The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2034-18

Advanced School and Conference on Knot Theory and its Applications to Physics and Biology

11 - 29 May 2009

Quantum & Mosaics

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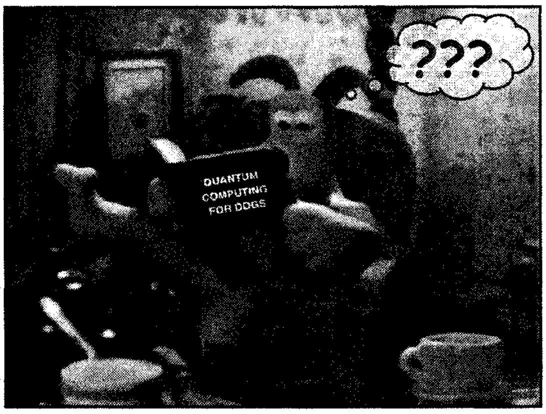

- Lecture I: A Rosetta Stone for Quantum Computing
  - ~~Three Quantum Algorithms~~
  - ~~Quantum Hidden Subgroup Algorithms~~
  - ~~Distributed Quantum Computing~~
  - ~~An Entangled Tale of Quantum Entanglement~~
  - ~~Introduction to Quantum Cryptography, or How Alice Outwits Eve~~
- Lecture II: Quantum Knots and Mosaics, or How to Play Quantum Knot Tile Rummy
- Lecture III: Quantum Knots & Lattices, or How to Teach a Quantum System to Do Rope Tricks

PowerPoint Lectures and Exercises can be found at:

www.csee.umbc.edu/~lomonaco

Quantum Knots & Mosaics

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L-O-O-P

How to Play Quantum Knot Tile Rummy

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L-O-O-P

Throughout this talk:

"Knot" means either a knot or a link

Outline

Knot Theory = Formal Rewriting System

⇩

Formal Rewriting System = Group Representation

⇩

Quantum Mechanics = Group Representation Theory

This talk is based on the paper:

Lomonaco and Kauffman, Quantum Knots and Mosaics, Journal of Quantum Information Processing, vol. 7, Nos. 2-3, (2008), 85-115. An earlier version can be found at: http://arxiv.org/abs/0805.0339

This talk was motivated by:

Lomonaco, Samuel J., Jr., The modern legacies of Thomson's atomic vortex theory in classical electrodynamics, AMS PSAPM/51, Providence, RI (1996), 145 - 166.

Kauffman and Lomonaco, Quantum Knots, SPIE Proc. on Quantum Information & Computation II (ed. by Donkor, Pirich, & Brandt), (2004), 5436-30, 268-284. http://xxx.lanl.gov/abs/quant-ph/0403228

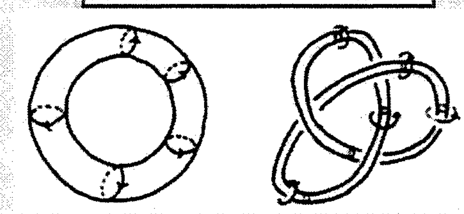
This talk was also motivated by:

Kitaev, Alexei Yu, Fault-tolerant quantum computation by anyons, http://arxiv.org/abs/quant-ph/9707021

Rasetti, Mario, and Tullio Regge, Vortices in He II, current algebras and quantum knots, Physica 80 A, North-Holland, (1975), 217-2333.

What Motivated This Talk ?

Classical Vortices in Plasmas



Lomonaco, Samuel J., Jr., The modern legacies of Thomson's atomic vortex theory in classical electrodynamics, AMS PSAPM/51, Providence, RI (1996), 145 - 166.

Knots Naturally Arise in the Quantum World as Dynamical Processes

Examples of dynamical knots in quantum physics:
Knotted vortices

- In supercooled helium II
- In the Bose-Einstein Condensate
- In the Electron fluid found within the fractional quantum Hall effect

Reason for current intense interest:
A Natural Topological Obstruction to Decoherence

Objectives

- We seek to create a quantum system that simulates a closed knotted physical piece of rope.
- We seek to define a quantum knot in such a way as to represent the state of the knotted rope, i.e., the particular spatial configuration of the knot tied in the rope.
- We also seek to model the ways of moving the rope around (without cutting the rope, and without letting it pass through itself.)

Rules of the Game

Find a mathematical definition of a quantum knot that is

- Physically meaningful, i.e., physically implementable, and
- Simple enough to be workable and useable.

Aspirations

We would hope that this definition will be useful in modeling and predicting the behavior of knotted vortices that actually occur in quantum physics such as

- In supercooled helium II
- In the Bose-Einstein Condensate
- In the Electron fluid found within the fractional quantum Hall effect

Overview

Part 0. Quick Overview of Knot Theory

Part 1. Mosaic Knots

We reduce tame knot theory to a formal system of string manipulation rules, i.e., string rewriting systems.

Part 2. Quantum Knots

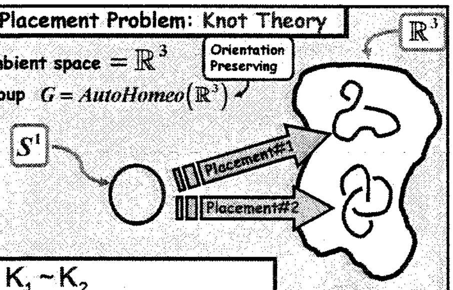
We then use mosaic knots to build a physically implementable definition of quantum knots.

