



**The Abdus Salam
International Centre for Theoretical Physics**



2025-45

Satellite Navigation Science and Technology for Africa

23 March - 9 April, 2009

Satellite Navigation

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Satellite Navigation Science and Technology for Africa

The Abdus Salam International Center for Theoretical Physics, Trieste, Italy

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GPS Satellite (Block IIF)

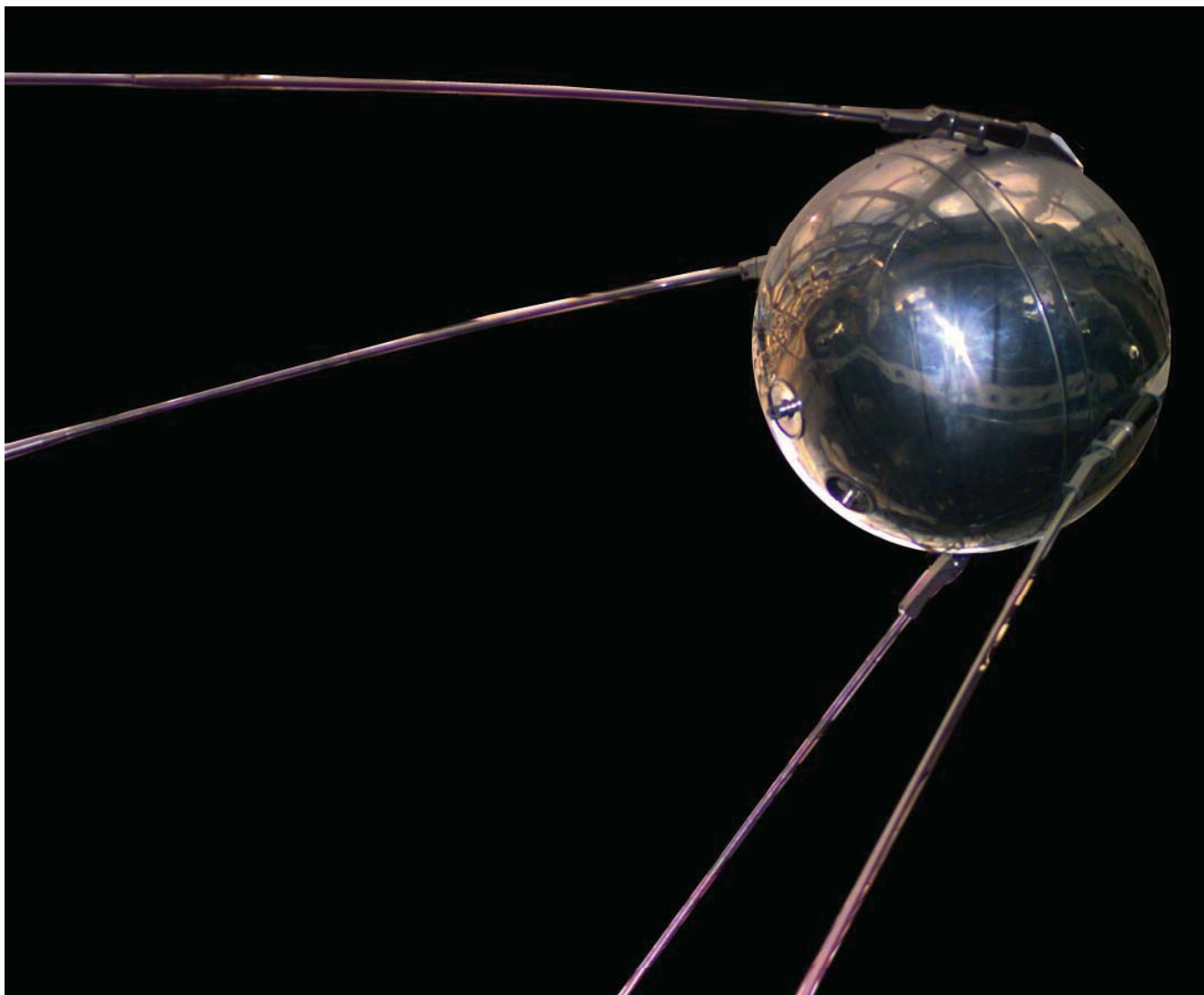
An Overview of Satellite Navigation

Pratap Misra

Objectives



- **To convey:**
 - **A broad understanding of the scientific and engineering principles of satellite navigation**
 - **The rudiments of GPS:**
 - **System**
 - **Signals and measurements**
 - **Performance**
 - **An outline of global navigation satellite systems under development: GLONASS, Galileo, Beidou**
- **Comprehensive discussions of these topics (and more) to follow later this week**

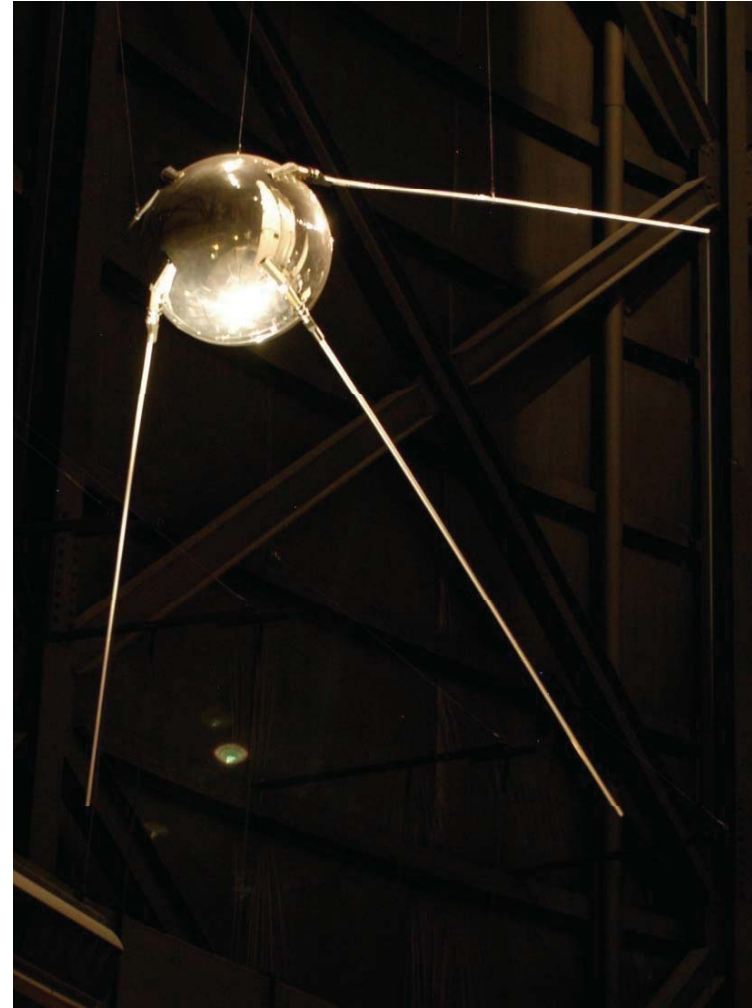


?

Простейший Спутник-1



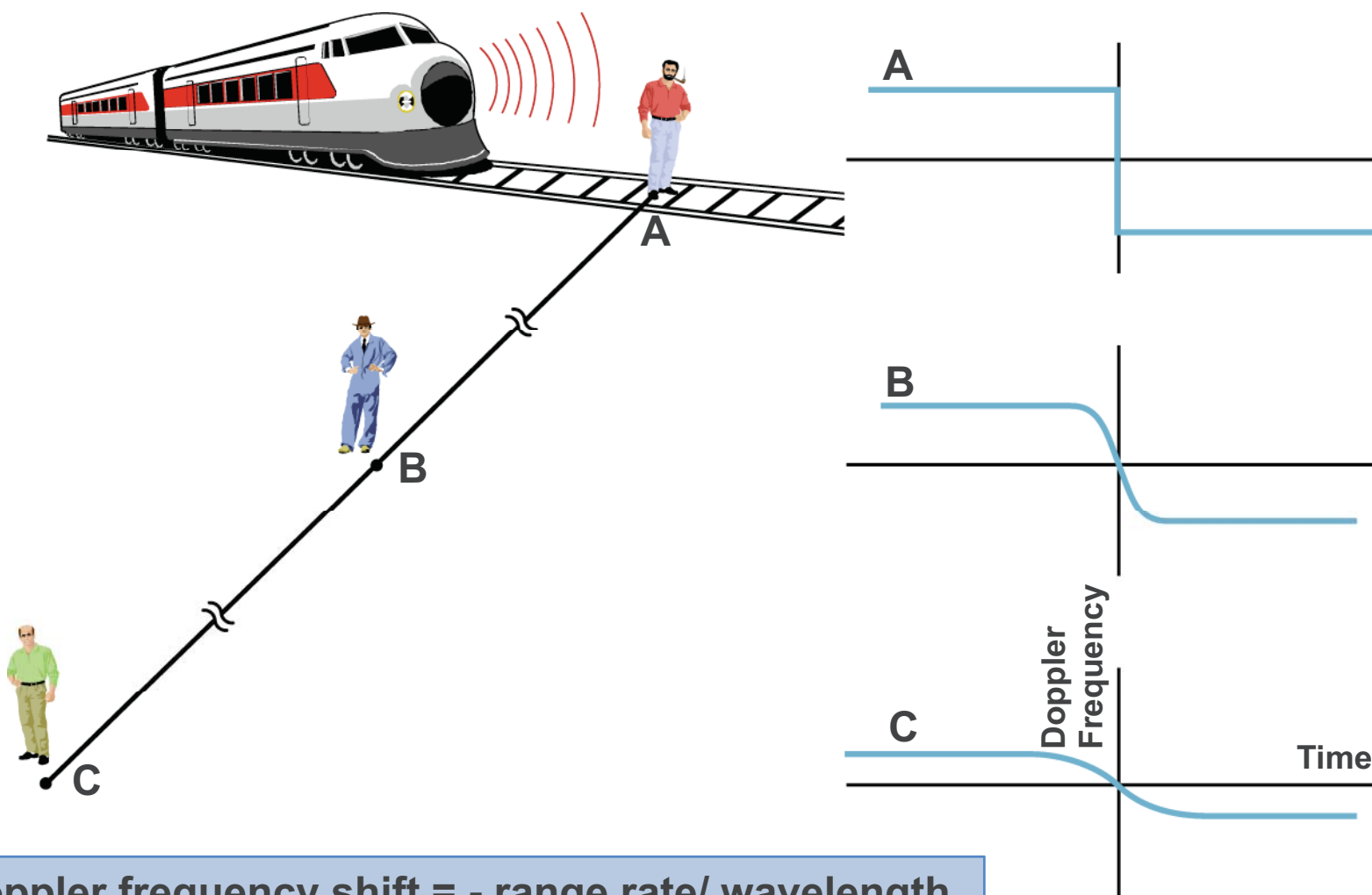
- Space age began with the launch of Sputnik I by the Soviet Union on 4 October 1957
- 'Beeps' heard on short-wave radios tuned to 20 MHz or 40 MHz, Doppler shifted as the satellite moved in the sky*
- Within days, the idea of using radio signals from space for positioning on the earth was born



*<http://history.nasa.gov/sputnik>



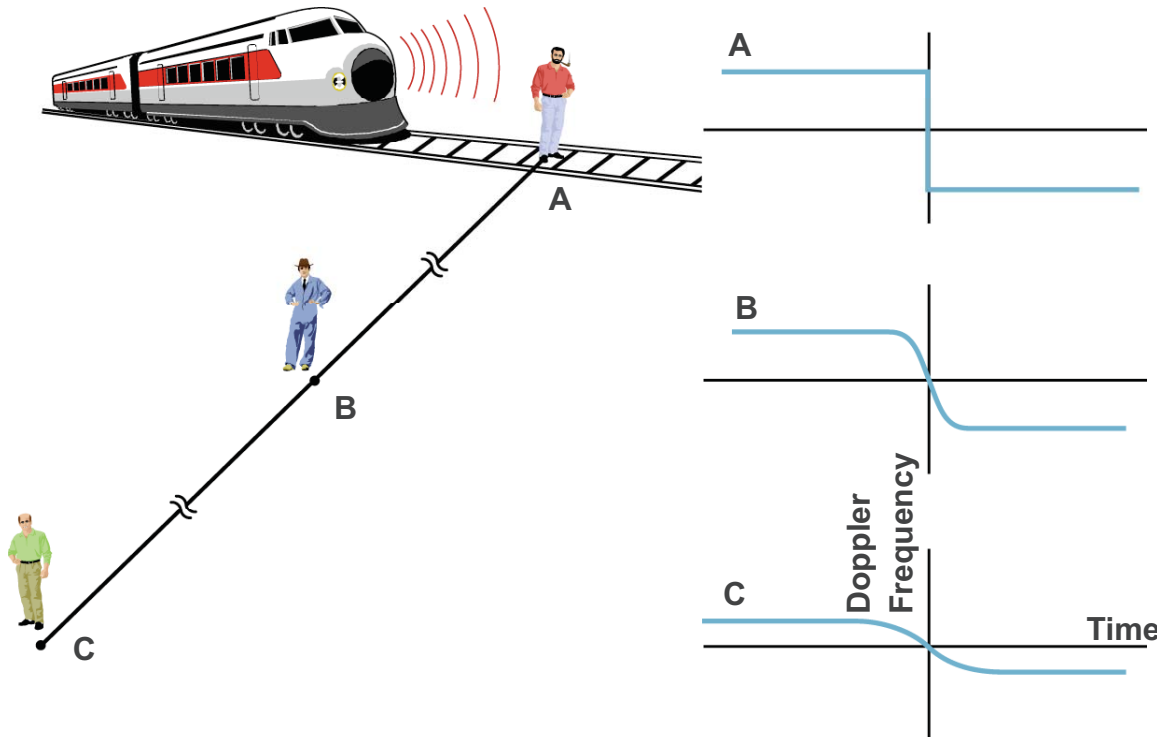
Doppler Effect



$$\text{Doppler frequency shift} = - \text{range rate} / \text{wavelength}$$

Doppler Positioning

A Conceptual Exercise



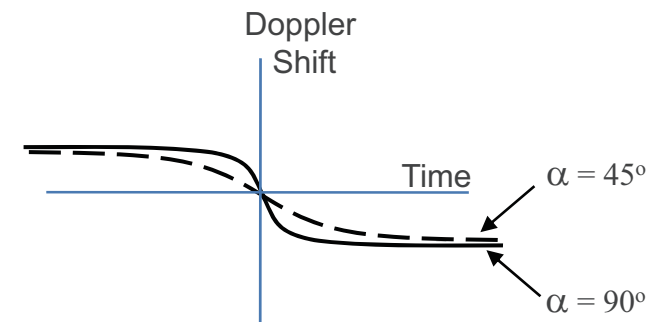
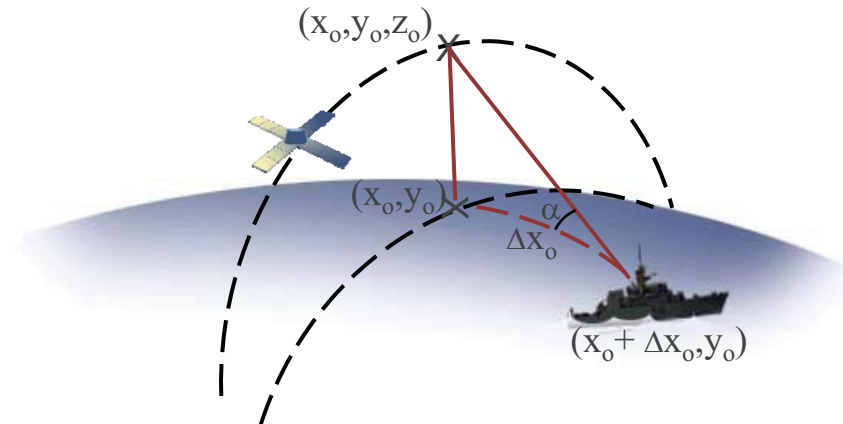
- Record time when Doppler shift went through zero
- **Along-track position**
 - From train's schedule
 - Error sources: Watch off, train off schedule
- **Cross-track position**
 - From Doppler profile
 - Ambiguity: Which side of the track?

$$\text{Doppler frequency shift} = - \text{range rate} / \text{wavelength}$$

A Global Satellite Navigation System based on Doppler Positioning



- **Satellite transmits**
 - Frequency-stable signal
 - Time, orbital parameters, clock parameters
- **Receiver measures Doppler frequencies and records transmitted data for an entire pass**
 - Determine coordinates of the point on the ground track corresponding to the point of closest approach
 - Determine offset from the ground track
- **Error Sources**
 - Satellite clock frequency stability over 10-20 min
 - User velocity

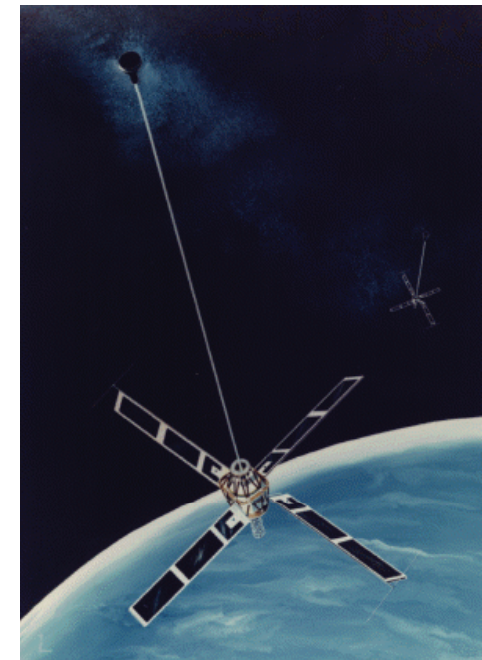
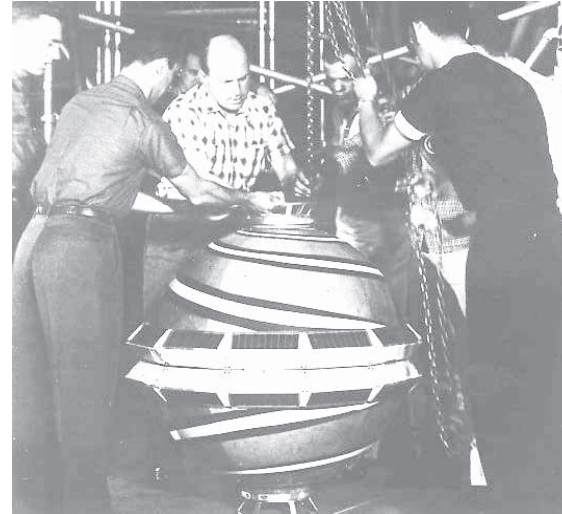


Adapted from *Marine Electronic Navigation*
by Appleyard et al.

