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**Joint ICTP/IAEA Workshop on Atomic and Molecular Data for
Fusion**

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The Use of Atomic Data in Collisional-Radiative Modeling

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THE USE OF ATOMIC DATA IN COLLISIONAL-RADIATIVE MODELING

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Workshop on Atomic and Molecular Data for
Fusion Energy Research

April 20 – 24, 2009

ICTP, Trieste, Italy



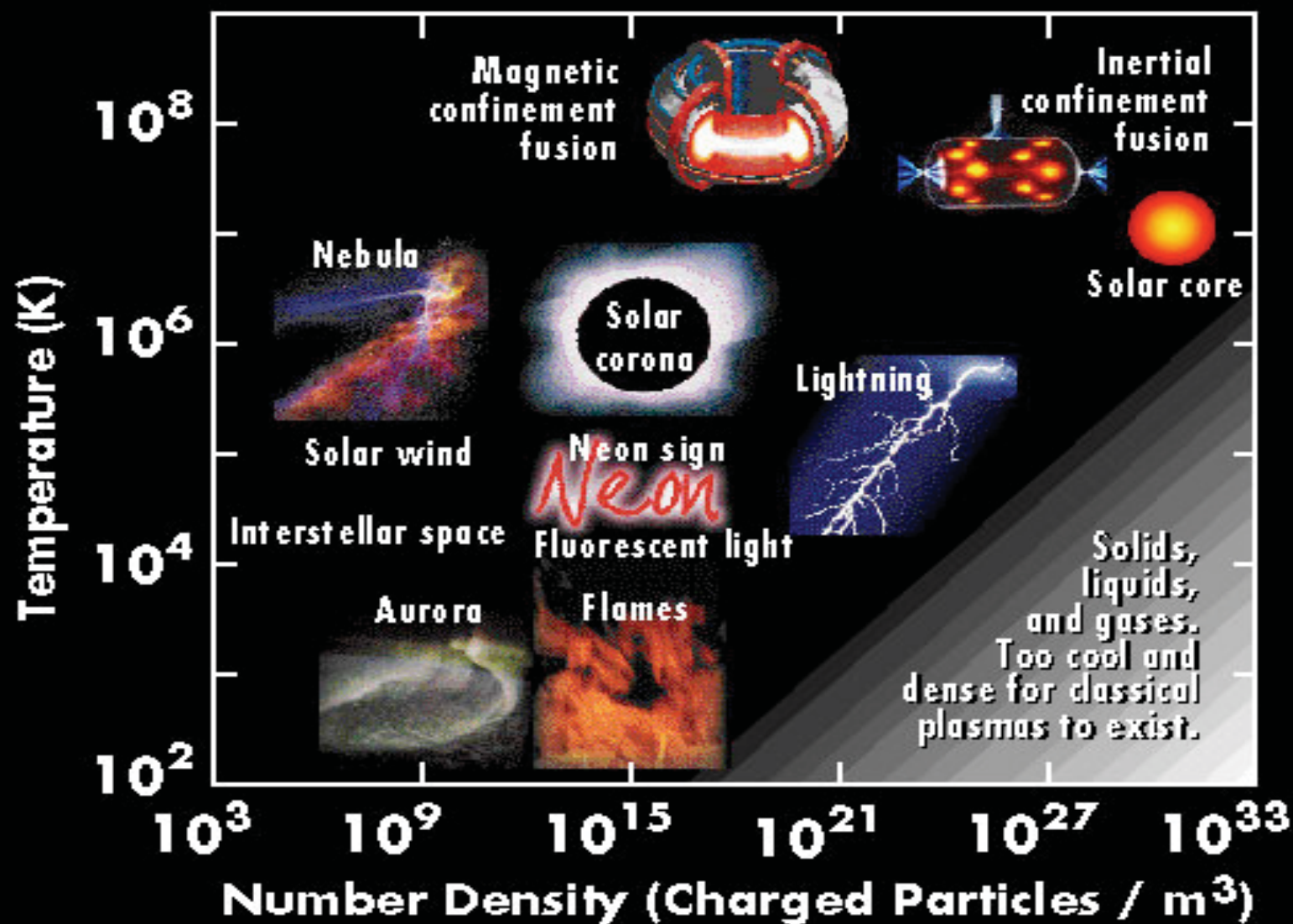
THE USE OF ATOMIC DATA IN COLLISIONAL- RADIATIVE MODELING

- Introduction and motivation
- Atomic Physics Data
- Collisional-Radiative Modeling-Methods
- Radiation Properties of Plasmas
- Opacity

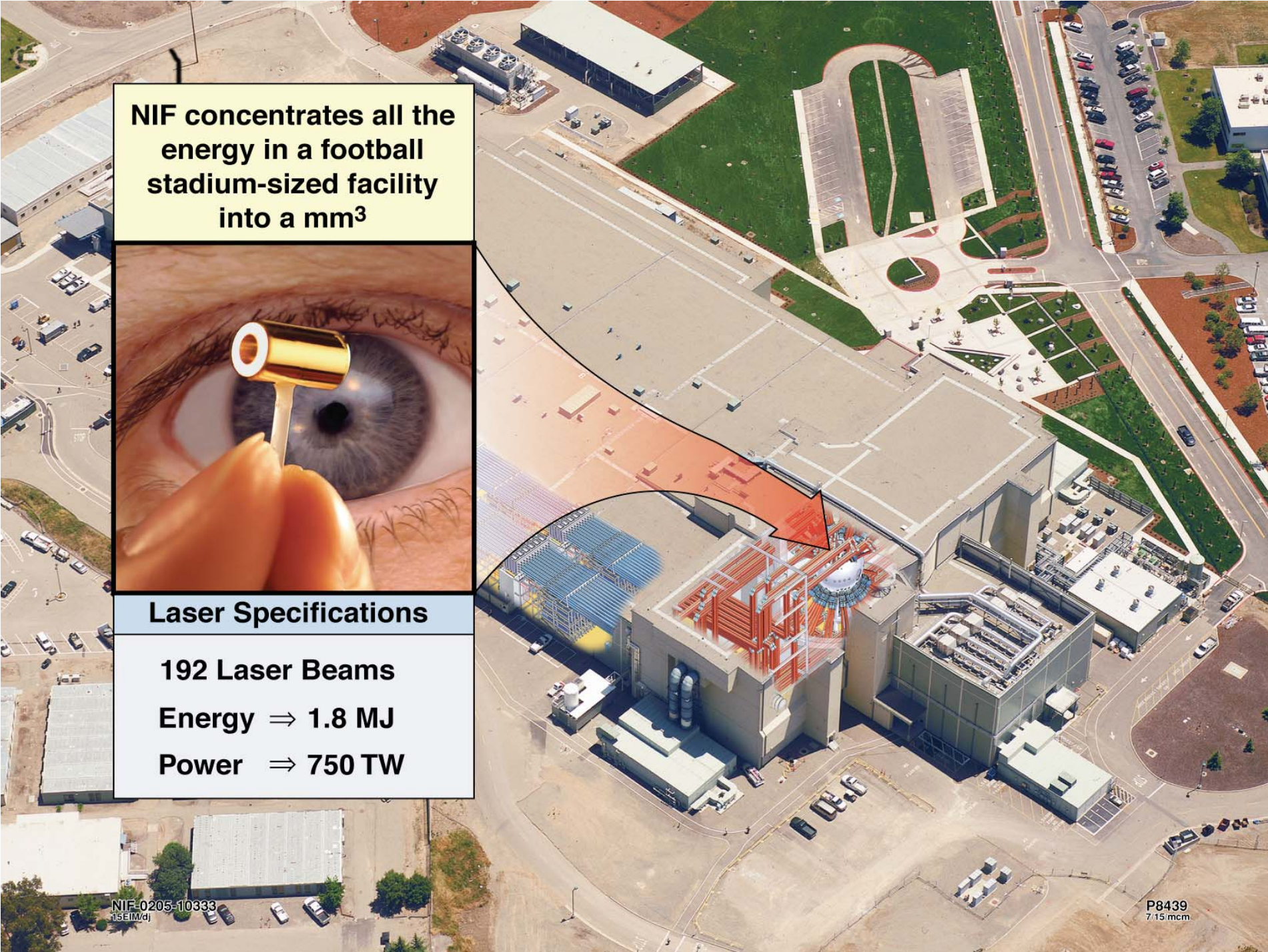


What are plasmas?

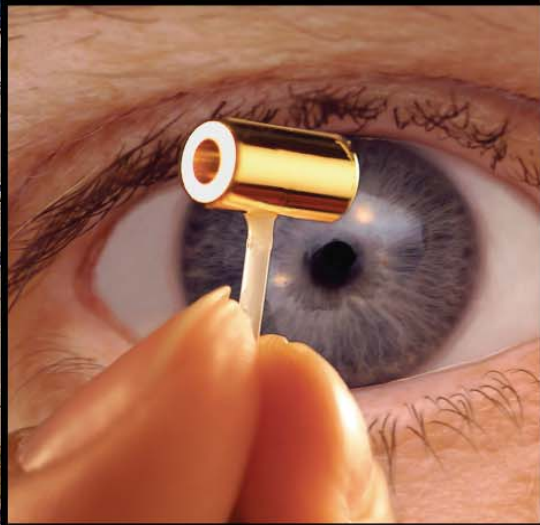
Solid	Liquid	Gas	Plasma
Example Ice H_2O	Example Water H_2O	Example Steam H_2O	Example Ionized Gas $\text{H}_2 \rightarrow \text{H}^+ + \text{H}^+ + 2\text{e}^-$
Cold $T < 0^\circ\text{C}$	Warm $0 < T < 100^\circ\text{C}$	Hot $T > 100^\circ\text{C}$	Hotter $T > 100,000^\circ\text{C}$ $I > 10$ electron Volts
			
Molecules Fixed in Lattice	Molecules Free to Move	Molecules Free to Move, Large Spacing	Ions and Electrons Move Independently, Large Spacing



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Images courtesy of DOE fusion labs, NASA, and Steve Albers.



**NIF concentrates all the
energy in a football
stadium-sized facility
into a mm³**



Laser Specifications

192 Laser Beams

Energy $\Rightarrow$ 1.8 MJ

Power $\Rightarrow$ 750 TW

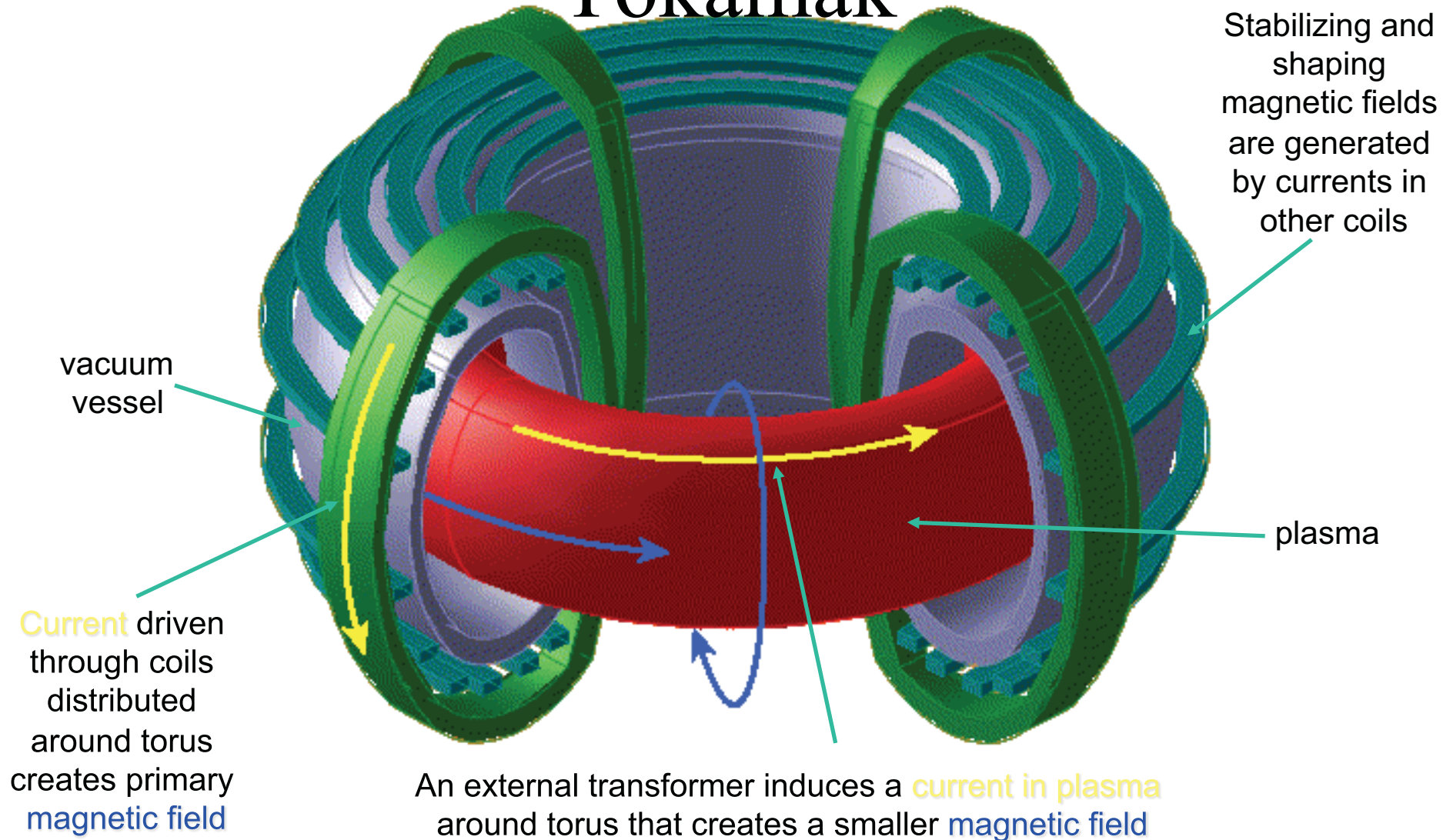
**NIF is 80%
complete**



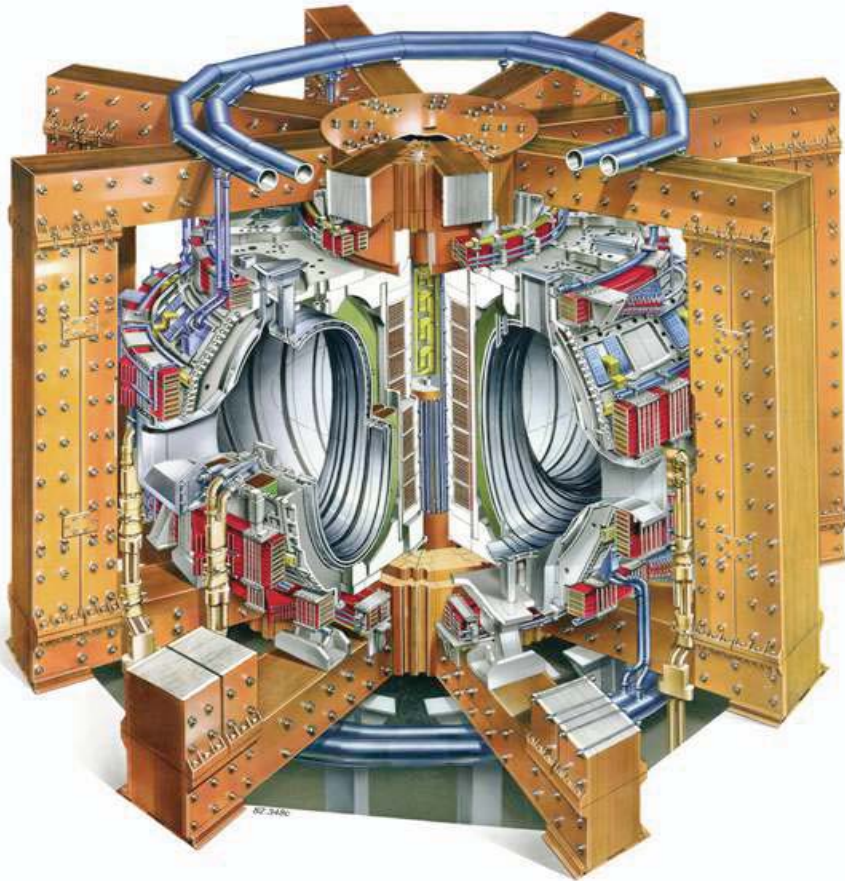
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01EIM/dj

E. Moses - WP3.2 P9687

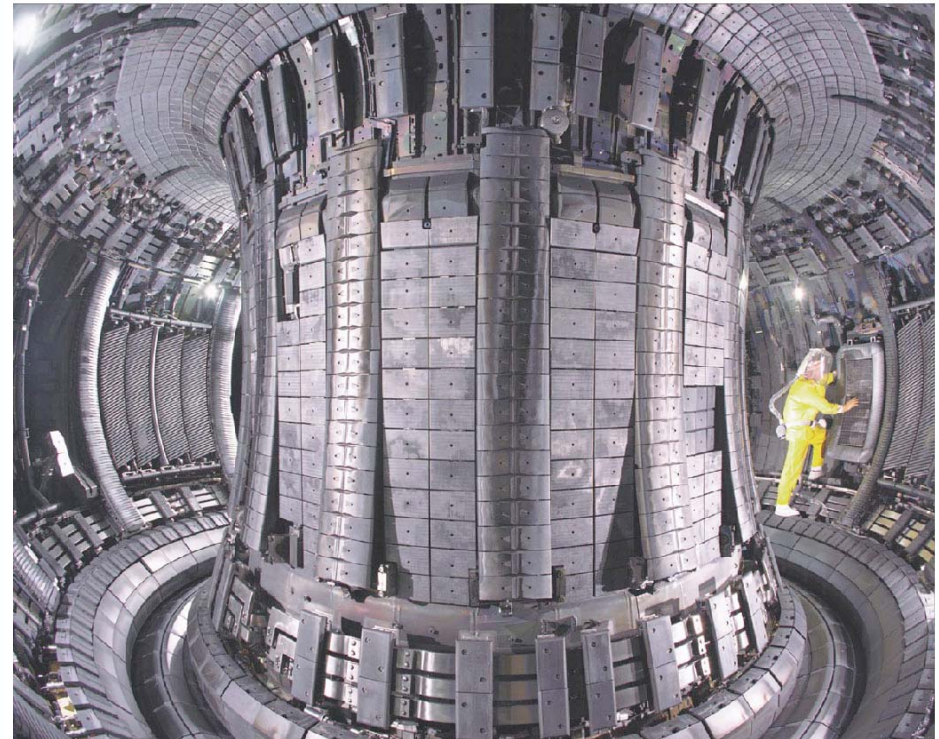
The Most Successful Magnetic Confinement Configuration is the Tokamak



The Largest Tokamak Is the Joint European Torus in England



cutaway drawing



view of inside during
maintenance

Working Tokamaks Are Usually Hidden by Support Systems & Diagnostics





The 20million Amp current provided by the Z accelerator
enables this research



Z accelerator

40 m



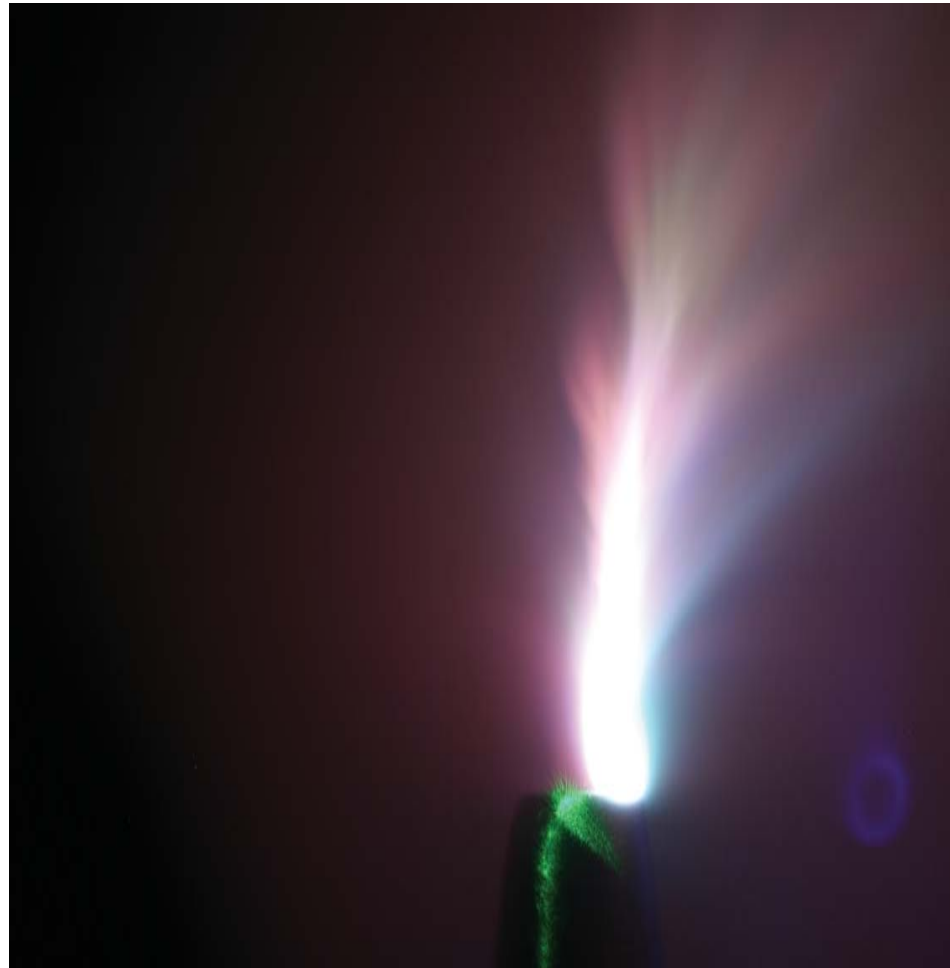
LLNL Electron Beam Ion Trap (EBIT):
Atomic Physics Experiments on highly ionized heavy atoms



LAPD/UCLA



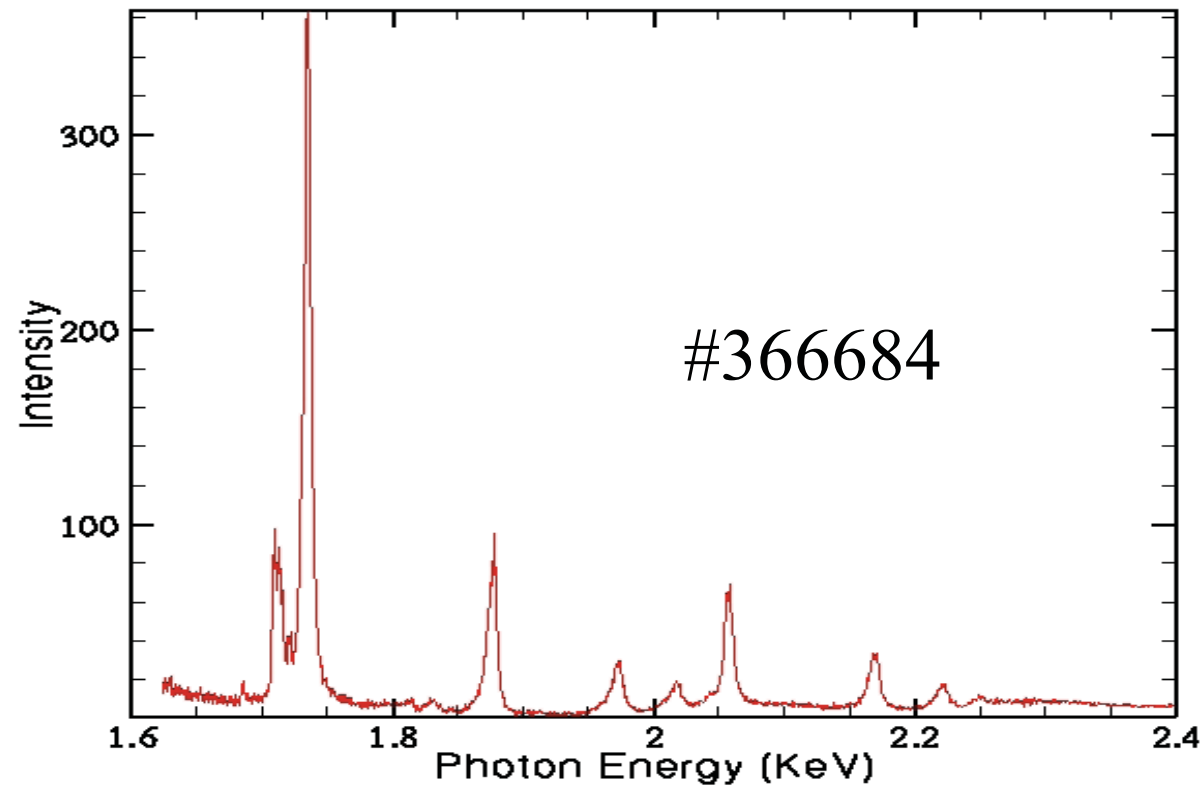
He Gas jet experiments, *TRIDENT LASER*



Why Collisional-Radiative (CR) Modeling?

