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Ionospheric Data Processing and Analysis

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Ionospheric Data Processing and Analysis

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Workshop on Satellite Navigation Science and Technology for Africa

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Outline

This talk is a tutorial on Total Electron Content (TEC) estimation using a GPS receiver

- Describe the GPS observables and the various linear combinations used to estimate TEC
- Demonstrate estimation and removal of the instrumental biases using several techniques
  - A simple technique (setting the minimum value of TEC at night)
  - A least squares approach (minimizing the variance of the vertical-equivalent TEC)
  - Kalman filter estimation of total TEC
- Discuss the influence of ionospheric structure on the GPS TEC
- Discuss the influence of the plasmasphere on the GPS TEC
  - A model for the plasmaspheric contribution to the total TEC
  - Kalman filter estimation of ionospheric and plasmaspheric TEC
- Concluding remarks

The GPS Observables and TEC

The Pseudorange Observation Equations

The Pseudorange Observation Equations:

$$P_1 = \rho + c\left(\Delta t_r - \Delta t_s\right) + I_1 + T + b_{1r}^P + b_{1s}^P + m_1^P + \varepsilon_1^P$$

$$P_2 = \rho + c\left(\Delta t_r - \Delta t_s\right) + I_2 + T + b_{2r}^P + b_{2s}^P + m_2^P + \varepsilon_2^P$$

For each signal broadcast:

1 – L1 (1575.42 x 10^6 Hz)

2 – L2 (1227.6 x 10^6 Hz)

Symbols:

P – Pseudorange (m)

ρ – Geometric range (m)

I – Ionospheric delay (m)

T – Tropospheric delay (m)

b – Instrumental bias for receiver and satellite

Δt_r – Receiver clock error (s)

Δt_s – Satellite clock error (s)

m^P – Multipath (m)

ε^P – thermal noise (m)

1 – L1 (1575.42 x 10^6 Hz)

2 – L2 (1227.6 x 10^6 Hz)

Forming the difference P2-P1 and neglecting multipath and thermal noise gives:

$$P_2 - P_1 = I_2 - I_1 + \left(b_{2r}^P - b_{1r}^P\right) + \left(b_{2s}^P - b_{1s}^P\right)$$
$$= I_2 - I_1 + b_r^P + b_s^P$$

(The geometric range, clock error, & tropospheric delay cancel)

The Carrier-Phase Observation Equations

The Carrier-Phase Observation Equations:

$$\Phi_1 = \rho + c(\Delta t_r - \Delta t_s) + I_1 + T + b_{1r}^{\Phi} + b_{1s}^{\Phi} + \lambda_1 N_1 + m_1^{\Phi} + \varepsilon_1^{\Phi}$$

$$\Phi_2 = \rho + c(\Delta t_r - \Delta t_s) + I_2 + T + b_{2r}^{\Phi} + b_{2s}^{\Phi} + \lambda_2 N_2 + m_2^{\Phi} + \varepsilon_2^{\Phi}$$

For each signal broadcast:

1 – L1 (1575.42 x 10^6 Hz)

2 – L2 (1227.6 x 10^6 Hz)

Symbols:

Φ – Carrier-phase (m)

ρ – Geometric range (m)

I – Ionospheric delay (m)

T – Tropospheric delay (m)

b – Instrumental bias for receiver and satellite

Δt_r – Receiver clock error (s)

Δt_s – Satellite clock error (s)

λ– Wavelength (m)

N – Phase cycle-ambiguity

m^{Φ} – Multipath (m)

ε^{Φ}– thermal noise (m)

Forming the difference P2-P1 and neglecting multipath and thermal noise gives:

$$\begin{aligned}\Phi_1 - \Phi_2 &= I_1 - I_2 + (b_{1r}^{\Phi} - b_{2r}^{\Phi}) + (b_{1s}^{\Phi} - b_{2s}^{\Phi}) + (\lambda_1 N_1 - \lambda_2 N_2)\\ &= I_1 - I_2 + b_r^{\Phi} + b_s^{\Phi} + (\lambda_1 N_1 - \lambda_2 N_2)\end{aligned}$$

(The geometric range, clock error, & tropospheric delay cancel)

Derivation of the Pseudorange TEC Observable

Ionospheric delay:

$$I_f = \frac{40.30}{f^2} TEC$$

f – Signal frequency (Hz)

I_f – Ionospheric delay (m)

TEC – total electron content (e^-/m^2)

Substituting ionospheric delay into the pseudorange observation equation gives:

$$P_2 - P_1 = I_2 - I_1 + b_r^P + b_s^P = 40.30\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right) TEC + b_r^P + b_s^P$$

Solving this for the TEC yields:

$$TEC = \frac{1}{40.30}\left[\frac{f_1^2 f_2^2}{f_1^2 - f_2^2}\right]\left[\left(P_2 - P_1\right) - \left(b_r^P + b_s^P\right)\right] = 9.52\times 10^{16}\left[\left(P_2 - P_1\right) - \left(b_r^P + b_s^P\right)\right]$$

We define the pseudorange TEC without the bias terms in units of TECU:

$TEC_P \equiv 9.52\left(P_2 - P_1\right)$ where 1 TECU = 10^{16} e^-/m^2

unambiguous but noisy and therefore an imprecise observable

Derivation of the Carrier-Phase TEC Observable

Ionospheric phase advance:

$$I_f = \frac{40.30}{f^2} TEC$$

f – Signal frequency (Hz)

I_f – Ionospheric delay (m)

TEC – total electron content (e^-/m^2)

Substituting phase advance into the carrier-phase observation equation gives:

$$\Phi_1 - \Phi_2 = I_1 - I_2 + 40.30\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right)TEC + b_r^\Phi + b_s^\Phi + \left(\lambda_1 N_1 - \lambda_2 N_2\right)$$

Solving this for the TEC yields:

$$TEC = \frac{1}{40.30}\left[\frac{f_1^2 f_2^2}{f_1^2 - f_2^2}\right]\left[\left(\Phi_1 - \Phi_2\right) - \left(b_r^\Phi + b_s^\Phi\right) - \left(\lambda_1 N_1 - \lambda_2 N_2\right)\right]$$

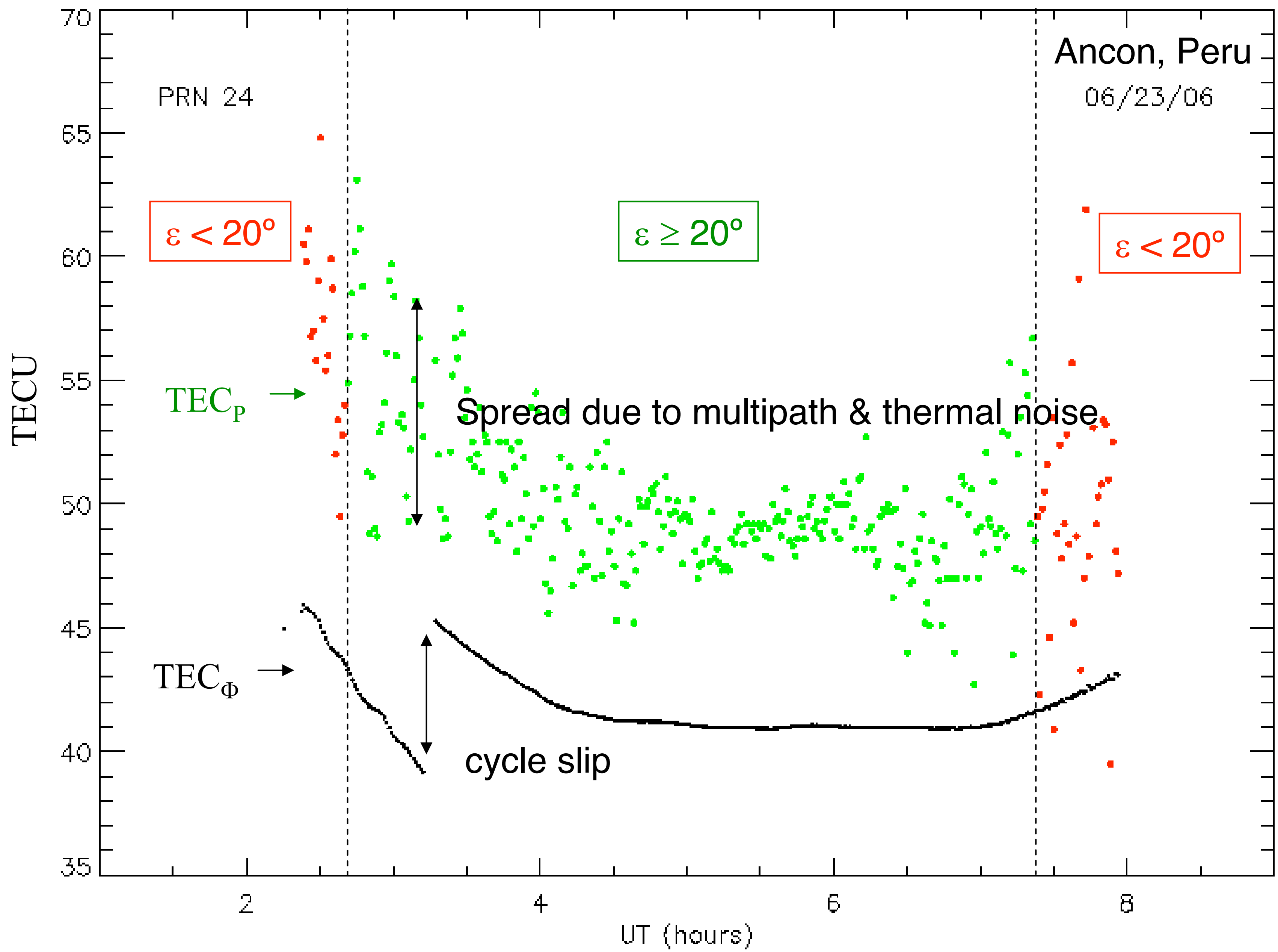
$$= 9.52\times 10^{16}\left[\left(\Phi_1 - \Phi_2\right) - \left(b_r^\Phi + b_s^\Phi\right) - \left(\lambda_1 N_1 - \lambda_2 N_2\right)\right]$$

We define the carrier phase TEC in units of TECU:

$TEC_\Phi \equiv 9.52\left(\Phi_1 - \Phi_2\right)$ where 1 TECU = 10^{16} e^-/m^2

Precise but ambiguous observable. Biases and ambiguities will be estimated using pseudoranges

Behavior of the Pseudorange and Carrier-Phase Measures of TEC



Measurement of Cycle Slips Using Least Squares

Approximate TEC_Φ as a superposition of a quadratic polynomial and a Heaviside step function at the slip location

$$\sum_j V_{ij} t_j = TEC_\Phi^j$$

Basis vectors:

$V_1(t)=1$ $V_3(t)=t^2$

$V_2(t)=t$ $V_4(t)=H_{ts}(t)$

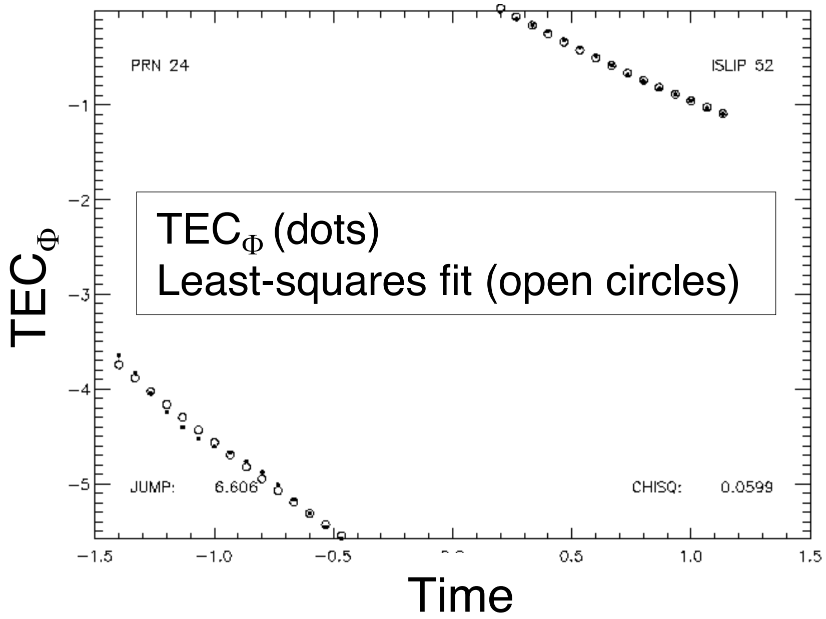
Find solution to this over-determined system via least squares:

$$\left(\mathbf{V}^T\mathbf{V}\right)\mathbf{T} = \mathbf{V}^T\mathbf{\Phi}$$

Solve via singular-value decomposition

Coefficient of the Heaviside step function (V_4) is the size of the slip

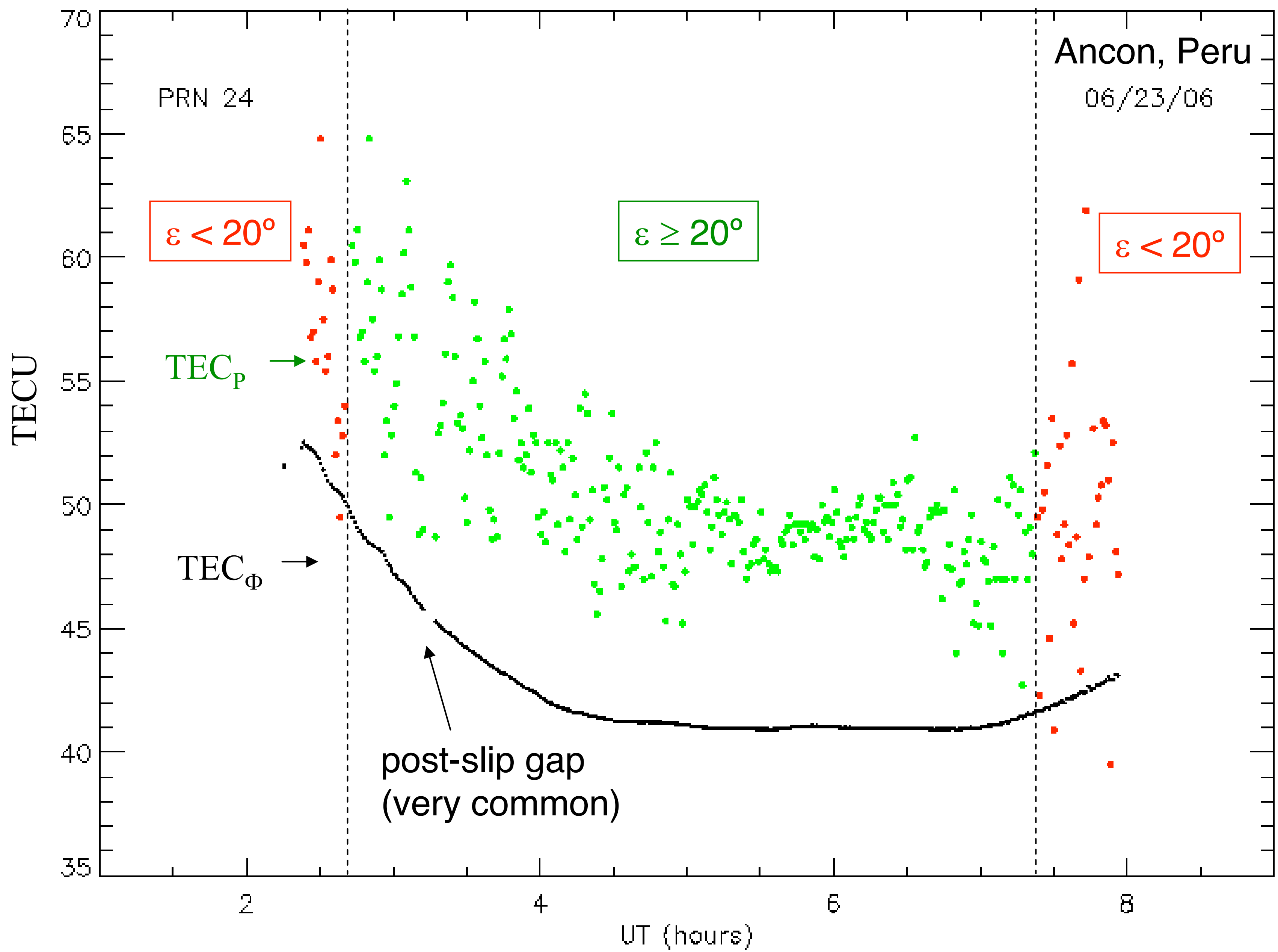
A cycle slip in TEC_Φ



Slip not corrected if $\chi^2 > 1$ TECU

Advantage over predictor methods – uses data on both sides of slip, can de-weight (or omit) data immediately surrounding the slip, if noisy

