



2035-4

#### **Conference on Superconductor-Insulator Transitions**

18 - 23 May 2009

Critical behavior near the superconductor-insulator transitions in two-dimensions

A. Kapitulnik

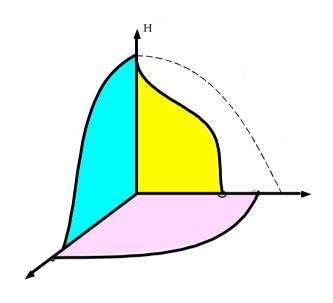
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## Some Aspects of the Superconductor-Insulator Transition





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Myles Steiner

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Steve Kivelson

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# Importance of phase fluctuations for two-dimensional superconductors:

The superconducting order parameter:  $\Psi(ec{r}) = |\Psi(ec{r})| e^{i\phi(ec{r})}$ 

For BCS - Eliashberg theory  $\phi$  is not important to determine the transition -  $T_{\rm c0}$ .

#### When are phase flctuations important?

\* Introduce the temperature  $T_\phi$  that gives the scale of "phase stiffness"

\*The Hamiltonian  ${\cal H}$  governing the effects of long wavelength phase fluctuations is the kinetic energy of the superfluid

$${\cal H}=rac{1}{2}m^{st}|\Psi|^{2}\!a\!\int dec{r}v_{s}^{2}(ec{r})$$

There is also a short distance cutoff - a

with: 
$$v_s = rac{\hbar}{2m^*} 
abla \phi$$

$$T_{\phi}$$
 is given by:

$$k_B T_\phi pprox rac{\hbar^2}{4m} |\Psi|^2 a = rac{(\hbar c)^2 a}{16\pi e^2 \lambda^2}$$

If  $T_{\phi}/T_{c0}\gg 1$  phase fluctuations are not important

If 
$$T_{\phi}/T_{c0}\ll 1$$
 phase fluctuations are important

#### **Examples:**

- lacktriangle Pb  $T_{\phi}/T_{c0} \sim 10^5$
- lacktriangle High-Tc  $T_{\phi}/T_{c0}$  ~ 10 100
- $\diamond$  2-D superconductors  $T_{\phi}/T_{c0} \sim 1$

phase fluctuations are important in 2-D

What about quantum phase fluctuations? These are associated with the uncertainty:

$$\Delta n_s \Delta \phi \ge 1$$



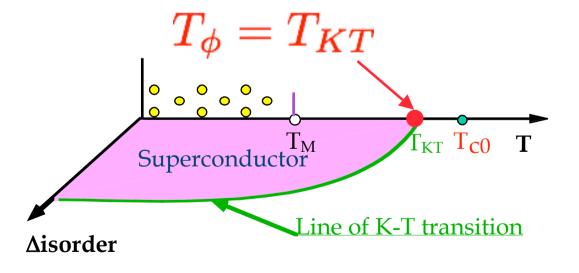
Phase coherence implies large coulomb energies (unless screening)

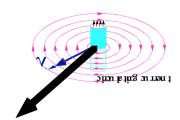
## Zero Magnetic-Field

Superconductivity is established below the Kosterlitz-Thouless transition

$$k_B T_\phi \approx \frac{\hbar^2}{4m} |\Psi|^2 a = \frac{(\hbar c)^2 a}{16\pi e^2 \lambda^2}$$

Setting 
$$a = d$$
 (film thickness):  $T_{KT} = k_B^{-1} \left(\frac{\hbar c}{2e}\right)^2 \frac{2d}{(4\pi\lambda)^2}$ 



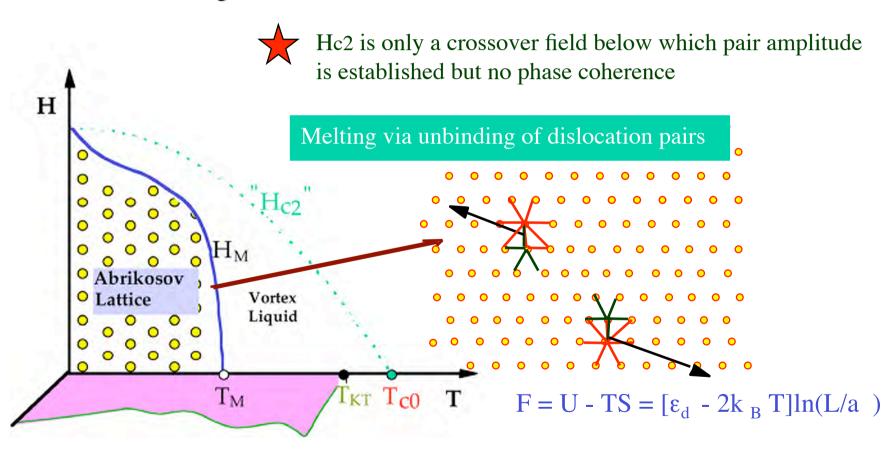


- J. M. Kosterlitz & D. J. Thouless, Journal of Physics C: Solid State Physics 6, 1181-1203 (1973).
- S. Doniach and B.A. Huberman, Phys. Rev. Lett. 42, 1169 (1979).

## Finite Magnetic Field

(no disorder)

Superconductivity is established below the melting transition: again a **Kosterlitz-Thouless Transition\*** 



B.A. Huberman and S. Doniach, Phys. Rev. Lett. 43, 950 (1979).D. S. Fisher, Phys. Rev. B 22, 1190 (1980).

