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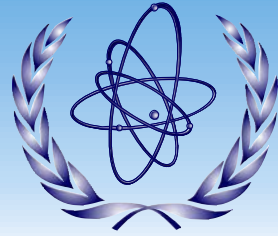
**Joint ICTP/IAEA Workshop on Atomic and Molecular Data for  
Fusion**

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**Calculation of Atomic Data for Plasma Modeling:  
Ionization Processes**

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**International Atomic Energy Agency**

# **Calculation of Atomic Data for Plasma Modeling: Ionization Processes**

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# Electron impact ionization

- **Process similar to excitation, incoming free electron, outgoing free electron with different energy**
- **However, now there are two outgoing free electrons, causes complication**
- **Total energy after collision is  $E_T = E_i - \Delta E$ , where  $E_T$  total energy after collision,  $E_i$  is the impact energy and  $\Delta E$  is the ionization energy**

# Electron impact ionization

- This energy is distributed between the two outgoing electrons
- For a given angular momentum of one outgoing electron, there is a range of angular momenta possible for the other, depending on the angular momentum of the target state
- These considerations give rise to a very large number of possible free electron wave functions needed to solve problem

# Common approximations

- **Lotz formula. Well-known. Not bad accuracy for well-studied systems.**
- **Scaled Coulomb Born. Extension of the excitation method. Fits available. Accuracy acceptable for many systems.**
- **Distorted wave. Problem is now in two outgoing free wave functions, split of total energy between the two. Large increase of computational time**
- **Convergent close coupling (CCC) has been successfully applied to ionization**

# Scaled hydrogenic method

Reduced hydrogenic cross section :

$$Q_{nl}^{HR} = \left( \frac{Z}{n} \right)^4 \frac{Q_{nl}^H}{\pi a_0^2}$$

For complex ion,

$$Q(u) = \pi a_0^2 \left( \frac{n}{Z_{eff}} \right)^4 r_{nl} Q_{nl}^{HR}$$

## Scaled hydrogenic method

Note for hydrogenic ion, ionization energy is :

$$E^I (Ryd) = \left( \frac{Z}{n} \right)^2$$

Assume similar for complex ion to arrive at :

$$Q(u) = \frac{\pi a_0^2}{[E^I (Ryd)]^2} r_{nl} Q_{nl}^{HR} (u)$$

# Scaled hydrogenic method

- If scaled hydrogenic cross section,  $Q_{nl}^{HR}(u)$  is known, then cross section for arbitrary ion can be calculated
- Scaled cross sections calculated for 1s through 6g have been fitted as function of shell and impact energy
- Result is fit formula for scaled hydrogenic cross section
- Allows calculation of approximate cross section for any ion



# Scaled Hydrogenic Method

Fit form used :

$$Q_{nl}^{HR}(u) = \frac{1}{u} \left[ \left( C_1 + \frac{C_2 + C_3 l}{n} \right) \ln(u) + \left( C_4 + \frac{C_5 + C_6 l}{n} \right) \left( 1 - \frac{1}{u} \right) + \left( C_7 + \frac{C_8 + C_9 l}{n} \right) \left( 1 - \frac{1}{u} \right)^2 \right]$$

## Scaled hydrogenic

- In previous,  $u$  is the electron impact energy in threshold units
- Result is cross section for one shell of hydrogenic
- Possible to include angular coupling effects as well as mixing of target states
- The  $C_i$  are the fitting coefficients with the values:

# Fitting parameters

**$C_1$  1.5369**

**$C_2$  0.99656**

**$C_3$  -0.61916**

**$C_4$  2.4463**

**$C_5$  -2.4773**

**$C_6$  3.2151**

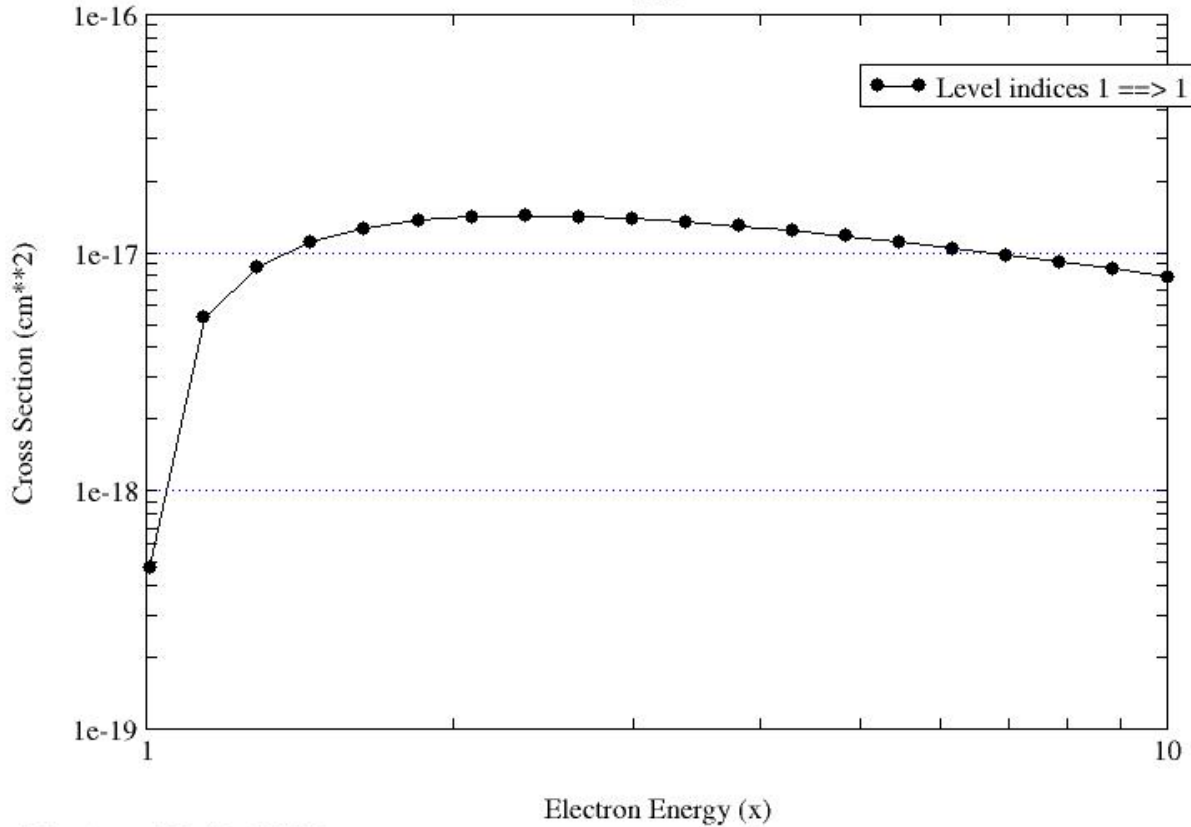
**$C_7$  -1.4512**

**$C_8$  1.7230**

**$C_9$  -0.47075**

### Scaled