIONS ON $S^2$ = SPHERE AT $\infty$

$\mathcal{E}$ → PRIMARY OPERATOR OF DIMENSION 3 ($\int dy^- T_{--}$ HAS DIM 3)

SYMMETRIES

- $B_{x^+x^-}$ ⟶ DILATATION IN $y^\mu$.
- $D_x$ ⟶ $B_{y^+y^-}$ IN $y$ COORDINATES
- $SO(1,3)_x$ ⟶ $SL(2,\mathbb{C})$ IN $y_\perp$ PLANE ≅ SPHERE $S^2$ BY USUAL RIEMANN MAP.

(E5) • CONCLUSION

• CONFORMAL TRANSFORMATION

ENERGY CORRETATORS (ON $S^2$ AT $\infty$). $\longrightarrow$ $\boxed{\int dy^- T_{--} \text{ DISTRIBUTIONS ON } y^- \text{ (DENSITY)}}$

• SIMPLE IN LIGHT CONE GAUGE:

$y^+ =$ TIME $\qquad \int dy^- T_{--} = \rho_-(y_\perp, y^+) =$ DENSITY OF $P_-$ in $Y_\perp$

• RECALL: IN LIGHT-CONE GAUGE WE HAVE A NON-RELATIVISTIC PROBLEM:

$$H \sim -P_+ \sim \frac{P_\perp^2}{(-P_-)} \qquad \left( \text{DLCQ: } P_- \sim N/R \ldots\ldots \right).$$

• WE ARE MEASURING THE $P_-$ DENSITY OR MASS DENSITY ( IN THE NON RELATIVISTIC SETTING

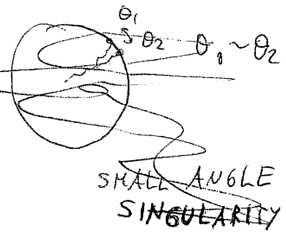
• IS POSITIVE (NOTE THAT $T_{--}(x^-)$ IS NOT POSITIVE

• $\mathcal{E}$ IS A POSITIVE OPERATOR. (WILL IMPOSE CONSTRAINTS).

• $\mathcal{E}(y)$ $\mathcal{E}(y')$ COMMUTE FOR $y \neq y'$

• VERY "LORENTZIAN" OBSERVABLE.

~~• WE WILL STUDY $\mathcal{E}(\vec{y}_\perp)\mathcal{E}(\vec{y}'_\perp)$ AS $y_\perp \to y'_\perp$~~




• WE CAN THINK OF DIAGONALIZING $\mathcal{E}(y)$.

STATE $|\psi\rangle \rightarrow \rho[\mathcal{E}(\theta)]$

$$\langle \mathcal{E}(\theta_1) \ldots \mathcal{E}(\theta_n) \rangle = \int \mathcal{D}\mathcal{E}(\theta)\, \rho[\mathcal{E}(\theta)].$$

(E6)

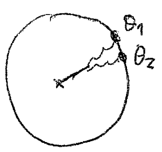
- <u>IN PRINCIPLE:</u>
  OTHER INCLUSIVE VARIABLES. CAN BE COMPUTED IF WE KNEW ALL ENERGY CORRELATORS.
  - $\mathcal{F}(\mathcal{E}(\theta))$.

$$\frac{d\sigma}{dF} \sim \int \mathcal{D}\mathcal{E}(\theta)\, \rho[\mathcal{E}(\theta)] \;\; \delta\big(\mathcal{F}(\mathcal{E}(\theta)) - F\big)$$

- <u>IN PRACTICE</u>
  - ONLY A FEW ENERGY CORRELATORS ARE COMPUTED
  - SO THEY ARE DIFFERENT OBSERVABLES.

  (LEP GRAPHS...)

SMALL ANGLE SINGULARITIES:



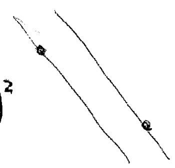

- DOMINATED BY COLINEAR RADIATION.

$$\longrightarrow \quad \mathcal{E}(\vec{y}_1)\,\mathcal{E}(\vec{y}_2) \qquad \text{AS } \vec{y}_1 \to \vec{y}_2$$

$$\downarrow$$

$$\int dy^- T_{--} \quad \int dy^- T_{--}$$

- THE DISTANCE IS SPACELIKE:
  ANY 2-POINTS $\to$ SAME DISTANCE : $d^2 = (\vec{y}_1 - \vec{y}_2)^2$



- LIGHT-CONE OPE:

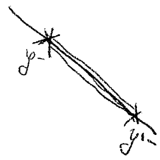
TWIST $= \Delta - S$

TWIST $[y^-] = 0$.
TWIST $[\mathcal{E}] = 2$.