- Plasmas emit and absorb electromagnetic radiation.
- **CR** models provide atom/ ion state populations that are required to simulate radiation properties of plasmas.
- **CR** calculations provide diagnostic information about plasma conditions, e.g. , density and temperature by comparison with experiment.
- **CR** include electron collisions which are generally responsible for populating excited states.

He-like and H-like Al lines from a laser produced plasma



Notation for Electron Configurations

set of orbitals $(nl)^w$

n = principle quantum number

$n = 1, 2, 3, \dots$

l = angular momentum quantum number

$l = 0, 1, 2, 3, \dots, n$

designated by spectroscopic notation $spdf$

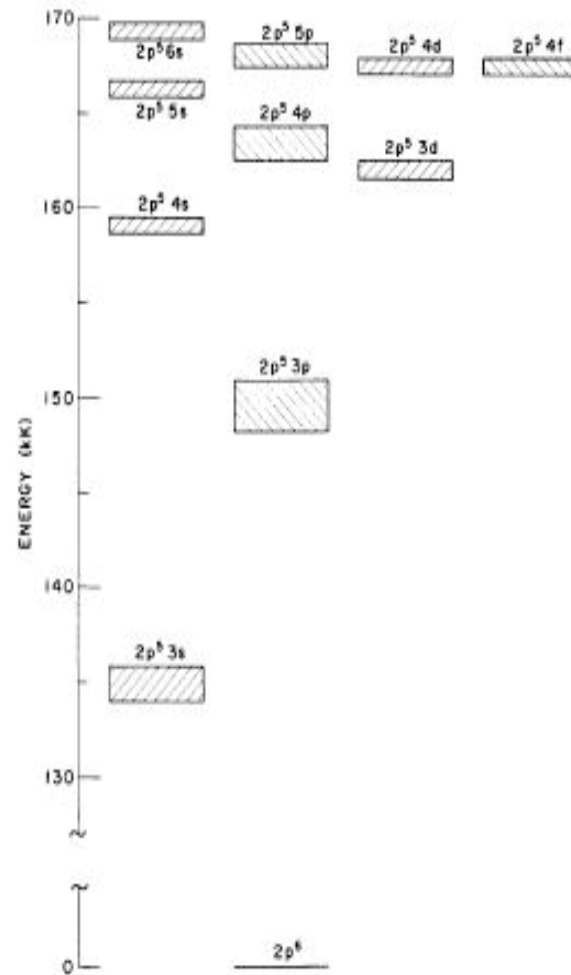
w = occupation number (number of electrons in orbital)

maximum value of $4l + 2$

Example, ground configuration of nitrogen :

$1s^2 2s^2 2p^3$

Block Diagram NeI



Energy level structure of pd configuration

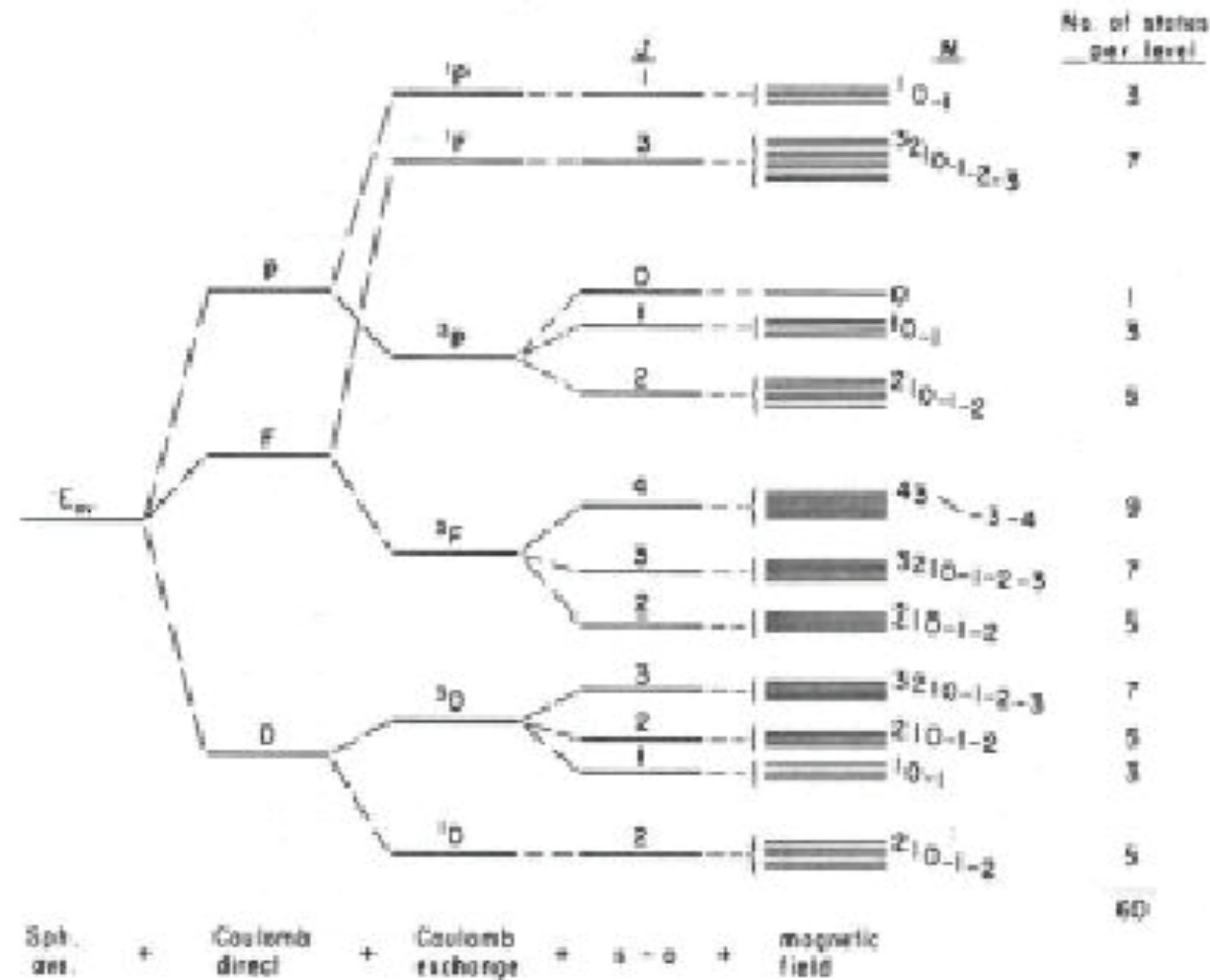


TABLE 4-2. PERMITTED LS TERMS FOR s^* , p^* , d^* , and f^* SUBSHELLS*. The subscripts indicate the number of different terms having the same values of LS. (Odd-parity superscripts omitted for brevity.)

				Total Number
s	2S			1
s^2	1S			1
p, p^3	2P			1
p^2, p^4	$^1(SD)$	3P		3
p^3	$^2(PD)$	4S		3
d, d^9	2D			1
d^2, d^8	$^1(SDG)$	$^3(PF)$		5
d^3, d^7	$^2(PD,FGH)$	$^4(PF)$		8
d^4, d^6	$^1(S_2D_2FG_2I)$	$^3(P_2DF_2GH)$	5D	16
d^5	$^2(SPD_3F_2G_2HI)$	$^4(PDFG)$	6S	16
f, f^{13}	2F			1
f^2, f^{12}	$^1(SDGI)$	$^3(PFH)$		7
f^3, f^{11}	$^2(PD_2F_2G_2H_2IKL)$	$^4(SDFGI)$		17
f^4, f^{10}	$^1(S_2D_4FG_4H_4I_4KL_2N)$	$^3(P_3D_2F_4G_3H_4I_2K_2LM)$	$^5(SDFGI)$	47
f^5, f^9	$^2(P_4D_3F_7G_6H_7I_2K_3L_3M_2NO)$	$^4(SP_2D_3F_4G_4H_3I_3K_2LM)$	$^6(PFH)$	73
f^6, f^8	$^1(S_4PD_6F_4G_6H_4I_4K_3L_4M_2N_2Q)$	$^3(P_6D_3F_6G_6H_6I_6K_6L_3M_3NO)$	$^5(SPD_3F_2G_2H_2I_2KL)$	7F 119
f^7	$^2(S_2P_3D_7F_{10}G_{10}H_9I_9K_7L_3M_4N_2OQ)$	$^4(S_2P_2D_6F_3G_7H_5I_3K_3L_3MN)$	$^6(PDFGHI)$	8S 119

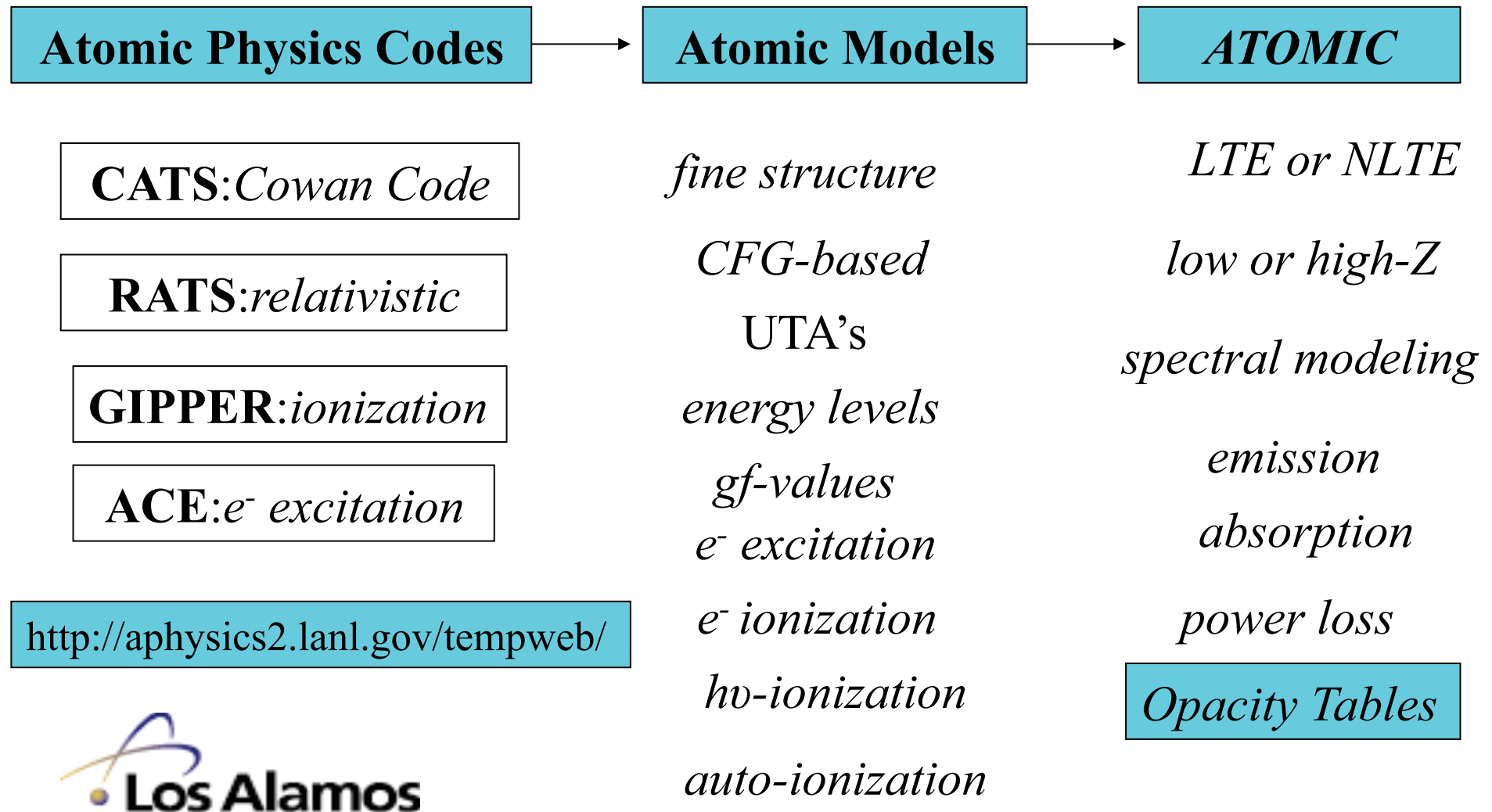
*H. N. Russell, Phys. Rev. 29, 782 (1927); R. C. Gibbs, D. T. Willber, and H. E. White, Phys. Rev. 29, 790 (1927).

Rate Equations and Definitions

$$\frac{dN_{il}}{dt} = \sum_{jm} A_{iljm} N_{jm}$$

- N_{il} = Population per unit volume(cm^{-3}) of level l of ion species i
- A_{iljm} = Rate matrix(sec^{-1})
- Rate coefficients for processes which alter level populations, integrated atomic cross sections.
- These include electron collisions, photon collisions and spontaneous processes
- Depends on N_e , the number of free electrons per unit volume (cm^{-3}), for processes involving electron collisions, cross sections for the various processes, electron energy distribution, and radiation field.

*Another Theoretical **Opacity** Modeling Integrated Code*



Electron Energy Distribution Function(EEDF)

- Describes the distribution of free electrons as a function of electron energy. The fraction of electrons per unit energy interval. Unit is (energy)⁻¹, usually eV .
- Described by the Maxwellian function for a given electron temperature kT_e (see below) for thermal plasmas.
 $\int_0^\infty F(E)dE = 1$
- kT_e expressed in energy units, usually eV .
- $kT_e = 11605 \text{ }^\circ K$

$$F(E) = \frac{2\sqrt{E}e^{-E/kT_e}}{\sqrt{\pi}(kT_e)^{3/2}}, \quad \int_0^\infty F(E)dE = 1$$

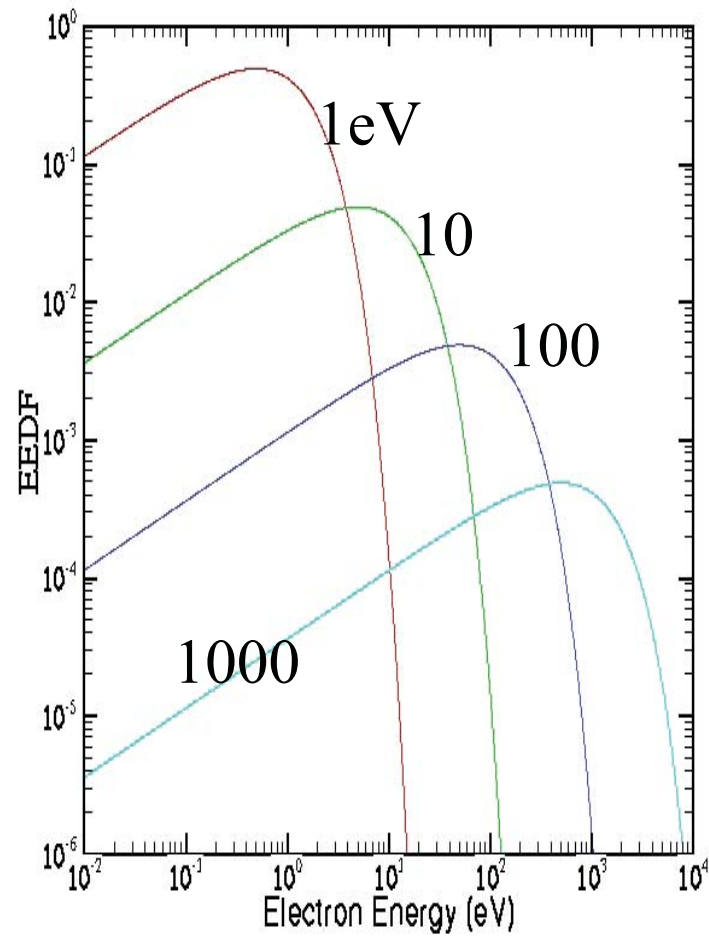
Radiation field

- The number of photons per unit energy interval per unit volume between $h\nu$ and $h\nu + d h\nu$, units of (volume)⁻¹(energy)⁻¹, usually cm⁻³eV⁻¹.
- kT_r in eV.
- For thermal plasmas the radiation field is given by the Planck function.

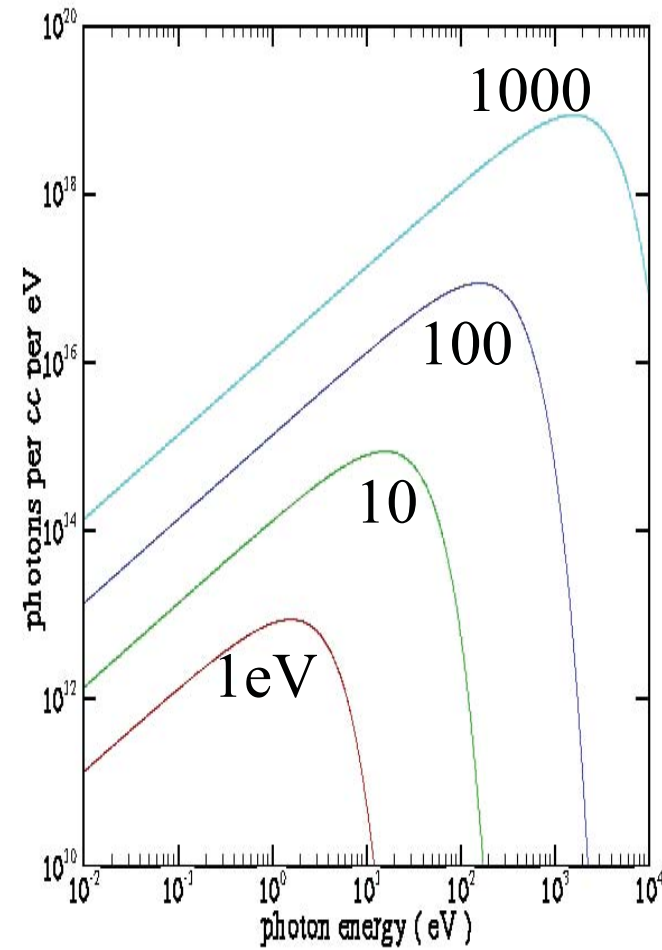
$$G(h\nu) = \frac{8\pi(h\nu)^2}{h^3 c^3 (e^{h\nu/kT_r} - 1)}$$

Electron and photon distributions at various temperatures

Maxwellian



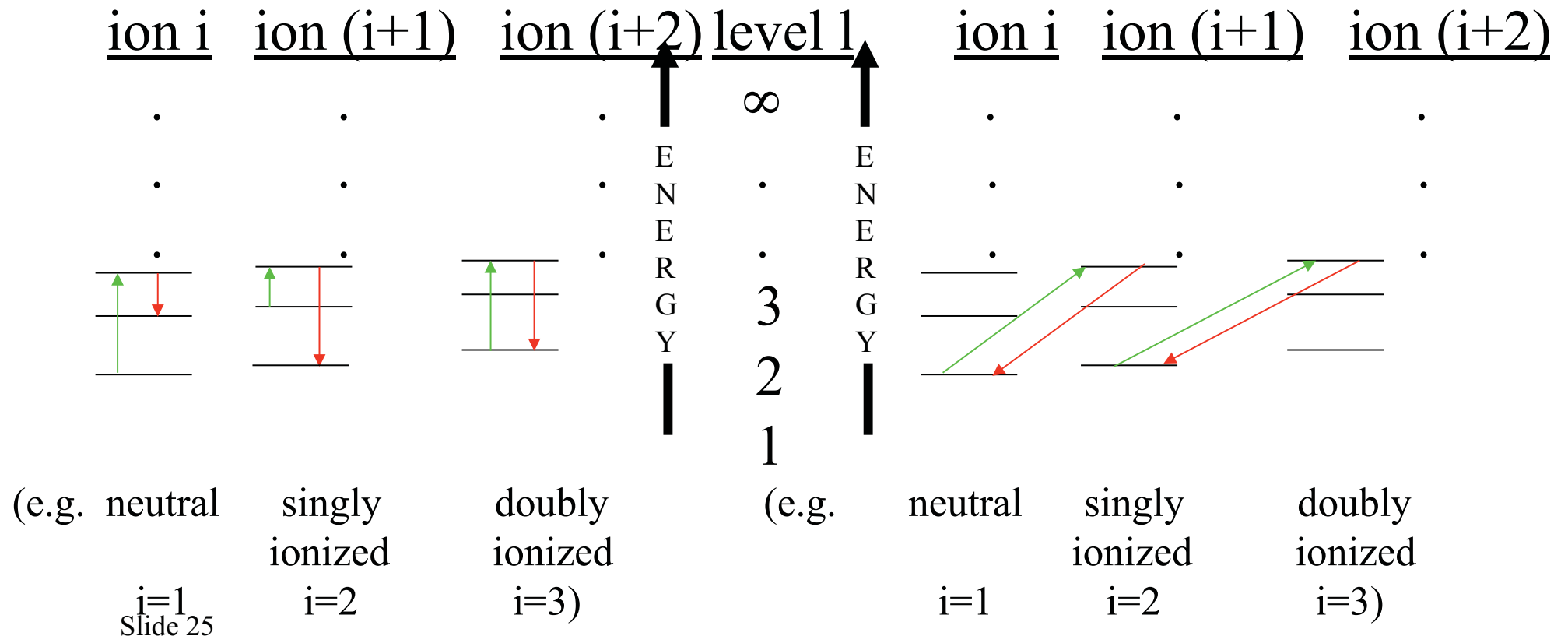
Planckian



Atomic processes in plasmas induced by electrons and photons

excitation/de-excitation

ionization/recombination



Processes

- Electron impact excitation/de-excitation:



- Radiative excitation/spontaneous decay:



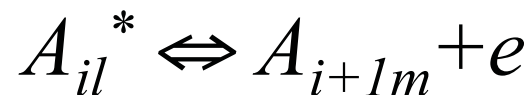
- Electron impact ionization / 3 body recombination:



- Photo-ionization / radiative recombination:



- Auto-ionization / di-electronic capture:



Example: integrated rate coefficient

$$e^{-}(E) + A_{il} \leftrightarrow e^{-} + A_{im}(E')$$

$$E = E' + (E_{im} - E_{il}) = E' + E_0$$

number of excitations per volume per second between E and $E + dE$

$$= N_{il} N_e F(E) v(E) \sigma_{i \rightarrow m}(E) dE$$

the total number of excitation per volume per second is then

$$= N_{il} N_e s$$

where,

$$s = \int_{E_0}^{\infty} F(E) v(E) \sigma(E) dE = \text{rate coefficient}$$

LTE - Saha Equations

$$\frac{N_{i+1}N_e}{N_i} = \frac{Z_e Z_{i+1} e^{-\phi_i / kT}}{Z_i}$$

$$N_e = \sum_{il} q_i N_i$$

$$Z_i = \sum_i g_{il} e^{-E_{il} / kT}$$

$$Z_e = 2 \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2}$$

$$\frac{N_{il}}{N_i} = \frac{g_{il} e^{-E_{il} / kT}}{Z_i}$$

ϕ_i = ionization energy of ion i

q_i = charge of ion i

E_{il} = energy of level l

g_{il} = statistical weight of level l

N_i = population of ion i

N_e = electron density

N_{il} = population of level l ion I

Z_i, Z_e = partition functions

Principle of Detailed Balance

- Under LTE conditions the forward rate of each process is balanced by the inverse rate
- Therefore, the cross section for the inverse process may be derived from the cross section for the forward process
- This is required for the solution to approach the LTE limit

Detailed balance for excitation / de-excitation

$$e^{-}(E) + A_{il} \leftrightarrow e^{-} + A_{im}(E')$$

$$E = E' + (E_{im} - E_{il})$$

excitation rate = de - excitation rate at LTE

number of excitations per volume per second between

E and $E + dE$

$$N_{il} N_e F(E) v(E) \sigma_{i \rightarrow m}(E) dE = N_{im} N_e F(E') v(E') \sigma_{m \rightarrow i}'(E') dE'$$

Therefore σ' can be calculated from σ using LTE properties

$$\text{relationships for } N_{il}, N_e, F(E), v = \sqrt{2E / m_e}$$

Derive an expression for the de - excitation cross section σ' in

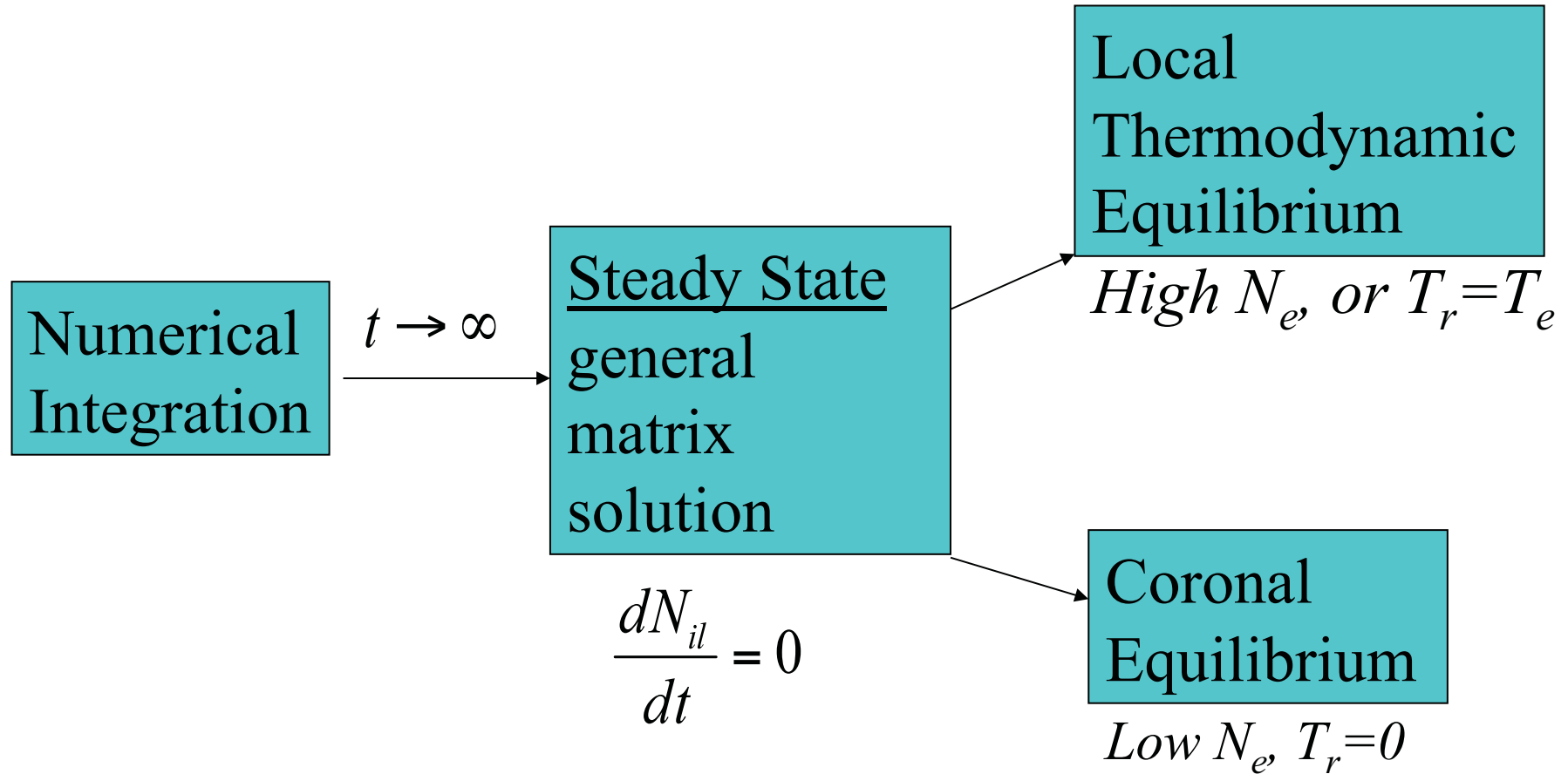
terms of the excitation cross section σ .

Coronal Equilibrium

$$\frac{dN_i}{dt} = I_{i-1}N_{i-1} - (R_i + I_i)N_i + R_{i+1}N_{i+1} = 0$$

- I_i =net ionization rate coefficient from from ground state of ion i including inner shell excitation/autoionization
- R_i =net recombination rate from ground state of ion i including recombination into excited states of ion i-1
- Low density limit
- Solar corona model
- Only ground states significantly populated
- Radiative decay rates $\gg$ e⁻ impact excitation rates

Solution of Rate Equations



e^- Impact Excitation $\leftrightarrow$ De-Excitation

$$e^-(E) + A_{il} \leftrightarrow e^-(E') + A_{im}$$

$$E' = E - E_0$$

$$E_0 = E_{im} - E_{il}$$

$$\frac{dN_{il}}{dt} = -sN_eN_{il} + tN_eN_{im}$$

$$s = \int_{E_0}^{\infty} F(E, kT_e) v(E) \sigma(E) dE$$

$$t = \frac{g_{il}}{g_{im}} e^{E_0/kT_e} s,$$

$$s \text{ and } t \text{ in } cm^3 / sec$$

E = impact energy

E' = final energy

E_0 = threshold energy

N_{il} = state population

N_e = electron density

T_e = electron temperature

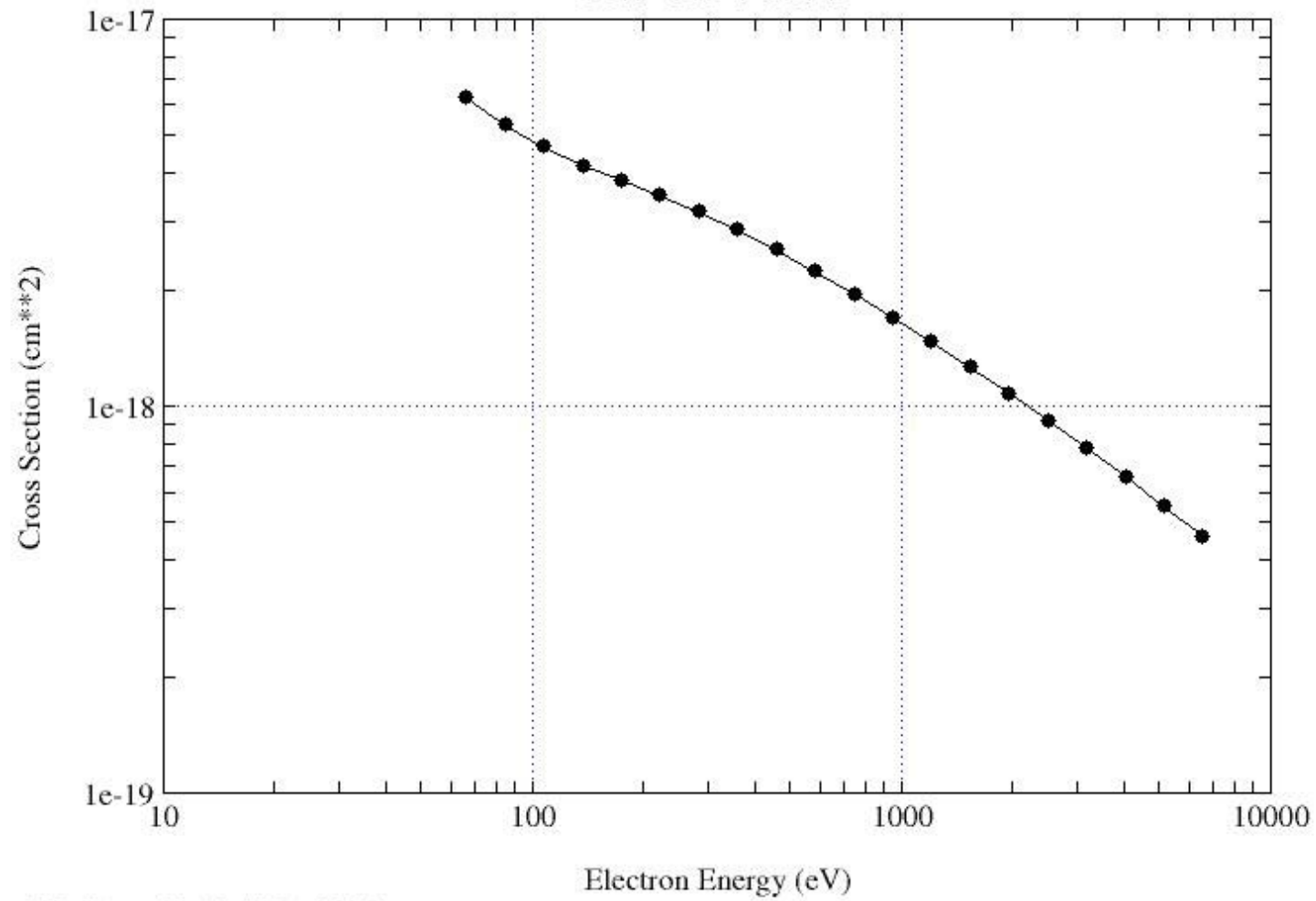
s = excitation rate coefficient

t = de - excitation rate coefficient

Ex : $e^- + 1s^2 \leftrightarrow e^- + 1s2p$

DWA Excitation

Fe+5 3d3--> 3d2 4f



Wed Aug 23 13:43:31 2006

$$h\nu + A_{il} \leftrightarrow A_{im}$$

$$h\nu_0 = E_{im} - E_{il}$$

Photo-Excitation $\leftrightarrow$ Decay

$$\frac{dN_{il}}{dt} = -uN_{il} + yN_{im} + zN_{im}$$

$$u = \frac{he^2\pi}{m_e} \frac{(gf)_{ilm}}{g_{il}} G(h\nu_0, kT_r)$$

$$y = \frac{8\pi^2 e^2}{h^2 c^3 m_e} \frac{(gf)_{ilm}}{g_{im}} (h\nu_0)^2$$

$$y = 4.338 \times 10^7 \frac{(gf)_{ilm}}{g_{im}} (h\nu_0)^2$$

$$z = \frac{(gf)_{ilm}}{g_{im}} u$$

$$u, y, z \text{ in } \text{sec}^{-1}, h\nu_0 \text{ in } eV$$

$h\nu$ = photon energy

$h\nu_0$ = transition energy

N_{il} = state population

kT_r = radiation temperature

u = excitation rate coefficient

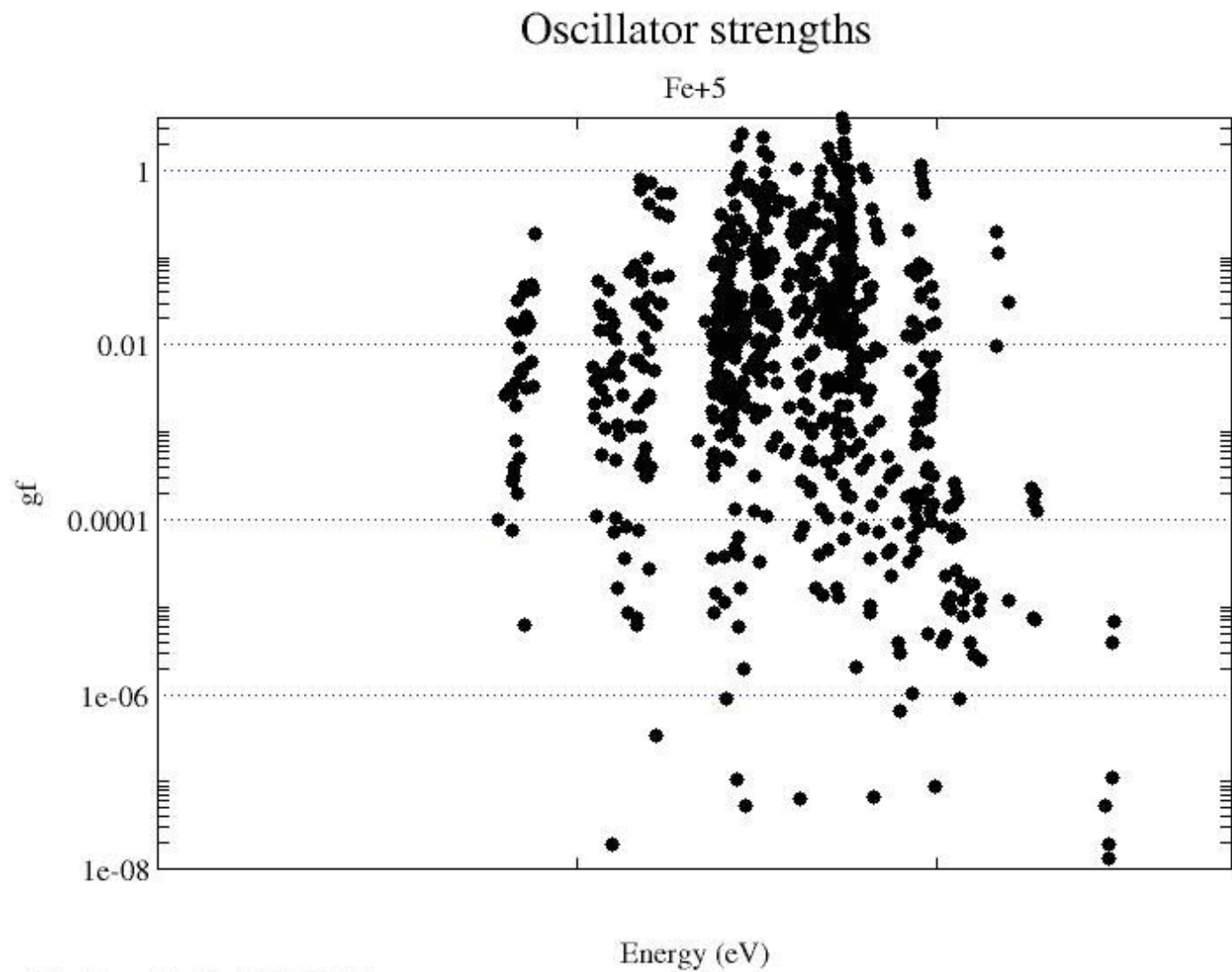
y = spontaneous decay rate coefficient
(Einstein A coefficient)

z = stimulated decay rate coefficient

$u = z = 0$ with no radiation field

Ex : $h\nu + 1s^2 \leftrightarrow 1s^1 2p^1$

$\text{Fe}^{+5}: 3d^3-3d^24f$



Wed Aug 23 12:46:50 2006

Photo-Ionization $\leftrightarrow$ Radiative Recombination

$$h\nu + A_{il} \leftrightarrow e^{-}(E) + A_{jm}$$

$$h\nu = E_{jm} - E_{il} + E = h\nu_0 + E$$

$$\frac{dN_{il}}{dt} = -pN_{il} + rN_e N_{jm} + qN_e N_{jm}$$

$$p = \int_{h\nu_0}^{\infty} G(h\nu, kT_r) \sigma_{iljm}(h\nu) c d h\nu$$

$$r = \frac{g_{il}}{g_{jm}} \frac{1}{2^{1/2} \frac{m_e}{c}^2} \int_0^{\infty} F(E, kT_e) \frac{(h\nu_0 + E)^2}{E^{1/2}} \sigma_{iljm}(h\nu_0 + E) dE$$

$$r + q = 1.66 \times 10^{-22} \frac{g_{il}}{g_{jm}} \frac{e^{h\nu_0/kT}}{kT^{3/2}} p$$

$$p \text{ is } \text{sec}^{-1}, r, q, \text{ in } \text{cm}^3 / \text{sec}, kT, E, h\nu_0 \text{ in eV.}$$

$h\nu$ = photon energy

$h\nu_0$ = ionization energy

u = photoionization rate coefficient

r = radiative recombination rate coefficient

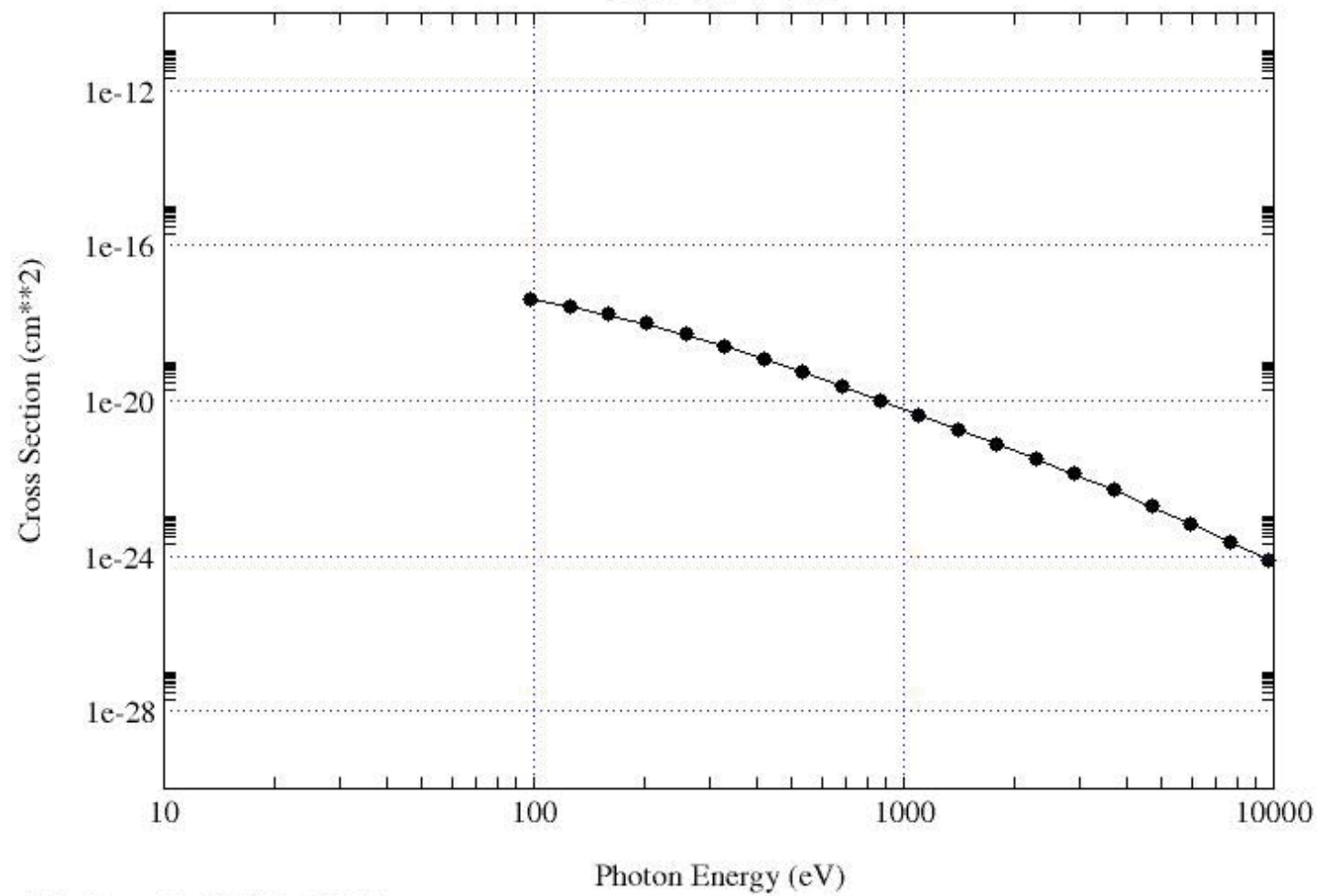
$r + q$ = radiative recombination rate coefficient
with stimulated emission, $kT_e = kT_r$

$q = 0$ with no radiation field

Ex : $h\nu + 1s^2 \leftrightarrow 1s^1 + e^{-}$

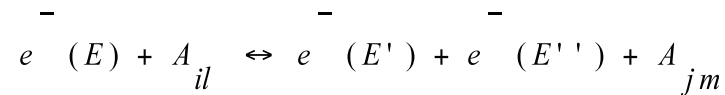
Photoionization Cross Sections

Fe+5 3d3--> 3d2



Wed Aug 23 13:52:24 2006

e^- Impact Ionization $\leftrightarrow$ Three-Body Recombination



$$E = E_{jm} - E_{il} + E' + E'' = E_0 + E' + E''$$

$$\frac{dN_{il}}{dt} = -cN_e N_{il} + bN_e^2 N_{jm}$$

$$c = \int_{E_0}^{\infty} F(E, kT_e) v(E) Q(E) dE$$

$$b = 1.66 \times 10^{-22} \frac{g_{il}}{g_{jm}} \frac{e^{E_0/kT_e}}{(kT_e)^{3/2}} c$$

$$c \text{ in cm}^3 / \text{sec}, b \text{ in cm}^6 / \text{sec}$$

$$kT_e \text{ and } E_0 \text{ in eV}$$

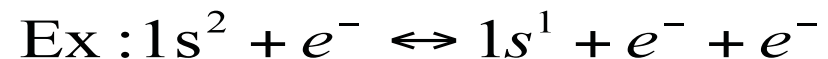
$E = e^-$ impact energy.

