2024-12

Spring School on Superstring Theory and Related Topics

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String theory and QCD

S. Sugimoto
IPMU
Japan

Trieste Spring School

3/22 ~ 31 / 2009

String theory and QCD

4 hours (3/30・31)

S. Sugimoto (IPMU)

Plan

1. Introduction
2. SUGRA description of Yang-Mills theory
3. Construction of QCD
4. holographic description of QCD
5. meson spectrum
6. pion effective action
7. interaction with vector mesons
8. baryon

ref hep-th/0412141, 0507073) with T. Sakai

① Introduction

new description of hadrons
~~powerful tool~~ to analyse

String theory ⇄ QCD

provides some evidence
of the gauge / string duality

"holographic QCD"

Out line

1. Construct QCD using D-brane
2. Use the idea of gauge / string duality

   ⇒ holographic description of QCD

   ↓

   5 dim Yang-Mills-Chern-Simons theory

   as a meson effective theory

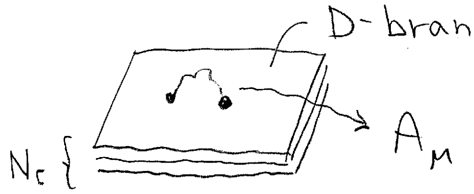
3. enjoyable calculation

   using 5 dim YM-CS theory

4. Compare with experimental data
5. Smile ☺

② SUGRA description of YM theory

★ gauge / string duality



D-brane ⇝ corresponding curved background

$\Downarrow$ $\qquad$ $\Downarrow$

U(N_c) gauge theory	$\xleftrightarrow{\text{dual!}}$	String theory in a curved b.g.

★ Consider

D4-brane × N_c wrapped on S^1

$\tau \sim \tau + 2\pi M_{KK}^{-1}$

• massless modes in 4 dim

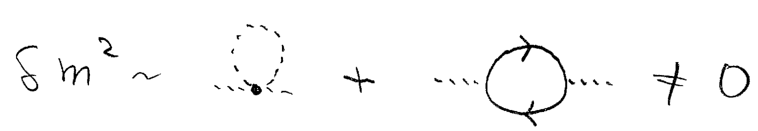
$$\underset{\substack{\uparrow \\ 0\sim3}}{A_\mu}, \ \underbrace{A_\tau, \ \underset{\text{scalar}}{\Phi \times 5}, \ \underset{\text{fermion}}{\psi \times 4}}_{\text{unwanted}}$$

→ impose $\underset{\substack{\uparrow \\ \text{fermion}}}{\Psi}(x^\mu, \tau + 2\pi M_{KK}^{-1}) = \underset{\sim}{-} \Psi(x^\mu, \tau)$

⇒ ψ become massive

⇒ ~~SUSY~~

⇒ scalar fields A_τ, $\Phi \times 5$ acquire mass via quantum effect.



⇒ 4 dim pure YM at low energy ($E \ll M_{KK}$)