<sup>\*</sup> Experimentally this was shown by: A. Yazdani, W.R. White, M.R. Hahn, M. Gabay, M.R. Beasley, and A. Kapitulnik, Phys. Rev. Lett. 70 (1993), 505.

#### Effect of disorder:

#### \* Larkin, 1970; Larkin & Ovchinikov, 1979

With disorder (pinning), positional long range order is lost: New length scale that describes by how much the disorder perturbes the positional order R

At  $R_c$  the cumulative displacement is  $\sim \xi$ 

#### \* Feigelman et al., 1991

For dislocations in 2-D the relevant length scale is:

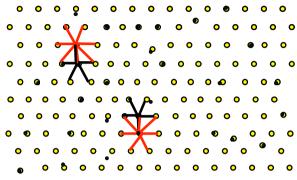
$$R_d \sim R_c \left(\frac{a_0}{\xi}\right) \gg R_c$$

At Rd the cumulative displacement is  $\sim a_0$ 

#### \* Tonner, 1992

Orientational order is also lost with disorder

In the presence of disorder: for H > 0 and finite T there is no true superconducting phase

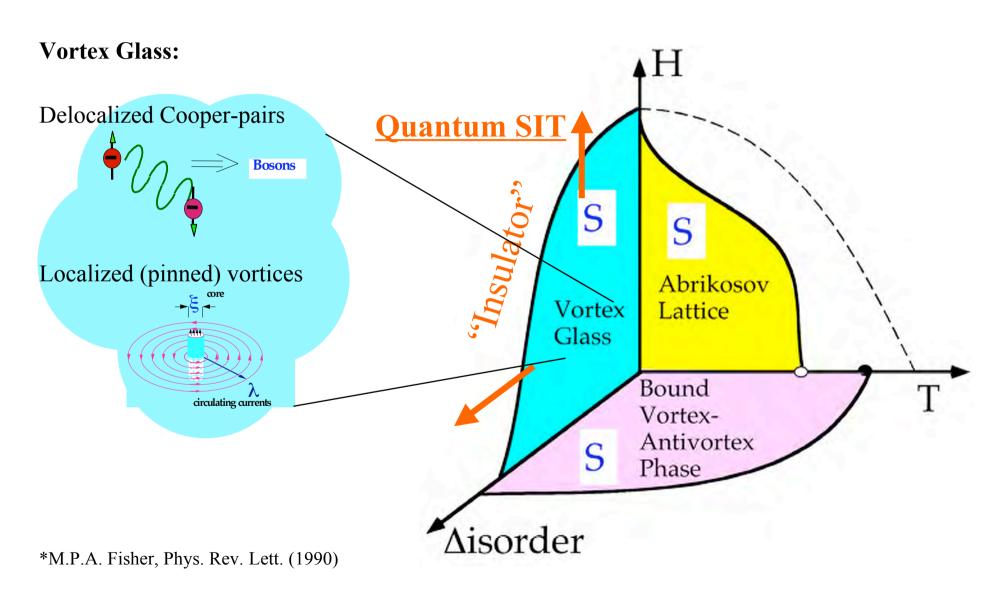


Vortex lattice gets distorted due to pinning - dislocation pairs are induced

See: T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. 72, 1530 (1994).

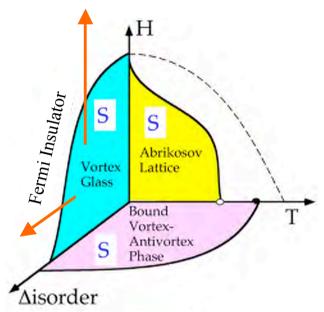
## Summary Phase Diagram:

Superconducting phases exist only on the planes (H-T,  $\Delta$ -T, H-  $\Delta$ )



### Two possible Superconductor-"Insulator" Transition at T=0:

#### **Superconductor** → **Fermi-Insulator**

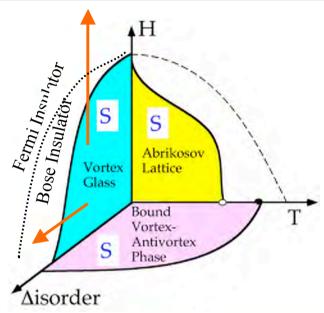


1) Superconductivity is destroyed by disappearance of Cooper pairs altogether. Cooper attraction is reduced due to Large Coulomb interaction. \*

\*A.M. Finkelstein, JETP Lett. 45, 46 (1987).

This model however neglects quantum fluctuations of the Bosonic field! **Free electrons exist**!

#### **Superconductor** → **Bose-Insulator**



2) Bosons in a random potential. Pairs can become localized due to coulomb repulsion.

Equivalent to array of Josephson-Junctions ( $E_J$  vs.  $E_C$ ). \*

\* M.P.A. Fisher, Phys. Rev. Lett. (1990).

No free electrons exist!