E_0 = ionization energy.

$c = e^-$ impact ionization rate coefficient.

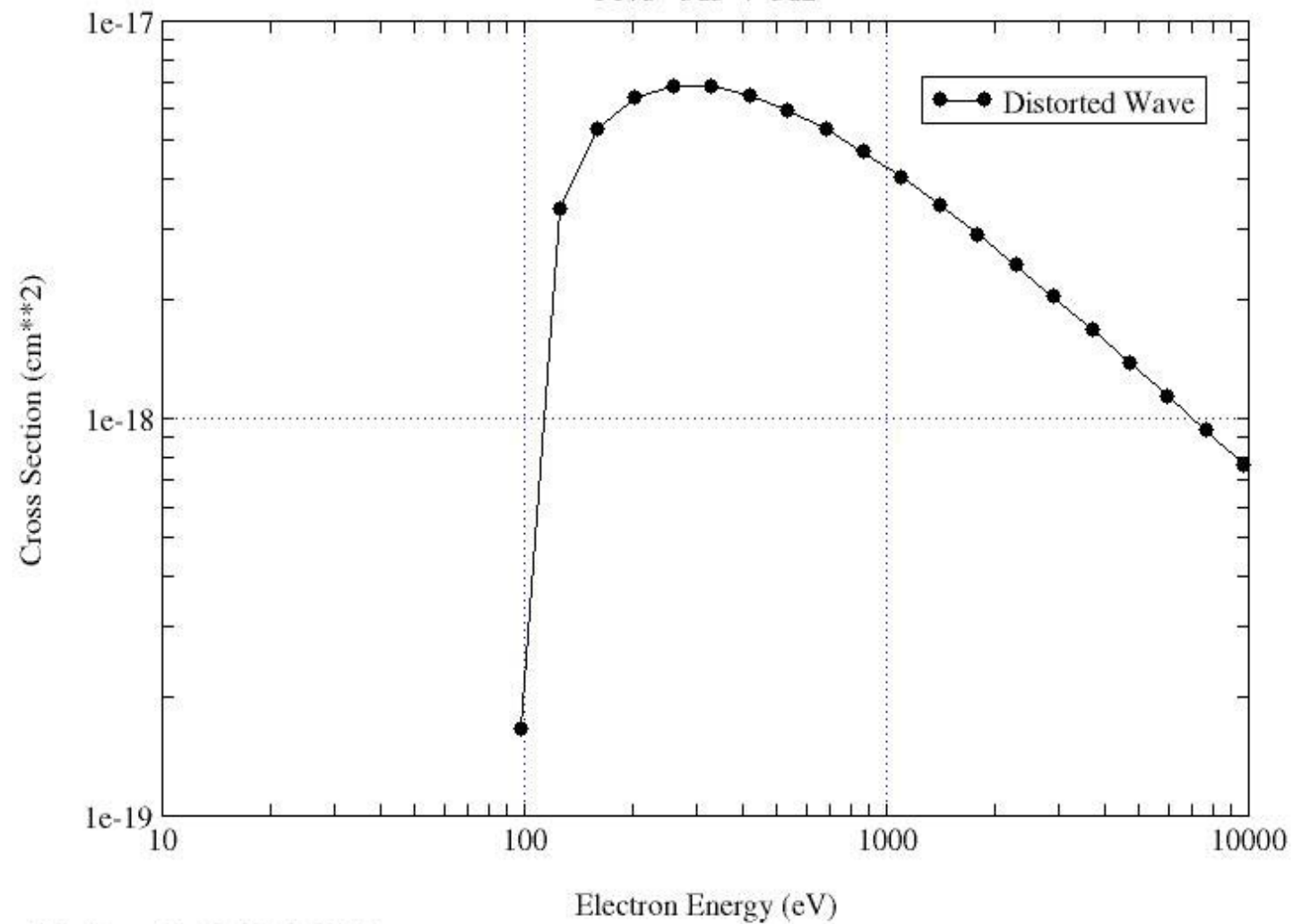
b = three - body recombination rate coefficient,
Maxwellian only.

$Q = e^-$ impact ionization cross section.



Electron impact ionization

Fe+5 3d3--> 3d2



Wed Aug 23 13:38:13 2006

Auto-Ionization $\leftrightarrow$ Di-Electronic Capture

$$A_{il} \leftrightarrow e^{-}(E_0) + A_{jm}$$

$$E_0 = E_{jm} - E_{il}$$

$$\frac{dN_{il}}{dt} = -aN_{il} + dN_e N_{jm}$$

$$a = A_{iljm}^a$$

$$d = 1.66 \times 10^{-22} \frac{g_{il}}{g_{jm}} \frac{e^{-E_0/kT_e}}{(kT_e)^{3/2}} a$$

$$a \text{ in sec}^{-1} \text{ and } d \text{ in cm}^3 / \text{sec}$$

$$E_0 \text{ and } kT_e \text{ in eV}$$

E_0 = ionization energy.

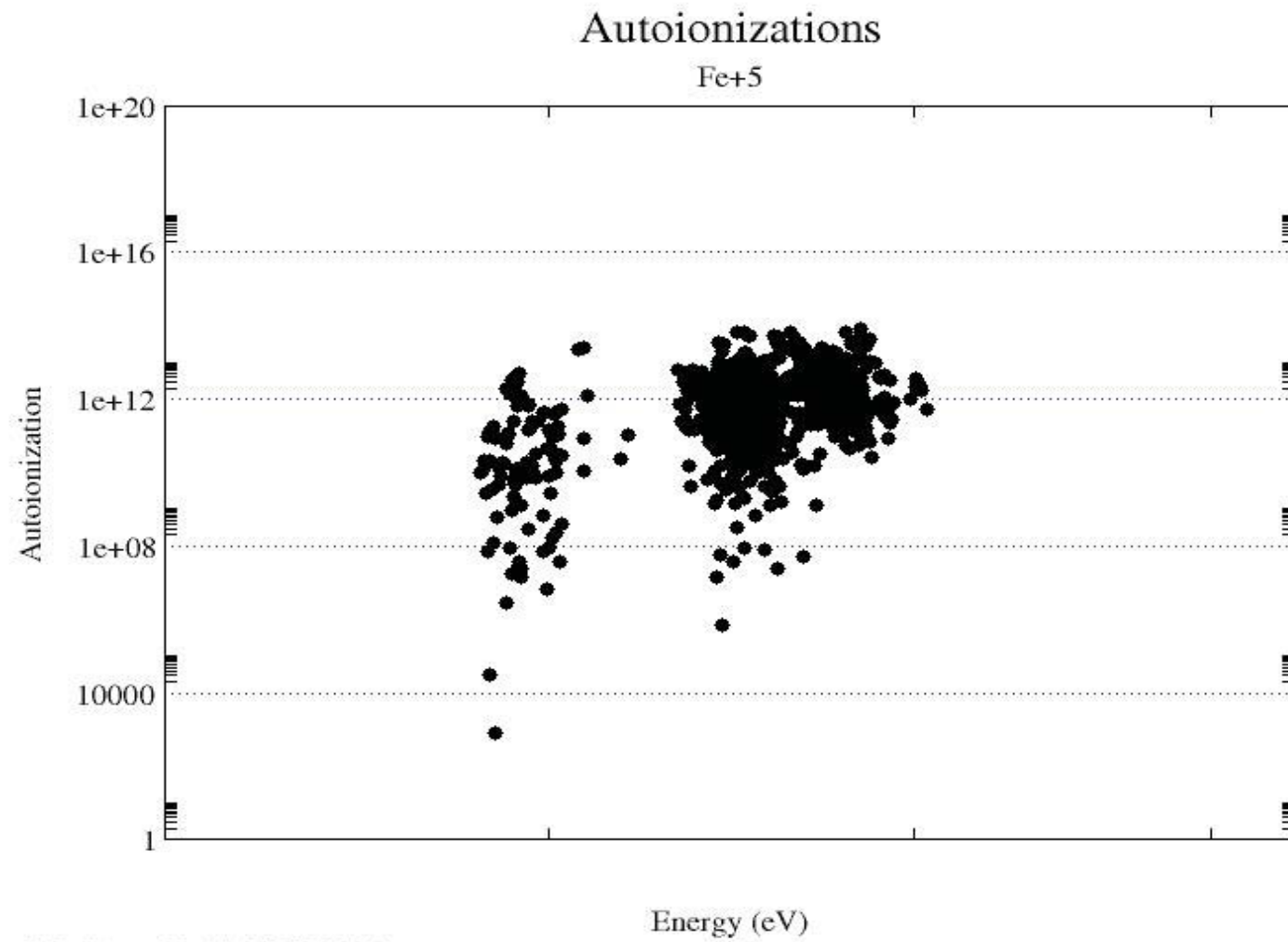
a = auto - ionization rate.

d = di - electronic capture
rate coefficient,

Maxwellian only.

Ex : He - like $2s^2 \leftrightarrow 2s^1 + e^{-}$

$$\text{Fe}^{+5}:3d^14f^2-3d^2$$



Wed Aug 23 14:25:55 2006

Emission spectra

- **Line or bound-bound** contribution:
- The number of emitting photons per second per volume per unit energy interval per transition:

$$N_{im} \frac{8\pi^2 e^2}{h^2 c^3 m_e} \frac{(gf)_{ilm}}{g_{im}} (h\nu_0)^2 \Phi_{ilm}(h\nu)$$

$$h\nu_0 = E_{im} - E_{il}$$

$$\Phi_{ilm}(h\nu) = \text{line shape}$$

$$\int_0^\infty \Phi_{ilm}(h\nu) d h\nu = 1$$

$$\text{e.g. } \Phi = \frac{w/\pi}{(h\nu - h\nu_0)^2 + w^2} \text{ or}$$

$$\Phi = \sqrt{\ln 2 / (\pi w^2)} e^{-\ln 2 (h\nu - h\nu_0)^2 / w^2}$$

$$w = HWHM$$

Emission spectra, lines, cont.

- The total amount radiated energy per second per volume per unit energy interval, sum over all transitions:

$$\sum N_{im} \frac{8\pi^2 e^2}{h^2 c^3 m_e} \frac{(gf)_{ilm}}{g_{im}} (h\nu_0)^3 \Phi_{ilm}(h\nu)$$

Emission spectra

- **Recombination or free-bound continuous** contribution
- The total amount radiated energy per second per volume per unit energy interval, sum over all transitions:

$$\sum N_{jm} N_e \frac{g_{il}}{g_{jm}} \frac{1}{2^{1/2} m_e^{3/2} c^2} F(E, kT_e) \frac{(h\nu)^3}{E^{1/2}} \sigma_{iljm}(h\nu)$$

$$h\nu = E_{jm} - E_{il} + E = h\nu_0 + E$$

Emission spectra

- free-free or Bremsstrahlung continuous contribution
- free electron decelerates in plasma and emits a photon
- simple hydrogenic formula
- The total amount radiated energy per second per volume per unit energy interval, sum over all transitions:

$$9.585552 \times 10^{-14} \frac{N_e e^{-h\nu/kT_e}}{(kT_e)^{1/2}} \sum N_i q_i^2$$

N_i = population of ion i

q_i = charge of ion i

**Power Loss Is Obtained by Integrating Emission Over
ALL Photon Energies**

$$P_{tot} = P_{bb} + P_{bf} + P_{ff}$$

$$P_{bb} = \sum_{im} N_{im} (y_{ilm} + z_{ilm}) (E_{im} - E_{il})$$

$$P_{bf} = \sum_{im} N_{jm} N_e (r_{iljm} + q_{iljm}) (E_{jm} - E_{il} + (3/2)kT_e)$$

$$P_{ff} = 9.55 \times 10^{-14} (kT_e)^{1/2} N_e \sum_i q_i^2 N_i$$

P in $eV/(sec\ cm^3)$, $1J=6.24 \times 10^{18} eV$, $1W=1J/sec$

Multi-Element Kinetics Algorithm*

- **Specify:** $N_a^{(1)}, N_a^{(2)}, \dots, N_a^{(N)}$ and δN_e
- **Initialize:** $Guess - {}^{(0)}N_e$
- **For** $i \geq 0$: Solve the set of rate equations to obtain the $\bar{Z}^{(s)}$ for each species,

$$\begin{aligned} {}^{(i)}N_e; kT &\rightarrow {}^{(i)}\bar{Z}^{(1)} \\ {}^{(i)}N_e; kT &\rightarrow {}^{(i)}\bar{Z}^{(2)} \\ &\vdots \\ {}^{(i)}N_e; kT &\rightarrow {}^{(i)}\bar{Z}^{(N)} \end{aligned} \quad (1)$$

$${}^{(i)}\bar{Z}^{(1)}N_a^{(1)} + {}^{(i)}\bar{Z}^{(2)}N_a^{(2)} + \dots + {}^{(i)}\bar{Z}^{(N)}N_a^{(N)} = {}^{(i+1)}N_e \quad (2)$$

if ${}^{(i+1)}N_e \leq {}^{(i)}N_e + \delta N_e$ **then**
 Multi-element calc. converged!
 EXIT For Loop
else
 Increment $i : i = i + 1$
 Return to beginning of For Loop
end if

- **End For Loop**

* *M.E. Sherrill et al, Phys. Rev. E 76 05640 (2007)*

Solar Corona Calculations

- Atomic structure and collisional data were computed for all ion stages of the 15 most abundant solar coronal elements:

PERIODIC TABLE
Atomic Properties of the Elements

Frequently used fundamental physical constants
For the most accurate values of these and other constants, visit physics.nist.gov/constants
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum c 299 792 458 m s⁻¹ (exact)
Planck constant h 6.626 069 3 × 10⁻³⁴ J s (exact)
elementary charge e 1.602 176 634 × 10⁻¹⁹ C (exact)
electron mass m_e 9.109 383 56 × 10⁻³¹ kg
proton mass m_p 1.672 621 9 × 10⁻²⁷ kg
fine-structure constant α 1/137.035 999 084 (exact)
Rydberg constant R_∞ 10 973 731.568 160 (exact)
Boltzmann constant k_B 1.380 658 × 10⁻²³ J K⁻¹

Legend:
Solids (blue)
Liquids (green)
Gases (yellow)
Artificially Prepared (pink)

Inset for Cerium (Ce):
Atomic Number: 58
Ground-state Level: $[\text{Xe}]4f^75s^2$
Symbol: Ce
Name: Cerium
Atomic Weight: 140.116
Ground-state Configuration: $[\text{Xe}]4f^75s^2$
Ionization Energy (eV): 5.5387

Footnote: Based upon ^{12}C . () indicates the mass number of the most stable isotope.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2003)

- Collisional data was computed using *PWB* and *FOMBT/DW* approaches, to test the sensitivity of the radiative losses to the quality of the excitation data.

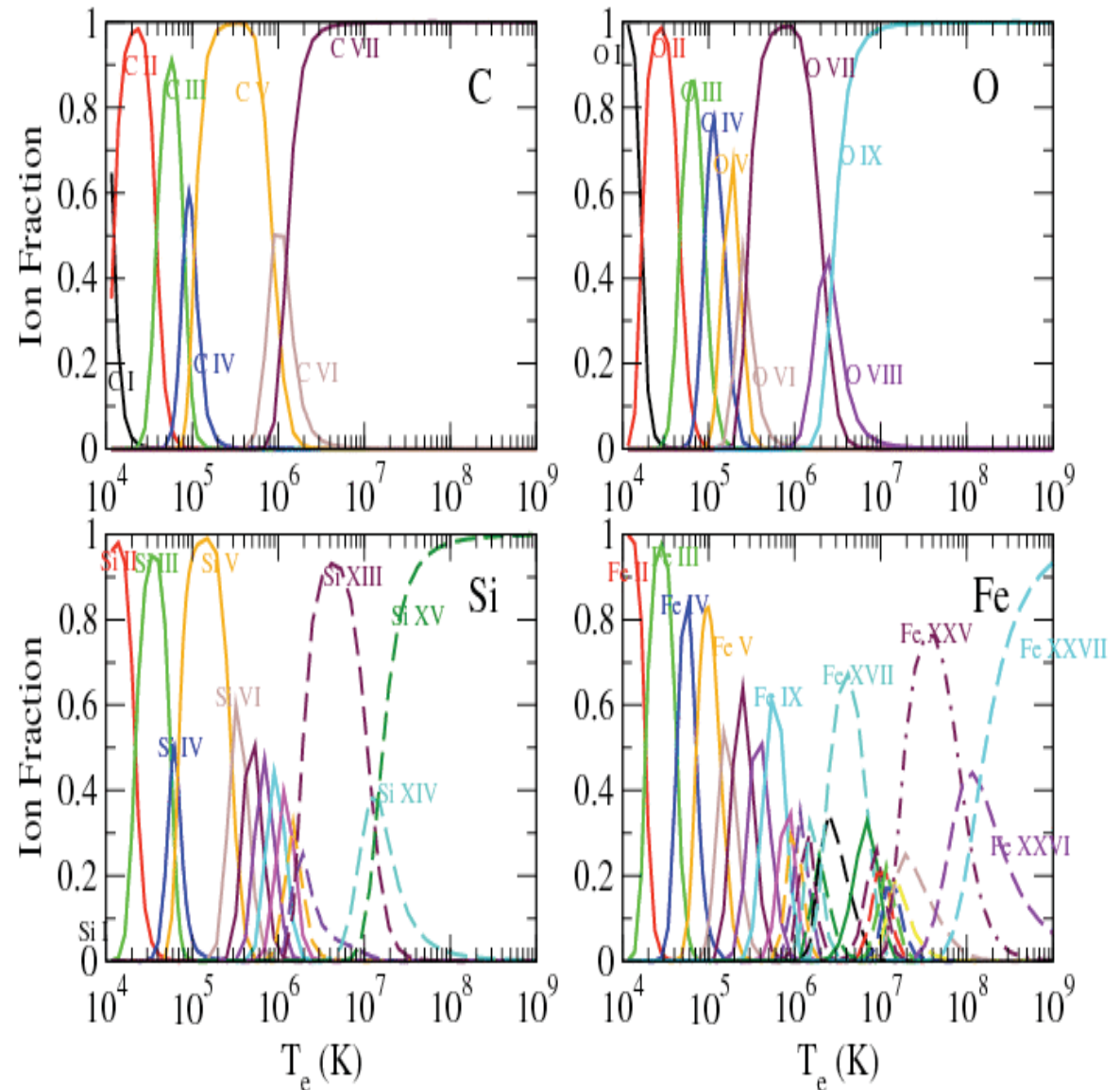
Ion balance is calculated consistently with power loss

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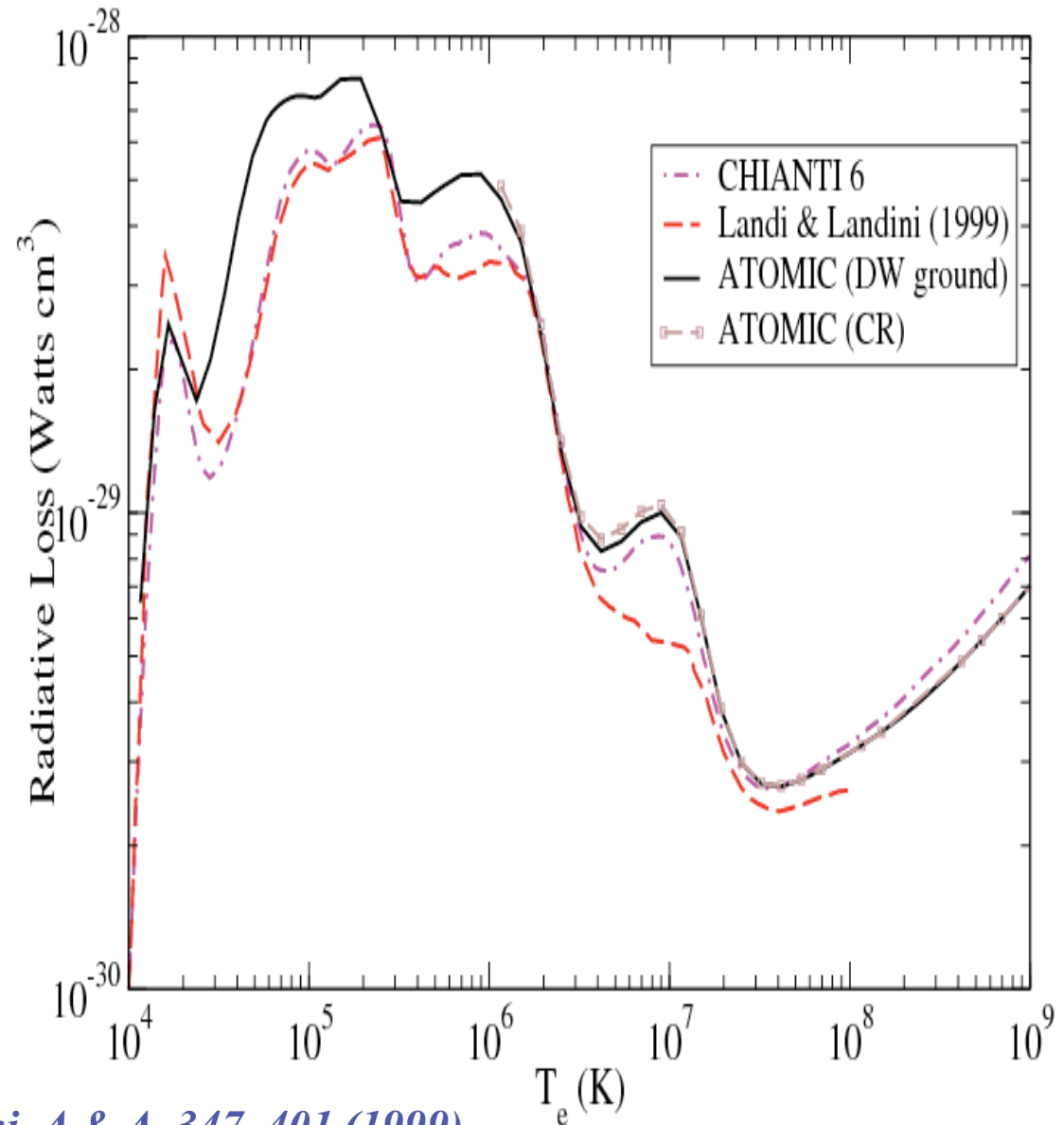