☆ SUGRA side

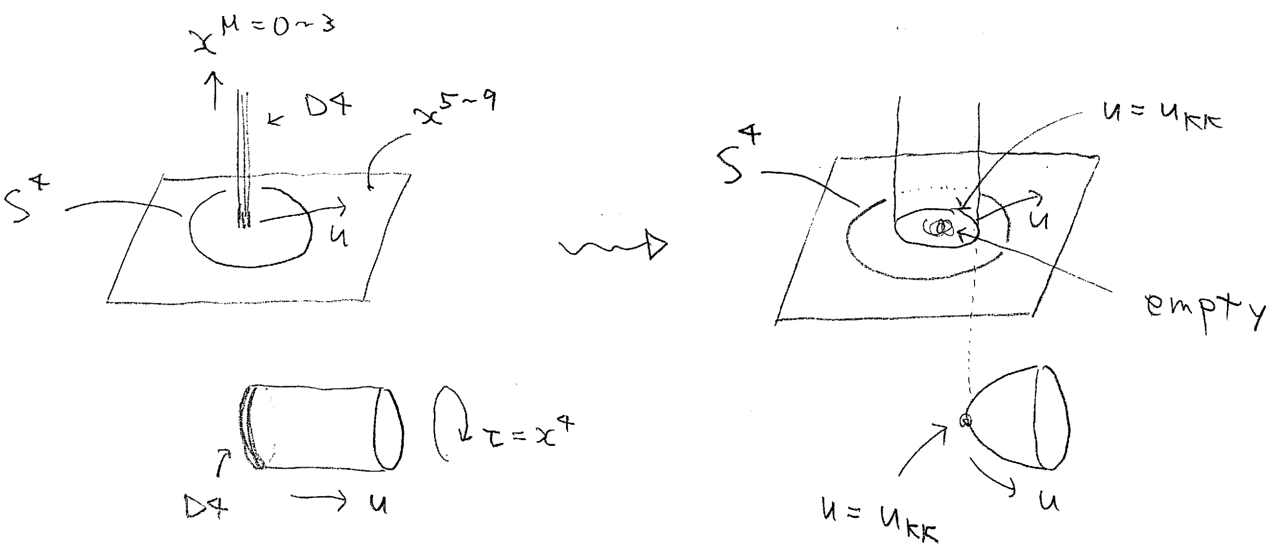
the corresponding SUGRA sol. is

$$ds^2 = H(u)^{-1/2}\left(\eta_{\mu\nu}\,dx^{\overset{0\sim 3}{\mu}}dx^\nu + f(u)\,d\overset{S^1}{\tau^2}\right) + H(u)^{1/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

$$H(u) = \frac{Q_4}{u^3} \qquad Q_4 = \pi g_s N_c l_s^3$$

$$f(u) = 1 - \frac{u_{KK}^3}{u^3} \qquad u_{KK} = \frac{4}{9} Q_4 M_{KK}^2 \qquad \boxed{\text{restricted to } u \geq u_{KK}}$$

$$e^\phi = g_s H(u)^{-\frac{1}{4}} \quad , \quad \frac{1}{2\pi}\int_{S^4} dC_3 = N_c$$



the metric looks singular at $u=u_{KK}$

but the geometry is smooth everywhere

◎ The topology of the background is

$$\boxed{\underset{\underset{x^{0\sim3}}{\uparrow}}{\mathbb{R}^{1,3}} \times \underset{\underset{x^4,\,u}{\uparrow}}{\mathbb{R}^2} \times S^4}$$

See hep-th/9905111 Aharony - Gubser - Maldacena - Ooguri - Oz

③ Construction of QCD

* outline

QCD = YM + quarks

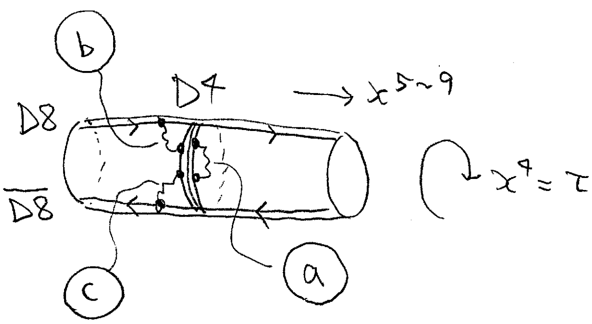
- YM → D4 on S^1 discussed in ②
- quark → add probe D8-branes

* brane configuration

$\tau \in S^1$ // x^4

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4 × N_c	○	○	○	○	○	×	×	×	×	×
D8-$\overline{D8}$ × N_f	○	○	○	○	×	○	○	○	○	○

↑ extended ↖ localized



chiral symmetry

massless modes	D4 $U(N_c)$	D8 $U(N_f)_L$	$\overline{D8}$ $U(N_f)_R$
ⓐ ⇒ A_M : gluon	adj.	1	1
ⓑ ⇒ ψ_L) quark	N_c	N_f	1
ⓒ ⇒ ψ_R) quark	N_c	1	N_f

⇒ 4 dim $U(N_c)$ QCD with N_f massless quarks is realized!

④ holographic description of QCD

* Let's apply the idea of gauge / string duality

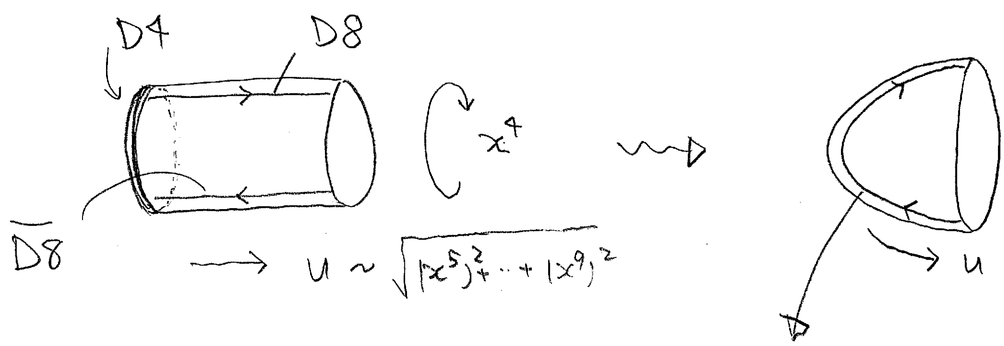
fortunately SUGRA solution corresponding
to the D4 is known (⇒ ②)

<u>Strategy</u>

ⓐ replace D4 × N_c with
the corresponding curved background.

