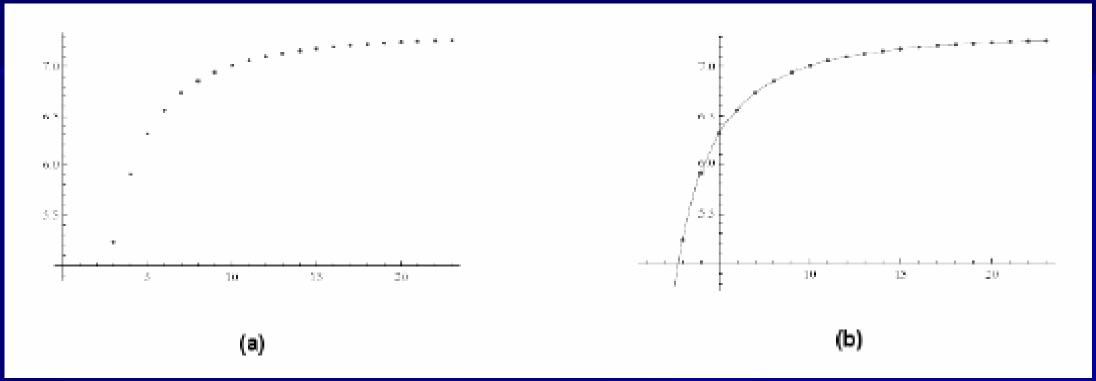
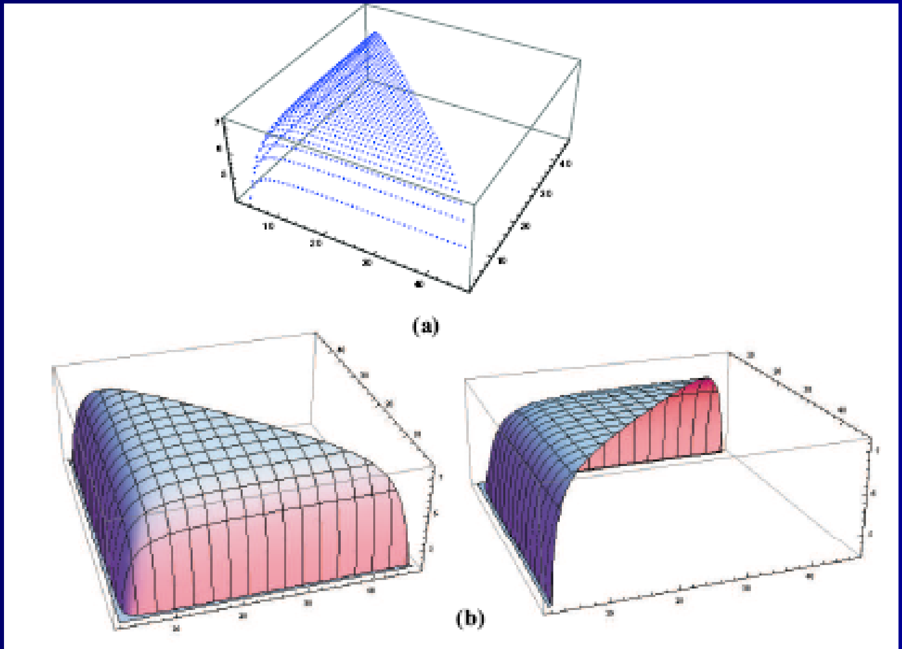
Knots, computers, conjectures

Slavik Jablan

Hyperbolic volumes

Family *p q*

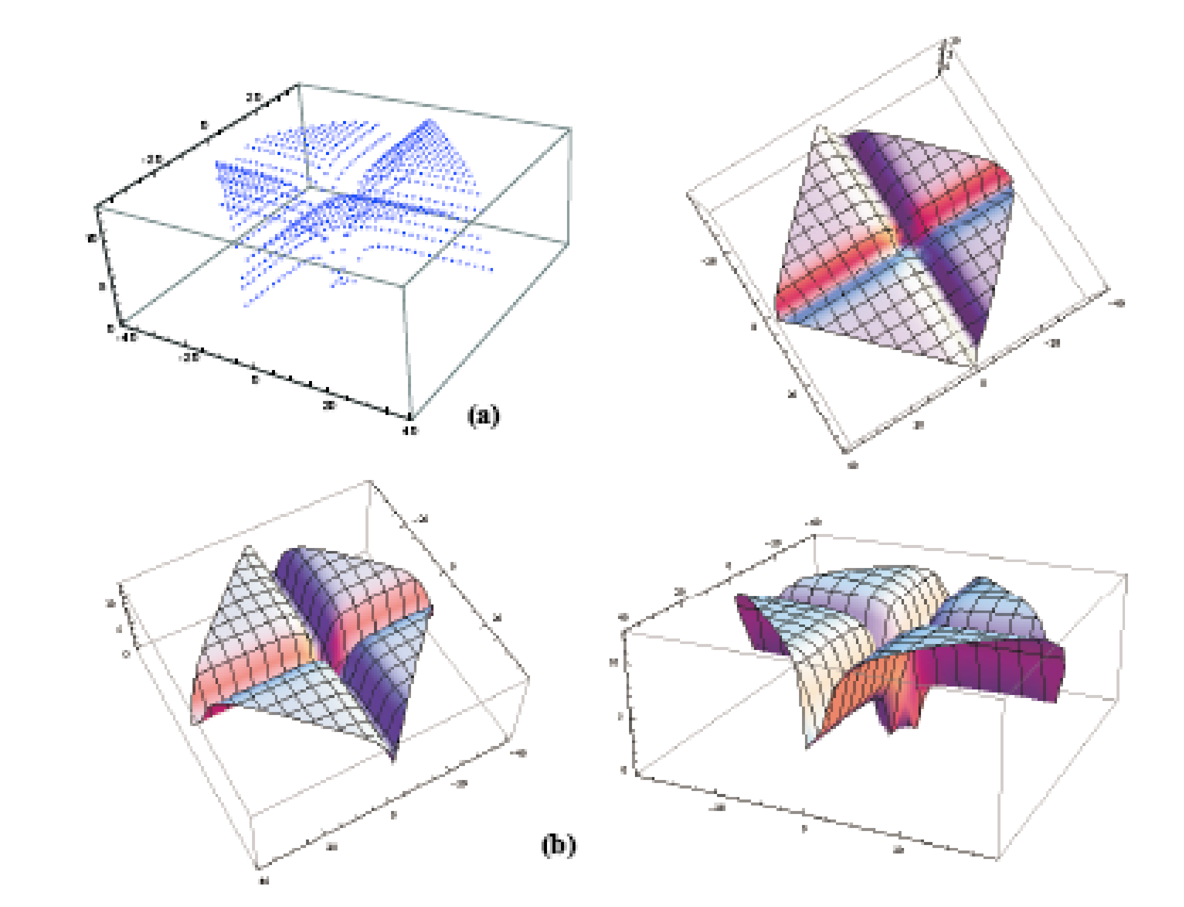




(joint work with Lj. Radovic)

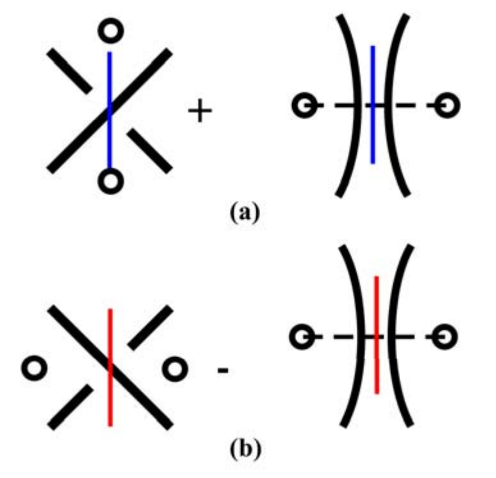
Hyperbolic volumes

Family of
Lorenz knots
6*-(2*p*+1).(2*q*).-2.2.-2



Adequacy: markers and state diagrams

In every crossing of a diagram D we can introduce $+$ or $-$ marker (Fig. 1). The state diagram obtained from D by introducing in all vertices $+$ or $-$ markers will be denoted by $s_+(D)$, and $s_-(D)$, respectively. The circles obtained in $s_+(D)$, and $s_-(D)$ are called *state circles*.

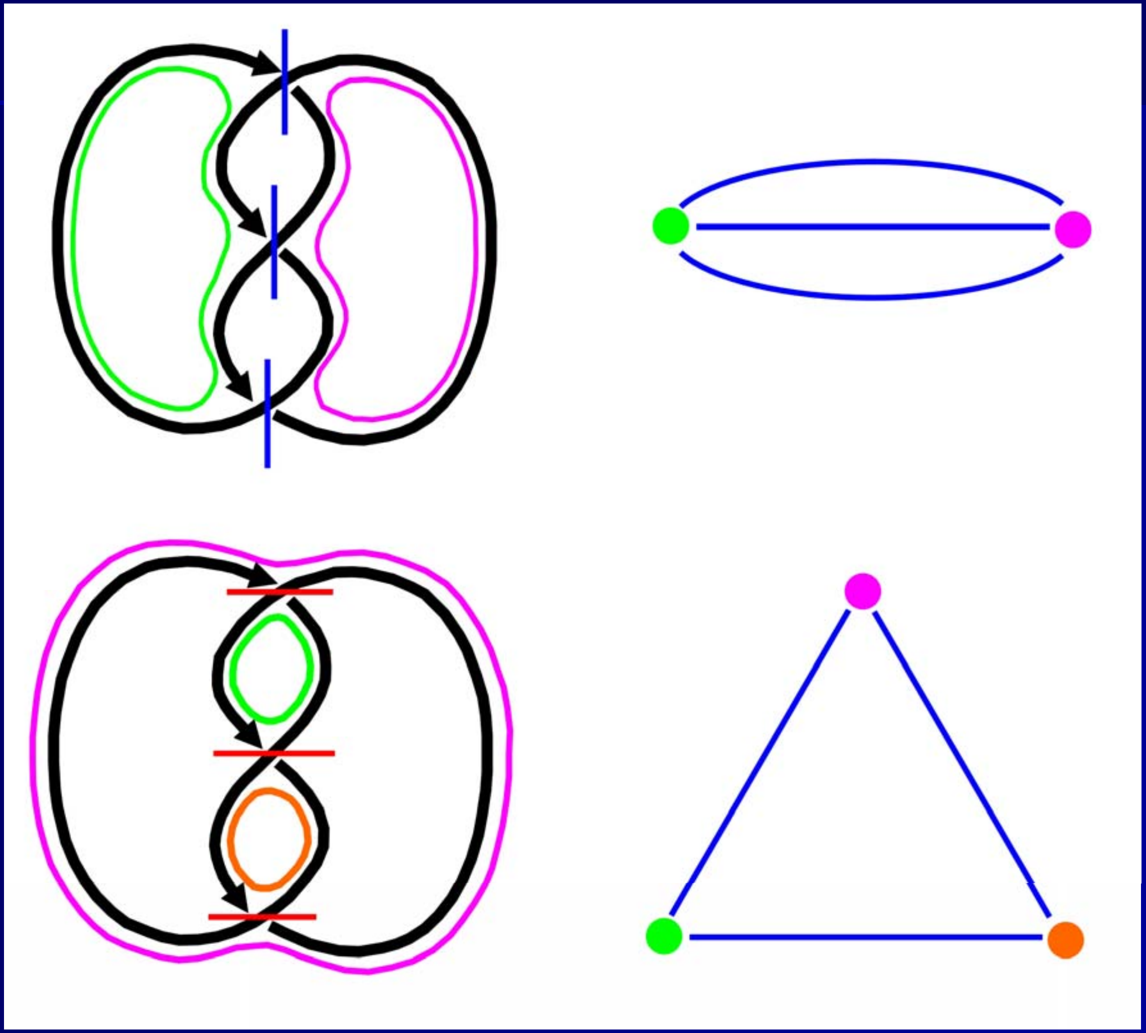


(a) +marker; (b) -marker. The broken lines represent the edges of the associated graph *G_s* connecting state circles represented by dots.

State circles **State graph**

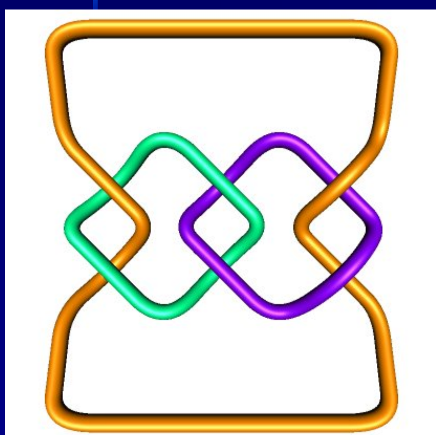
S_+

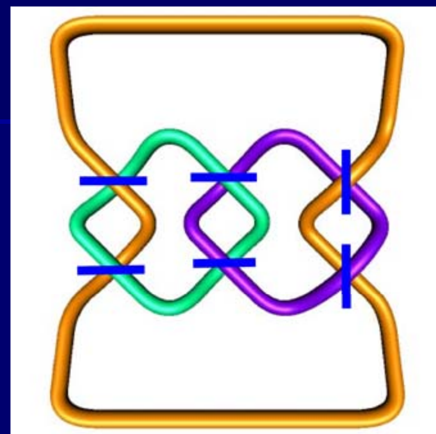
S_-

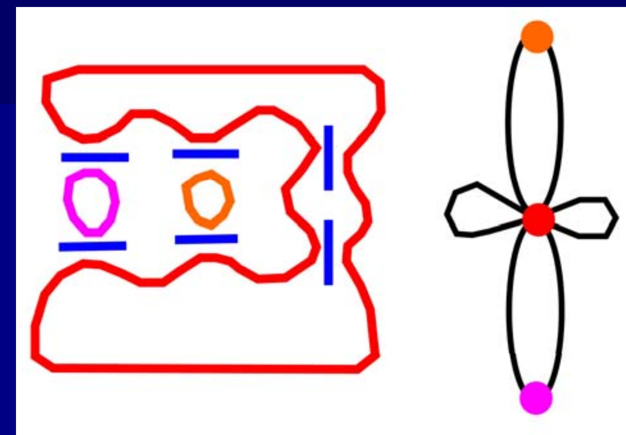


Definition 1. *A diagram D is $+$adequate if, at each crossing, the two segments of $s_+(D)$ that replace the crossing are in different state circles. Similarly, D is $-$adequate if the segments of $s_-(D)$ are in different circles. If a diagram is neither $+$adequate nor $-$adequate it is called* $inadequate$. *If a diagram is both $+$adequate and $-$adequate, it is called* $adequate$, *and if it is only $+$adequate or $-$adequate, it is called* $semi$-$adequate$.