Results – comparisons to previous work

We compare the ATOMIC calculations to previous work of Landi & Landini (1999) and from the CHIANTI database

Substantial differences are found with Landi & Landini (1999) over most of the temperature range

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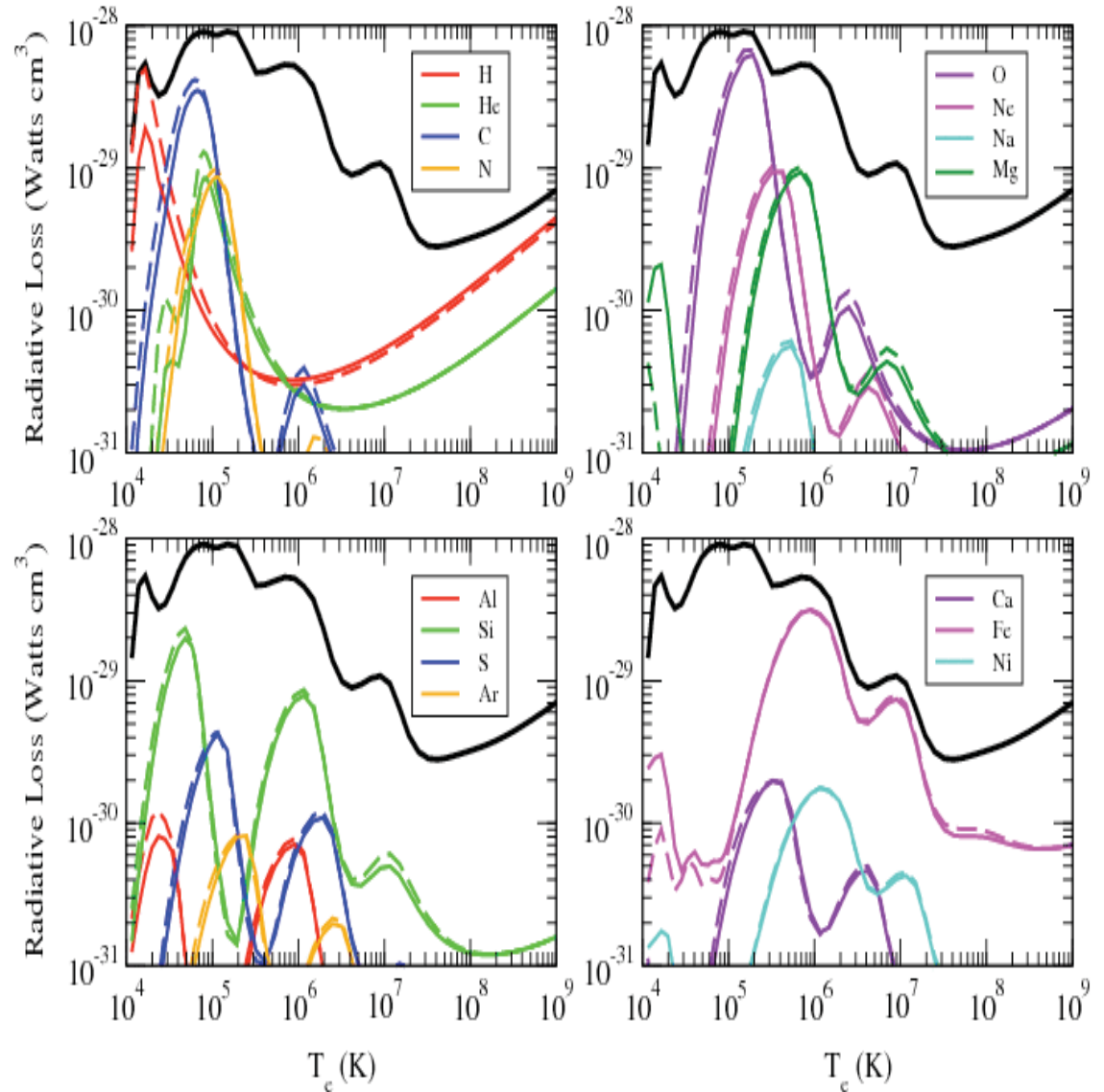
Landi & Landini, A & A, 347, 401 (1999).

Radiative losses from the individual elements

We present the contribution of the radiative losses from the individual 15 elements included in our calculation

At low temperatures **H**, **He**, **C**, and **O** are the dominant contributors

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Colgan et al, Astrophysical Journal, accepted (2008).

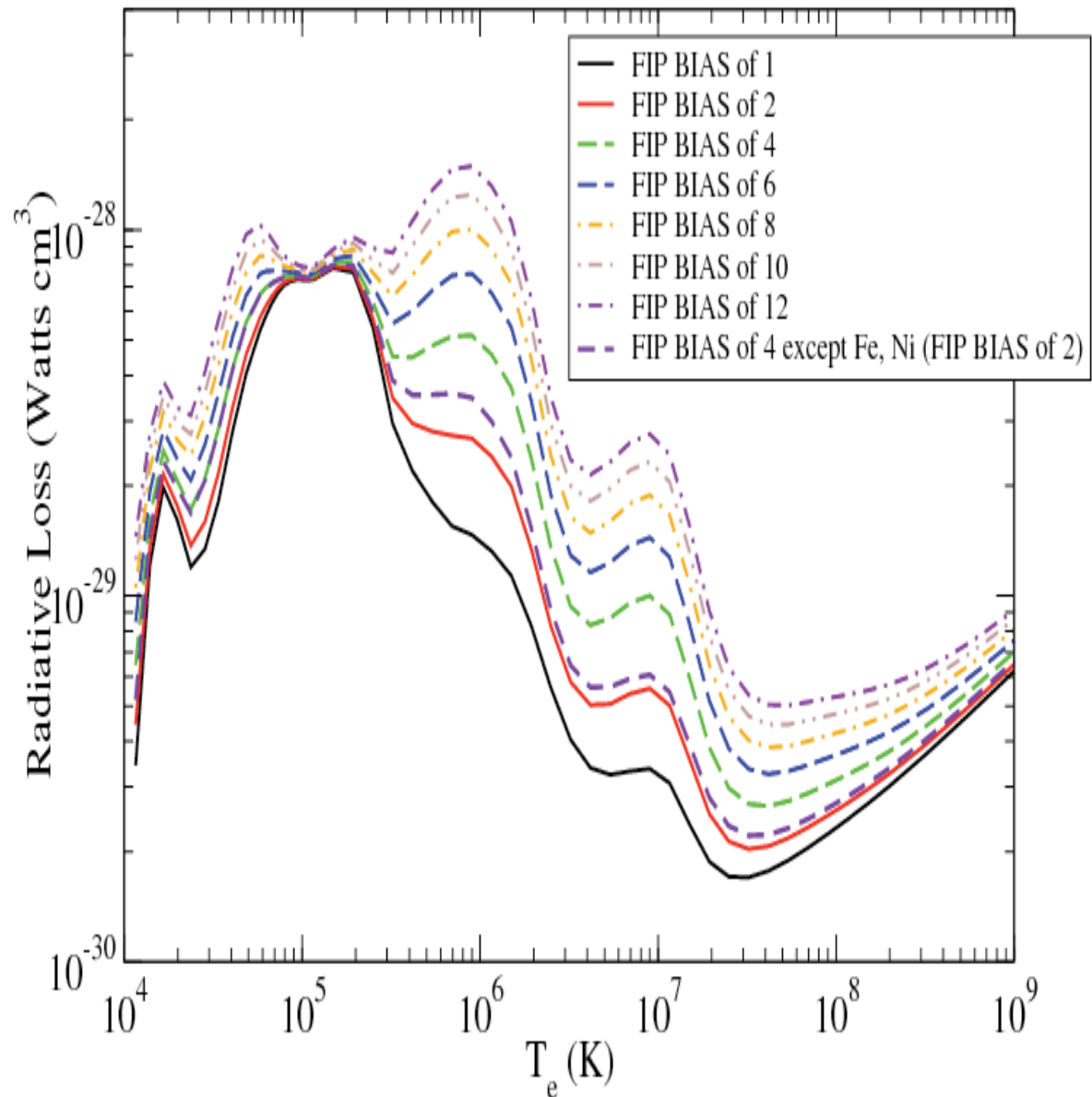
Sensitivity to elemental abundances

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These include the elements Na, Mg, Al, Si, Ca, Fe, and Ni

Thus we present the radiative losses for a variety of FIP bias values (relative to the photospheric abundances)



Multi-Element Kinetics Algorithm*

- **Specify:** $N_a^{(1)}, N_a^{(2)}, \dots, N_a^{(N)}$ and δN_e
- **Initialize:** $Guess - {}^{(0)}N_e$
- **For** $i \geq 0$: Solve the set of rate equations to obtain the $\bar{Z}^{(s)}$ for each species,

$$\begin{aligned} {}^{(i)}N_e; kT &\rightarrow {}^{(i)}\bar{Z}^{(1)} \\ {}^{(i)}N_e; kT &\rightarrow {}^{(i)}\bar{Z}^{(2)} \\ &\vdots \\ {}^{(i)}N_e; kT &\rightarrow {}^{(i)}\bar{Z}^{(N)} \end{aligned} \tag{1}$$

$${}^{(i)}\bar{Z}^{(1)}N_a^{(1)} + {}^{(i)}\bar{Z}^{(2)}N_a^{(2)} + \dots + {}^{(i)}\bar{Z}^{(N)}N_a^{(N)} = {}^{(i+1)}N_e \tag{2}$$

if ${}^{(i+1)}N_e \leq {}^{(i)}N_e + \delta N_e$ **then**
 Multi-element calc. converged!
 EXIT For Loop
else
 Increment $i : i = i + 1$
 Return to beginning of For Loop
end if

- **End For Loop**

* *M.E. Sherrill et al, Phys. Rev. E 76 05640 (2007)*

Solar Corona Calculations

- Atomic structure and collisional data were computed for all ion stages of the 15 most abundant solar coronal elements:

PERIODIC TABLE
Atomic Properties of the Elements

Frequently used fundamental physical constants
For the most accurate values of these and other constants, visit physics.nist.gov/constants
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum c 299 792 458 m s⁻¹ (exact)
Planck constant h 6.626 070 15 × 10⁻³⁴ J s (exact) ($h = h/2\pi$)
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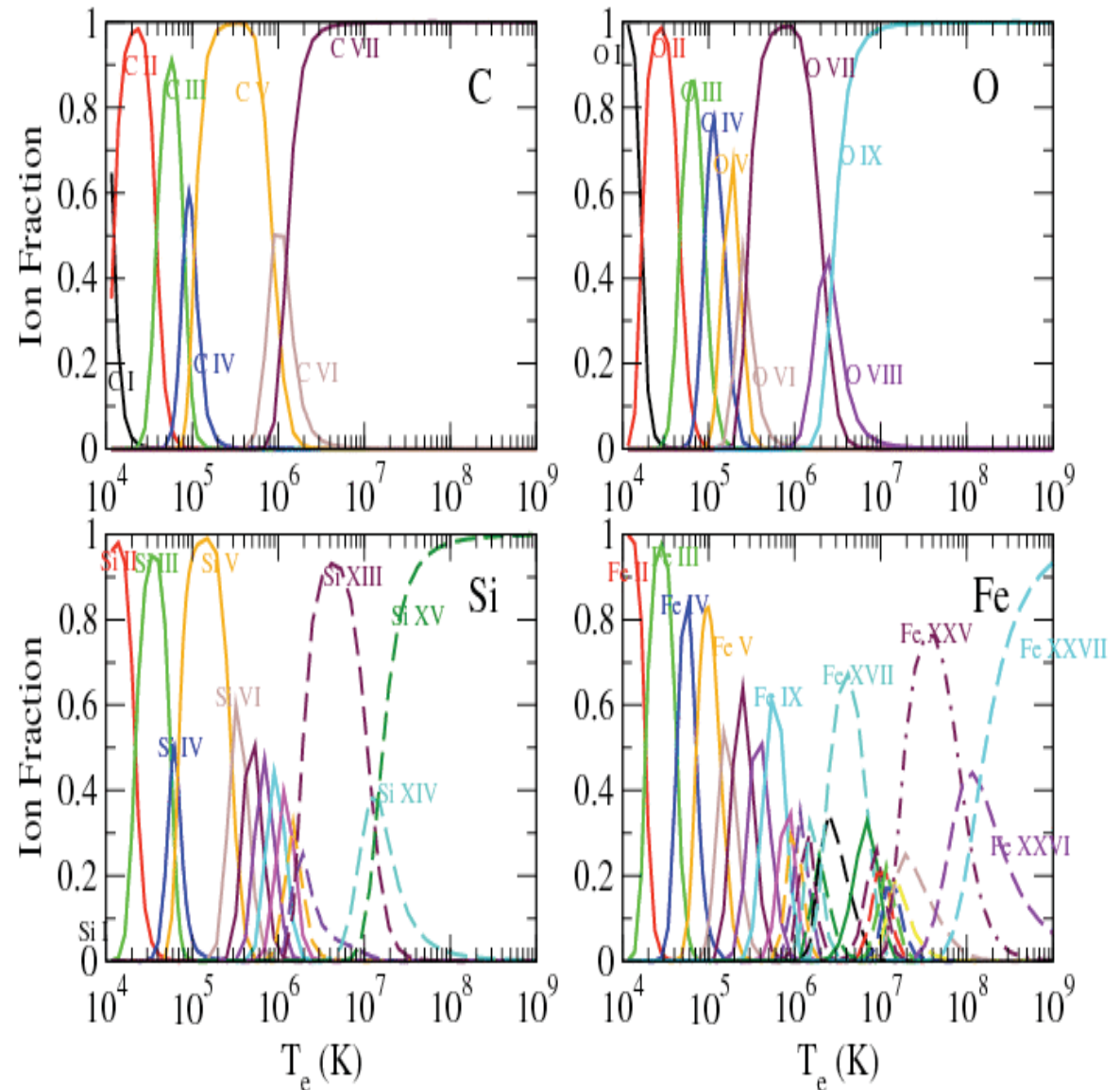
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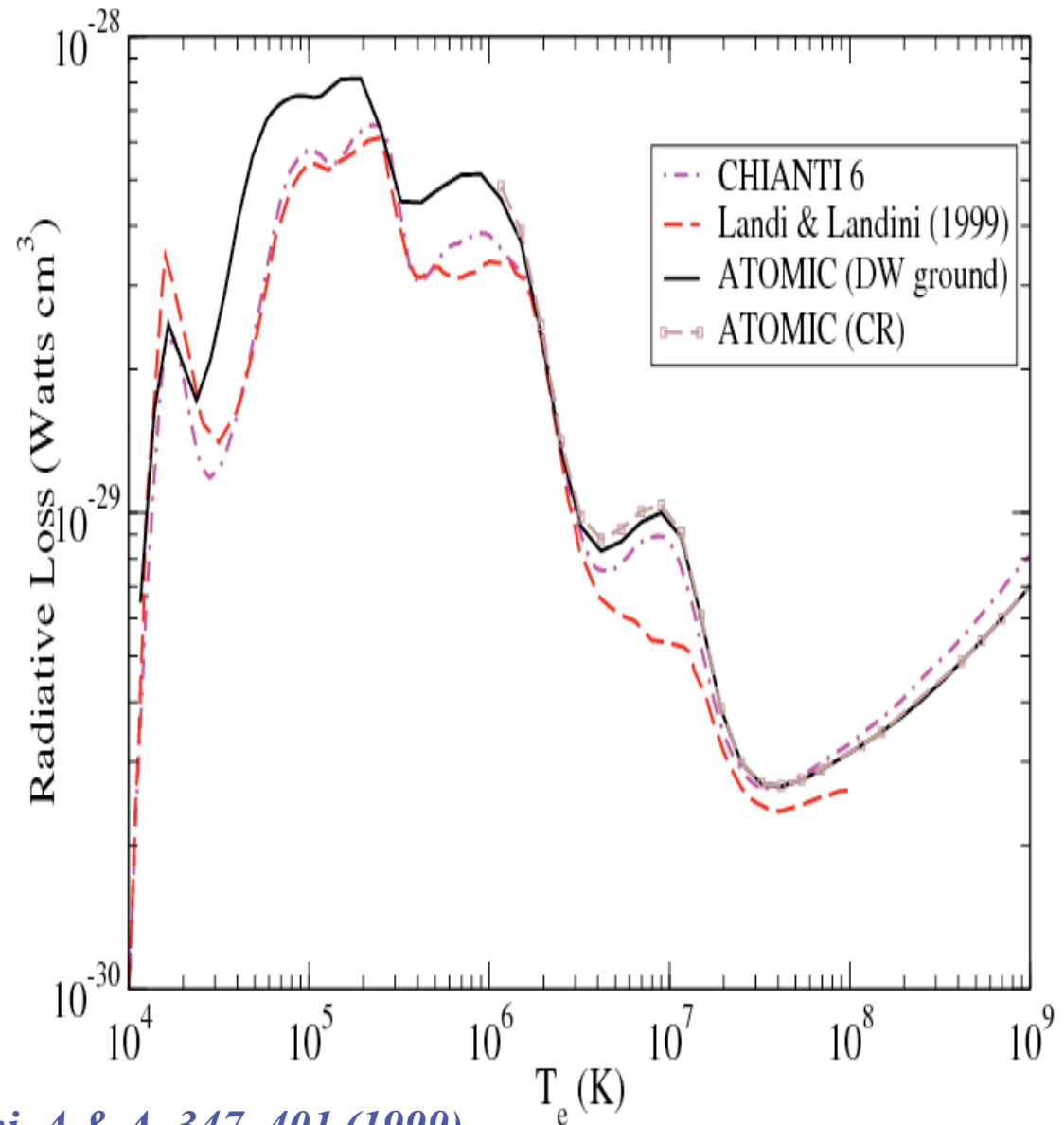


