Flux rope dynamos
(an exploratory model)

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Outline

- Motivation: magnetic reconnection in the context of dynamo action
- A kinematic model of a turbulent flow
- Exploratory reconnection model
- Dynamo based on magnetic reconnections

- Collisional plasma, MHD:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \ ,$$

dynamo constrained by diffusion time $\tau_\eta \propto L^2 \eta^{-1}$

- Hyperdiffusion facilitates dynamo action:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + (-1)^{n-1} \eta_n \nabla^{2n} \mathbf{B} \ ,$$

with $n = 2$, helical $\mathbf{V}$: larger growth rates,

larger steady-state B (Brandenburg & Sarson, PRL 2002)

- Rarefied, hot plasmas (galactic & solar coronae, …): fast reconnection time is independent of η, $\tau_{\text{rec}} \sim L/V_A$

- How the non-diffusive reconnection would affect dynamo action and plasma heating?

A kinematic model of a turbulent flow

$$\mathbf{V}(\mathbf{x},t) = \sum_{n=1}^{N} \left(\mathbf{A}_n \times \mathbf{k}_n \cos\psi_n + \mathbf{B}_n \times \mathbf{k}_n \sin\psi_n\right),$$

$$\psi_n = \mathbf{k}_n \cdot \mathbf{x} + \omega_n t\,, \qquad \omega_n = k_n v_n,$$

$$\nabla \cdot \mathbf{V} = 0\,,$$

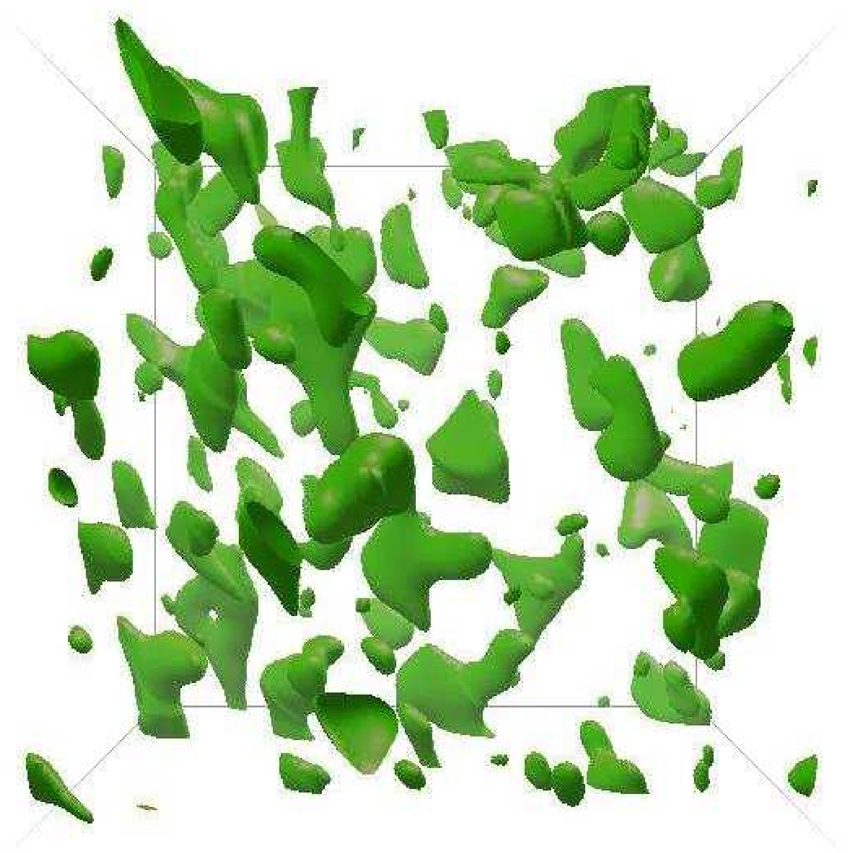
$$A_n,\ B_n: \qquad E_k \propto k^{-s}, \qquad \text{e.g., } s = 5/3\,.$$

Randomly chosen $\widehat{\mathbf{k}}_n,\ \widehat{\mathbf{A}}_n,\ \widehat{\mathbf{B}}_n$.

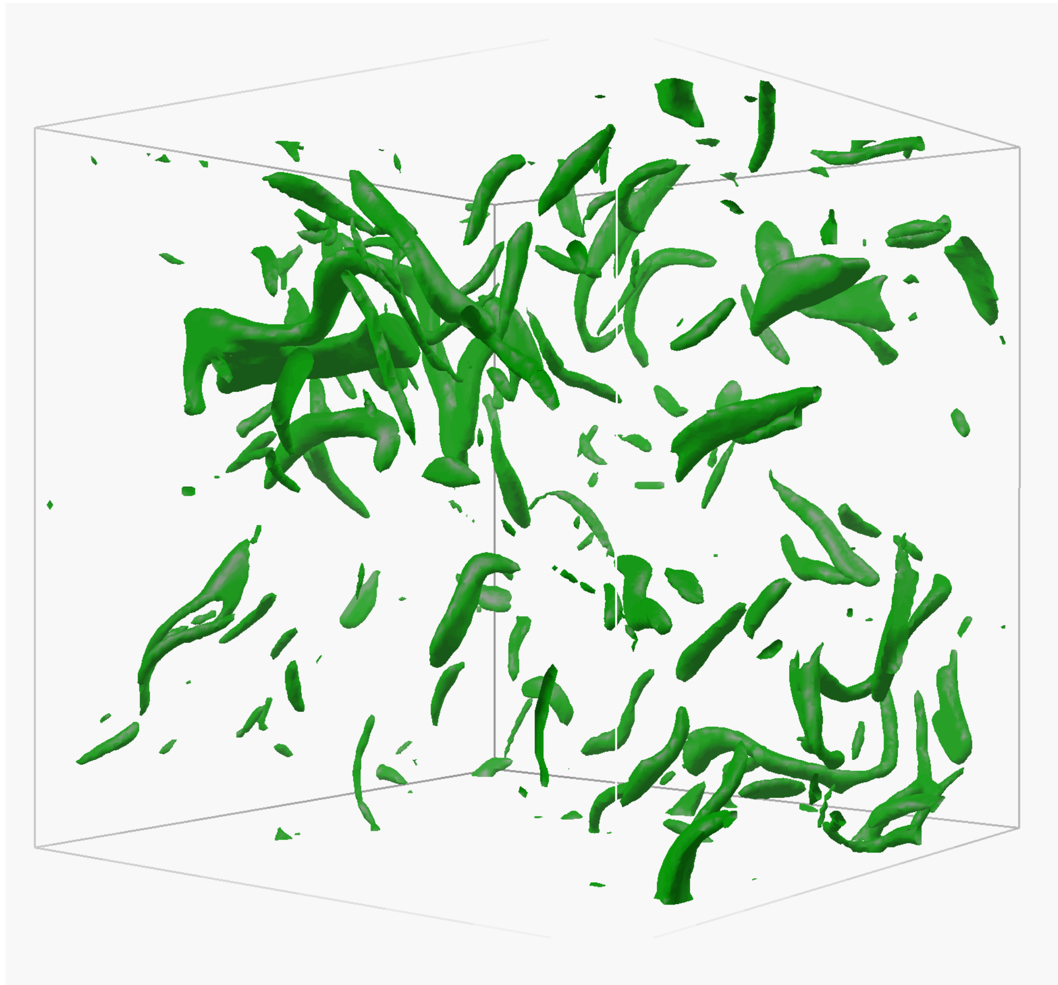
Chaotic (parameters fixed) or random (parameters varying at random)

Wilkin et al. (PRL 2007): this flow is a dynamo.

$1.75V_{\mathrm{rms}}$ isosurface of $|\mathbf{V}|$



$2.75B_{\mathrm{rms}}$ isosurface of $|\mathbf{B}|$



- All (?) existing dynamo models rely on the MHD approximation (diffusive time scales)

- How can fast reconnections affect the dynamo action?