Quick Overview to Knot Theory

Skip to mosaic knots

Placement Problem: Knot Theory

- Ambient space $= \mathbb{R}^3$
- Group $G = AutoHomeo(\mathbb{R}^3)$



Def. $K_1 \sim K_2$ if $g \in G$ s.t. $gK_1 = K_2$

Problem. When are two placements the same ?

$$K_1 \sim K_2 \ ?$$

What is a knot invariant ?

Def. A knot invariant I is a map

$I : Knots \to Mathematical\ Domain$

that takes each knot K to a mathematical object I(K) such that

$$K_1 \sim K_2 \Rightarrow I(K_1) = I(K_2)$$

Consequently,

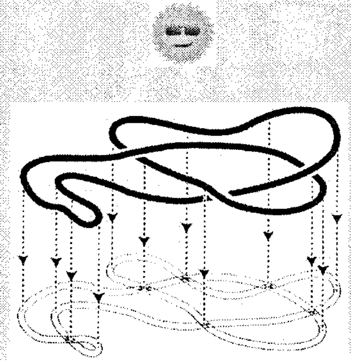
$$I(K_1) \neq I(K_2) \Rightarrow K_1 \nsim K_2$$

The Jones polynomial is a knot invariant.

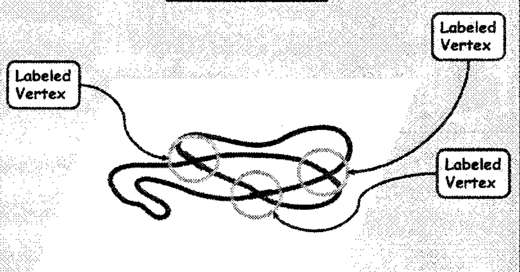
Knot Diagrams:

A fundamental tool in knot theory.

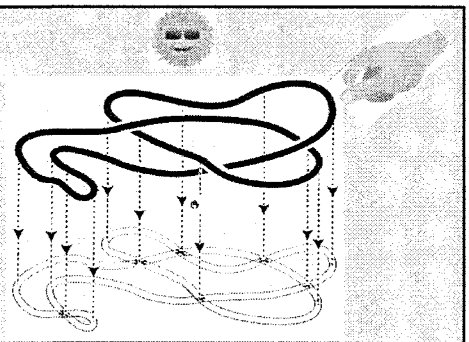
Knot Projections



Knot Diagram

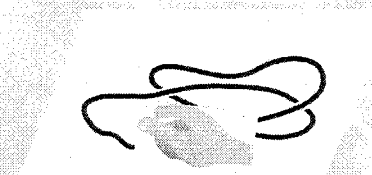


- Planar four valent graph with
- Labeled vertices



Question: If we locally move the rope, what does its shadow (knot diagram) do ???

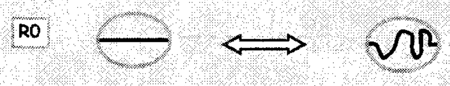
Planar Isotopy Moves



In this case, we have not changed the topological type of the knot diagram

This is a <u>planar</u> <u>isotopy</u> move denoted by R0

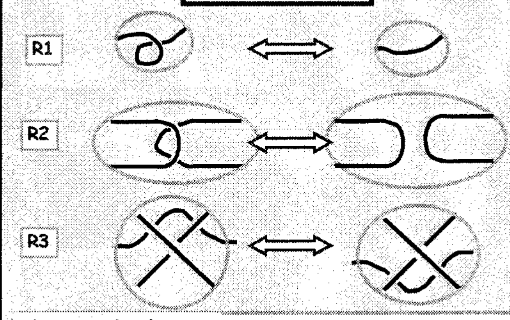
Planar Isotopy Moves



This is a <u>local</u> move !

It does not change the topological type of the knot diagram.

Reidemeister Moves

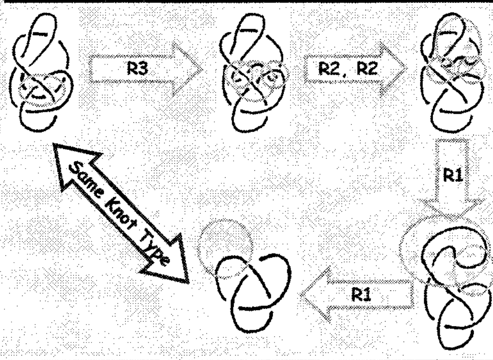


These are <u>local</u> moves that change the topological type of the knot diagram !

When do two Knot diagrams represent the same or different knots ?

<u>Theorem</u> (Reidemeister). Two knots diagrams represent the same knot iff one can be transformed into the other by a finite sequence of Reidemester moves (and planar isotopy rules).