$$\mathcal{E}(\vec{y}_1)\;\mathcal{E}(\vec{y}_2) = \sum_n |\vec{y}_1 - \vec{y}_2|^{\tau_n - 2 - 2}\;\mathcal{O}_n(\vec{y}_2)$$

(E 7)

- $\mathcal{O}_n$ = • NON-LOCAL IN $y^-$
  - LOCAL IN $y^+$, $\vec{y}_\perp$
  - ROUGLY




$$\mathcal{O}_n \sim \int dy^- \int dy'^- \; f(y^-, y'^-) \; \mathrm{Tr}\left[ F_{-i}(y) \, W(y^-, y'^-) \, F_{-i}(y'^-) \right]$$

WILSON LINE

(TWIST 2 AT LEADING ORDER).

• ~~WHAT ARE THE PROPERTI~~

OBSERVATIONS

- THESE OPERATORS ARE NATURAL IN LORENTZIAN SITUATIONS
- FAST MOVING PARTICLES..., ETC.
- APPEAR ALL OVER THE PLACE.
- IS THE NEW THING RELATIVE TO ORDINARY EUCLIDEAN OPE'S.

• PROPERTIES OF THESE OPERATORS.

Müller Beilitsky Korchemsky.

- DEFINED ALONG A LINE
  - TRANSFORMATIONS THAT LEAVE THE LINE FIXED
  - COLINEAR CONFORMAL GROUP

$$P_- \;, K^- \;, \; D + B_{+-} \;\to\; SL(2,R).$$

$$D - B_{+-} = \text{TWIST}.$$
$$= \ell - s$$

(COLINEAR TWIST → ONLY SPIN IN +- DIRECTIONS, NOT TOTAL SPIN).

E 8

- USE THESE SYMMETRIES TO CLASSIFY THE OPERATORS.

- ~~Diags~~ DIAGONALIZE THE CONFORMAL SPIN $j = \frac{l+s}{2}$.

- LOWEST TWIST @ LOWEST ORDER.

2 INSERTIONS $(\phi(x^-)\ \phi(x'^-))$

$\rightarrow$ DECOMPOSE INTO $SL(2,\mathbb{R})$ REPRESENTATIONS $\rightarrow$ 1-FOR EACH SPIN $j$.

(NOT HIGHEST WEIGHT REPRESENTATIONS).

- FOR EACH REPRESENTATION $\rightarrow$ FIX MOMENTUM ALONG $x^-$ FOR CM.

IN OUR CASE IT WILL BE ZERO.

SIDE REMARK

- e.g.:

$$\int dx^- \int dx'^-\ \bar{q}(x^-) e^{i(xP_-)(x^- - x'^-)}\ q(x'^-) = \hat{f}(x).$$

PDF: $f(x) = \langle P|\ \hat{f}\ |P\rangle$

- EVOLUTION $\rightarrow$ AP EQN $\rightarrow$ KERNEL $\rightarrow$ DIAGONALIZED BY GOING TO BASIS WITH FIXED $SL(2)$ SPIN.

- MOMENTS OF PDF'S $\rightarrow$ ~~SPARSE~~ OPERATORS w/ DEFINITE SPIN $\rightarrow$ (NOT NECESARILY EVEN INTEGERS.).

E 9

- BACK TO OUR CASE

$$\mathcal{E}(\vec{y})\,\mathcal{E}(0) \sim \sum_n |y|^{\tau_n - 2 - 2}\, \mathcal{O}_n.$$

SPIN IN $y^+ y^-$ PLANE : $\int dy^- T_{--} \to$ SPIN 1.

$1+1 = 2.$

- TOTAL SPIN OF $\mathcal{O} = 2.$ $\to$ INTEGRAL OF SPIN 3 OPERATOR.

$$\mathcal{O} = \int dx^- \underbrace{\mathcal{O}_{---}}$$

NOTHING LOCAL CAN BE CONSTRUCTED

NON-LOCAL $\sim \displaystyle\int_0^\infty \frac{d\alpha^-}{(\alpha^-)^2}\, \mathrm{Tr}\left[F_{-i}(\alpha^-) F_{-i}(-\alpha^-)\right]$

- $\tau_n$ = TWIST OF THE SPIN 3 OPERATOR

- AMONG THE TWIST 2 OPERATORS $\to$ ~~ALL~~ $\tau_n \approx 2 + g^2 N \times \underbrace{\text{coef}}_{>0}$

  - ONLY A FEW WITH SPIN 3.

