The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2024-6

Spring School on Superstring Theory and Related Topics

23 - 31 March 2009

Holographic Superconductors and Superfluids

Lecture 2

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Field Theory Techniques for Strongly Interacting Systems

(Transport Coefficients)

- set up a framework in which we can use the AdS/CFT results

- consider the response of a system to the presence of weak external fields $\{\phi_i(x)\}$ coupled to a set of operators $\{\hat{\mathcal{O}}^i(x)\}$

$$\delta \hat{H} = - \int d^3\vec{x}\ \phi_i(t,\vec{x})\, \hat{\mathcal{O}}^i(t,\vec{x})$$

- worthwhile exercise (starting from Schrödinger's eqn.) to show

$$\delta \langle \hat{\mathcal{O}}^i(x) \rangle = \int d^4x'\ G_R^{ij}(x,x')\, \phi_j(x') + O(\phi^2)$$

$$G_R^{ij}(x,x') = i\,\theta(t-t') \langle [\hat{\mathcal{O}}^i(x), \hat{\mathcal{O}}^j(x')] \rangle$$

- Fourier transformed version

$$\delta \mathcal{O}^i(k) = \tilde{G}_R^{ij}(k)\, \tilde{\phi}_j(k) + O(\phi^2)$$

$$\tilde{G}_R^{ij}(k) = \int d^4x\ e^{-ikx}\, G_R^{ij}(x,0)$$

- think about Ohm's Law in this context (for a spatially homogenous current, $\vec{k} = 0$)

$\hat{\mathcal{O}}^i(k) \to \vec{J}(\omega)$, $\tilde{\phi}_i(k) \to \vec{A}(\omega)$

$\Rightarrow J^i(\omega) = \tilde{G}_R^{ij}(\omega,\vec{0}) \cdot A_j(\omega)$

vs.
$$J^i(\omega) = \sigma^{ij}(\omega) \cdot E_j(\omega)$$

- choose a gauge $A_t = 0$, $E_x = -\partial_t A_x$
- make a Fourier decomposition, $A_x \sim e^{-i\omega t}$

$$E_x = i\omega A_x$$

$$\Rightarrow \sigma^{ij}(\omega) = \frac{\tilde{G}_R^{ij}(\omega,\vec{0})}{i\omega}$$

σ related to $\langle JJ \rangle$

$\hat{O}^i(x)$	$\phi_i(x)$	
J^μ	A_μ	Ohm's Law
$T^{\mu\nu}$	$g_{\mu\nu}$	heat conductivity

emphasize: weak externally imposed fields
no dynamics – no photon, no graviton

how do we get heat conductivity?
response of a system to a thermal gradient $\partial_i T$

- in Euclidean signature $0 \le \tau < \frac{1}{T}$, $g_{00} = -\frac{T_0^2}{T(x)^2}$

$$\partial_i g_{00} = 2\frac{T_0^2}{T^2}\frac{\partial_i T}{T} \approx 2\frac{\partial_i T}{T}$$

- consider an infinitesimal coordinate transformation

$$\delta g_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \quad \text{s.t. } \partial_i(g_{00} + \delta g_{00}) = 0$$

set $\xi_i = 0 \Rightarrow \partial_i g_{00} + 2\partial_i\partial_0 \xi_0 = 0 \Rightarrow \partial_i \xi_0 = \frac{\partial_i T}{i\omega T}$

$$\delta g_{0i} = \partial_i \xi_0 = \frac{\partial_i T}{i\omega T}$$

$$\langle T^{0j}\rangle = \tilde{G}_R^{0j,0i}(\omega)\,\delta g_{0i}(\omega) = \frac{\tilde{G}_R^{0j,0i}}{i\omega}\frac{\partial_i T}{T}$$

identify $K_{ij} = \frac{-\tilde{G}_R^{0i,0j}}{i\omega T}$ w/ a heat conductivity