ⓑ D8-$\overline{D8}$ × N_f are treated as probes.

"probe approximation" justified when $N_c \gg N_f$



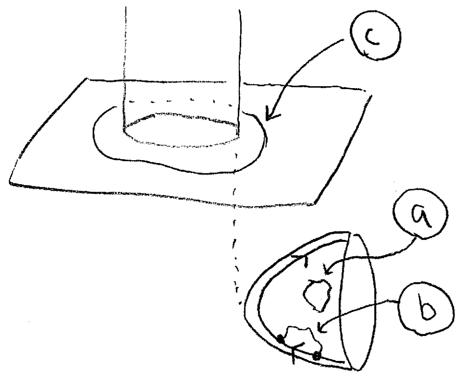
D8 and $\overline{D8}$ are
smoothly connected

⇙

Interpreted as chiral symmetry breaking !

$$\underset{D8}{U(N_f)_L} \times \underset{\overline{D8}}{U(N_f)_R} \rightarrow \underset{\text{connected } D8}{U(N_f)}$$

★ particles in the model



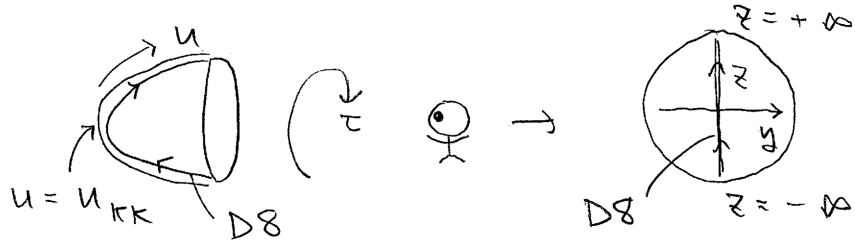
ⓐ closed string ⇒ glueballs

ⓑ open string ⇒ mesons → ⑤

ⓒ D4 wrapped on S^4

⇒ baryons → ⑥

★ effective theory of open strings on D8



$$\left(\frac{u}{u_{KK}}\right)^3 = 1 + y^2 + z^2$$

- world-volume of D8 : $\{ | x^{\mu=0\sim3}, z) \} \times S^4$
- massless modes

gauge field — scalar — fermions

A_μ, A_z, A_I, Φ, Ψ

μ: $0\sim3$, I: S^4

(adj. rep of $U(N_f)$)

[note: $SO(5) \curvearrowright S^4$

quark, gluons are invariant under $SO(5)$

⇒ we are interested in $SO(5)$ inv. modes]

$\boxed{A_\mu(x^\mu, z),\ A_z(x^\mu, z),\ \Phi(x^\mu, z)}$ ⇒ 5 dim $U(N_f)$ gauge theory

We ignore this for simplicity

※ $SO(5)$ non-inv. modes are unwanted artifacts.

- effective action

$$S_{D8} = S_{D8}^{DBI} + S_{D8}^{CS}$$

$$S_{D8}^{DBI} = T_{D8} \int d^9x \, e^{-\phi} \sqrt{-\det \left(g_{MN} + (2\pi\alpha') F_{MN} \right)}$$

$$\simeq \circledcirc \int d^9x \, e^{-\phi} \sqrt{-\det g} \, g^{MN} g^{PQ} \underbrace{F_{MP} F_{NQ}}_{\mathrm{Tr}(F_{MP}F_{NQ})}$$

integrate over S^4

insert SUGRA sol.