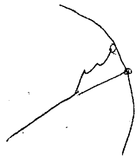
- OTHER SYMMETRIES.
  - SPIN IN $y_1, y_2$ PLANE $\to$ ALLOWS $S_{12} = 0, \pm 2$
    $\downarrow \frac{1}{|y|}$ $\hookrightarrow \frac{f(y_i)}{|y| \cdots}$
  - $X^\pm \to -X^\pm \to$ CPT (EVEN OPERATORS ONLY).

- IN $\mathcal{N}=4$ SYM.

  $\mathrm{Tr}[\phi\,\phi]$ $\mathrm{Tr}[\psi\,\psi]$ $\mathrm{Tr}[F_{-i} F_{-i}]$ $\to S_{12} = 0$

  $S_{12} = \pm 2,$ $\mathrm{Tr}[F_{-(i} F_{-j)}]$

E10



IN PERT THEORY

$$\mathcal{E}(\theta)\ \mathcal{E}(0) \sim \alpha\ \frac{\theta^{\gamma}}{\theta^{2}}\ \mathcal{O}_{\text{spectr}}$$

$\uparrow$

$\mathcal{O}\,p^2 \sim$ IR SAFE VERSION OF $\mathcal{E}^2$

$$\gamma \sim c\alpha \quad ; \quad c > 0$$

$$\sim \alpha\ \frac{\theta^{\alpha c}}{\theta^{2}}$$

$\theta$

$$\int_0^\delta d^2\theta\ \mathcal{E}(\theta)\ \mathcal{E}(0) \sim \alpha \int_0^\delta \frac{d^2\theta}{\theta^2}\ \theta^{\alpha c} \sim \frac{1}{c}\,\delta^{\alpha c} \sim \text{cst}$$

$\uparrow$ NOT DIVERGENT AT ZERO

THE CONSTANT IS SUCH THAT WE ARE SIMPLY GETTING

$$\cancel{\int d^2\theta\ \mathcal{E}(\theta)\ \mathcal{E}(0) \sim \langle \mathcal{E}(0)\rangle^2}$$

THE TOTAL ENERGY OF THE JET.

- OPENING ANGLE:

FRACTION $f<1$ OF TOTAL ENERGY $\qquad f \sim \delta^{\alpha c}$

$$\sim \delta \sim f^{\frac{1}{\alpha c}} \sim e^{-\frac{1}{\alpha c}\log f}$$

KONISHI
OKAWA
VENEZIANO
$\rightarrow$ 1-LOOP ANALYSIS IN QCD

MORE PRECISE: $\rightarrow$ SEVERAL OPERATORS WITH SIMILAR DIMENSIONS
$\bar{q}q$, FF

L4

E10.5

- Start with some simple remarks about 1-pt functions
- Theoretical application of ~~collider~~ energy correlation functions

E 11

## 1- POINT FUNCTIONS

$\langle 0 | \mathcal{O}^{\dagger} \ \mathcal{E}(\theta) \ \mathcal{O} | 0 \rangle$

4- MOMENTUM OF OPERATOR $\mathcal{O}$ $\quad q^{\mu} = (q^0, 0, 0, 0)$.

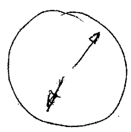
$e^+ e^- \longrightarrow \mathcal{O} \longrightarrow$ STATE IN CFT

( CFT $\approx$ UNPARTICLES )

SCALAR OPERATOR $\mathcal{O}$

. SO(3) INVARIANCE $\longrightarrow$ $\mathcal{E}(\theta) = \frac{q^0}{4\pi}$

UNIFORM.



. S- WAVE

. 2- POINT FUNCTION WOULD NOT NECESSARILY BE $\left(\frac{q^0}{4\pi}\right)^2$.

(AT WEAK COUPLING IT IS NOT! ).

. CURRENT OPERATOR $\mathcal{O} \sim \vec{q} \cdot \vec{J}$

$\rightarrow \quad \langle \mathcal{E}(\theta) \rangle \sim \frac{q_0}{4\pi} \left[ 1 + a_2 \left( \cos^2\theta - \frac{1}{3} \right) \right]$

INTEGRAL IS ZERO

. 1- UNDETERMINED COEFFICIENT $a_2$

$\rightarrow$ BOSONS $\rightarrow$ $\sin^2\theta$

FERMIONS $\rightsquigarrow$ $\cos^2\theta$

$\rightarrow$ DETERMINED BY 3pt FUNCTIONS OF

$\langle J J T \rangle \rightarrow 2$ POSSIBLE STRUCTURES.

E12

. 1 OF THEM FIXED BY THE WARD IDENTITY OF THE STRESS TENSOR (ENERGY CONSERVATION).