Transit (1964-1996)



- 4-7 satellites in 1100-km, circular, polar orbits
- One satellite in view at a time
- A satellite pass lasted 10-20 min; up to 100-min wait between passes
- Satellite weight: 50 Kg (160 Kg)
- Signals at 150 MHz & 400 MHz
- Signal power: 1 watt
- 2-D Positioning accuracy (for a stationary users): 25 m
- ~ 10,000 receiver sets in 1980, cost: ~\$25,000



Satellite Navigation Overview

Outline

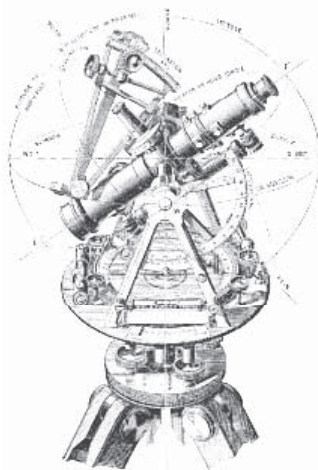


- **Principles of Satellite Navigation**
- **GPS Overview: System, Signals and measurements, Performance**
- **Applications and Performance Metrics**
- **Potential Partners/Rivals: GLONASS, Galileo, BeiDou/Compass, ...**

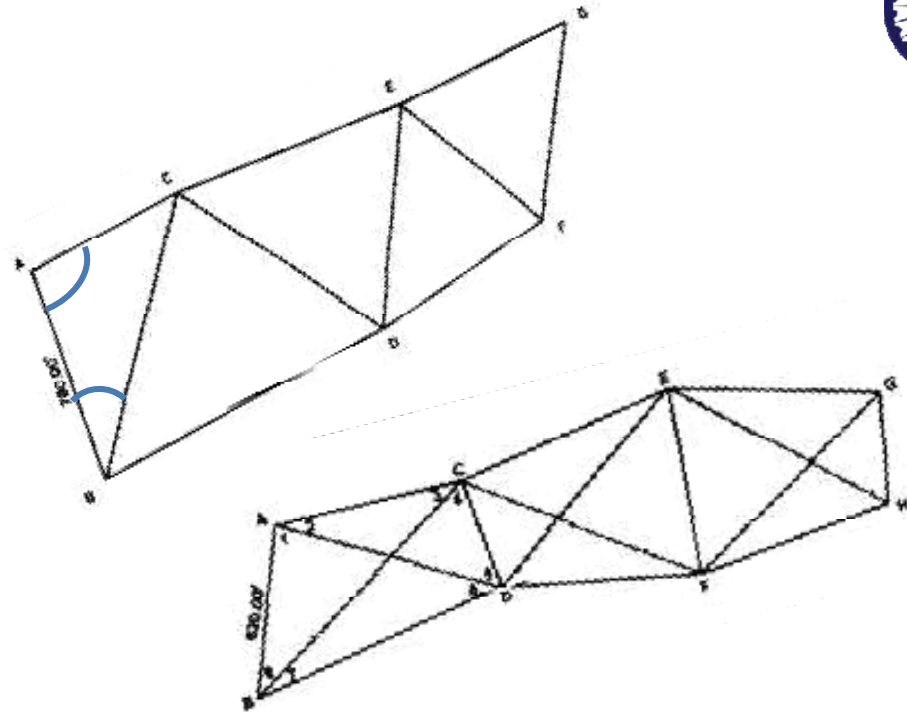


Triangulation

Method of determining the position of a fixed point from the angles to it from two fixed reference points a known distance apart



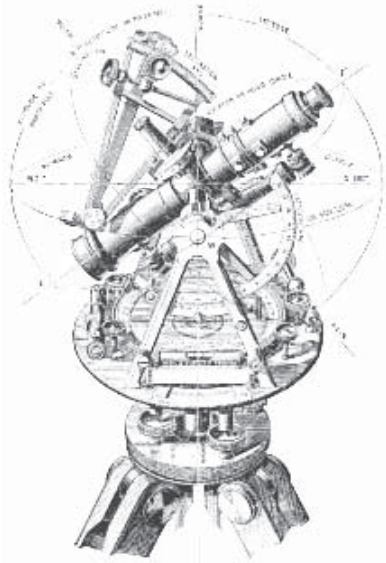
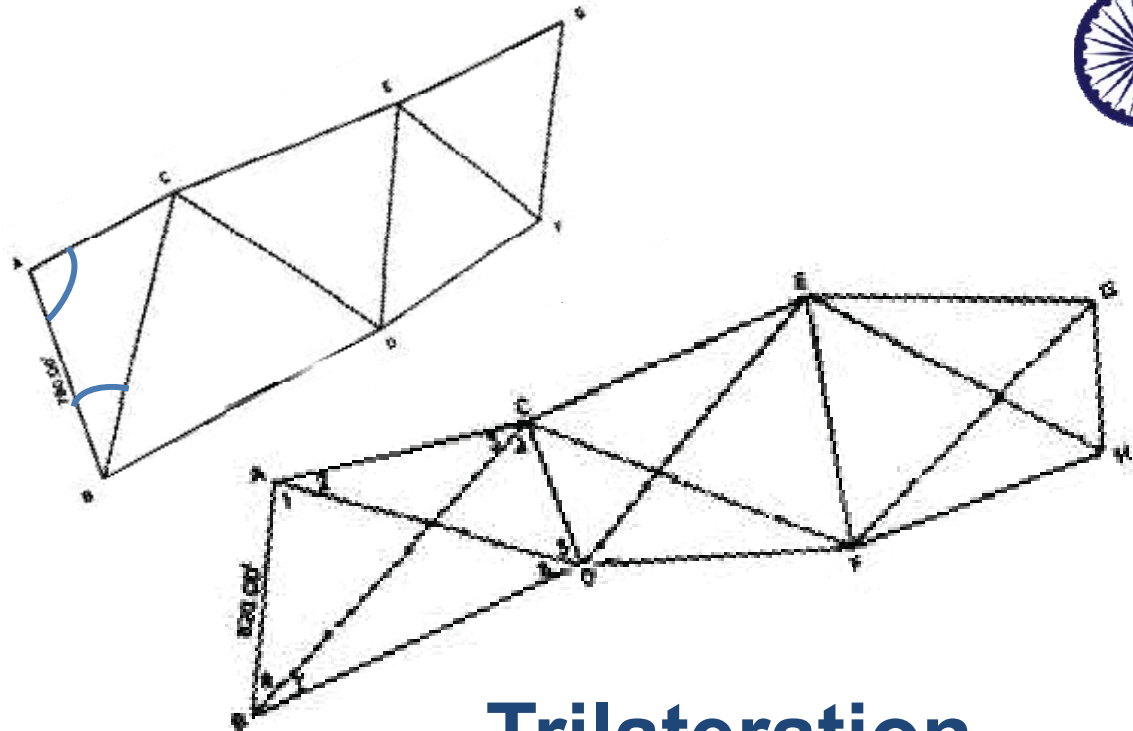
from *Trigonometry Surveying and Navigation* by G.A. Wentworth



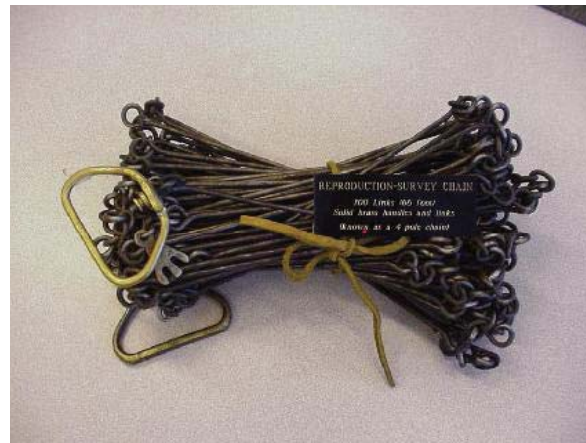


Triangulation

Method of determining the position of a fixed point from the angles to it from two fixed reference points a known distance apart



from *Trigonometry Surveying and Navigation* by G.A.Wentworth



Surveyor's chain
from www.landsurveyinghistory.ab.ca

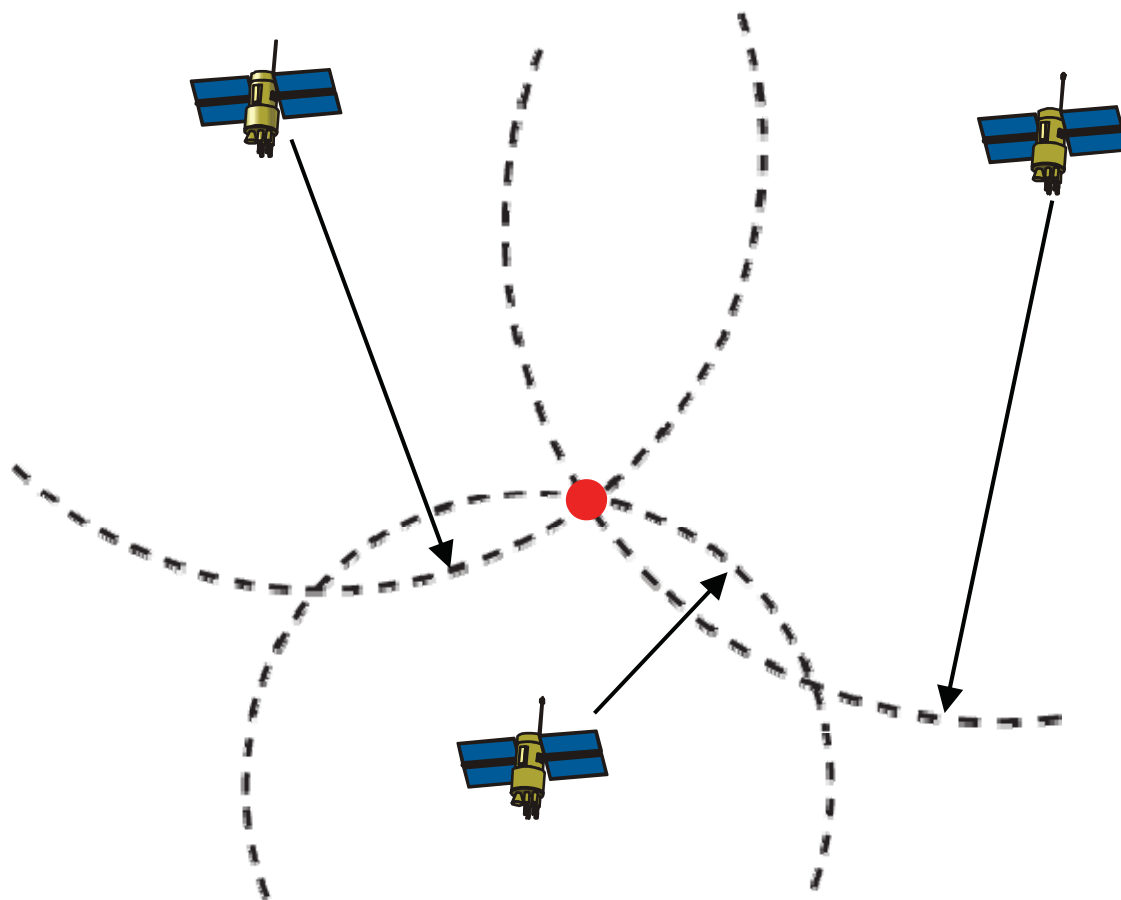
Trilateration

Measure lengths of the sides of a triangle rather than angles

A chain = 100 links = 66 feet long,
80 chains make a mile.
A "rod" or "pole" is 1/4 of a chain, or
16-1/2 feet long. Thus "40 rods" is 10
chains, or 1/8 of a mile.

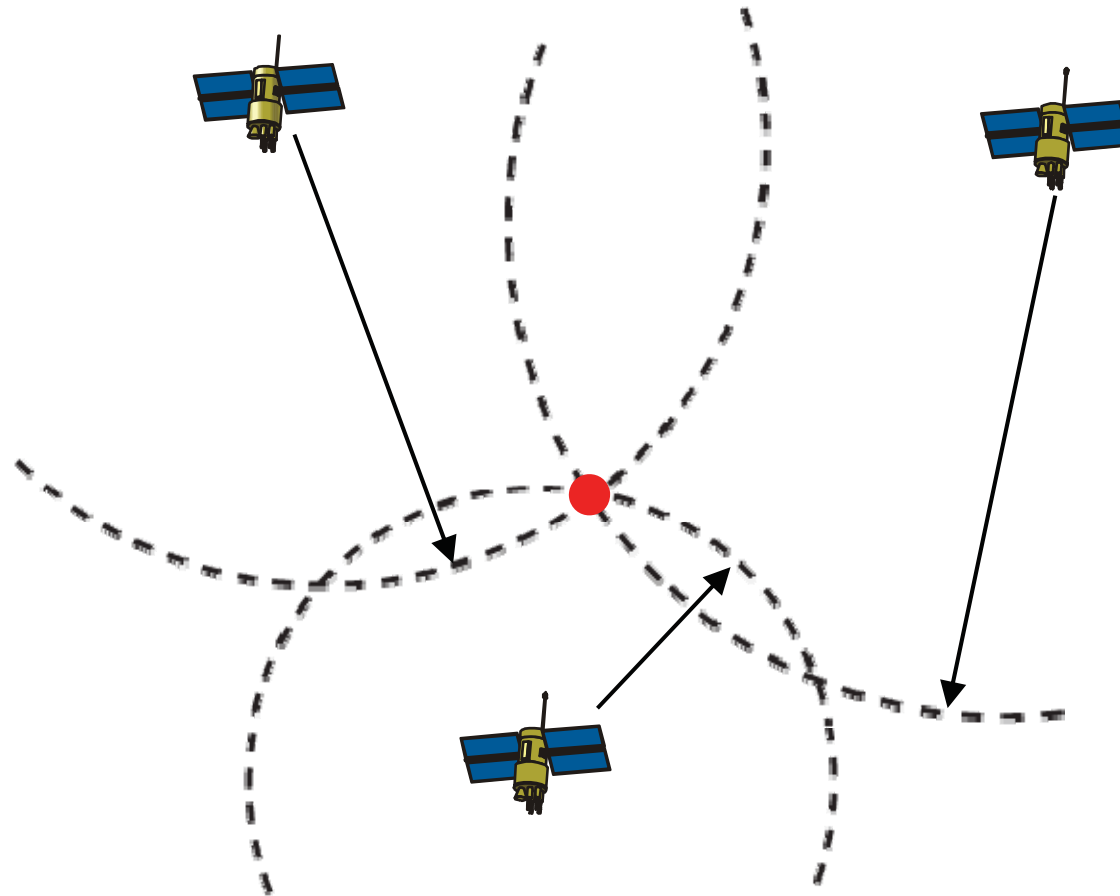


2-D Trilateration





Trilateration

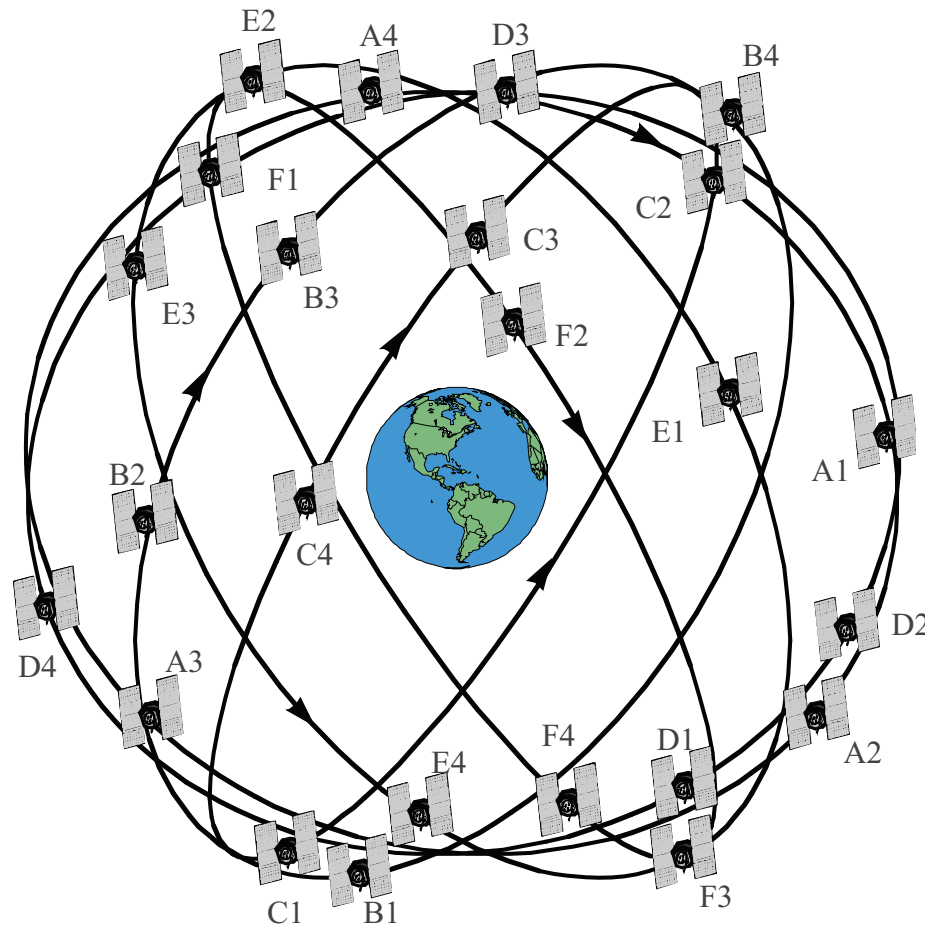


Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits

GPS Baseline Satellite Constellation

since 1980

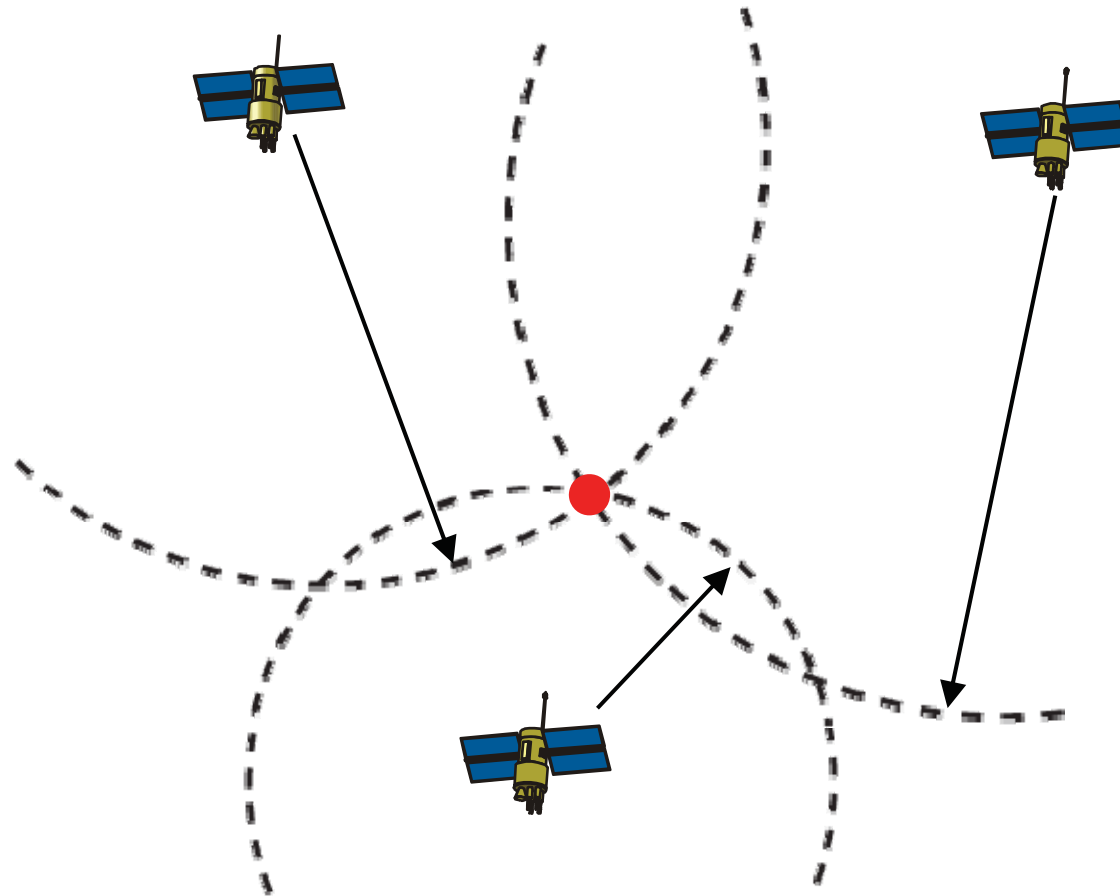


- Satellites: 24
- Orbital planes: 6
- Inclination: 55 deg
- Altitude: 20,000 km
- Period: 11 h, 58 min

- Actual number of satellites has exceeded 24 since 1995, and is currently 29
- U.S. Government intends to maintain at least 22 satellites in their nominal slots



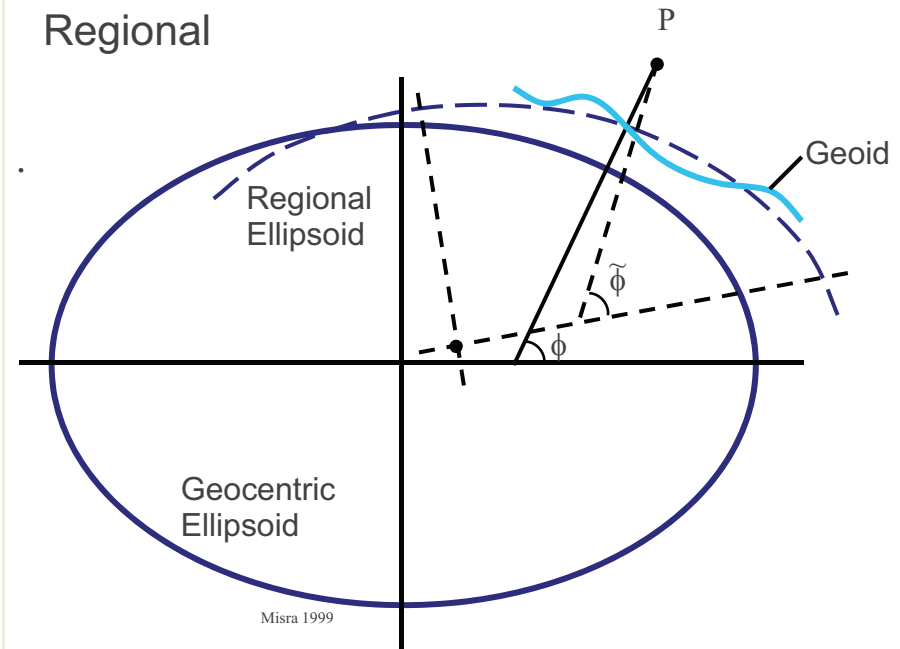
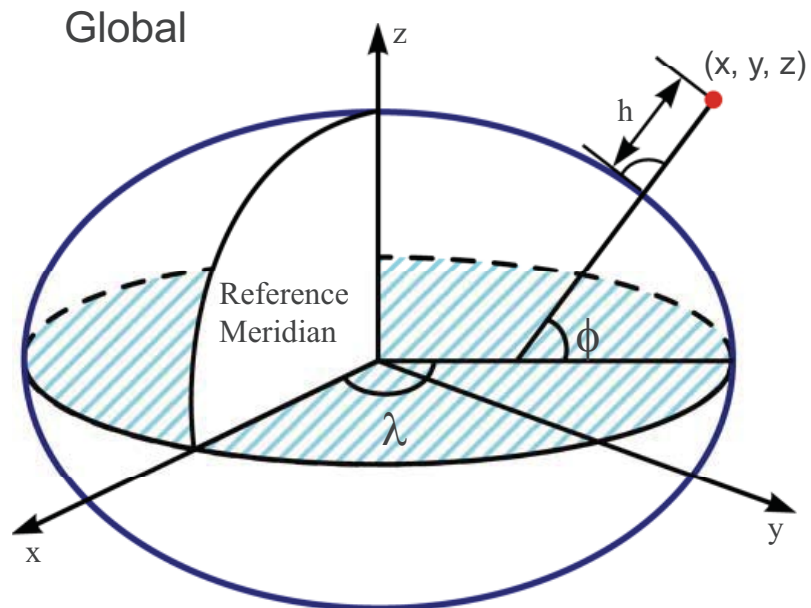
Trilateration



Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)

Global and Regional Coordinate Frames

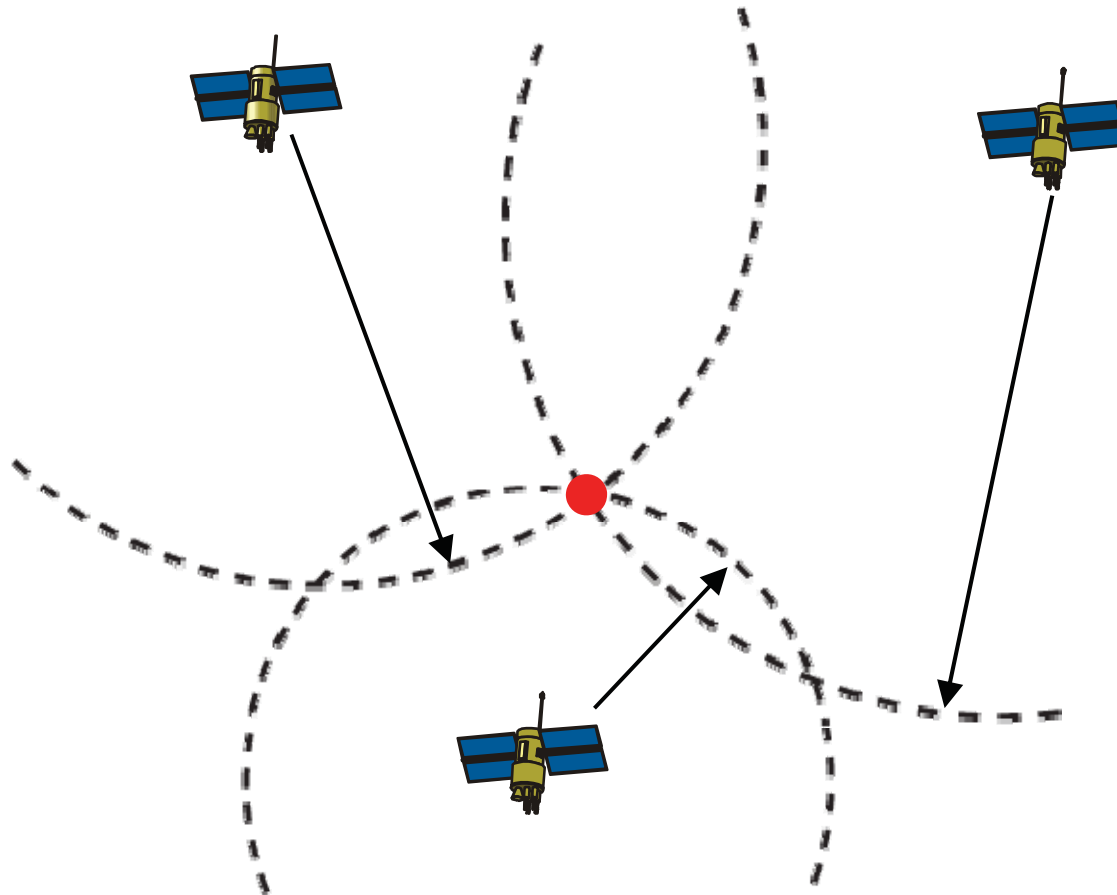


ϕ Geodetic latitude defined relative to a geocentric ellipsoid

$\tilde{\phi}$ Geodetic latitude defined relative to a regional ellipsoid



Trilateration



Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)
- Ultra-stable clocks aboard satellites to transmit synchronized signals

Frequency stability of 1 part in 10^{13} /day:

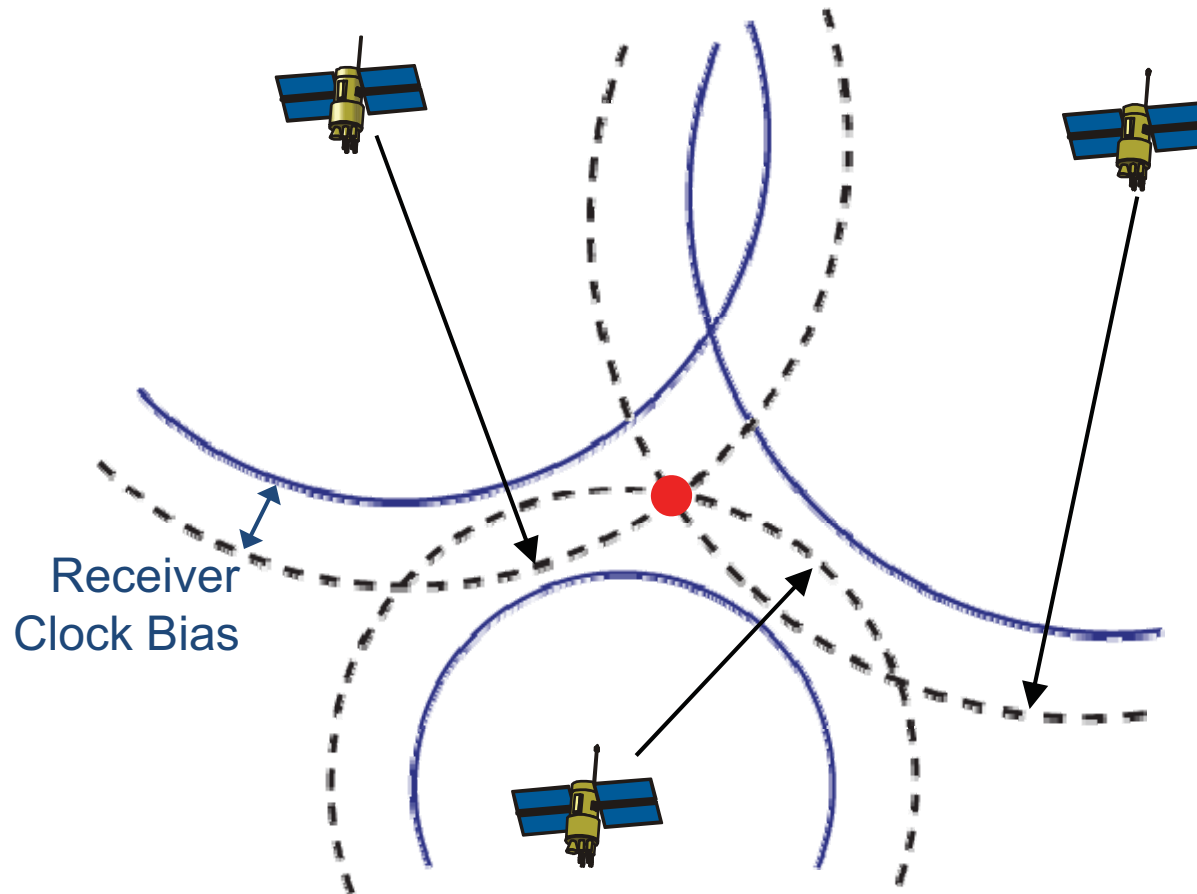
$$\frac{\Delta f}{f} = 10^{-13}$$

$$= -\frac{\text{Timekeeping error}}{\text{Time interval}}$$

Timekeeping error : ~10 ns/day



Trilateration



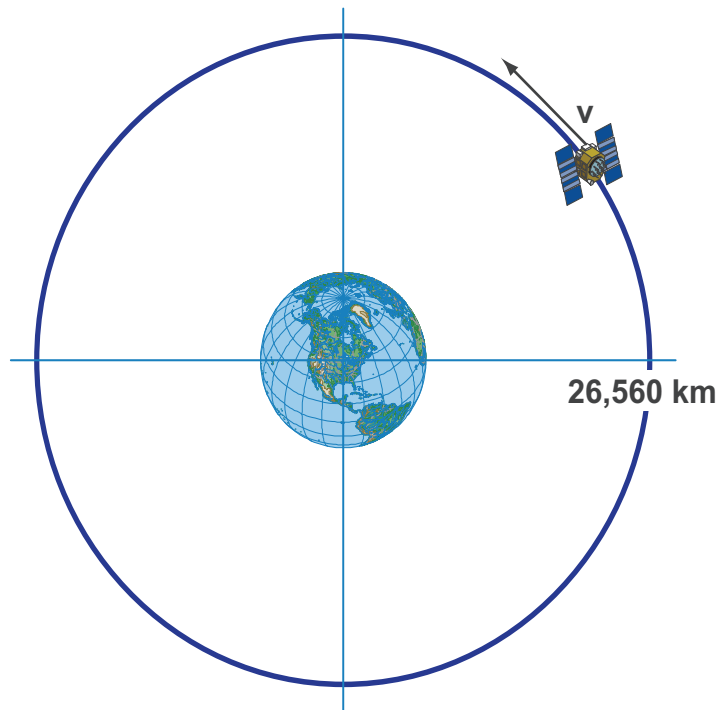
Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)
- Ultra-stable clocks aboard satellites to transmit synchronized signals, **but** inexpensive clocks in receivers

Relativistic Effects: Circular 12-h Orbit



Atomic clock drift: $\frac{\Delta f}{f} \approx 10^{-13}$



- Second-order Doppler shift (time dilation)

$$\frac{\Delta f}{f} = \frac{v^2}{2c^2} \approx 1 \times 10^{-10} \text{ (negative)}$$

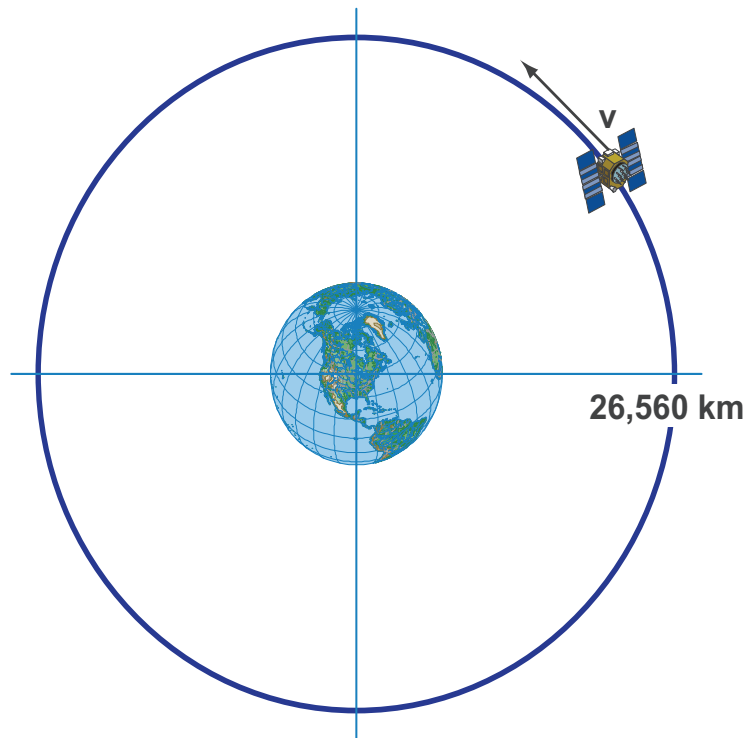
- Gravitational frequency shift

$$\frac{\Delta f}{f} = \frac{\Delta \Phi}{c^2} \approx 5 \times 10^{-10} \text{ (positive)}$$

Relativistic Effects: Circular 12-h Orbit



Atomic clock drift: $\frac{\Delta f}{f} \approx 10^{-13}$



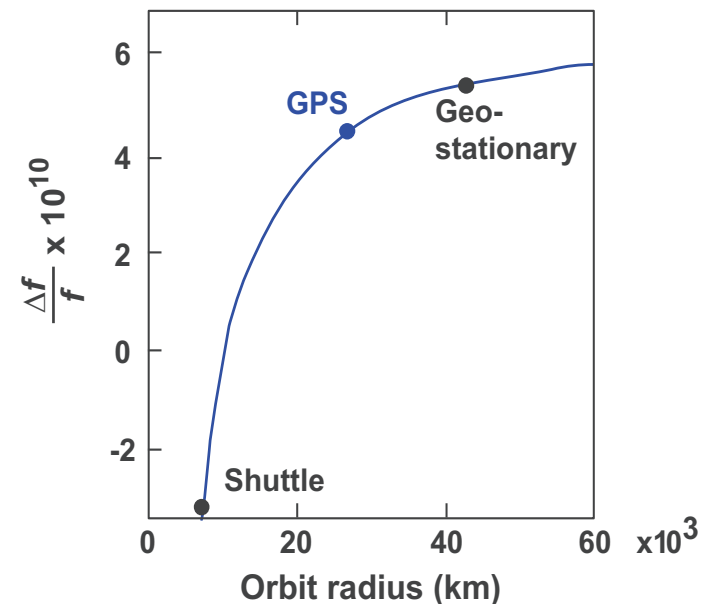
Combined effect accounted for by
“factory offset” of satellite clock by
 $\frac{\Delta f}{f} = -4.4645 \times 10^{-10}$

- Second-order Doppler shift (time dilation)

$$\frac{\Delta f}{f} = \frac{v^2}{2c^2} \approx 1 \times 10^{-10} \text{ (negative)}$$

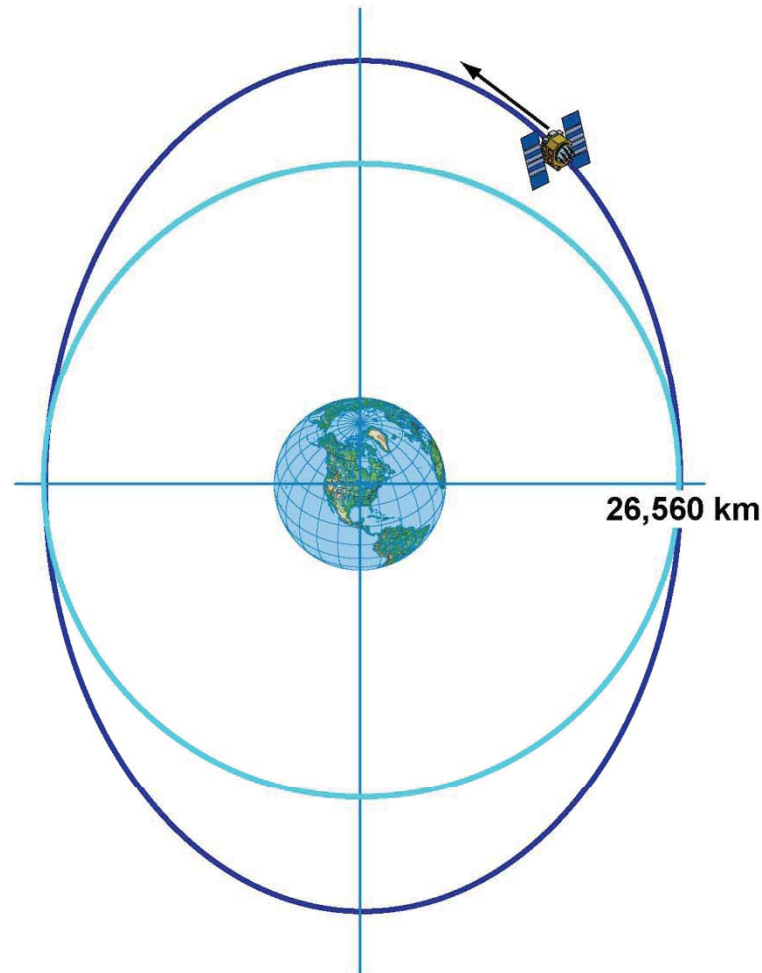
- Gravitational frequency shift

$$\frac{\Delta f}{f} = \frac{\Delta \Phi}{c^2} \approx 5 \times 10^{-10} \text{ (positive)}$$





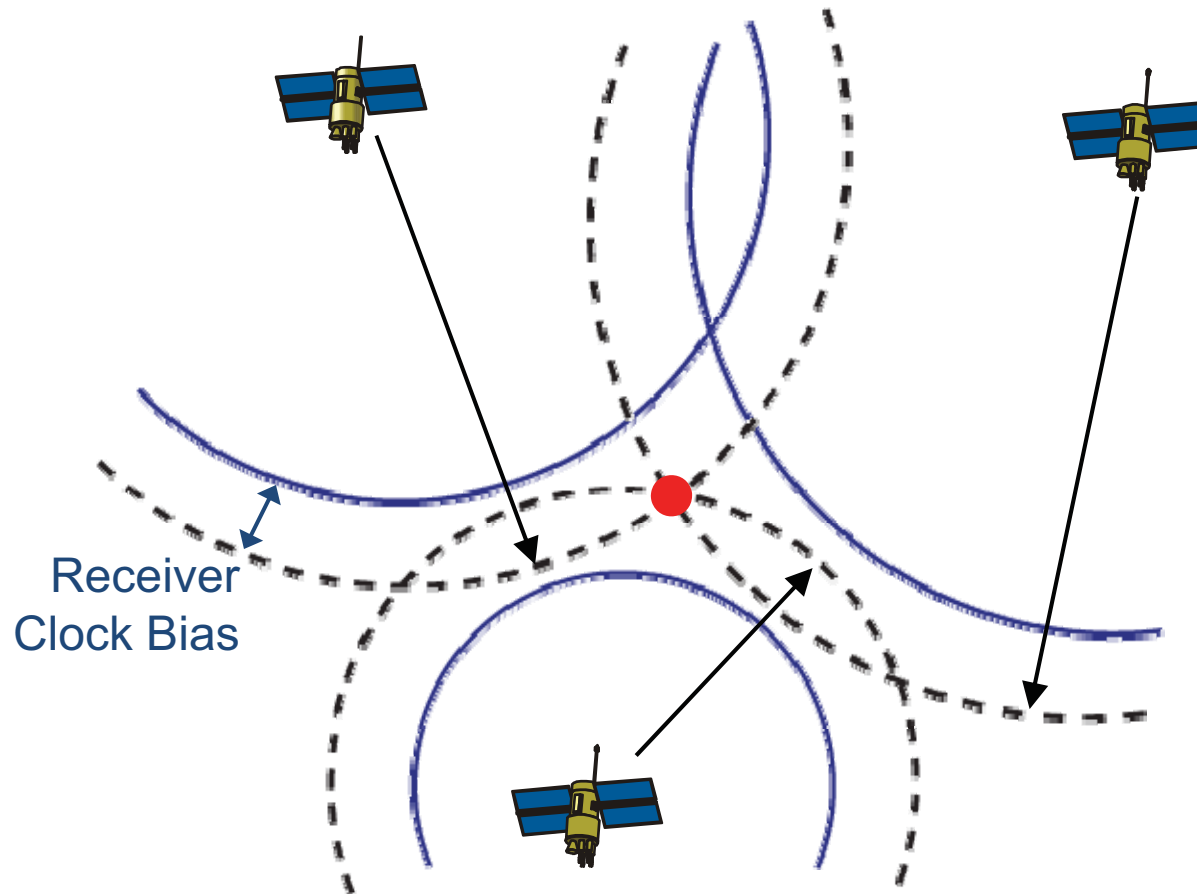
Relativistic Effects: Elliptical Orbit



- **Eccentricity effect**
 - Periodic shift of clock rate with 12-h period
 - 1% eccentricity: Periodic error with amplitude 28 ns in signal transit time (~ 10 m)
 - Accounted for in GPS receivers
- **Smaller Effects, generally neglected but compensated for automatically in differential GPS**
 - Sagnac effect
 - Shapiro delay
 - Nonspherical gravity potential
 - Tidel effects from sun and moon
 - Lense-Thirring drag



Trilateration



Satellite Navigation Enabling Technologies

- Stable space platforms with predictable orbits
- Global coordinate frame (Earth-centered, Earth-fixed)
- Ultra-stable clocks aboard satellites to transmit synchronized signals, but inexpensive clocks in receivers
- Integrated circuits: Compact, light, inexpensive receivers

Evolution of GPS Receivers

from 10 Kg to 100 g, 100 watts to 1 watt, \$ 100k to \$ 100



Early 1980s



USAF photo

2005

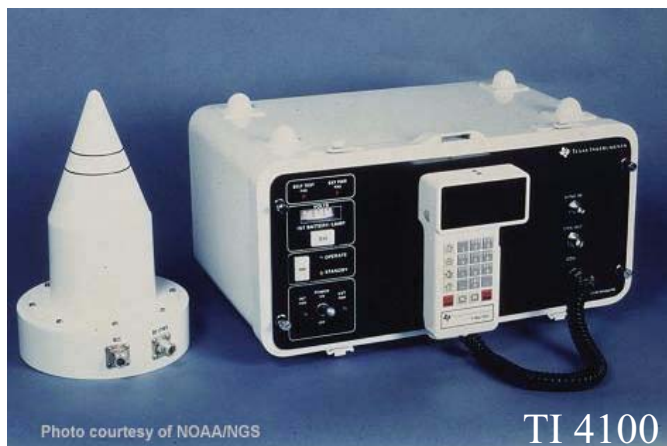
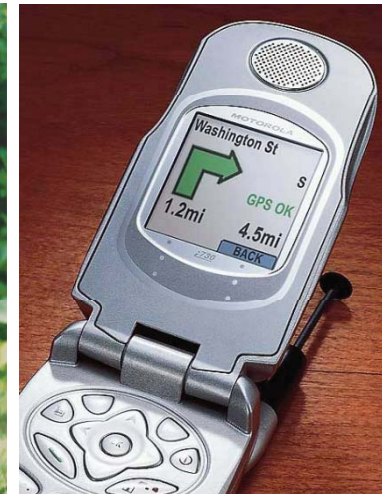
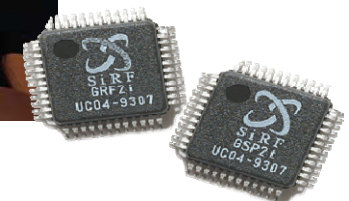