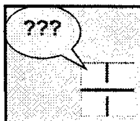
Example of Application of Reidemeister Moves

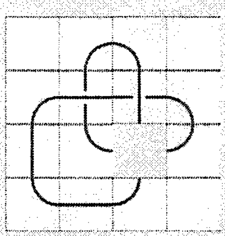


Part 1

Mosaic Knots

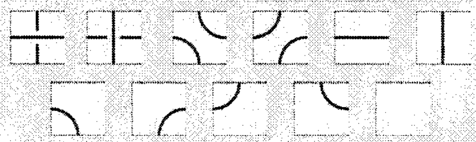
Mosaic Knots





Mosaic Tiles

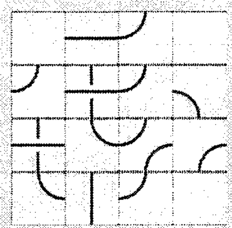
Let $T^{(u)}$ denote the following set of 11 symbols, called mosaic (unoriented) tiles:



Please note that, up to rotation, there are exactly 5 tiles

Definition of an n-Mosaic

An n-mosaic is an $n \times n$ matrix of tiles, with rows and columns indexed $0, 1, \ldots, n-1$



An example of a 4-mosaic

Tile Connection Points

A connection point of a tile is a midpoint of a tile edge which is also the endpoint of a curve drawn on the tile. For example,

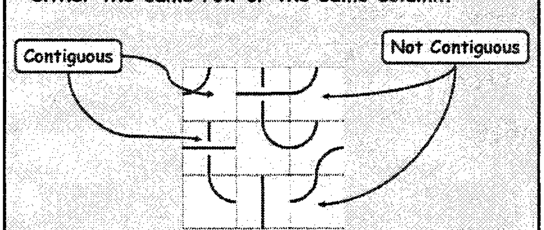
0 Connection Points

2 Connection Points

4 Connection Points

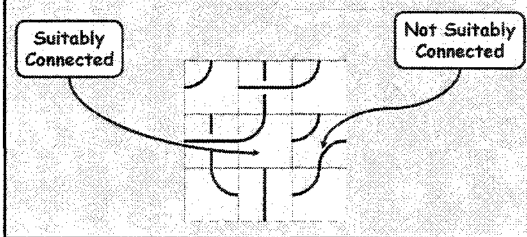
Contiguous Tiles

Two tiles in a mosaic are said to be contiguous if they lie immediately next to each other in either the same row or the same column.



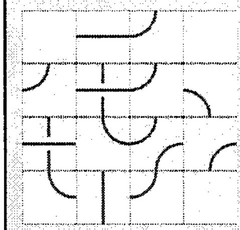
Suitably Connected Tiles

A tile in a mosaic is said to be Suitably Connected if all its connection points touch the connection points of contiguous tiles. For example,

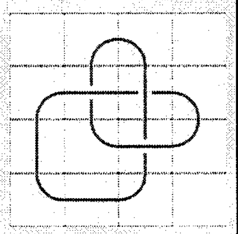


Knot Mosaics

A <u>knot</u> <u>mosaic</u> is a mosaic with all tiles suitably connected. For example,

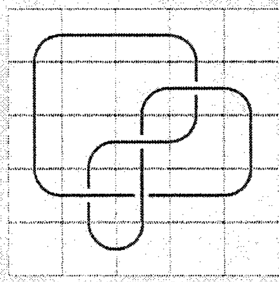


Non-Knot 4-Mosaic

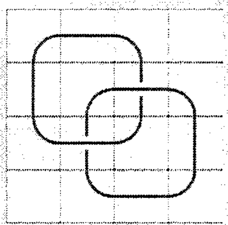


Knot 4-Mosaic

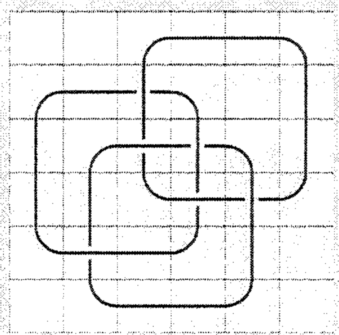
Figure Eight Knot 5-Mosaic



Hopf Link 4-Mosaic



Borromean Rings 6-Mosaic



Notation

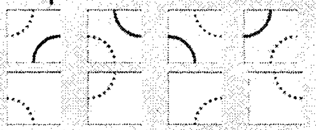
$M^{(n)} =$ Set of n-mosaics

$K^{(n)} =$ Subset of knot n-mosaics

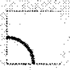
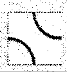
Planar Isotopy Moves

Non-Determistic Tiles

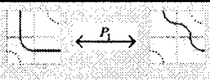
We use the following tile symbols to denote one of two possible tiles:

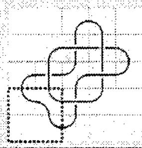


For example, the tile  denotes either

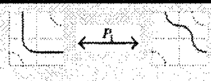
 or 

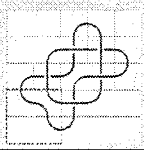
A Planar Isotopy (PI) Move on Mosaics



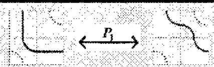


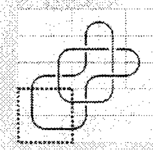
A Planar Isotopy (PI) Move on Mosaics



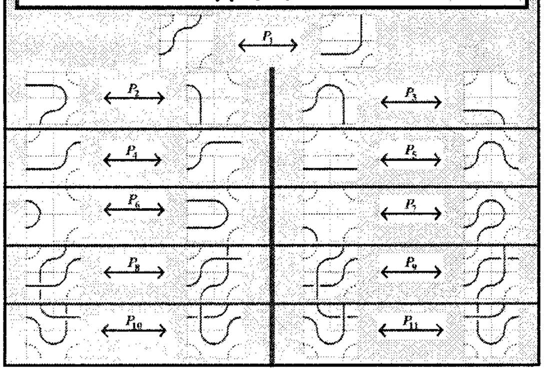


A Planar Isotopy (PI) Move on Mosaics



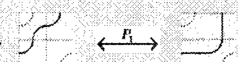


11 Planar Isotopy (PI) Moves on Mosaics

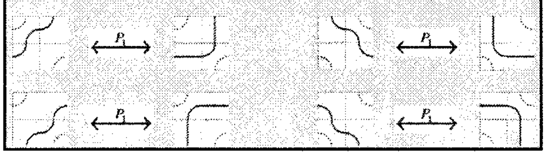


Planar Isotopy (PI) Moves on Mosaics

It is understood that each of the above moves depicts all moves obtained by rotating the 2×2 sub-mosaics by 0, 90, 180, or 270 degrees.

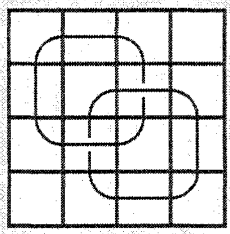
For example, 

represents each of the following 4 moves:



Planar Isotopy (PI) Moves on Mosaics

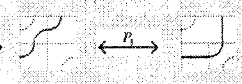
Each of the PI 2-submosaic moves represents any one of the $(n-2+1)^2$ possible moves on an n-mosaic



Terminology: k-Submosaic Moves

Def. A k-submosaic move on a mosaic M is a mosaic move that replaces one k-submosaic in M by another k-submosaic.

All of the PI moves are examples of 2-submosaic moves. I.e., each PI move replaces a 2-submosaic by another 2-submosaic

For example, 

Planar Isotopy (PI) Moves on Mosaics

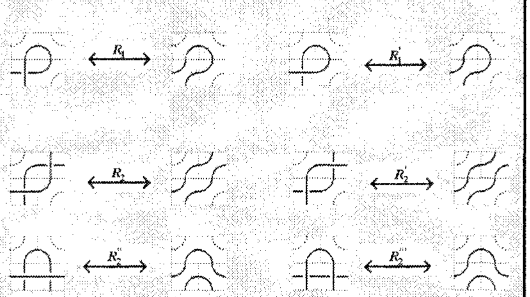
Each PI move acts as a local transformation on an n-mosaic whenever its conditions are met. If its conditions are not met, it acts as the identity transformation.

Ergo, each PI move is a permutation of the set of all knot n-mosaics $K^{(n)}$

In fact, each PI move, as a permutation, is a product of disjoint transpositions.

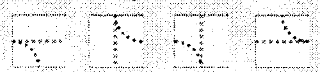
Reidemeister Moves

Reidemeister (R) Moves on Mosaics

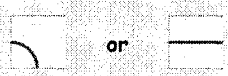


More Non-Deterministic Tiles

We also use the following tile symbols to denote one of two possible tiles:

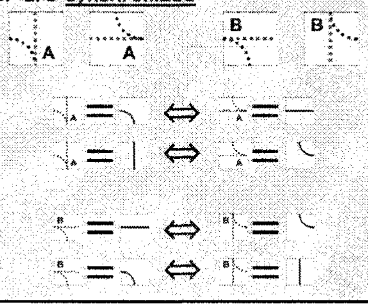


For example, the tile  denotes either

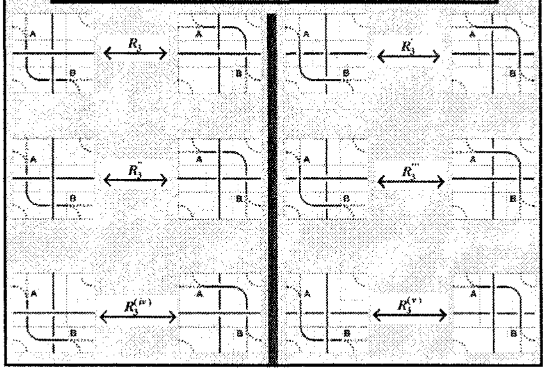


Synchronized Non-Determistic Tiles

Nondeterministic tiles labeled by the same letter are synchronized:



Reidemeister (R) Moves on Mosaics



Reidemeister (R) Moves on Mosaics

Just like each PI move, each R move is a permutation of the set of all knot n-mosaics $K^{(n)}$

In fact, each R move, as a permutation, is a product of disjoint transpositions.

The Ambient Group $A(n)$

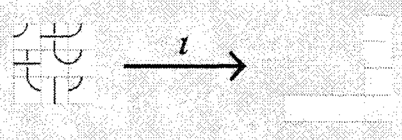
We define the ambient isotopy group $A(n)$ as the subgroup of the group of all permutations of the set $K^{(n)}$ generated by the all PI moves and all Reidemeister moves.