. $2^{nd}$ DEPENDS ON THE THEORY

. DEMANDING THAT $\mathcal{E}(\theta) > 0$

WE GET SOME BOUNDS ON $a_2$

$\rightarrow$ BOUNDS ON PROPERTIES OF OPE'S $\left( -\frac{3}{2} \leq a_2 \leq 3 \right)$

BOSONS. FERMIONS

. STRONG COUPLING $\rightarrow a_2 = 0$ (UNIFORM).

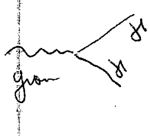
. $j$ = GLOBAL SYMMETRY OF SUSY THEORY $\rightarrow$ UNIFORM. $a_2 = 0$. ALL COUPLING

. $j$ = R- CURRENT OF $\mathcal{N}=1$ SUSY THEORY.

$\Rightarrow a_2 \propto a - c$ $\quad a, c. \rightarrow$ ANOMALY COEFFICIENTS

$\langle T_\mu{}^\mu \rangle \sim c (\text{WEYL})^2 + a (\text{EULER})$

GRAVITY DUAL - I WILL GIVE A MORE PRECISE DESCRIPTION LATER




2- POSSIBLE ON SHELL COUPLINGS. IN THE BULK

$\text{Tr}[F^2]$ $\quad$ $\text{Tr}[RFF]$ ↖ Higher derivative ↗ IF PRESENT → CAN'T TRUST GRAVITY

↳ must give uniform

(E 13) • SIMILAR STORY W/ STRESS TENSOR.

$\mathcal{O} = \xi_{ij} T_{ij}$ $\longrightarrow$ GRAVITON ~~SCATTERED~~ INTERMEDIATE STATE

$$\langle 0|\mathcal{O}^{\dagger} \; \mathcal{E} \; \mathcal{O}_{\xi_{ij}}|0\rangle \sim \frac{q_0}{4\pi}\Big[1 + \tilde{a}_2 \, \varepsilon_{ij}\varepsilon^{*}_{ik} n_j n_k + \tilde{a}_4 \, \varepsilon_{ij}\varepsilon_{lk} n_i n_j n_l n_k \Big]$$

• Bosons
Fermions $\leftrightarrow$ POSITIVITY CONDITIONS.
Vectors

• IN $\mathcal{N}=1$ SUSY THEORIES $\tilde{a}_4 = 0$.

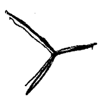
AND $\tilde{a}_2 \sim \frac{a-c}{c}$ $\qquad \left|\frac{a-c}{c}\right| \leq \frac{1}{2}$

$\rightarrow$ BOUND : SATURATED BY PURE VECTORS OR PURE CHIRAL MULTIPLETS.

• GRAVITY:

~~3 STRU~~ GRAV

3 TYPES OF ON SHELL GRAVITON VERTICES.

 $\sqrt{g}\,R + \sqrt{g}\,R^2 + \sqrt{g}\,R^3$

CFT: 3 TYPES OF CONFORMAL STRUCTURES.

E14

## . CURRENTS.

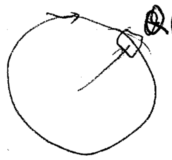
3-PT FUNCTIONS:

$\langle j\, j\, j \rangle$ → PARITY CONSERVING → 2 STRUCTURES

→ PARITY VIOLATING → 1 STRUCTURE

→ ANOMALY.

## . CHARGE CORRELATORS.



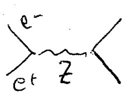
$Q(\theta)$ ← CHARGE EXPECTATION VALUE AT THIS POINT

$$\langle 0 | \mathcal{O}\; Q(\theta)\; \mathcal{O} | 0 \rangle$$

$$\langle 0 | J^\dagger \int J \; J | 0 \rangle \sim \mathrm{Im}\left[ \epsilon^{ijk} \varphi_i \varphi_j^* n^k \right]$$

ANOMALY COEFFICIENT

$\sim \sin\theta$




$\langle 0 | J_z^\dagger \; Q_{em} \; J_z | 0 \rangle \sim$ ANOMALOUS PIECE IF WE RESTRICT TO LEPTONS ONLY

→ FORWARD-BACKWARDS ASYMMETRIES → RELATED TO ANOMALIES

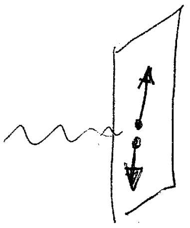
→ NO CHARGE ASYMMETRIES IF WE HAD QUARKS.