hydrogenic ionization

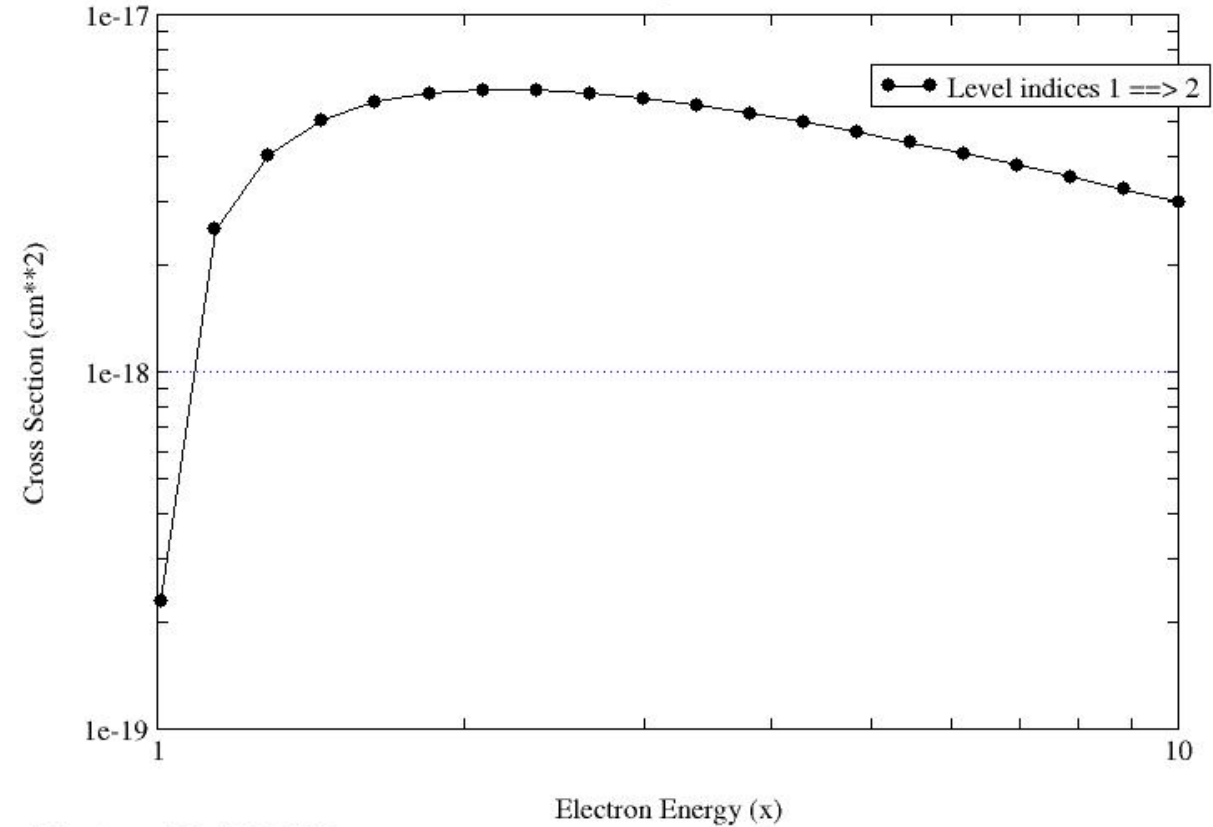
C+2



Mon Apr 6 02:47:16 2009

### Scaled hydrogenic ionization

Mg+2



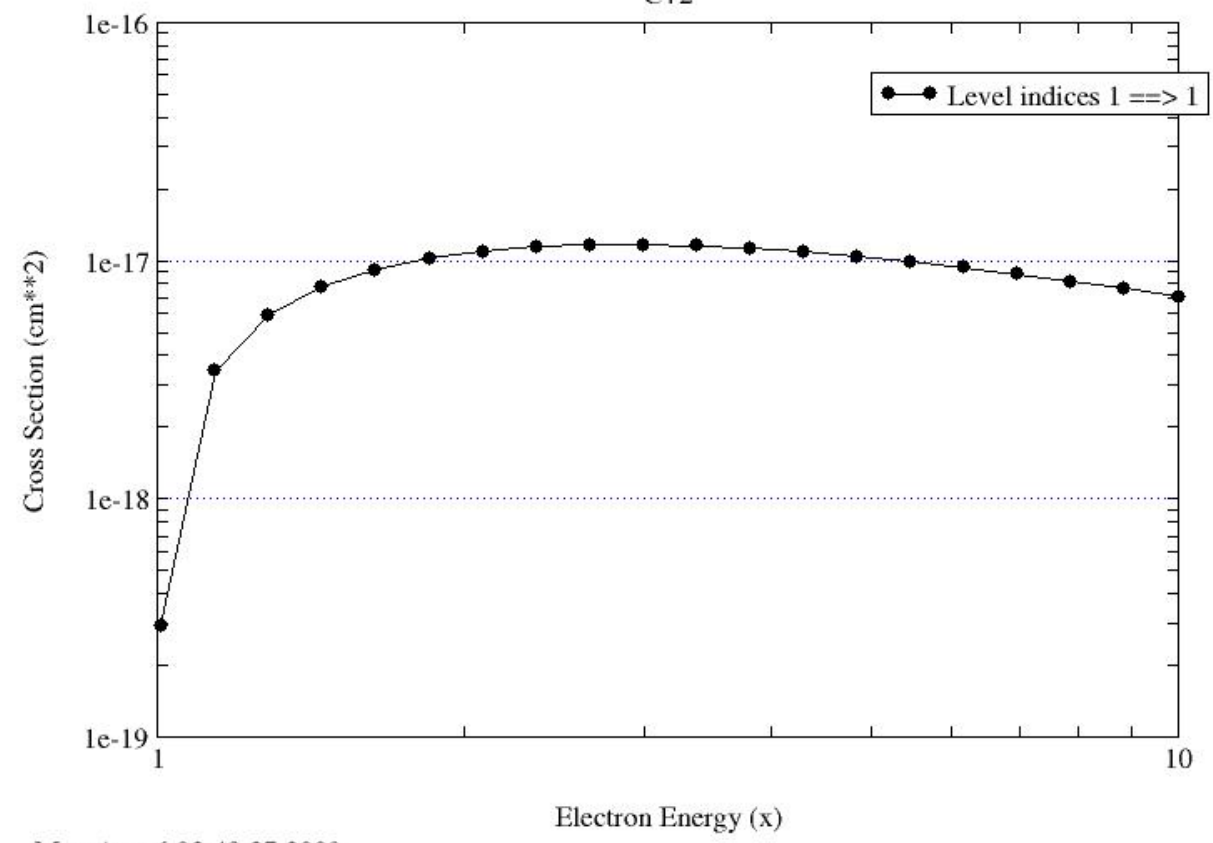
Mon Apr 6 02:54:34 2009

## Distorted wave

- **Similar to calculation for excitation**
- **Difficulty is now there are two outgoing free electrons**
- **Energy splits between two, very many free electron orbitals needed**
- **Perform quadrature over outgoing electron energy split**
- **Computational time is significant**