- Are kinetic energy ⇒ magnetic field ⇒ heat transformations affected by the nature of magnetic dissipation?

Yes, of course --- but then, how?

Modelling reconnecting magnetic flux tubes

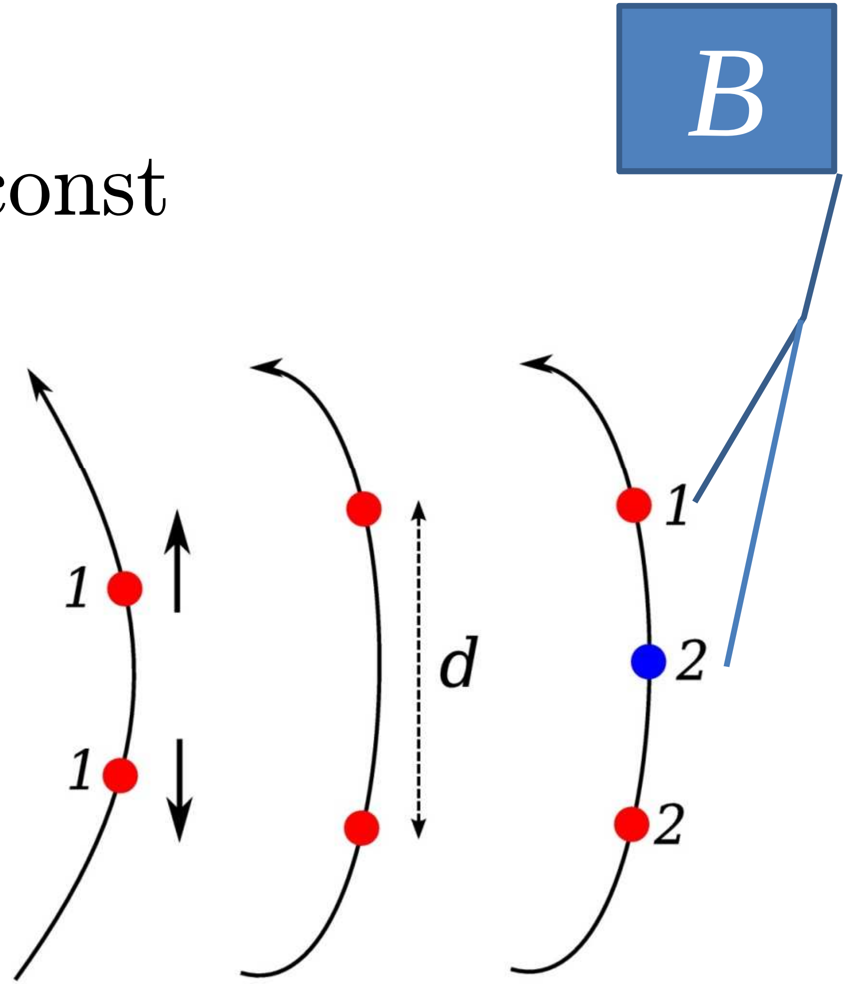
Evolving closed magnetic loops (flux tubes):

$$\dot{\mathbf{x}} = \mathbf{V}(\mathbf{x}, t),$$

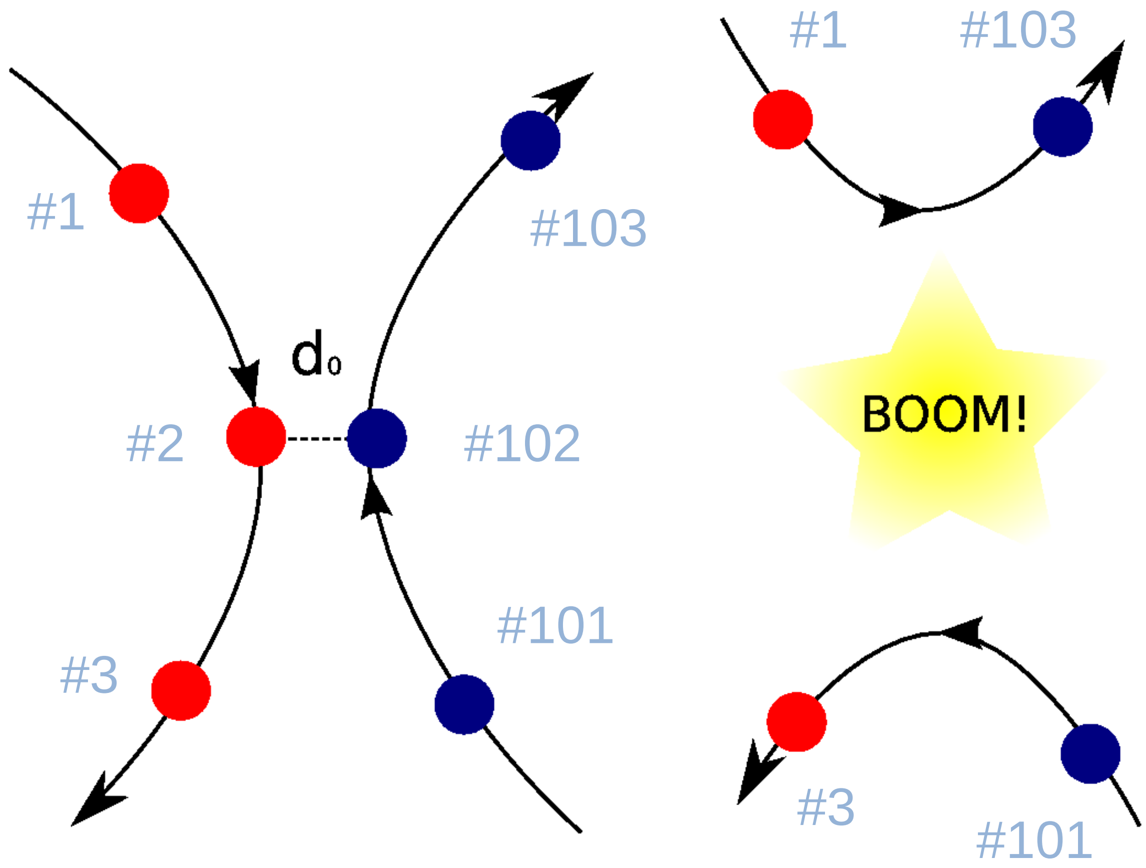
$$\Phi = BS = \text{const} \ , \qquad LS = \text{const}$$

$$\Rightarrow B \propto L$$

Tracer particles positioned along magnetic lines. New particles introduced (removed) when their separation exceeds (reduces below) d; magnetic field enhanced by stretching.