$\mathrm{Tr}(F_{MP}F_{NQ})$ for $N_f \geq 2$

$$\boxed{S_{D8}^{DBI} \simeq \kappa \int d^4x dz \, \mathrm{Tr}\left(\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2 \right)}$$

κ: const $\quad K(z) = 1 + z^2 \quad (M_{KK} = 1 \text{ unit})$

$C = C_1 + C_3 + \cdots$ RR fields

C_3: non-trivial

$F = \frac{1}{2} F_{MN} dx^M \wedge dx^N$

$$S_{D8}^{CS} = \int_{9\,\mathrm{dim}} C \wedge \mathrm{Tr}\, e^{F/2\pi}$$

$$= \int_{9\,\mathrm{dim}} C_3 \frac{1}{3!(2\pi)^3} \underbrace{\mathrm{Tr}\, F \wedge F \wedge F}_{= d W_5(A)}$$

$$\frac{1}{2\pi} \int_{S^4} dC_3 = N_c \quad (\star)$$

integrate over S^4 & $(\star)$

$$\boxed{S_{D8}^{CS} = \frac{N_c}{24\pi^2} \int_{5\,\mathrm{dim}} W_5(A)}$$

$$W_5(A) = \mathrm{Tr}\left(AF^2 - \frac{1}{2} A^3 F + \frac{1}{10} A^5 \right)$$

CS 5-form

- <u>Claim</u> This 5 dim YM-CS theory is the effective theory of mesons.

5 meson spectrum

Here we assume
$A_M \to 0$ at $z \to \pm\infty$
We will consider non-zero bdry values in 7

* mode expansion

$$\star \begin{cases} A_\mu(x^\mu, z) = \sum_n B_\mu^{(n)}(x^\mu)\, \psi_n(z) \\ A_z(x^\mu, z) = \sum_n \varphi^{(n)}(x^\mu)\, \phi_n(z) \end{cases}$$

complete sets of normalizable functions.

$\star$

5 dim YM-CS theory $\rightsquigarrow$ 4 dim meson theory

◎ To diagonalize $\left(\begin{array}{l} \text{kinetic terms} \\ \text{mass terms} \end{array} \right.$ of $B_\mu^{(n)}, \varphi^{(n)}$,

we choose $\{\psi_n\}_{n \geq 1}$ and $\{\phi_n\}_{n \geq 0}$

$$\text{s.t.} \begin{cases} -K^{1/3} \partial_z (K \partial_z \psi_n) = \lambda_n \psi_n \\ \kappa \int dz\, K^{-1/3} \psi_n \psi_m = \delta_{nm} \\ \phi_n = \frac{1}{\sqrt{\lambda_n}} \partial_z \psi_n \quad (n \geq 1) \\ \phi_0 = \frac{1}{\sqrt{\kappa\pi}} \frac{1}{K(z)} \end{cases}$$

eigenvalue $0 < \lambda_1 < \lambda_2 < \cdots$

$\Rightarrow \kappa \int dz\, K \phi_n \phi_m = \delta_{nm}$

5 dim YM-CS
$\downarrow$

$$\Rightarrow S_{D8} \simeq \int d^4x \sum_{n \geq 1} \mathrm{Tr}\left[\frac{1}{2} (\partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)})^2 + \lambda_n \left(B_\mu^{(n)} - \frac{1}{\sqrt{\lambda_n}} \partial_\mu \varphi^{(n)} \right)^2 \right]$$
$$+ \int d^4x\, \mathrm{Tr}\left((\partial_\mu \varphi^{(0)})^2 \right) + \left(\begin{array}{c} \text{interaction} \\ \text{terms} \end{array} \right)$$

- $B_\mu^{(n)}$ are massive vector meson $m_n^2 = \lambda_n M_{KK}^2$
- $\varphi^{(n)}$ $(n \geq 1)$ are eaten by $B_\mu^{(n)}$
- $\varphi^{(0)}$ is a massless scalar meson

◎ Parity, charge conj.

$$\left(\begin{array}{l} P: (x^1, x^2, x^3, z) \to (-x^1, -x^2, -x^3, \underset{\sim}{-z}) \\ C: A_M \to -A_M^T \quad \& \quad z \to \underset{\sim}{-z} \end{array} \right.$$

	$\varphi^{(0)}$	$B_\mu^{(odd)}$	$B_\mu^{(even)}$
J^{PC}	0^{-+} pseudo-scalar	1^{--} vector	1^{++} axial-vector

(<u>note</u> $\psi_n(-z) = \begin{cases} +\psi_n(z) & \text{for } n = \text{odd} \\ -\psi_n(z) & \text{for } n = \text{even} \end{cases}$)

☆ We interprete

$$\begin{array}{cccccc} \varphi^{(0)} & B_\mu^{(1)} & B_\mu^{(2)} & B_\mu^{(3)} & B_\mu^{(4)} & \cdots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ \pi & \rho & a_1 & \rho' & a_1' & \end{array}$$

J^{PC} is consistent.