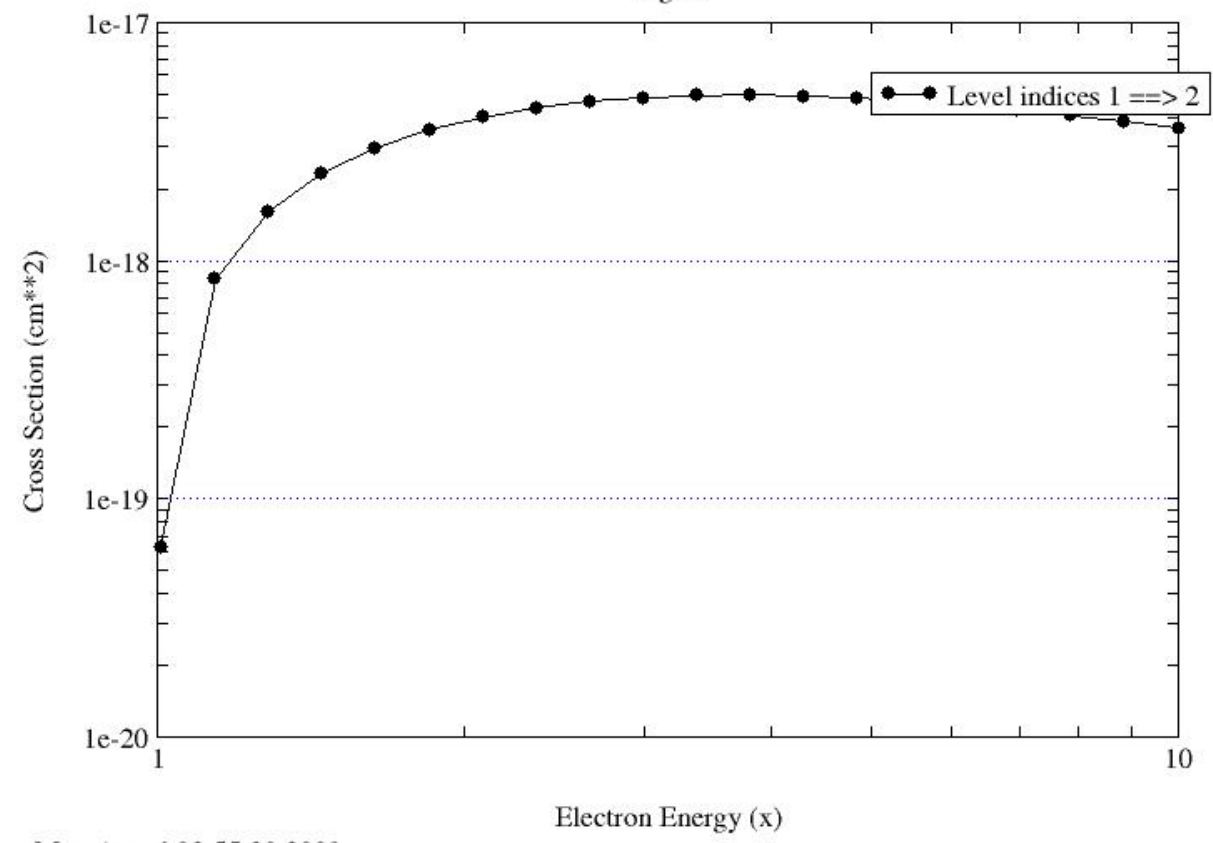
### Distorted wave ionization

C+2



Mon Apr 6 02:48:27 2009

### Distorted wave ionization Mg+2



Mon Apr 6 02:55:29 2009



# Close coupling

- **Application now made to ionization**
- **Convergent close coupling technique developed by Bray and Fursa has wide range of applications**
- **Still under development**
- **Difficulty handling partial filled shells**

# Photoionization

- **Photon causes ejection of electron**
- **Not very likely in fusion plasmas**
- **Inverse process is radiative recombination**
- **Competes with inverse of electron impact ionization, three body recombination**
- **Radiative recombination is often dominant recombination mechanism**

# Autoionization

- **May have ion suffer inner shell excitation**
- **Result may lie above continuum**
- **Result is autoionization**
- **Inverse process is di-electronic recombination**
- **Both may be very important in fusion plasmas**
- **May be calculated with DW theory, or automatically generated with CC methods**

Levels of Mg + 1

Energy (eV)	Stat. Wt.	Level
0.0000e+00	2	$(2p^6 1S)^1S (3s^1 2S)^2S_{0,5}$
4.4725e+00	2	$(2p^6 1S)^1S (3p^1 2P)^2P_{0,5}$
4.4812e+00	4	$(2p^6 1S)^1S (3p^1 2P)^2P_{1,5}$
9.1187e+00	4	$(2p^6 1S)^1S (3d^1 2D)^2D_{1,5}$
9.1187e+00	6	$(2p^6 1S)^1S (3d^1 2D)^2D_{2,5}$
4.9418e+01	4	$(2p^5 2P)^2P (3s^2 1S)^2P_{1,5}$
4.9689e+01	2	$(2p^5 2P)^2P (3s^2 1S)^2P_{0,5}$
5.2834e+01	4	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4S_{1,5}$
5.3212e+01	8	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4D_{3,5}$
5.3275e+01	6	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4D_{2,5}$
5.3341e+01	4	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4D_{1,5}$
5.3401e+01	2	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4D_{0,5}$
5.3539e+01	6	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4P_{2,5}$
5.3613e+01	4	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4P_{1,5}$
5.3659e+01	2	$(2p^5 2P)^2P (3s^1 2S)^3P (3p^1 2P)^4P_{0,5}$
5.3768e+01	4	$(2p^5 2P)^2P (3s^1 2S)^1P (3p^1 2P)^2D_{1,5}$
5.3863e+01	6	$(2p^5 2P)^2P (3s^1 2S)^1P (3p^1 2P)^2D_{2,5}$
5.3895e+01	2	$(2p^5 2P)^2P (3s^1 2S)^1P (3p^1 2P)^2P_{0,5}$