Carrier-Phase TEC Corrected for Cycle-Slips



Leveling the Carrier-Phase TEC to the Pseudorange TEC

Relative TEC (leveled phases):

$$TEC_R = TEC_\Phi + \underbrace{\langle TEC_P - TEC_\Phi \rangle_{arc}}_{Offset}$$

Take difference between pseudorange TEC and phase TEC

$$x_i = TEC_P^i - TEC_\Phi^i$$

Weighted average of x gives the offset:

$$Offset = \frac{\sum_i w_i x_i}{\sum_i w_i}$$

Summation is taken over all samples (i) in the same phase connected arc with $\varepsilon_i > 20^\circ$

Weighted standard deviation, σ, provides estimate of the leveling error:

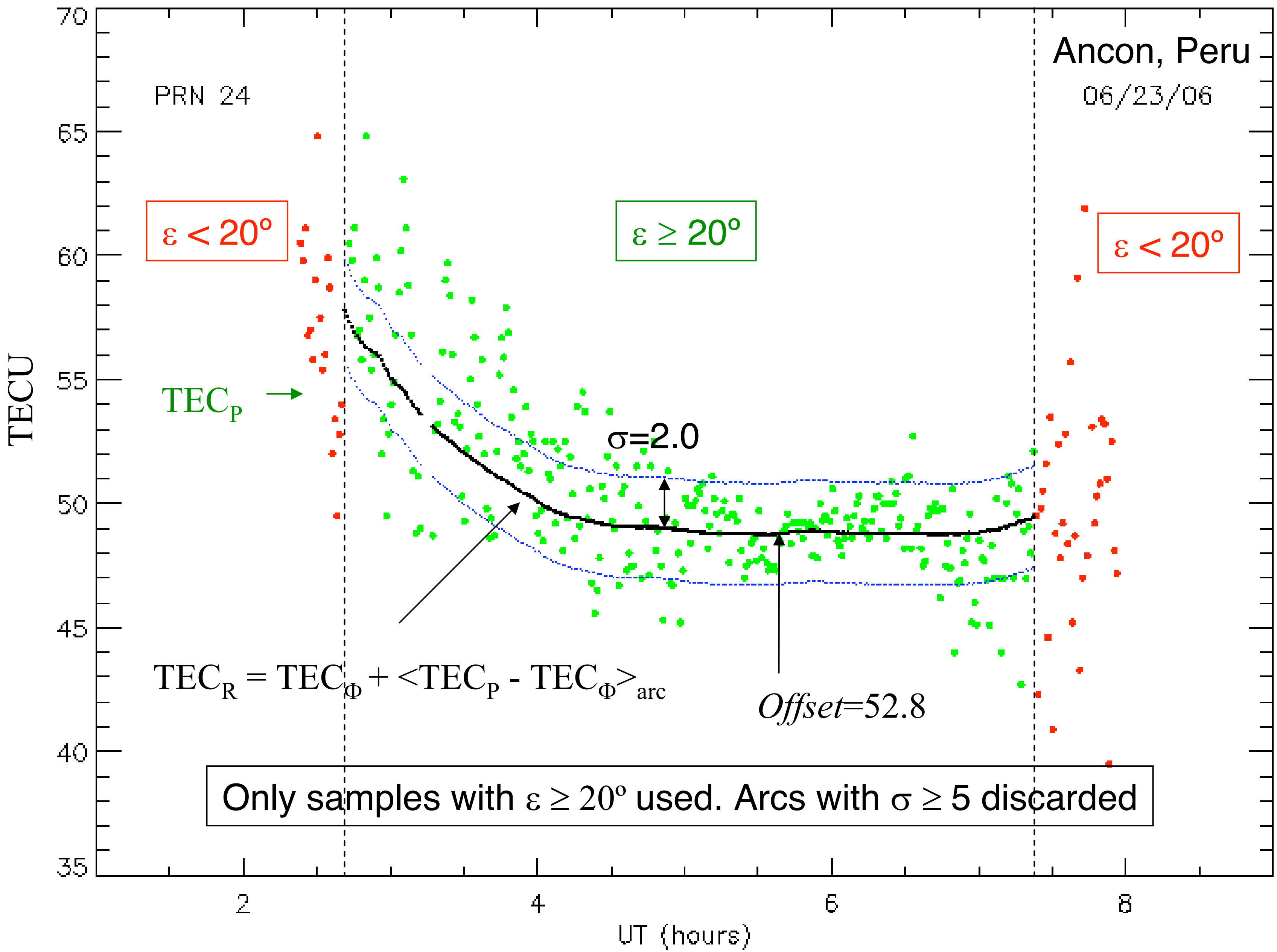
$$\sigma^2 = \frac{\left[\sum_i w_i x_i^2\right]\left[\sum_i w_i\right] - \left[\sum_i w_i x_i\right]^2}{\left[\sum_i w_i\right]^2 - \sum_i w_i^2}$$

Weighting chosen is usually the sine of the satellite elevation, ε:

$$w_i = \sin(\varepsilon_i)$$

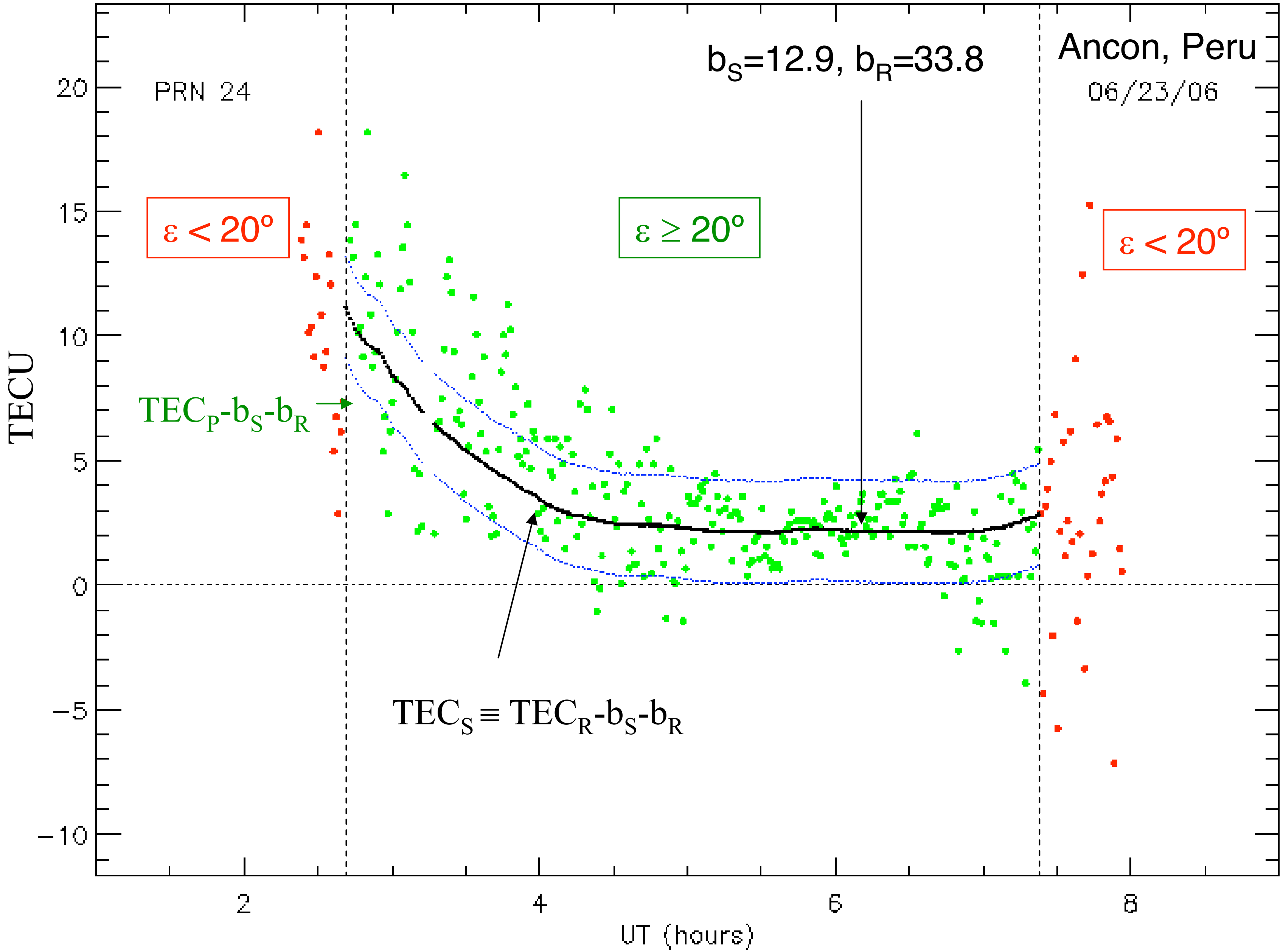
Quality control: entire phase connected arc is discarded if $\sigma > 5$ TECU

Relative TEC and the Estimated Phase Leveling Error



The Calibrated (Unbiased) Slant TEC

Once the instrumental biases are known, they can be subtracted from relative TEC measurements to give the calibrated (unbiased) slant TEC



Estimation and Removal of the GPS Instrumental Biases

Estimation of GPS Instrumental Biases from the Measurements

To estimate the instrumental biases from the measurements themselves, we must make assumptions about the (real) TEC we are trying to measure:

TEC must be non-negative

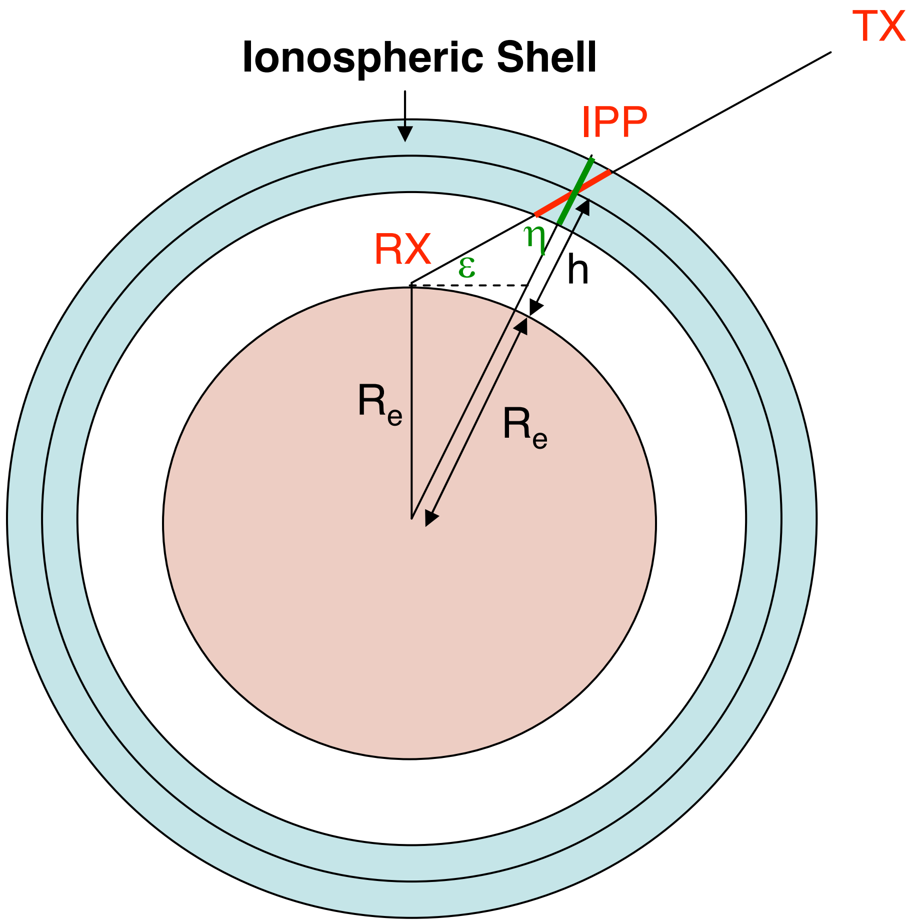
- and -

The structure of the TEC is assumed to satisfy one or more of these …

- We assume a value for the TEC attained at night
- Spatial gradients in TEC assumed negligible (at night)
- TEC well approximated by a polynomial of order N (higher order derivatives assumed negligible)
- TEC in the ionosphere well approximated by a polynomial and TEC in the plasmasphere by a model (generally structured according to dipole field-lines)

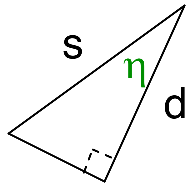
These techniques work by exploiting the fact that slant TEC depends on elevation (since the path length through ionized region is longer) while the biases do not

A Useful Tool for Bias Estimation and Visualization: Computing the Vertical-Equivalent TEC



The standard geometric mapping function, *M,* is the projection of slant distance onto zenith distance at the IPP:

$$M \equiv s/d = \sec(\eta)$$



The zenith angle at the IPP, η, can be expressed in terms of shell height, h, and Earth radius R:

$$(R_e + h)\sin(\eta) = R_e \sin(90° + \varepsilon) = R_e \cos(\varepsilon)$$

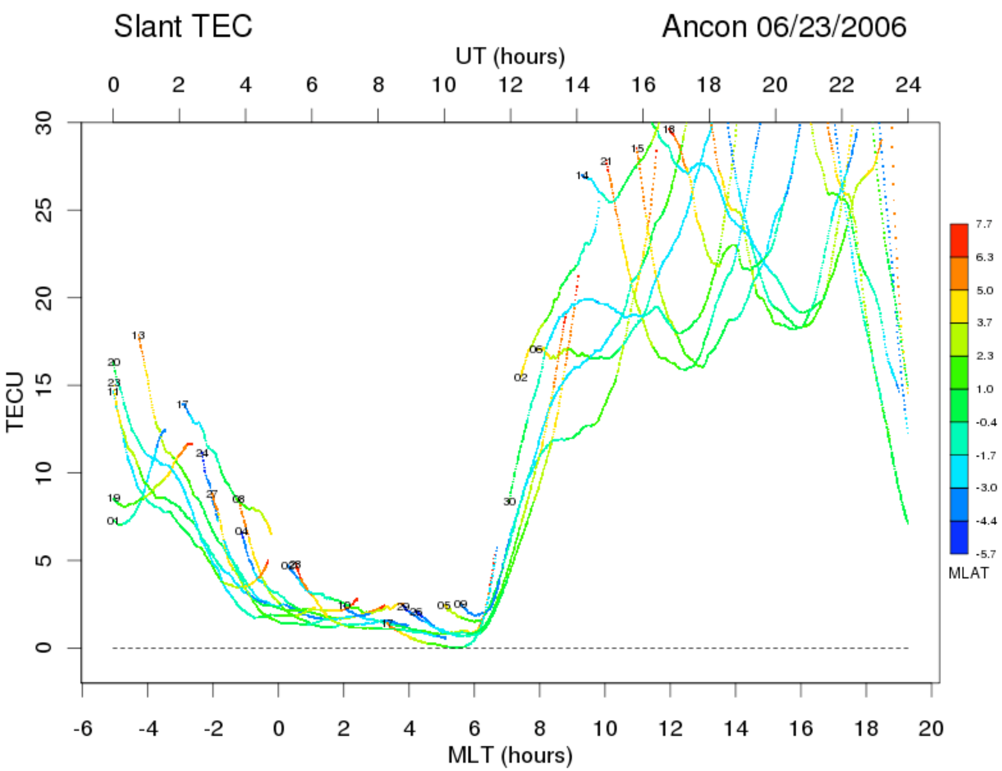
This gives the mapping function in terms of the satellite elevation, ε:

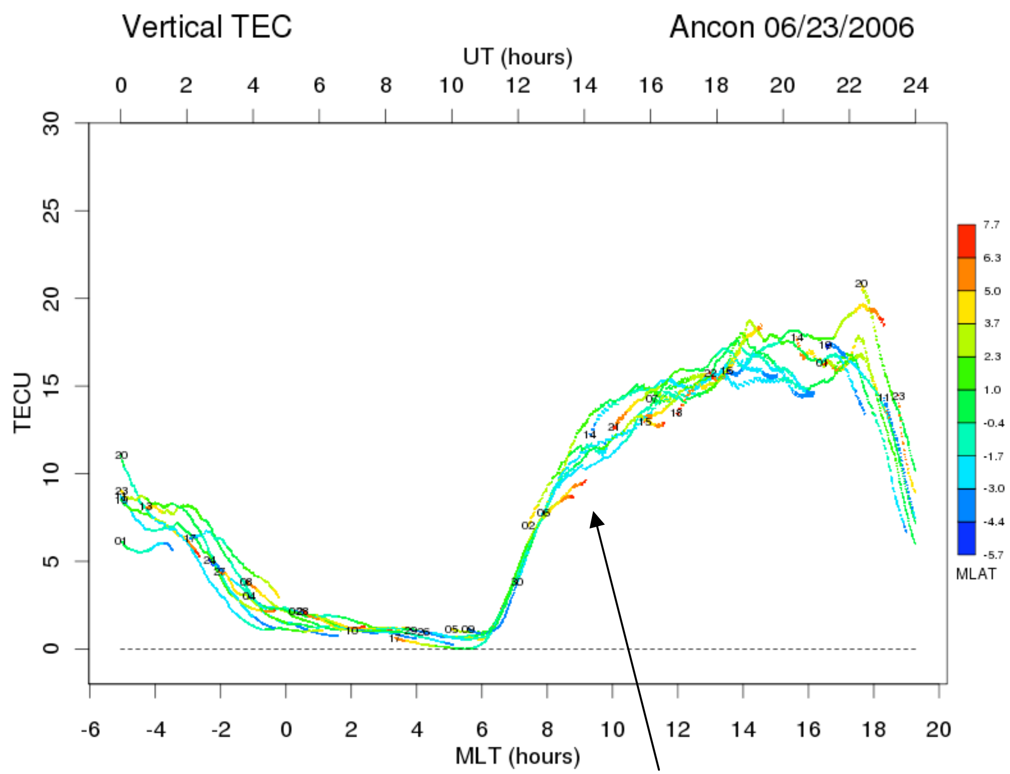
$$M(\varepsilon) = \sec\left[\sin^{-1}\left(\frac{R_e}{R_e + h}\cos(\varepsilon)\right)\right]$$

Application of mapping function to slant TEC gives the vertical-equivalent TEC:

$$TEC_V = TEC_s / M(\varepsilon)$$

Calibrated Slant and Vertical-Equivalent TEC (All Satellites)





Magnetic local time at the IPP

Curves colored by magnetic latitude

Characteristics of well-calibrated TEC:

- TEC is non-negative
- TEC curves collapse well (especially at night and during post-sunrise ramp up)

There are noteworthy Exceptions to this rule!

Removing the Correct GPS Satellite Biases

The technique used by the receiver to measure TEC_P dictates the type of satellite instrumental biases that must be removed.

Receiver Model	Method used to measure the DPR	Type of Satellite Bias to Remove
Ashtech Z-12	L2(P2) - L1(P1)	P1P2 bias
Ashtech µZ-CGRS	L2(P2) - L1(P1)	P1P2 bias
NovAtel GSV 4004B	L2(P2) - L1(CA)	P1P2 bias minus the P1C1 bias

Files containing monthly estimates for the P1P2 and P1C1 biases can be downloaded from http://www.aiub.unibe.ch/download/CODE/.

These satellite differential codes bias are not absolute timing biases, instead they average to zero. The unknown offset is immaterial in that it will be lumped together and removed along with the receiver bias.

A Very Simple Technique for Approximate TEC Calibration

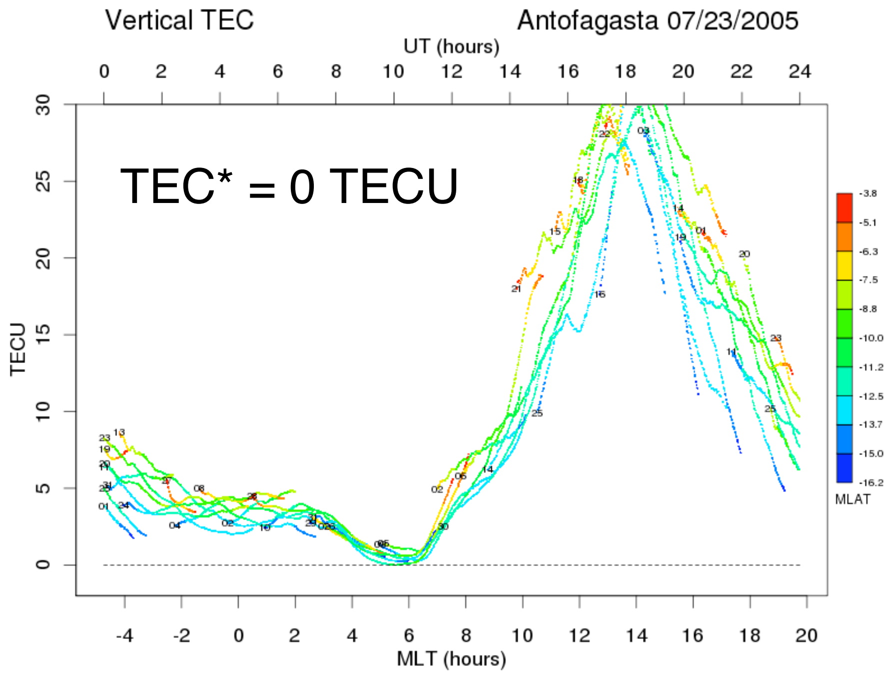
Procedure:

- Assume the minimum TEC (generally attained during nighttime) is known, e.g. zero
- Download estimates of the satellite biases from CODE. Multiply the biases by -2.85 TECU/ns to convert the reported biases from units of nanoseconds to TECU.
- Select the receiver bias to enforce that min(TECS) = TEC*:

$$b_R = \min(TEC_R) + b_S - TEC^*$$

- Compute the calibrated slant TEC:

$$TEC_S = TEC_R - b_S - b_R$$

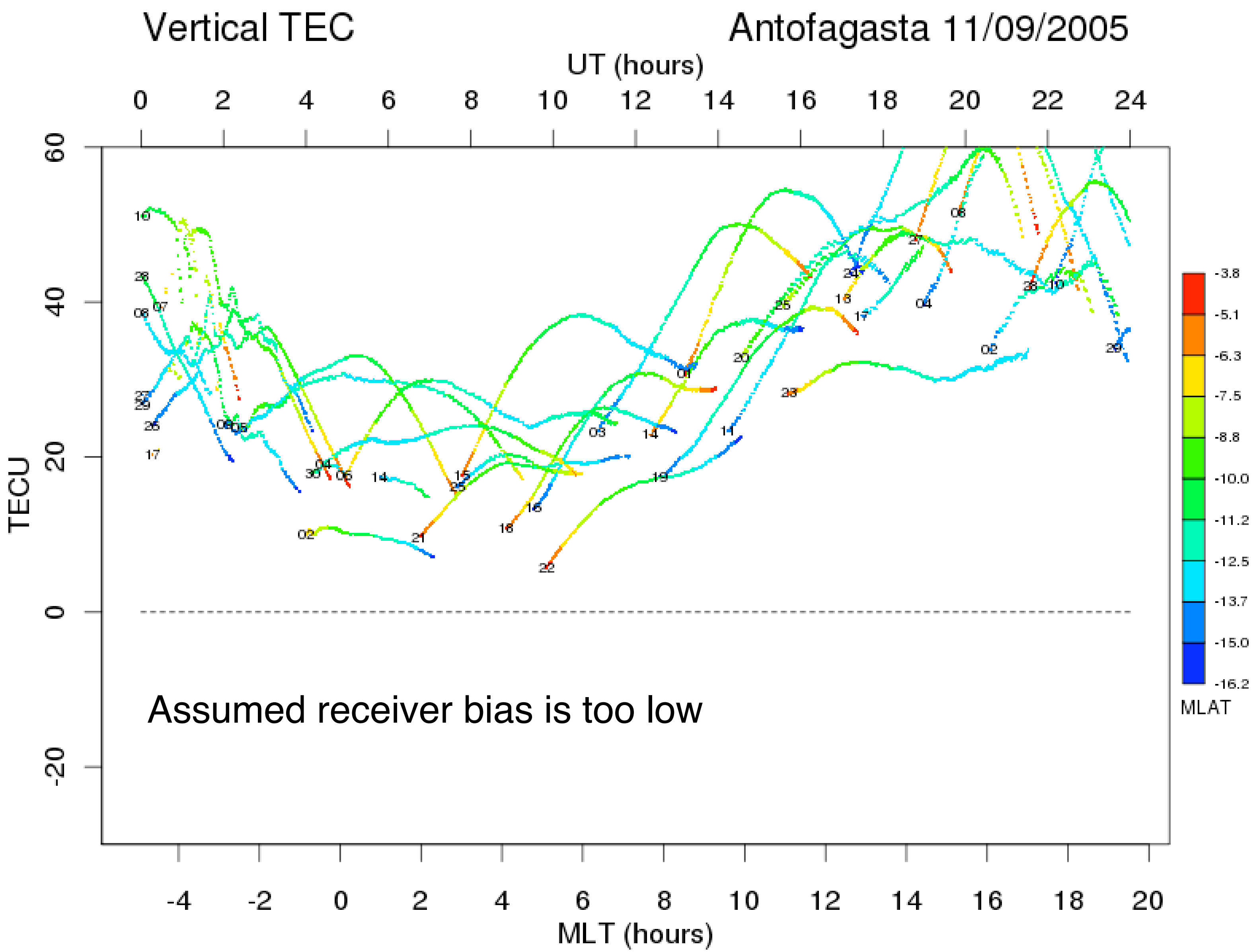


A Better Technique for TEC Calibration (Performed Manually for Illustration)

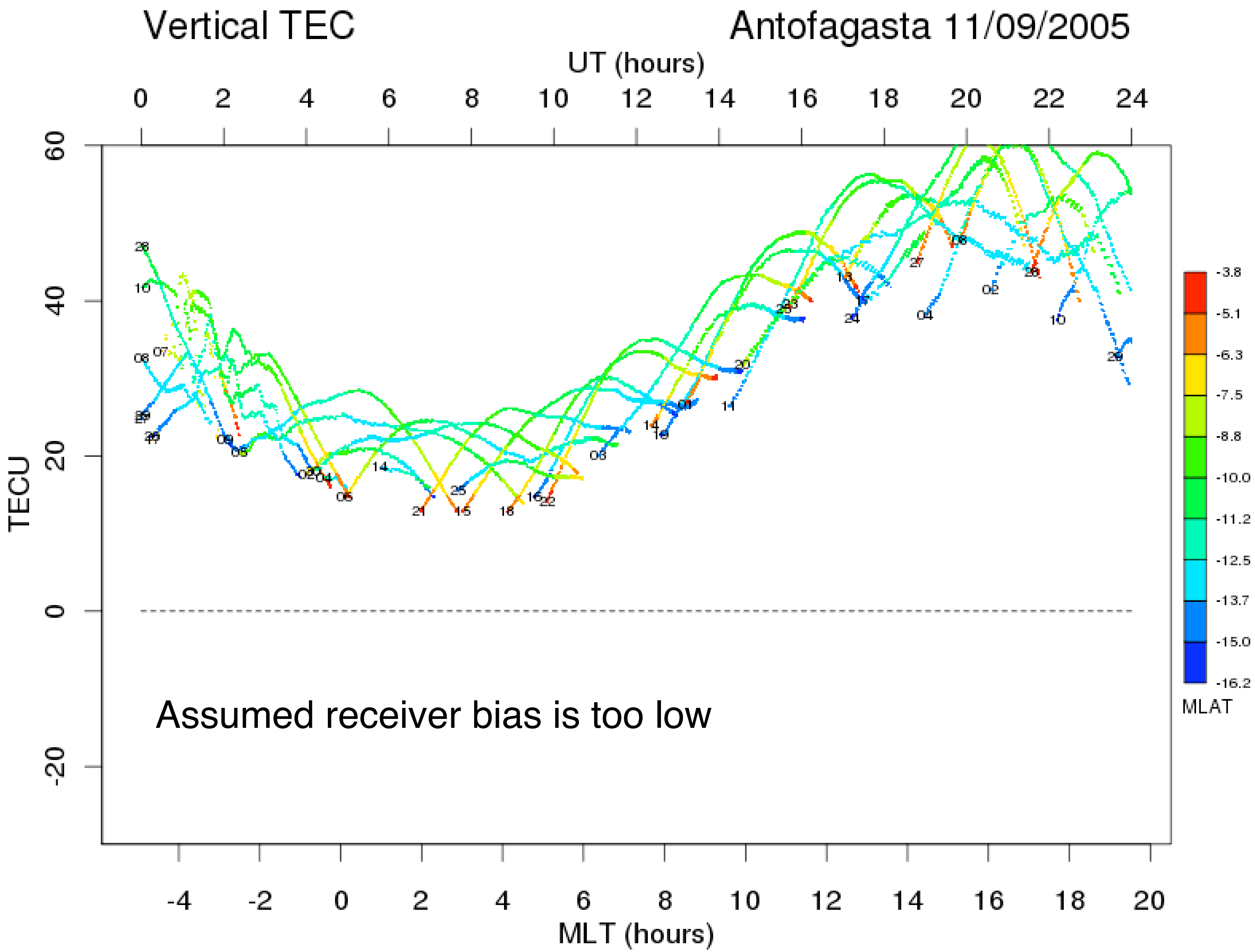
Procedure:

- If ionosphere is uniformly distributed in a thin slab (no spatial gradients) then the vertical-equivalent TEC estimates should the same for all satellites.
- Download estimates of the satellite biases from CODE. Multiply the biases by -2.85 TECU/ns to convert the reported biases from units of nanoseconds to TECU.
- Manually change the assumed value of the receiver bias until the vertical-equivalent curves collapse most closely together (at least during nighttime hours)

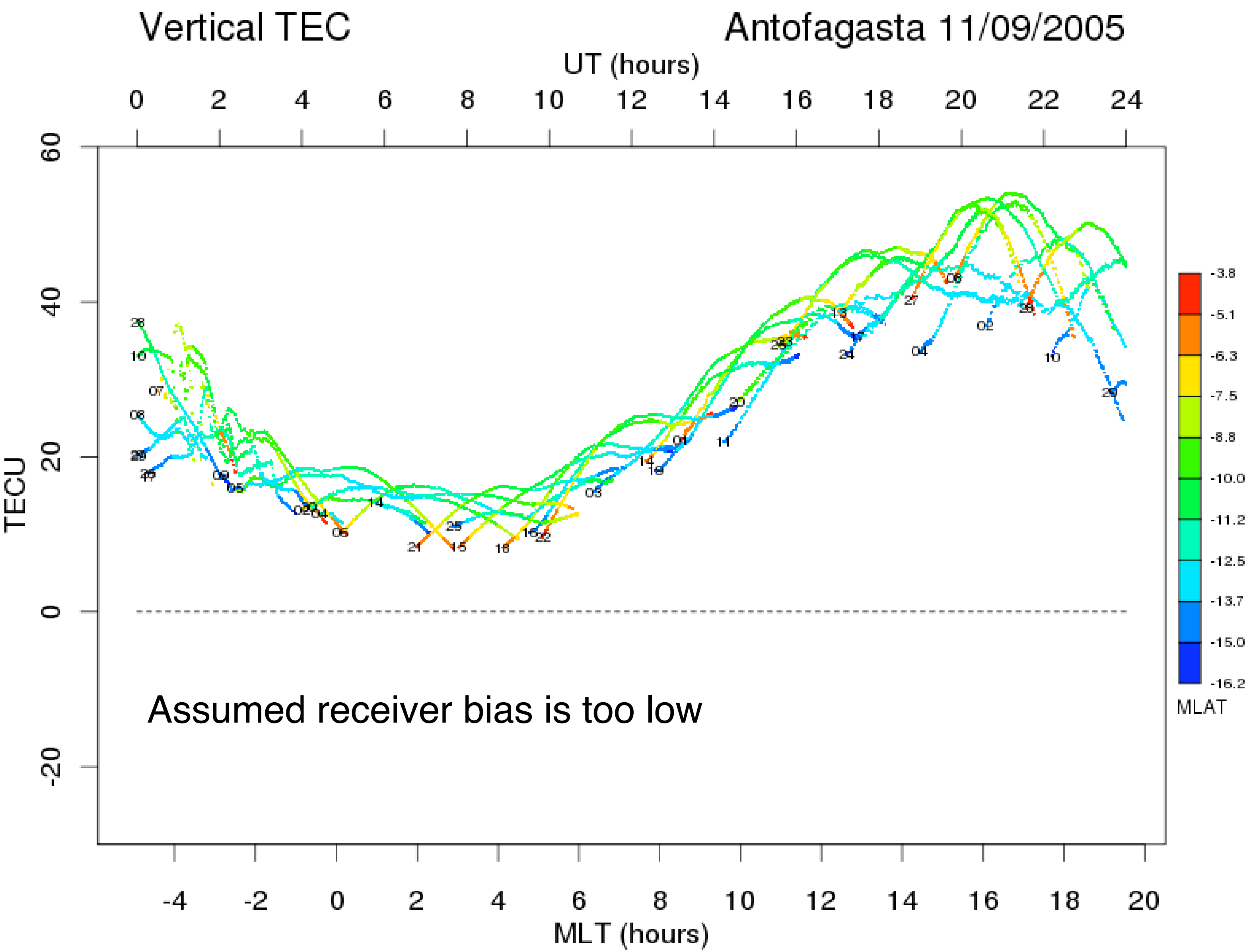
Vertical-Equivalent TEC (no Biases Removed)