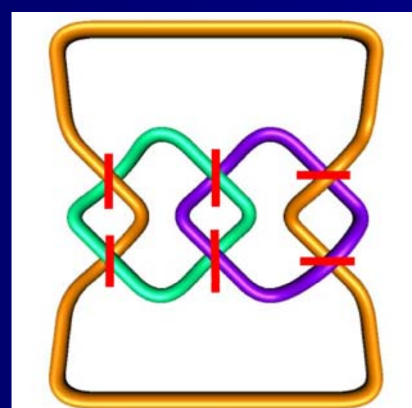
Semiadequate link 2,2,-2

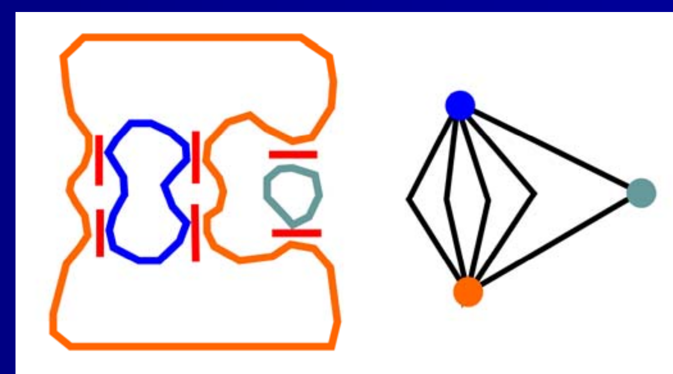






s_+ inadequate





s_- adequate

We can also consider the other states different from $s_+(D)$, and $s_-(D)$, which have markers of different signs. To every state we of a diagram D we can associate the graph $G_s(D)$, whose vertices are state circles of $s(D)$, and edges are lines connecting state circles via smoothed crossings in D. The graph $G_s(D)$ is $adequate$ if it has no loops.

Definition 2. *A link is $adequate$ if it has an adequate (+adequate and −adequate) diagram. A link is $semi$-$adequate$ if it has a + or −adequate diagram. A link is $inadequate$ if it is neither + or −adequate.*

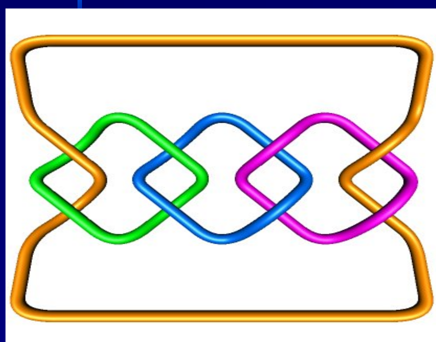
- Theorem: Every alternating diagram without nugatory crossings is adequate (Thistlethwaite, Lickorish, 1988).
- Hence, alll alternating links are adequate

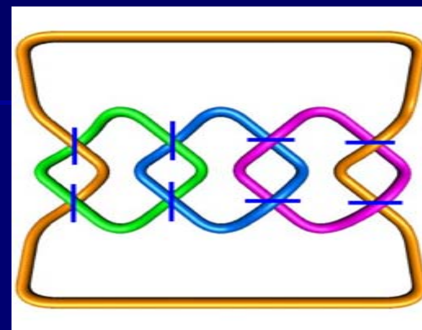
- Theorem: An adequate diagram has minimal crossing number (Lickorish, Thistlethwaite, 1988).

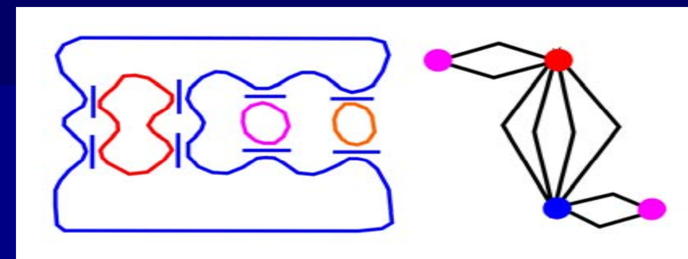
- This theorem can be used to prove minimality of some
- non-alternating link diagrams.

- Problem: recognition of semi-adequate links.

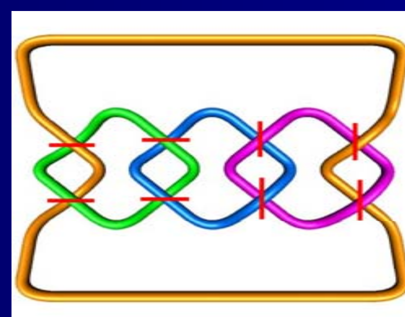
First adequate non-alternating link: 2,2,-2,-2

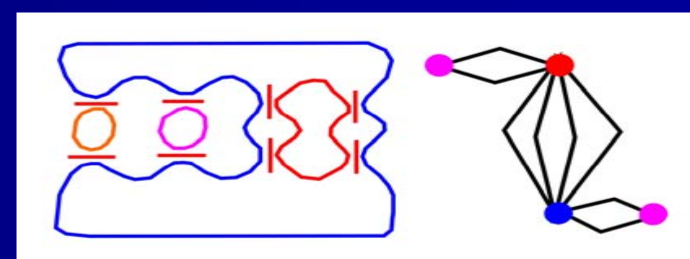




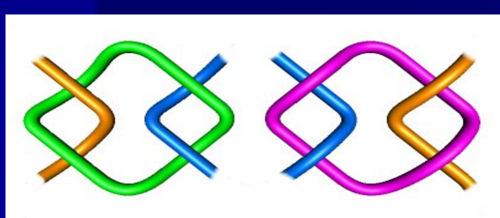


s_+ adequate



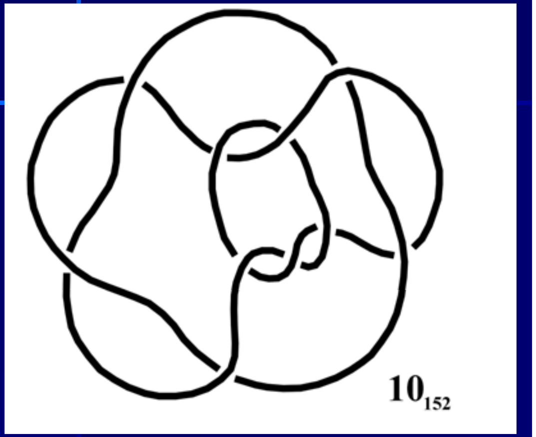


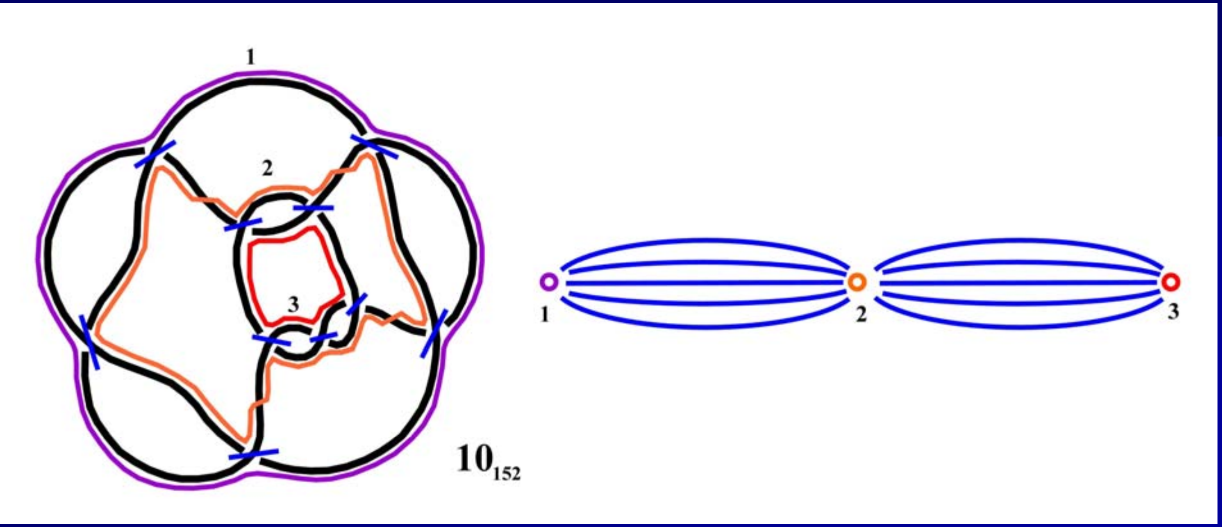
s_- adequate

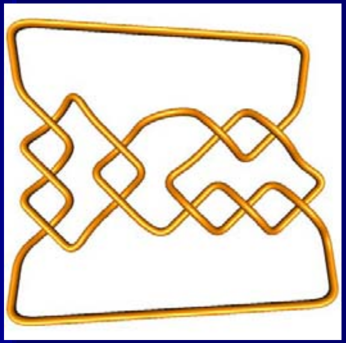


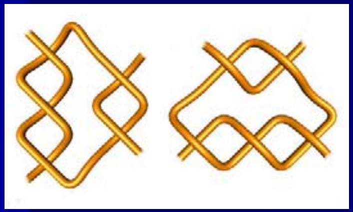
Non-alternating sum of strongly alternating tangles

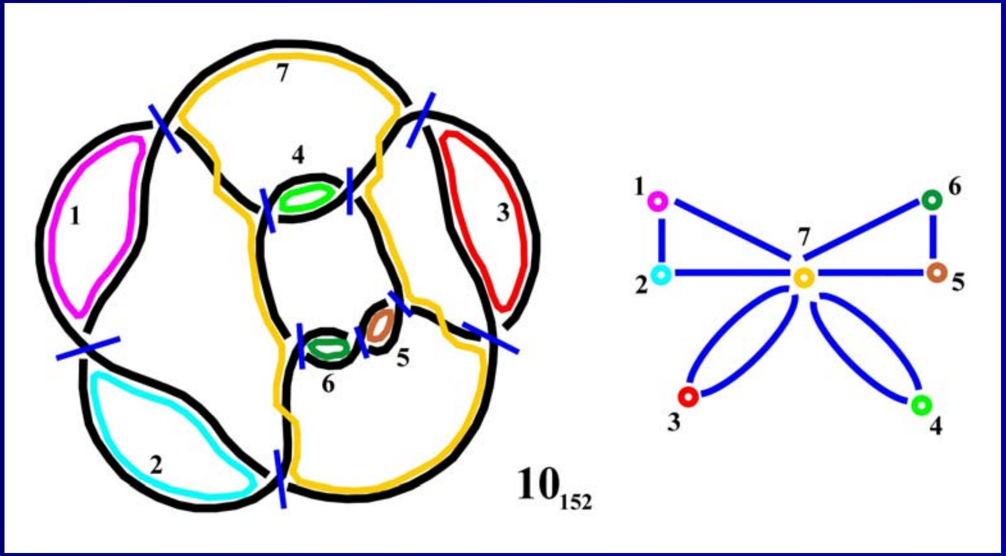
First non-alternating adequate knot 10_{152} (3,2) –(3,2)











Non-alternating sum of strongly alternating tangles

Minimality and adequacy

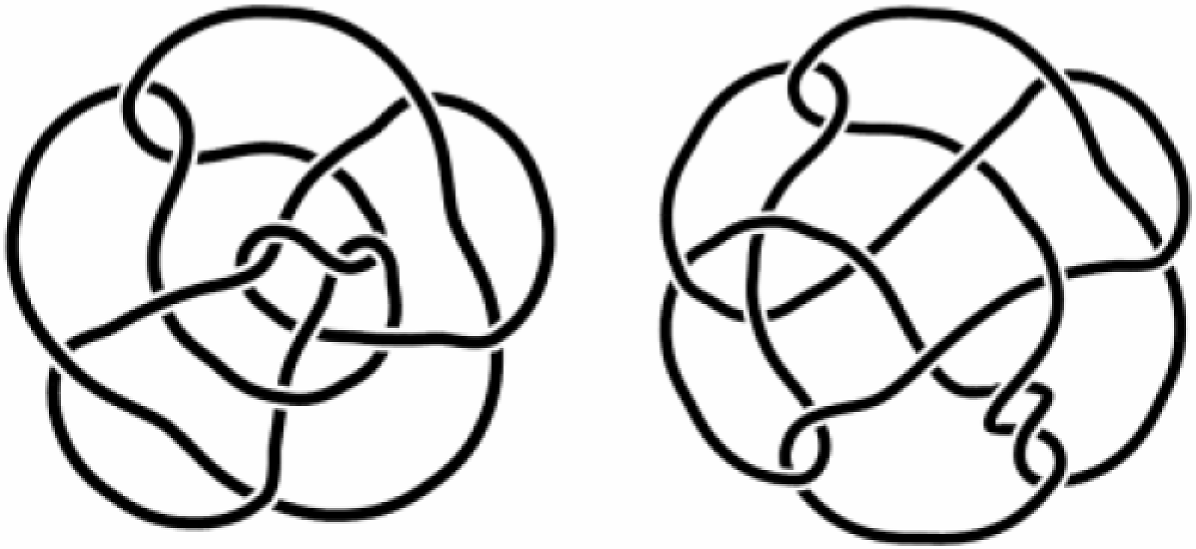
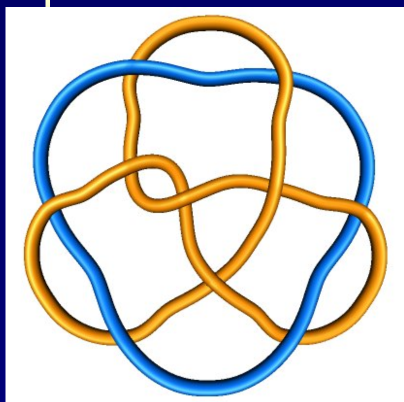
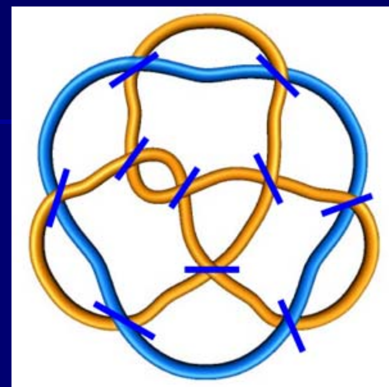
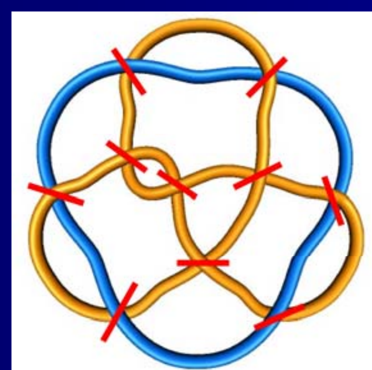


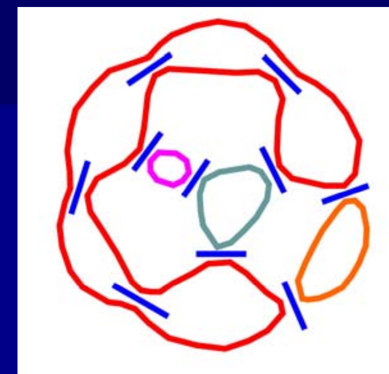
Figure 3: Semi-adequate knot $15n_{164563}$ which has only minimal diagram $10^{**}-1.-2\,0.2\,0::.2\,0.2\,0.-2\,0$ which is inadequate and non-minimal 16-crossing diagram $11^{*}2\,0.-1.-2.-1.3\,0.-1.2\,0::-1$ which is semi-adequate [Stoi3].