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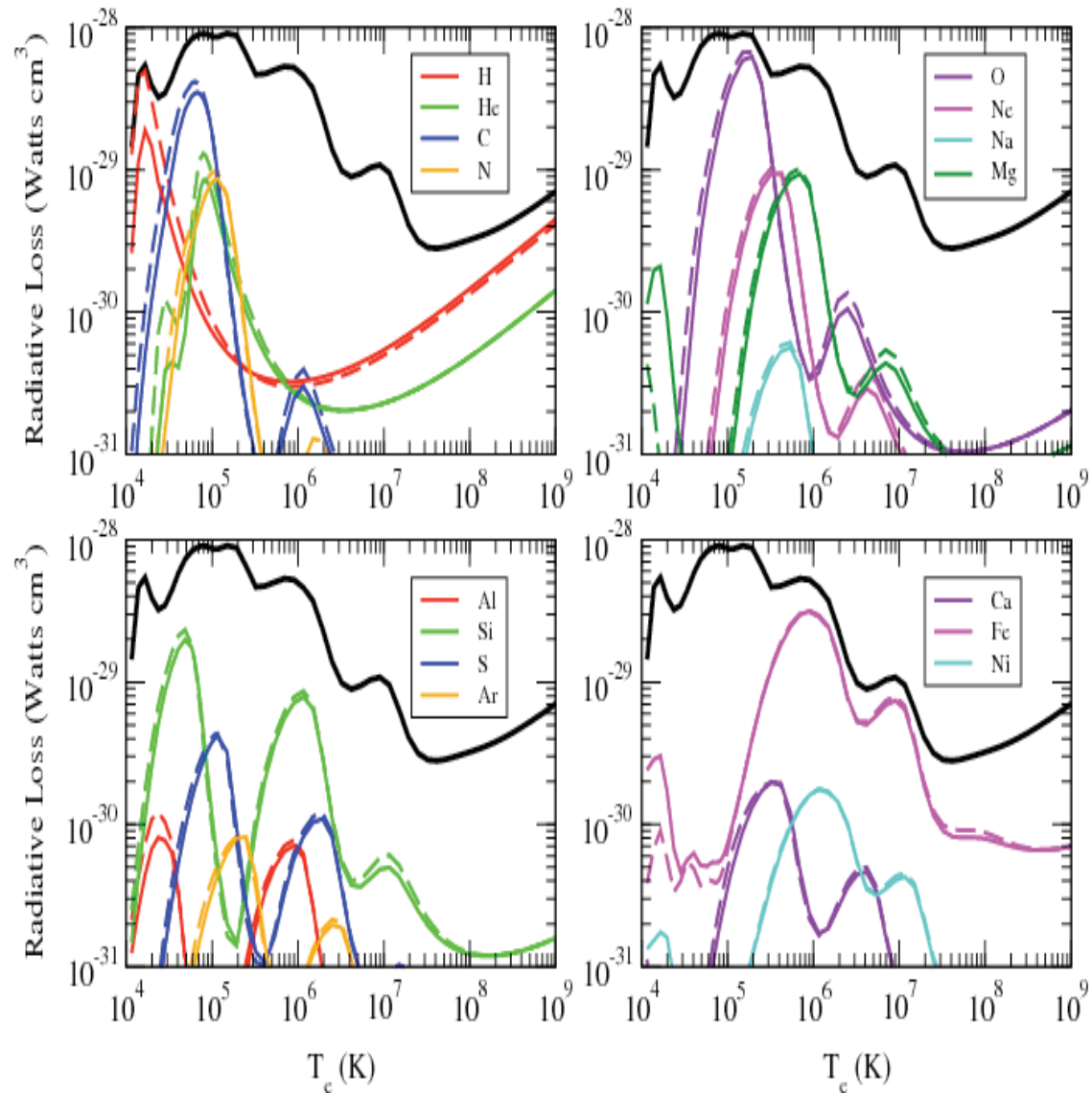
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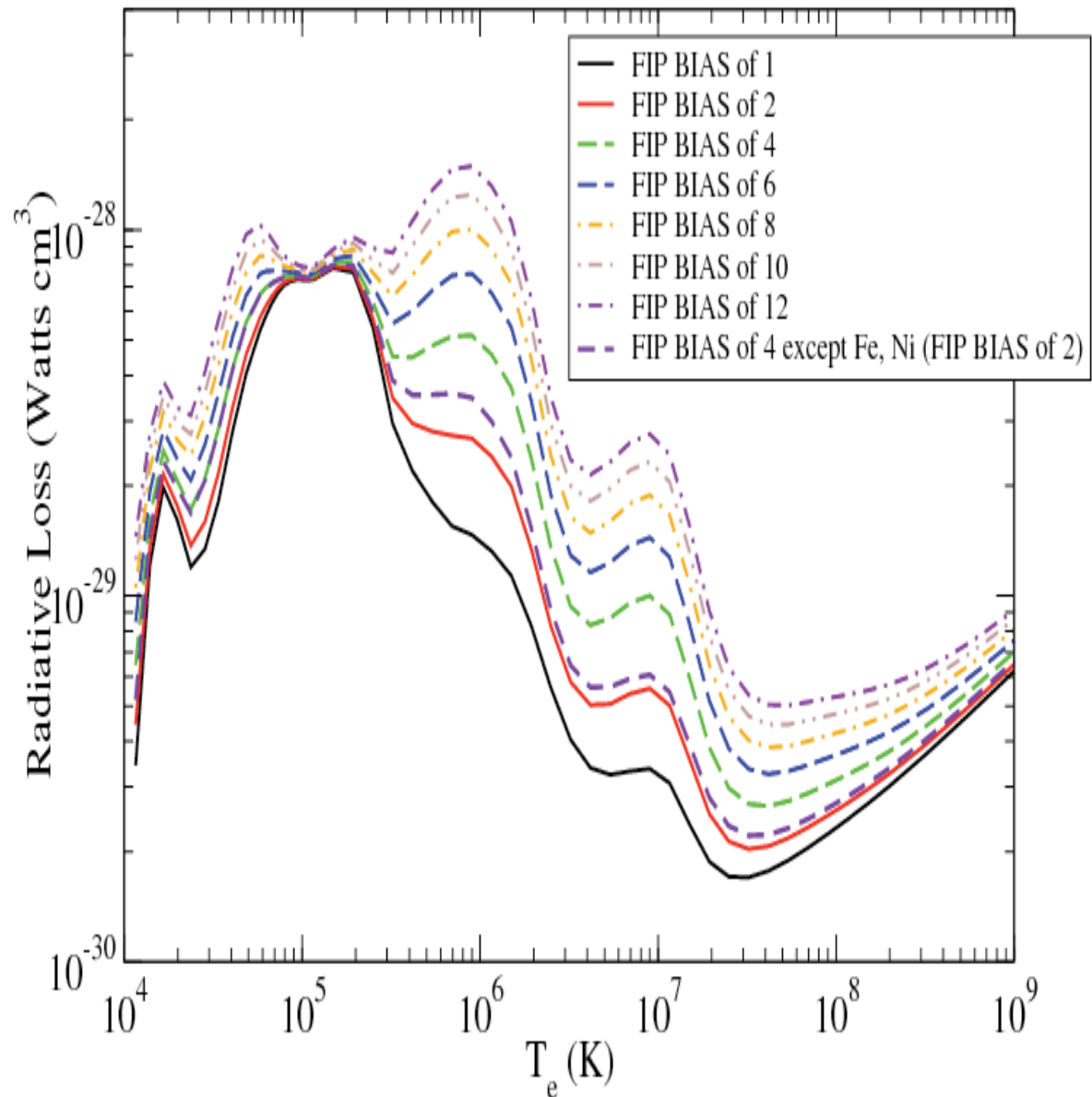
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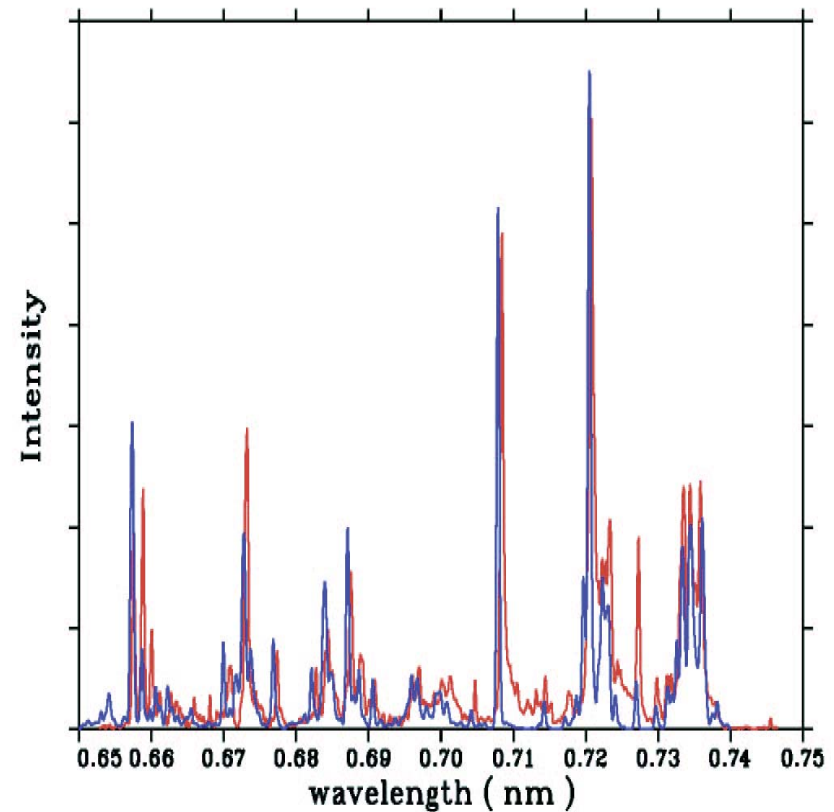
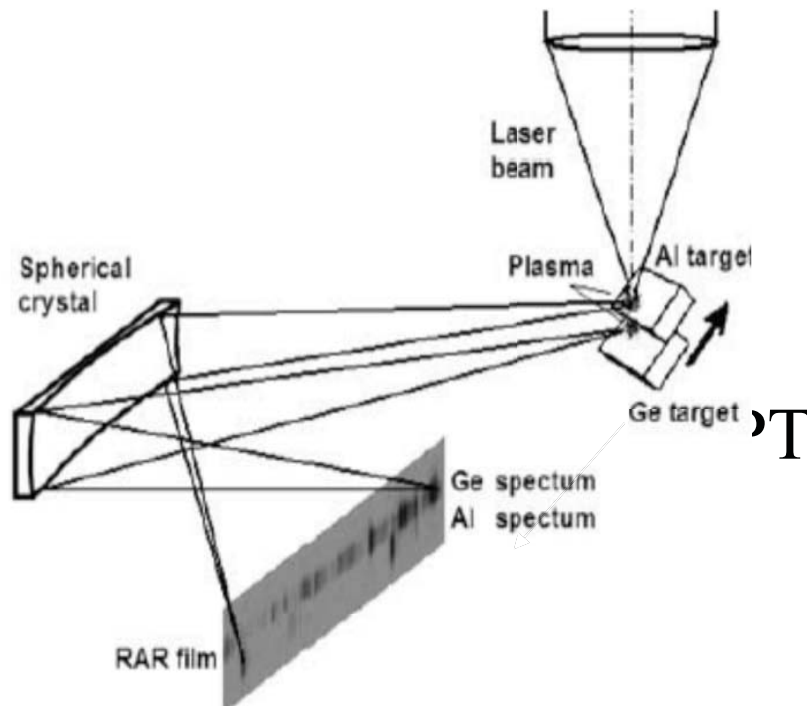
Let's model L-shell Ge spectra

- Choose configurations
- Run CATS, ACE, and Gipper for energy levels and cross sections
- Guess likely plasma conditions, density and temperature
- Run ATOMIC code
- Iterate

Choosing configurations

- Looking at spectra for excitations from the L shell ($n=2$).
- For neon-like choose: $2p^6$, $2p^5nl$, $2p^43lnl$, $2s^12p^63l$, $2s^12p^53lnl$, $2s^02p^63lnl$, $n \leq 7$, about 200 configurations.
- Choose similar configurations for Mg-like, Na-like, F-like, and O-like.

CR modeling can be used to diagnose plasma conditions. Ge laser produced plasma, Milan, $\langle z \rangle \sim 22$, Ne-like. $KT = 560\text{eV}$, $N_e = 10^{21}$. ABDALLAH et. al., Laser and Particle Beams (2007), 25, 245–252, blue calculation, red experiment.



Individual spectral lines can be identified

Table 1. *Transition lines (angstroms)*

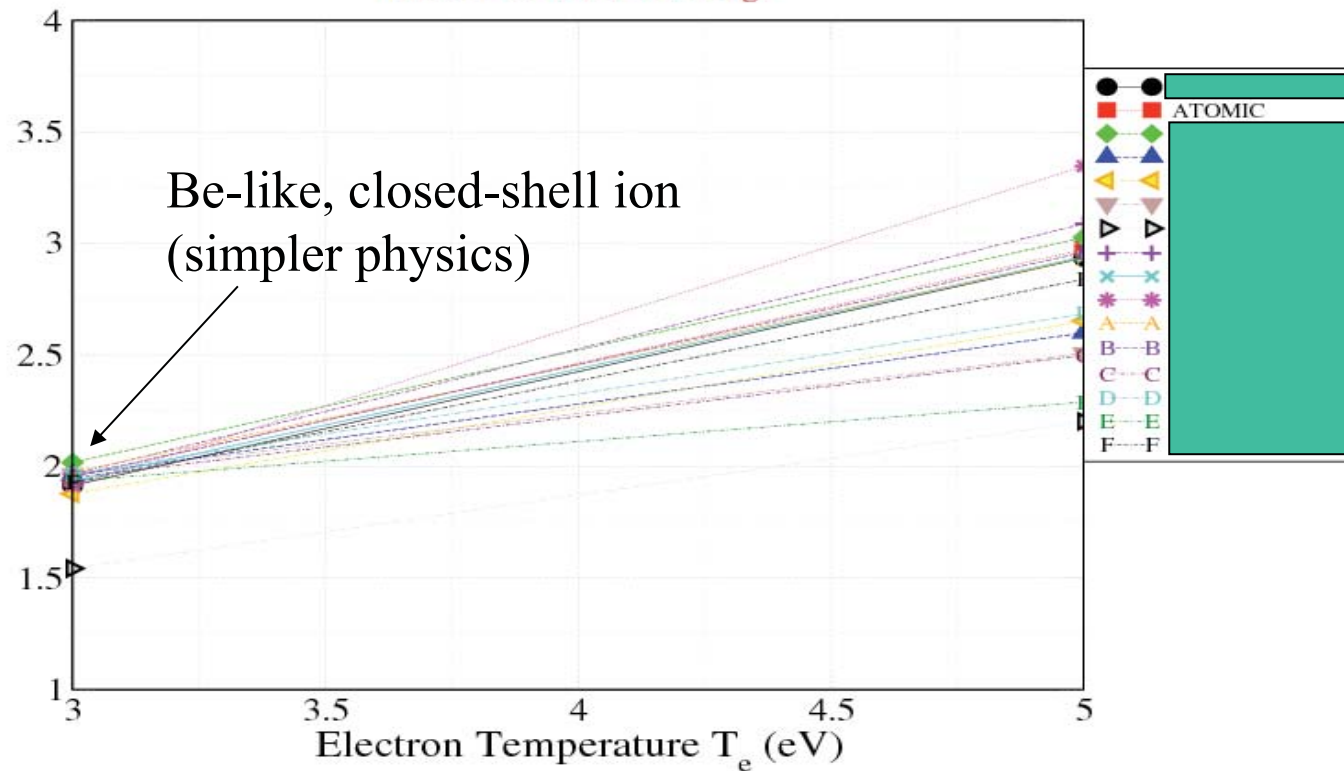
	Theory	Experiment
1s) 1s 0.0 2s2 2p6 – 2p) 1p 1.0 2s2 2p5 5d1	6.5717	6.5754
1s) 1s 0.0 2s2 2p6 – 2s) 1p 1.0 2s1 2p6 4p1	6.5744	6.5883
2p) 2p 1.5 2s2 2p6 3p1 – 3p) 2d 2.5 2s2 2p5 3p1 5d1	6.6060	6.6008
2p) 2p 0.5 2s2 2p6 3p1 – 3d) 4f 1.5 2s2 2p5 3p1 5d1	6.6126	6.6120
2d) 2d 2.5 2s2 2p6 3d1 – 3p) 2f 3.5 2s2 2p5 3d1 5d1	6.6223	6.6268
1s) 1s 0.0 2s2 2p6 – 2p) 1p 1.0 2s2 2p5 5s1	6.6548	6.6597
2p) 2p 0.5 2s2 2p5 – 1s) 2d 1.5 2s2 2p4 4d1	6.6993	6.7031
2s) 2s 0.5 2s2 2p6 3s1 – 1p) 2p 1.5 2s2 2p5 3s1 5d1	6.7087	6.7077
2p) 2p 1.5 2s2 2p5 – 1d) 2s 0.5 2s2 2p4 4d1	6.7294	6.7324
2p) 2p 1.5 2s2 2p5 – 1d) 2p 1.5 2s2 2p4 4d1	6.7282	
2p) 2p 1.5 2s2 2p5 – 1d) 2f 2.5 2s2 2p4 4d1	6.7261	
2s) 2s 0.5 2s1 2p6 – 1p) 2p 0.5 2s1 2p5 4d1	6.7367	6.7444
2s) 2s 0.5 2s1 2p6 – 1p) 2p 1.5 2s1 2p5 4d1	6.7391	
2p) 2p 1.5 2s2 2p5 – 3p) 2f 2.5 2s2 2p4 4d1	6.7679	6.7739
2p) 2p 1.5 2s2 2p5 – 3p) 4p 2.5 2s2 2p4 4d1	6.7694	6.7791
2p) 3d 2.0 2s2 2p5 3p1 – 1s) 1f 3.0 2s2 2p4 3p1 4d1	6.8124	6.8048
2p) 2p 1.5 2s2 2p5 – 1s) 2d 2.5 2s2 2p4 4d1	6.8212	6.8227
2p) 2p 0.5 2s2 2p5 – 1d) 2d 1.5 2s2 2p4 4d1	6.8405	6.8408

Carbon mean ion charge:

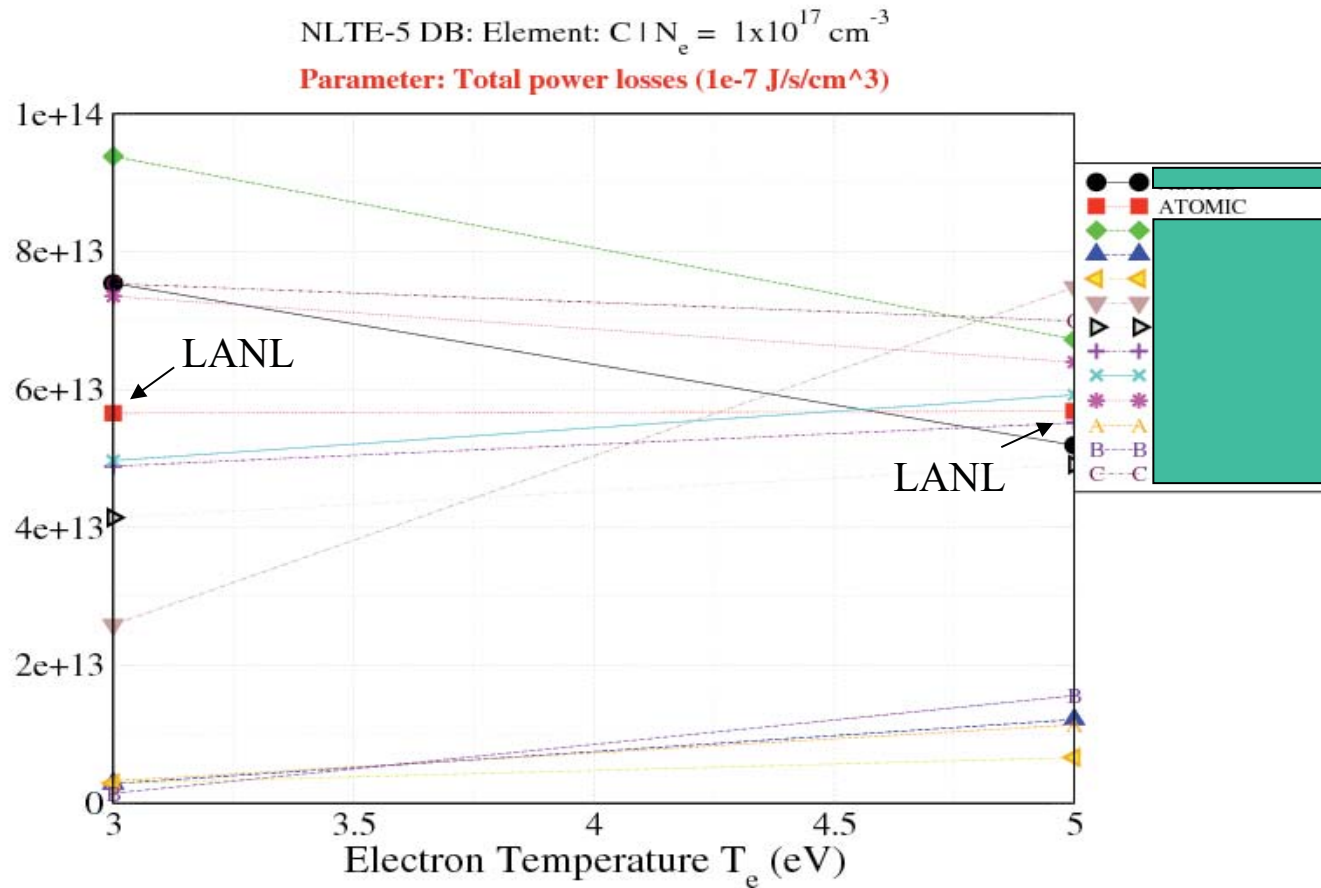
$$N_e = 10^{17} \text{ cm}^{-3}$$

NLTE-5 DB: Element: C | $N_e = 1 \times 10^{17} \text{ cm}^{-3}$

Parameter: Mean ion charge



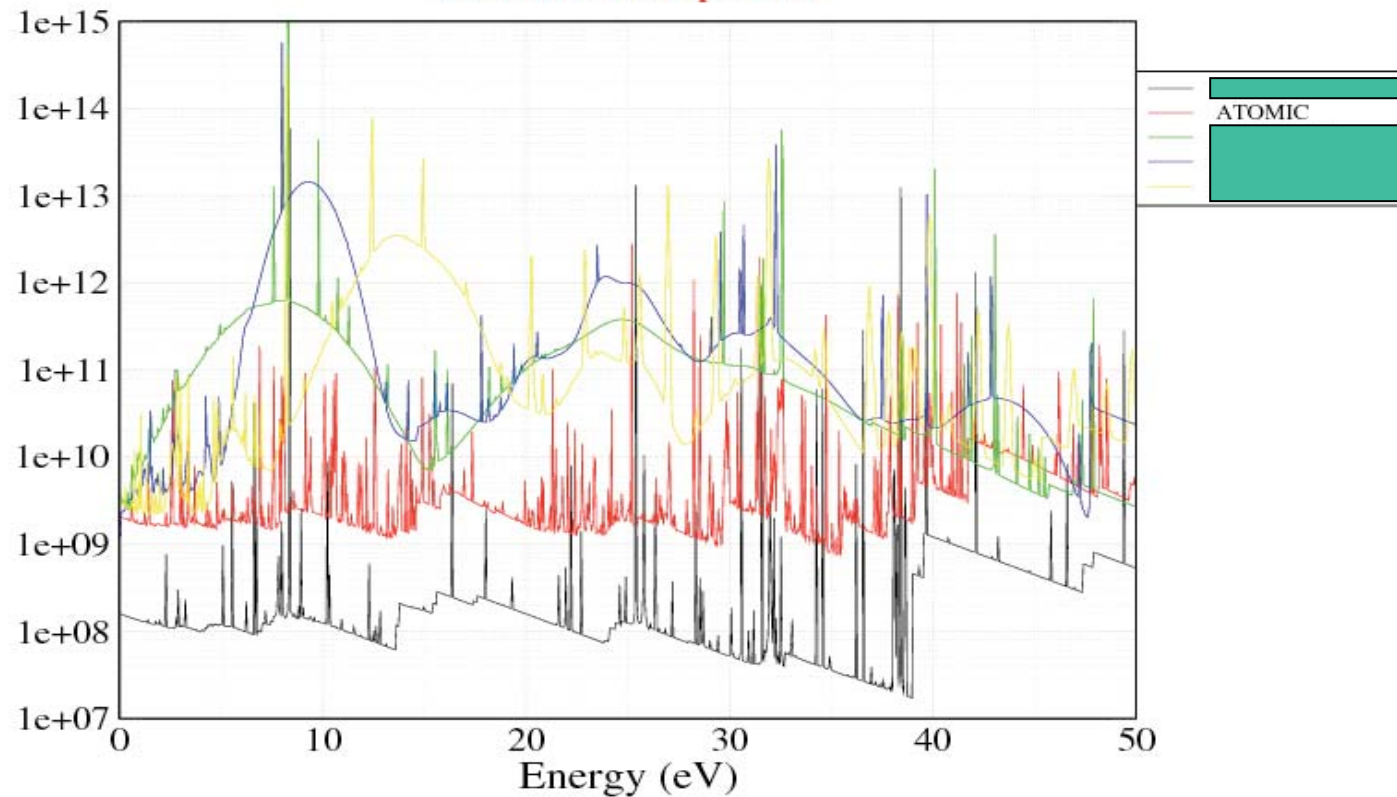
Carbon power loss: $N_e=10^{17}\text{cm}^{-3}$