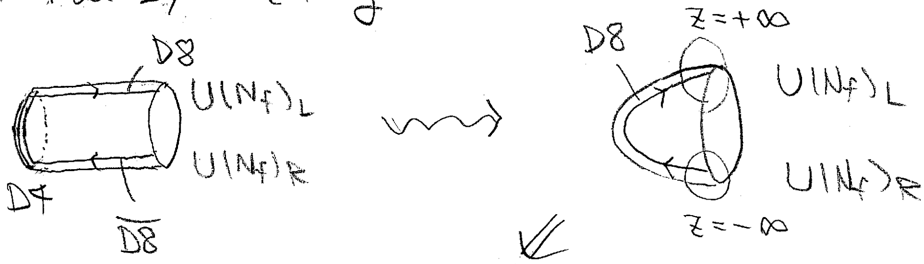
☆ mass : $m_n^2 = \lambda_n M_{KK}^2$ $\quad \lambda_n \overset{\text{Mathematica}}{\simeq} 0.669, 1.57, 2.87, 4.55 \cdots$

	ρ	a_1	ρ'	a_1'	$\cdots$
exp [MeV]	776	1230	1465	1640	
$\sqrt{\lambda_n} M_{KK}$	[input]	1189	1607	2023	
ratio	[1]	1.03	0.911	0.811	

Good! But don't trust too much.

⑥ Pion effective action

- chiral symmetry



gauge sym at $z \to \pm\infty$ is interpreted as chiral sym

$$g_\pm := \lim_{z\to\pm\infty} g(x^\mu, z) \qquad (g_+, g_-) \in U(N_f)_L \times U(N_f)_R$$

- consider

$$\boxed{U(x^\mu) \equiv P\, e^{-\int_{-\infty}^{\infty} dz' A_z(x^\mu, z')}} \in U(N_f)$$

gauge tr. $A_M \to A_M^g \equiv g A_M g^{-1} + g \partial_M g^{-1}$

$$\Rightarrow \quad \underbrace{U(x) \to g_+ U(x^\mu) g_-^{-1}}$$

interpreted as the pion field in chiral Lagrangian

$$U(x) \equiv e^{2i\Pi(x)/f_\pi} \qquad f_\pi : \text{pion decay const.}$$

note $\quad A_z = \varphi^{(0)}(x) \underbrace{\phi_0(z)}_{= \frac{1}{\sqrt{\pi\kappa}} \cdot \frac{1}{1+z^2}} + \varphi^{(1)}(x) \underbrace{\phi_1(z)}_{\propto \partial_z \psi_1(z)} + \cdots$

$$\Rightarrow \int dz\, A_z = \varphi^{(0)}(x) \sqrt{\frac{\pi}{\kappa}}$$

$$\Rightarrow \quad \Pi(x) = i\varphi^{(0)}(x) + (\text{non-linear term})$$

with $\quad f_\pi = 2\sqrt{\frac{\kappa}{\pi}}\, M_{KK}$

• $A_z = 0$ gauge

Consider ...

$$A_M^g \equiv g A_M g^{-1} + g \partial_M g^{-1} \quad \text{with } g^{-1}(x^\mu, z) = P e^{-\int_{-\infty}^{z} dz' A_z(x,z')}$$

$\uparrow_{0\sim 3, z}$

$$\Rightarrow \quad A_z^g = 0$$

Then $$\underset{\uparrow 0\sim 3}{A_\mu^g} \longrightarrow \begin{cases} U^{-1}\partial_\mu U & (z \to +\infty) \\ 0 & (z \to -\infty) \end{cases}$$

mode expansion with this b.c.

$$A_\mu^g (x^\mu, z) = U^{-1}\partial_\mu U (x^\mu) \psi_+(z) + \sum_{n\geq 1} B_\mu^{(n)}(x) \psi_n(z)$$

$$\psi_+(z) \equiv \frac{1}{2} + \frac{1}{\pi}\arctan z \to \begin{cases} 1 & (z \to +\infty) \\ 0 & (z \to -\infty) \end{cases}$$

$$S_{D8}^{DBI} \sim \int d^4x \, \mathrm{Tr}\left(\frac{f_\pi^2}{4}(U^{-1}\partial_\mu U)^2 + \frac{1}{32 e_S^2}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 + \cdots \right)$$

$$f_\pi^2 = \frac{4}{\pi}\kappa M_{KK}^2, \quad e_S^{-2} = 16\kappa \int dz\, K^{-\frac{1}{3}} \psi_+^2 (\psi_+ - 1)^2 \simeq 2.51\kappa$$

