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Spring School on Superstring Theory and Related Topics

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Holography and strongly coupled model building

Lecture 2

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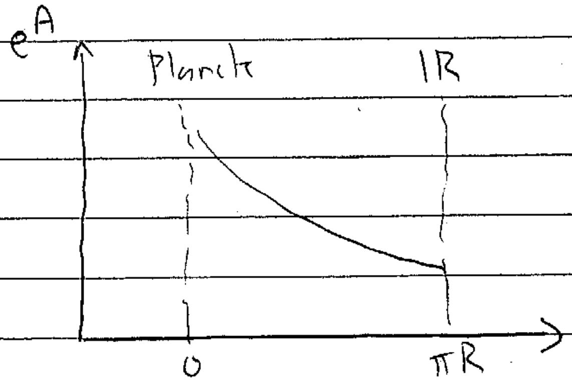
Trieste '09, Lecture II — Kachru

Today, we want to discuss how this picture



$$ds^2 = e^{-2kX_5}\eta_{\mu\nu}dX^\mu dX^\nu + dX_5^2$$

can be roughly realized in IIB string theory.

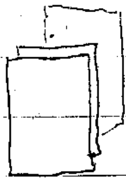
- Bulk will be SUSY
- We'll put $\cancel{\text{SUSY}}$ on IR brane, eventually

II. A "confining" gravity dual

To realize this picture, we need to deal with both the IR cutoff & the UV cutoff.

a) AdS/CFT at the conifold

We already know that



$$\text{N D3s @ smooth pt} \xrightarrow[\text{at low E}]{\text{open strings}} \mathcal{N}=4 \text{ SYM}$$

$$\text{N D3s @ smooth pt} \xrightarrow[\text{horizon}]{\text{near}} AdS_5 \times S^5$$

<u>DUAL</u>

How to get <u>OTHER EXAMPLES</u>? <u>Any</u> smooth pt looks the <u>same</u> in the near-horizon limit. So, must place D3s @ a singular pt!

Perhaps the simplest next case is to consider D3s @ <u>orbifold</u> singularities. But more useful for us will be the <u>conifold</u>: cone over $S^3 \times S^2$

$$\sum z_i^2 = 0 \subset \mathbb{C}^4$$

This arises in many compact Calabi-Yau spaces.

Change of variables $\Rightarrow$

$$(\checkmark) \qquad z_1 z_2 - z_3 z_4 = 0$$

We can <u>solve</u> $(\checkmark)$:

$z_1 = A_1 B_1 \qquad z_2 = A_2 B_2 \qquad z_3 = A_1 B_2 \qquad z_4 = A_2 B_1$

But you get the <u>same</u> z_i if you act by:

$$A_k \to \lambda A_k, \quad B_k \to \lambda^{-1} B_k \qquad \forall \lambda \in \mathbb{C}^*$$

- Writing $\lambda = s e^{i\alpha}$ $\quad s \in \mathbb{R}^+$,

can fix s by setting (<u>away</u> from $z_i = 0$ node)

$(\square)$ $$|A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2$$

- To get the conifold, must // by the $U(1)$

$(\bigcirc)$ $$A_k \to e^{i\alpha} A_k \qquad B_k \to e^{-i\alpha} B_k$$

- $\exists$ an $SU(2) \times SU(2)$ symmetry: one for As, one for Bs

<u>Dual QFT to N D3 @ Conifold</u> (Klebanov, Witten)

Consider the $N=1$ SUSY gauge theory with $U(1)$ gauge group + matter content:

Field	Charge
$A_{1,2}$	$+1$
$B_{1,2}$	-1

D-term eqn for moduli space: $|A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2$ same as (D)

Then $\mathcal{M}_{vacua} = \{D = 0\} / U(1) \longrightarrow$ same as (0)

So this gauge theory gives conifold as its moduli space of vacua.