We will argue below that in disordered thin films the transition will always be phase-dominated. (analyze data in terms of Superconductor to Bose-Insulator Transition)

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#### A note about the disorder axis:

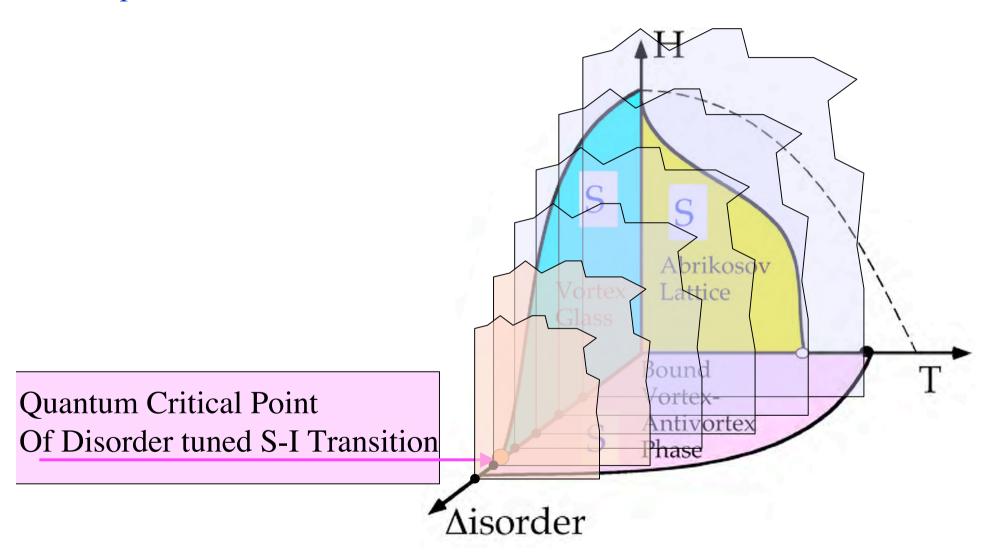
While in general the disorder axis is taken as proportional to  $R_{/square}$ , we note that since vortices are involved, this **may not** be an accurate measure of the disorder.

\*for example, a very homogeneous, large-R<sub>/square</sub> film will pin vortices very poorly and may therefore overestimate the disorder. Similarly, lower-R<sub>/square</sub> films that are granular in nature may underestimate the disorder because of their ability for strong pinning.

With this observation, we will argue that all films, whether granular or uniform (and become granular due to phase separation and puddle formation), are in the same universality class with respect to the SIT and the behavior near it.

#### Disorder tuned S-I Transition

In experiment one fabricate films where the disorder is fixed



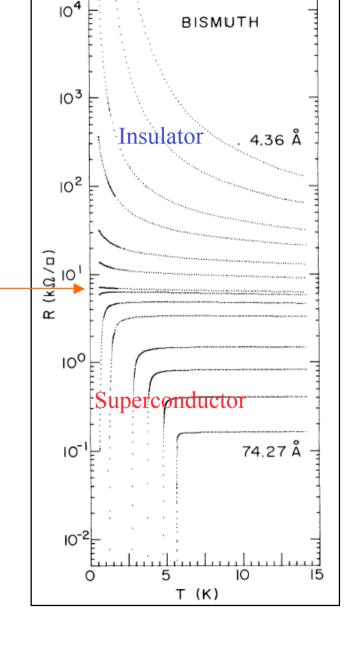
Onset of Superconductivity in the Two-Dimensional Limit



Experiments on Bi layer grown on amorphous Ge

## Disorder is varied by changing film thickness

 $R_c$ 



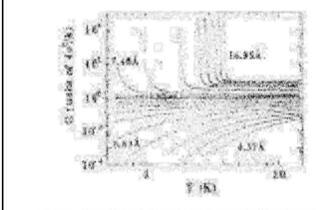
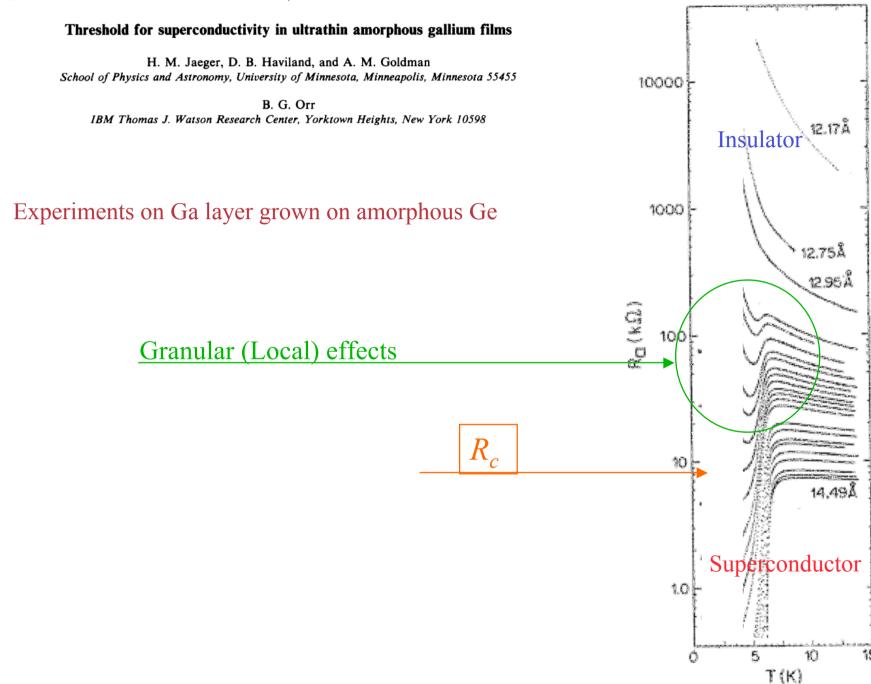
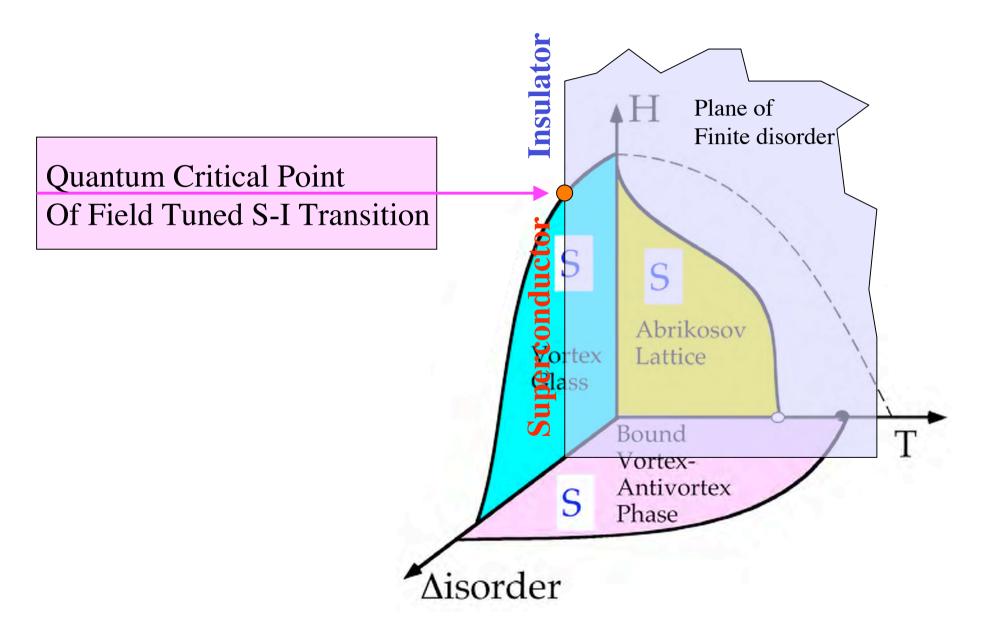