Knot Type

The Mosaic Injection $\iota : M^{(n)} \to M^{(n+1)}$

We define the mosaic injection $\iota : M^{(n)} \to M^{(n+1)}$

$$M^{(n+1)}_{i,j} = \begin{cases} M^{(n)}_{i,j} & \text{if } 0 \le i,j < n \\ \square & \text{otherwise} \end{cases}$$



Mosaic Knot Type

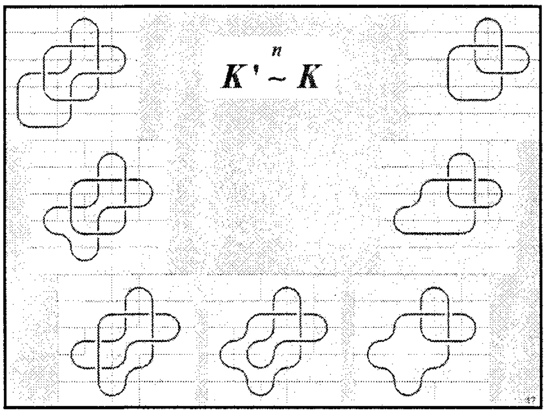
Def. Two n-mosaics M and M' are of the same knot n-type, written

$$M \overset{n}{\sim} M'$$

provided there exists an element of the ambient group $A(n)$ that transforms M into M' .

Two n-mosaics M and M' are of the same knot type if there exists a non-negative integer k such that

$$i^k M \overset{n+k}{\sim} i^k M'$$

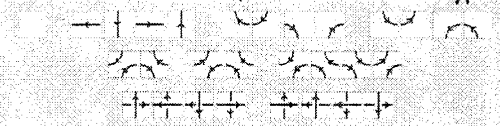


$$K' \overset{n}{\sim} K$$

Oriented Mosaics

Oriented Mosaics and Oriented Knot Type

In like manner, we can use the following oriented tiles to construct oriented mosaics, oriented mosaic knots, and oriented knot type



There are 29 oriented tiles, and 9 tiles up to rotation. Rotationally equivalent tiles have been grouped together.

Part 2

Quantum Knots & Quantum Knot Systems

The Hilbert Space $\mathcal{M}^{(n)}$ of n-mosaics

Let $\mathcal{H}$ be the 11 dimensional Hilbert space with orthonormal basis labeled by the tiles

T_0 T_1 T_2 T_3 T_4 T_5 T_6 T_7 T_8 T_9 T_{10}

We define the Hilbert space $\mathcal{M}^{(n)}$ of n-mosaics as

$$\mathcal{M}^{(n)} = \bigotimes_{k=0}^{n^2-1} \mathcal{H}$$

This is the Hilbert space with induced orthonormal basis

$$\left\{ \otimes_{k=0}^{n^2-1} \left| T_{\ell(k)} \right\rangle \; : \; 0 \le \ell(k) < 11 \right\}$$

The Hilbert Space $\mathcal{M}^{(n)}$ of n-mosaics

We identify each basis ket $\bigotimes_{k=0}^{n^2-1} |T_{\ell(k)}\rangle$ with a ket $|M\rangle$ labeled by an n-mosaic M using row major order.

For example, in the 3-mosaic Hilbert space $\mathcal{M}^{(3)}$, the basis ket

$$|T_2\rangle \otimes |T_5\rangle \otimes |T_4\rangle \;\otimes\; |T_9\rangle \otimes |T_2\rangle \otimes |T_1\rangle \;\otimes\; |T_5\rangle \otimes |T_8\rangle \otimes |T_3\rangle$$

is identified with the 3-mosaic labeled ket

$$\left| \begin{matrix} T_2 & T_5 & T_4 \\ T_9 & T_2 & T_1 \\ T_5 & T_8 & T_3 \end{matrix} \right\rangle$$

Identification via Row Major Order

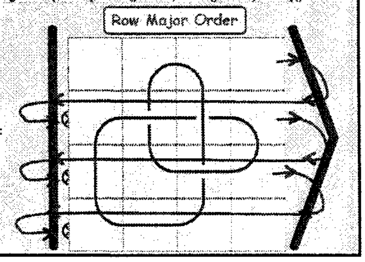
Let $\mathcal{H}$ be the 11 dimensional Hilbert space with orthonormal basis labeled by the tiles

$T_0 \; T_1 \; T_2 \; T_3 \; T_4 \; T_5 \; T_6 \; T_7 \; T_8 \; T_9 \; T_{10}$

Construct Mosaic Space

$$\mathcal{H}^{\otimes n^2} = \bigotimes_{0\le i,j<n} |T_{k(i,j)}\rangle =$$

Select Basis Element



The Hilbert Space $\mathcal{K}^{(n)}$ of Quantum Knots

The Hilbert space $\mathcal{K}^{(n)}$ of quantum knots is defined as the sub-Hilbert space of $\mathcal{M}^{(n)}$ spanned by all orthonormal basis elements labeled by knot n-mosaics.