Keep track of the magnetic field direction and reconnect magnetic lines when their separation is $< d_0 \, (\approx d\,)$:



Energy released in the reconnection event: $\propto B^2_{\#2} + B^2_{\#102}$

A conservative model of reconnection:

$$V_{\rm rec} = V(d_0) \ll V(l_0) = V_A$$

Nonlinear effects can be included:

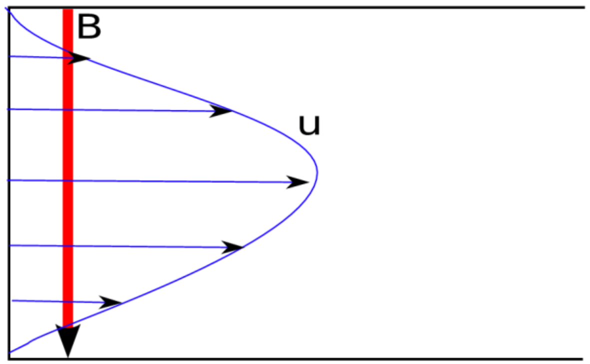
$$\mathbf{V}(t+\delta t) = \mathbf{V}(t) + \delta t\,(\mathbf{B}\cdot\nabla)\mathbf{B},$$

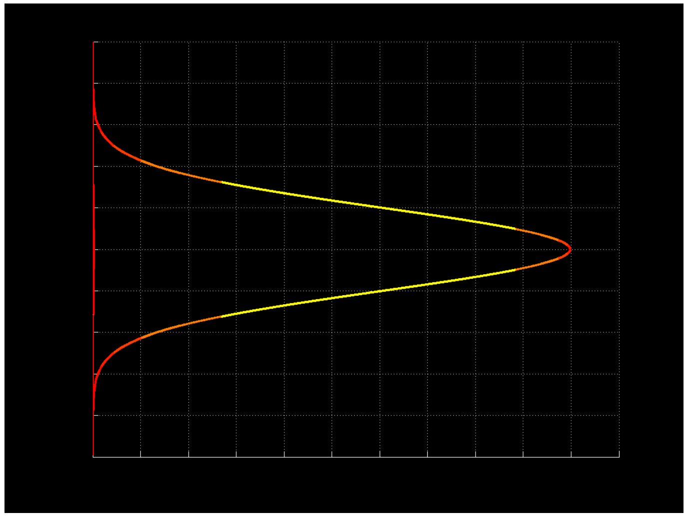
$$(\mathbf{B}\cdot\nabla)\mathbf{B} = B_s\frac{\partial \mathbf{B}}{\partial s}.$$