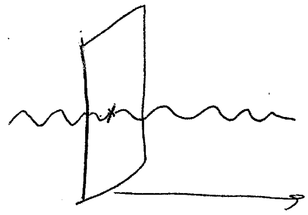
• ~~(Some theoretical applications of energy correlators~~ —
$\mathcal{N}=4$

E 14.5

• YANG MILLS vs. gravity

gluons $\Leftrightarrow$ strings.





AdS

- Why are these pictures so different?
- What is the difference precisely
  (is it simply that we have an S-wave for the $q\bar{q}$ ?)
- Is there any kind of transition between one picture and the other

× ←— gluon $\sim \frac{g^2 N}{r}$ $\quad p \sim 1/r$.
heavy adjoint source
↗ WINS.

transition

LARGE COUPLING → × ← GLUON FALLS IN.

¿How does the energy distribution change?

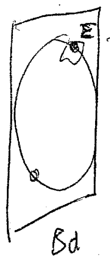
- Intuition: → LARGE AMOUNTS OF RADIATION
  → UNIFORM DISTRIBUTION OF ENERGY.

uniform energy

E 15

~~Compute~~

. COMPUTATION OF ENERGY CORRELATION FUNCTIONS ~~IN AdS~~ AT STRONG COUPLING in $\mathcal{N}=4$ SYM.




$$\lim_{r\to\infty} r^2 \int dt\, T_{0i} n^i = \mathcal{E}(\theta).$$

COORDINATE CHANGE

$$\mathcal{E}(\vec{y}) = \int dy^- \, T_{--}(y^-)$$

AT $y^+=0$.

. $\mathcal{E}(\vec{y}) \to$ GRAVITON INSERTION IN THE BULK.

$g_{\mu\nu}+h_{\mu\nu}$

$$h_{++} \sim \frac{z^2}{\left((y-y')^2+z^2\right)^4} \longrightarrow \delta h = \frac{\delta(y^+)\; z^2 \; (dy^+)^2}{\left((\vec{y}-\vec{y}') + z^2\right)^3} \longleftrightarrow \mathcal{E}(\vec{y}')$$

SHOCK WAVE $\to$ PROBES THE STATE AS IT CROSSES $y^+=0$

$y^+, y^-, \vec{y}, z$ = BULK COORDINATES

$$-W_{(-1)}^2 - W_0^2 + W_1^2 + \dots + W_3^2 + W_4^2 = -1$$

GO BACK & FORTH BETWEEN VARIOUS COORDINATE

E16

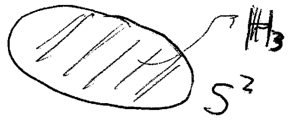
SYSTEMS BY EXPRESSING $W^{\mu}$ IN DIFFERENT WAYS.

- IN SOME COORDINATES:

$$\delta(y^+) \to \delta(w^+) \to -(W^0)^2+(W^1)^2+\dots+(W_4^2)^2=-1$$

$$\downarrow \mathbb{H}_3.$$

$$\mathcal{E}(\vec{m}) \Rightarrow \frac{\delta(W^+)\,(dW^+)^2}{(W^0-\vec{W}\cdot\vec{m})^3} \approx h_{MN}dx^M dx^N$$




$S^2$ = SPHERE AT $\infty$ IN POINCARE COORDINATES

- $\mathbb{H}_3 \times W^-$ $\to$ HORIZON OF ADS IN USUAL POINCARE PATCH

$$\left( \frac{dx^2+dr^2}{r^2} \quad (r=\infty). \right)$$

-

(E 17)

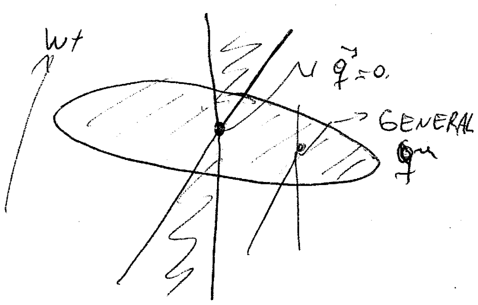
- STATE FALLING IN:

$$\mathcal{O}_q |0\rangle_{CFT} \longrightarrow \quad \mathcal{O}_q = \int d^4x \, e^{iq\cdot x} \, \mathcal{O}(x).$$

IF WE HAVE A STATE WITH FIXED 4-MOMENTUM:

- IT CROSSES $W^+=0$, AT A PARTICULAR POINT IN $W^\mu$ & $P_- = \frac{\partial}{\partial W^-}$ WILL BE RELATED TO $q^2$.

$$P_\mu \Big|_{W^+=0} \approx W_\mu \underbrace{\frac{\partial}{\partial W^-}}_{P_-} - \cancel{W^+ \frac{\partial}{\partial W^\mu}}$$



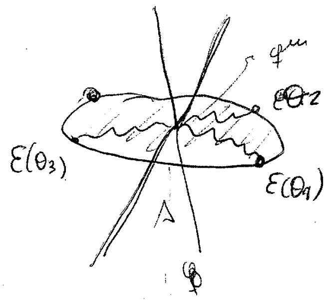

- IF $q^\mu = (q^0, \vec{q}=0)$

$\rightarrow$ CROSSES AT THE "CENTER" OF $\mathbb{H}^3$.