Carbon emissivity: $N_e = 10^{17} \text{ cm}^{-3}$,

NLTE-5 DB: Element: C | $T_e = 5 \text{ eV}$; $N_e = 1e+17 \text{ cm}^{-3}$

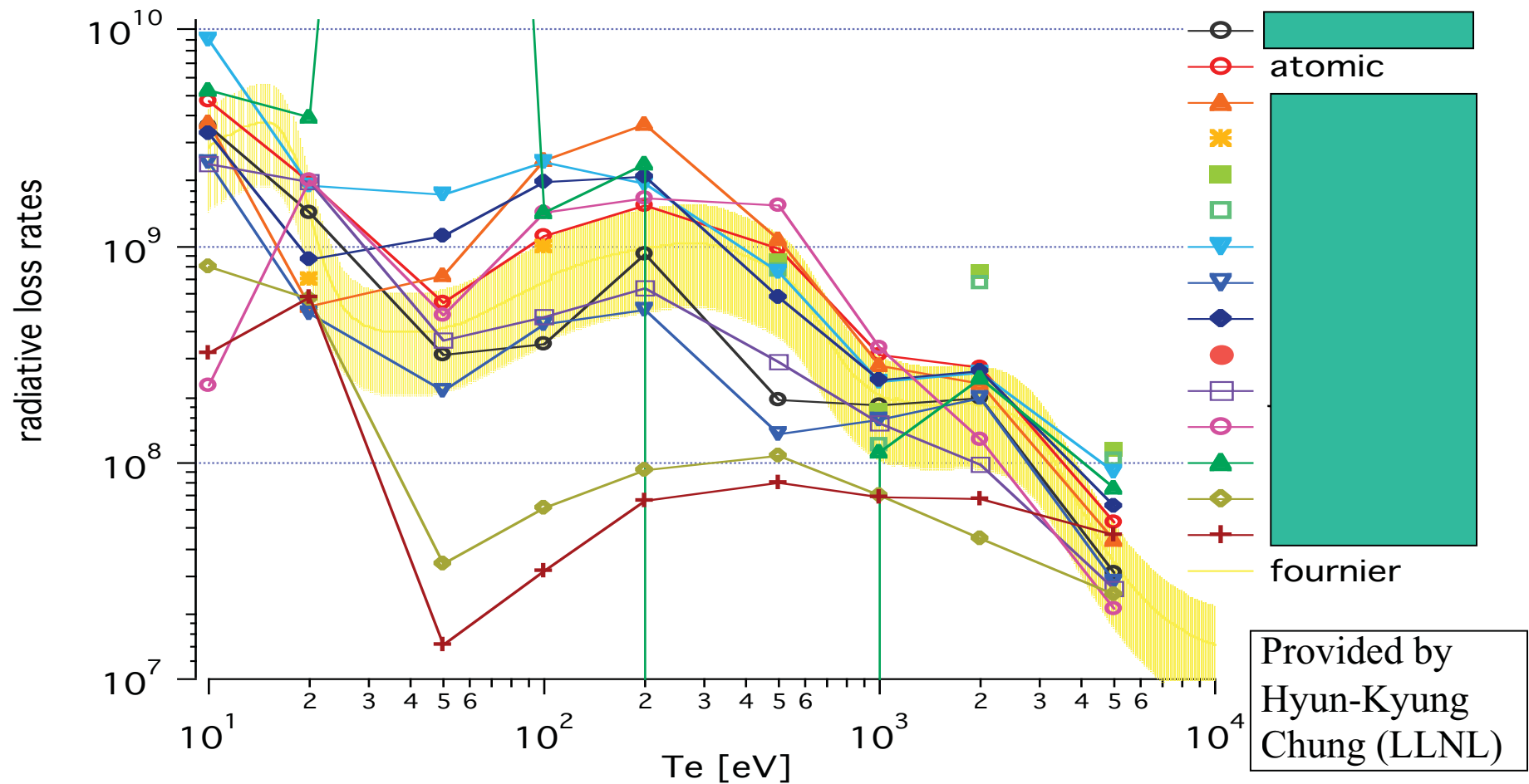
Parameter: Total spectrum



Krypton: radiative loss rates at $N_e=10^{14} \text{ cm}^{-3}$

LANL provides only results within error bars

Experimental tokamak data from Fournier et al. Nucl. Fusion 40, 847 (1999)



Tungsten:

*Nuclear Fusion, June 2007:
Progress in ITER Physics Basis*

5 possible ITER wall material

500 MW of fusion energy for extended periods of time (x10 more than needed to sustain plasma)

Duration: > 400 s

Currently: **JET 16 MW**

Cost: **5 G€** construction + **5 G€** for 20-year operation

Radius: **6.2 m**

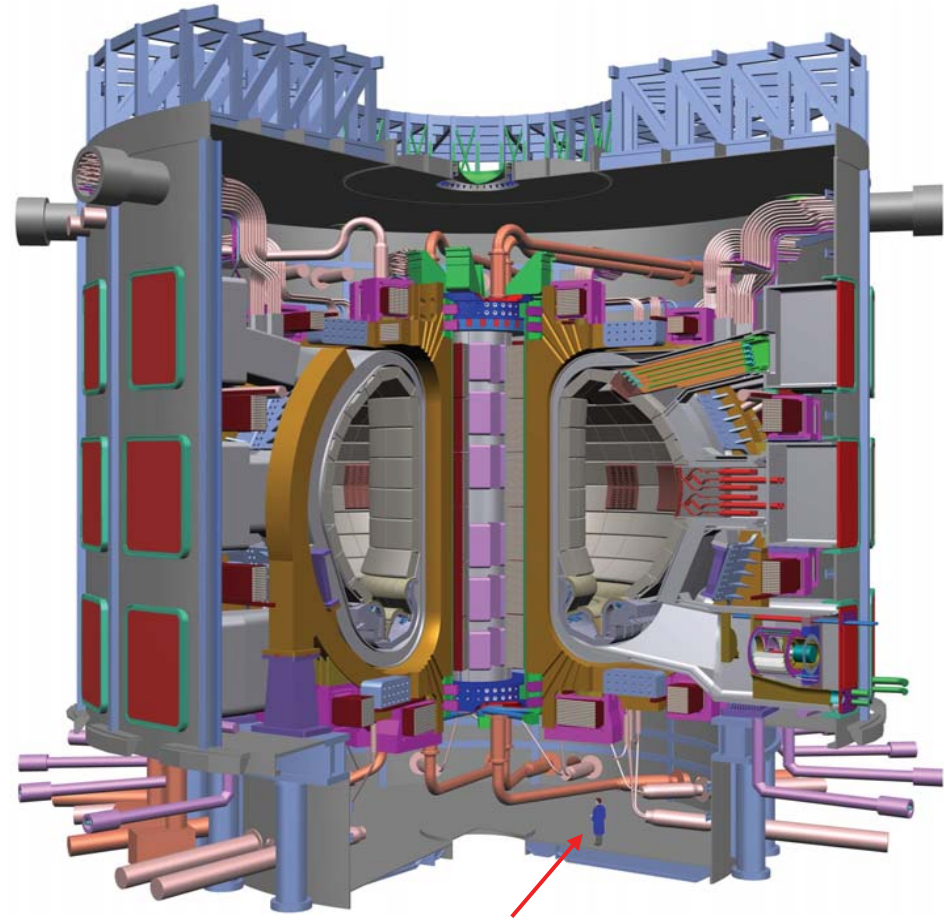
Volume: **830 m³**

Current: **15 MA**

Density: **10¹⁴ cm⁻³**

Temperature: **20 keV**

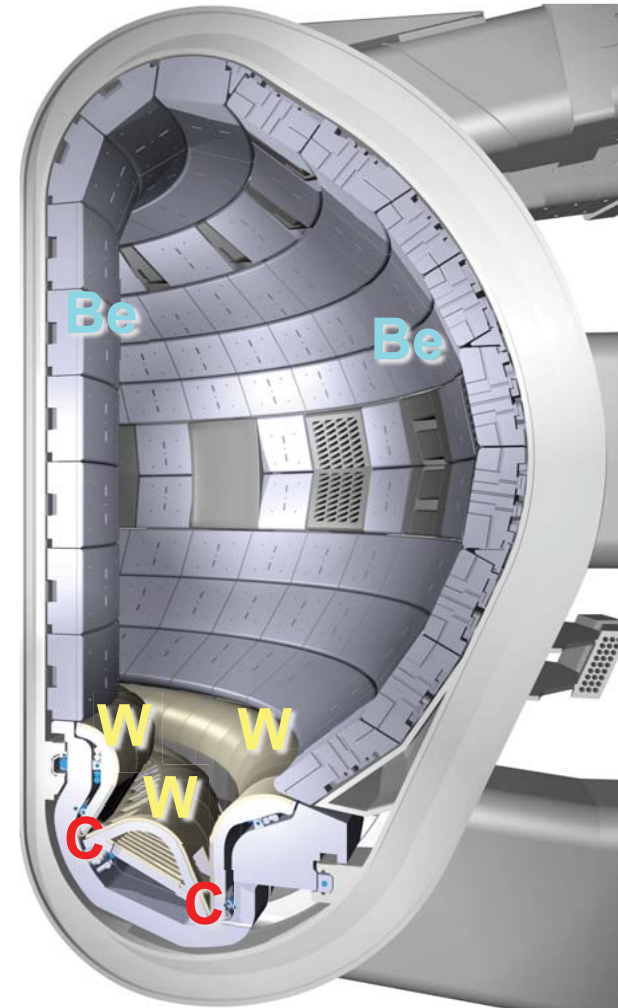
First plasma: 2016



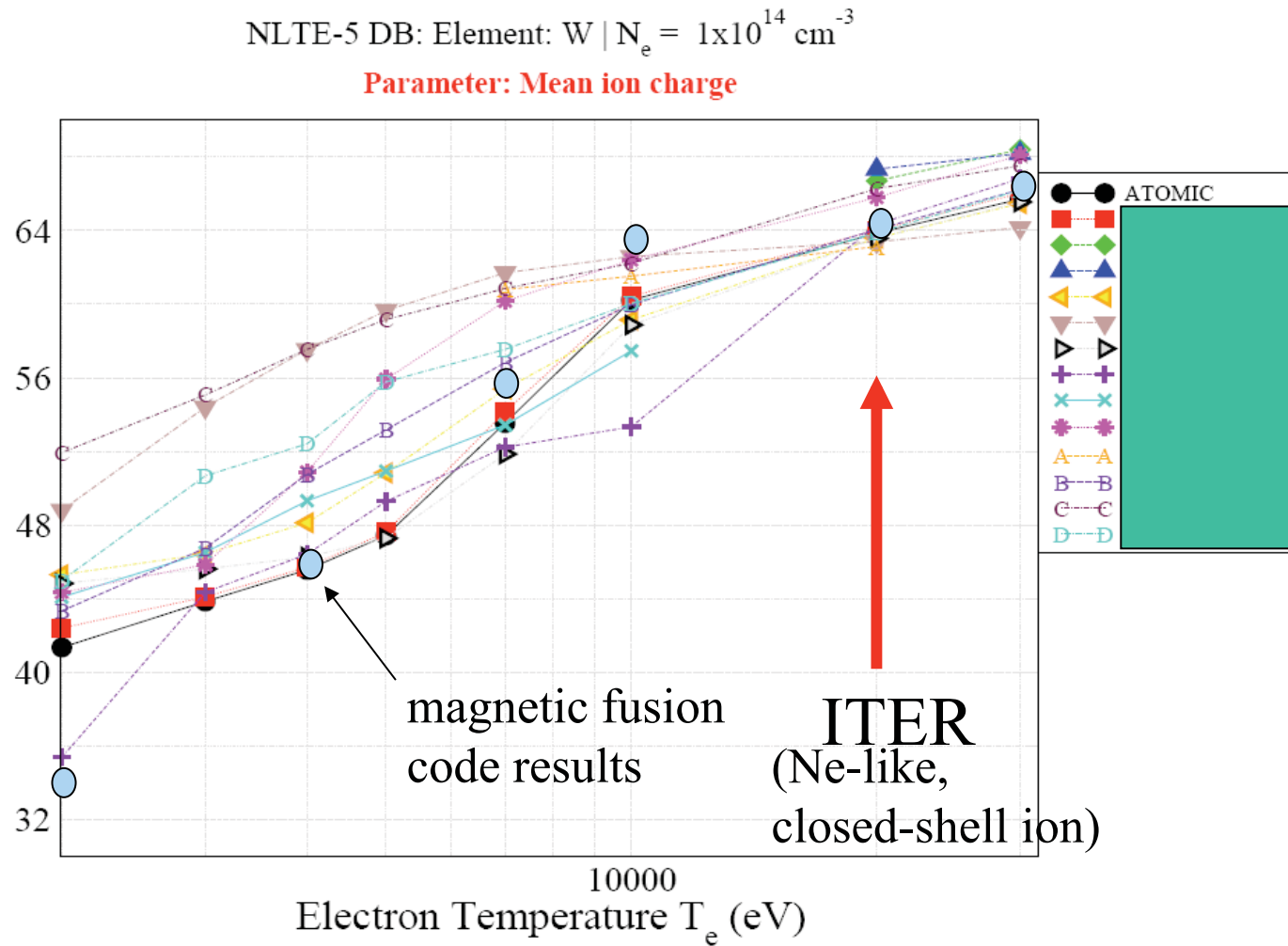
Cross-section of the vessel

Provided by
Yuri Ralchenko (NIST)

- **Be**
 - Low radiative losses
 - Low plasma contamination
 - Low bulk tritium inventory
 - **BAD: low melt temperature**
- **W**
 - Low erosion rate (high sputtering threshold)
 - Low tritium retention
 - **BAD: high radiative losses**
- **CFC**
 - High melting temperature
 - High thermal shock and thermal fatigue resistance
 - **BAD: high tritium retention**

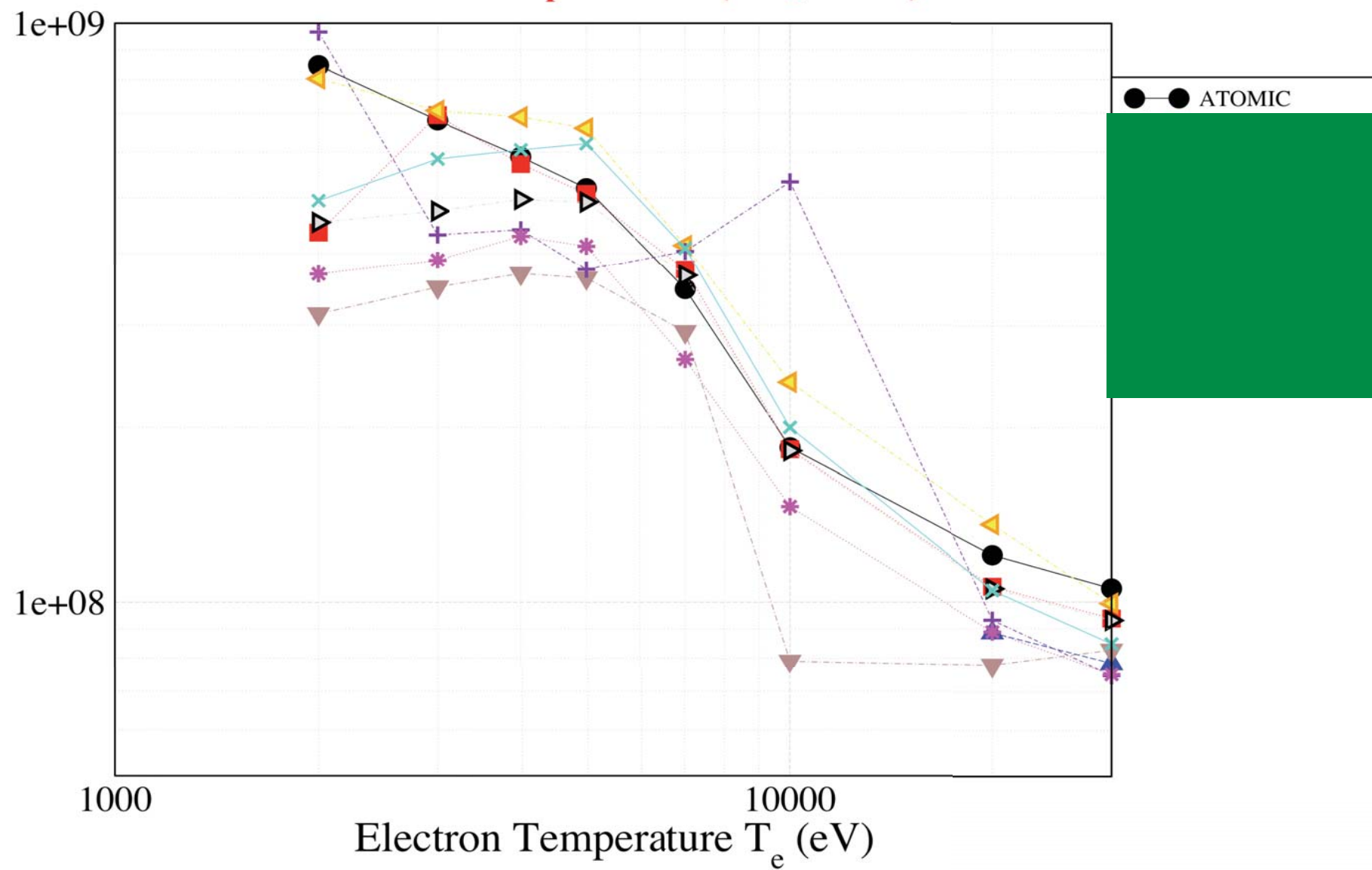


Tungsten mean ion charge for ITER conditions

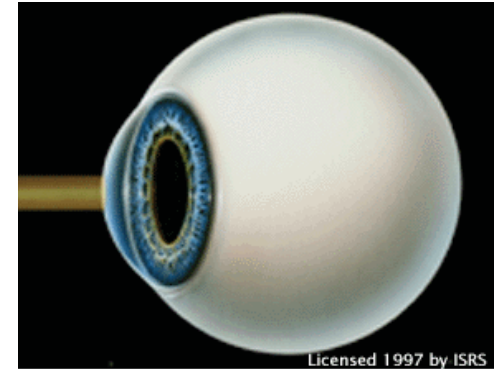
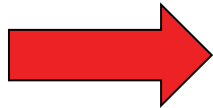
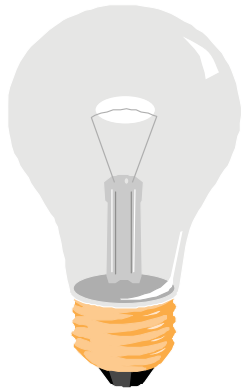


NLTE-5 DB: Element: W | $N_e = 1 \times 10^{14} \text{ cm}^{-3}$

Parameter: Total power losses (10^{-7} J/s/cm^3)



What is opacity?



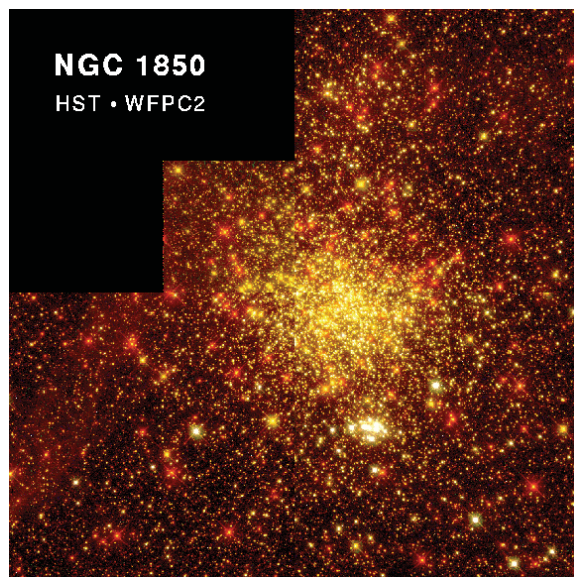
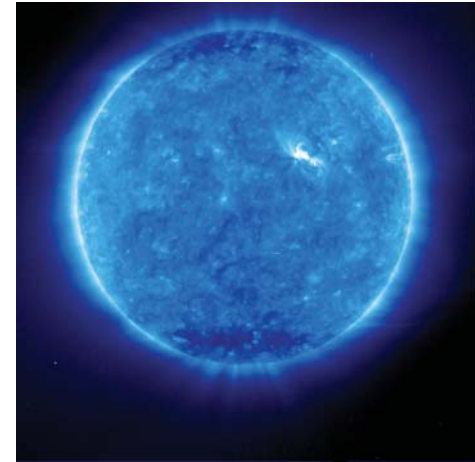
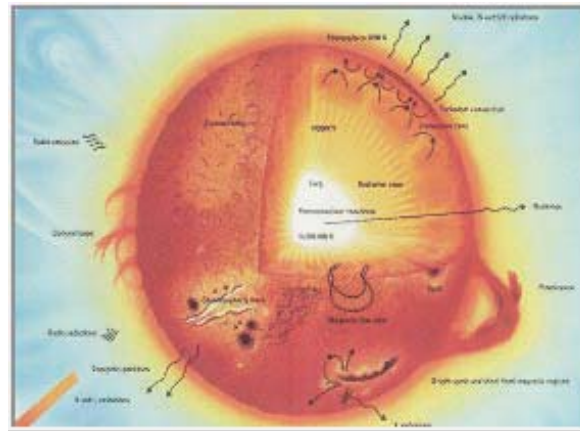
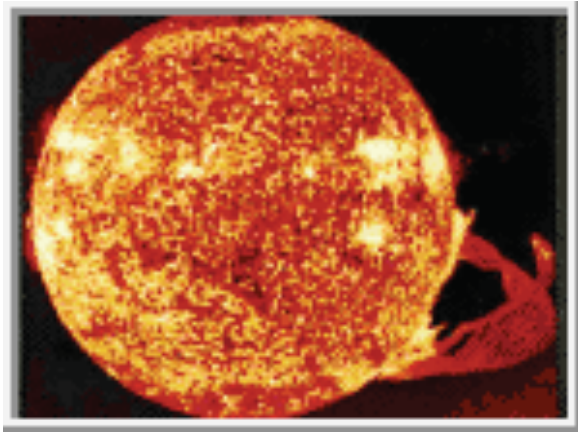
Licensed 1997 by ISRS
Artwork (c)1997 Stephen F. Gordon

$$\frac{\partial I_{\omega}(x)}{\partial x} = -\kappa_{\omega} \rho I_{\omega}(x)$$