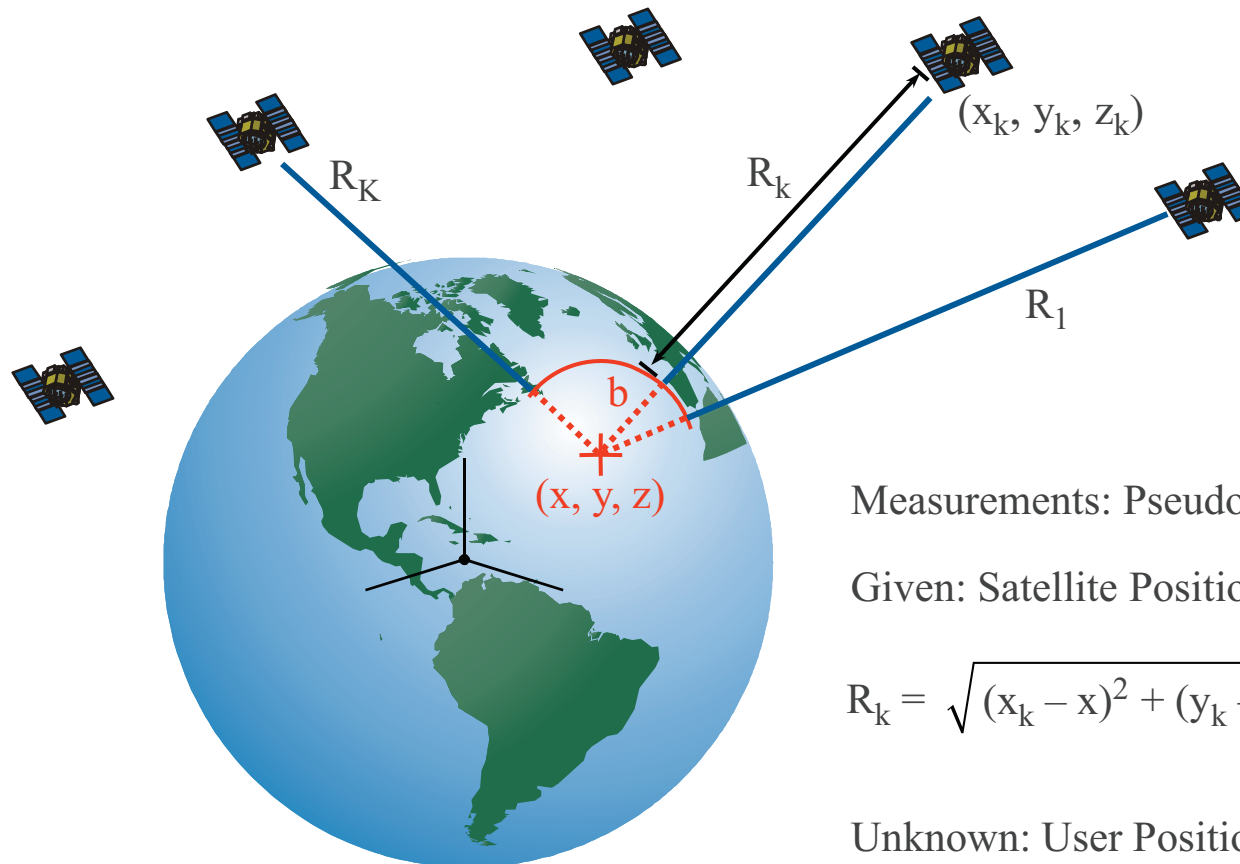
Photo courtesy of NOAA/NGS

TI 4100



Satellite Navigation

Position Estimation by Trilateration



Error Sources

- Ephemeris
- Satellite Clock
- Propagation through
 - Ionosphere
 - Troposphere
- Multipath
- Receiver Noise

Measurements: Pseudoranges $\{R_k\}$

Given: Satellite Positions $\{(x_k, y_k, z_k)\}$

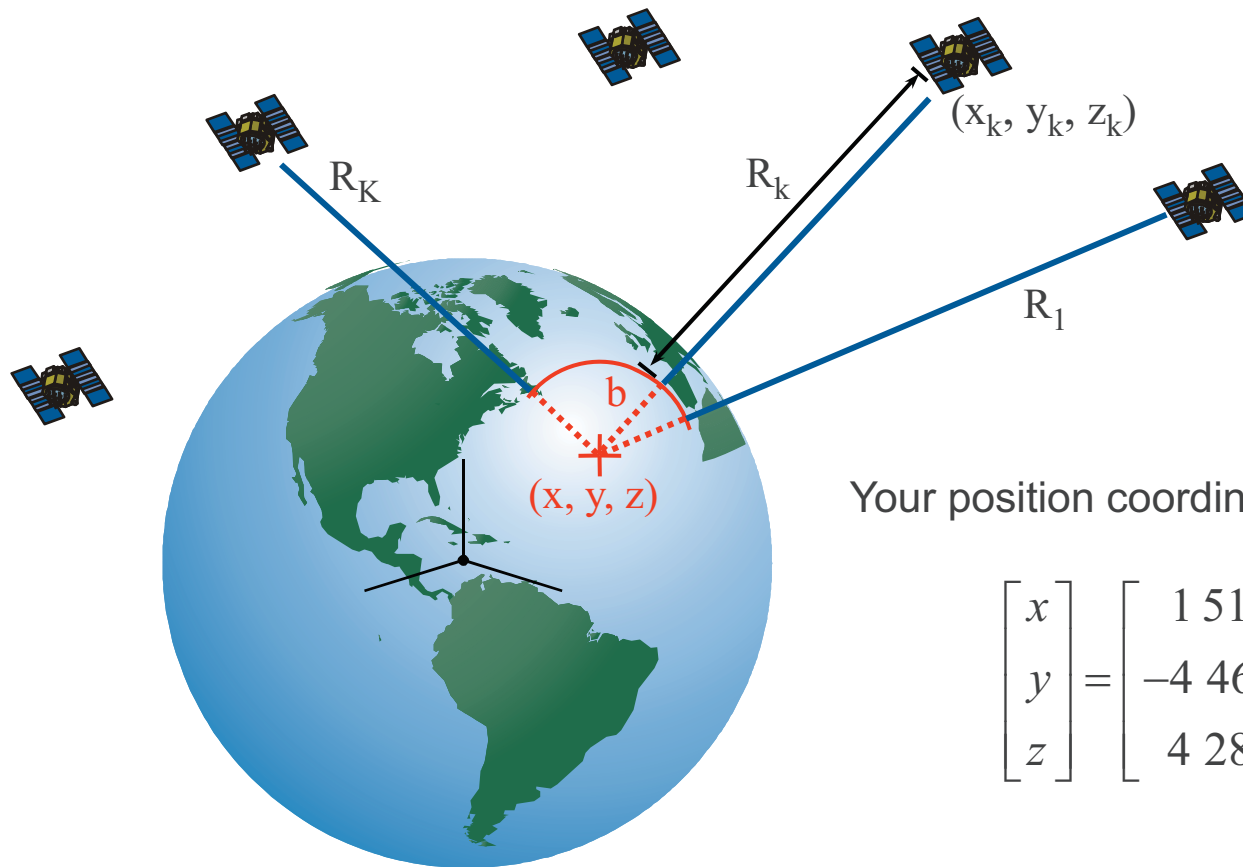
$$R_k = \sqrt{(x_k - x)^2 + (y_k - y)^2 + (z_k - z)^2} - b, \quad k = 1, 2, \dots, K$$

Unknown: User Position (x, y, z)

Receiver Clock Bias b

Satellite Navigation

Position Estimation by Trilateration



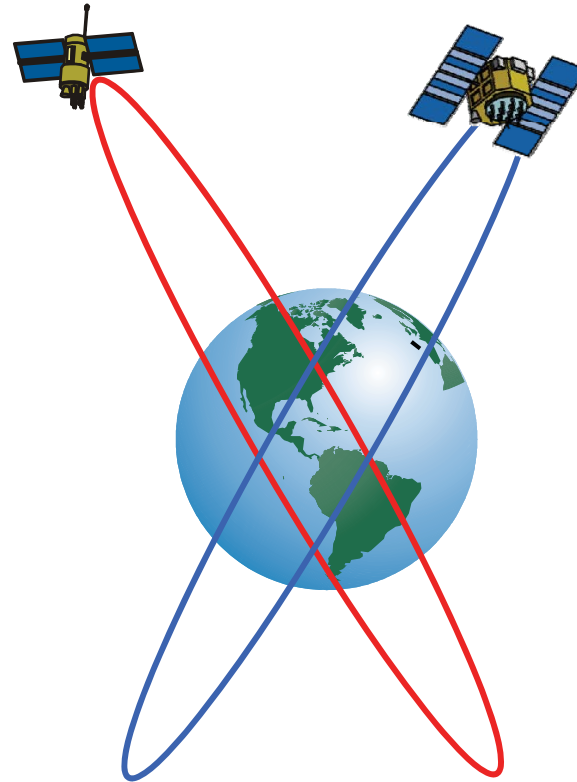
Your position coordinates are (in meters):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1\,510\,885.12 \\ -4\,463\,460.45 \\ 4\,283\,906.78 \end{bmatrix}$$



Satellite Navigation Objectives

- To provide estimates of
 - Position [~ 10 m]
 - Velocity [~ 0.1 m/s]
 - Time [~ 0.1 ms]
- Instantaneously
- Continuously
- Globally
- Cheaply, etc.
- To any number of users

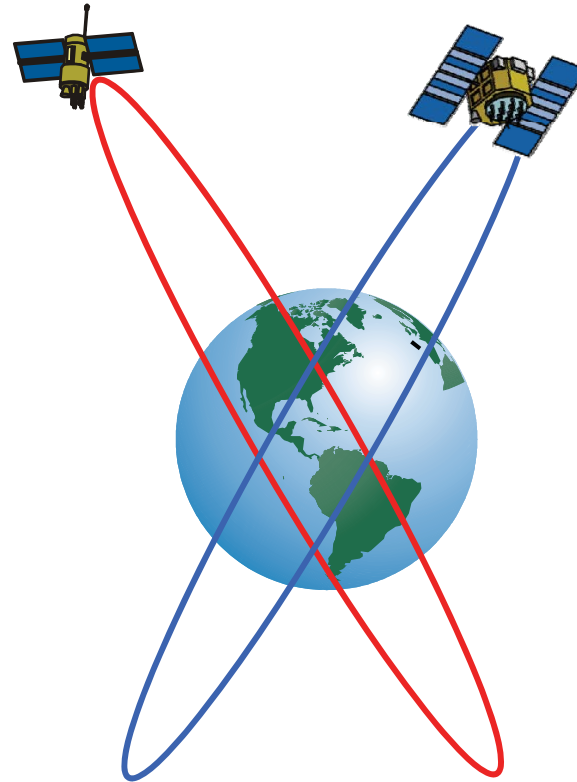




Satellite Navigation Objectives

- To provide estimates of
 - Position [**~ 10 m**]
 - Velocity [**~ 0.1 m/s**]
 - Time [**~ 0.1 μ s**]
- Instantaneously,
continuously,
globally,
cheaply, etc.
- To any number of users

Misra 1999



GPS Joint Program Office motto (Ca. 1975)

“The mission of this Program is: (1) Drop 5 bombs in the same hole, and (2) build a cheap set that navigates (< \$10,000), and don't you forget it!”

Satellite Navigation Overview

Outline



- Principles of Satellite Navigation



- **GPS Overview: System, Signals and measurements, Performance**
- **Applications and Performance Metrics**
- **Potential Partners/Rivals: GLONASS, Galileo, BeiDou/Compass, ...**



GPS at a Glance

- **Development began in early 1970s**
 - First prototype satellite launched in 1978
 - Estimated number of receivers required: 27,000 (!)
 - Target cost of a receiver: \$10,000 (!)
- **Operational System**
 - First operational satellite launched in 1989
 - System declared operational in 1995
- **Expenditure**
 - U.S. taxpayer investment (through 2007): \$ 32b
 - Annual O&M costs: \$ 1b
- **Users: Millions**
 - Most widely used military radio, albeit one way
 - Civil receivers manufactured annually: > 1 million
- **Annual commerce in GPS products & services > \$10 b**

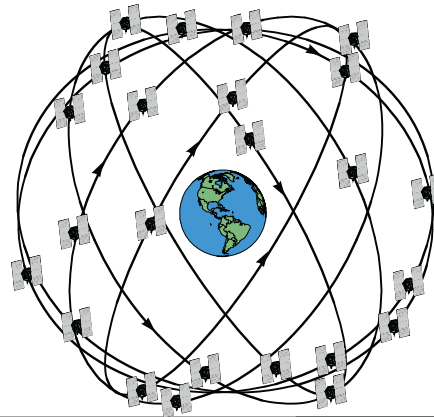


U.S. Policy on GPS

- **Services**
 - Standard Positioning Service (SPS) available to all
 - Precise Positioning Service (PPS) for “authorized” users
- **Selective Availability (SA)**
 - Purposeful degradation of the civil signal throughout 1990s, SPS horizontal positioning accuracy (95%): ~60 m
 - Discontinued by Presidential Order (2000)
 - Foresworn for GPS III by Presidential Order (2007)
- **Governance**
 - DoD (until 1996), Inter-Agency GPS Executive Board (1996-2004), U.S. National Space-Based Positioning, Navigation, and Timing Executive Committee (2004-)

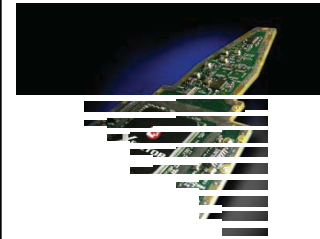


GPS Segments

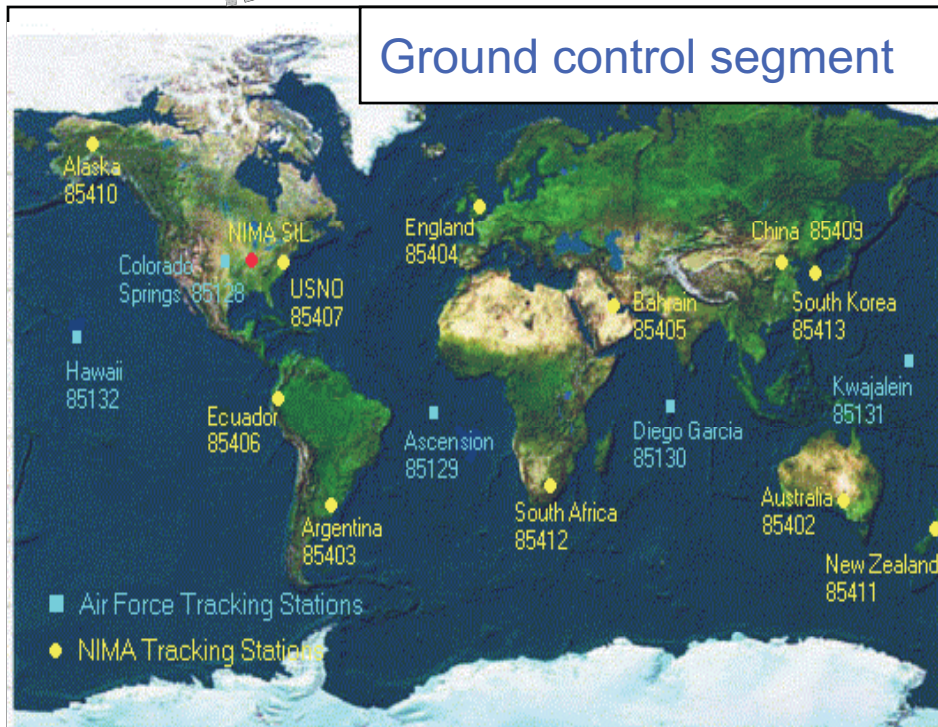


Space Segment

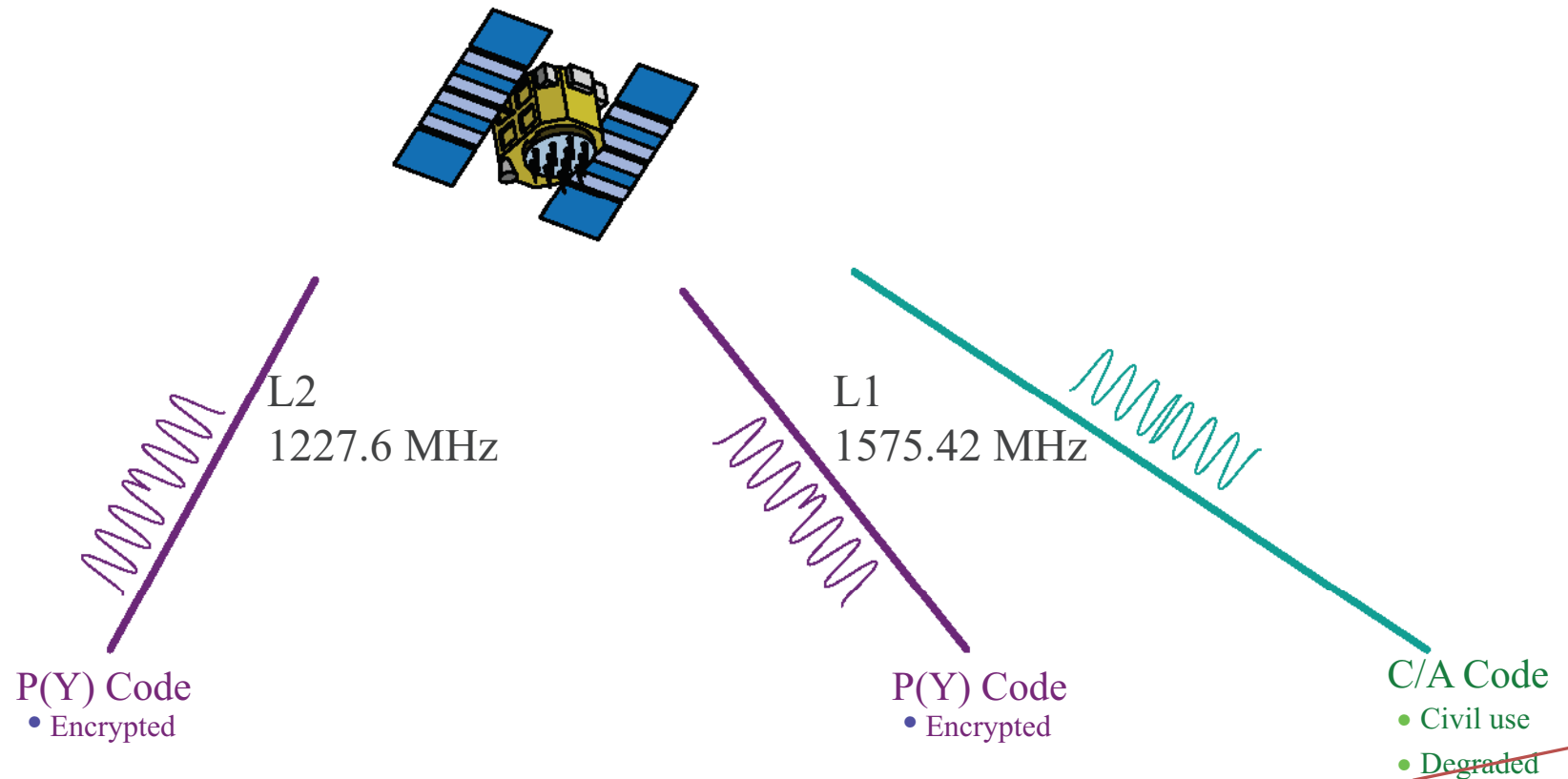
User segment: civil and military



Ground control segment



GPS Signals in mid-2005

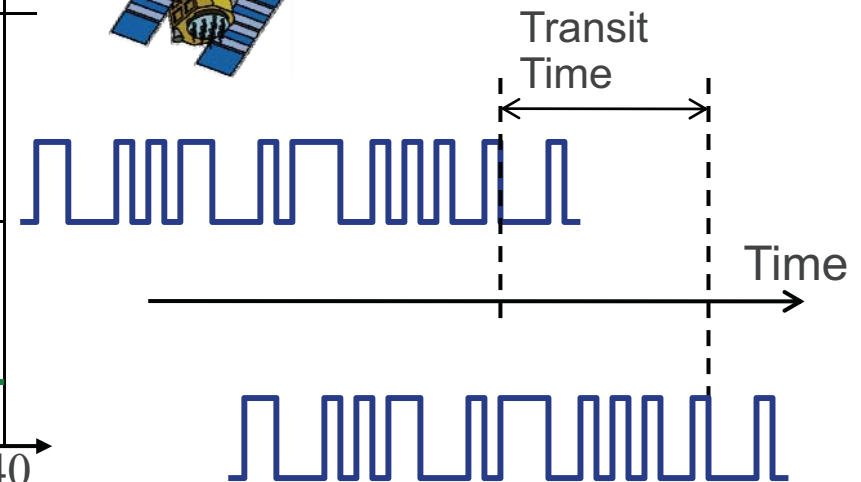
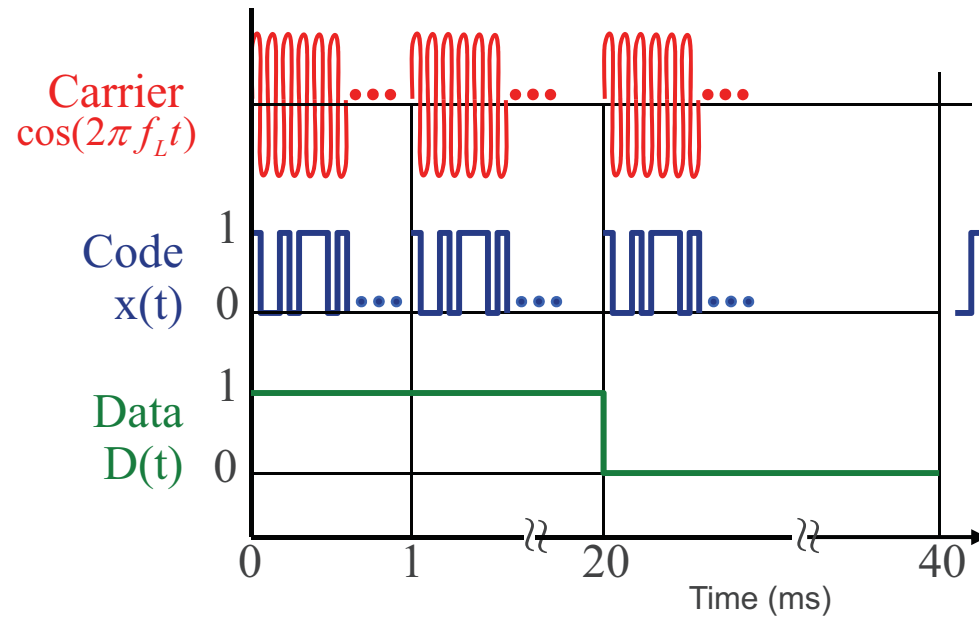