- chemical pot'al $\hat{H} \to \hat{H} - \mu\hat{N}$

for J^μ, A_μ, $\delta\hat{H} = -\int d^3\vec{x}\, A_\mu(x)\hat{J}^\mu(x)$

$\Rightarrow$ a const $A_t = \mu$ acts like a chemical pot'al

- more generally

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \overleftrightarrow{\sigma} & \overleftrightarrow{\alpha}\, T \\ \overleftrightarrow{\alpha}'\, T & \overleftrightarrow{\kappa}\, T \end{pmatrix} \begin{pmatrix} \vec{E} \\ -(\vec{\nabla} T)/T \end{pmatrix}$$

$Q^{\nu} = T^{0\nu} - \mu J^{\nu}$ is the heat current

$\overleftrightarrow{\alpha}$ thermoelectric coefficient

Onsager relations: $\overleftrightarrow{\alpha} = \overleftrightarrow{\alpha}'$ (symmetry under time reversal)

- there are related quantities that can be derived from $\overleftrightarrow{\sigma}, \overleftrightarrow{\alpha}, \overleftrightarrow{\kappa}$
  - Nernst effect $\vec{E}$ in response to $\vec{\nabla} T$ when $\vec{J} = 0$

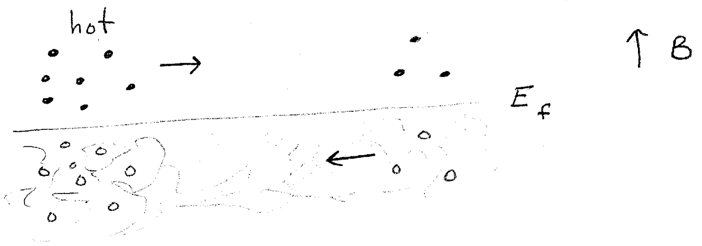
$$\overleftrightarrow{\theta} = -\overleftrightarrow{\sigma}^{-1} \cdot \overleftrightarrow{\alpha}$$

  - canonical heat conductivity $\vec{Q}$ in response to $\vec{\nabla} T$ w/ $\vec{J} = 0$

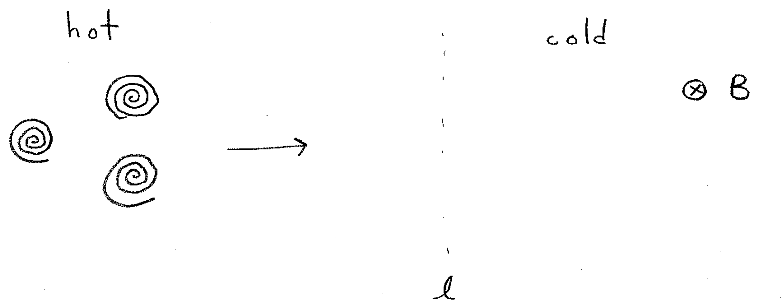
$$\overleftrightarrow{\kappa}' = \overleftrightarrow{\kappa} - T\, \overleftrightarrow{\alpha} \cdot \overleftrightarrow{\sigma}^{-1} \cdot \overleftrightarrow{\alpha}$$

Ong et al. have measured an anomalously large Nernst effect in the high T_c cuprate superconductors

- expect θ_{xy} to be very small due to Sondheimer cancellation



- if the principle degrees of freedom are vortices, Nernst effect can be much larger



- vortex cores have entropy which cause them to move in response to a thermal gradient
- a vortex moving past the line l causes a phase slip of 2π
- phase slippage induces a voltage $\partial_\mu \phi + A_\mu$

- if we are interested in $\overleftrightarrow{\sigma}, \overleftrightarrow{\alpha}, \overleftrightarrow{\kappa}$, we should calculate $\langle JJ \rangle$, $\langle TJ \rangle$, $\langle TT \rangle$
  - AdS/CFT will provide a method to do so
  - there are some constraints on the form of these correlators that follow from field theory considerations alone – Ward identities

in 2+1 dim.s, introducing $\sigma_\pm = \sigma_{xy} \pm i\sigma_{xx}$, $\alpha_\pm$, $\kappa_\pm$

$$\pm \alpha_\pm T\omega = (B \mp \mu\omega)\sigma_\pm - \rho$$

$$\pm \kappa_\pm T\omega = \left(\frac{B}{\omega} \mp \mu\right)\alpha_\pm T\omega - sT + mB$$

ρ charge density

s entropy density

$\Rightarrow$ knowing σ means we know α, κ, θ, and κ'!