This is Skyrme model !

$$\frac{1}{32 e_S^2}\mathrm{Tr}\left([U^{-1}\partial_\mu U, U^{-1}\partial_\mu U]^2\right) = L_1 P_1 + L_2 P_2 + L_3 P_3 \quad \text{for } N_f = 3$$

$$P_1 \equiv \left[\mathrm{Tr}\, \partial_\mu U^{-1} \partial^\mu U\right]^2, \; P_2 \equiv \left(\mathrm{Tr}\, \partial_\mu U \partial_\nu U^{-1}\right)^2, \; P_3 \equiv \mathrm{Tr}\left((\partial_\mu U \partial^\mu U^{-1})^2\right)$$

$$L_1 = \frac{1}{32 e_S^2}, \quad L_2 = \frac{1}{16 e_S^2}, \quad L_3 = -\frac{3}{16 e_S^2}$$

hep-ph/9502366 $\quad \simeq 0.584 \times 10^{-3} \quad \simeq 1.17 \times 10^{-3} \quad \simeq -3.51 \times 10^{-3}$

$\hookrightarrow$ exp. $L_1 \simeq (0.4 \pm 0.3) \times 10^{-3}, \; L_2 \simeq (1.4 \pm 0.3) \times 10^{-3}, \; L_3 \simeq (-3.5 \pm 1.1) \times 10^{-3}$

$(\mu = m_\rho)$

⑦ interaction with vector mesons

★ ρ-π-π , a_1-ρ-π coupling

$$S_{D8}^{DBI} \sim \kappa \int d^4x\, dz\, \mathrm{Tr}\Big(\cdots + K(z) \underbrace{F_{\mu z}^2}_{\downarrow} \Big)$$

$$[A_\mu, A_z](\partial^\mu A_z - \partial_z A^\mu)$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$B_\mu^{(n)}\psi_n \quad \varphi^{(0)}\phi_0 \quad \varphi^{(0)}\phi_0 \quad B_\mu^{(m)}\psi_m$$

$$\sim \int d^4x \Big(\cdots + 2 g_{v^n\pi\pi}\, \mathrm{Tr}\big(v^n_\mu [\pi, \partial^\mu \pi]\big) + 2i\, g_{a^m v^n \pi}\, \mathrm{Tr}\big(a^m_\mu [\pi, v^{n\mu}]\big) + \cdots \Big)$$

where

$$\pi(x) = i\,\varphi^{(0)}(x) , \quad v^n_\mu(x) \equiv B_\mu^{(2n-1)}(x), \quad a^n_\mu(x) \equiv B_\mu^{(2n)}(x)$$

vector — axial-vector

$$g_{v^n\pi\pi} = \kappa \int dz\, K \psi_{2n-1} \phi_0^2 = \frac{1}{\pi}\int dz\, K^{-1} \psi_{2n-1}$$

$$g_{a^m v^n \pi} = 2\kappa \int dz\, K \phi_0 \psi_{2m} \partial_z \psi_{2n-1} = f_\pi \int dz\, \psi_{2m} \partial_z \psi_{2n-1}$$

★ Coupling to external $U(N_f)_L \times U(N_f)_R$ gauge field

Consider non-zero boundary values

$$A_\mu(x^\mu, z) \longrightarrow \begin{cases} A_{L\mu}(x) & (z \to +\infty) \\ A_{R\mu}(x) & (z \to -\infty) \end{cases}$$

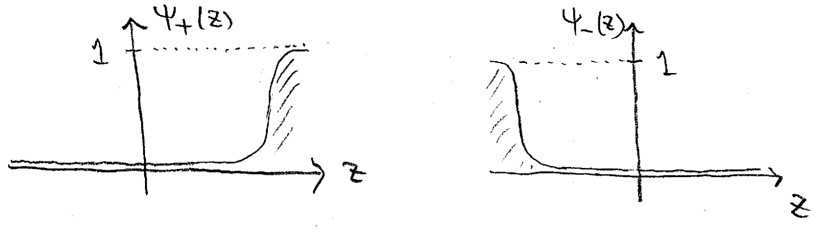
(A_L, A_R) is interpreted as $U(N_f)_L \times U(N_f)_R$ gauge field