Specifications	Precise Positioning Service (PPS)	Standard Positioning Service (SPS)
Horizontal Error (95%)	22 m	< 10* 100 m
Vertical Error (95%)	27	< 15* 156

* Since 2 May 2000 (empirical)

Satellite Signal

GPS C/A-Code



Satellite Signal: $[D(t) \oplus x(t)] \otimes \cos(2\pi f_L t)$

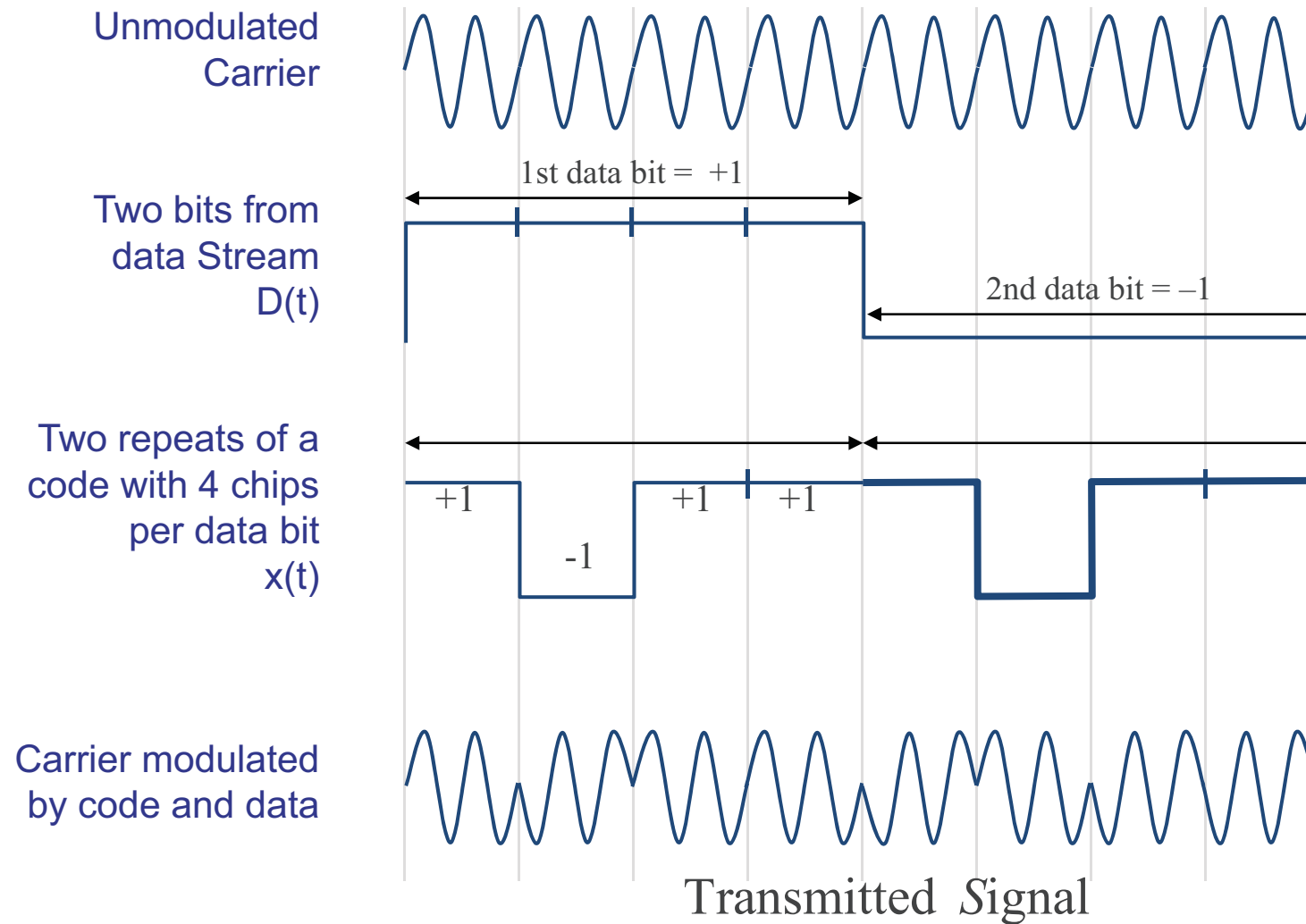
\oplus : Mod 2 Sum

\otimes : Biphase Modulation





Spread Spectrum Signaling



Source: Prof. Per Enge, Stanford University

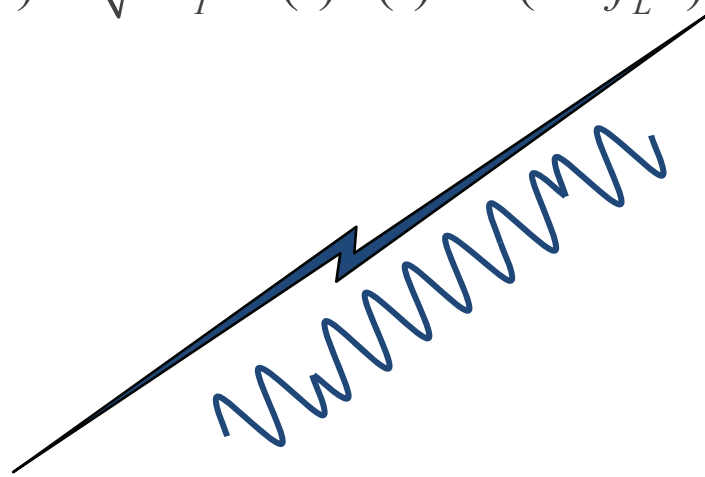
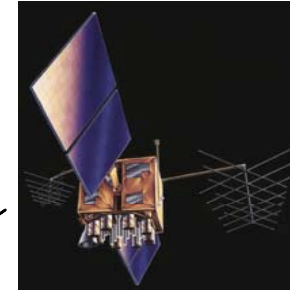
$$s_T(t) = \sqrt{2P_T} D(t) x(t) \cos(2\pi f_L t)$$



A Generic GPS Signal

Transmitted Signal

$$s_T(t) = \sqrt{2P_T} D(t) x(t) \cos(2\pi f_L t)$$



Received Signal

$$s_R(t) = \sqrt{2P_R} D(t - \tau) x(t - \tau) \cos(2\pi(f_L + f_D)t + \theta)$$

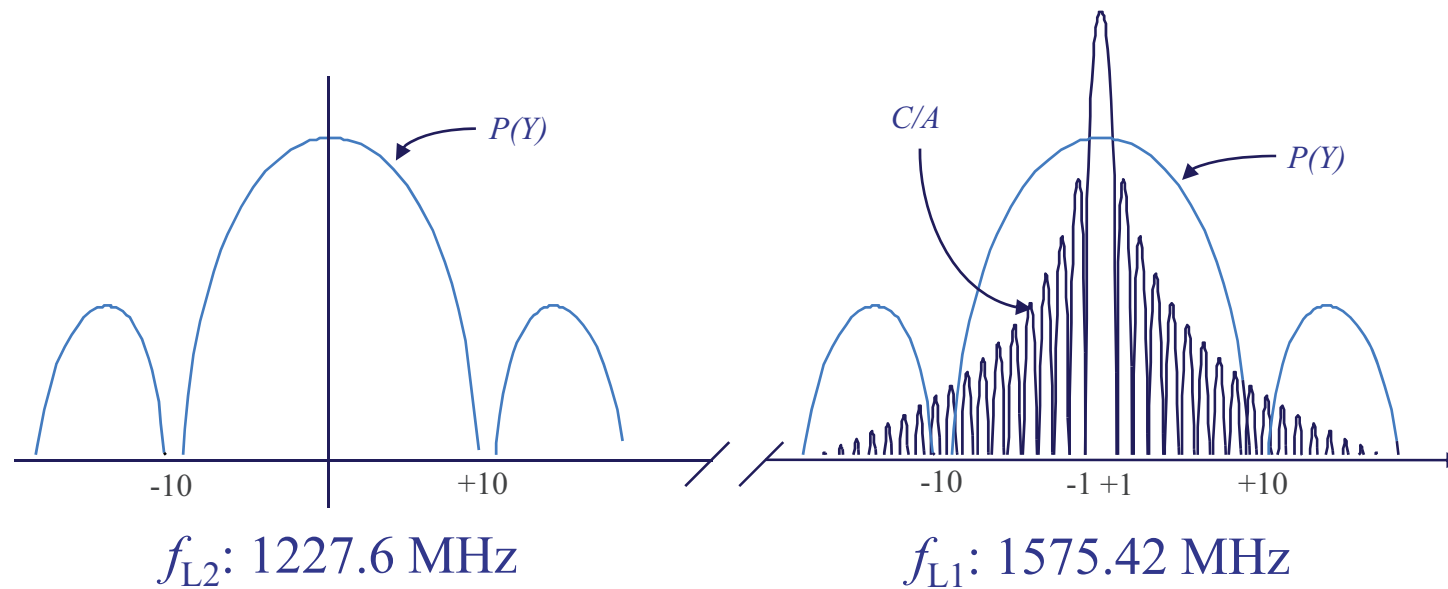
Estimate delay (τ) and Doppler (f_D)

Range = $c \cdot \tau$; Range rate = $\lambda \cdot f_D$

$D(t)$: Nav data (± 1), $x(t)$: PRN code (± 1), f_L : Carrier frequency, f_D : Doppler frequency

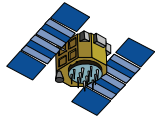
Amplitude Spectrum of GPS Signals

1 November 2005



Source: Prof. Per Enge, Stanford University

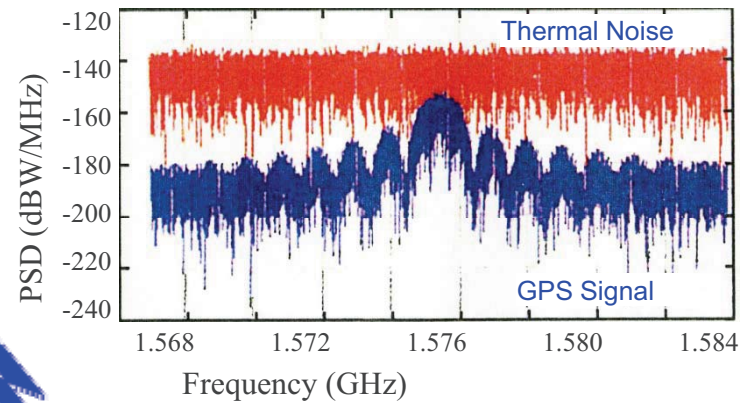
GPS Signals are Extremely Weak-1



Freq: 1563-1587 MHz (L1)

Power: ~ 27 W (C/A-Code)

~ 20,000 km

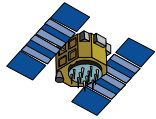


- 160 dBW
(10^{-16} W)

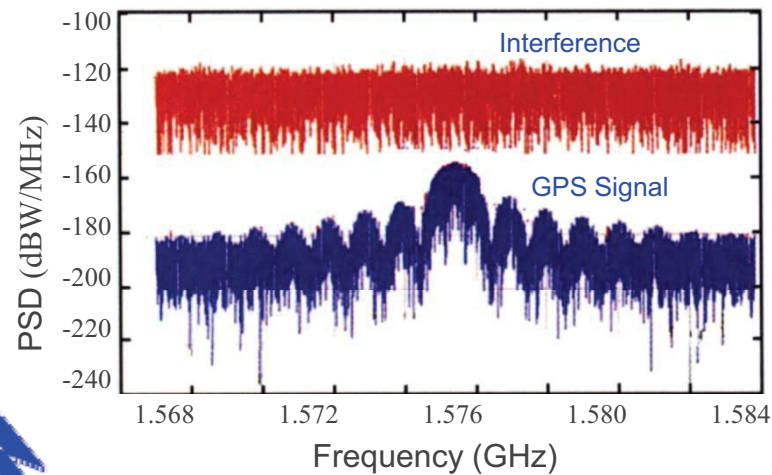
$$P_R = \frac{P_T G_T}{4\pi R^2} \cdot \frac{G_R \lambda^2}{4\pi}$$



GPS Signals are Extremely Weak-2



Freq: 1563-1587 MHz (L1)
Power: ~27 W (C/A-Code)



~ 20,000 km

- 160 dBW
(10^{-16} watts)

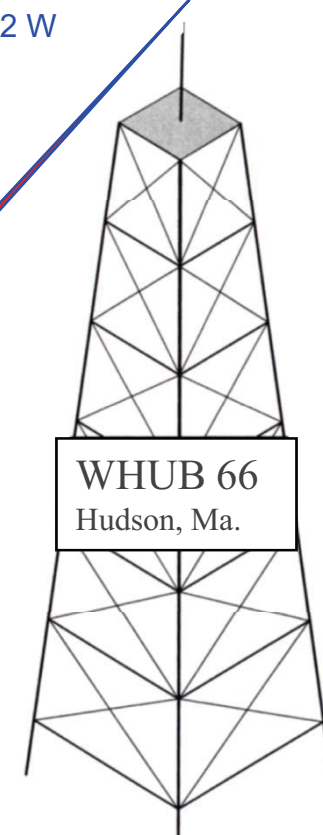
- 120 dBW
(10^{-12} watts)

$$P_R = \frac{P_T G_T}{4\pi R^2} \cdot \frac{G_R \lambda^2}{4\pi}$$



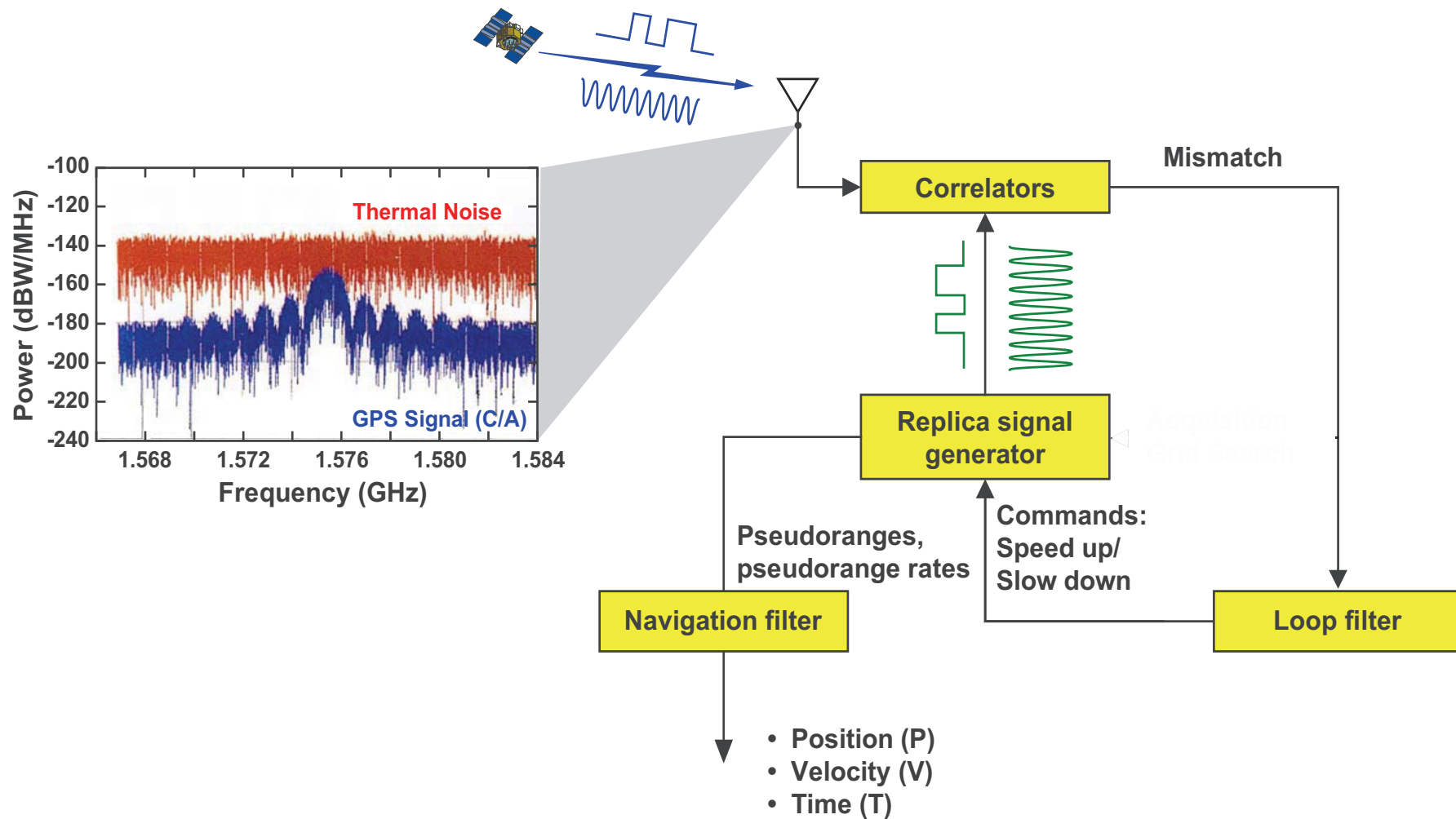
Freq: 762-788 MHz
Power: 2000 kW
FCC: Out-of-band < - 60 dB
Second harmonics:
1564-1576 MHz
Power: ~ 2 W

~ 10 km

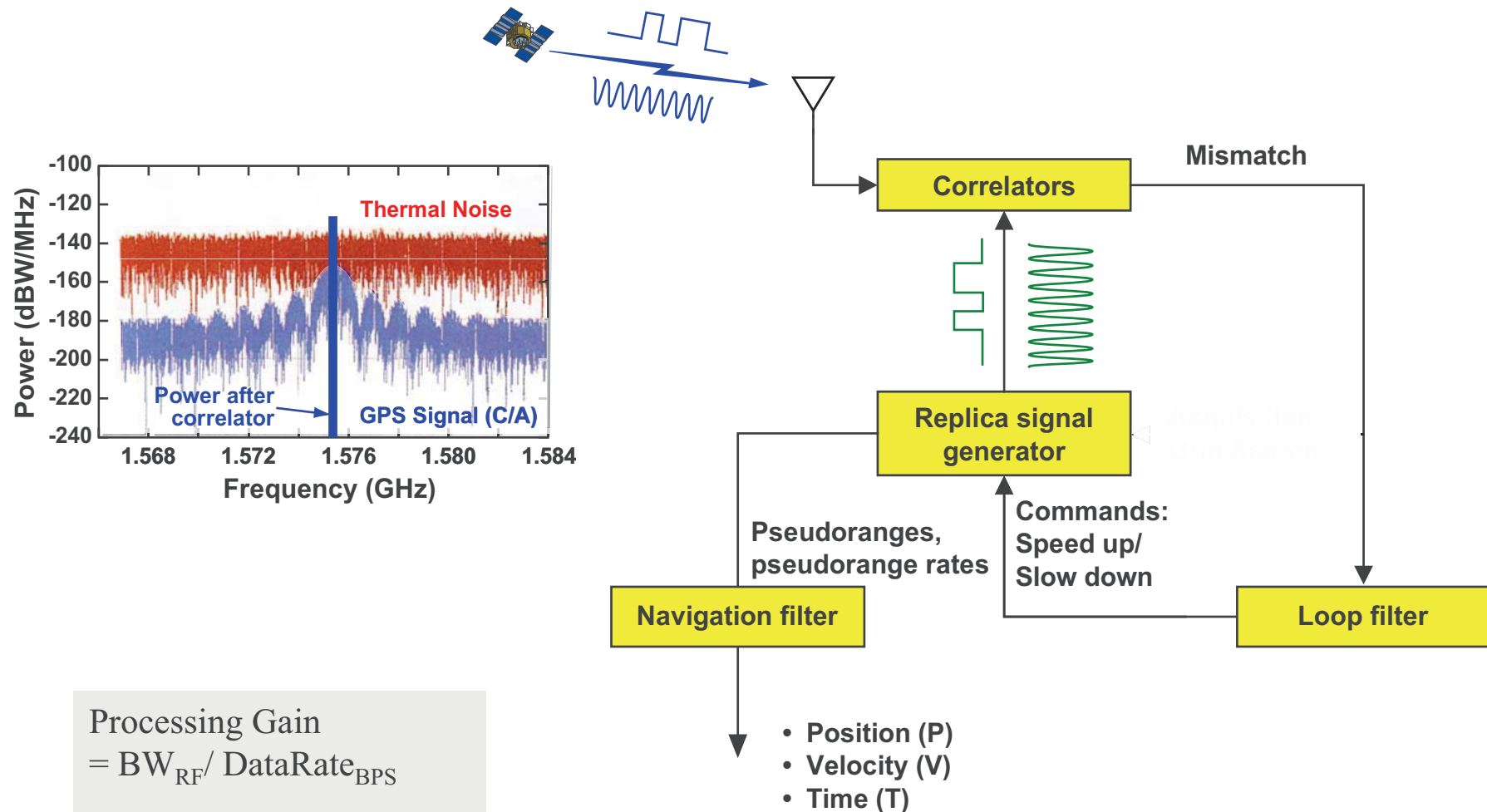


Based on paper by Philip W. Ward, P.E.