Quantum Knots

We define the Hilbert space $\mathcal{M}^{(n)}$ of n-mosaics as

$$\mathcal{M}^{(n)} = \bigotimes_{k=0}^{n^2-1} \mathcal{H}$$

This is the Hilbert space with induced orthonormal basis

$$\left\{ \bigotimes_{k=0}^{n^2-1} |T_{\ell(k)}\rangle \;:\; 0 \le \ell < n^2 \right\}$$

We identify each basis element $\bigotimes_{k=0}^{n^2-1} |T_{\ell(k)}\rangle$ with the mosaic labeled ket $|M\rangle$ via the bijection

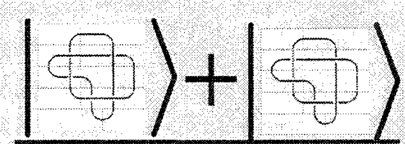
$$T_\ell \leftrightarrow M_{i,j}$$

Row major order

where $\begin{cases} i = \lfloor \ell / n \rfloor \\ j = \ell - n\lfloor \ell / n \rfloor \end{cases}$ and $\ell = ni + j$

An Example of a Quantum Knot

$|K\rangle =$



$\sqrt{2}$

The Ambient Group $A(n)$ as a Unitary Group

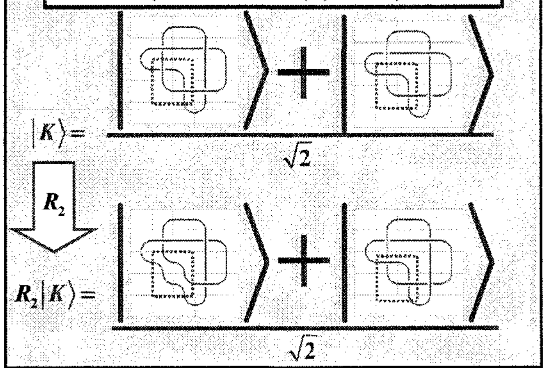
We identify each element $g \in A(n)$ with the linear transformation defined by

$$\begin{aligned} \mathcal{K}^{(n)} &\rightarrow \mathcal{K}^{(n)} \\ |K\rangle &\mapsto |gK\rangle \end{aligned}$$

Since each element $g \in A(n)$ is a permutation, it is a linear transformation that simply permutes basis elements.

Hence, under this identification, the ambient group $A(n)$ becomes a discrete group of unitary transfs on the Hilbert space $\mathcal{K}^{(n)}$.

An Example of the $A(n)$ Group Action



The Quantum Knot System $\left(\mathcal{K}^{(n)}, A(n)\right)$

Def. A quantum knot system $\left(\mathcal{K}^{(n)}, A(n)\right)$ is a quantum system having $\mathcal{K}^{(n)}$ as its state space, and having the Ambient group $A(n)$ as its set of accessible unitary transformations.

The states of quantum system $\left(\mathcal{K}^{(n)}, A(n)\right)$ are quantum knots. The elements of the ambient group $A(n)$ are quantum moves.

$$\left(\mathcal{K}^{(1)}, A(1)\right) \xrightarrow{\iota} \cdots \xrightarrow{\iota} \left(\mathcal{K}^{(n)}, A(n)\right) \xrightarrow{\iota} \left(\mathcal{K}^{(n+1)}, A(n+1)\right) \xrightarrow{\iota} \cdots$$

Physically Implementable | Physically Implementable | Physically Implementable

The Quantum Knot System $\left(\mathcal{K}^{(n)}, A(n)\right)$

$$\left(\mathcal{K}^{(1)}, A(1)\right) \xrightarrow{\iota} \cdots \xrightarrow{\iota} \left(\mathcal{K}^{(n)}, A(n)\right) \xrightarrow{\iota} \left(\mathcal{K}^{(n+1)}, A(n+1)\right) \xrightarrow{\iota} \cdots$$

Physically Implementable | Physically Implementable | Physically Implementable

Choosing an integer n is analogous to choosing a length of rope. The longer the rope, the more knots that can be tied.

The parameters of the ambient group $A(n)$ are the "knobs" one turns to spacially manipulate the quantum knot.

Quantum Knot Type

Def. Two quantum knots $|K_1\rangle$ and $|K_2\rangle$ are of the same knot n-type, written

$$|K_1\rangle \sim_n |K_2\rangle,$$

provided there is an element $g \in A(n)$ s.t.

$$g|K_1\rangle = |K_2\rangle$$

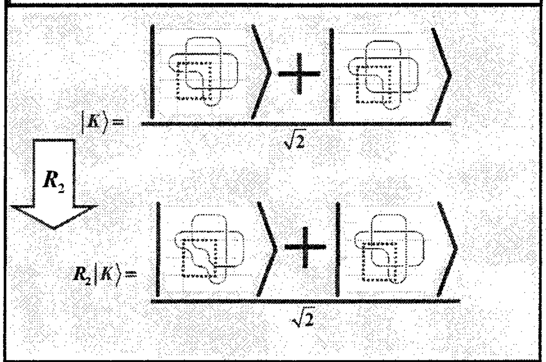
They are of the same knot type, written

$$|K_1\rangle \sim |K_2\rangle,$$

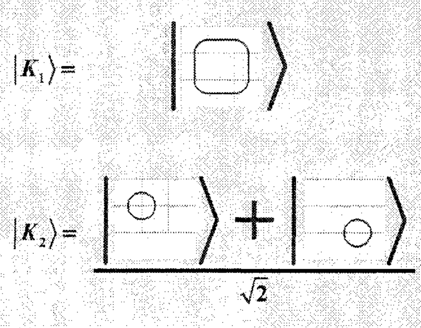
provided there is an integer $m \geq 0$ such that

$$\iota^m|K_1\rangle \sim_{n+m} \iota^m|K_2\rangle$$

Two Quantum Knots of the Same Knot Type



Two Quantum Knots NOT of the Same Knot Type



Hamiltonians
of the
Generators
of the
Ambient Group

Hamiltonians for $A(n)$

Each generator $g \in A(n)$ is the product of disjoint transpositions, i.e.,

$$g = \left(K_{\alpha_1}, K_{\beta_1}\right)\left(K_{\alpha_2}, K_{\beta_2}\right)\cdots\left(K_{\alpha_\ell}, K_{\beta_\ell}\right)$$