FIG. 2. Evolution for his bless of the electronic constraines of the section of 4x2/A as a function of outprojector. To the thicknesses of a less extracted from the independent black can durantee and constraintly and taken and in the discount of the date of the exposure of these in alternative extraction between the superior of these in alternative extraction by the date popular.



### Field tuned S-I Transition



#### Magnetic-Field-Tuned Superconductor-Insulator Transition in Two-Dimensional Films

## A. F. Hebard and M. A. Paalanen AT&T Bell Laboratories, Murray Hill, New Jersey 07974 (Received 26 February 1990)

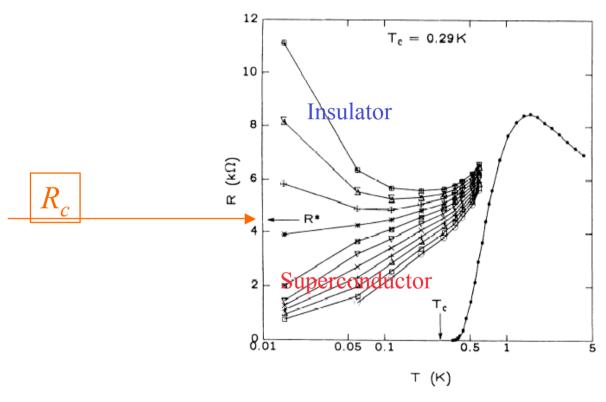
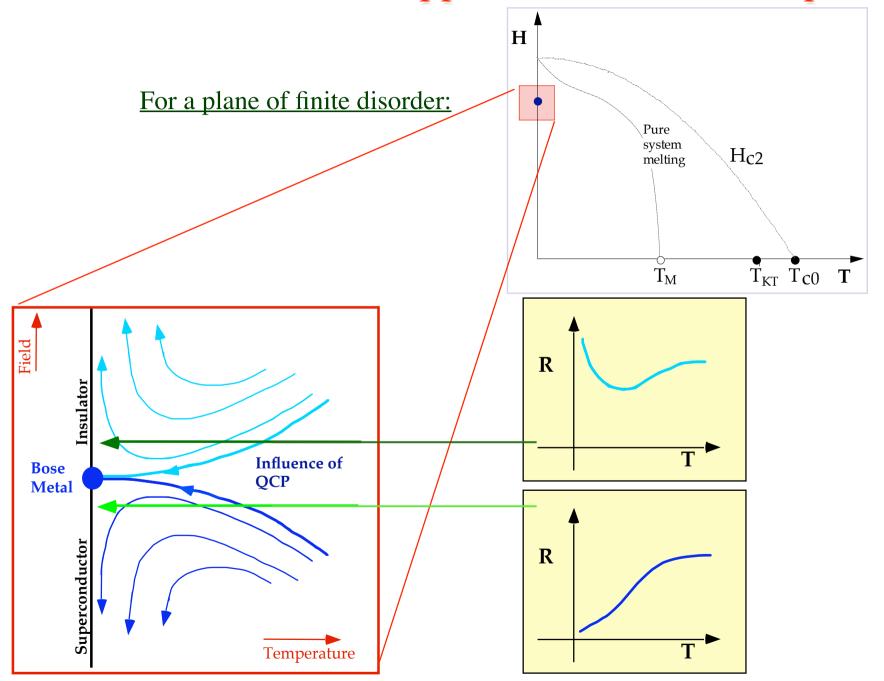


FIG. 1. Logarithmic plots of the resistance transitions in zero field ( $\bullet$ ) and nonzero field (open symbols) for a film with  $T_c = 0.29$  K. The isomagnetic lines range from B = 4 kG ( $\circ$ ) to B = 6 kG ( $\circ$ ) in 0.2-kG steps. The horizontal and vertical arrows identify  $R^*$  and  $T_c$ , respectively.