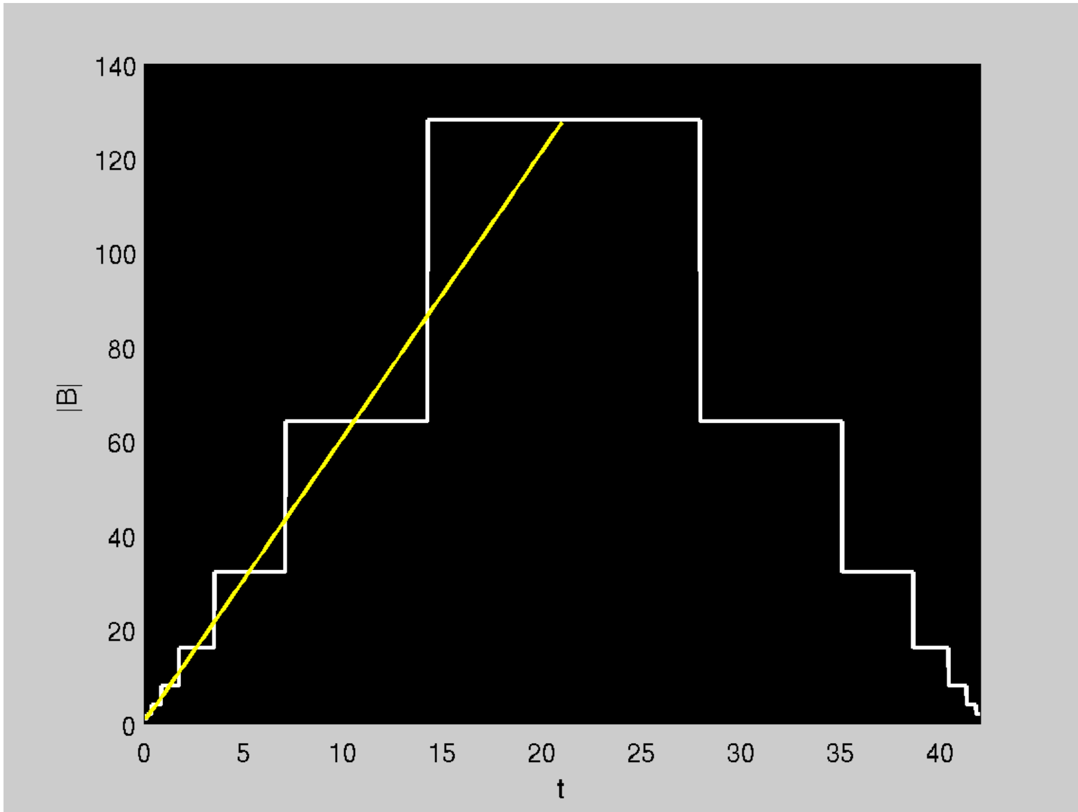
Test 1: shear flow

$$\mathbf{V} = (V_x, 0, 0)\ , \quad V_x = V_0 e^{-y^2/2}$$

$$\eta = 0 \quad \Rightarrow \quad B = B_0 \sqrt{1 + V_0 y^2 e^{-y^2} t^2}$$

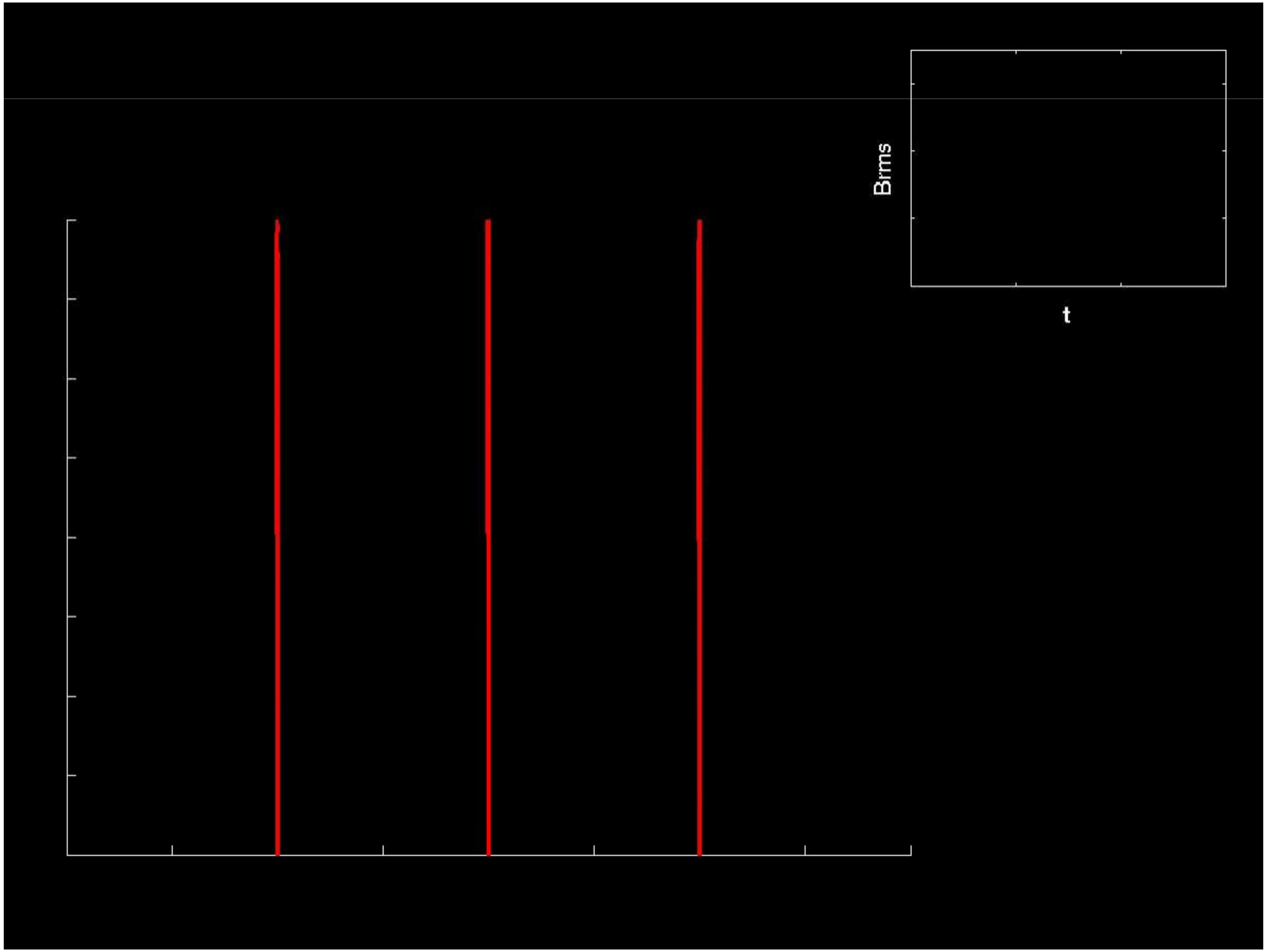




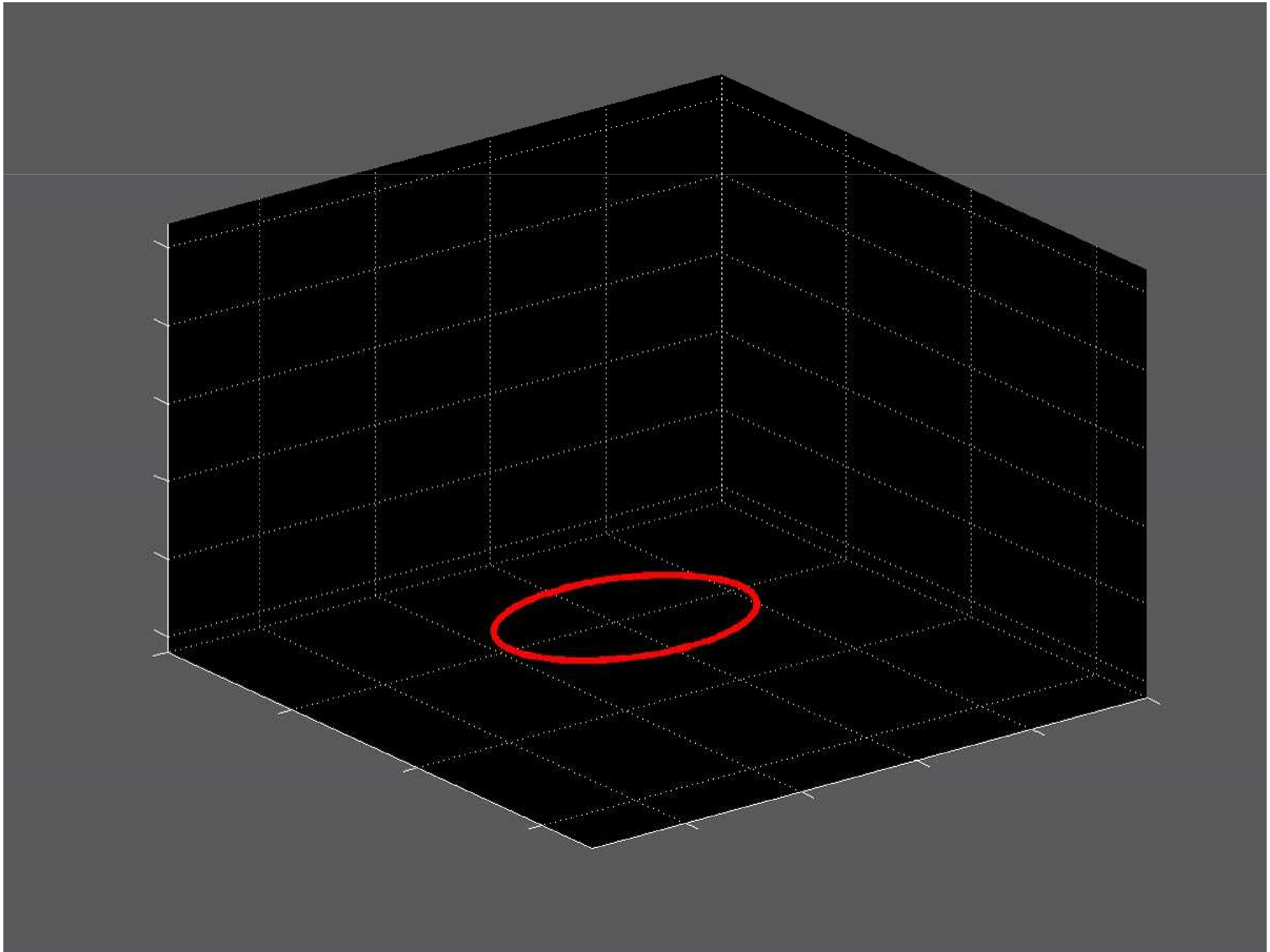


Test 2: flux expulsion from a vortex

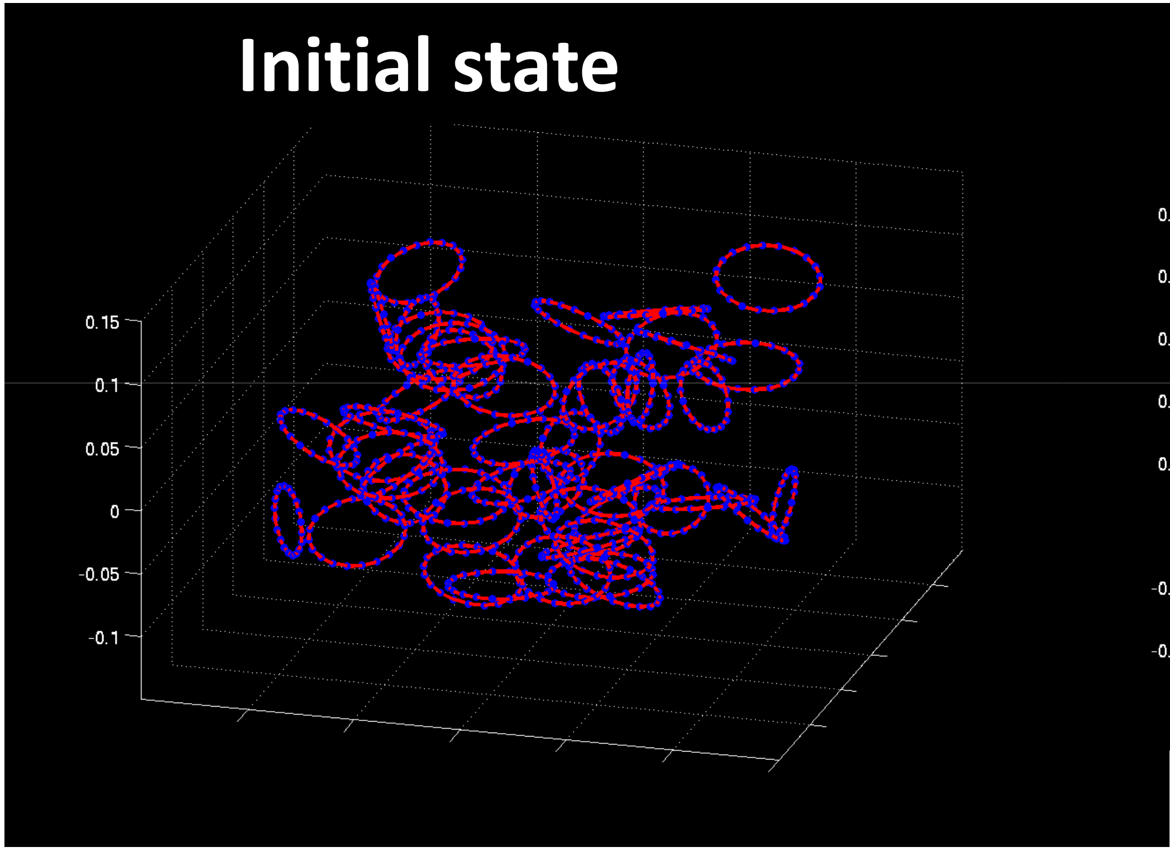
$$\Omega_z = e^{-r^2}\ , \qquad \mathbf{B}\big|_{t=0} = (0, 1, 0)\ .$$