Choose a permutation η so that

$$\eta^{-1} g \eta = \left(K_1, K_2\right)\left(K_3, K_3\right)\cdots\left(K_{\ell-1}, K_\ell\right)$$

Hence,

$$\eta^{-1} g \eta = \begin{pmatrix} \sigma_1 & & & & \\ & \sigma_1 & & \mathbf{0} & \\ & & \ddots & & \\ & \mathbf{0} & & \sigma_1 & \\ & & & & I_{n-2\ell} \end{pmatrix}$$

, where $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Hamiltonians for $A(n)$

Also, let $\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and note that

$$\ln\left(\sigma_1\right) = \frac{i\pi}{2}\left(2s+1\right)\left(\sigma_0 - \sigma_1\right), \quad s \in \mathbb{Z}$$

For simplicity, we always choose the branch $s = 0$.

$$H_g = -i\eta \ln\left(\eta^{-1} g \eta\right)\eta^{-1}$$

$$= \frac{\pi}{2}\eta \begin{pmatrix} I_\ell \otimes \left(\sigma_0 - \sigma_1\right) & 0 \\ 0 & 0_{(n-2\ell)\times(n-2\ell)} \end{pmatrix} \eta^{-1}$$

Log of a matrix

The Log of a Unitary Matrix

Let U be an arbitrary finite rxr unitary matrix.

Then eigenvalues of U all lie on the unit circle in the complex plane.

Moreover, there exists a unitary matrix W which diagonalizes U, i.e., there exists a unitary matrix W such that

$$WUW^{-1} = \Delta\left(e^{i\theta_1}, e^{i\theta_2}, \ldots, e^{i\theta_r}\right)$$

where $e^{i\theta_1}, e^{i\theta_2}, \ldots, e^{i\theta_r}$ are the eigenvalues of U.

The Log of a Unitary Matrix

Then

$$\ln\left(U\right) = W^{-1}\Delta\left(\ln(e^{i\theta_1}), \ln(e^{i\theta_2}), \ldots, \ln(e^{i\theta_r})\right)W$$

Since $\ln\left(e^{i\theta_j}\right) = i\theta_j + 2\pi i n_j$, where $n_j \in \mathbb{Z}$ is an arbitrary integer, we have

$$\ln\left(U\right) = iW^{-1}\Delta\left(\theta_1 + 2\pi n_1, \theta_2 + 2\pi n_2, \ldots, \theta_r + 2\pi n_r\right)W$$

where $n_1, n_2, \ldots, n_r \in \mathbb{Z}$

The Log of a Unitary Matrix

Since $e^A = \sum_{m=0}^{\infty} A^m / (m!)$, we have

$$e^{\ln(U)} = e^{W^{-1}\Delta(\ln i\theta_1, \ldots, \ln i\theta_r)W}$$

$$= W^{-1} e^{\Delta(\ln i\theta_1, \ldots, \ln i\theta_r)} W$$

$$= W^{-1}\Delta\left(e^{\ln i\theta_1}, \ldots, e^{\ln i\theta_r}\right)W$$

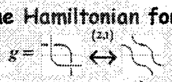
$$= W^{-1}\Delta\left(e^{i\theta_1 + 2\pi i n_1}, \ldots, e^{i\theta_r + 2\pi i n_r}\right)W$$

$$= W^{-1}\Delta\left(e^{i\theta_1}, \ldots, e^{i\theta_r}\right)W = U$$

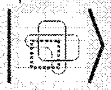
Back

Hamiltonians for $A(n)$

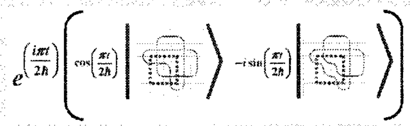
Using the Hamiltonian for the Reidemeister 2 move



and the initial state



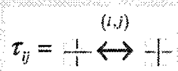
we have that the solution to Schroedinger's equation for time t is



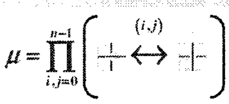
Some Miscellaneous Unitary Transformations Not in $A(n)$

Misc. Transformations

The crossing tunneling transformation

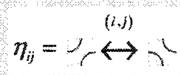


The mirror image transformation

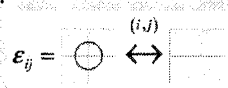


Misc. Transformations

The hyperbolic transformation



The elliptic transformation



Observables which are Quantum Knot Invariants

Observable Q. Knot Invariants

<u>Question</u>. What do we mean by a physically observable knot invariant ?

Let $(\mathcal{K}^{(n)}, A(n))$ be a quantum knot system. Then a quantum observable Ω is a Hermitian operator on the Hilbert space $\mathcal{K}^{(n)}$.

Observable Q. Knot Invariants

Question. But which observables Ω are actually knot invariants ?