Field-tuned transition: Approach to the critical point:



## Inhomogeneous nature of the transition

- I. Granular films are inherently inhomogeneous Equivalent to phase fluctuations and strong disorder
- II. Homogeneous films: Amplitude fluctuations + Disorder Island formation due to strong fluctuations of △

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B. Spivak and F. Zhou, Phys. Rev. Lett. 74, 2800 (1995); F. Zhou and B. Spivak, Phys. Rev. Lett. 80, 5647 (1998).
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V. M. Galitski and A. I. Larkin, Phys. Rev. Lett. 87, 087001 (2001).

→ Adding quantum fluctuations result in a phase transition

M. A. Skvortsov, M. V. Feigel'man, Phys. Rev. Lett. 95, 057002 (2005).

→ Mesoscopic fluctuations in large dimensionless conductance

A. Ghosal, M. Randeria and N. Trivedi, Phys. Rev. Lett. 81, 3940 (1998); Phys. Rev. B 65, 14501 (2001).

→ Solving B-dG in the presence of disorder- show island formation

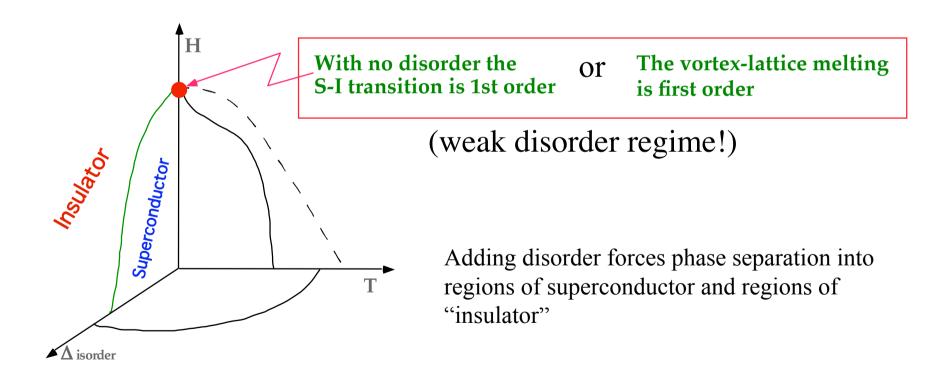
Yonatan Dubi, Yigal Meir, and Yshai Avishai, Phys. Rev. B 78, 024502 (2008).

→ Using a locally self-consistent numerical solution of the B-dG equations, show **island formation** and evolution with magnetic field.

#### III. Homogeneous films: Phase fluctuations + weak disorder

Island formation due to proximity to quantum melting + disorder (Imry & Ma mechanism)

•E. Shimshoni, A. Auerbach and A. Kapitulnik, Phys. Rev. Lett. 80 (1998), 3352.

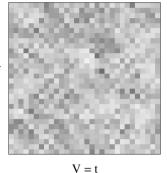


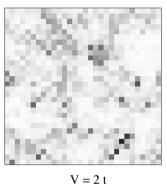
#### Numerical examples:

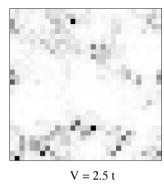
A. Ghosal, M. Randeria and N. Trivedi, Phys. Rev. Lett. 81, 3940 (1998); Phys.

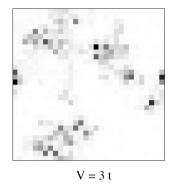
Rev. B 65, 14501 (2001).