※ The kinetic terms of these fields are not normalizable ⇒ considered as external fields

• mode expansion satisfying the boundary cond.

$$A_\mu(x,z) = A_{L\mu}(x)\,\psi_+(z) + A_{R\mu}\,\psi_-(z) + \sum_{n\geq 1} B^{(n)}_\mu(x)\,\psi_n(z)$$

$$\psi_\pm(z) \to \begin{cases} 1 & (z\to\pm\infty) \\ 0 & (z\to\mp\infty)\end{cases}$$



$$S^{DBI}_{D8} \sim \kappa \int d^4x\, dz\, \mathrm{Tr}\left(\cdots + \underbrace{K(z)}_{=\,1+z^2}\, \underbrace{F_{\mu z}^2}_{\partial_z A_\mu\, \underbrace{\partial_z A^\mu}_{B^{(n)\mu}\underbrace{\partial_z\psi_n}_{\sim\, O(z^{-2})}} + \cdots}\right)$$

$$\sim \int d^4x \left(\cdots + 2\, g_{v^n}\, \mathrm{Tr}(v^n_\mu\, \mathcal{V}^\mu) + \cdots\right)$$

$$\mathcal{V}_\mu \equiv \frac{1}{2}(A_{L\mu} + A_{R\mu}), \quad g_{v^n} = \kappa\left[K\,\partial_z\psi_{2n-1}\right]_{z=-\infty}^{z=+\infty}$$

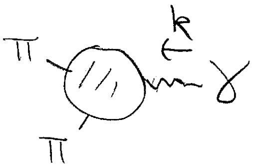
★ numerical values

	$g_{\rho\pi\pi}$	$g_{a_1\rho\pi}$	g_ρ
exp.	5.99	2.8 ~ 4.2 GeV	0.121 GeV^2
our model	4.81	4.63 GeV	0.164 GeV^2

here $m_\rho = \sqrt{\lambda_1}\, M_{KK} \leftrightarrow 776$ MeV, $f_\pi = 2\sqrt{\frac{\kappa}{\pi}}\, M_{KK} \leftrightarrow 92.4$ MeV

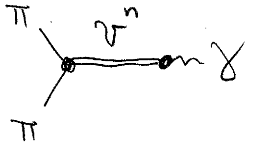
are used to fix M_{KK} and κ

- Pion form factor



$= F_\pi(k^2)$

It turns out that this is given entirely by vector meson exchange diagrams.

$F_\pi(k^2) =$ 

$= \sum_{n\geq 1} \frac{g_{v^n} g_{v^n\pi\pi}}{k^2 + m_{v^n}^2}$

⇒ vector meson dominance !

(proposed in the '60s

$F_\pi(k^2) = \frac{g_\rho g_{\rho\pi\pi}}{k^2+m_\rho^2}$ agrees well with experiments)

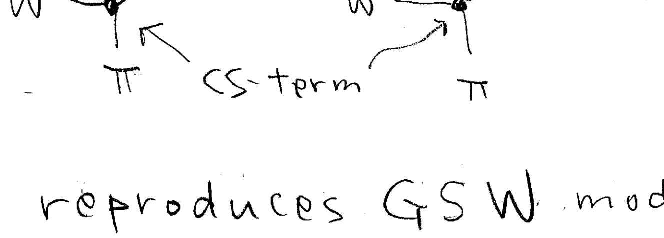
charge radius

$$\langle r^2\rangle \equiv -6 \frac{\partial}{\partial(k^2)} F_\pi(k^2)\Big|_{k^2=0} \simeq \begin{cases} (0.690\,\mathrm{fm})^2 & \text{our model.} \\ (0.672\,\mathrm{fm})^2 & \text{exp} \end{cases}$$

- $\omega \to \pi^0\gamma$, $\omega\to\pi^0\pi^+\pi^-$

$$\rho_M = \frac{1}{\sqrt2}\begin{pmatrix} \frac{1}{\sqrt2}(\omega_M+\rho_M^0) & \rho_M^+ \\ \rho_M^- & \frac{1}{\sqrt2}(\omega_M-\rho_M^0)\end{pmatrix}$$

relevant diagrams



(No direct couplings

)

⇒ reproduces GSW model (1962) !