- This $U(1)$ is Higgsed, but a D3 would have an unbroken worldvol. $U(1)$ $\rightarrow$ want

	$U(1)$	$U(1)$
$A_{1,2}$	$+1$	-1
$B_{1,2}$	-1	$+1$

COM $U(1)$ decouples; difference as above. ✓

This gives 1 D3 moduli space on conifold.

- N D3s? Natural guess

	$U(N)$	$U(N)$
$A_{1,2}$	N	$\bar{N}$
$B_{1,2}$	$\bar{N}$	N

} ~~has the $SU(2)\times SU(2)$ symmetry~~ ??

- No renormalizable superpot W possible $\to$ $W \overset{??}{=} 0$

- Diagonalize A, B w/ distinct eigenvalues $\to$ family of vacua w/ N D3s @ distinct pts on conifold; and $G = U(1)^N$. But, $\exists$ massless charged chirals. So we need a W!

- Lowest order single Tr $SU(2)^2$ invariant:

$$\boxed{W = \frac{\lambda}{2}\,\epsilon^{ij}\epsilon^{kl}\,\mathrm{Tr}\,A_i B_k A_j B_l}$$

This does the job.

The CFT is strongly coupled: $R_A = R_B = 1/2$

$\Rightarrow$ $\Delta(A) = \Delta(B) = 3/4$, not 1 (large anomalous dimension)

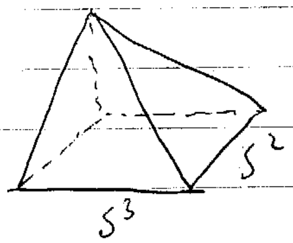
Moduli of CFT: Gauge thy λ of $\Lambda_1, \Lambda_2, \lambda$

String theory: τ_{IIB} (axio-dilaton)

conifold = cone over $S^3 \times S^2$ ← periods of B_2, C_2

b) 'Perturbing' to get confining theory (Klebanov, Strassler)

Since the conifold is a cone over $S^3 \times S^2$



} Can consider having N D3s, M D5s on S^2

Resulting gauge theory:

	$SU(N+M)$	$SU(N)$
$A_{1,2}$	$N+M$	$\bar{N}$
$B_{1,2}$	$\overline{N+M}$	N

(W as before in conifold theory)

Dynamics?

QFT Side: "Cascade of Seiberg dualities"

Seiberg duality:

$$SU(N_c) \xrightarrow[\text{coupling}]{\text{strong}} SU(N_f - N_c) + \text{"mesons"}$$

N_f flavors $\qquad\qquad$ N_f flavors

Here the gauge factor that runs to strong coupling is the $SU(N+M)$ [relatively less N_f than N_c compared to other factor]

$$\text{"}N_c\text{"} = N + M \qquad \text{"}N_f\text{"} = 2N$$

$\rightarrow$ dual group $= SU(2N - (N+M)) = SU(N-M)$

So $\quad SU(N+M) \times SU(N) \rightarrow SU(N) \times SU(N-M)$

You can check that the field content is also self-similar, as is W ("mesons" massive!) at each step $\rightarrow$ cascade w/ N to $N-M$ at each step.

End of cascade? Say $\underline{N = kM}$.

Then eventually, you reach a step:

$$SU(2M) \times SU(M) \rightarrow SU(M)$$

Final "N_f" $= 0$ → pure gauge theory!

Pure $SU(M)$ $\mathcal{N}=1$ QFT →

- $W = \Lambda^3_{SU(M)}$
- M vacua (Witten index M), related by phase rotations of Λ

<u>Gravity Side</u>:

KS worked out metric etc (we'll see later), but essential physics is as follows.

<u>Conifold</u>: $\sum_{i=1}^{4} z_i^2 = 0$

<u>Deformed Conifold</u>: $\sum_{i=1}^{4} z_i^2 = \varepsilon^2$

M D5s had $\int_{S^3} F_3^{RR} = M$

around D5s

They undergo a "Geometric transition". The deformed conifold has a finite sized S^3 A:

$$A: \quad \sum_{i=1}^{4} (\mathrm{Re}\, z_i)^2 = \varepsilon^2$$

+ its dual (non-compact) B-cycle.