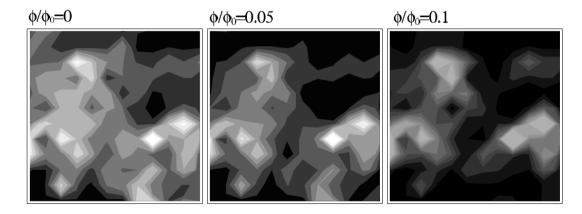
#### Zero-magnetic-field







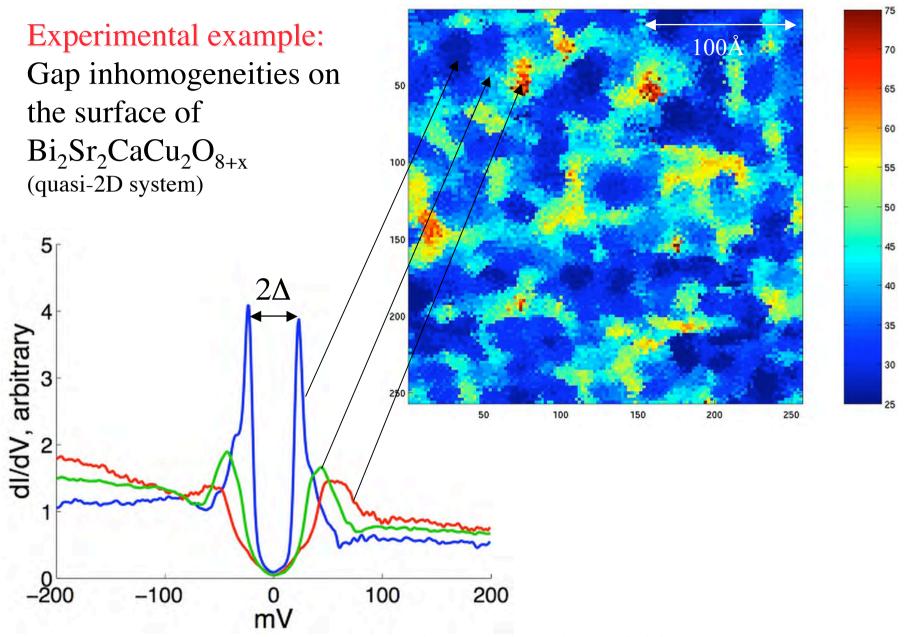




Yonatan Dubi, Yigal Meir, and Yshai Avishai, arXiv:0712.4398: cond-mat

FIG. 1: Spatial distribution of the order parameter amplitude  $|\Delta|$  for different values of the orbital magnetic field,  $\phi/\phi_0 = 0, 0.05, 0.1$ . Bright regions, corresponding to large values of  $|\Delta|$ , constitute "superconducting islands", separated by regions of small  $|\Delta|$  (dark regions). Increasing magnetic field leads to attenuation of the SC order parameter and even to the vanishing of SC order in some areas of the sample. Calculation is performed on a  $12 \times 12$  size lattice with electron density  $\langle n \rangle = 0.875$ , disorder strength W = 1 and interaction strength U = 2 (this value for U is maintained throughout).

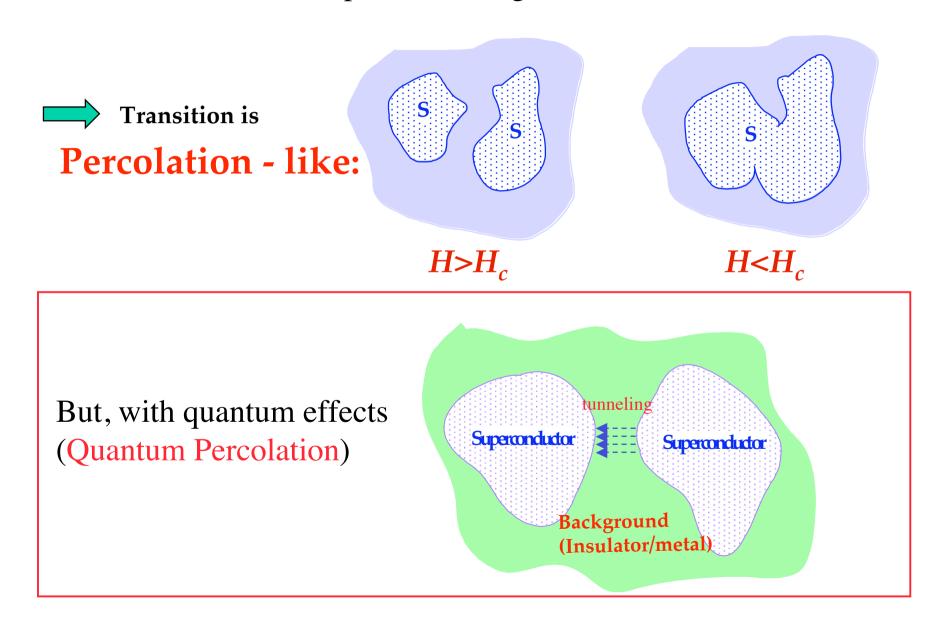
#### Finite-magnetic-field



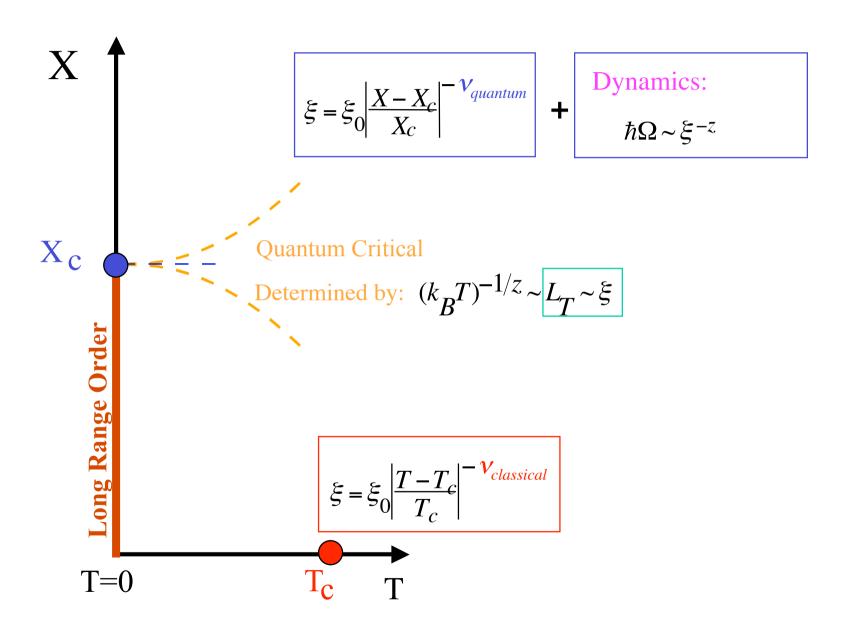
C. Howald, P. Fournier, and A. Kapitulnik, Phys. Rev. B 64 (2001), 100504.

A. C. Fang, L. Capriotti, D.J. Scalapino, S.A. Kivelson, and A. Kapitulnik, Phys. Rev. Lett. 96 (2006), 017007.

Modeling the inhomogenous nature of superconductivity near the transition to the superconducting state:



## Quantum Critical Point



Approach to the critical point: Н For a plane of finite disorder: Pure system  $H_{c2}$ melting  $T_{KT}$   $T_{CO}$  T $\overset{\circ}{T}_{M}$ Field R Influence of Bose QCP Metal Superconductor R Temperature

## Field-tuned SIT at T=0: Scaling Theory

\* M.P.A. Fisher, Phys. Rev. Lett. (1990).