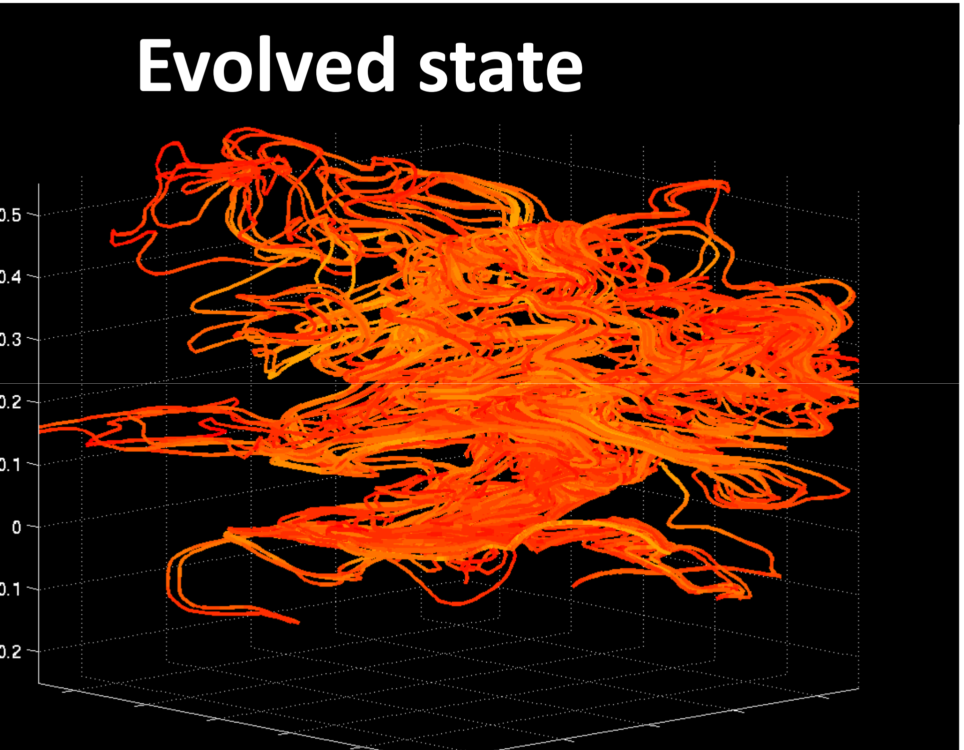


Multiscale flow, single loop, $E_k \propto k^{-5/3}$

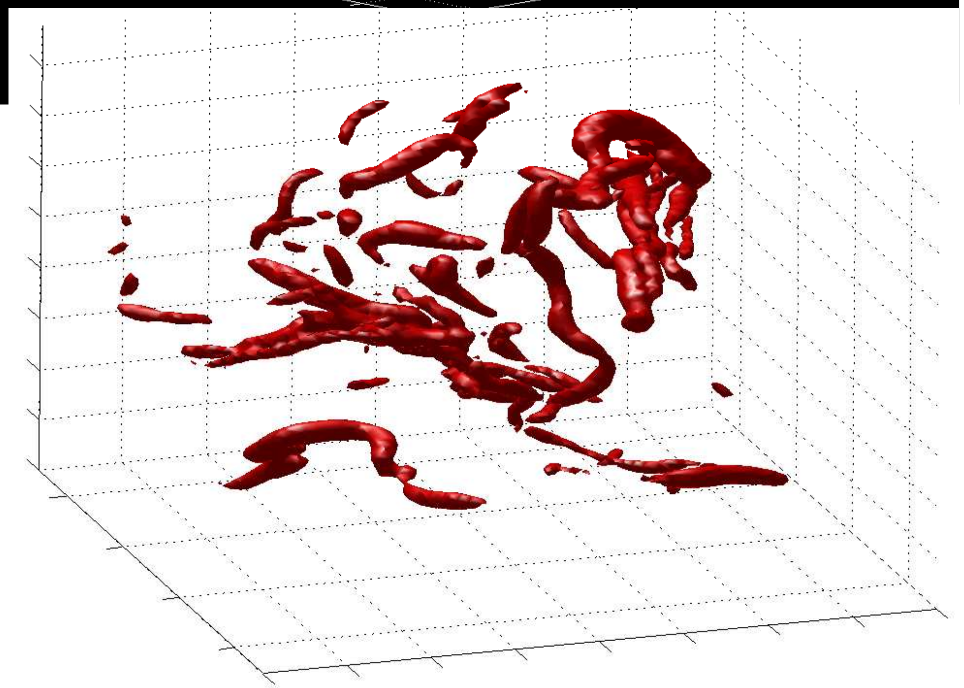


Fluctuation dynamo with reconnections





B^2 = const,
Gaussian smoothing $\Rightarrow$



Compare solutions of the induction equation with those obtained from the reconnection model, with the same $\mathbf{V}(\mathbf{x},t)$

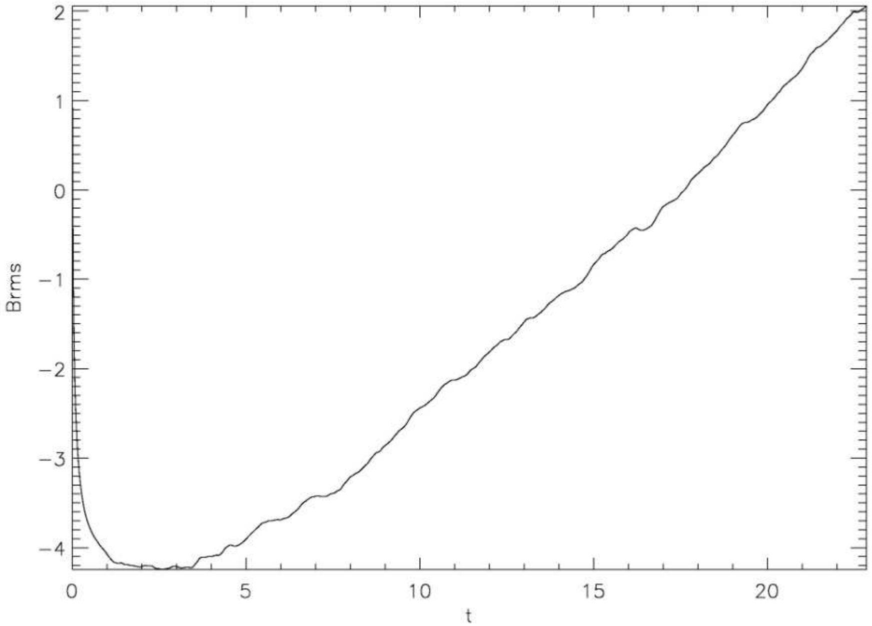
Reconnection dynamo: larger magnetic field growth rate under comparable conditions