Basic GPS Receiver Architecture-1



Basic GPS Receiver Architecture-2



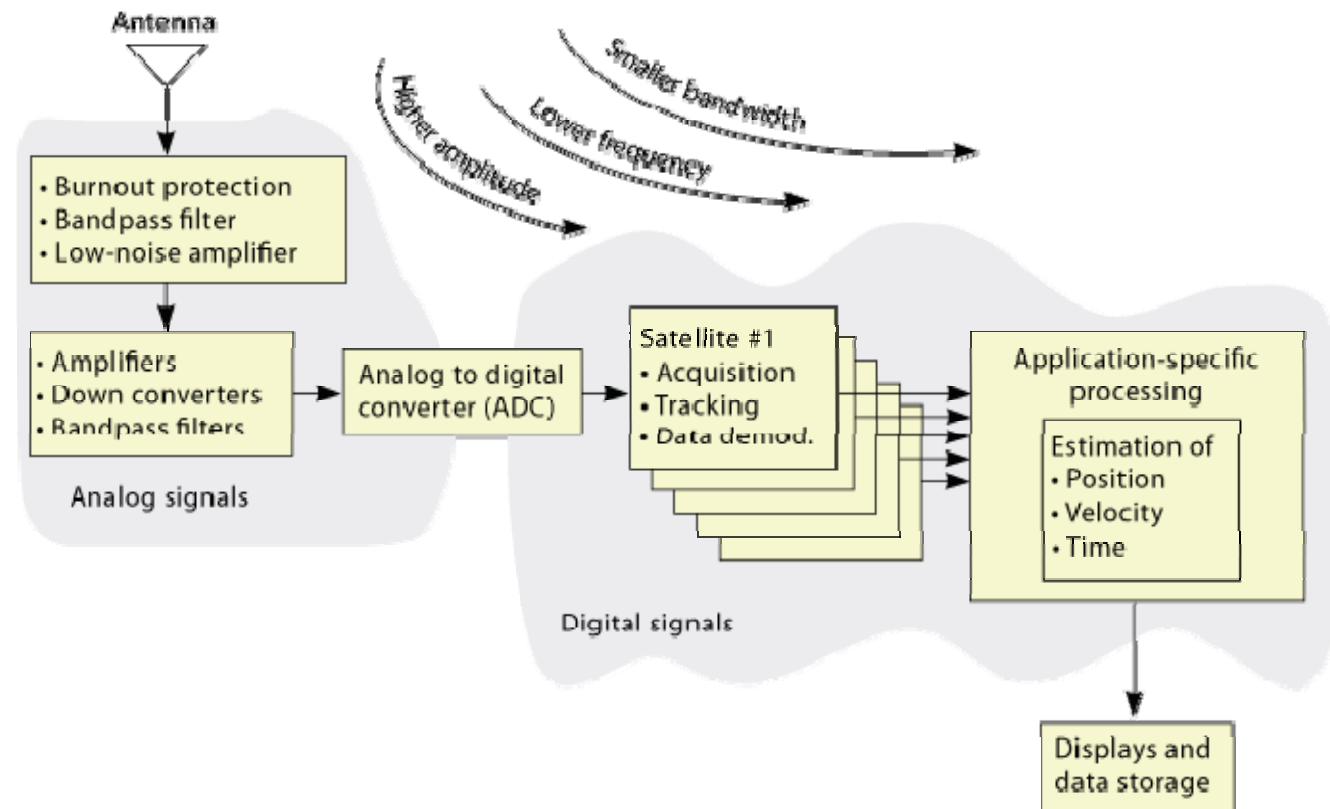


Receiver Functions



- **Condition input signal**
 - Bandpass filter to suppress OOB interference
 - Down-convert
 - Digitize (A/D conversion)
- **Separate signals from individual SVs**
- **Acquire and Track Signals**
- **Demodulate navigation data**
- **Calculate position, velocity, and time (PVT)**
- **Report results through user interface**

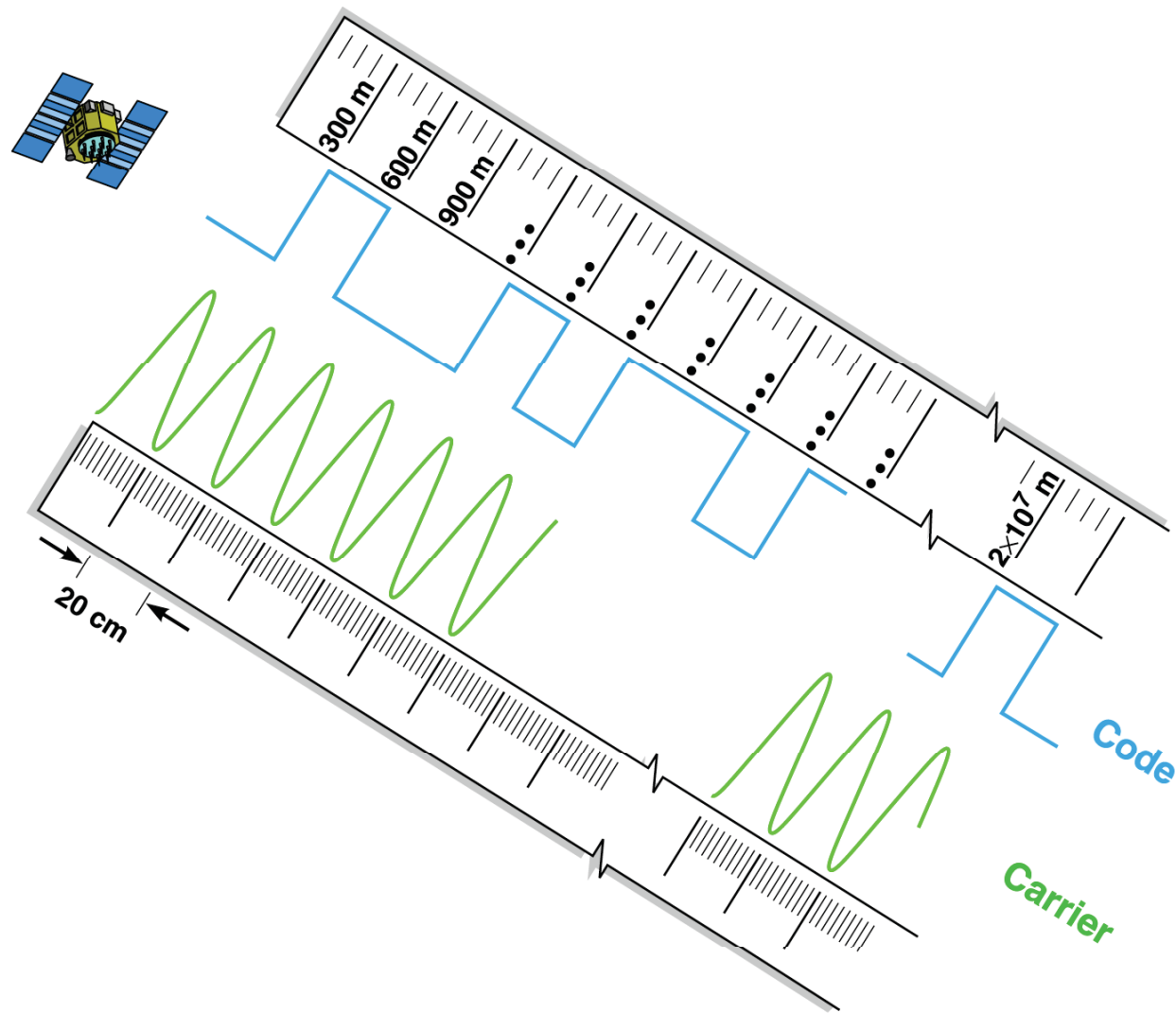
GPS Receiver Functional Diagram



Courtesy: Prof. Per Enge, Stanford University

Code and Carrier Phase Measurements

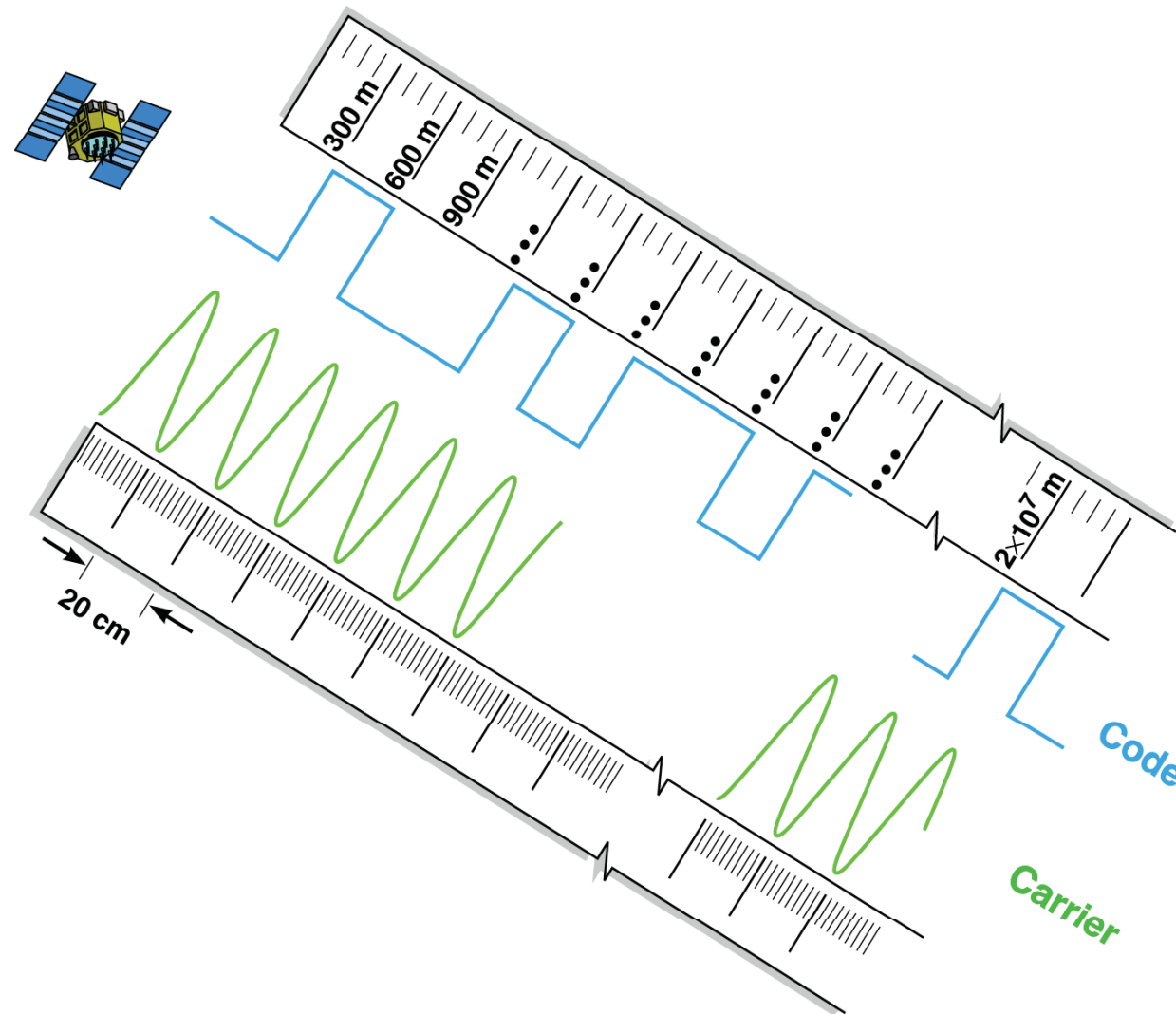
A Conceptual Exercise



Misra/1998

Code and Carrier Phase Measurements

Precision vs. Accuracy



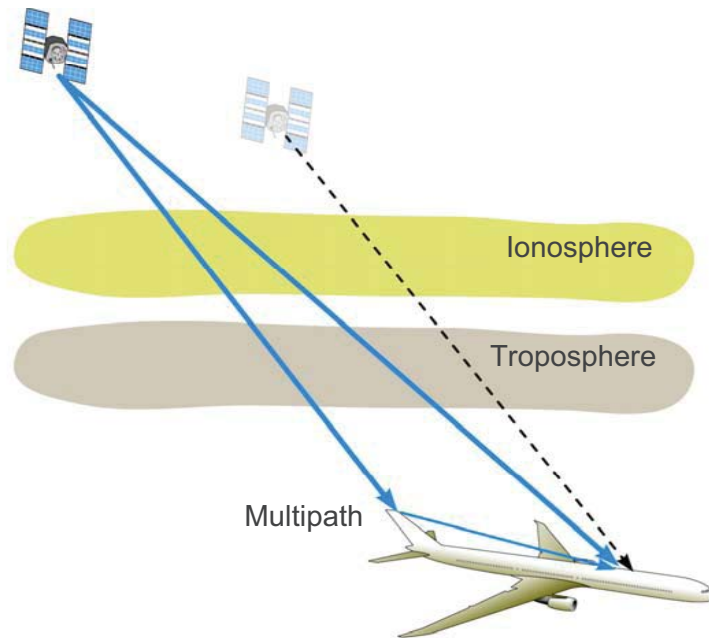
- Carrier phase can be measured with a precision of millimeters, code phase with decimeters
- Pseudoranges from each are affected by the same error sources, and the error in each can be several meters



Misra/1998



GPS Error Sources



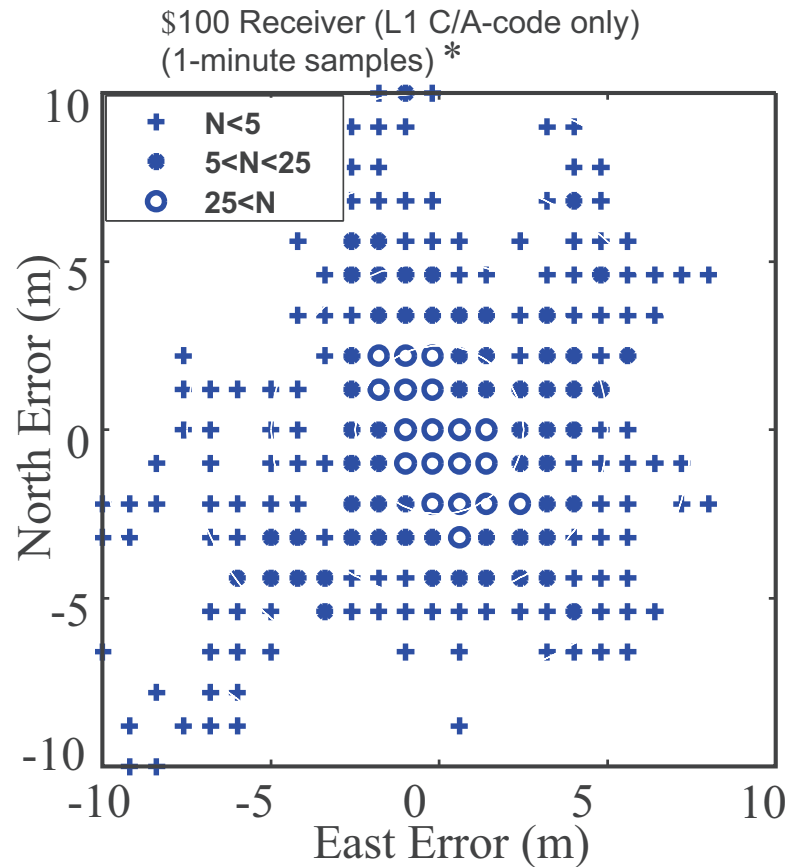
Pseudorange Error Source	Size (typical)
▪ Satellite clock/orbit error	1– 2 m
▪ Mis-modeled ionospheric delay	0– 3
▪ Mis-Modeled tropospheric delay	1
▪ Multipath	1– 3
▪ Receiver noise	< 1

Pseudorange Error: 2– 5 m

Horizontal position Error: 2– 5 m

GPS-based Position Estimates (SPS)

Sampled over 24 hours (post-SA)*



N: # points in a cell

Error: 95% (empirical)

Horizontal position	10 m
Vertical position	15 m
Time	30 ns

*Source: MIT Lincoln laboratory

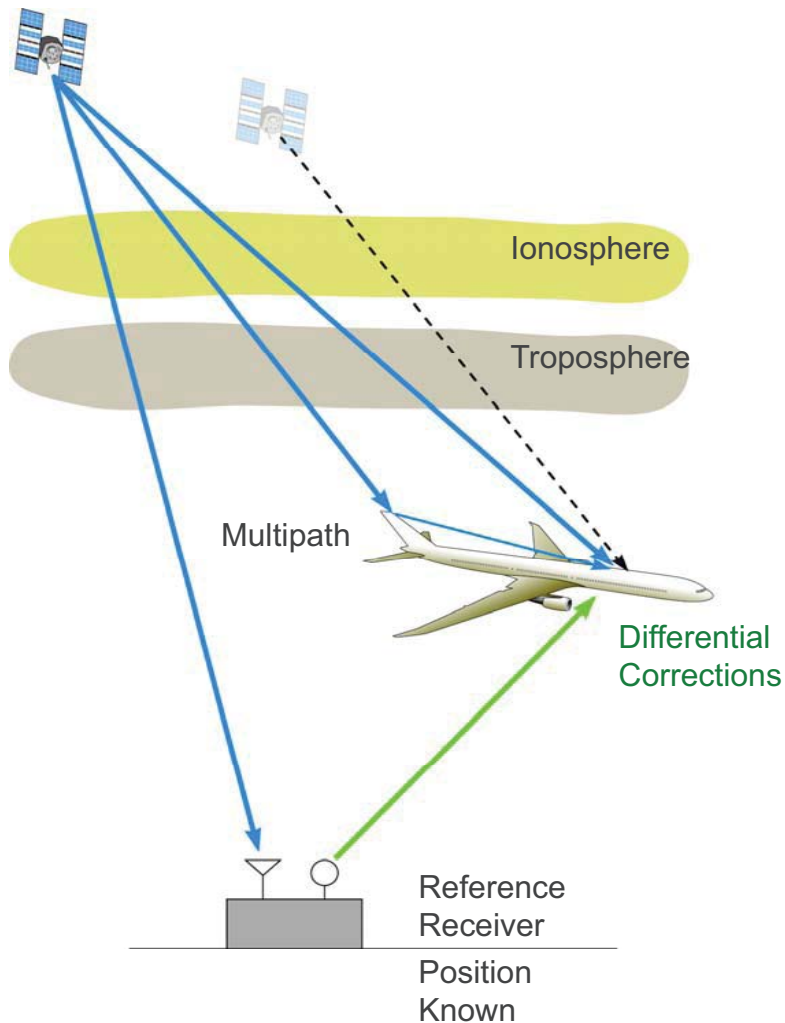


GPS Augmentations

- **Why augment?**
 - **For better accuracy: Mitigate measurement errors**
 - **For robustness: Mitigate effects of**
 - RFI (intentional or not)
 - signal attenuation due to blockage (e.g., by foliage or building), or temporary loss of signal (e.g., going under a bridge or through a tunnel)
- **How augment?**
 - **Transmit corrections for errors that are correlated spatially and temporally**
 - Local Area Differential GPS
 - Space-Based Augmentation Systems (SBAS): WAAS, EGNOS, MSAS
 - **Assist GPS receiver with complementary technologies (e.g., inertial), signals of opportunity (e.g., eLoran), or by offloading some functions (e.g., to a cell tower in E911)**