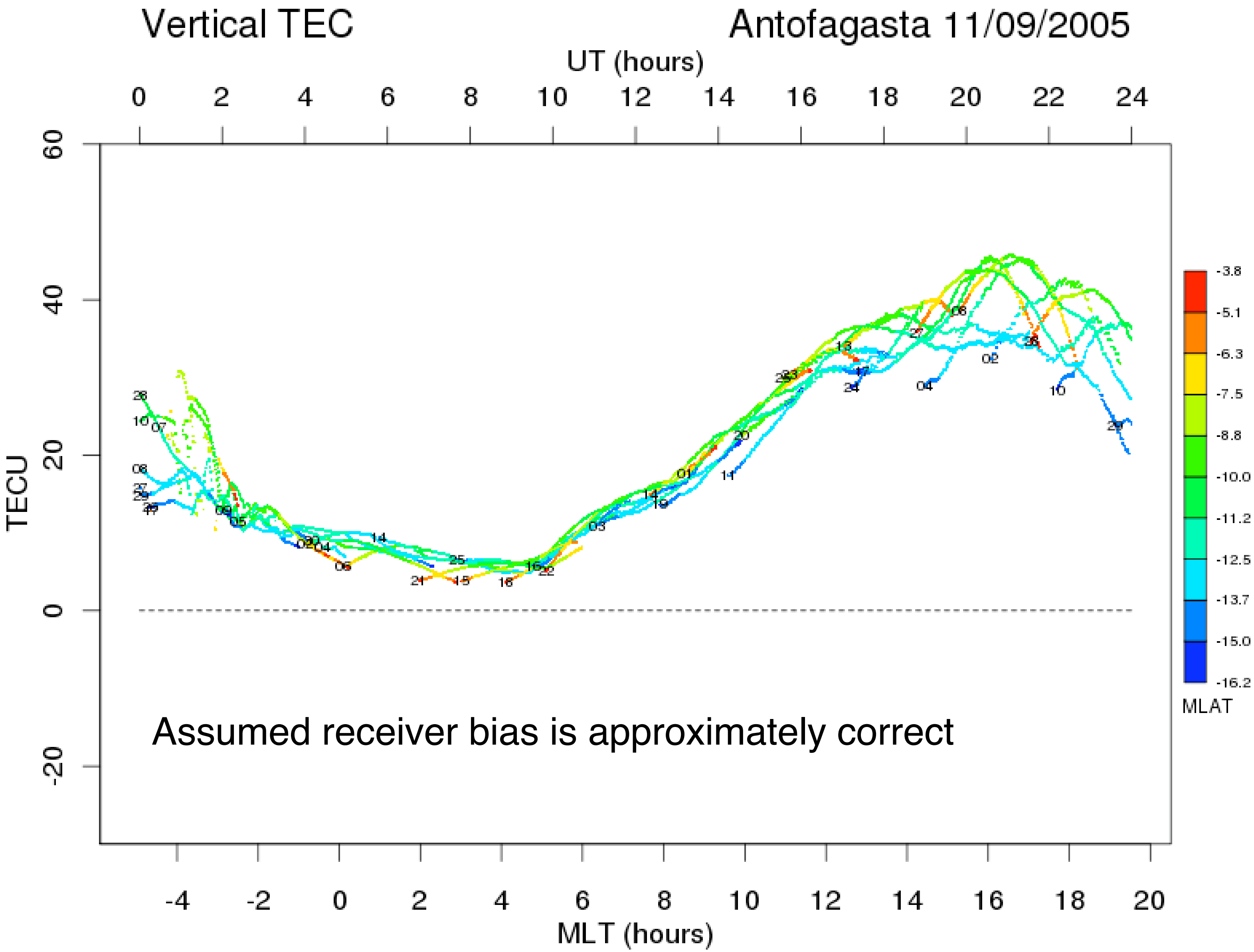
Vertical-Equivalent TEC (Satellite Biases Removed, b_R = 0 TECU)



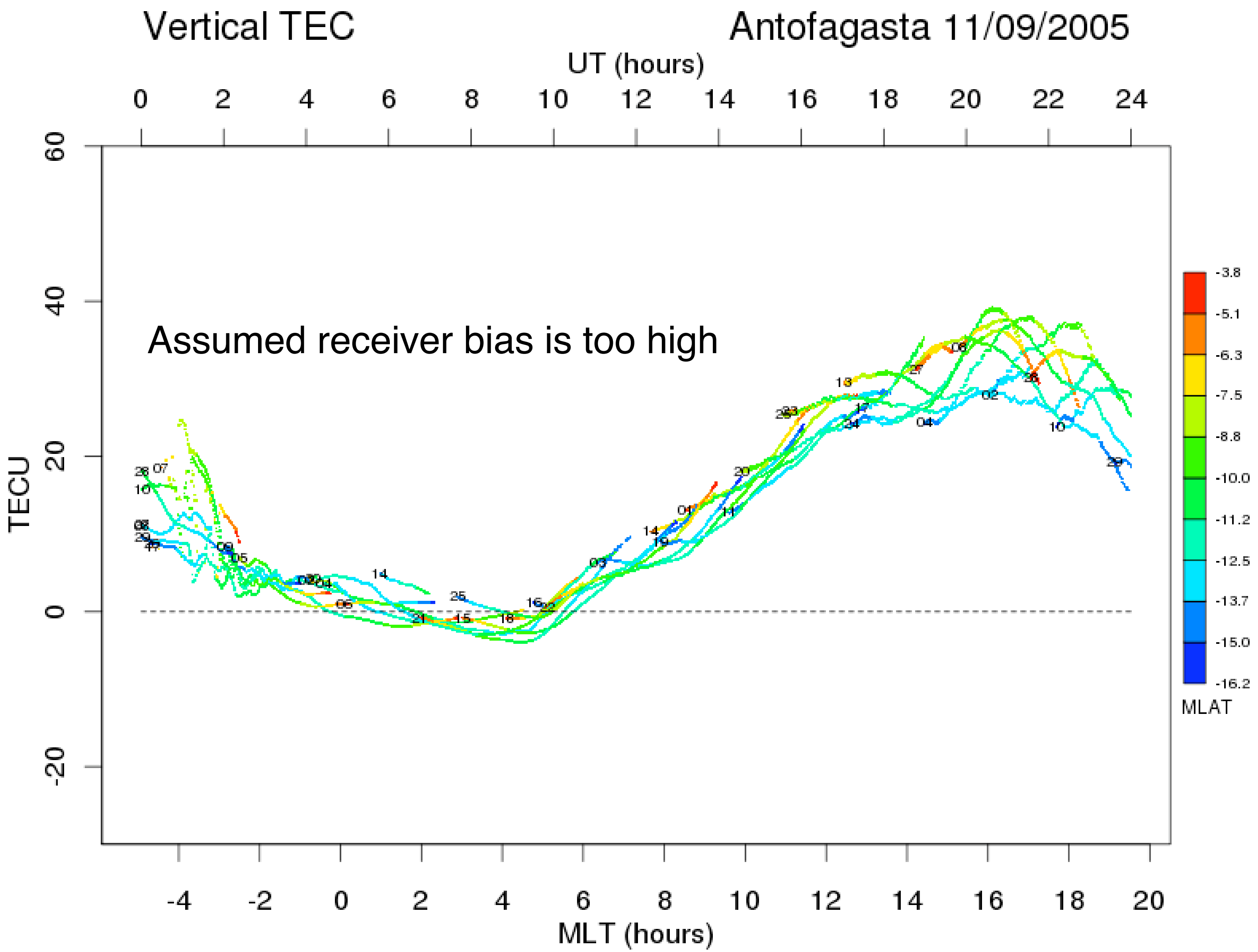
Vertical-Equivalent TEC (Satellite Biases Removed, b_R = 10 TECU)



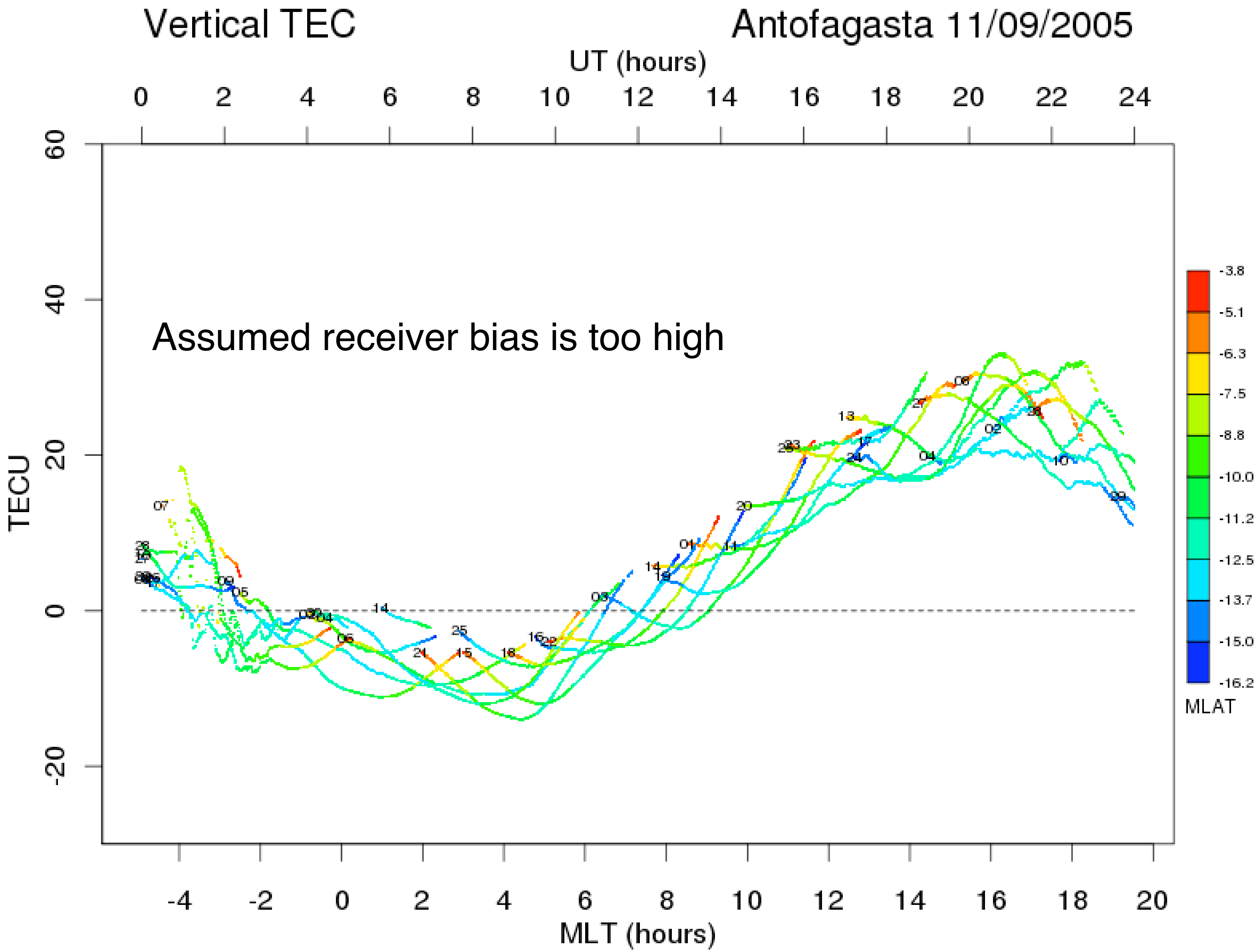
Vertical-Equivalent TEC (Satellite Biases Removed, b_R = 20 TECU)



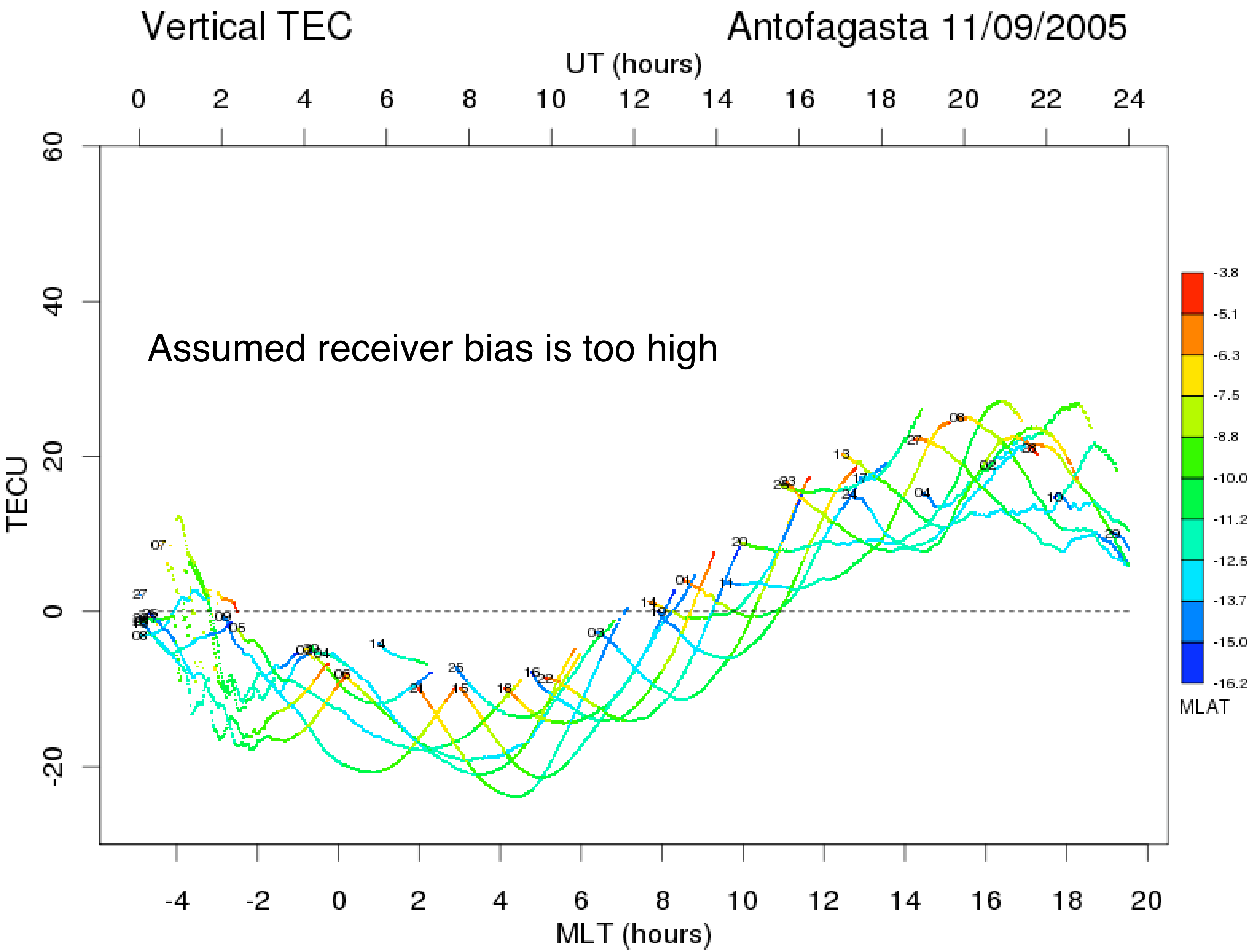
Vertical-Equivalent TEC (Satellite Biases Removed, b_R = 30 TECU)