$SL(2,\mathbb{C}) \simeq SO(1,3)$ BOOSTS

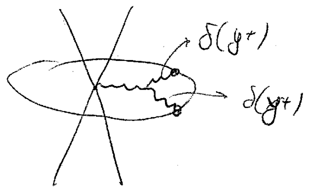
$\rightarrow$ ISOMETRIES OF $\mathbb{H}^3$ $\rightarrow$ MORE GENERAL $q^\mu$

E 18

- 


- PROBE THE STATE AT $y^+=0$

- CHECK THAT IT MAKES SENSE

① 

$\sim (\delta(y^+))^2 \to$ WOULD GIVE $\infty$.

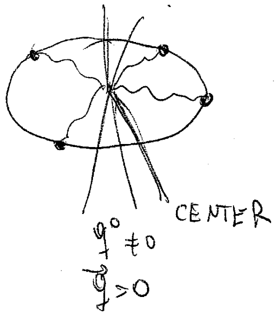
(a) : IN GRAV → PLANE WAVES → EXACT SOLUTIONS
→ NO SUCH DIAGRAMS.

REAL REASON (b) → IN STRING THEORY → $(\delta(y^+))^2$ → REPLACED
BY SOMETHING SMOOTH
& REGGE BEHAVIOR $\sim \frac{1}{s^{2+\frac{\alpha' t}{2}}}$

$\delta(y^+) \to \int ds\, A(s,t).$

E19

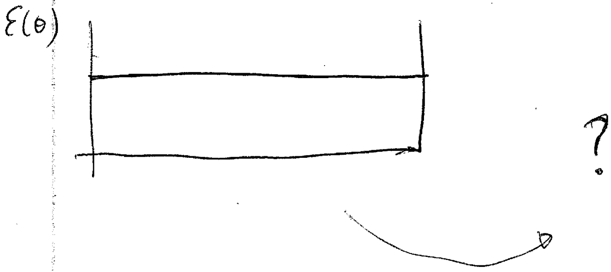
- GRAVITY RESULT:

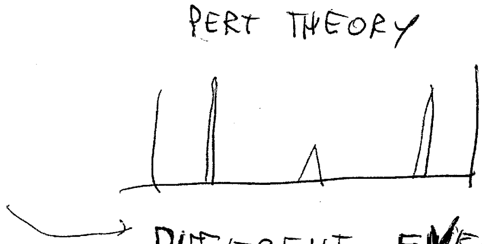



- ALL INDEPENDENT
- EACH $\mathcal{E}(\theta) \rightarrow$ INDEP OF $\theta$.

GRAVITY

- COMPLETELY **UNIFORM** WITH <u>NO</u> FLUCTUATIONS




DIFFERENT EVENTS LOOK DIFFERENT

- COMPLETELY INDEPENDENT OF THE **SPIN** OF THE PARTICLE

- **LOCALIZED** PARTICLE (POINTLIKE) IN BULK $\rightarrow$ UNIFORM (& DELOCALIZED) DISTRIBUTION ON THE BOUNDARY

(MOVING POINT $\rightarrow$ BOOSTING $\hookrightarrow SL(2,\mathbb{C})$ )

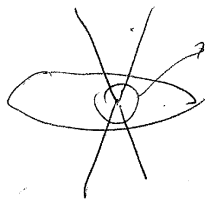
- WHAT GIVES A NON-UNIFORM DISTRIBUTION?

E20

• STRINGY CORRECTIONS.

OBS:

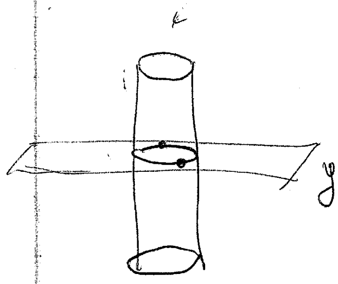
ⓐ



LOCALIZ~~ZA~~ED

↓

ZOOM → 10-d SPACE

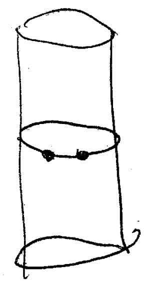



. ↑FALLING SRING

. LIGHT-CONE GAUGE $y^+ = \tau$.