- $$\int_A F_3 = M \quad \text{after transition.}$$

- Also, $\exists$ N units of D3 charge before.

$$F_5 = dC_4 + B_2 \wedge F_3 - C_2 \wedge H_3$$

$$dF_5 \simeq N_{D3} + H_3 \wedge F_3$$

So if we let $B \sim S^2 \times$ radial direction of conifold cone

$$\int_A F_3 = M \rightarrow \text{if} \int_B H_3 = k \qquad (N = kM)$$

We also see D3 charge matches. (if $N \neq kM$ for $k \in \mathbb{Z}^+$, must be probe D3s left ...).

Physics of $\int_A F_3 = M$, $\int_B H_3 = k$?

Fluxes $\Rightarrow$ superpot. for moduli of the Calabi-Yau.

Suppose Ω = holomorphic 3-form

$$\int_A \Omega = z \qquad \int_B \Omega = \frac{z}{2\pi i}\log(z) + \text{regular}$$

[$\to$ conifold monodromy: as $z \to e^{2\pi i} z$, $A \to A$, but $B \to B + A$ ...]

Here $z \sim \varepsilon^{\text{power}}$; A is a vanishing cycle as one approaches conifold pt $\varepsilon \to 0$, $z \to 0$.

Then: $W = \int (F - \tau H_3) \wedge \Omega \Rightarrow$

$$W = k\tau z + \frac{M}{2\pi i}\left[z \log(z) + \cdots\right]$$

$D_z W = 0$ (using $K \sim -\log\left(\int \Omega \wedge \bar{\Omega}\right) + \cdots$

$$\Rightarrow \boxed{z \simeq \exp\left[-\frac{2\pi k}{g_s M}\right]}$$

} actually M vacua related by phase of z

Z = A-cycle volume $\Rightarrow$ near conifold pt in moduli space! (But, warp factor keeps the SUGRA approximation valid...).

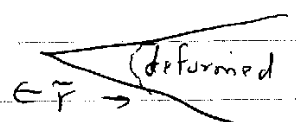
Fluxes backreact on metric & → <u>warping</u>. How much? If

$$ds^2 = e^{2A(y)} \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{-2A} \tilde{g}_{mn}(y)\, dy^m dy^n$$

N D3s $\rightarrow$ $e^{-4A} = \dfrac{4\pi g_s N}{\tilde{r}^4}$ ← distance to D3s in $\tilde{g}$ metric

A <u>fact</u> about the conifold geometry is that the distance from the tip, $\tilde{r}$, satisfies

$$\tilde{r}_{min} \simeq Z^{1/3}$$ } Candelas, de la Ossa



$$\simeq e^{-2\pi K/3g_s M}$$

$$\Rightarrow \quad e^A\Big|_{min} \simeq e^{-2\pi K/3g_s M}.$$

Warped IR scale = scale of gluino condensate in the $\mathcal{N}=1$ pure YM @ end of cascade!

c) The IR geometry

So, what is the IR SUGRA sol'n?

- The metric on the conifold is deformed by fluxes:

$$\sum z_i^2 = \varepsilon^2$$

$$\bullet \quad ds^2 = \overset{\frac{\varepsilon^{4/3}}{g_s M}}{a_0^2}\, dX_\mu\, dX^\mu + g_s M\, \underset{O(1)}{b_0^2} \left(\tfrac{1}{2} dr^2 + d\Omega_3^2 + r^2\, \underset{\substack{S^2 \text{ collapses} \\ \text{at tip}}}{d\tilde{\Omega}_2^2} \right)$$

And,

$$\int_{S^3} F = M \quad \rightarrow \quad (\text{big } S^3)$$

$$F_3 \sim f\, \epsilon_{ijk}$$

where
$$f \simeq \frac{\alpha}{\sqrt{g_s^3 M}\; b_0}$$

Next time: ~~SUSY~~ states in this geometry, in open & closed string descriptions.