Vertical-Equivalent TEC (Satellite Biases Removed, b_R = 40 TECU)



Vertical-Equivalent TEC (Satellite Biases Removed, b_R = 50 TECU)



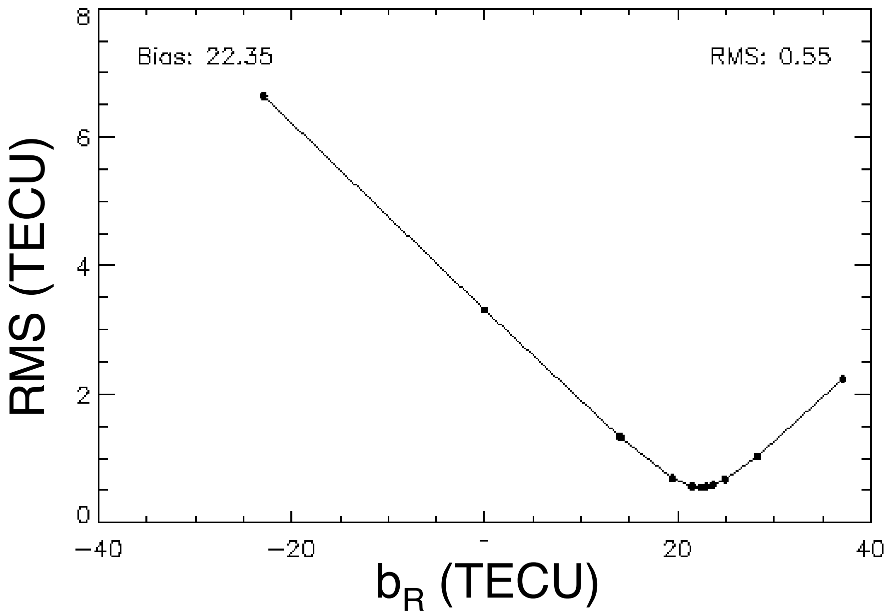
Automated Receiver Bias Determination by Least Squares

Assumption: In absence of spatio-temporal density gradients, the verticalized calibrated TEC measured by all satellites should be the same.

Given the satellite biases and *h*, TEC_V can be expressed as a function of b_R:

$$TEC_V(b_R) = [\ TEC_R - b_R + b_S\ ] \,/\, M(\varepsilon, h)$$

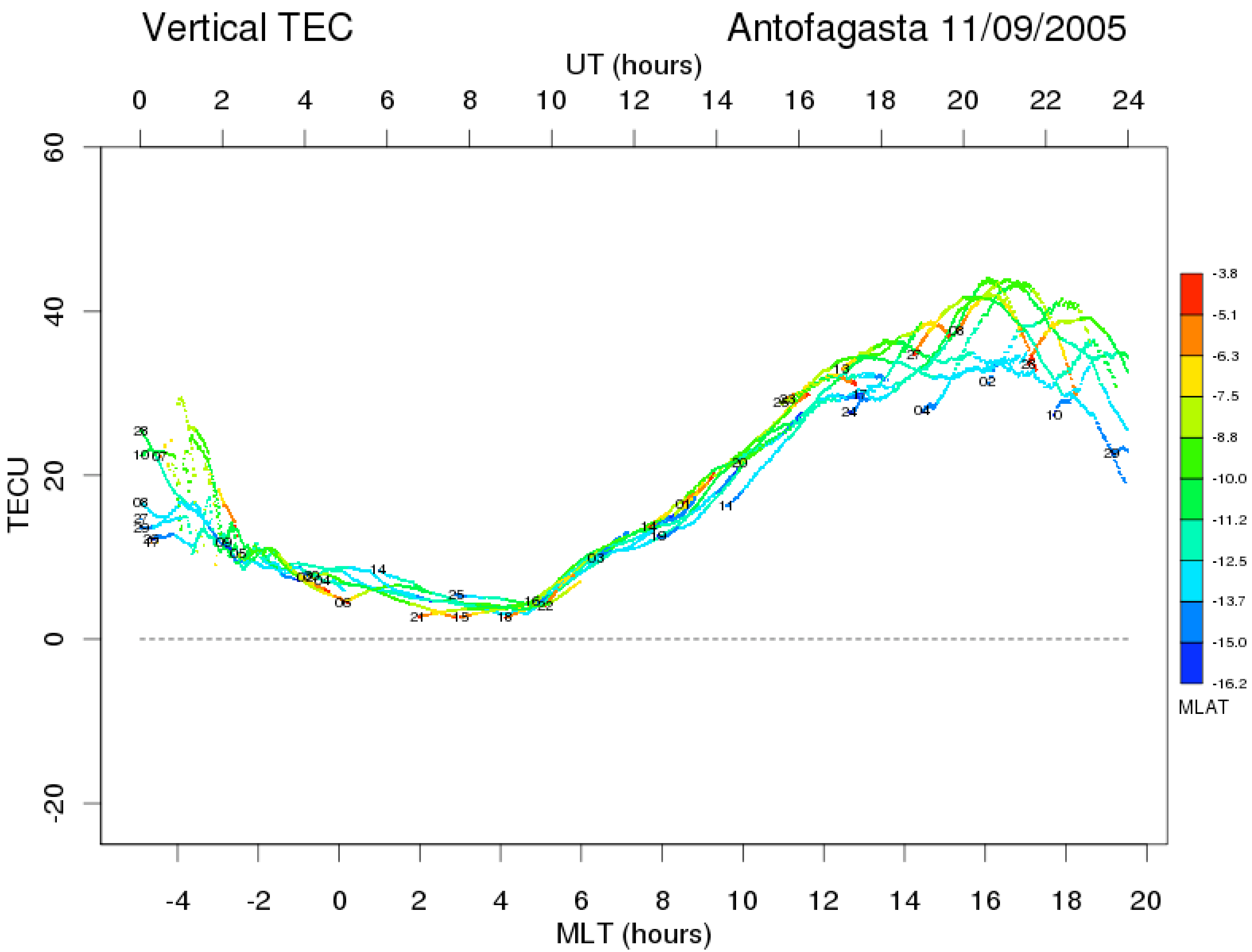
Single layer mapping function



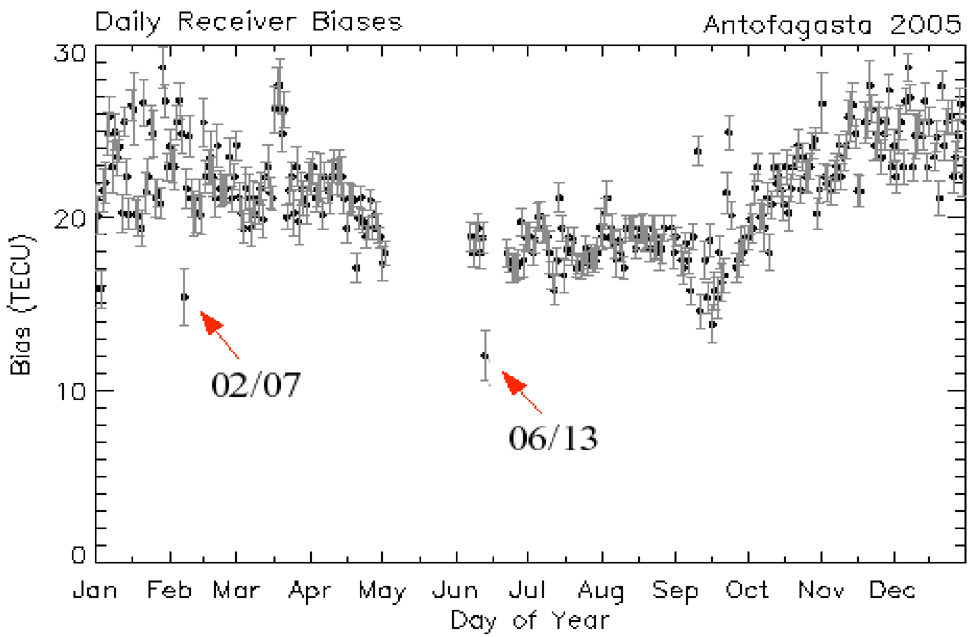
We calculate the b_R that minimizes $Var(TEC_V)$ late at night when gradients are smallest

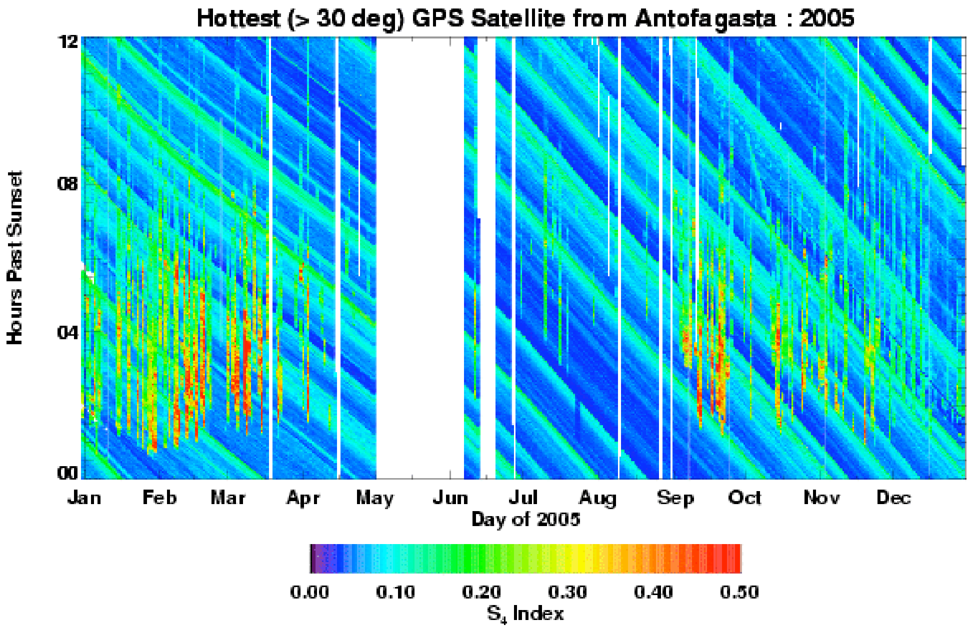
TEC Calibration by Least-Squares (Results)

Final calibrated TEC result, using estimated value of 22.3 TECU for receiver bias



Variability of the Receiver Bias Estimates at Antofagasta

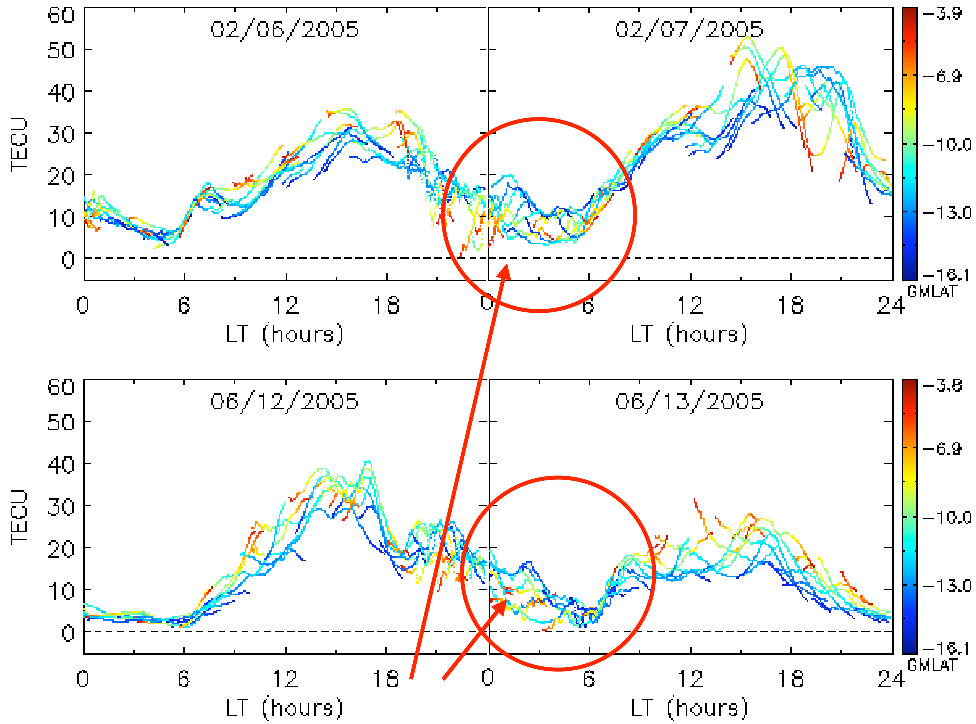




Largest deviations from trend occur when TEC is structured late at night

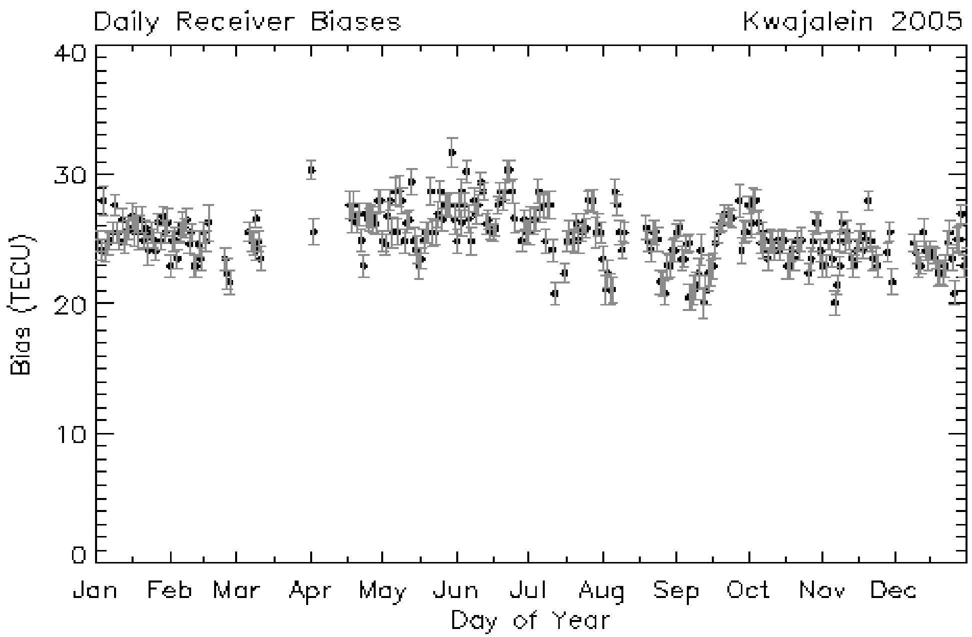
These nights often correlate with the occurrence of scintillation

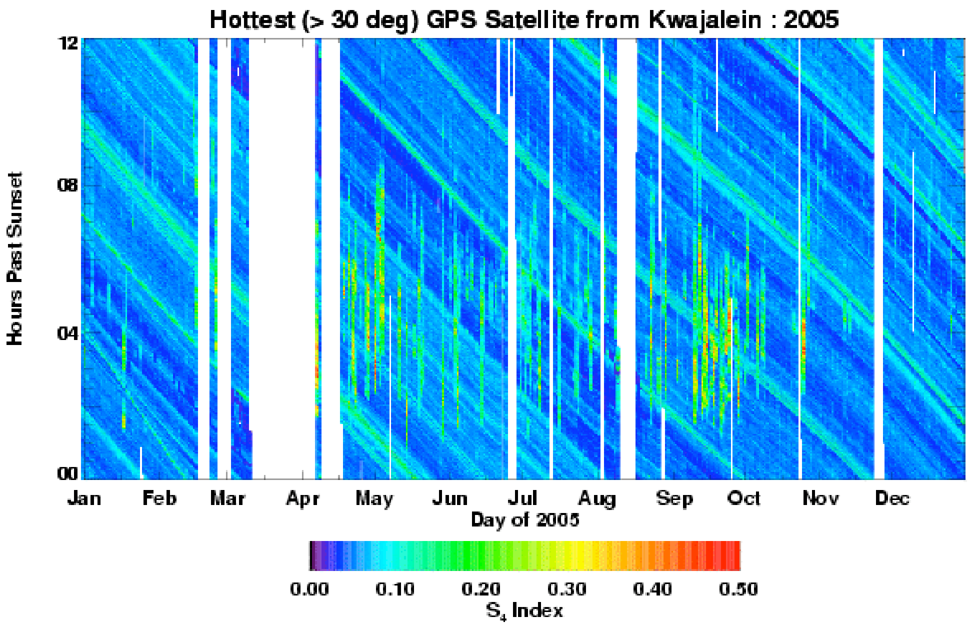
A closer look at two outliers:



Late-night structure in TEC

Variability of the Bias Estimates at Kwajalein





This station (Kwajalein) experienced weaker GPS scintillations than Antofagasta in 2005

Deviations in the receiver bias from the trend are correspondingly smaller

Less structure at night generally means more accurate TEC calibration

Kalman Filter Estimation of Total TEC

The Kalman Filter Estimation

Observation equation (for the i^{th} GPS receiver-satellite pair):

Measured slant TEC	Bilinear fit to ionospheric TEC_V	Instrumental biases

$$TEC_{RS}^{i} = M\left(\varepsilon_{RS}^{i}\right)\left[a_{0,R}^{i} + a_{1,R}^{i} \cdot d\lambda_{RS}^{i} + a_{2,R}^{i} \cdot d\varphi_{RS}^{i}\right] + b_{R}^{i} + b_{S}^{i}$$

ε – Elevation

α – Azimuth

$d\lambda$ – Difference between MLT at ionospheric penetration point and station

$d\varphi$ – Difference between MLAT at ionospheric penetration point and station

b_R, b_S – Receiver and satellite instrumental biases

Thin shell mapping function

$$M\left(\varepsilon\right) = \sec\left[\arcsin\left(\frac{R_e}{R_e + h}\right)\cos(\varepsilon)\right]$$

The Kalman Filter Implementation

Kalman state vector (unknowns)

Ionospheric fit parameters — Instrumental biases

$$\mathbf{X}_k = \begin{bmatrix} \underbrace{a_{0,R}^k \quad a_{1,R}^k \quad a_{2,R}^k}_{\text{Ionospheric fit parameters}} & \underbrace{b_R^k \quad b_{S_1}^k \quad b_{S_2}^k \cdots b_{S_N}^k}_{\text{Instrumental biases}} \end{bmatrix}^T$$

Measurement vector (knowns)

Measured slant TEC

$$\mathbf{y}_k = \begin{bmatrix} \underbrace{TEC_{RS_1}^k \quad TEC_{RS_2}^k \quad \cdots \quad TEC_{RS_N}^k}_{\text{Measured slant TEC}} \end{bmatrix}^T$$

Kalman process to be estimated

$$\mathbf{X}_k = \mathbf{\Phi}_{k,k-1} \cdot \mathbf{X}_{k-1} + \mathbf{w}_{k-1}$$
$$\mathbf{y}_k = \mathbf{H}_k \cdot \mathbf{X}_k + \mathbf{v}_k$$

- Identity state transition matrix
- Zero-mean white Gaussian process noise $\mathbf{w}_k$ and measurement noise $\mathbf{v}_k$
- Kalman updates performed every 60 seconds (each new data epoch)

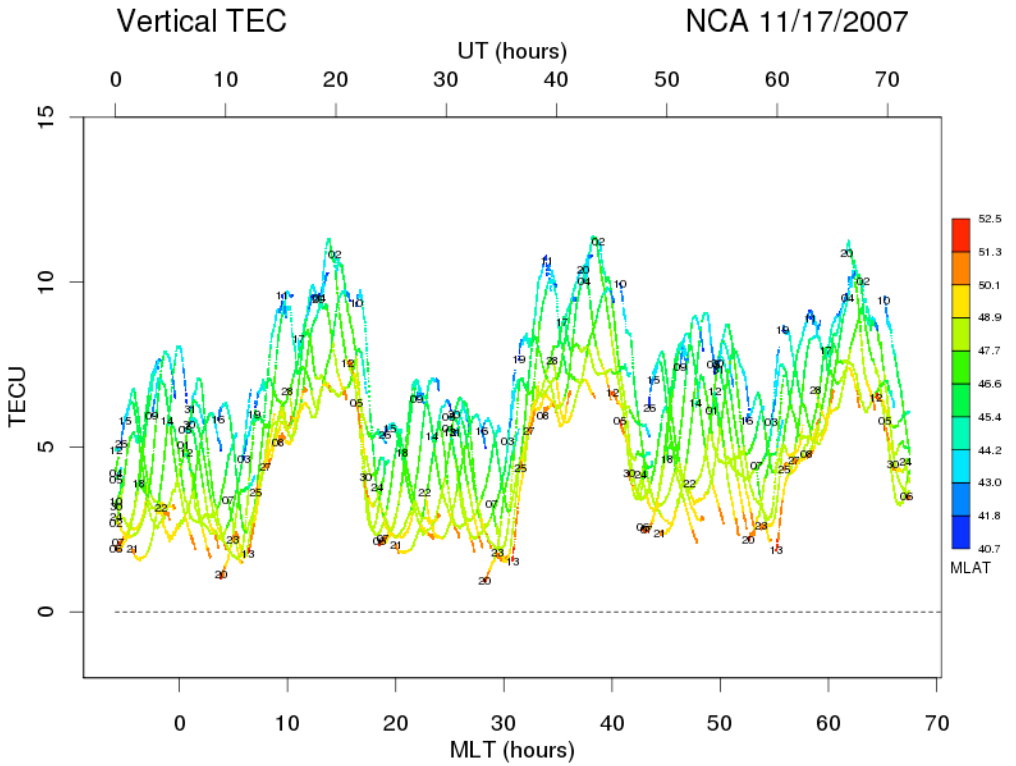
Kalman Filter Estimation of TEC in both the Ionosphere and Plasmasphere

Plasmaspheric Signatures in the Estimated TEC

- Gradients from the plasmasphere cause an apparent "spread" in the vertical-equivalent TEC which violates our assumption that spatial gradients are small.

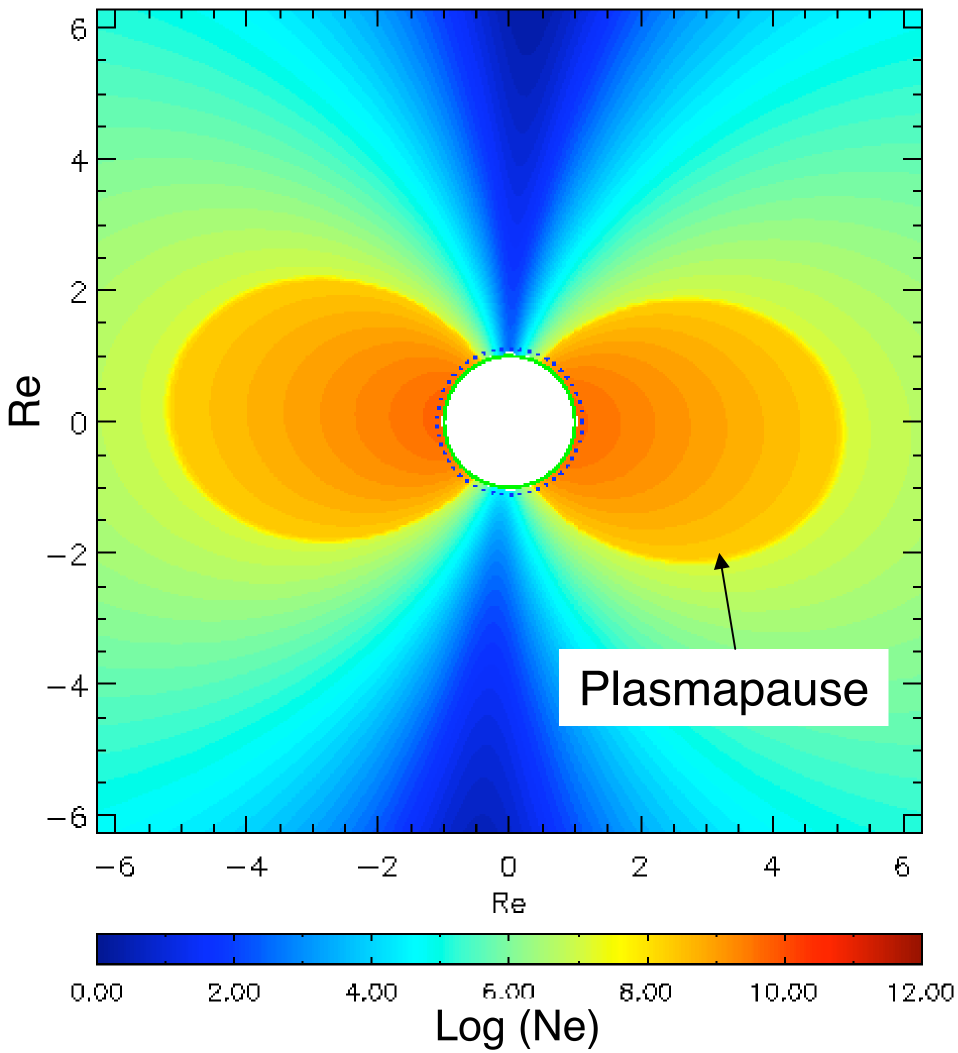
- Moreover, the thin-shell approximation commonly used for the ionosphere is not a suitable representation for the plasmaspheric contribution to the TEC

- This effect is most evident during periods of very low solar activity such the one we are currently experiencing.

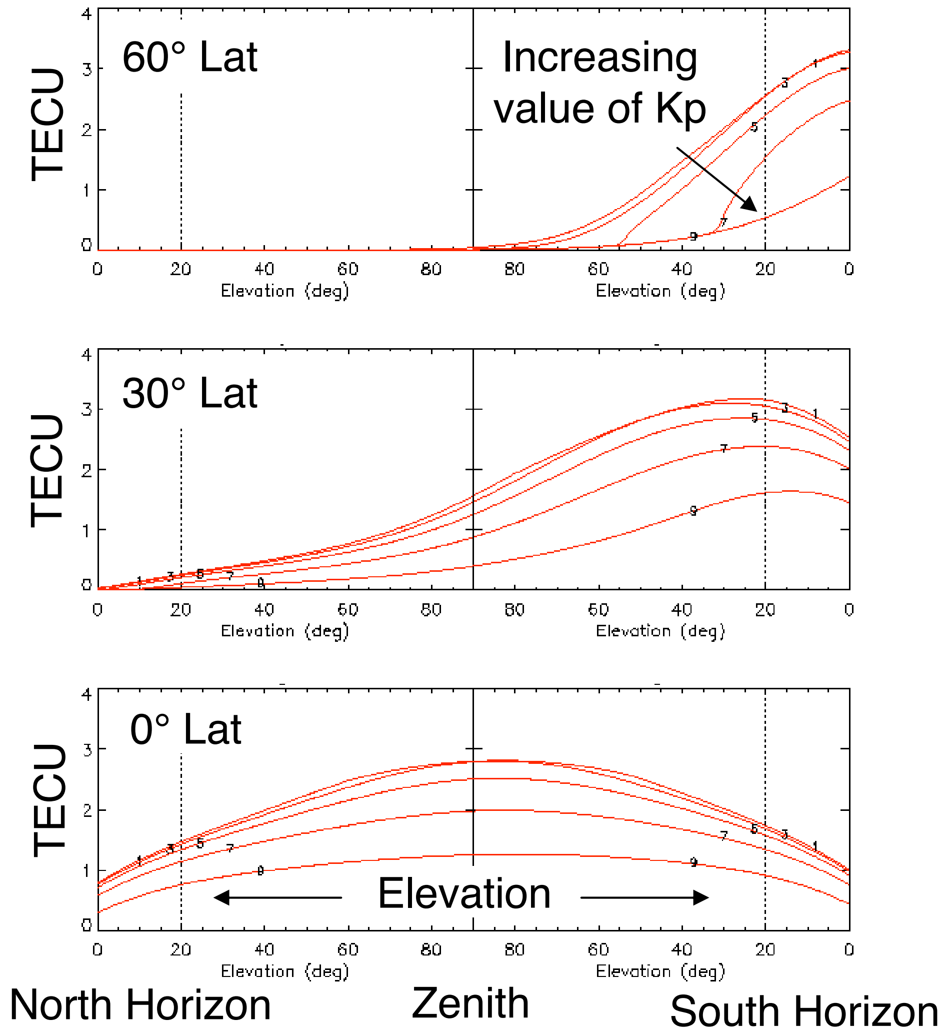


GPS Signal Paths through the Plasmasphere

Electron Density in the Plasmasphere



PTEC between 700 km and 20200 km



Plasmaspheric contribution to the TEC depends on location, azimuth, and elevation.

Plasmapause location has strong influence on PTEC encountered at high to mid latitudes.

Carpenter-Anderson Plasmasphere Model

- Carpenter and Anderson [1992] model for the electron density in the inner plasmasphere:

$$\log n_e^i = -0.3145L + 3.9043 + \left[0.15\cos\frac{2\pi(d+9)}{365} - 0.075\cos\frac{4\pi(d+9)}{365} + 0.00127\overline{R} - 0.0635\right]e^{-\frac{(L-2)}{1.5}}$$

- Location of the plasmapause:

$$L_p = 5.6 - 0.46Kp,_{\max}$$

- Width of the plasmapause (Gallagher et al. [2000]), neglecting local time dependence:

$$L_w = 0.14$$

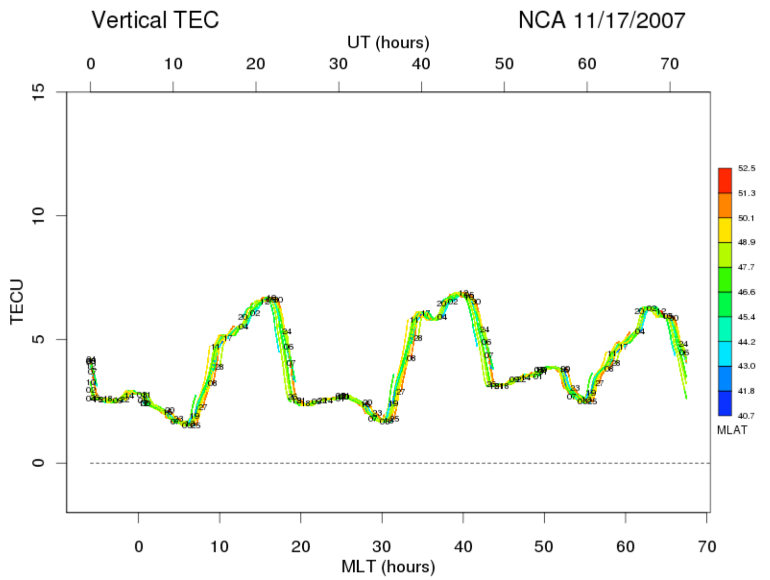
- Electron density in the trough (Sheeley et al. [2001]) neglecting local time dependence:

$$n_e^t = 124(3/L)^4$$

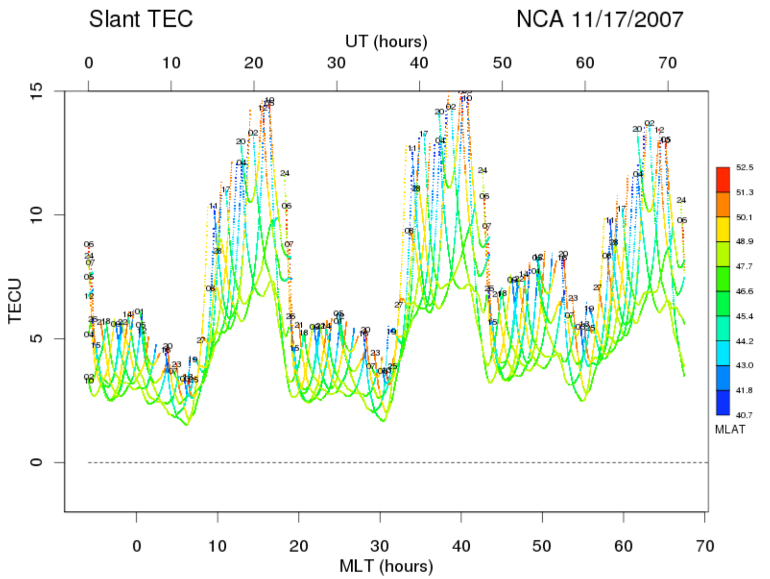
- Regions spliced together using tanh step function
- Integration of electron density from 700 km to 20,200 km along signal path gives $P(\alpha, \varepsilon)$
- Model very simple, but Kalman filter will scale the results to best fit the measurements