. PROBED AT $y^+=0$ OR $\tau=0$

. VERTEX OPERATORS → WAVEFUNCTION OF GRAVITONS → LOCALIZED AT $y^+=0$, $\tau=0$.



2-pt FN. TRANSVERSE 3-d SPACE $\vec{k}=0$

$$\int d\sigma_1 \int d\sigma_2 \langle \underbrace{p_- e^{i k_1 X(\sigma_1)} \; p_- e^{i k_2 X(\sigma_2)}}_{p_-^2 \, |\sin(\sigma_1-\sigma_2)|^{\alpha' \vec{k}_1 \cdot \vec{k}_2}} \rangle$$

$$\delta(y^+)(\partial_\tau y^+)^2 e^{ik\cdot x} \xrightarrow{y^+ = p_- \tau} p_- \, \delta(\tau) \, e^{ikx}$$

(E21) • FACTORS OF $P_-$ → TOTAL SPIN IN $y^+y^-$ PLANE → (CONSERVED...)

→ DO INEGRALS.

$$\frac{\Gamma\left(\frac{1}{2}+\frac{\alpha' k_1 \cdot k_2}{2}\right)}{\Gamma\left(1+\frac{\alpha' k_1 \cdot k_2}{2}\right)} 2^{\alpha' k_1 \cdot k_2}$$

$$\approx 1 + \frac{\pi^2}{24}\left(\alpha' k_1 \cdot k_2\right)^2.$$

FIRST CORRECITION:

INSERT IN FULL WAVE FUNCTIONS & INTEGRATE.

→ GET:

$$\langle \mathcal{E}(\theta_1)\, \mathcal{E}(\theta_2) \rangle \sim \left(\frac{q_0}{4\pi}\right)^2 \left[ 1 + \alpha'^2 \left(\cos\theta^2 - \frac{1}{3}\right) + \ldots \right]$$

$$\frac{1}{\lambda} = \frac{1}{g^2 N}$$

→ COMPUTE HIGHER POINT FUNCTIONS.

E 27

**OPE**

$$\langle \mathcal{E}(\theta_1)\, \mathcal{E}(\theta_2) \rangle$$

AS $\theta_1 \to \theta_2$

→ POLES IN $\Gamma$ FUNCTIONS $\frac{\alpha' k_1 \cdot k_3}{2} \sim$

$$t = -(k_1+k_2)^2 = \frac{2+4M}{\alpha'}$$

• → NO ON SHELL CLOSED STRINGS

( • VENEZIANO $\int ds\, A_4(s,t) \sim \int ds \frac{1}{s^{2}} s^{\frac{\alpha' t}{2}}$ ).

• WORLDSHEET OPE :

$$P_-^2\, e^{ik_1\cdot X_1}\, e^{ik_1\cdot X_2} \sim P_-^2\, |\sigma|^{\overbrace{\alpha' k_1\cdot k_2}^{-1}}\, e^{i(k_1+k_2)\cdot X}$$

$$P_-^2 \to \delta(y^+) \frac{(\partial_\sigma y^+)^3}{(\partial_\alpha y^+ \partial_\alpha y^+)^{3/2}}$$

→ SPIN 3 STATE

& NON-LOCAL → NON-LOCAL $\overbrace{\text{OPERATORS}}^{\text{LIGHT RAY}}$ IN STRING THEORY

(E 23)

- MASS SHELL CONDITION.

$$\Delta \sim mR \; ; \; m^2 \sim -k^2$$

$$\Delta \simeq \sqrt{2}\sqrt{j-2}\;\lambda^{1/4}$$

$\Rightarrow$ WE SEE THAT WE GET THIS LARGE DIMENSION IF $\lambda \gg 1$

$$\mathcal{E}(\theta)\,\mathcal{E}(\theta) \sim \theta^{-2+\underbrace{(\tau-2)}_{\text{LARGE}}} \rightarrow \text{NON SINGULAR}$$