G: Gell-Mann, S: Sharpe, W: Wagner

8 baryon

☆ Skyrme (1961)

$$\boxed{\text{Baryon} \simeq \text{soliton in Skyrme model (Skyrmion)}}$$

Consider static configuration

pion field $U(\vec{x}) : S^3 = \underset{\vec{x}}{\underset{\Psi}{\mathbb{R}^3}} \cup \{\infty\} \to U(N_f)$

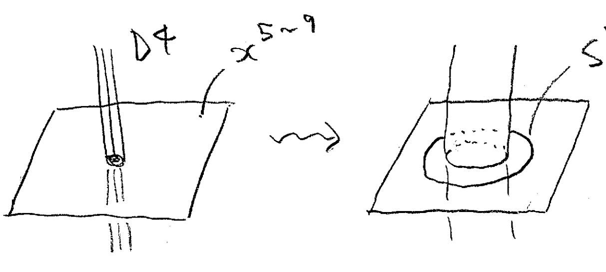
$\Rightarrow$ classified by $\pi_3(U(N_f)) \simeq \mathbb{Z}$

winding number $= \frac{1}{24\pi^2} \int_{S^3} \mathrm{Tr}((UdU^{-1})^3)$

$\rightsquigarrow$ interpreted as baryon number

☆ In our model,

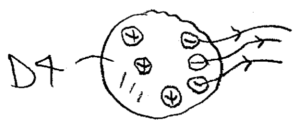
$$\boxed{\text{Baryon} \simeq \text{D4 wrapped on } S^4}$$



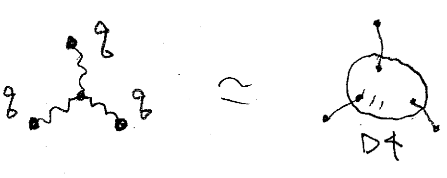
$$S^{CS}_{D4} = \int_{\mathbb{R}\times S^4} C_3 \wedge \frac{F^{D4}}{2\pi} = N_c \int_{\mathbb{R}} A^{D4}$$

$\Downarrow$

Source of N_c ele charge



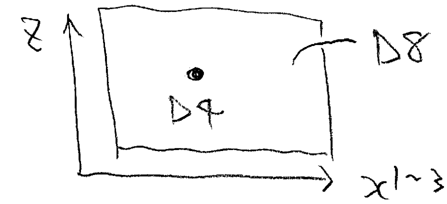
} N_c F1 should be attached

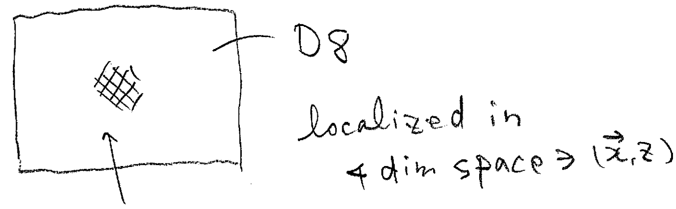


interpreted as a bound state of N_c quarks.

※ This D4 is embedded in D8.

D4 within D8 = "instanton" on D8





$$\frac{1}{8\pi^2}\int \mathrm{Tr}\, F\wedge F = 1$$

$\Rightarrow$ $\boxed{\text{Baryon} = \text{"instanton" in 4 dim space in the 5 dim YM-CS theory}}$

$\#$ baryon $= \#$ D4 $= \frac{1}{8\pi^2}\int \mathrm{Tr}\, F^2$: instanton $\#$

◎ relation to Skyrmion

Recall Skyrme model is derived in $A_z = 0$ gauge

with $A^g_\mu \to \begin{cases} U^{-1}\partial_\mu U & (z\to+\infty) \\ 0 & (z\to-\infty)\end{cases}$ (See ⑥)

$$\Rightarrow \frac{1}{8\pi^2}\int_{S^3\times\mathbb{R}} \underbrace{\mathrm{Tr}\, F^2}_{\parallel \atop dw_3} = \frac{1}{8\pi^2}\int_{S^3\times\mathbb{R}} dw_3 = \frac{1}{8\pi^2}\int_{S^3} w_3\Big|_{z=+\infty} = \frac{1}{24\pi^2}\int_{S^3}\mathrm{Tr}\left((UdU^{-1})^3\right) \quad //$$

$$w_3 = \mathrm{Tr}\left(AF - \tfrac{1}{3}A^3\right) : \text{CS 3-form}$$

$\therefore$

Baryon

Skyrmion $\simeq$ instanton on D8
$\qquad\qquad\qquad\qquad$ $\wr\wr$
$\qquad\qquad\qquad$ D4 on S^4 $\simeq$ bound state of N_c quarks