A Numerical Experiment: Idealized Ionosphere Plus Plasmasphere

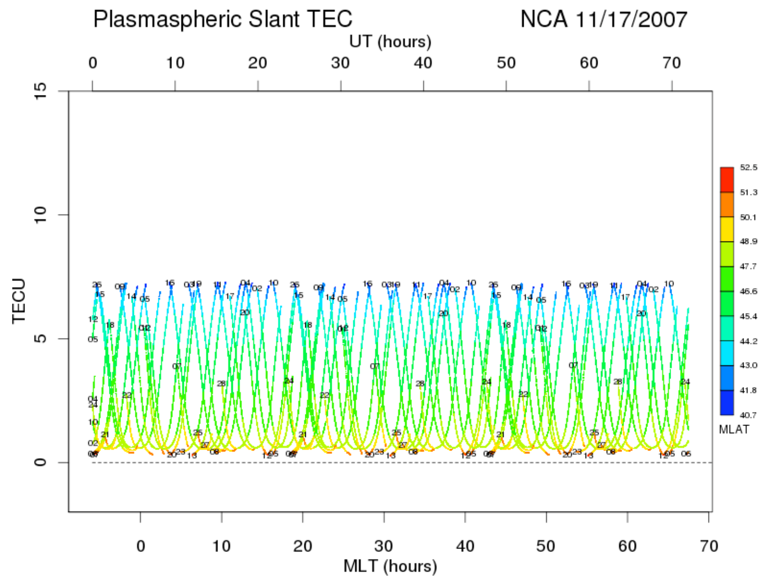
Assume ionosphere is an idealized thin slab



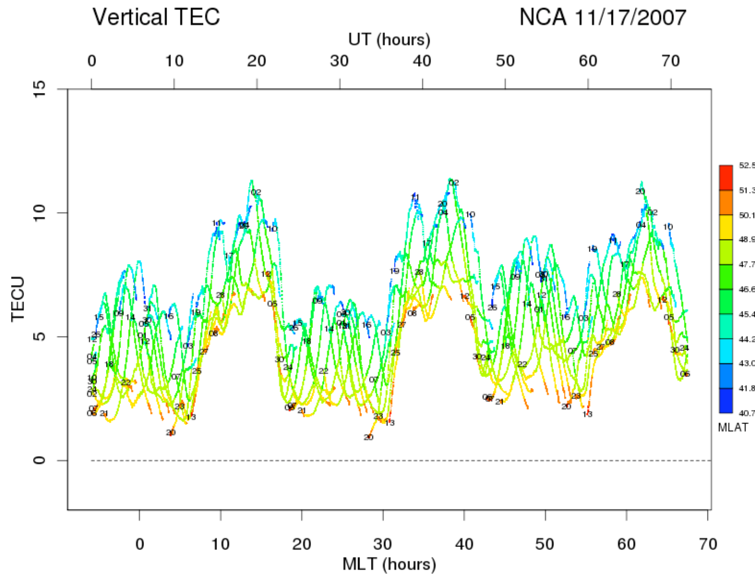
Construct slant TEC via thin-shell mapping fn



Add slant TEC through model plasmasphere

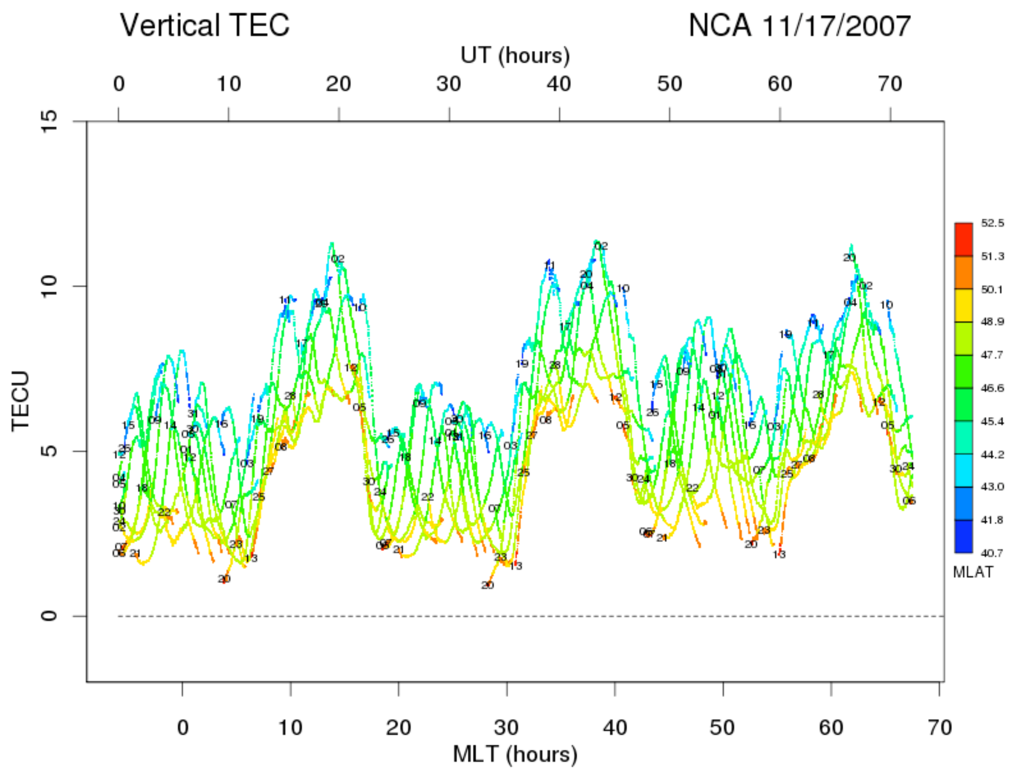


Verticalize the results

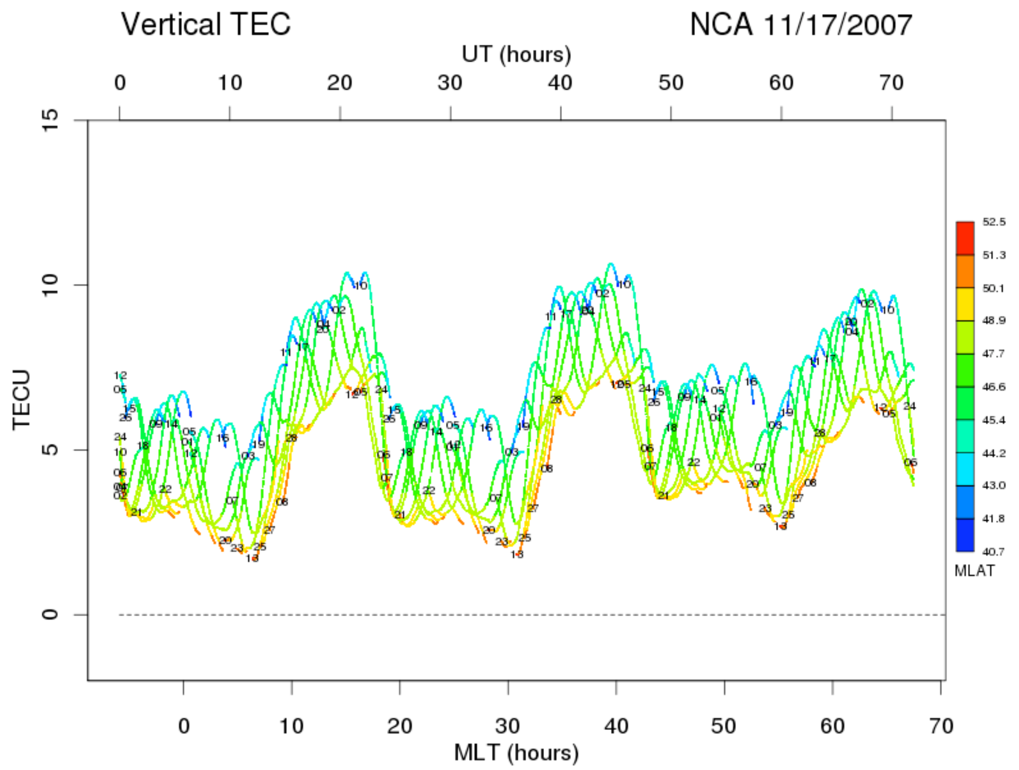


Comparing the Idealized and Estimated Total TEC

TEC When Neglecting Plasmasphere



Idealized Ionosphere and Plasmasphere



The Kalman Filter Implementation (with Plasmasphere)

Observation equation (for the i^{th} GPS receiver-satellite pair):

Measured slant TEC	Bilinear fit to ionospheric TEC_V	Plasmaspheric slant TEC	Instrumental biases

$$\underbrace{TEC_{RS}^{i}} = M\left(\varepsilon_{RS}^{i}\right)\left[\underbrace{a_{0,R}^{i} + a_{1,R}^{i}\cdot d\lambda_{RS}^{i} + a_{2,R}^{i}\cdot d\varphi_{RS}^{i}}\right] + \underbrace{a_{3,R}^{i}\cdot P\left(\alpha_{RS}^{i},\varepsilon_{RS}^{i}\right)} + \underbrace{b_{R}^{i} + b_{S}^{i}}$$

ε – Elevation

α – Azimuth

$d\lambda$ – Difference between MLT at ionospheric penetration point and station

$d\varphi$ – Difference between MLAT at ionospheric penetration point and station

$P(\alpha, \varepsilon)$ – PTEC from Carpenter-Anderson et. al [1992] (scaled to fit observations)

b_R, b_S – Receiver and satellite instrumental biases

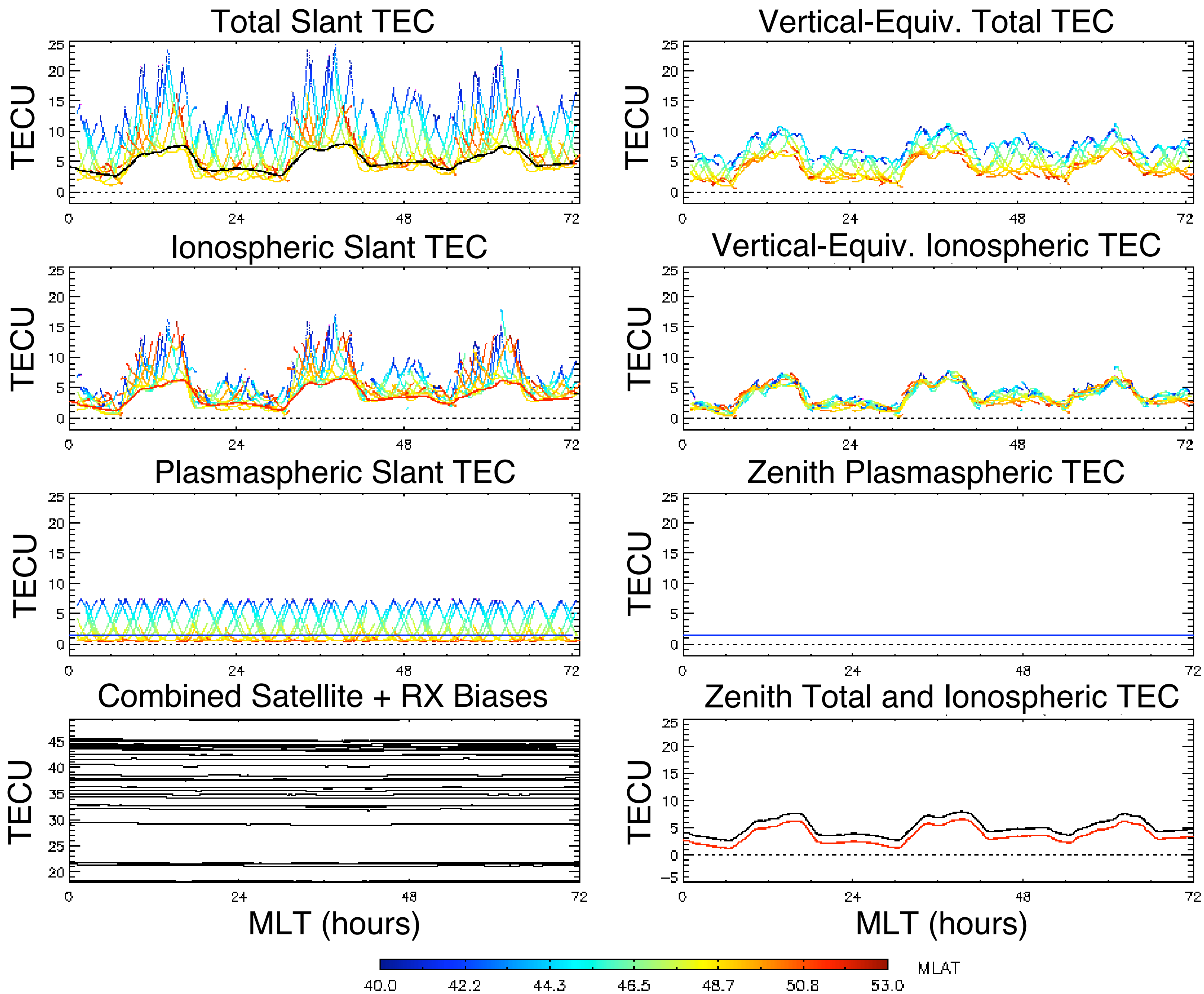
Thin shell mapping function
(for the ionosphere only)

$$M(\varepsilon) = \sec\left[\arcsin\left(\frac{R_e}{R_e + h}\right)\cos(\varepsilon)\right]$$

The Kalman Filter Implementation (with Plasmasphere)

Kalman state vector (unknowns)	Ionospheric fit parameters: $a_{0,R}^k\ a_{1,R}^k\ a_{2,R}^k$; Plasmaspheric scaling: $a_{3,R}^k$; Instrumental biases: $b_R^k\ b_{S_1}^k\ b_{S_2}^k \cdots b_{S_N}^k$ $\mathbf{X}_k = \left[a_{0,R}^k \quad a_{1,R}^k \quad a_{2,R}^k \quad a_{3,R}^k \quad b_R^k \quad b_{S_1}^k \quad b_{S_2}^k \cdots b_{S_N}^k \right]^T$
Measurement vector (knowns)	Measured slant TEC: $\mathbf{y}_k = \left[TEC_{RS_1}^k \quad TEC_{RS_2}^k \quad \cdots \quad TEC_{RS_N}^k \right]^T$
Kalman process to be estimated	$\mathbf{X}_k = \mathbf{\Phi}_{k,k-1} \cdot \mathbf{X}_{k-1} + \mathbf{w}_{k-1}$ $\mathbf{y}_k = \mathbf{H}_k \cdot \mathbf{X}_k + \mathbf{v}_k$ • Identity state transition matrix • Zero-mean white Gaussian process noise $\mathbf{w}_k$ and measurement noise $\mathbf{v}_k$ • Kalman updates performed every 60 seconds (each new data epoch)

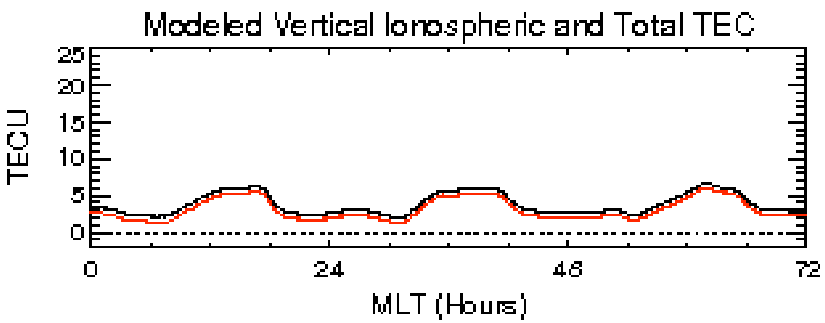
Results at Greensboro (36°, 280°) on 17-19 Nov 2007