6.0575e+01	6	$(2p^5 2p)^2 P (3s^1 2s)^1 P (3d^1 2D)^2 F_{2.5}$
6.0620e+01	4	$(2p^5 2p)^2 P (3s^1 2s)^1 P (3d^1 2D)^2 P_{1.5}$
6.0662e+01	8	$(2p^5 2p)^2 P (3s^1 2s)^1 P (3d^1 2D)^2 F_{3.5}$
6.0709e+01	2	$(2p^5 2p)^2 P (3s^1 2s)^1 P (3d^1 2D)^2 P_{0.5}$
6.0724e+01	6	$(2p^5 2p)^2 P (3s^1 2s)^1 P (3d^1 2D)^2 D_{2.5}$
6.0724e+01	4	$(2p^5 2p)^2 P (3s^1 2s)^1 P (3d^1 2D)^2 D_{1.5}$
6.1111e+01	2	$(2p^5 2p)^2 P (3p^2 3p)^2 P_{0.5}$
6.1167e+01	4	$(2p^5 2p)^2 P (3p^2 3p)^2 P_{1.5}$
6.2601e+01	8	$(2p^5 2p)^2 P (3s^1 2s)^3 P (3d^1 2D)^2 F_{3.5}$
6.2732e+01	6	$(2p^5 2p)^2 P (3s^1 2s)^3 P (3d^1 2D)^2 F_{2.5}$
6.2762e+01	4	$(2p^5 2p)^2 P (3p^2 1S)^2 P_{1.5}$
6.2936e+01	4	$(2p^5 2p)^2 P (3s^1 2s)^3 P (3d^1 2D)^2 D_{1.5}$
6.2970e+01	6	$(2p^5 2p)^2 P (3s^1 2s)^3 P (3d^1 2D)^2 D_{2.5}$
6.2991e+01	2	$(2p^5 2p)^2 P (3p^2 1S)^2 P_{0.5}$
6.3217e+01	2	$(2p^5 2p)^2 P (3s^1 2s)^3 P (3d^1 2D)^2 P_{0.5}$
6.3350e+01	4	$(2p^5 2p)^2 P (3s^1 2s)^3 P (3d^1 2D)^2 P_{1.5}$

Levels of Mg + 2

Energy (eV)	Stat. Wt.	Level
1.5354e+01	1	$(2p^6 1S)^1 S_{0,0}$

Current element is Mg (atomic number 12) with charge of + 1

[Help on configuration energies](#)

Shell energies of Mg + 1 Energy (eV) Shell	Shell energies of Mg + 2 Energy (eV) Shell
Shell energies for 3s 1.3469e+03 1s <sup>2</sup> 1.1259e+02 2s <sup>2</sup> 7.1853e+01 2p <sup>6</sup> 1.5213e+01 3s Total configuration energy is 0.0000e+00 Stat. Wt. is 2	Shell energies for 1.3581e+03 1s <sup>2</sup> 1.2342e+02 2s <sup>2</sup> 8.2845e+01 2p <sup>6</sup> Total configuration energy is 1.5198e+01 Stat. Wt. is 1
Shell energies for 3p 1.3488e+03 1s <sup>2</sup> 1.1434e+02 2s <sup>2</sup> 7.3786e+01 2p <sup>6</sup> 1.0751e+01 3p Total configuration energy is 4.4510e+00 Stat. Wt. is 6	
Shell energies for 3d 1.3519e+03 1s <sup>2</sup> 1.1721e+02 2s <sup>2</sup> 7.6638e+01 2p <sup>6</sup> 6.2349e+00 3d Total configuration energy is 8.9633e+00 Stat. Wt. is 10	

AUTOIONIZATION

[JPEG file\(new window\)](#)