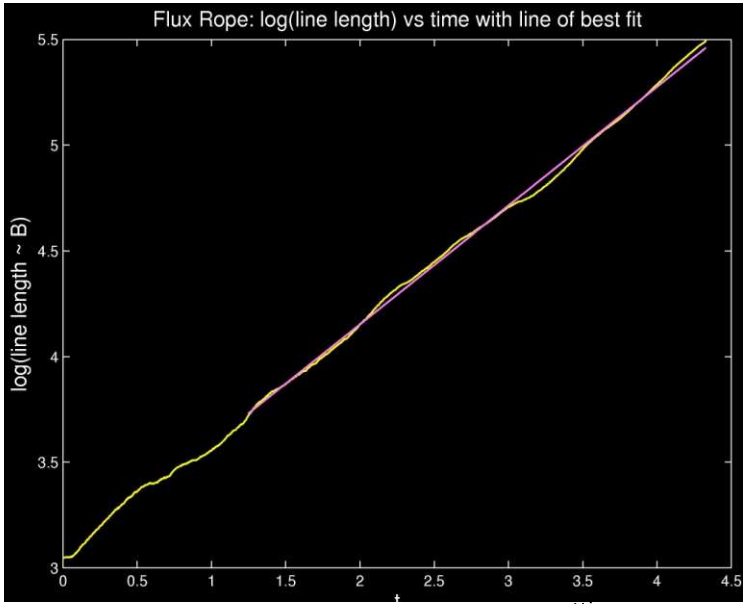
Compare runs with similar growth rates

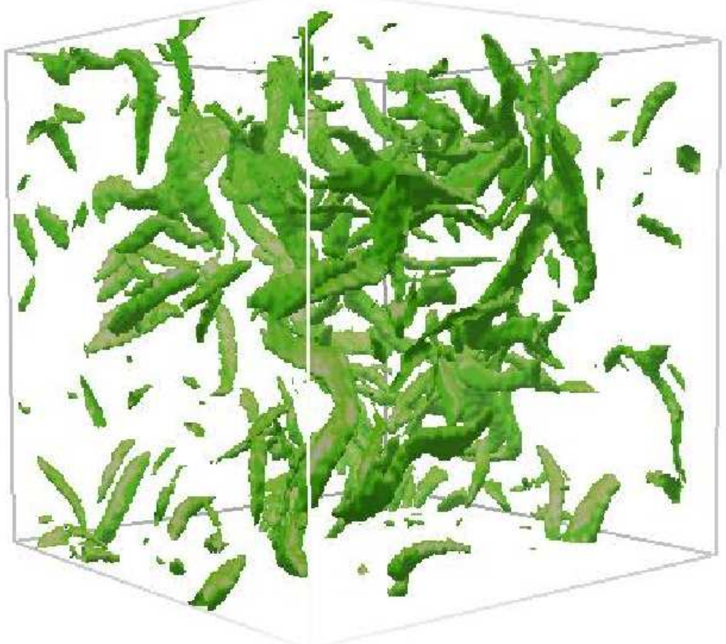
Induction equation

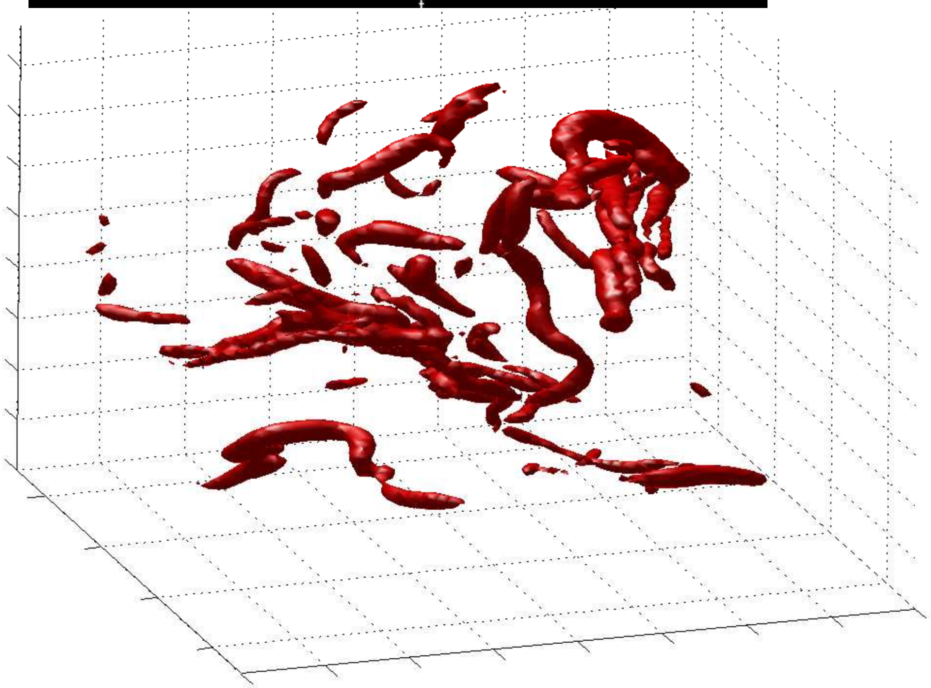
Reconnection model

Kinematic stage









Induction equation

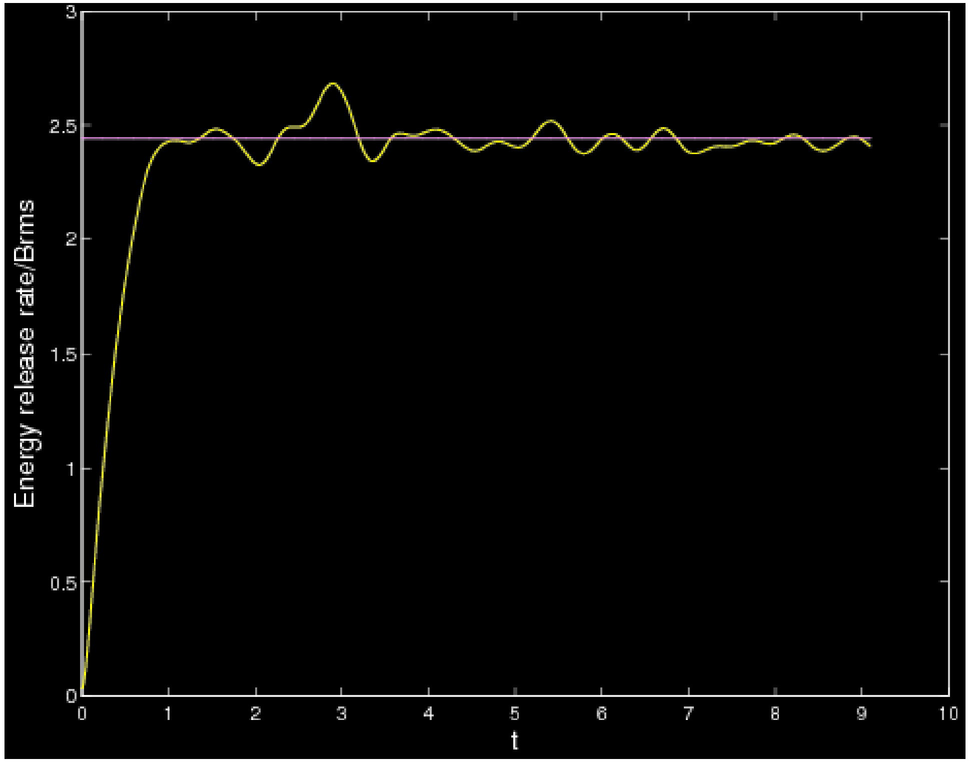
Reconnection model

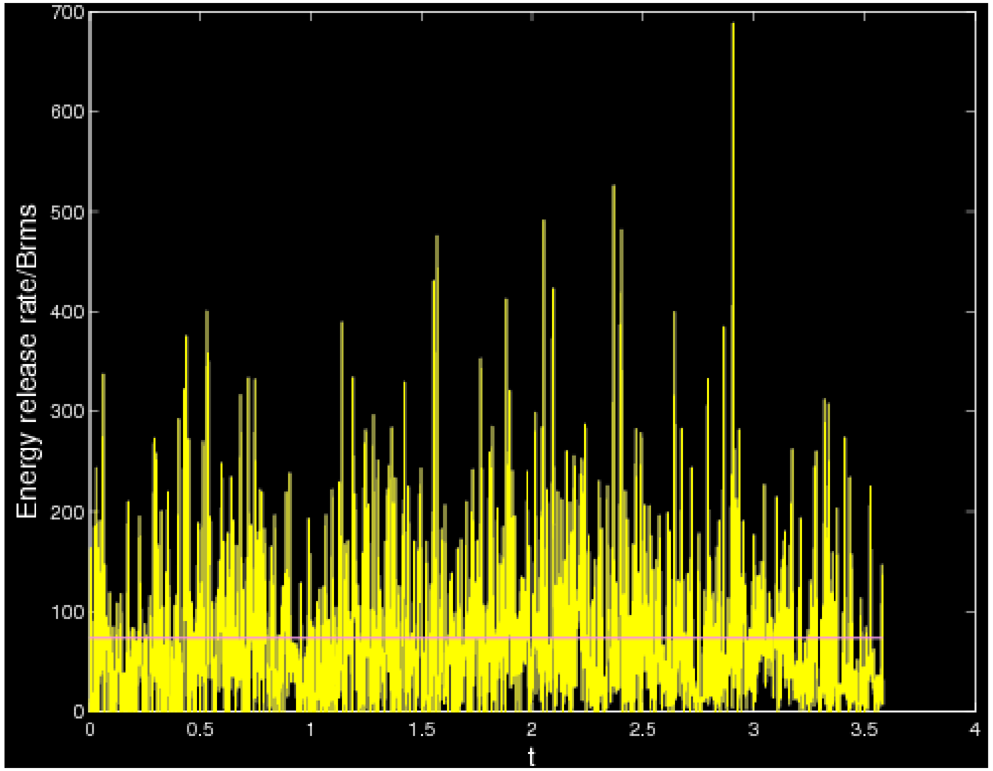
Energy release rate