Assume a continuous quantum transition at the critical field  $H_c$ . The correlation length near the transition is:

$$\xi \sim \left| \frac{H - H_c}{H_c} \right|^{-\nu}$$

The dynamics of the transition is described by the vanishing frequency as the transition is approached:

$$\Omega \sim \xi^{-z}$$

$$(\xi \sim \Omega^{-1/z})$$

From vortex displacement argument the current density must enter the scaling relation with the combination:

$$\frac{(\hbar/2e)J\xi}{\hbar\Omega}$$

From Cooper pair displacement argument the electric field E will appear in the scaling function with the combination:

$$\frac{2eE\xi}{\hbar\Omega}$$

Thus, the J-E relation near the Superconductor-Insulator transition should be:

$$E=rac{\hbar\Omega}{2e\xi}\mathcal{E}_{\pm}\left(rac{J\xi}{2e\Omega}
ight)$$

## Finite temperature and electric field:

At finite temperature,  $k_BT$  will determine the frequency scale:  $L_T \sim (k_BT)^{-1/z}$ 

Similarly, for the electrostatic energy:  $L_E \sim E^{-1/(z+1)}$ 

General scaling function: 
$$R(H, T, E) = R_c \mathcal{F}\left(\frac{C_a|H - H_c|}{T^{1/z\nu}}, \frac{C_b|H - H_c|}{E^{1/(z+1)\nu}}\right)$$

The resistance near the transition in the Ohmic regime is given by:

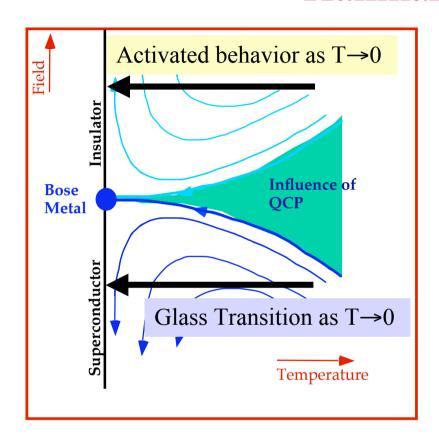
$$R = R_c \mathcal{F}_T \left( rac{c_T |H - H_c|}{T^{1/z
u}} 
ight)$$
  $c_T$  is a constant

And the non-linear resistance at **fixed temperature\***:

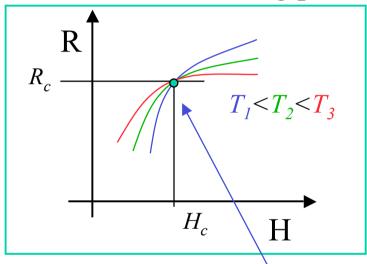
$$R = R_c \mathcal{F}_E \left( rac{c_E |H - H_c|}{E^{1/(z+1)
u}} 
ight)$$
  $c_E$  is a constant

<sup>\*</sup> Provided that heating is not present  $(T > T_{\phi})$  A. Yazdani and A. Kapitulnik, Phys. Rev. Lett. 74, 3037 (1995).

#### Hallmarks of the SIT



Isotherms show a crossing point!



$$R_c = \frac{h}{(2e)^2} \approx 6.5 \ k\Omega/\Box$$

$$R = R_c \mathcal{F}_T \left( \frac{c_T |H - H_c|}{T^{1/z\nu}} \right)$$

#### With the above arguments:

Critical behavior is expected to be in the same universality class as quantum percolation:  $\mathbf{v} \approx 2.3 \ (\approx 7/3)$ 

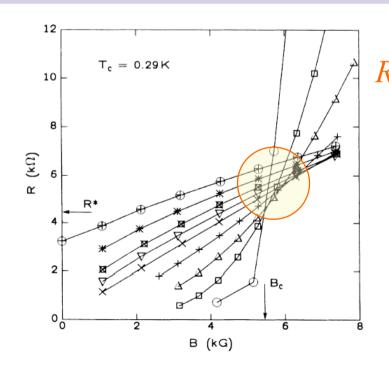
## Early work: Scaling - InOx

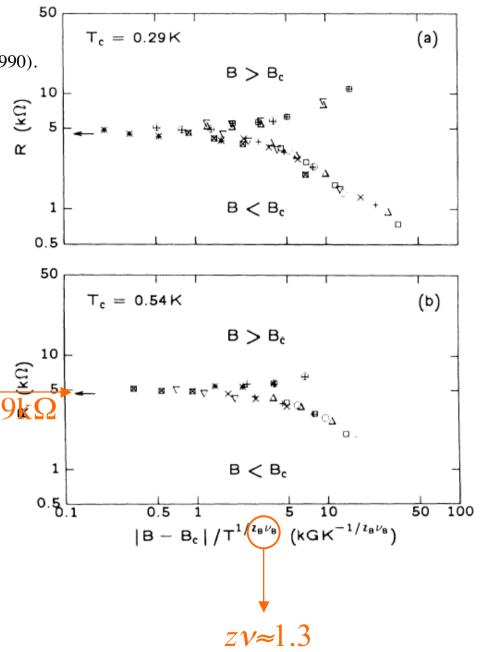
A. F. Hebard and M. A. Paalanen, Phys. Rev. Lett. 65, 927 (1990).