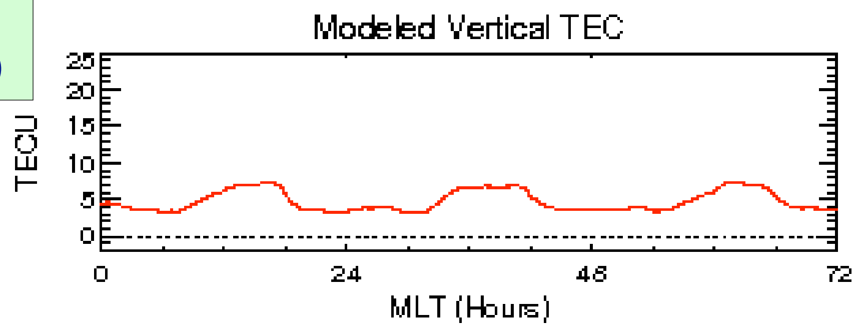


Impacts of Ignoring the Plasmasphere when Estimating TEC

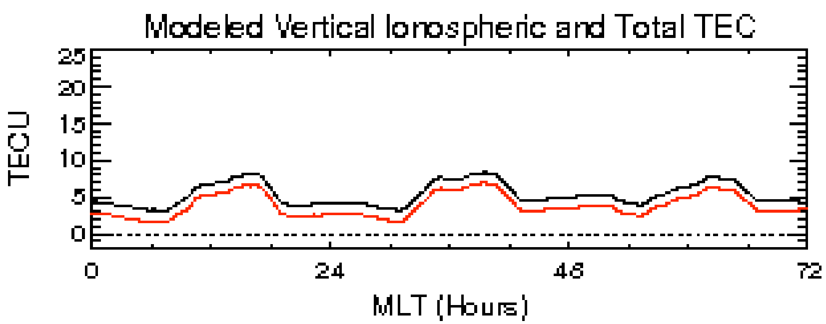
Accounting for Plasmasphere	Ignoring Plasmasphere

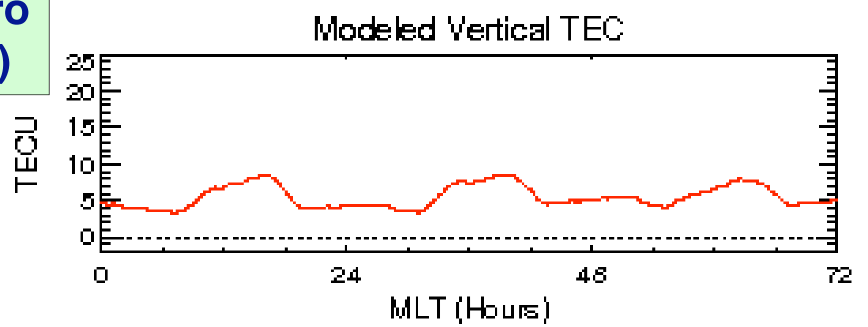
Haystack (43°, 288°)



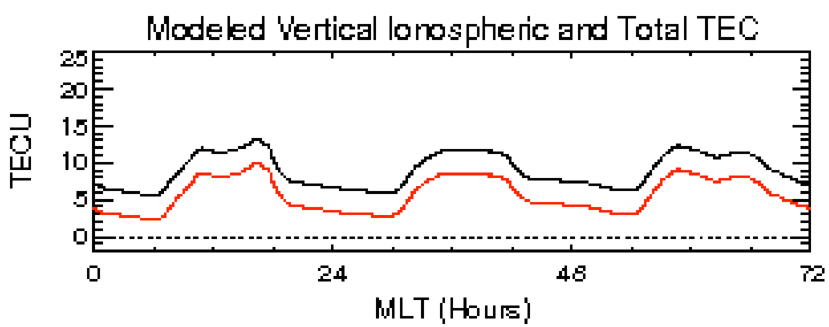


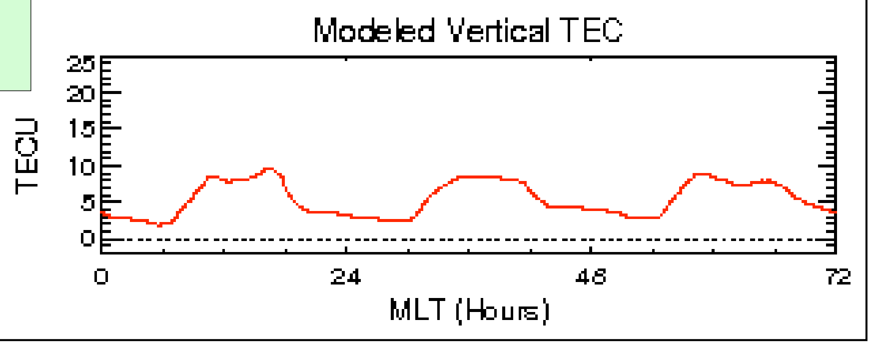
Greensboro (36°, 280°)





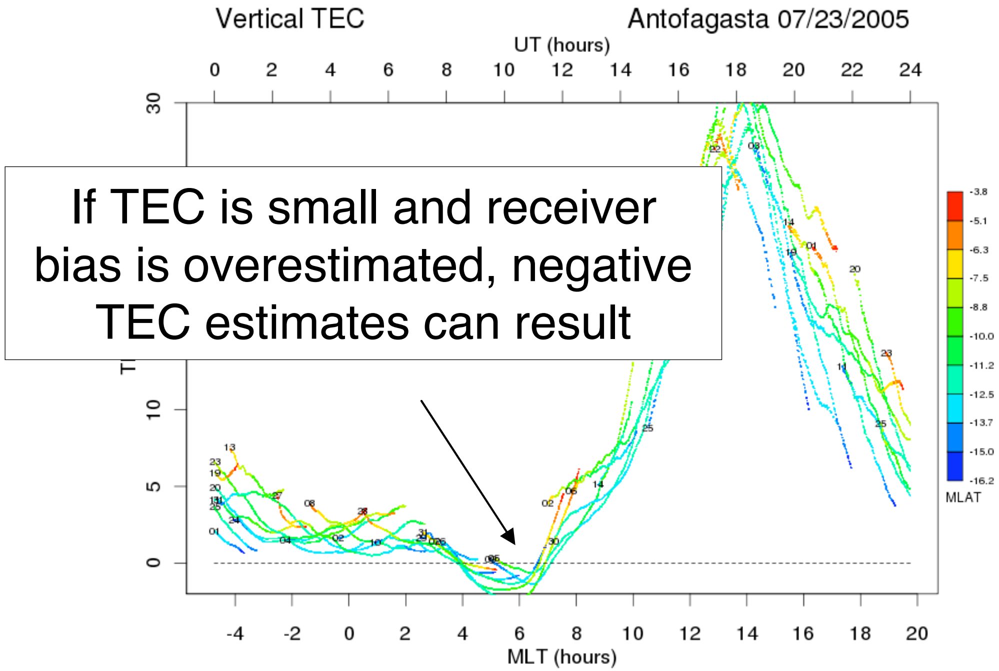
Roatan (16°, 273°)

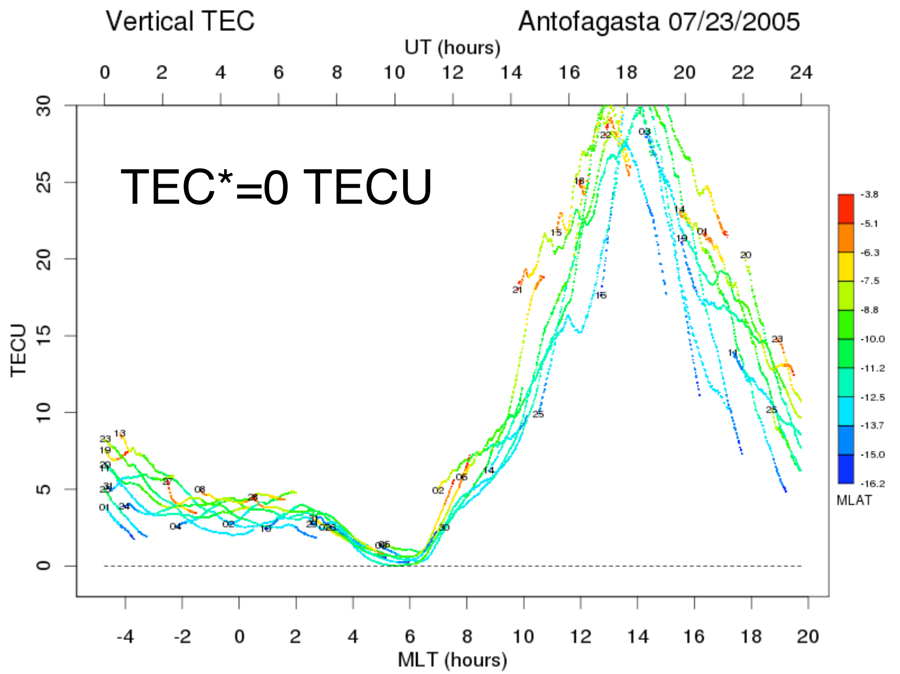




Neglecting the plasmasphere tends to cause overestimation of the total TEC at middle latitudes and underestimation at equatorial latitudes.

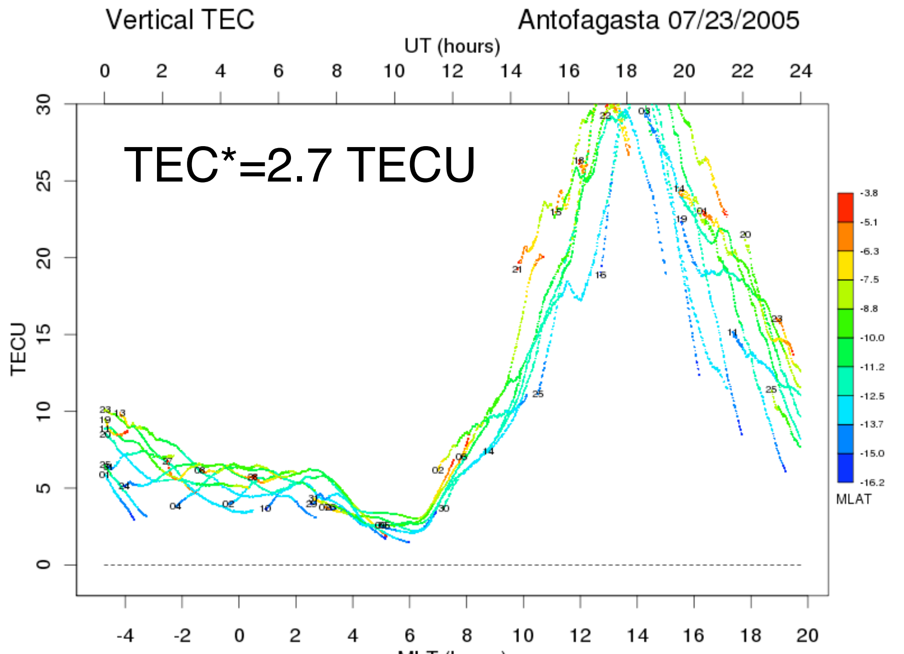
Why the Estimated TEC Can Be Negative and What to Do About It





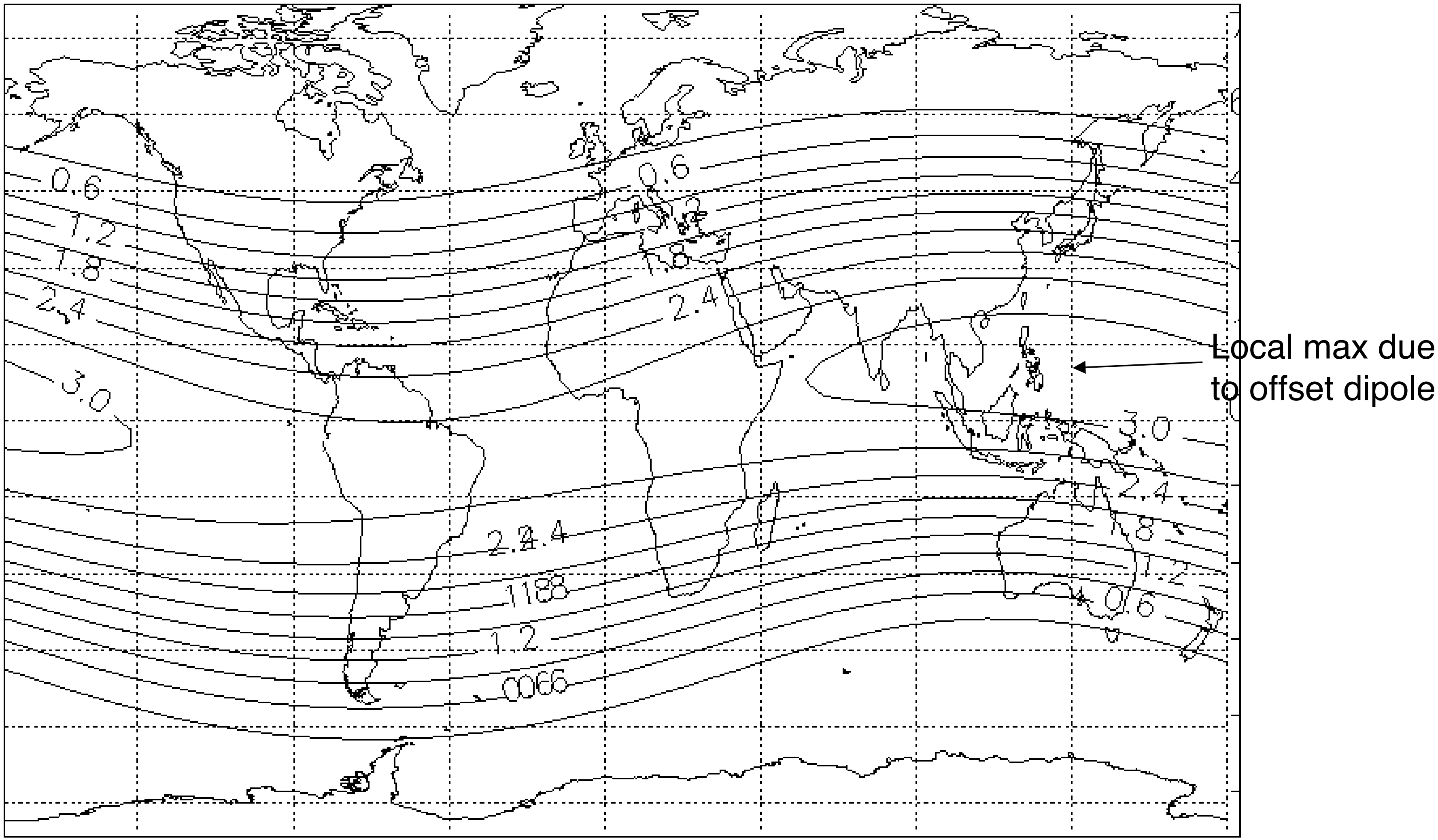
When this happens, we fall back on the simple approach: choose the bias to enforce that min(TECS) = TEC*

Now, however, we can make a more informed selection of TEC*. A reasonable value to use is the (zenith) plasmaspheric contribution according to the Carpenter-Anderson model.



Plasmaspheric Contribution to the GPS TEC According to the Carpenter-Anderson Plasmasphere Model

Simulation conditions: 13 month average solar flux = 7.9; Kp=1; Day of year = 1



Conclusions

- When estimating the GPS instrumental biases from the measurements we must make various assumptions about the structure of the ionized regions traversed by the signals
- Inaccuracies in estimation of the biases can be expected when these assumptions are violated. Phenomena that cause difficulty in estimating the biases include:
  - Ionospheric structure and scintillation
  - The contribution to the GPS TEC from the plasmasphere
- Neglecting the plasmasphere tends to cause overestimation of the total TEC at middle latitudes and underestimation at equatorial latitudes.
- Software to perform the calibrations using the Kalman filter approach (with and without the plasmasphere term) is available upon request. We will demonstrate this software during Wednesday's TEC calibration laboratory.
- A manuscript (draft) recently submitted to Radio Science describing the technique is also available upon request