Differential GPS (DGPS)

Mitigation of Error Sources



Pseudorange Error Source	Size (typical)	Spatially and Temporally Correlated?
Satellite clock/orbit error	1– 2 m	Yes
Mis-modeled ionospheric delay	1– 3	Yes
Mis-modeled tropospheric delay	1	Yes
Multipath	1– 3	No
Receiver noise	< 1	No

Pseudorange error: 2– 5 m

Horizontal position error: 2–5 m

Differentially Corrected

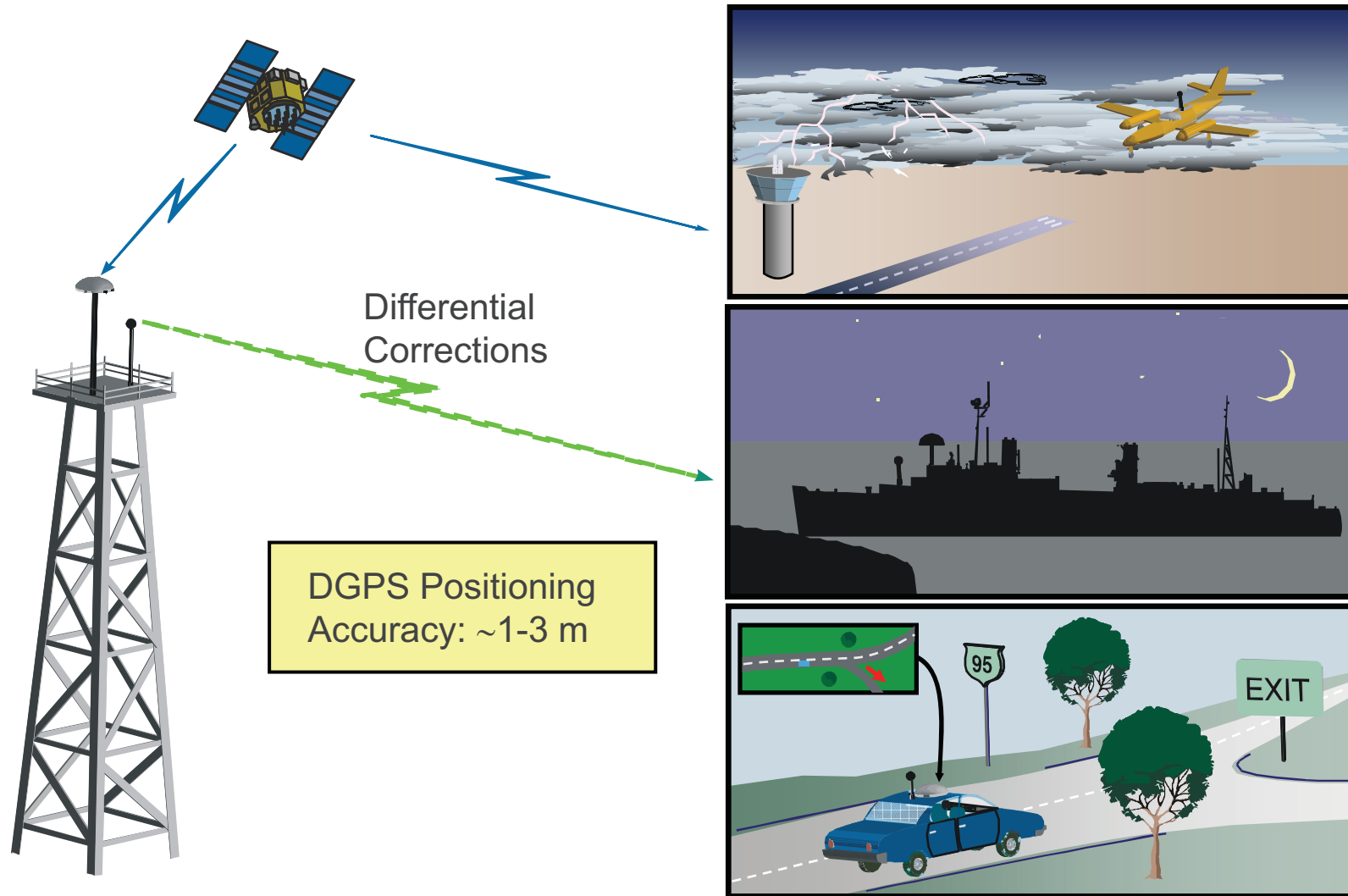
(~10 km from reference receiver)

Pseudorange error: < 1 m

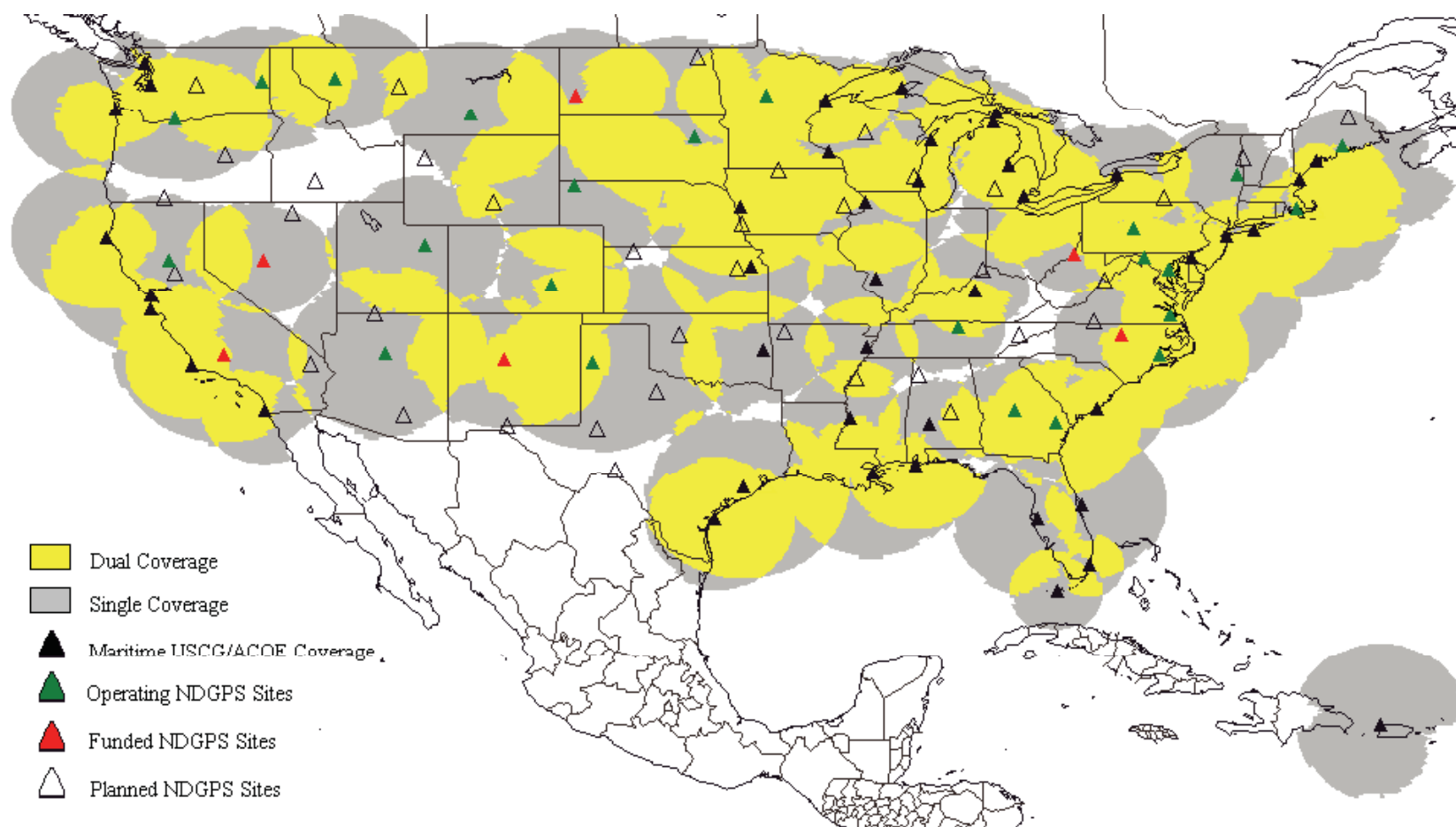
Horizontal position error: 1– 2 m

Local Area Differential GPS (DGPS)

Mitigation of Correlated Measurement Errors



National DGPS Coverage (2005)



In the Lower 48, single coverage: 87%, dual coverage: 55%

Satellite Navigation Overview

Outline



- Principles of Satellite Navigation
- GPS Overview: System, Signals and measurements, Performance
- ➔ • **Applications and Performance Metrics**
- **Potential Partners/Rivals: GLONASS, Galileo, BeiDou/Compass, ...**

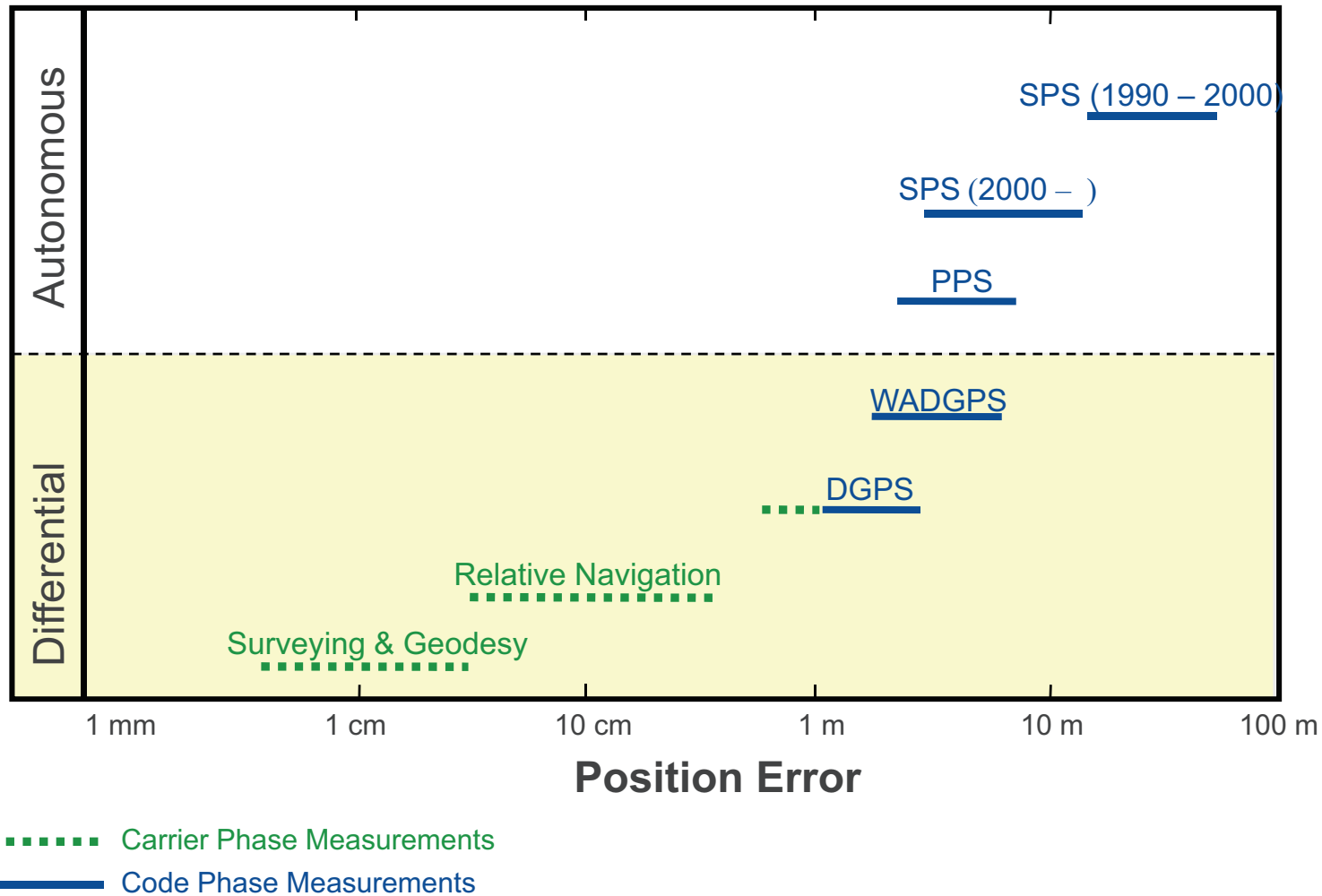


Performance Metrics

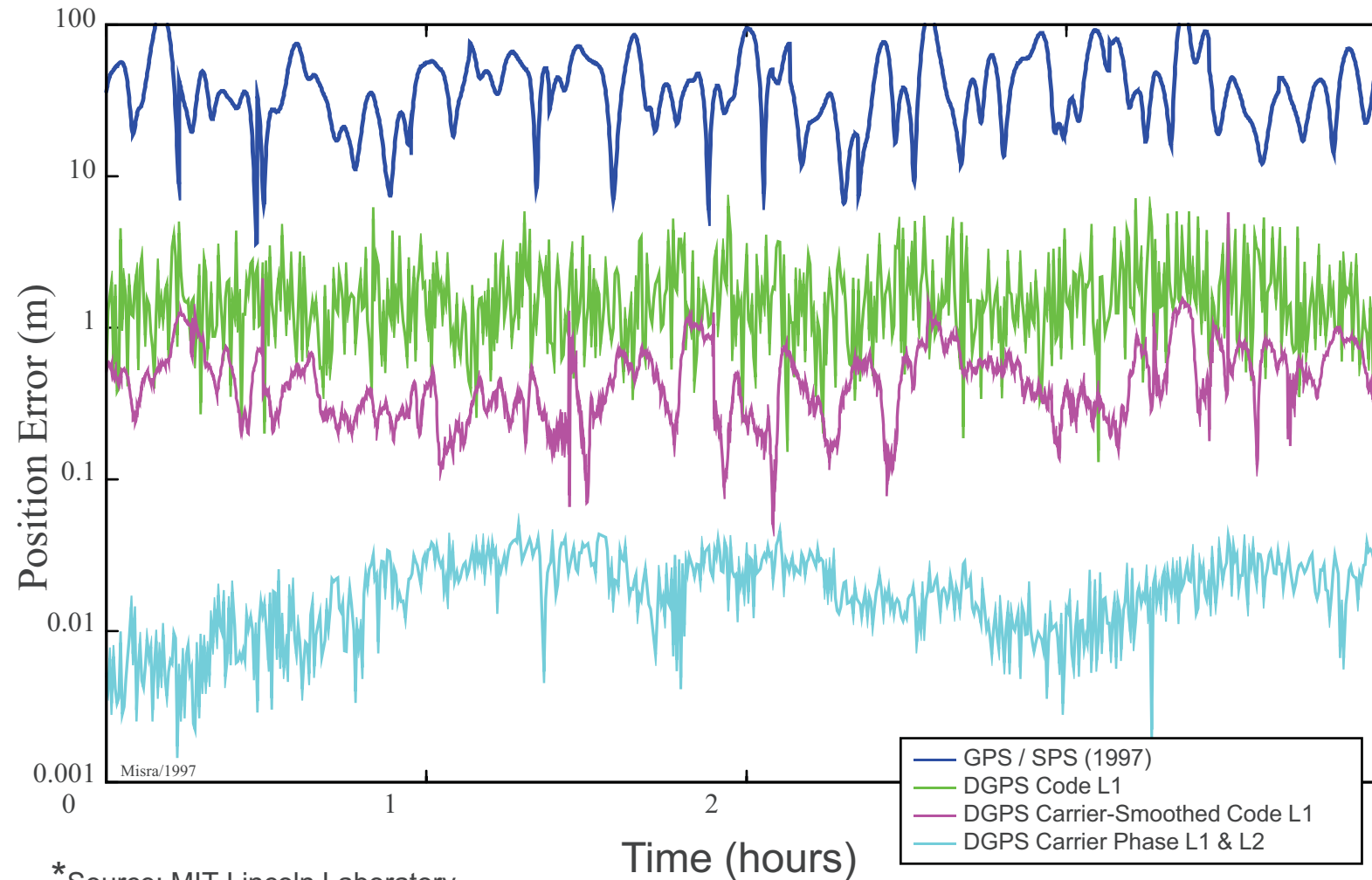
- **Accuracy**
 - How good are the estimates? RMS error
- **Error Bounds**
 - Your error is no worse than x (with probability 0.999...9)
- **Integrity of Signals**
 - The signals on which your estimates are based are genuine. (Probability)
- **Availability of Service**
 - Consistent with your requirements. (Probability)
- **Continuity of Service**
 - For the next x seconds, consistent with your requirements. (Probability)

Positioning Accuracy Hierarchy

GPS and Its Augmentations



Real-Time Position Estimates from GPS (1997)*



*Source: MIT Lincoln Laboratory

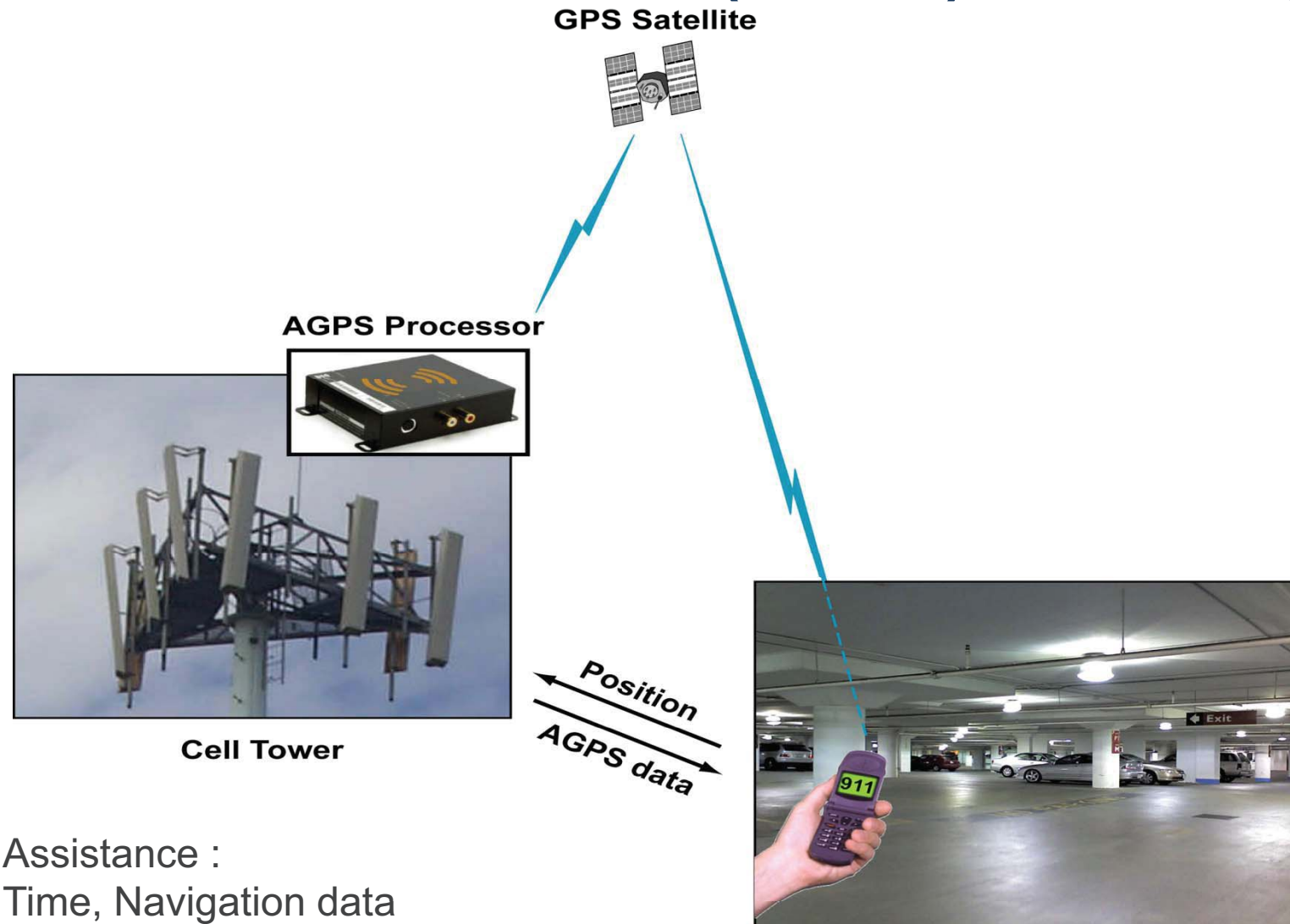


GPS Applications





Assisted GPS (AGPS)