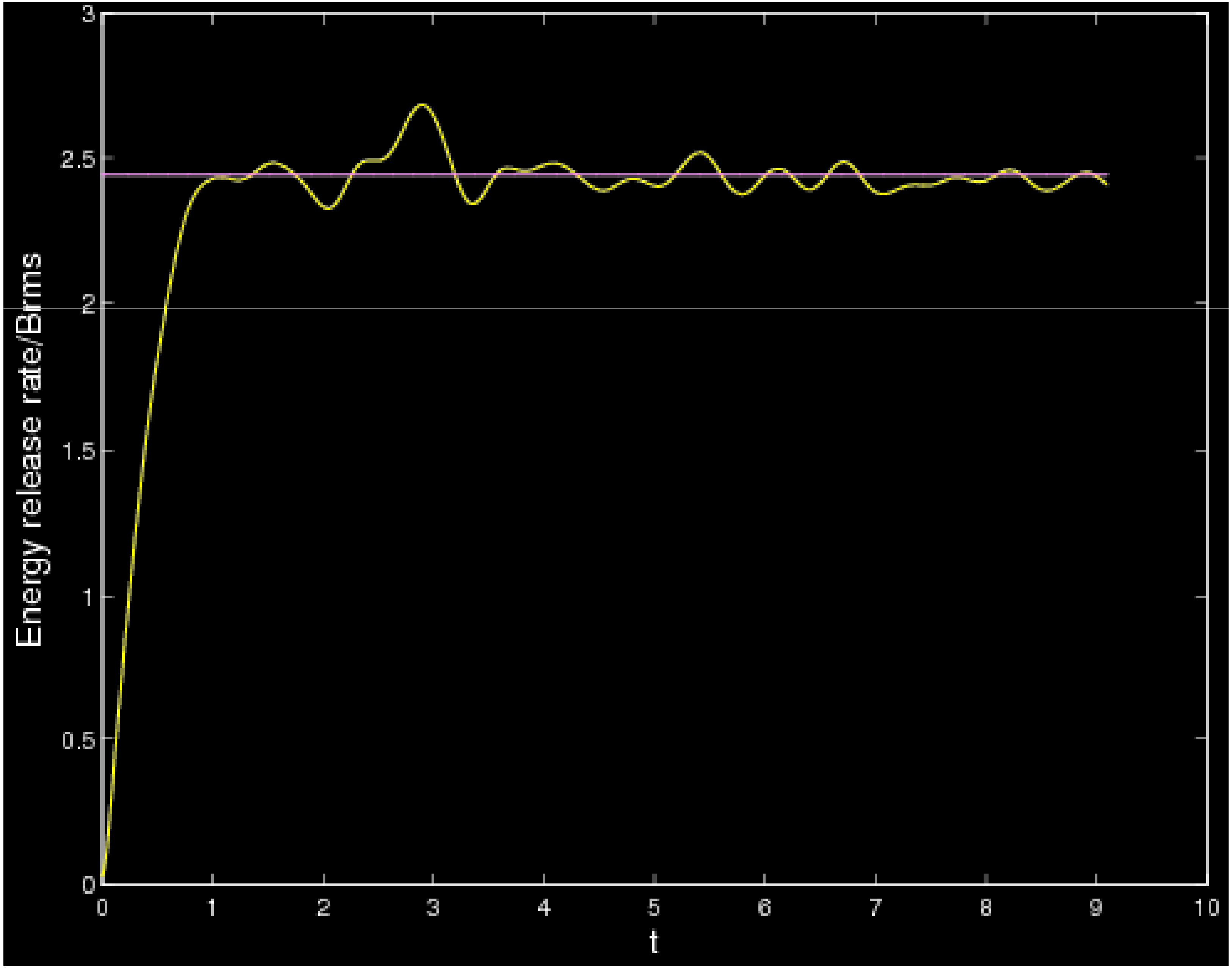
$$\tau_{\mathrm{E}}^{-1} = -\eta \frac{\langle \mathbf{B} \cdot \nabla^2 \mathbf{B} \rangle_V}{B_{\mathrm{rms}}^2}$$

$$\tau_{\mathrm{E,rec}}^{-1} = \frac{1}{B_{\mathrm{rms}}^2 \Delta t} \sum_{i=1}^{N_r} B_i^2 \,,$$

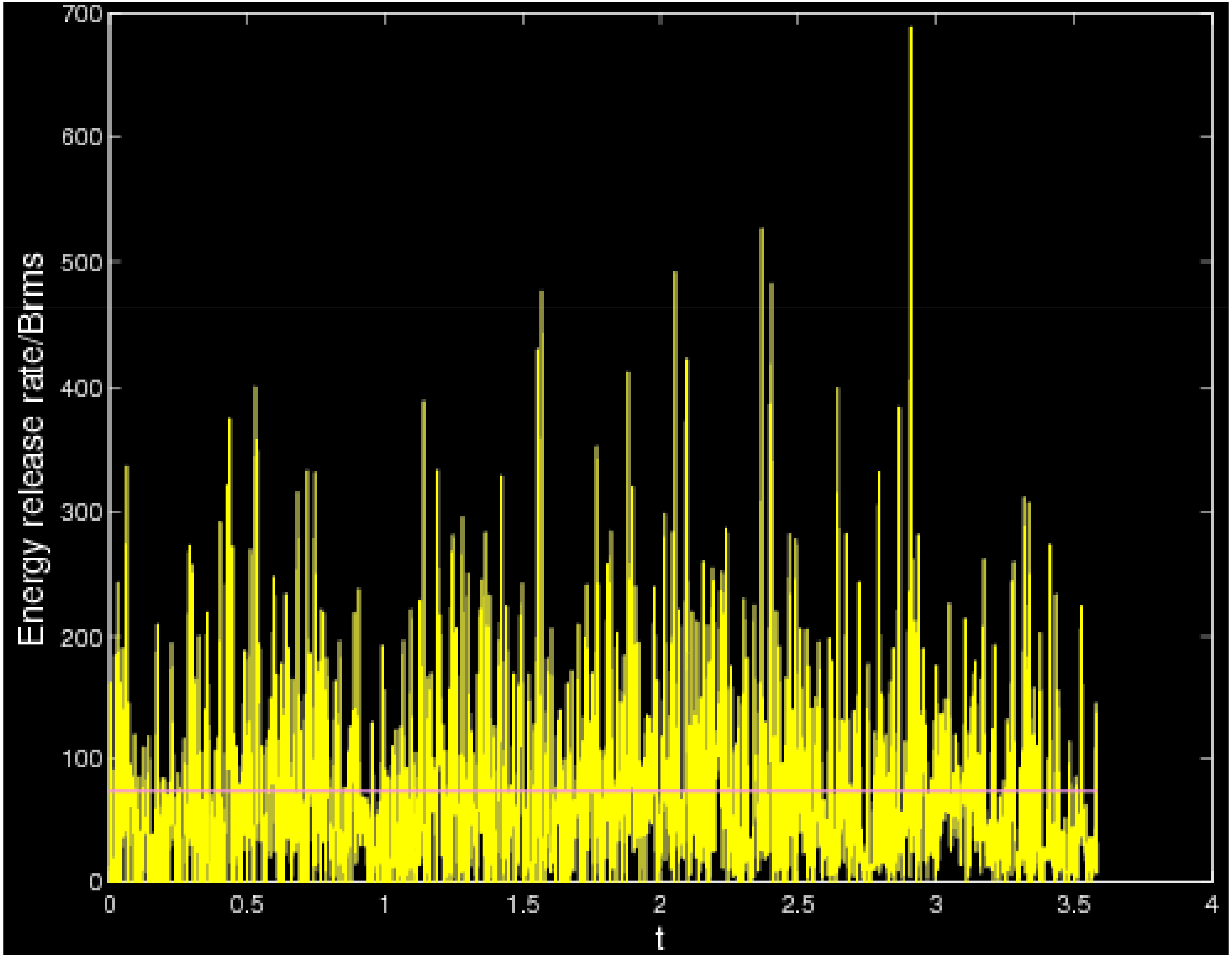
sum over segments removed in reconnection events