Ward identities

- start w/ a gen'ating fn'al for Euclidean time ordered correlators

$$e^{W[g,A]} \equiv Z[g,A] = \int D\phi \; e^{-S[\phi,g,A]}$$

g metric, A abelian vector field are external and non-dynamical

- define $\langle J^\mu(x)\rangle = \dfrac{\delta W}{\delta A_\mu(x)}$, $\langle T^{\mu\nu}\rangle = 2\,\dfrac{\delta W}{\delta g_{\mu\nu}(x)}$, etc.

  densities rather than fields

- assume $W[g,A]$ invariant under

$$x^\mu \to x^\mu + \xi^\mu \qquad \text{diffeo}$$

$$A_\mu \to A_\mu + \partial_\mu f \qquad \text{gauge}$$

- under diffeo

$$\delta g_{\mu\nu} = (\mathcal{L}_\xi g)_{\mu\nu} = g_{\mu\lambda}\,\xi^\lambda{}_{,\nu} + g_{\nu\lambda}\,\xi^\lambda{}_{,\mu} + g_{\mu\nu,\lambda}\,\xi^\lambda$$

$$\delta A_\mu = (\mathcal{L}_\xi A)_\mu = A_\nu\,\xi^\nu{}_{,\mu} + A_{\mu,\nu}\,\xi^\nu$$

$$0 = \int d^dx \left(\frac{\delta W}{\delta g_{\mu\nu}(x)} (\mathcal{L}_\xi g)_{\mu\nu} + \frac{\delta W}{\delta A_\mu(x)} (\mathcal{L}_\xi A)_\mu \right)$$

short exercise $\Rightarrow$ $g_{\nu\lambda}\left(\partial_\mu \langle T^{\mu\nu}(x)\rangle + \Gamma^\nu_{\mu\rho}\langle T^{\mu\rho}(x)\rangle\right) - F_{\mu\lambda}\langle J^\mu(x)\rangle = 0$

in flat space $\to$ $\partial_\mu\langle T^{\mu\nu}(x)\rangle = F^{\mu\nu}\langle J_\mu(x)\rangle$

similarly gauge invariance $\Rightarrow$ $\partial_\mu\langle J^\mu(x)\rangle = 0$

- the 2 pt. Ward identities are what we're after

$$0 = -k_\mu \tilde{G}_R^{\alpha,\mu\nu}(k) + i F_\mu{}^\nu \tilde{G}_R^{\alpha,\mu}(k) + k^\nu \langle J^\alpha \rangle - \eta^{\alpha\nu} k_\mu \langle J^\mu \rangle$$

$$0 = k_\mu \left(\tilde{G}_R^{\alpha\beta,\mu\nu}(k) + \eta^{\alpha\nu} \langle T^{\beta\mu} \rangle + \eta^{\beta\nu} \langle T^{\alpha\mu} \rangle - \eta^{\mu\nu} \langle T^{\alpha\beta} \rangle \right)$$
$$+ i \eta^{\beta\nu} F_\mu{}^\alpha \langle J^\mu \rangle + i \eta^{\alpha\nu} F_\mu{}^\beta \langle J^\mu \rangle - i F_\mu{}^\nu \tilde{G}_R^{\alpha\beta,\mu}(k)$$

- to get these id.s, we used the fact that $\tilde{G}_R(k)$ is the analytic continuation of $\tilde{G}_E(k)$

- to obtain the relations between σ, α, and K

set $k^\mu = (\omega, \vec{0})$ $\quad F_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix}$ $\quad J^\mu = (\rho, \vec{0})$

$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$ $\quad$ $A_t = \mu$