Satellite Navigation Overview

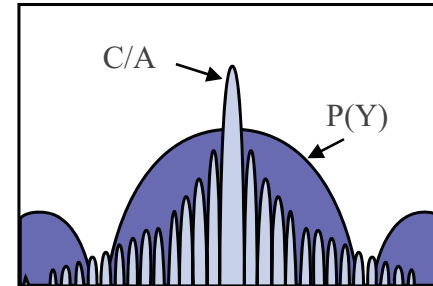
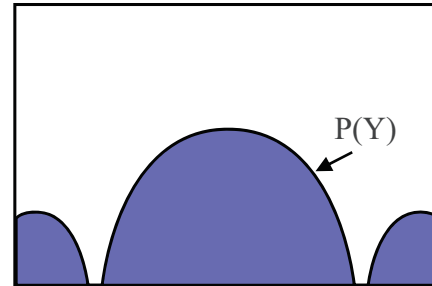
Outline



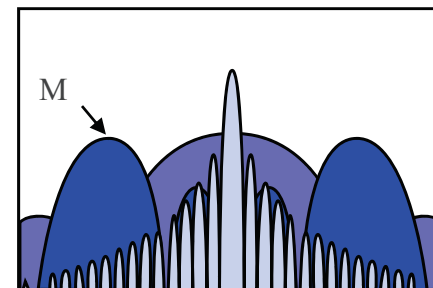
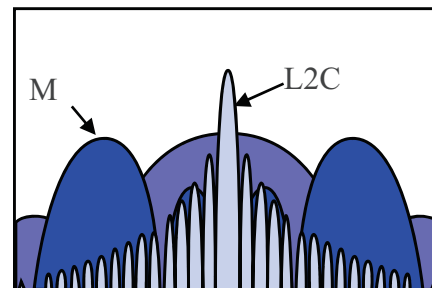
- Principles of Satellite Navigation
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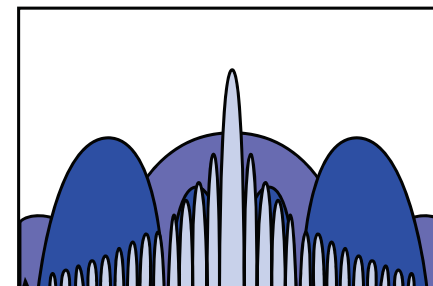
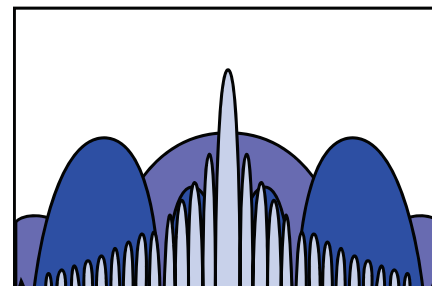
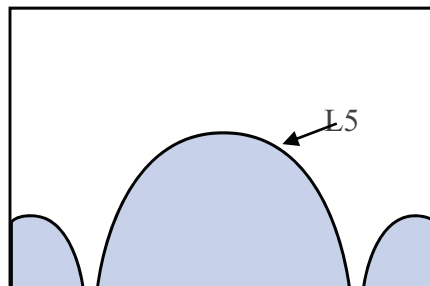
Evolution of GPS Signals



Block II / IIA /
IIR
1989 - 2005



Block IIR-M
Starting in
2005



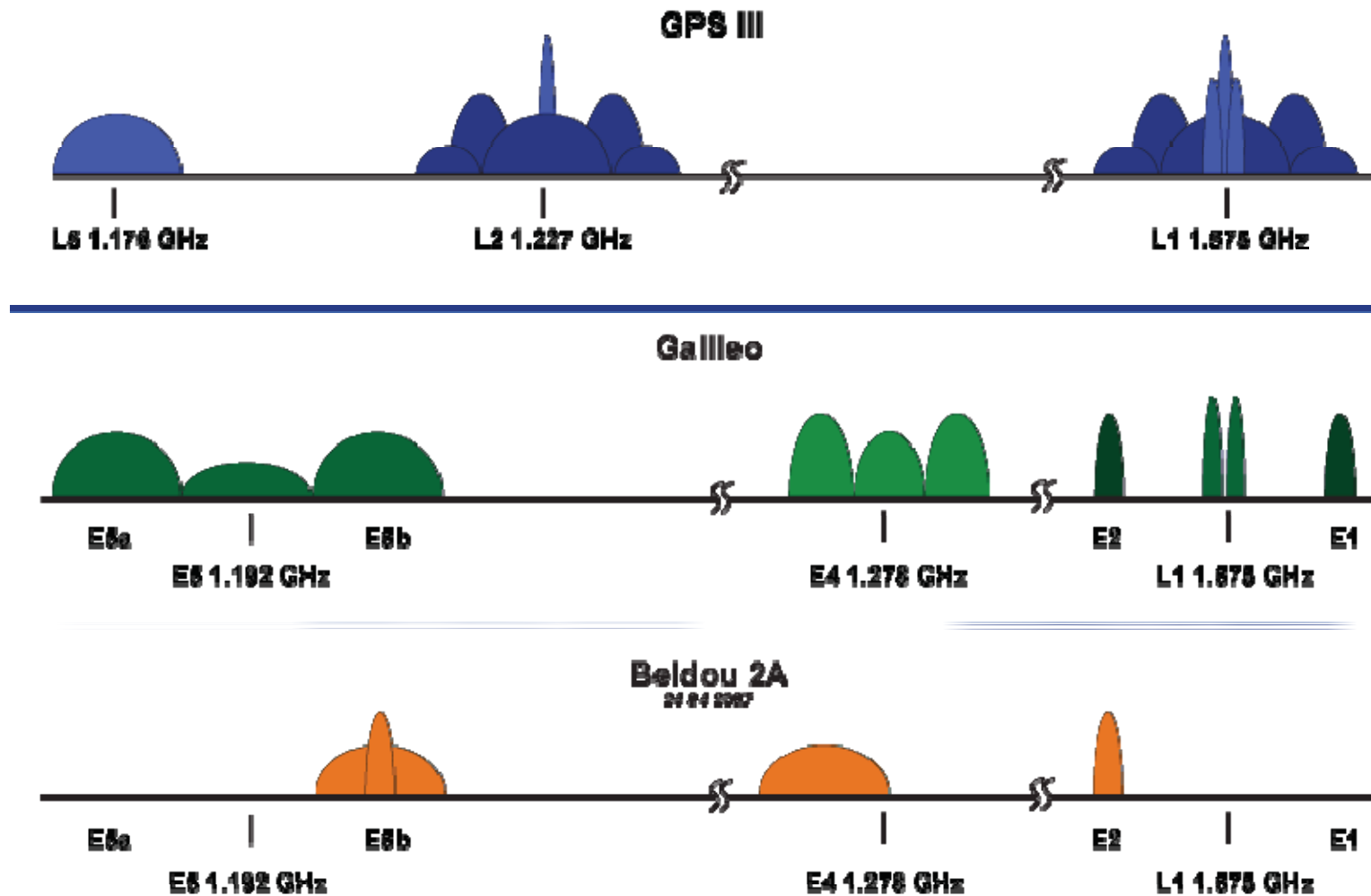
Block IIF
Starting in
2009

1176.45 MHz

1227.6 MHz

1575.42 MHz

Frequency Plans*



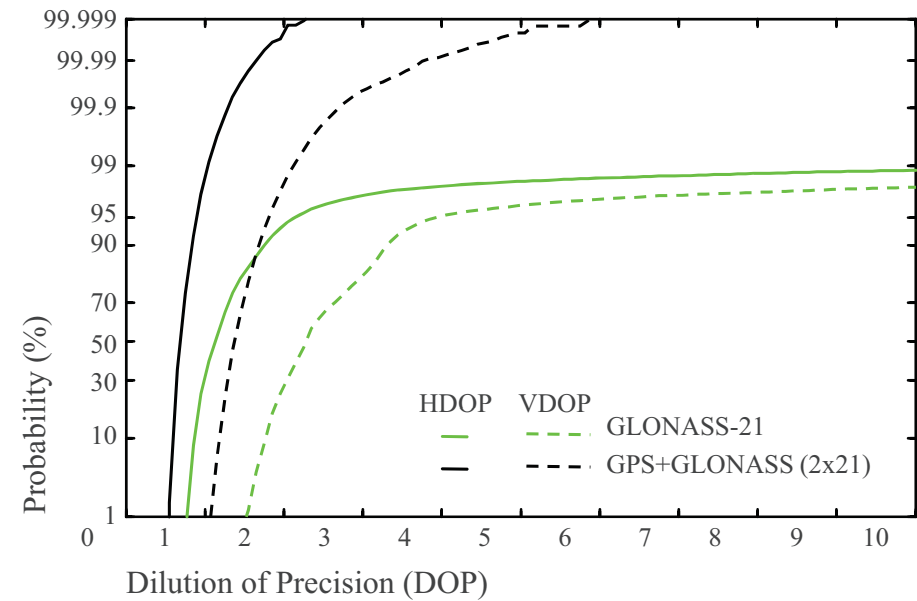
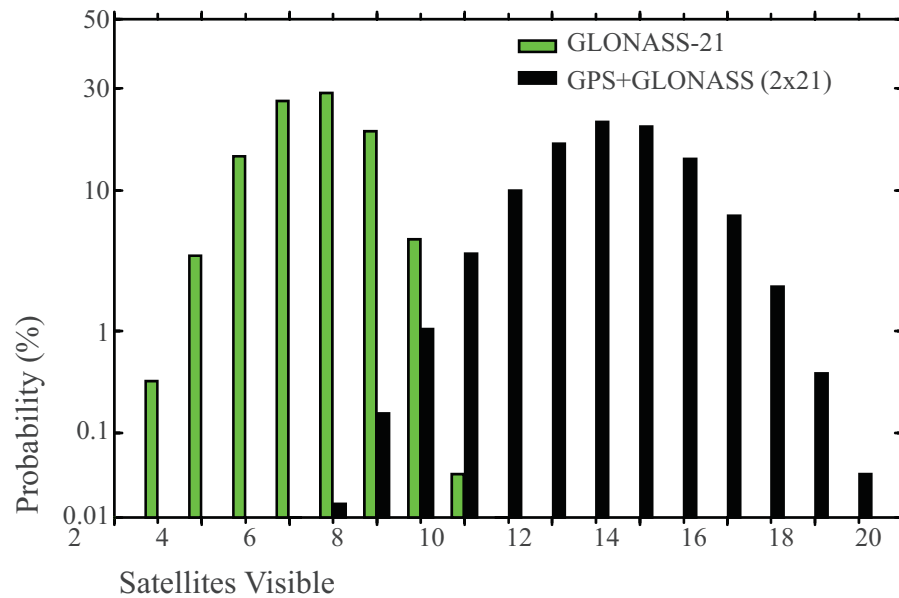
*Adapted from T. Grelrier et al., Inside GNSS, May/June 2007

GLONASS



- **History**
 - Developed by Soviet Union, first launch: 1982
 - Declined under Russia, but newly revived
 - Similar to GPS: Passive, one-way ranging
 - 10-12 working satellites over the past 5 of years, currently 16
 - No significant user base
- **Constellation**
 - 24 satellites in 3 orbital planes, 64.8° inclination
 - 19,100 km altitude, 11 ¼ hour period
- **Signals**
 - 3 allocated bands: G1 (1602 MHz), G2: (1245 MHz), G3 (?)
 - C/A-like code: 511 chips, 1 ms code period, 50 bps data
 - All SVs use same PRN with frequency division multiple access (FDMA) using 16 frequency channels, reused for antipodal SVs
- **Plans: 18 SVs in 2008, full constellation in 2011 (?)**

GPS+GLONASS Satellite Visibility

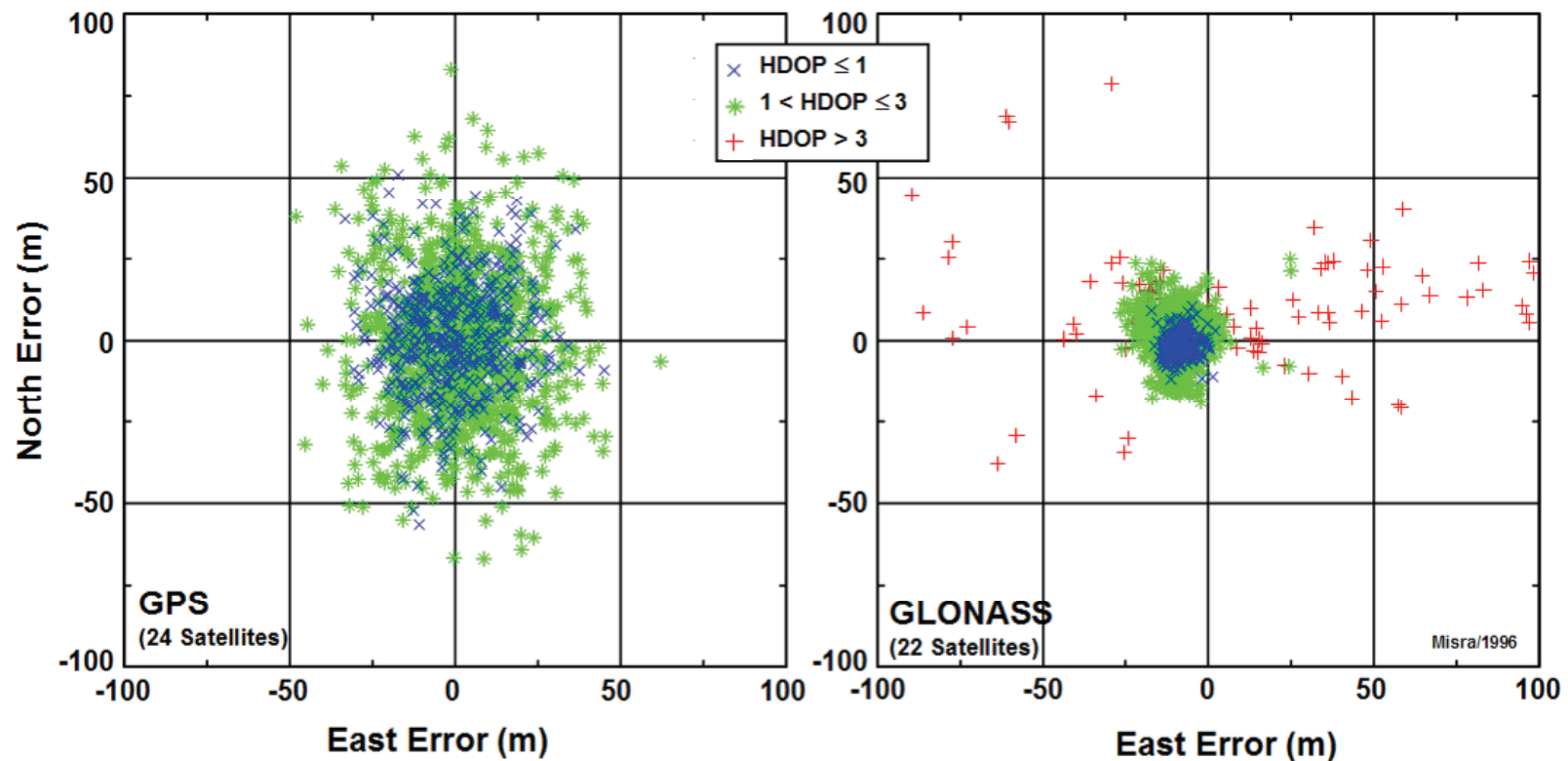


Misra/1997

GPS & GLONASS Position Estimates*



1-Minute Samples, 15 June 1996

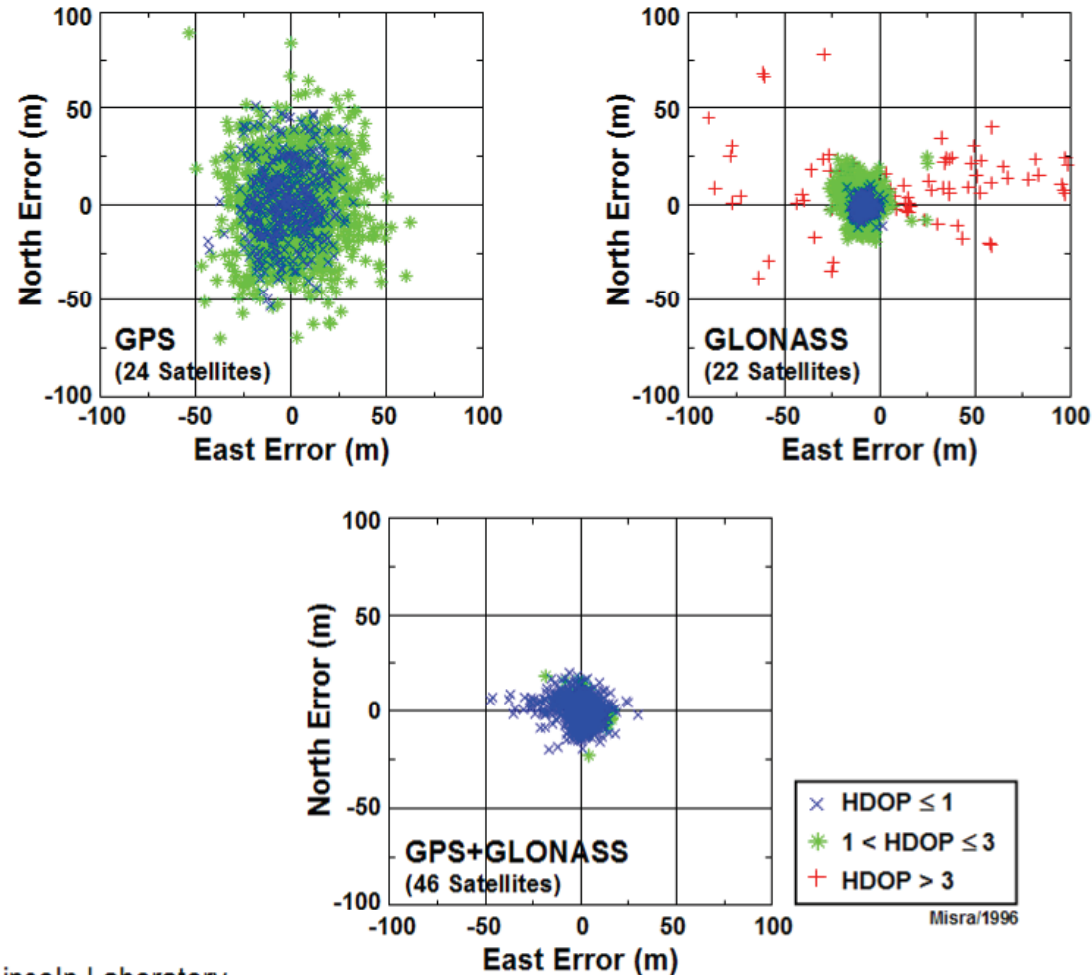


*Source: MIT Lincoln Laboratory

GPS & GLONASS Position Estimates*

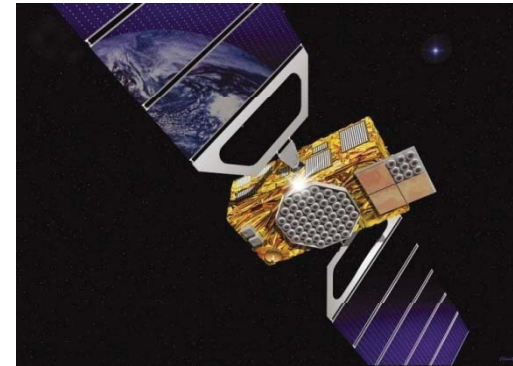


1-Minute Samples, 15 June 1996



* Source: MIT Lincoln Laboratory

Galileo



- **European-owned: planned public-private partnership didn't work out**
- **“Seen” as a civil system, but military role may emerge**
- **5 services**
 - **Free:** Open Service
 - **For a Fee:**
 - Commercial Service
 - Safety-of-Life Service
 - Public Regulated Service
 - Search & Rescue Service
- **30 MEOs in 3 planes inclined at 56°**
- **First experimental satellite launched in 2005**
- **Appears to have recovered from recent setbacks; system operational around 2013**

PROGRAM OBJECTIVES

- **Objectives of Galileo:**
 - Increased overall performance
 - Civil system in contrast to GPS
 - Independent **and** interoperable with GPS
 - Better robustness
 - Certified quality of services
 - Qualified for safety critical applications
 - ...

BeiDou/Compass



- **Chinese**
- **BeiDou: Regional System**
 - Active system
 - 2 - 3 geostationary satellites orbited in 2000 – 2003
- **Compass: GNSS**
 - 1 MEO launched in 2007
-





Summary: Take-Away Points

- Satellite navigation systems exploit basic properties of radio waves: Transit exploited the Doppler effect, GPS exploits the known speed of propagation
- GPS is based on the old idea of trilateration, but implemented with the technology of the second-half of the 20th century: space-based radio transmitters, ultra-stable clocks, and spread spectrum signals
- A GPS receiver measures pseudoranges to the satellites by measuring pseudo-transit times of radio signals. It takes 4 satellites (i.e., 4 pseudoranges) in order to estimate position (x, y, z) and time t
- With a clear view of the sky, it's easy to get positioning accuracy of several meters with a \$100 GPS receiver, or relative positioning accuracy of millimeters with a pair of \$1000 receivers.
- GPS satellites are 20-watt transmitters 20,000 km away, so the signals reaching the earth are very weak and, therefore susceptible to interference.
- The success and breadth of GPS applications is attributable largely to “the chip.” The VLSI revolution was well-timed for GPS