Energy eV	Transition	AI Rate
3.8047e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^4D_{0.5} -- (2p^6 1S)^1S_{0.0}$	1.3459e+09
3.8305e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^4P_{0.5} -- (2p^6 1S)^1S_{0.0}$	1.6541e+11
3.8541e+01	$(2p^5 2p)^2P (3s^1 2S)^1P (3p^1 2P)^2P_{0.5} -- (2p^6 1S)^1S_{0.0}$	5.4758e+10
3.9512e+01	$(2p^5 2p)^2P (3s^1 2S)^1P (3p^1 2P)^2S_{0.5} -- (2p^6 1S)^1S_{0.0}$	1.3030e+13
4.1598e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^2P_{0.5} -- (2p^6 1S)^1S_{0.0}$	3.3172e+12
4.2140e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^2S_{0.5} -- (2p^6 1S)^1S_{0.0}$	6.2930e+13
3.7987e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^4D_{1.5} -- (2p^6 1S)^1S_{0.0}$	1.5674e+09
3.8259e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^4P_{1.5} -- (2p^6 1S)^1S_{0.0}$	6.6580e+09
3.8413e+01	$(2p^5 2p)^2P (3s^1 2S)^1P (3p^1 2P)^2D_{1.5} -- (2p^6 1S)^1S_{0.0}$	3.2325e+10
3.8602e+01	$(2p^5 2p)^2P (3s^1 2S)^1P (3p^1 2P)^2P_{1.5} -- (2p^6 1S)^1S_{0.0}$	7.9359e+08
4.1343e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^2D_{1.5} -- (2p^6 1S)^1S_{0.0}$	2.2132e+12
4.1558e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^2P_{1.5} -- (2p^6 1S)^1S_{0.0}$	1.1169e+12
3.7921e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^4D_{2.5} -- (2p^6 1S)^1S_{0.0}$	1.1165e+09
3.8185e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^4P_{2.5} -- (2p^6 1S)^1S_{0.0}$	3.9190e+09
3.8509e+01	$(2p^5 2p)^2P (3s^1 2S)^1P (3p^1 2P)^2D_{2.5} -- (2p^6 1S)^1S_{0.0}$	3.2099e+10
4.1189e+01	$(2p^5 2p)^2P (3s^1 2S)^3P (3p^1 2P)^2D_{2.5} -- (2p^6 1S)^1S_{0.0}$	3.3343e+12
3.4335e+01	$(2p^5 2p)^2P (3s^2 1S)^2P_{n,s} -- (2p^6 1S)^1S_{n,n}$	1.2827e+12

## Data from archives

- **Choice is made on opening page**
- **Select element from list**
- **Select particular model, normally only one, can be more**
- **Select ion**
- **Can now view data, but cannot add to these files**







# Search LANL Atomic Data

Choose element

- Al
- Ar
- Fe
- He

Select next task to perform

- Select a set of data files
- EXIT from calculations

Continue

[Comments, suggestions and questions are welcome.](#)

Make data selections - Windows Internet Explorer

http://aphysics2.lanl.gov/cgi-bin/ION/runlan11.pl

File Edit View Favorites Tools Help

Google Search

Windows Live Live Search

What's New Profile Mail Photos Calendar MSN Share

Make data selections

# Make data selections

---

Current element is Ar (atomic number 18) with charge of + 16

---

Select next task to perform:

- Show energy levels
- Select levels for mixing coefficient display
- Show total configuration energies
- Show shell energies for all configurations
- Select configuration for graph of radial wave function
- Select radiative transitions
- Select PWB transitions
- Select ACE excitation transitions
- Select photoionization transitions
- Select scaled hydrogenic transitions
- Select autoionization transitions
- Change parameters
- Select a stage of ionization
- Select a set of data files
- Select an element from existing data files

Internet | Protected Mode: On 100%

REHC Make data selection... Microsoft PowerPoi... 5:56 PM

Select levels for autoionization transitions - Windows Internet Explorer

http://aphysics2.lanl.gov/cgi-bin/ION/runlan11.pl

File Edit View Favorites Tools Help

Google Search

Windows Live Live Search

Select levels for autoionization transitions

## Select levels for autoionization transitions

---

Current element is Ar (atomic number 18) with charge of + 16

---

There are options to select transitions: 1) from a single lower level to selected upper levels, 2) to a single upper level from selected lower levels or 3) in the case of relatively few transitions, to select all transitions.

Make a choice by checking the appropriate box.

- Select a specific level from first (doubly excited) ion stage for autoionization transitions
- Select a specific level from second ion stage for autoionization transitions
- All transitions (a total of 45 transitions)
- View data or continue calculations from previous calculation
- Change parameters
- Select a stage of ionization
- Select a set of data files
- Select an element from existing data files
- Reset
- EXIT from calculations

Continue

Done

Internet | Protected Mode: On 100%

REHC Select levels for aut... Microsoft PowerPoi...

5:57 PM

# Summary

- **Interface allows calculation of electron impact ionization, photoionization and autoionization**
- **Scaled hydrogenic, binary encounter of DW**
- **Graphical and files of data available**
- **Archived data available**