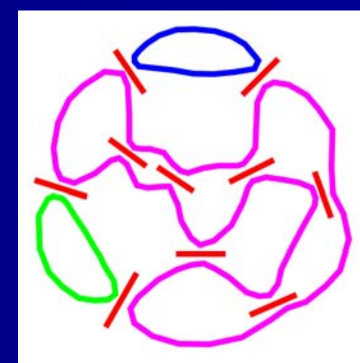
Inadequate link 2.-2 0.-2.2 0





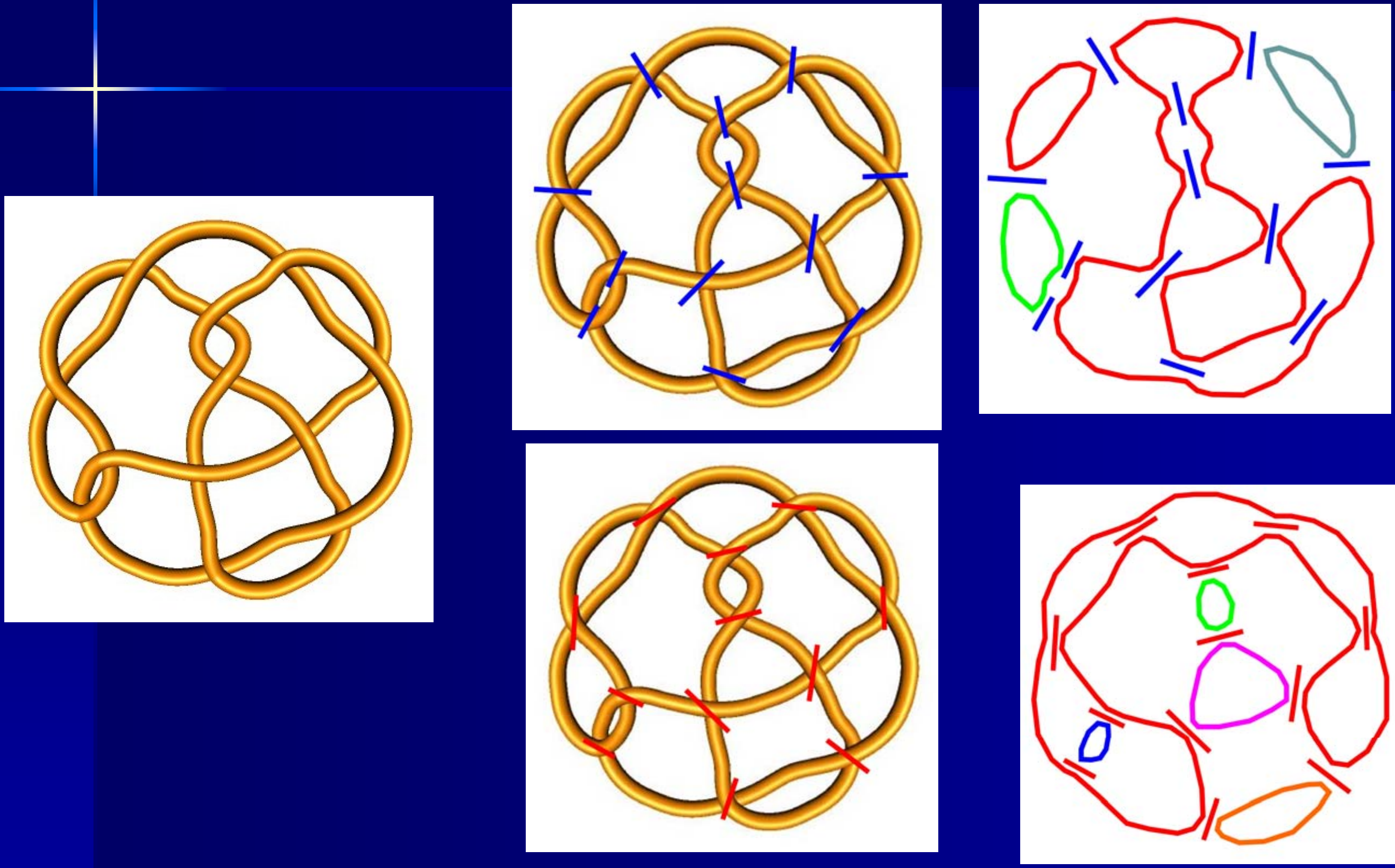






It is inadequate since its first and last coefficient of Jones polynomial is 2.

Inadequate knot 2 0.-2 1.-2 0.2



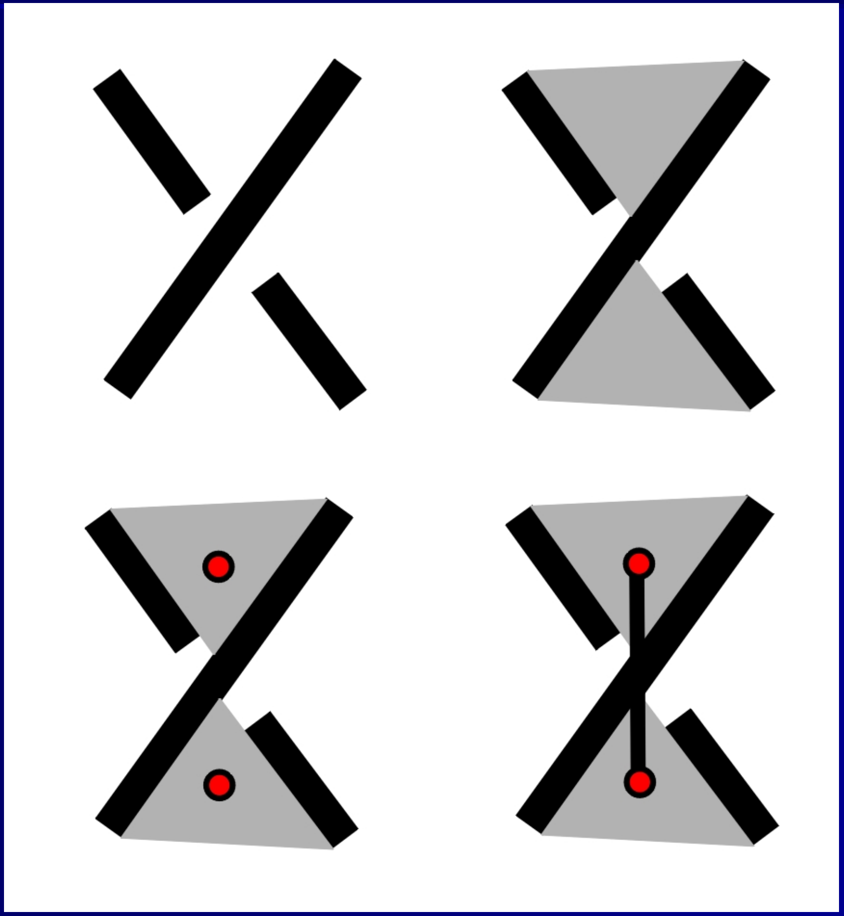
It is inadequate since its first and last coefficient of Jones polynomial is 2.

Lemma 1. *All minimal diagrams of the same alternating link have the same number of adequate states.*

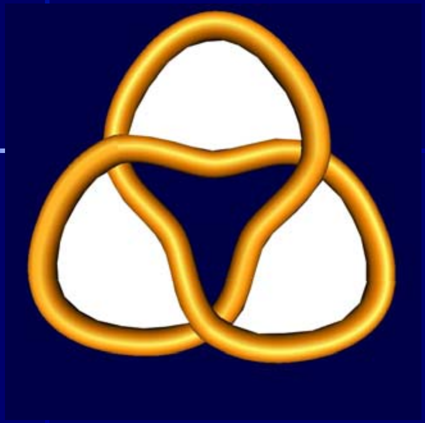
Lemma 2. *The number of adequate states* $a(L)$ *is the invariant of a family of alternating links* L *and it is realized in any minimal diagram of the link family.*

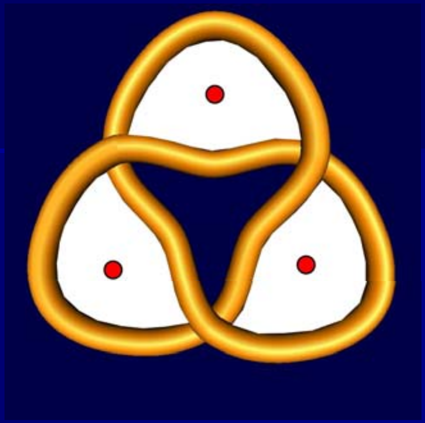
S. Jablan: Adequacy of Link Families (2008)

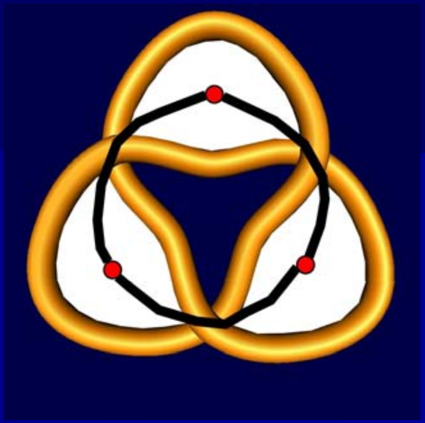
Knots and Graphs:
Tait (checkerboard) coloring



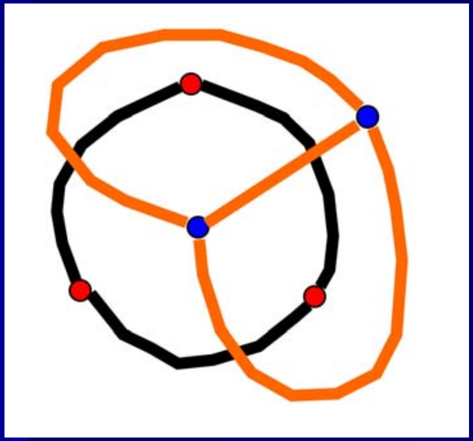
Trefoil



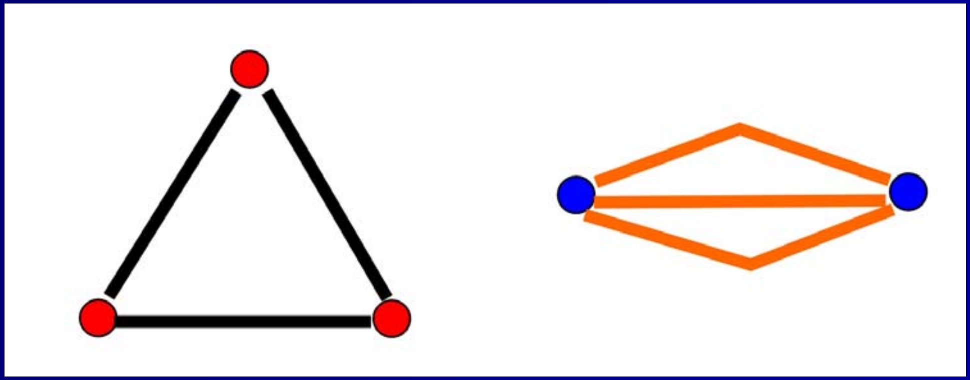




Graph of the trefoil

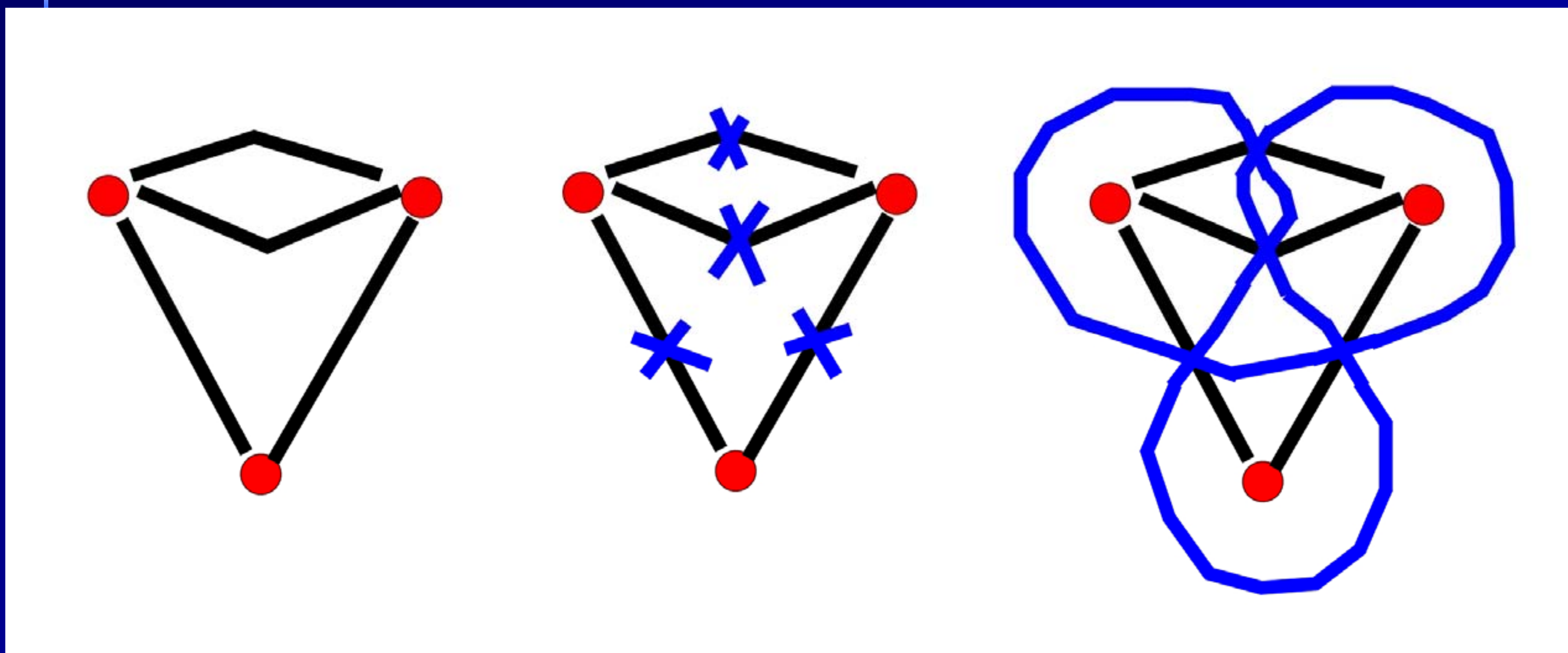


Dual graph



Non-isomorphic dual graphs

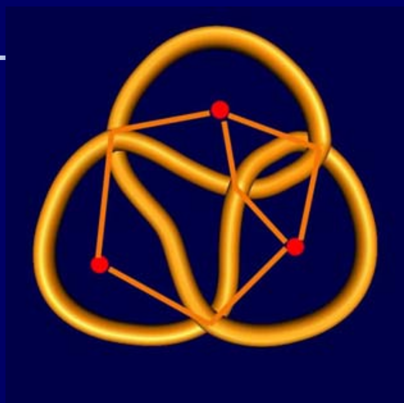
Link from graph- middle graph



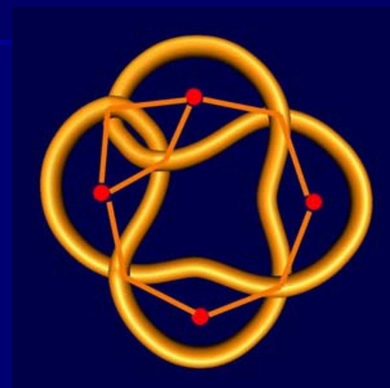
Graph of 2 2

Middle graph = 2 2

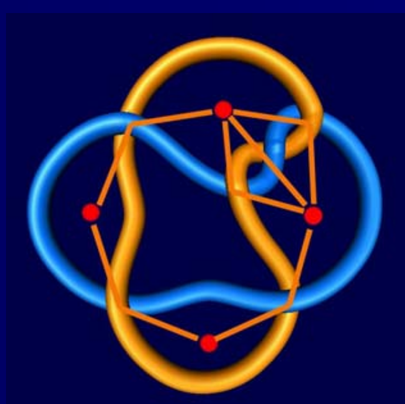
Familes of alternating links and families of graphs