Def. An observable Ω is an invariant of quantum knots provided $U\Omega U^{-1} = \Omega$ for all $U \in A(n)$

Observable Q. Knot Invariants

Question. But how do we find quantum knot invariant observables ?

Theorem. Let $\left(\mathcal{K}^{(n)}, A(n)\right)$ be a quantum knot system, and let

$$\mathcal{K}^{(n)} = \bigoplus_{\ell} W_{\ell}$$

be a decomposition of the representation

$$A(n) \times \mathcal{K}^{(n)} \to \mathcal{K}^{(n)}$$

into irreducible representations .

Then, for each ℓ , the projection operator P_{ℓ} for the subspace W_{ℓ} is a quantum knot observable.

Observable Q. Knot Invariants

Theorem. Let $\left(\mathcal{K}^{(n)}, A(n)\right)$ be a quantum knot system, and let Ω be an observable on $\mathcal{K}^{(n)}$. Let $St(\Omega)$ be the stabilizer subgroup for Ω , i.e.,

$$St(\Omega) = \left\{ U \in A(n) \ : \ U\Omega U^{-1} = \Omega \right\}$$

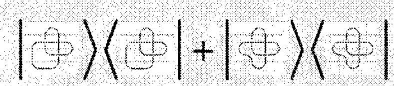
Then the observable

$$\sum_{U \in A(n)/St(\Omega)} U\Omega U^{-1}$$

is a quantum knot invariant, where the above sum is over a complete set of coset representatives of $St(\Omega)$ in $A(n)$.

Observable Q. Knot Invariants

The following is an example of a quantum knot invariant observable:

$\Omega =$ 

Future Directions & Open Questions

Future Directions & Open Questions

- What is the structure of the ambient group $A(n)$ and its direct limit $A = \varinjlim A(n)$? Can one find a presentation of this group ? Is $A(n)$ a Coxeter group?
- Unlike classical knots, quantum knots can exhibit the non-classical behavior of quantum superposition and quantum entanglement. Are quantum and topological entanglement related to one another ? If so, how ?

Future Directions & Open Questions

- How does one find a quantum observable for the Jones polynomial ? This would be a family of observables parameterized by points on the circle in the complex plane. Does this approach lead to an algorithmic improvement to the quantum algorithm created by Aharonov, Jones, and Landau ?

- How does one create quantum knot observables that represent other knot invariants such as, for example, the Vassiliev invariants ?

Future Directions & Open Questions

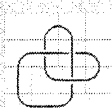
- What is gained by extenting the definition of quantum knot observables to POVMs ?

- What is gained by extending the definition of quantum knot observables to mixed ensembles ?

Future Directions & Open Questions

Def. We define the mosaic number of a knot k as the smallest integer n for which k is representable as a knot n-mosaic.

- The mosaic number of the trefoil is 4. In general, how does one compute the mosaic number of a knot? How does one find a quantum observable for the mosaic number?



- Is the mosaic number related to the crossing number of a knot?

Future Directions & Open Questions

Quantum Knot Tomography: Given many copies of the same quantum knot, find the most efficient set of measurements that will determine the quantum knot to a chosen tolerance $\varepsilon > 0$.

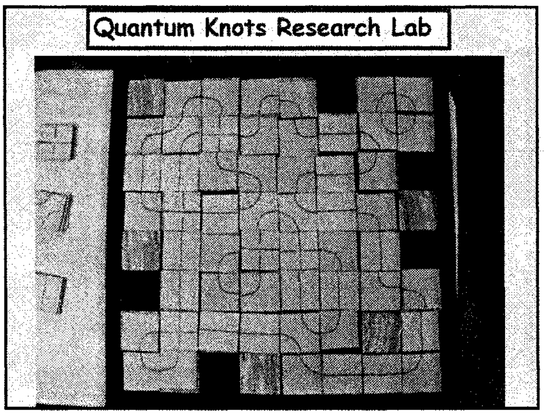
Quantum Braids: Use mosaics to define quantum braids. How are such quantum braids related to the work of Freedman, Kitaev, et al on anyons and topological quantum computing?

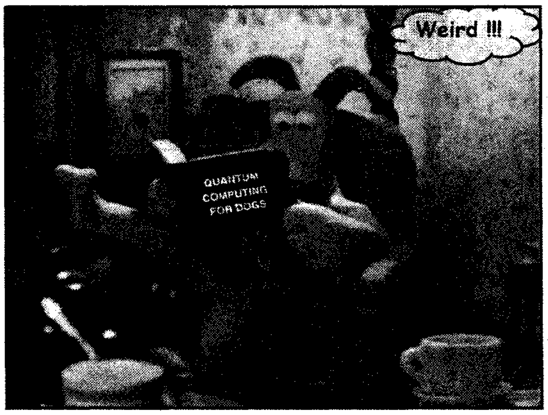
Future Directions & Open Questions

- Can quantum knot systems be used to model and predict the behavior of
  - Quantum vortices in supercooled helium 2 ?
  - Quantum vortices in the Bose-Einstein Condensate
  - Fractional charge quantification that is manifest in the fractional quantum Hall effect

UMBC Quantum Knots Research Lab

We at UMBC are very proud of our new state of the art Quantum Knots Research Laboratory.


Quantum Knots Research Lab


Weird !!!
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FOR DOGS