Energy release rate/Brms
t
0
0.5
1
1.5
2
2.5
3
0
1
2
3
4
5
6
7
8
9
10



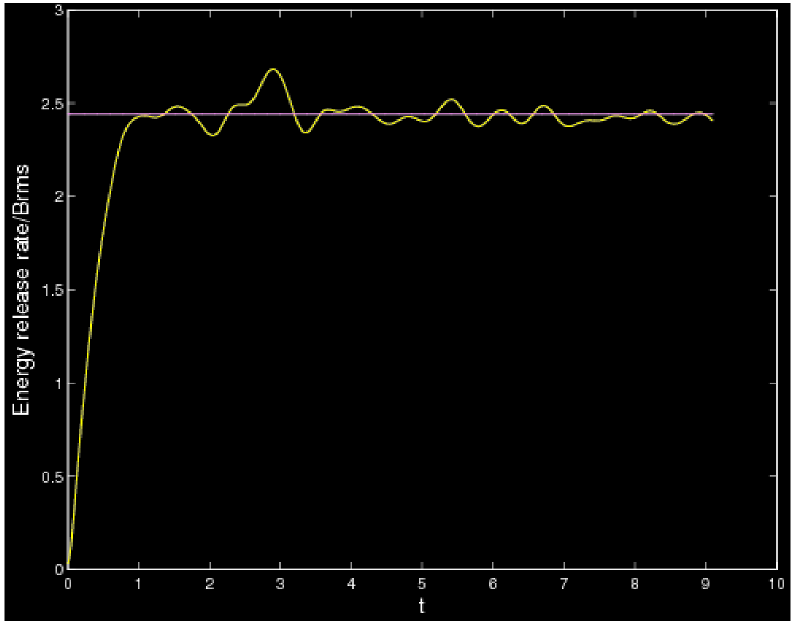
Energy release rate/Brms
t
0
100
200
300
400
500
600
700
0
0.5
1
1.5
2
2.5
3
3.5
4

Induction equation

Reconnection model

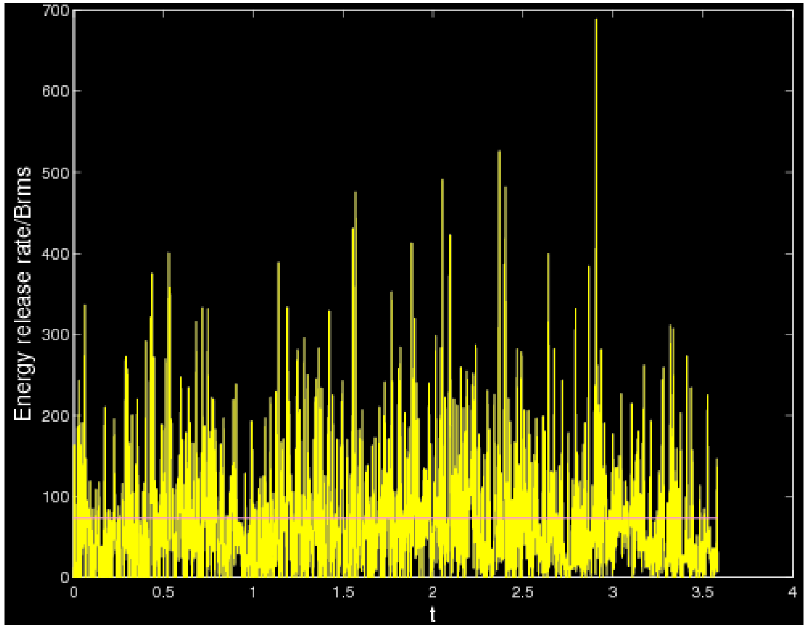
Energy release rate

$$\tau_{\mathrm{E}}^{-1} = \eta \frac{\langle \mathbf{B} \cdot \nabla^2 \mathbf{B} \rangle_V}{B_{\mathrm{rms}}^2}$$



$$\tau_{\mathrm{E,rec}}^{-1} = -\frac{1}{B_{\mathrm{rms}}^2 \Delta t} \sum_{i=1}^{N_r} B_i^2 \ ,$$

sum over segments removed in reconnection events



$$\frac{\tau_{\mathrm{E}}}{\tau_{\mathrm{E,rec}}} \simeq 30$$

Similar results with the ABC flow

Conclusions and prospects

- Magnetic field growth rate and B_{max} are sensitive to the dissipation mechanism.
- Reconnections: growth rates, B_{max} and energy release rates are larger than those from the induction equation (even if B_{rms} are similar).
- Reconnections: more efficient dynamos, faster conversion of kinetic energy into heat, stronger heating.
- Add nonlinearity & reconnection physics, e.g.,

$$\mathbf{V}(t+\Delta t) = \mathbf{V}(t) + \Delta t(\mathbf{B}\cdot\nabla)\mathbf{B} + \mathbf{V}_{\mathrm{rec}} \ , \quad \mathbf{V}_{\mathrm{rec}} = \mathbf{V}_A$$

Nonlinear effects

$$\mathbf{J} \times \mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{B} - \tfrac{1}{2}\nabla B^2$$

$$\tfrac{1}{2}\nabla B^2 + \nabla p/\rho = 0 \qquad \text{(flux tubes in lateral equilibrium)}$$

$$\Rightarrow \frac{\partial \mathbf{u}}{\partial t} = (\mathbf{B} \cdot \nabla)\mathbf{B} + \frac{\mathbf{u}_0 - \mathbf{u}}{\tau}, \qquad (\mathbf{B} \cdot \nabla)\mathbf{B} = B\frac{\partial \mathbf{B}}{\partial s}$$

$$\tau \gg 1 \Rightarrow \frac{\partial \mathbf{u}}{\partial t} = B\frac{\partial \mathbf{B}}{\partial s} \qquad \text{(Alfvén waves)}$$

$$\tau \ll 1 \Rightarrow 0 = B\frac{\partial \mathbf{B}}{\partial s} + \frac{\mathbf{u}_0 - \mathbf{u}}{\tau}$$

$$\mathbf{u} = \mathbf{u}_0 + \tau B\frac{\partial \mathbf{B}}{\partial s} \qquad \text{(slow evolution, e.g., dynamo)}$$

Alfvén waves, nonlinear interactions (& numerical noise)