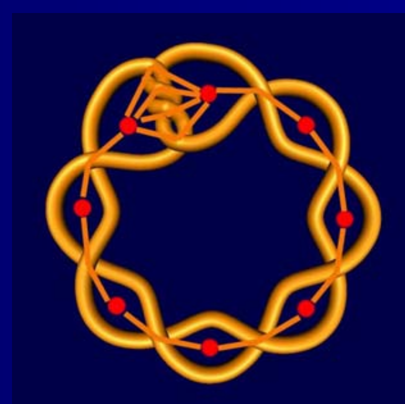
2 2



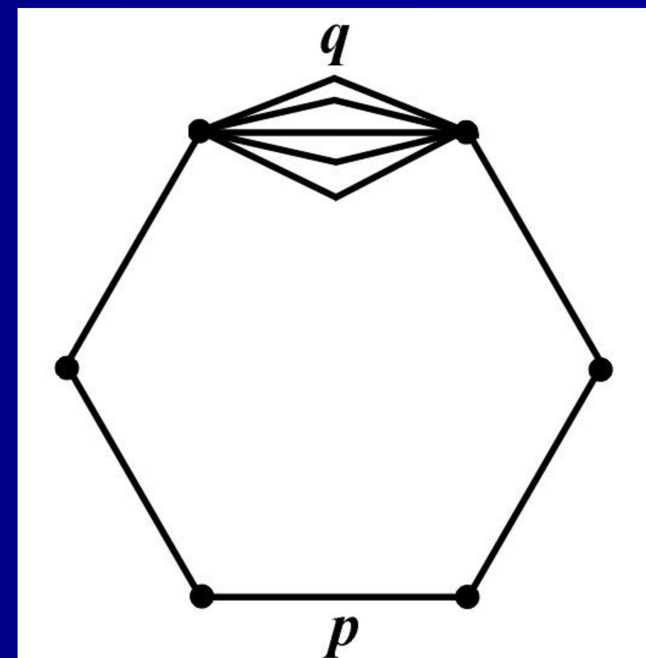
3 2



3 3



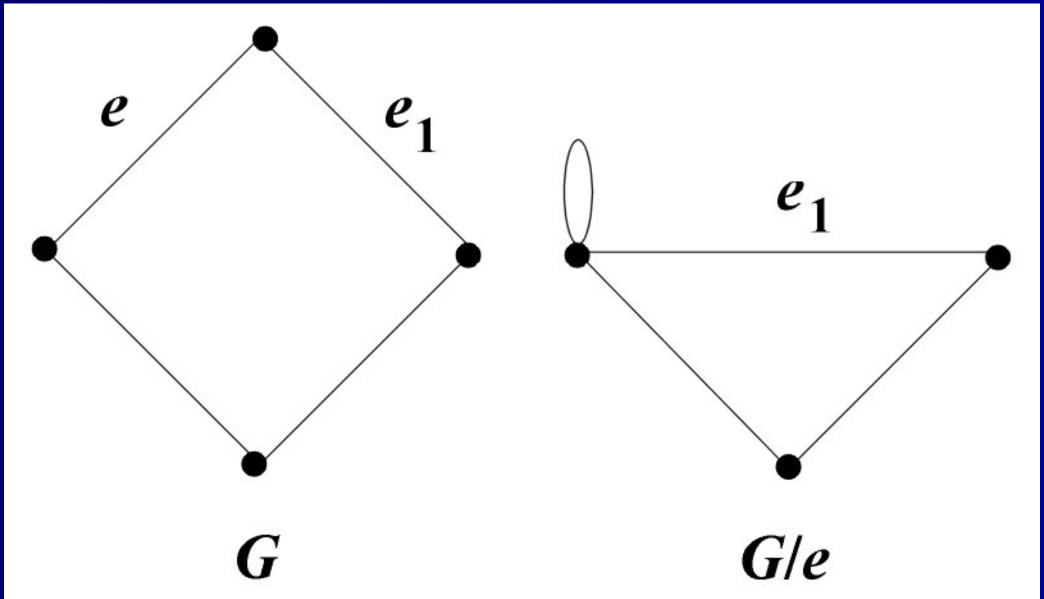
7 4



p q

Tutte polynomial

Two operations are essential to understanding the Tutte polynomial definition. These are: *edge deletion* denoted by $G - e$, and *edge contraction* G/e. The latter involves first deleting e, and then merging its endpoints as follows:



Tutte polynomial

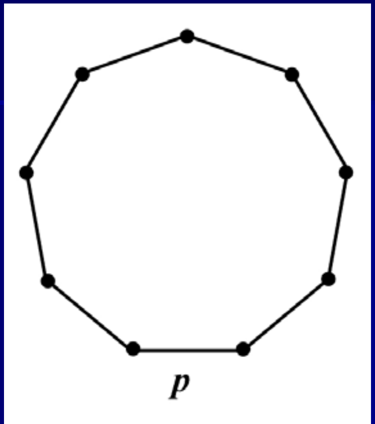
Definition The *Tutte polynomial* of a graph $G(V,E)$ is a two-variable polynomial defined as follows:

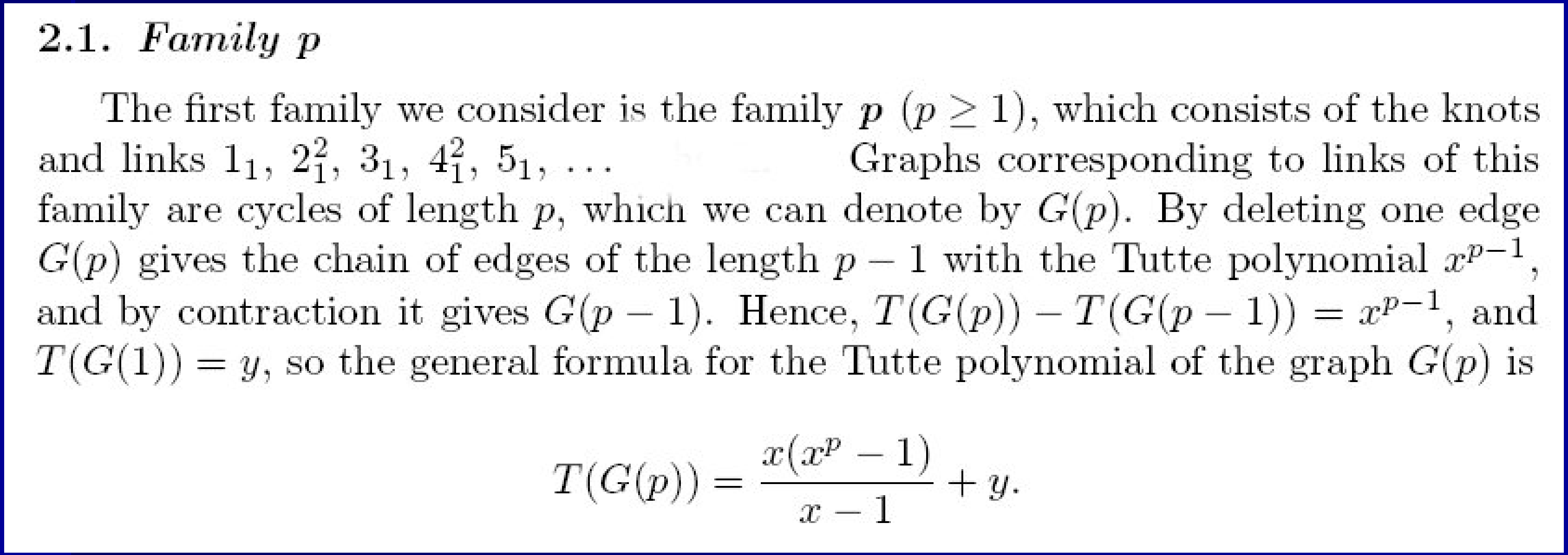
$$T(G) = \begin{cases} 1 & E(\emptyset) & (1) \\ xT(G/e) & e \in E \text{ and } e \text{ is a bridge} & (2) \\ yT(G-e) & e \in E \text{ and } e \text{ is a loop} & (3) \\ T(G-e) + T(G/e) & e \in E \text{ and } e \text{ is neither a loop or a bridge} & (4) \end{cases}$$

The definition of a Tutte polynomial outlines a simple recursive procedure for computing it, but the order in which rules are applied is not fixed.

Rem: An edge of a connected graph is a bridge iff it does not lie on any cycle.

Tutte polynomial- general formula





2.1. ***Family p***

The first family we consider is the family $\boldsymbol{p}$ ($p \geq 1$), which consists of the knots and links 1_1, 2_1^2, 3_1, 4_1^2, 5_1, ... Graphs corresponding to links of this family are cycles of length p, which we can denote by $G(p)$. By deleting one edge $G(p)$ gives the chain of edges of the length $p-1$ with the Tutte polynomial x^{p-1}, and by contraction it gives $G(p-1)$. Hence, $T(G(p)) - T(G(p-1)) = x^{p-1}$, and $T(G(1)) = y$, so the general formula for the Tutte polynomial of the graph $G(p)$ is

$$T(G(p)) = \frac{x(x^p - 1)}{x - 1} + y.$$

Tutte polynomial- general formula

2.2. ***Family p q***

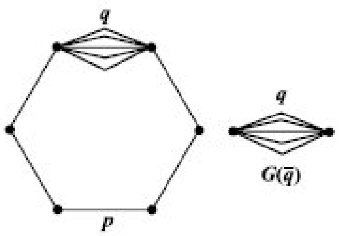


Figure 2: Graph $G(p\,q)$.

The link family $\boldsymbol{p\,q}$ gives the family of graphs, illustrated in Fig. 2, satisfying the relations

$$T(G(p\,q)) - T(G((p-1)\,q)) = x^{p-1}T(G(\overline{q})),$$

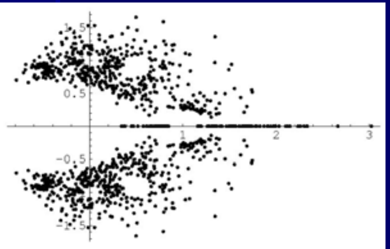
where $G(\overline{q})$ is the dual of the graph $G(q)$. Since the Tutte polynomial of the graph $G(0\,q)$ is $T(G(0\,q)) = y^q$, the general formula for the Tutte polynomial of the graphs $G(p\,q)$ is

$$T(G(p\,q)) = \frac{x(x^p-1)(x^q-1)}{(x-1)(y-1)} + x^p + y^q - 1.$$

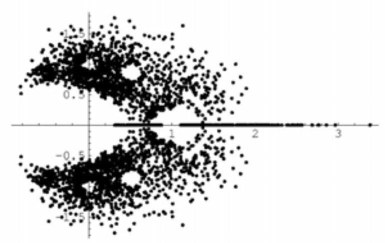
Jones polynomial

Thistlethwaite's Theorem Jones polynomial of an alternating link, up to a factor, can be obtained from Tutte polynomial by replacements: $x \to -x$ and $y \to -1/x$.

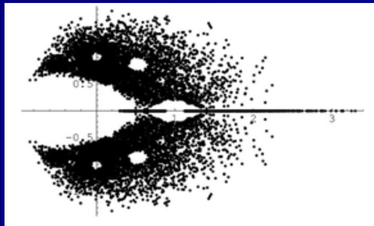
Consequence: From general formulas for Tutte polynomials we obtain general formulas for Jones polynomials.



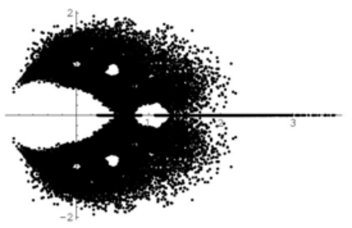
N=10. Number of knots: 123.



N=11. Number of knots: 367.



N=12. Number of knots: 1288.