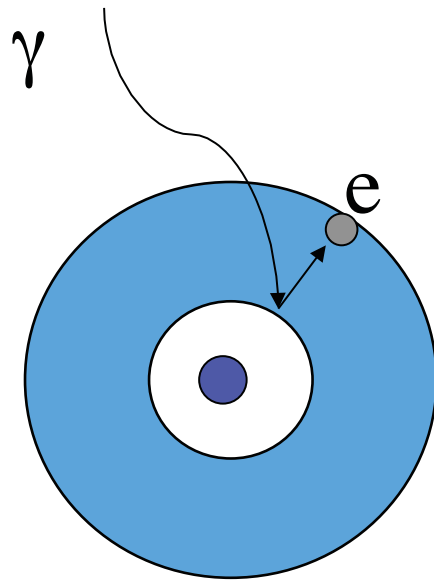
$$I_{\omega}(x) = I_0 \exp(-\kappa_{\omega} \rho x)$$



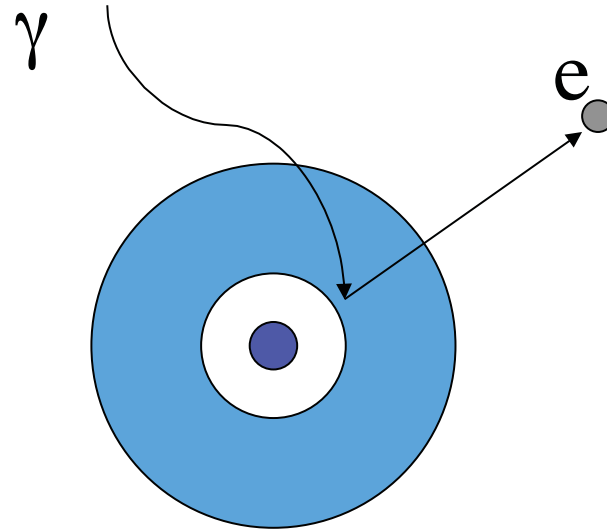
Applications of Opacities



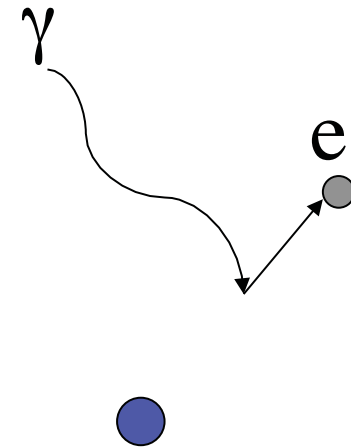
Physical processes contributing to the opacity:



bound-bound

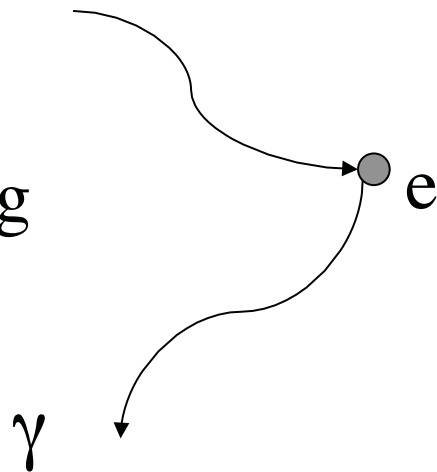


bound-free



free-free

scattering



These photon absorption and scattering processes
Combine to give the frequency dependent opacity:

$$\kappa_{\omega} = \left(\kappa_{bb} + \kappa_{bf} + \kappa_{ff} \right) \left(1 - \exp(-\hbar\omega/kT) \right) + \kappa_s$$

$$\kappa_{bb} = \frac{1}{\rho} \sum_{il} N_{il} \frac{he^2\pi}{m_e c} \frac{(gf)_{ilm}}{g_{il}} \Phi_{ilm}(h\nu)$$

$$\kappa_{bf} = \frac{1}{\rho} \sum_{il} N_{il} \sigma_{iljm}(h\nu)$$

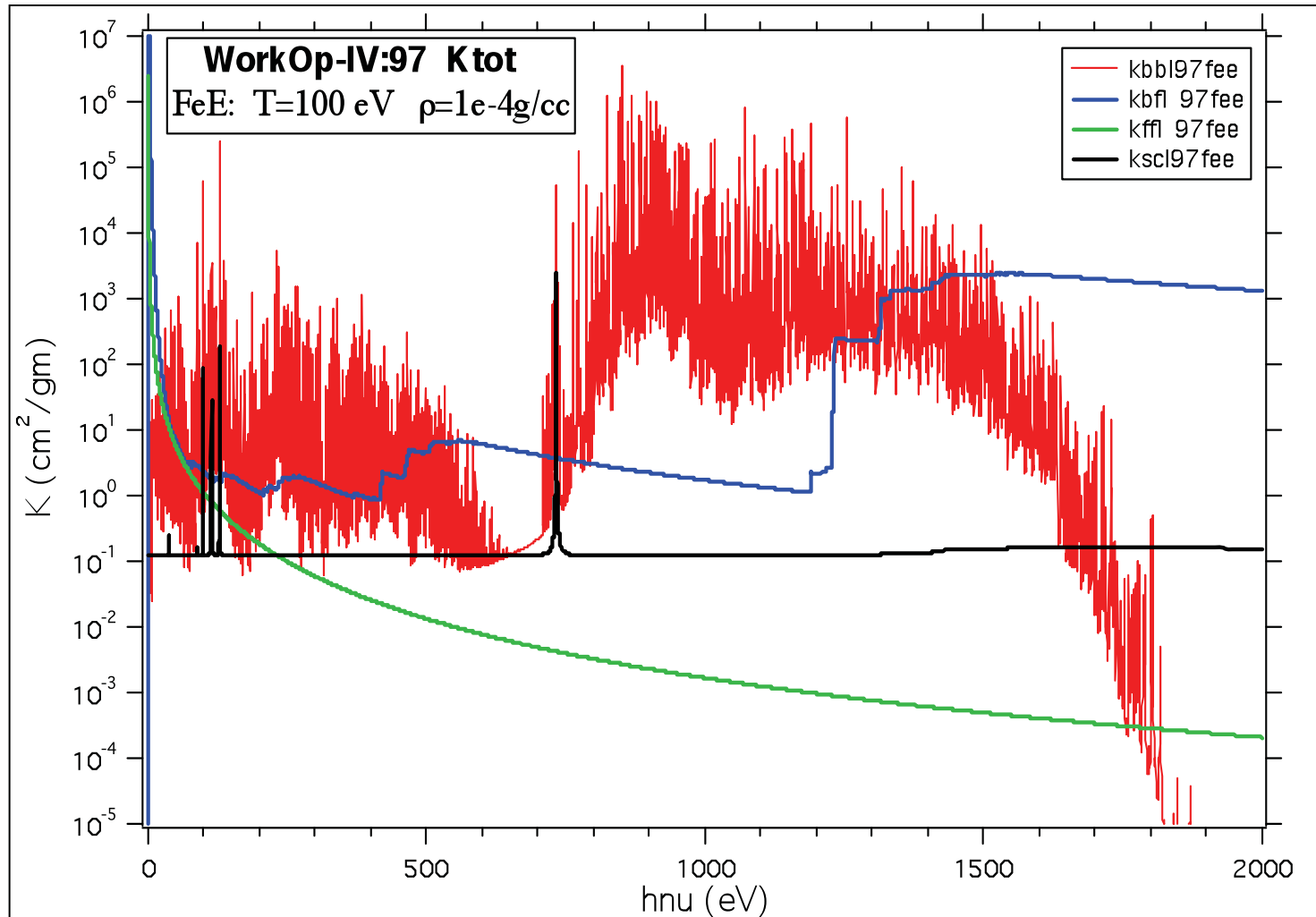
$$\kappa_{ff} = 4.1 \times 10^{-23} \frac{1}{\rho} \frac{N_e}{T_0^{7/2} (h\nu / kT_e)^3} \sum_i q_i^2 N_i$$

ρ = mass density $g(cm)^{-3}$

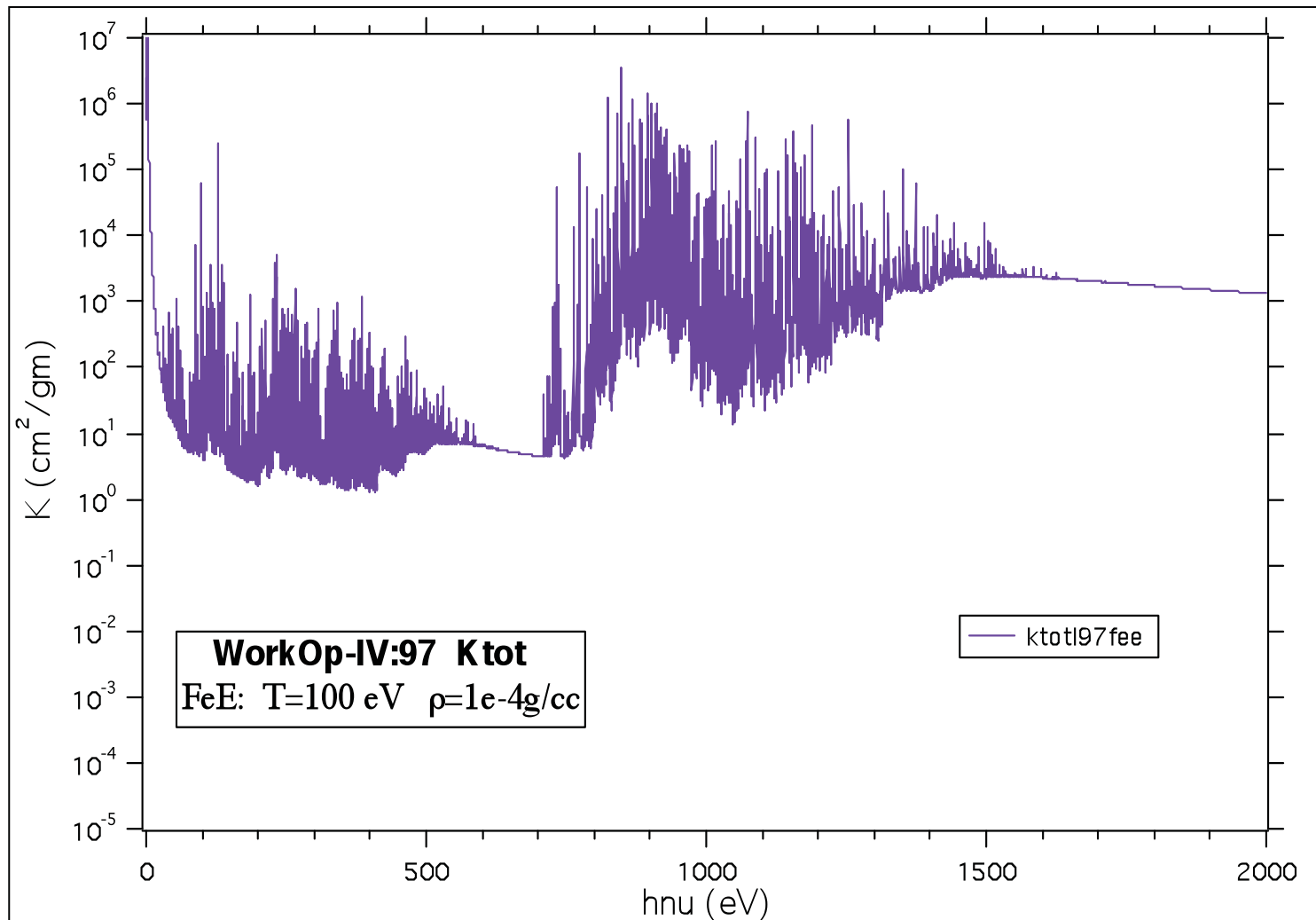
T_0 = temperature($^{\circ} Kelvin$)



Components of the frequency dependent opacity (bound-bound, bound-free, free-free and scattering)



The resultant total frequency dependent opacity





The 20million Amp current provided by the Z accelerator
enables this research



cm-scale laboratory
astrophysics experiment

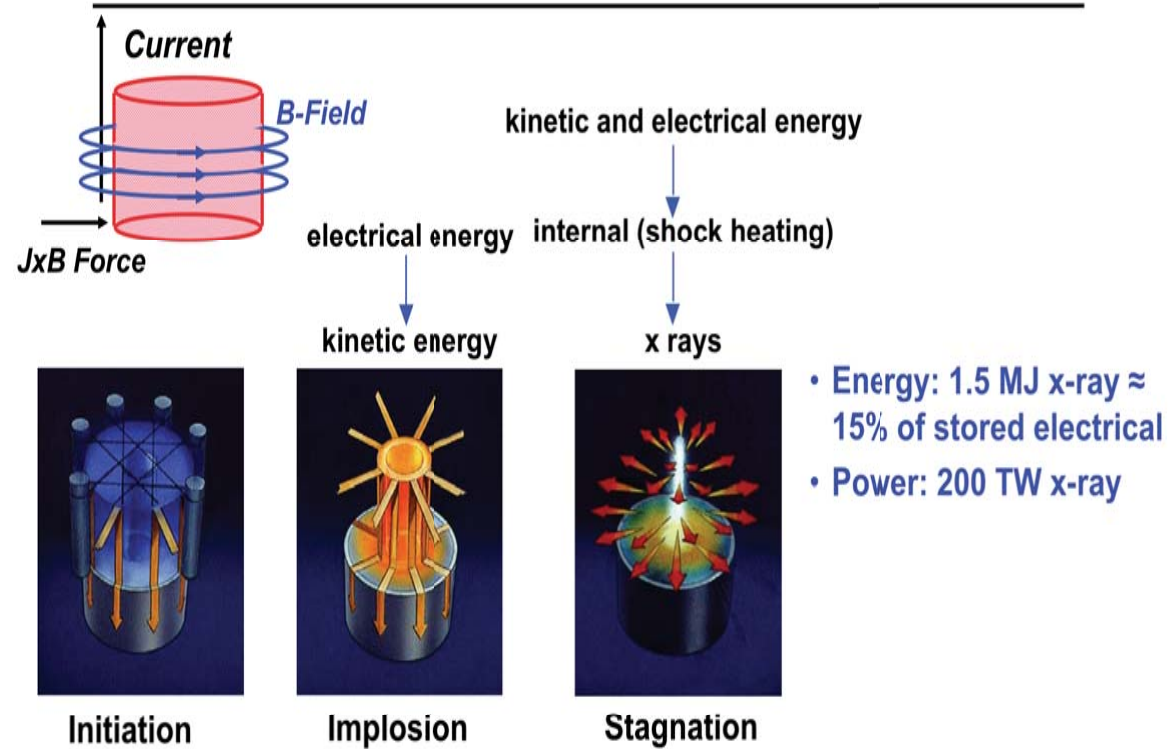
Z accelerator

40 m



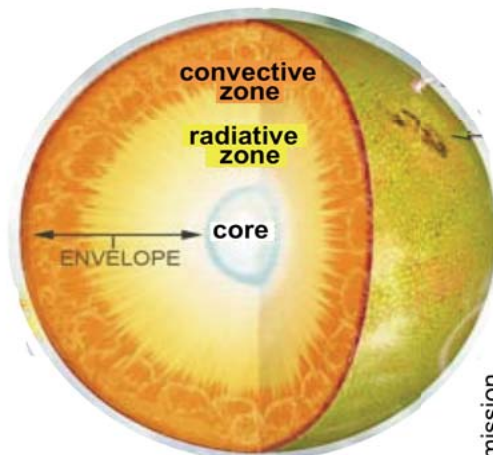


Magnetically-driven z-pinch implosions efficiently convert electrical energy into radiation





Laboratory experiments test opacity models that are crucial for stellar interior physics



2007 Don Dixon / cosmographica.com

Predictions of solar structure do not agree with observations

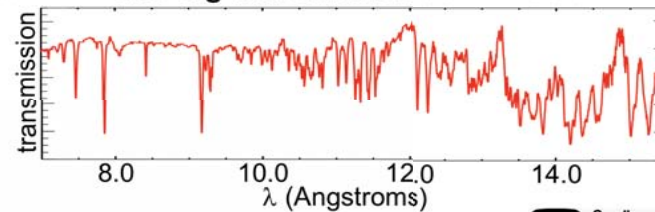
Solar structure depends on opacities that have never been measured

Challenge: create and diagnose stellar interior conditions on earth

Z opacity experiments reach $T \sim 156$ eV

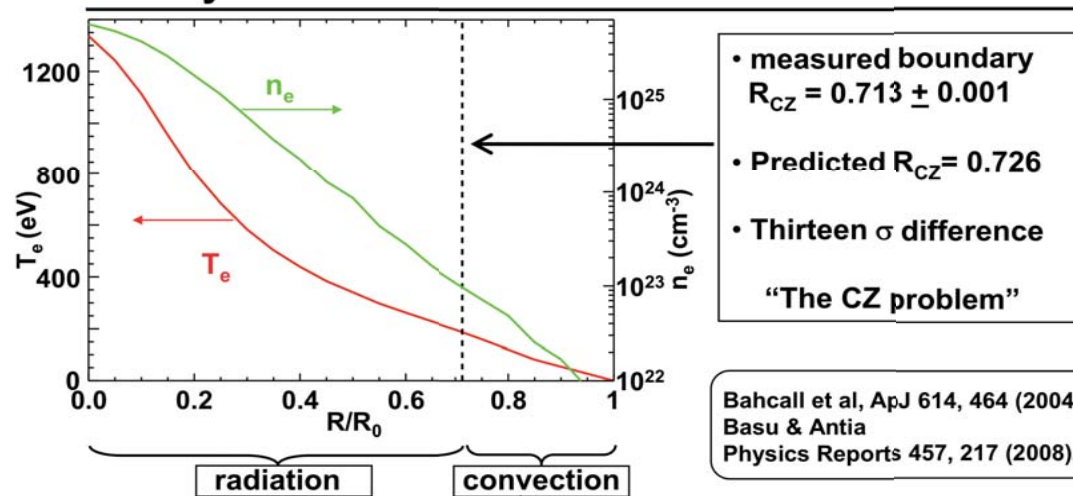
High T enables first studies of transitions important in stellar interiors

Fe / Mg transmission at $T \sim 156$ eV



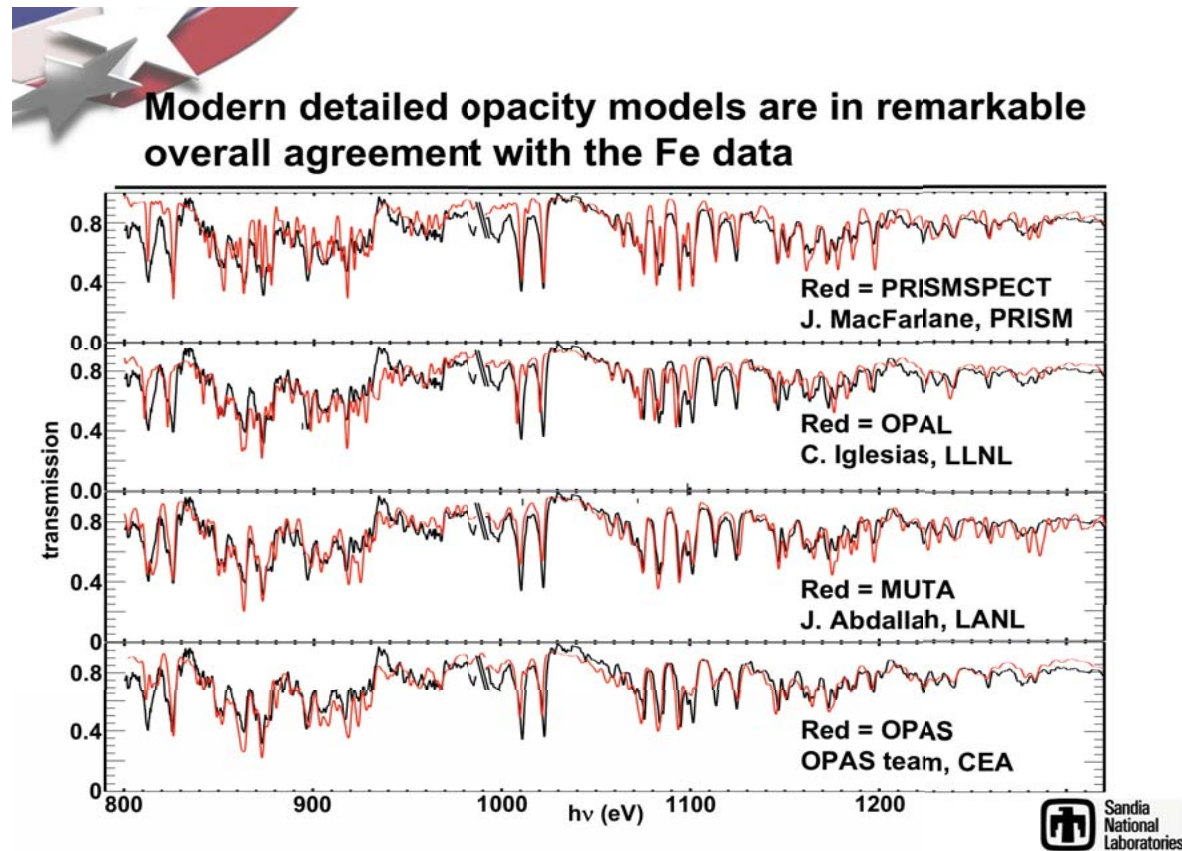


Modern solar models disagree with observations. Why?



- Boundary location depends on radiation transport
- A 10-20% opacity change solves the CZ problem.
- This accuracy is a challenge – experiments are needed to know if the solar problem arises in the opacities or elsewhere.

J.E. Bailey et. Al. Iron opacity measurements at temperatures above 150 eV, Physical Review Letters 99, 265002(2007).



Exercises

(I) Consider the 2 state system. Let N_1 and N_2 be the population density of the 2 states respectively. Let $N =$ the constant total number density. Let c be the rate coefficient for excitation from state 1 to state 2, d be the de-excitation rate coefficient from state 2 to state 1, and r be the spontaneous decay rate from state 2 to state 1. Let N_e denote a given electron density.

(1) Write down the rate equations for N_1 and N_2 and derive an expression for the steady-populations of both in terms of N .

(2) Derive a formula for N_1 and N_2 as a function of time(t) by directly solving the differential rate equations.

(3) Check the formula to if the steady-result is obtained as t gets large, also check the formula for N_e small (coronal limit), and for large N_e (LTE).

(4) Derive an expression to first order for the relaxation time, the time it takes for N_1 to reach its steady-state value.

Exercises

(II) Assume the 2-level system of the previous example to be the 1s and 2p configurations of H-like Lithium. This example is actually of interest in Lithography because of its wavelength. Calculate the power (Watts/cm³) emitted by the 2p-1s emission line at $N=?$, $N_e=?$ and $kT_e=?$. Use the LANL atomic physics website to evaluate the required atomic data.

(III) Use the LANL atomic physics website to identify the major lines observed in the following O spectra shown . Match the observed line position with calculated values. Use the assumption of LTE to estimate the temperature from selected line ratios.

Exercise III