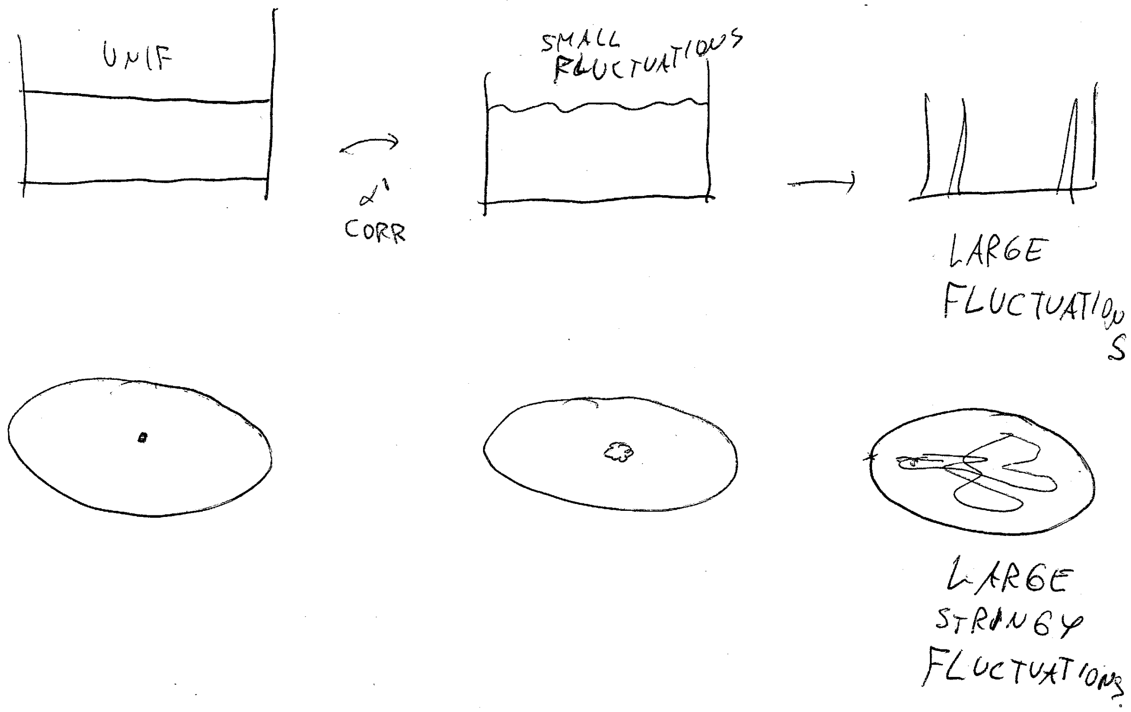
- LARGE ANOMALOUS DIMENSIONS

FOR ~~SPIN 2 OPS~~ HIGHER SPIN OPERATORS $\longleftrightarrow$ HOW UNIFORM THE DISTRIBUTION IS

$\updownarrow$

VALIDITY OF THE GRAVITY APPROXIMATION.

E 24




- RANDOM VARIABLES THAT DETERMINE THE FLUCTUATIONS.

  → WAVE FUNCTIONAL OF THE STRING.

  → POINT → PARTICLE → ONLY CM → FIXED BY TOTAL MOMENTUM REQUIREMENT

- MEASURING THE STATE AT ∞ IN THE GAUGE THEORY =

  = "TAKING A SNAPSHOT OF THE STRING AS IT FALLS THROUGH THE HORIZON_

E 25

- NON CONFORMAL / CONFINING.

→ FACTORIZATION?

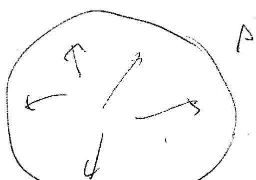
(a) → NOT VALID @ LARGE N IN CONFINING THEORIES

(b) ⇒ WOULD <u>NOT</u> BE VALID IN NATURE IF QUARKS WERE HEAVY $m_q \gtrsim$ FEW GeV.

→ PROVEN IN PERTURBATION THEORY

→ SEEMS TO WORK FOR N=3.

→ WHAT DO WE NEED?



PATTERN FIXED DURING CONFORMAL EVOLUTION.

NEED

RAPID TRANSITION **TO** HADRONS.

IN (a) & (b) → GET A STRING WHICH OSCILLATES A LOT → INITIAL PATERNS → GREATLY AFFECTED BY CONFINEMENT.

(E 26)

- COSMOLOGY ANALOGY
  - 2-d SPHERE → FLUCTUATIONS.
  - HADRONIZATION & HORIZON REENTRY

- ENERGY CORRELATIONS -

- BASHAM, BROWN, ELLIS & LOVE
  PRD 19, 2018 (1979)
  PRL 41, 1585 (1978)

[JETS IN QCD → STERMAN & WEINBERG. PRL 39, 1436 (1977)]

- STERMAN HEP-PH/0501270

- SMALL ANGLE OPE
  HOFMAN & J.M. 0803.1467

- ENERGY CORRELATORS @ STRONG COUPLING VIA GRAVITY

- SMALL ANGLE IN QCD (AT 1-LOOP)
  KONISHI, UKAWA, VENEZIANO → NPB 157, 45 (1979)