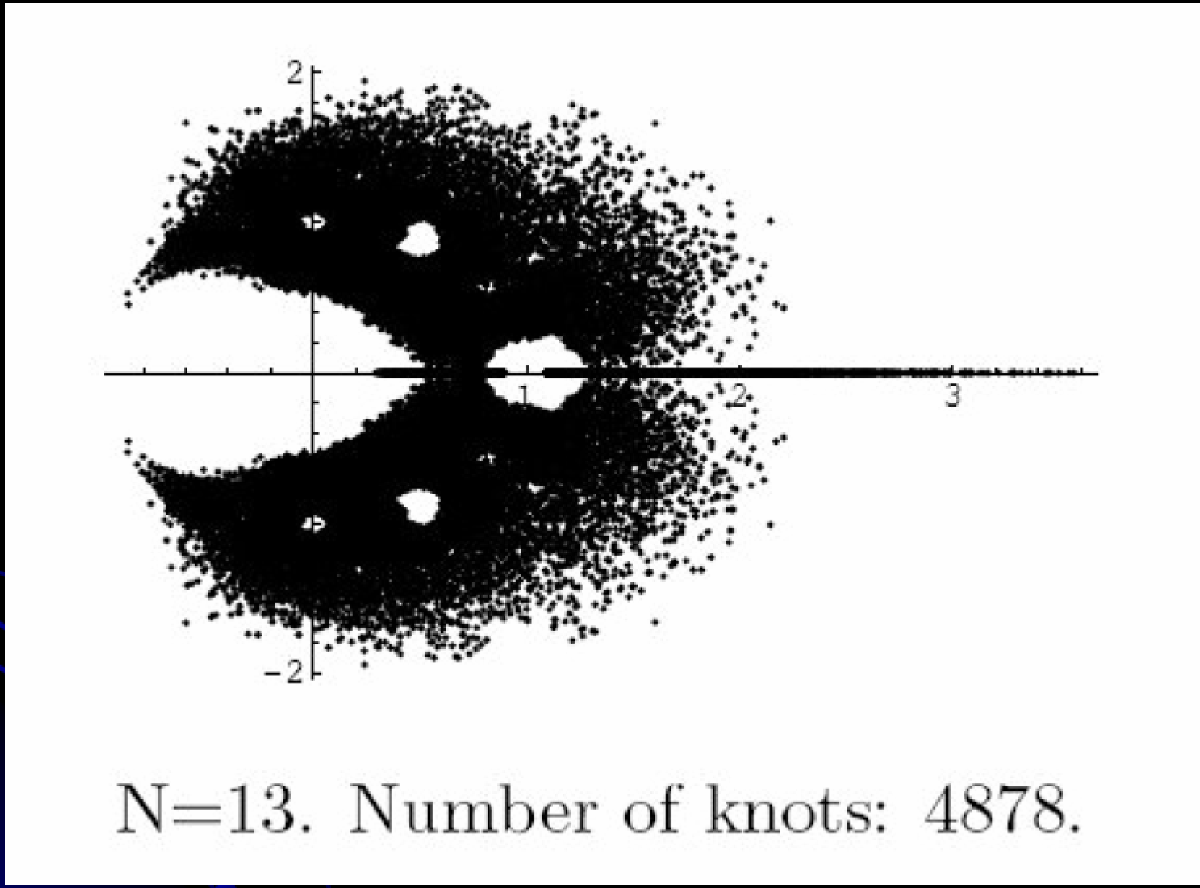


N=13. Number of knots: 4878.

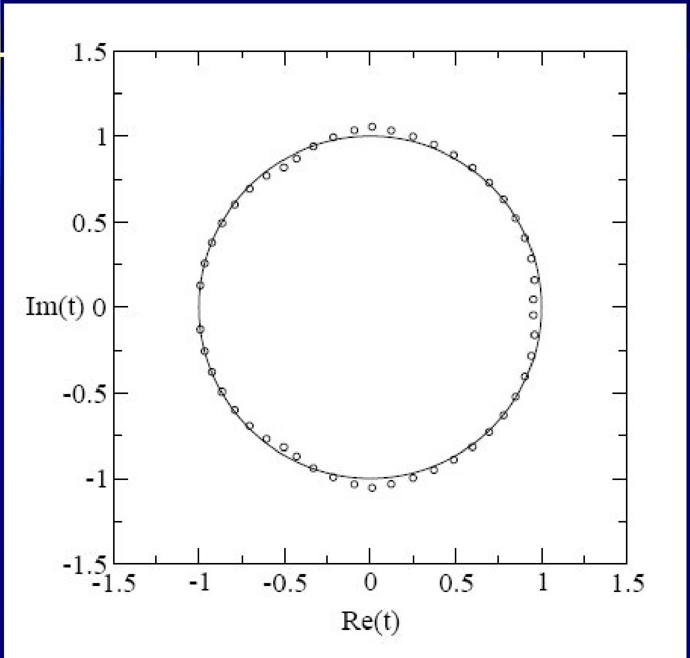
“Packman”

Xiao-Song Lin: Zeroes of Jones polynomial

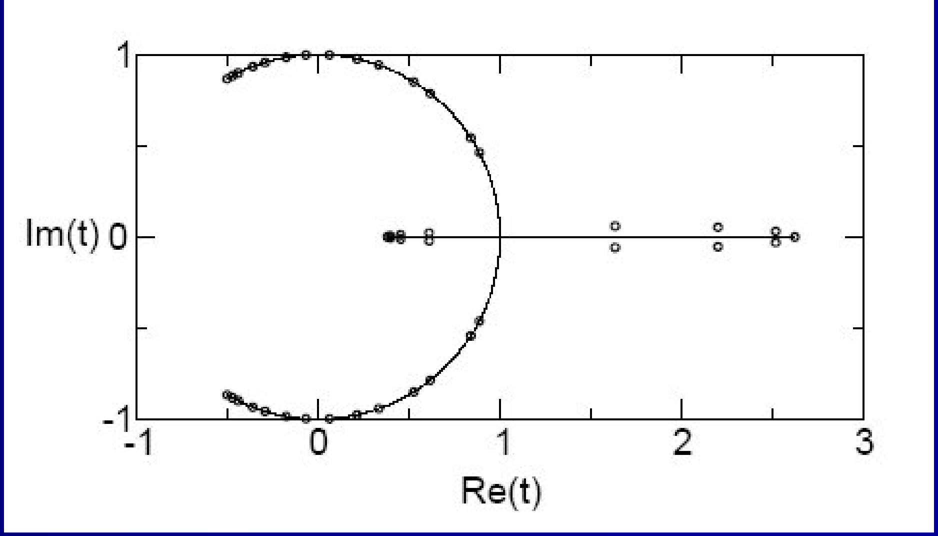
“Packman”



Zeroes of Jones polynomial



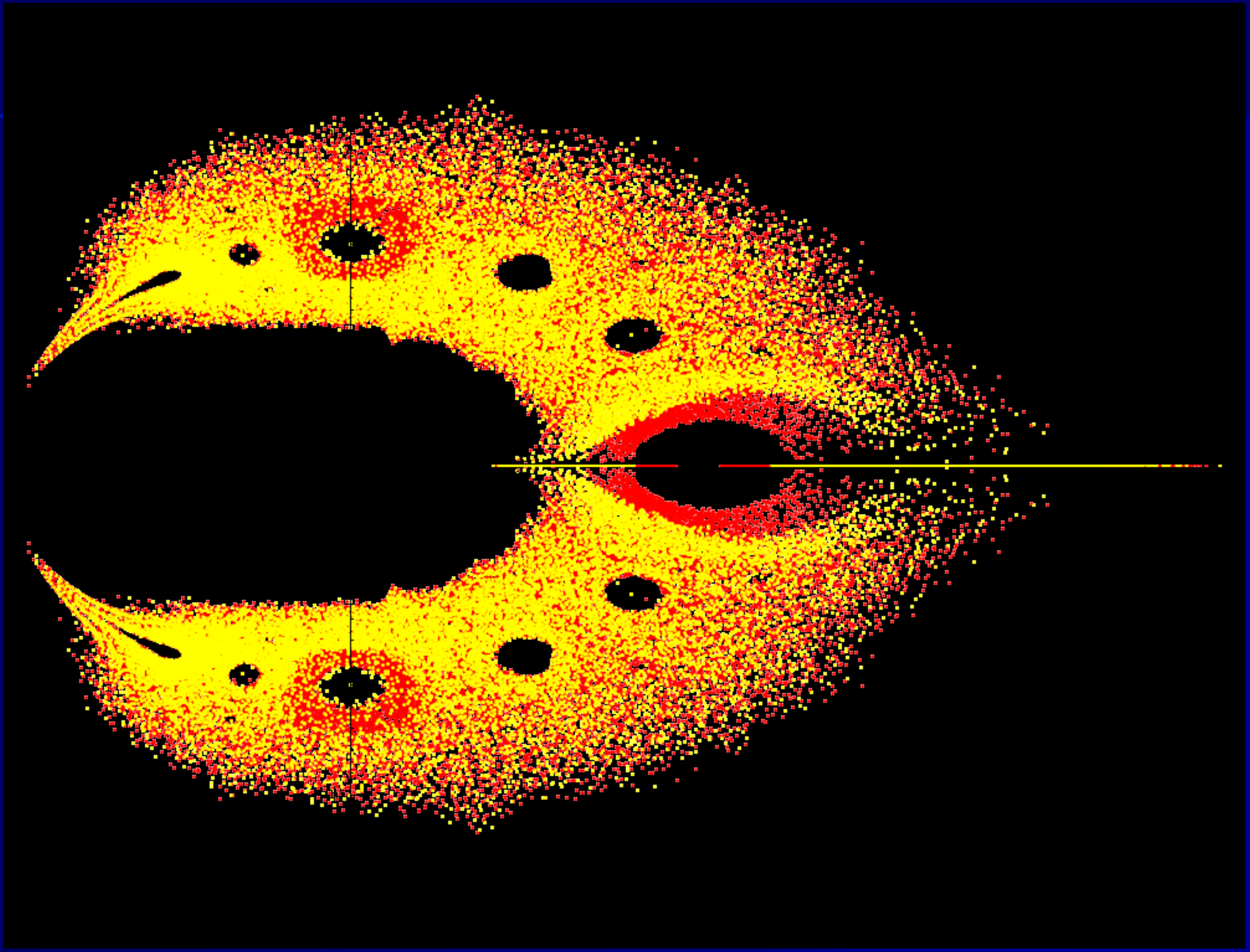
Family *p* 2



Basic polyhedra (2*n*)*
Wheel graphs *Wh*(*n*+1)

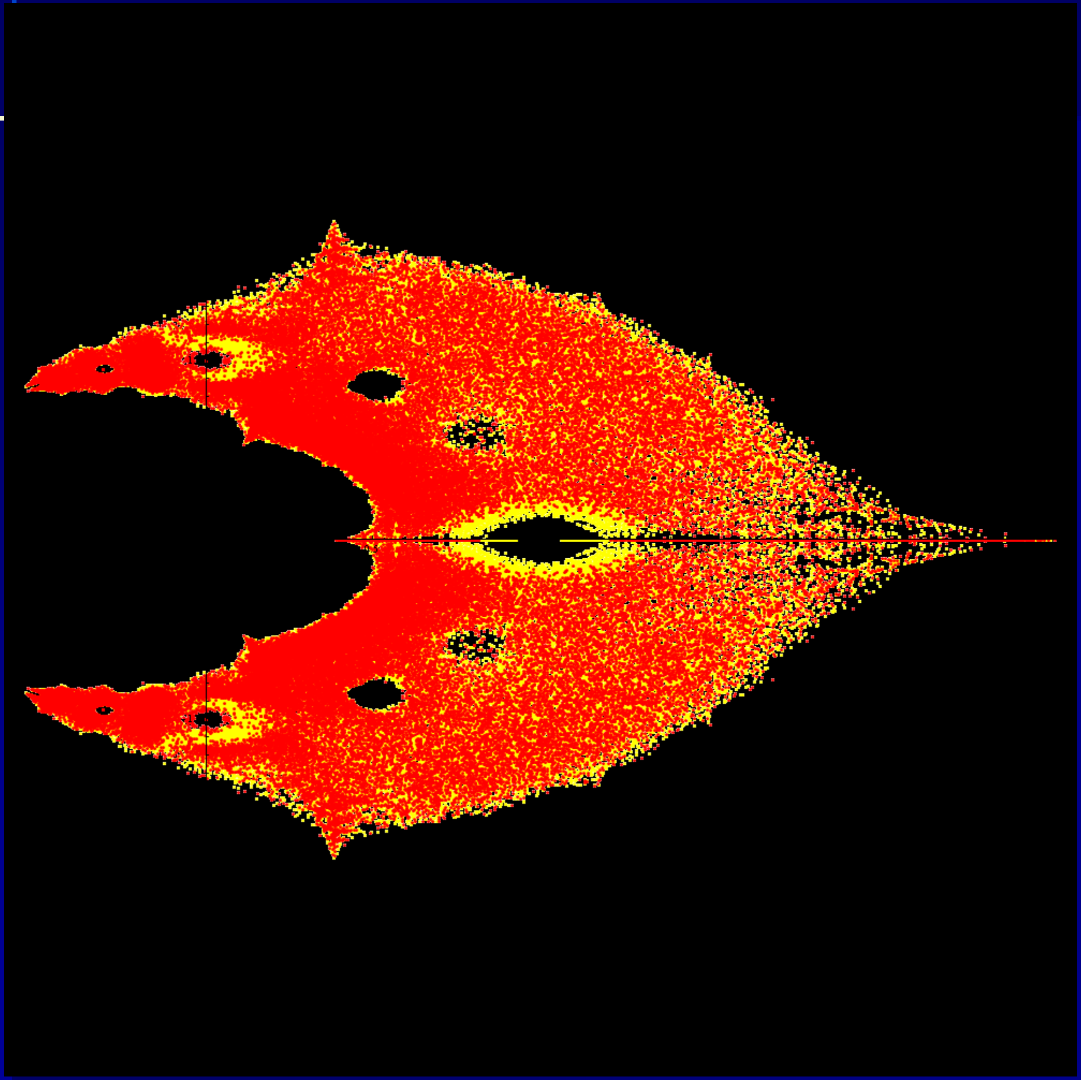
S.-C.Chang, R.Shrock: Zeroes of Jones polynomials for families of knots and links, 2001

Jones zeros



Rational knots and links, n<13

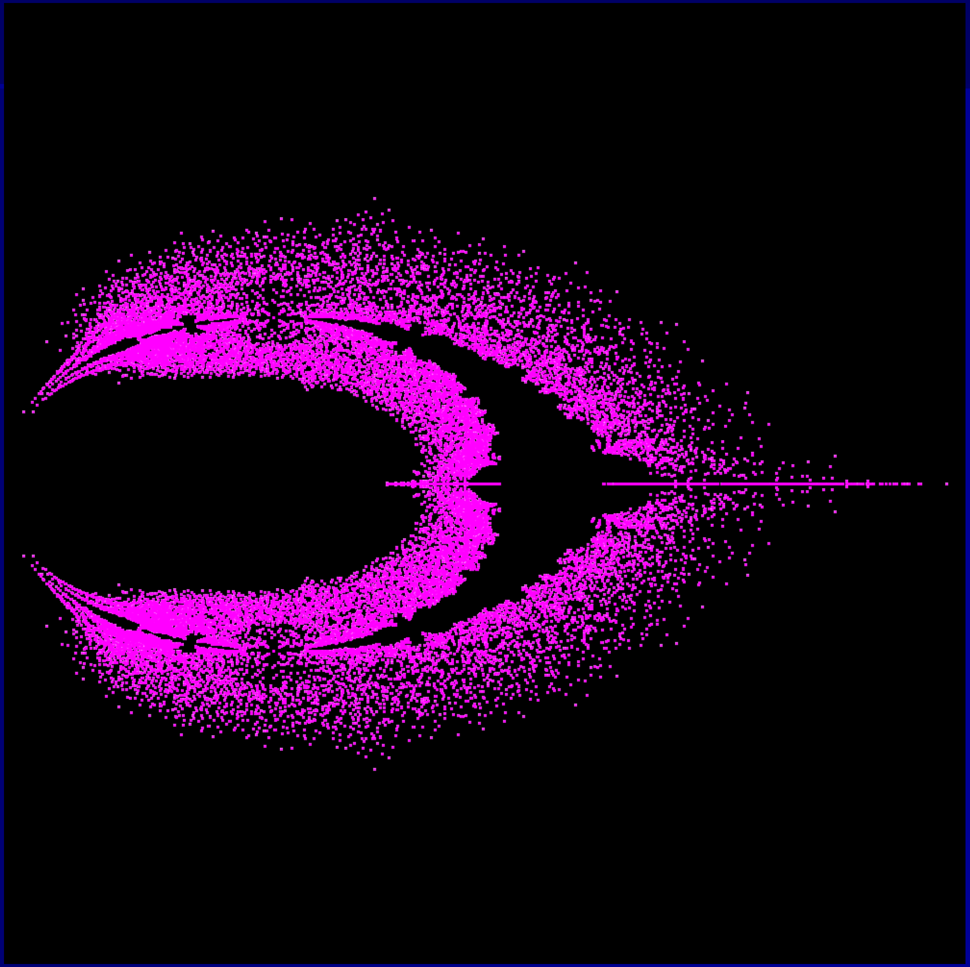
Jones zeros



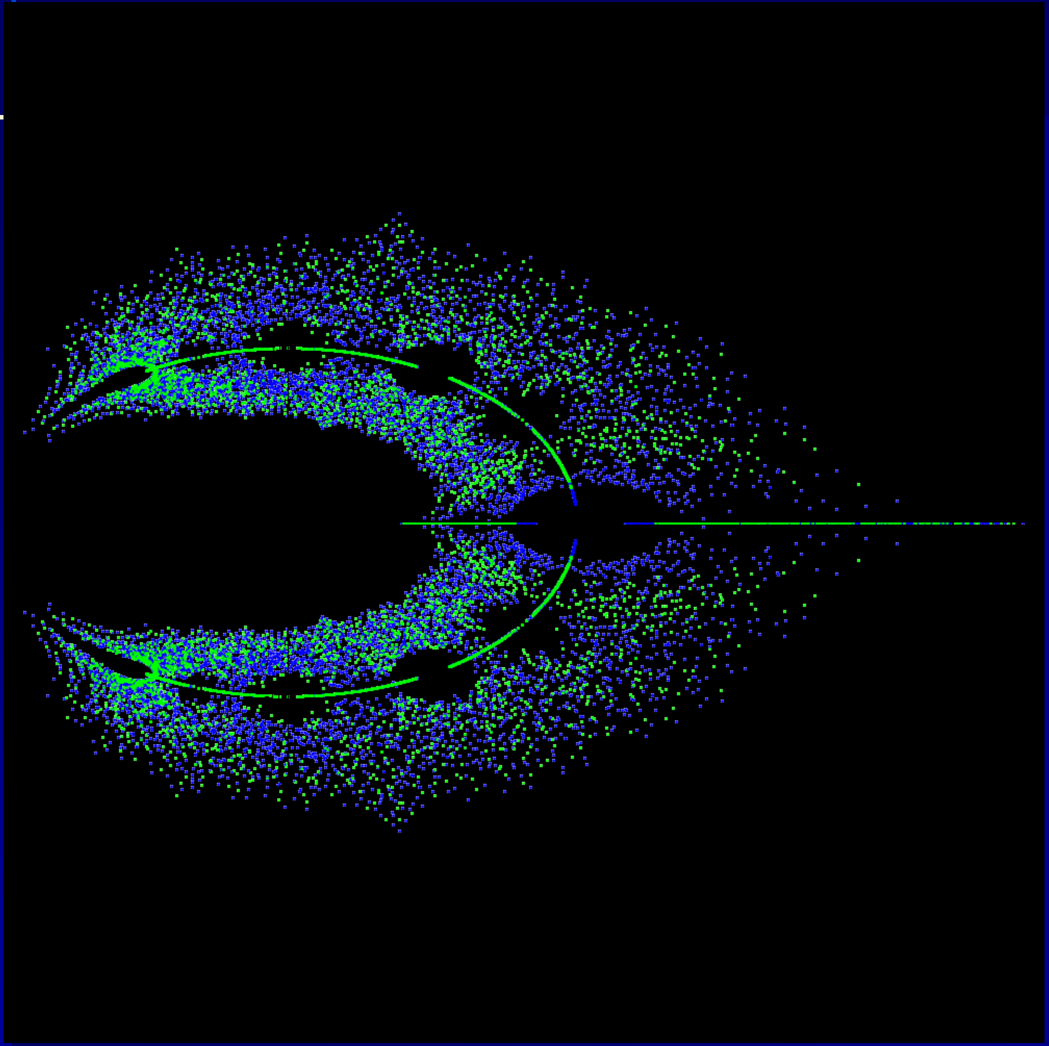
Rational generating knots and links

Jones zeros

Rational knots with unknotting number 1



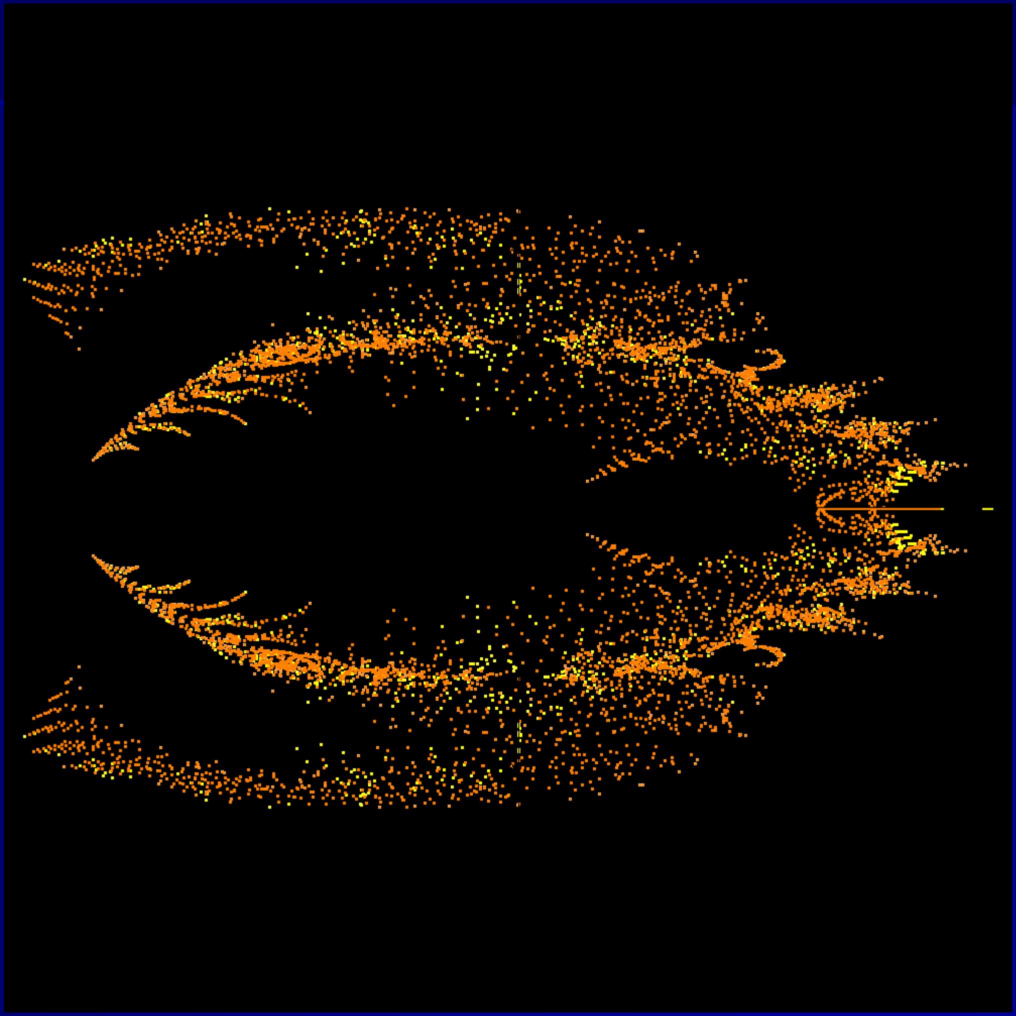
Jones zeros



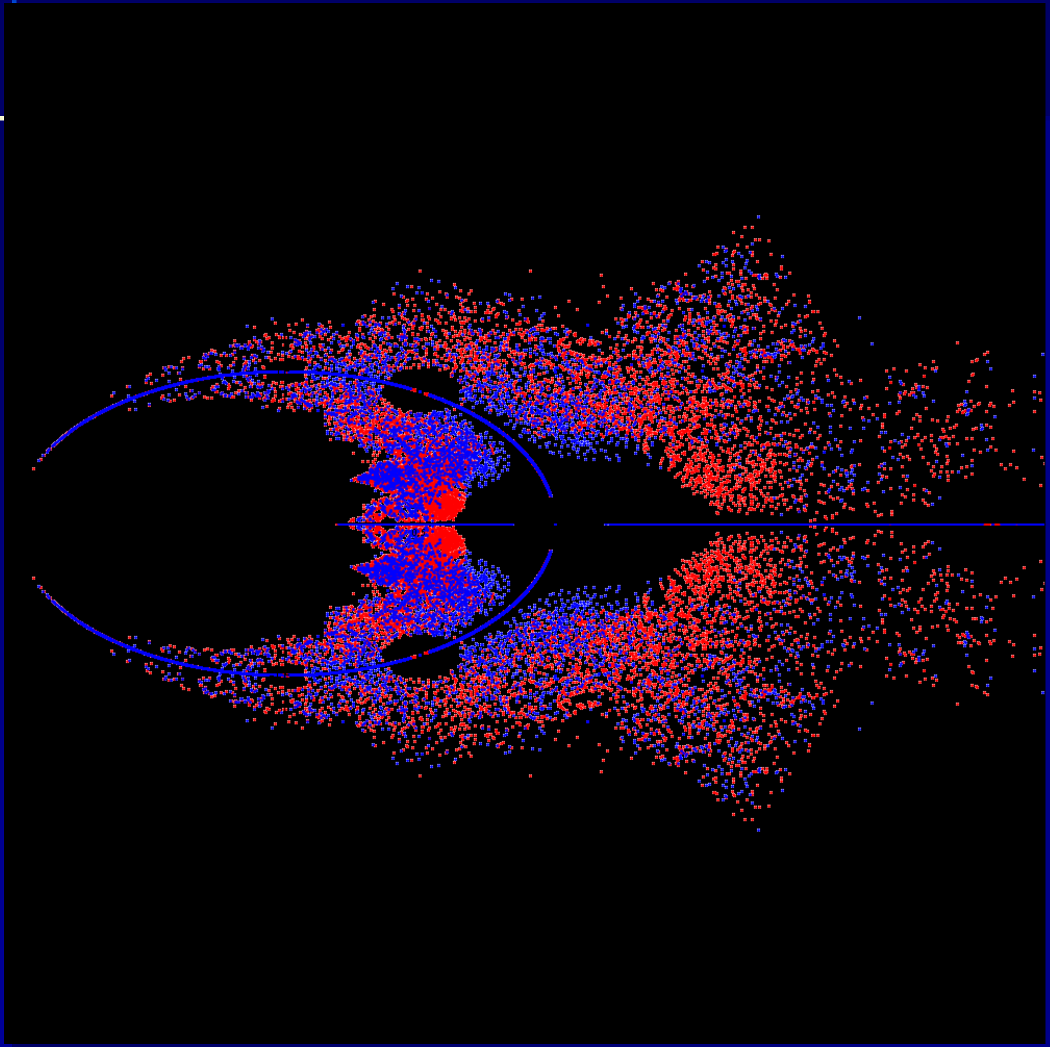
Rational amphicheiral knots and links

Jones zeros

Alternating pretzel knots and links

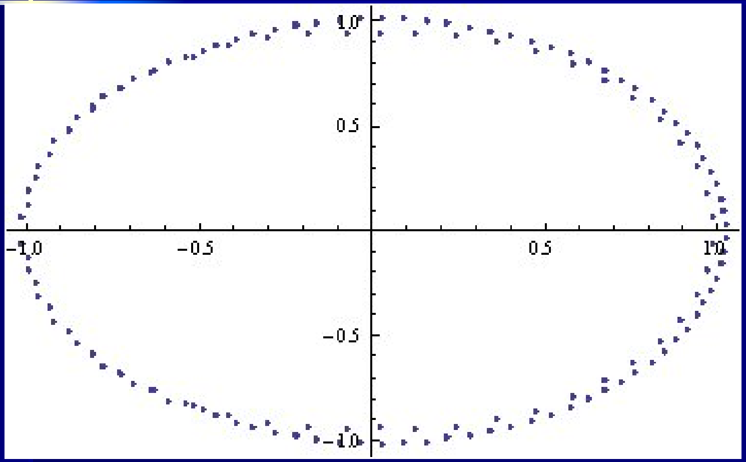


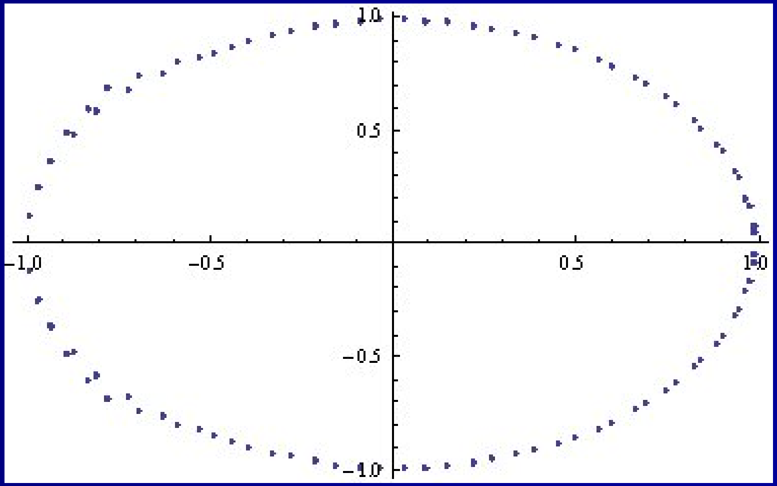
Alexander zeros



Rational knots
and links

Particular links



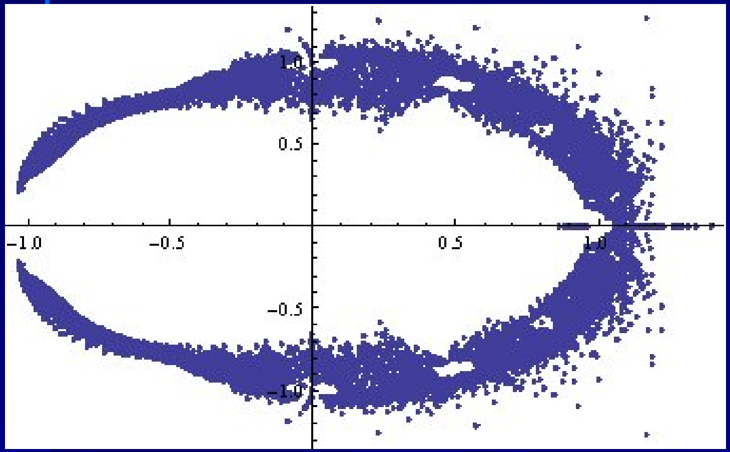


Alternating pretzel links *p,q,r*

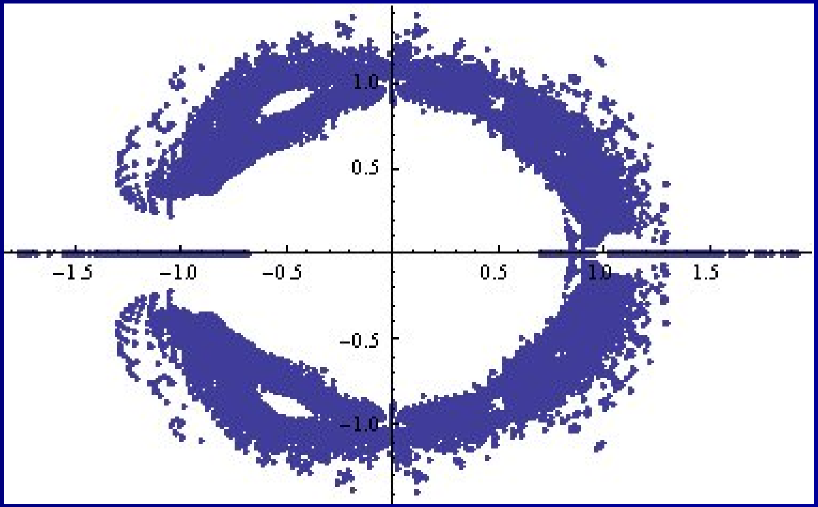
Non-alternating pretzel links *p,q,-r*

$p=q=r=50$

Link families



Alternating pretzel *p,q,r*
up to $p=q=r=15$



Non-alternating pretzel *p,q,-r*
up to $p=q=r=15$

Quasi-alternating links

Definition: The set Q of quasi-alternating links is the smallest set of links such that

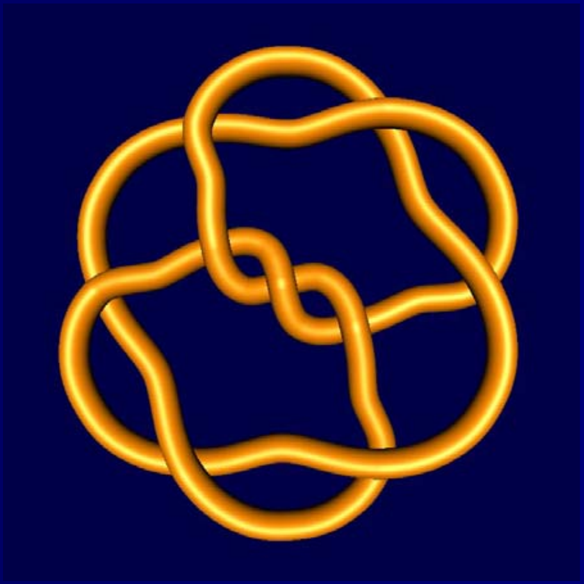
- the unknot is in Q;
- if the link L has a diagram D with a crossing c such that
  1. both smoothings of c, L_0 and L_∞, are in Q;
  2. $\det(L) = \det(L_0) + \det(L_\infty)$

Quasi-alternating links

- all alternating links are quasi-alternating;
- quasi-alternating links with a higher number of crossings can be obtained as extension of links which are already recognized as quasi-alternating: every quasi-alternating crossing *c* can be replaced by alternating rational tangle of the same sign as *c* (Champanarekar, Kofman, 2008). For the application, see T. Widmer (2008)
- Obstruction for quasi-alternating links:
- 1) Quasi-alternating links have H-thin Khovanov homology (over Z);
- 2) Quasi-alternating links have H-thin Heegaard-Floer homology (over $Z\mathbf{2}$) (C. Manolescu, P. Ozsváth, 2007). This theorem works for odd homology as well.
- Remark: in odd homology link is thick if its odd homological width $ohw(L)>1$ (including torsion).
- Problem: find candidates for homologically thin knots that are not quasi-alternating.

Quasi-alternating links

- For a long time, knots 9_{46} = 3,3,-3 and 10_{140} = 4,3,-3 have been the main candidates. However, according to A. Shumakovitch's computations by KhoHo both are odd-homology thick, so they are not quasi-alternating. The same property holds for all knots of the family *p*,3,-3 up to *n*=16 crossings: all knots of this family have 3-torsion.



	-6	-5	-4	-3	-2	-1	0
0							2
-2						1	1_3
-4					1		
-6				2			
-8			1				
-10		1					
-12	1						

```
K = "3,3,-3";
ho = fKhoHoOddHom[K]
```

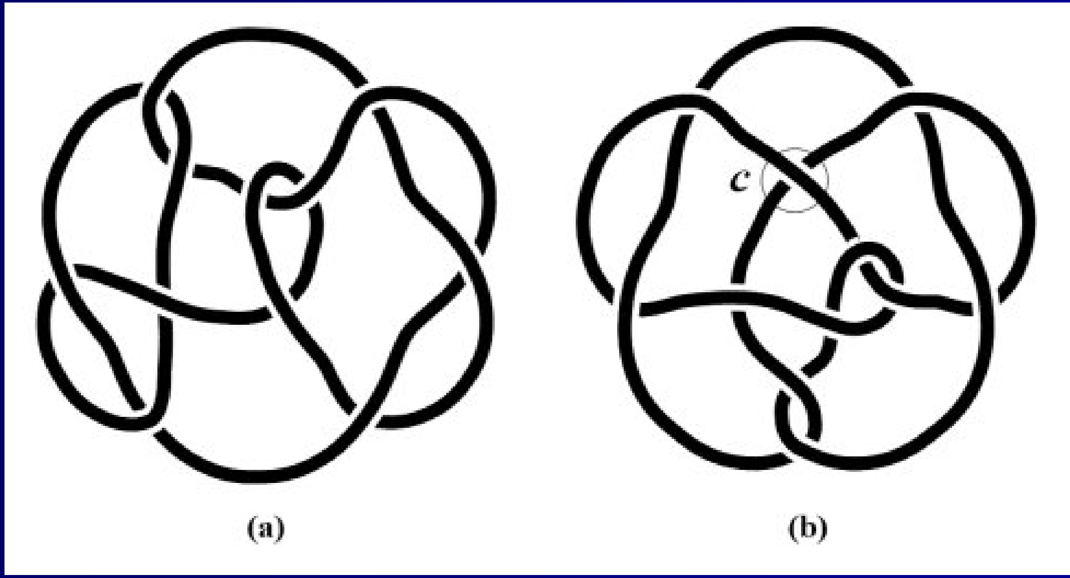
$$\left\{2+\frac{1}{q^{12}\,t^{6}}+\frac{1}{q^{10}\,t^{5}}+\frac{1}{q^{8}\,t^{4}}+\frac{2}{q^{6}\,t^{3}}+\frac{1}{q^{4}\,t^{2}}+\frac{1}{q^{2}\,t},\ \frac{T3}{Q^{2}}\right\}$$

```
K = "10,3,-3";
ho = fKhoHoOddHom[K]
```

$$\left\{1+\frac{1}{q^{26}\,t^{13}}+\frac{1}{q^{24}\,t^{12}}+\frac{1}{q^{22}\,t^{11}}+\frac{2}{q^{20}\,t^{10}}+\frac{1}{q^{18}\,t^{9}}+\frac{1}{q^{16}\,t^{8}}+\frac{1}{q^{14}\,t^{7}},\ \frac{T3+Q^{4}\,t^{2}\,T3+Q^{6}\,t^{3}\,T3+Q^{8}\,t^{4}\,T3+Q^{10}\,t^{5}\,T3+Q^{12}\,t^{6}\,T3+Q^{14}\,t^{7}\,T3}{Q^{16}\,t^{7}}\right\}$$

Quasi-alternating links

- The knot $11n_{65}$ has two minimal diagrams: (3,-2 1) (2 1,2) and 6*2.2 1.-2 0.-1.-2 The first diagram is not quasi-alternating, and the second is quasi-alternating. Moreover, all minimal knot diagrams of the family derived from knot $11n_{65}$, (3,-p 1) (2 1,2) and 6*2.2 1.-p 0.-1.-2 (p>1) which represent the same knot have this property: the first is not quasi-alternating, and the other is quasi-alternating.
- Open problem: find a quasi-alternating knot/link without a minimal quasi-alternating diagram.

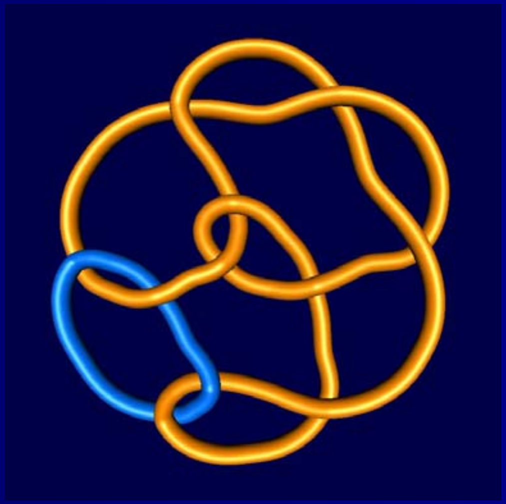


(3,-2 1) (2 1,2) 6*2.2 1.-2 0.-1.-2

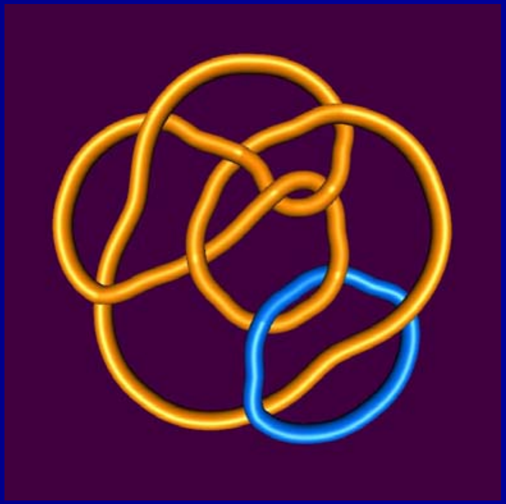
Thin non quasi-alternating links

- As the first candidate for a thin non-QA knot we proposed Montesinos knot $11n_{50}$ = -2 2,2 2,3 = *M*(0;(5,-2),(5,2),(3,1))= *M*(1;(5,3),(5,2),(3,1)) without quasi-alternating diagrams up to *n*=12 crossings. Three weeks later, Josh Greene at *Knots in Washington* proved that $11n_{50}$ is not QA.
- Among 11-crossing links we proposed two candidates: (2,2+) -(2 1,3), and 6*2.(2,-2):2 0. For the first two J. Greene proved that they are not QA.

-2 2,2 2,3



(2,2+) -(2 1,3)



6*2.(2,-2):2 0

For p>5 knots of the family -2 2,2 2, *p* have 5-torsion, but KLs of the family (*p*,2+)-(2 1,3) are thin up to 16 crossing, so this is maybe the first infinite family of thin KLs which are not QA..

Thin non QA knots and links

- Candidates for 12-crossing thin knots that are not QA

$12n_{139}$	$.2.(-2\,1,2).2$	$12n_{145}$	$-2\,2,2\,2,4$
$12n_{196}$	$-2\,2,2\,1\,2,3$	$12n_{331}$	$(3,2+)\ -\ (2\,1,3)$
$12n_{414}$	$-2\,1\,0.3.2.2\,0$	$12n_{768}$	$2:-3\,1\,0:3\,0$
$12n_{838}$	$-2.-2.-2\,0.2.2.2\,0$		

Candidates for 12-crossing thin links that are not QA:

(2 1,3),2,(2,-2), 6*2:.(-2 1,3) 0, 6*3.(2,-2):2 0,

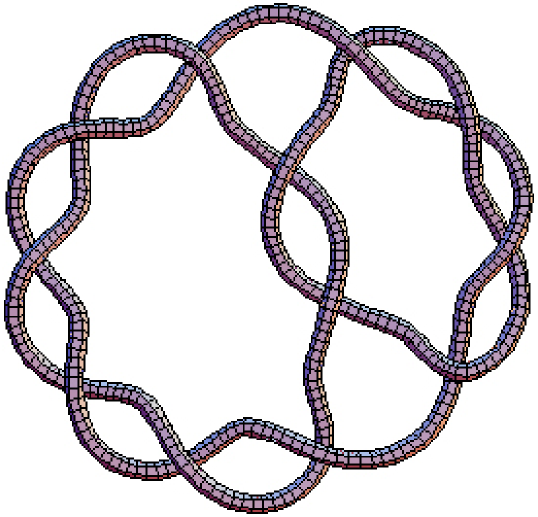
8*2.-2 1 0::2, 8*(2,-2)::-2 0

S. Jablan and R. Sazdanovic: Quasi-alternating links and odd homology: computations and conjectures (2009)

Input:
knots and links in Conway notation

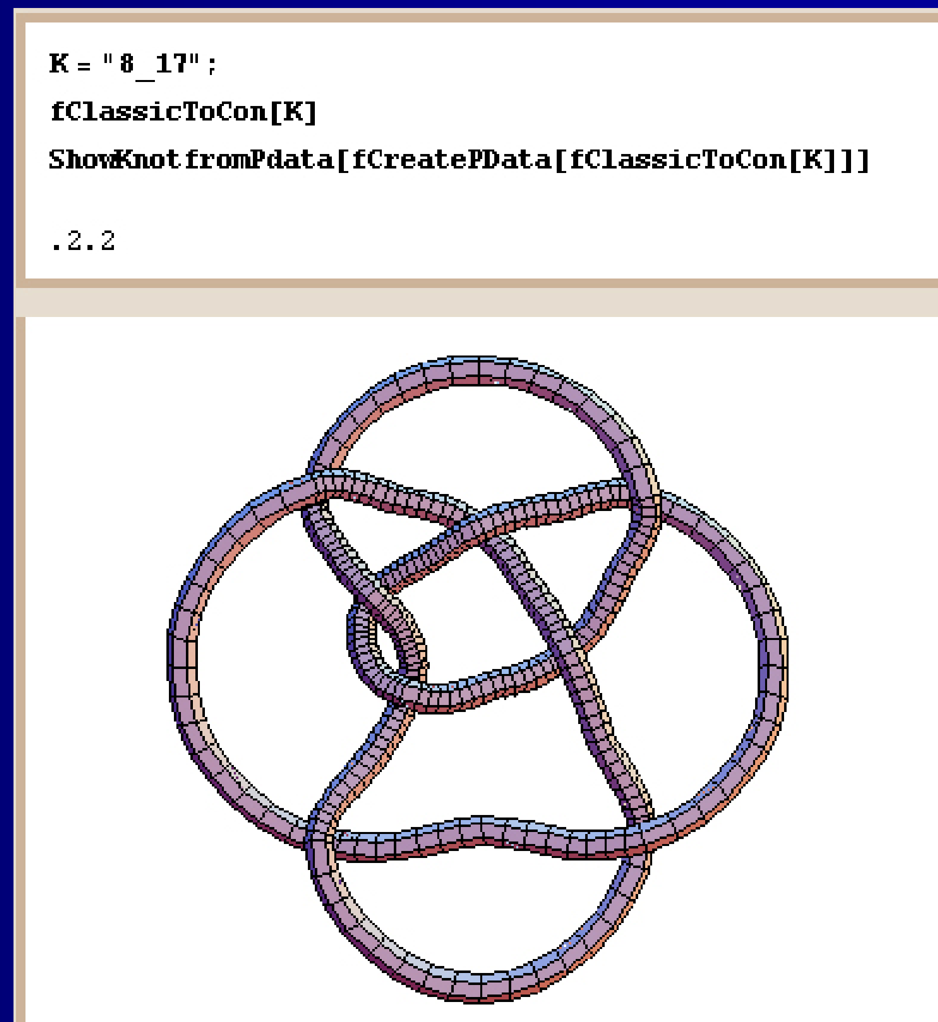
```
(* INPUTING KLs IN CONWAY NOTATION *)
K = "6,4,2";
pdata = fCreatePData[K]
ShowKnotfromPdata[pdata]

{{4, 5, 3}, {18, 10, 12, 22, 2, 4, 6, -20, -24, -16, -14, 8}}
```



Conversion functions

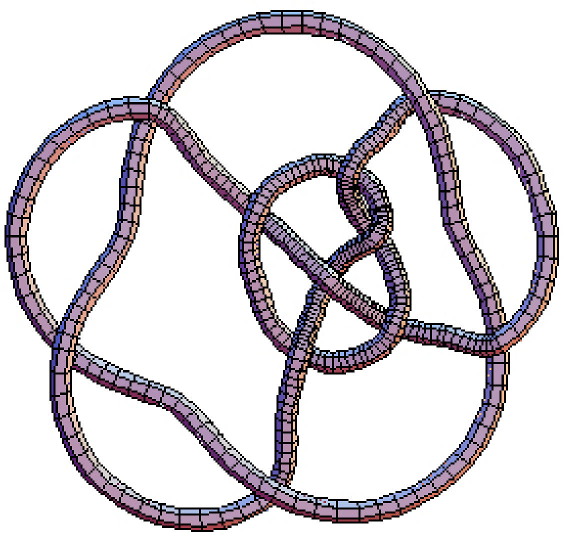
- fFindCon
- fClassicToCon
- fPdataFromDow
- fPDataFromDowker
- DowfromPD
- fKnotscapeDow
- fSignsKL

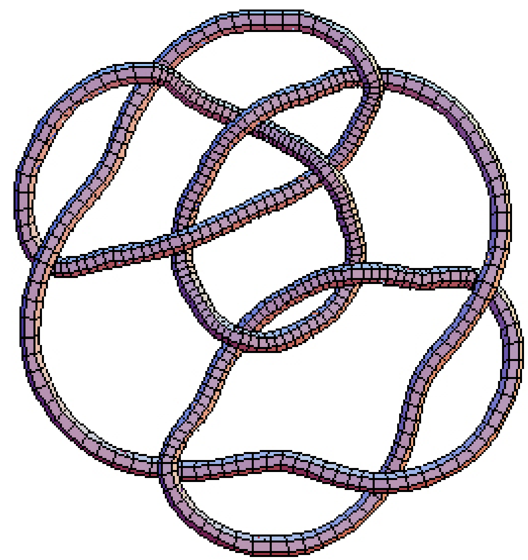


Minimal DT codes & Mutants

```
K2 = ".(2,3).2";
SameAltConKL[K1, K2]
ShowKnotfromPdata[fCreatePData[K1]]
ShowKnotfromPdata[fCreatePData[K2]]

0
```

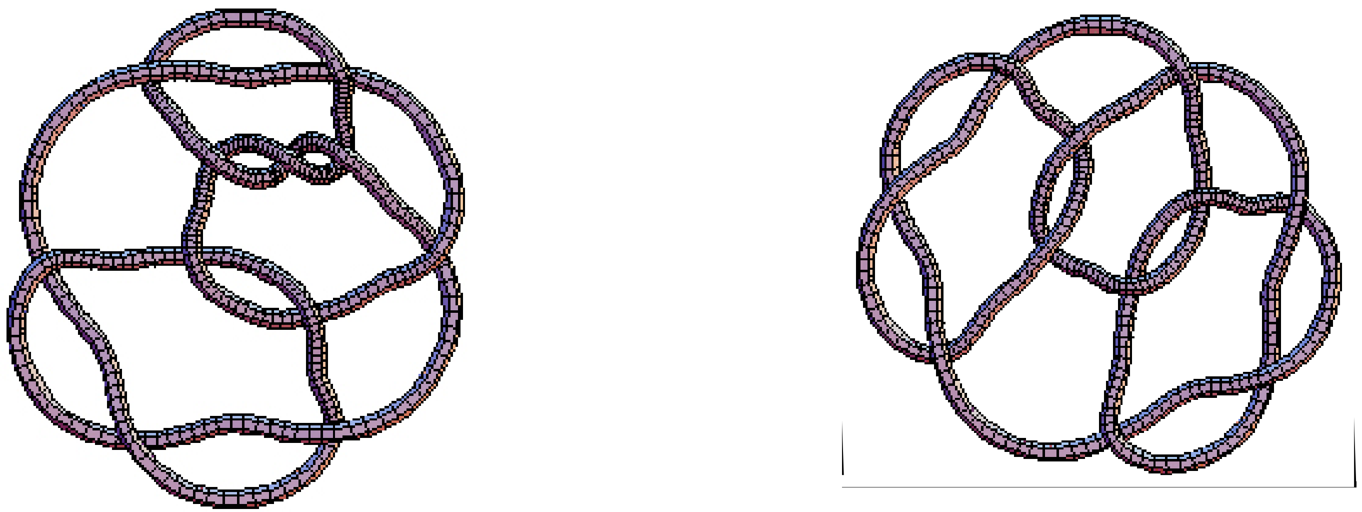




Different projections of knots and links

```
K = fDiffProjectionsAltKL["(3,2) 1 (3,2)"]
k1 = fCreatePData[K[[1, 1]]]; k2 = fCreatePData[K[[2, 1]]];
ShowKnotfromPdata[k1] ShowKnotfromPdata[k2]

{{((3,2),1) (3,2), {{11}, {6, 8, 12, 2, 18, 22, 4, 20, 10, 14, 16}}},
 {(1,(2,3)) (3,2), {{11}, {6, 8, 12, 2, 14, 18, 4, 20, 22, 10, 16}}}}
```



Torus KLs

```
m = 5
n = 4;
fTorusKL[m, n]

5
```

Braid word: abcabcabcabcabc

Pdata: {{15}, {24, 17, 10, 30, 23, 16, 6, 29, 22, 12, 5, 28, 18, 11, 4}}

Braid word: abcabcabcabcabc

Crossings: 15

Number of components: 1

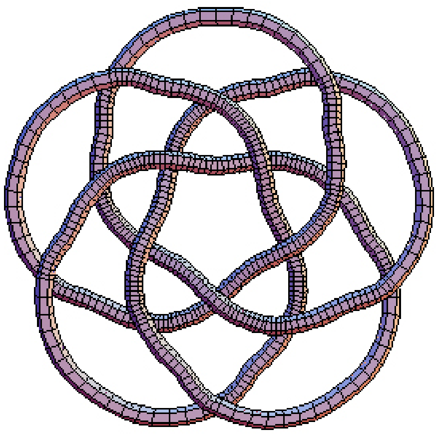
Unknotting number: 6

Bridge number: 4

Alexander polynomial: $1 - x + x^4 - x^6 + x^8 - x^{11} + x^{12}$

Signature: 8

Murasugi signature: 8



{{15}, {24, 17, 10, 30, 23, 16, 6, 29, 22, 12, 5, 28, 18, 11, 4}}

Rational KLs

RationalKL - computes all rational sa *n* crossings. Their number Is given b

2

```
n = 12;
RR = RationalKL[n]

272

{12, 6 6, 7 5, 8 4, 9 3, 10 2, 2 8 2, 3 6 3, 3 7 2, 4 4 4, 4 5 3, 4 6 2, 5 2 5, 5 3 4, 5 4 3, 5 5 2, 6 1 5, 6 2 4,
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```

UnKnotLink

Open problem:
Continued fractions and unlinking number of rational KLs

K2KC1NEW.nb *

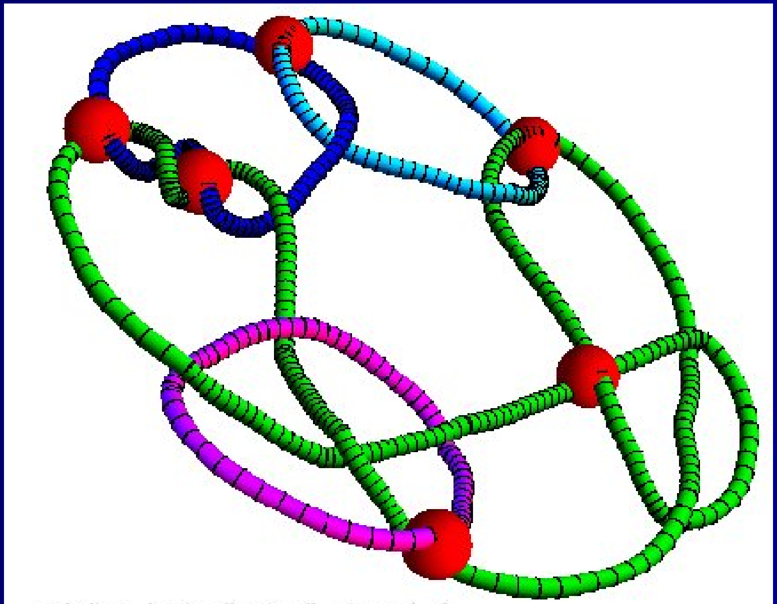
The function **UnKnotLink** calculates unknotting/unlinking No. of a given KL and if it coincides with half of the signature than it is given without question mark.

```
UnKnotLink["5 1 1 1 3"]

Unlinking number: 3?
```

```
UnKnotLink["6*"]

Unlinking number: 2
```

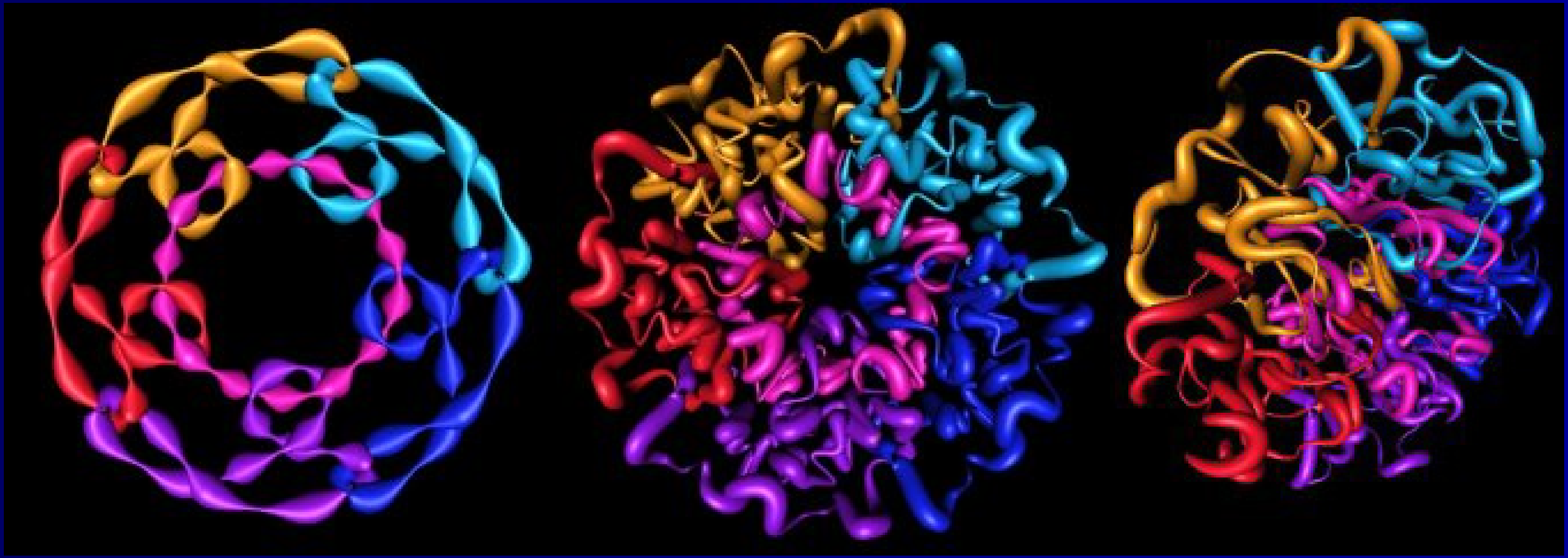
100%

VIRTUAL KNOTS AND LINKS IN CONWAY NOTATION



6*(i,1,i,1),(i,1),(i,1),2 i..i

Acknowledgements: K2K (M.Ochiai and N.Imafuji), Knot Theory (Dror Bar-Natan), basic polyhedra (B. McKay), extended bracket (Louis Kauffman), cabled Jones (Jeremy Green), Miyazawa polynomials (Naoko Kamada), undetectable knots (Heather Dye)…



webMathematica: http://math.ict.edu.rs