$$R = R_c \mathcal{F}_T \left( \frac{c_T |H - H_c|}{T^{1/z\nu}} \right)$$

#### Which implies:

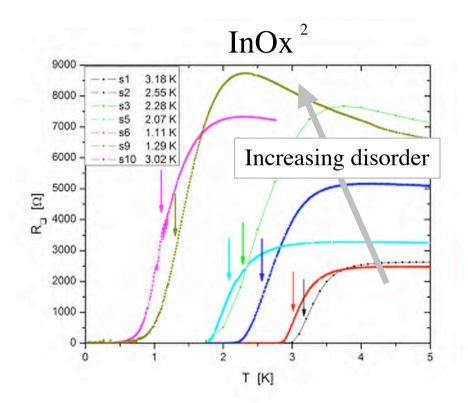
$$R(H = H_c) = R_c \mathcal{F}_T(0) \equiv R_c$$



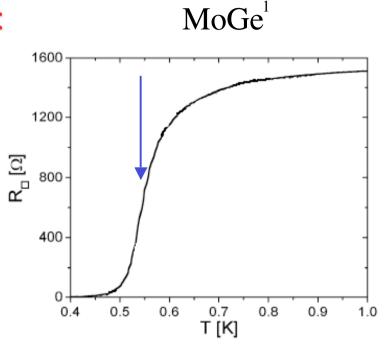


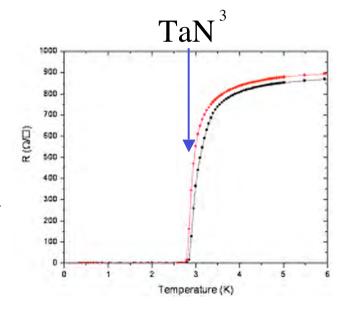
#### More recent results:

## Superconducting Transitions (H=0):



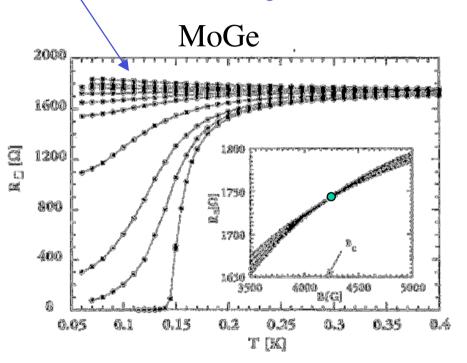
- 1. A. Yazdani and A. Kapitulnik, Phys. Rev. Lett. 74, 3037 (1995).
- 2. M. Steiner and A. Kapitulnik, Physica C 422, 16 (2005)
- 3. N. Brezney and A. Kapitulnik, (2008)

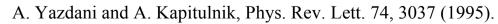




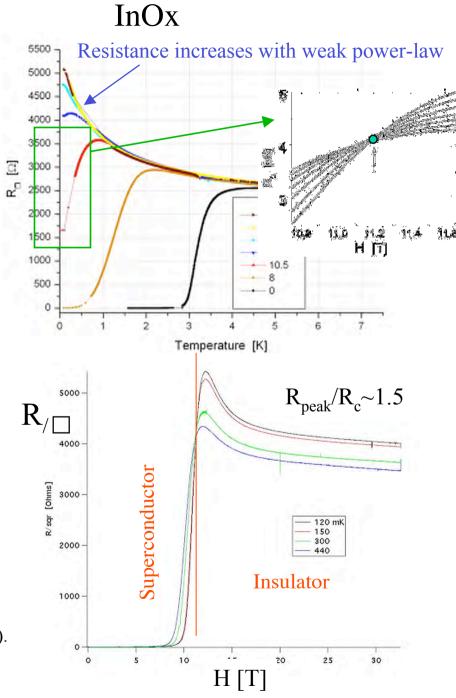
#### Weak disorder behavior

Resistance increases according to weak-localization



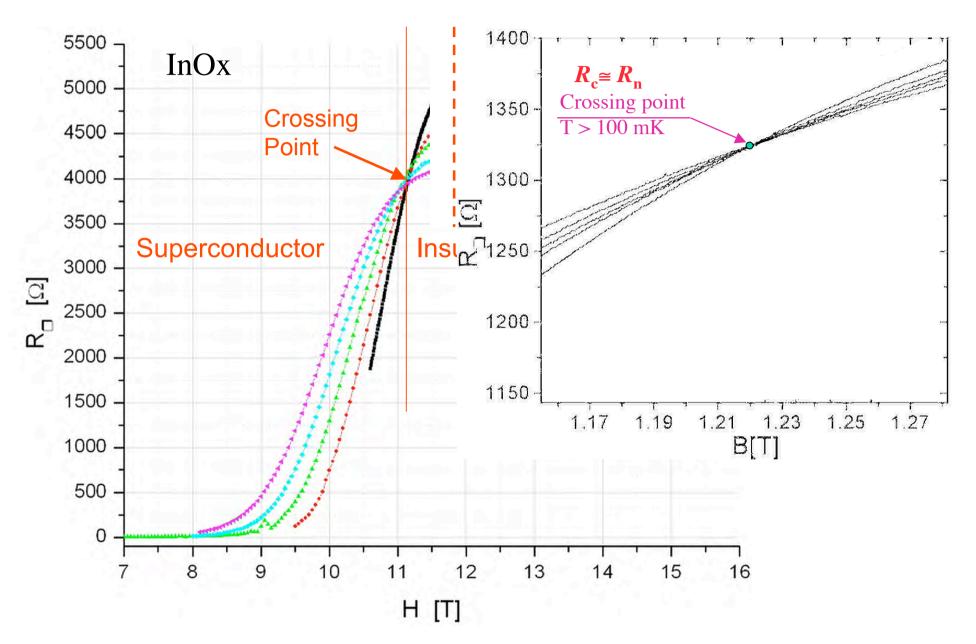


M. Steiner and A. Kapitulnik, Physica C 422, 16 (2005)

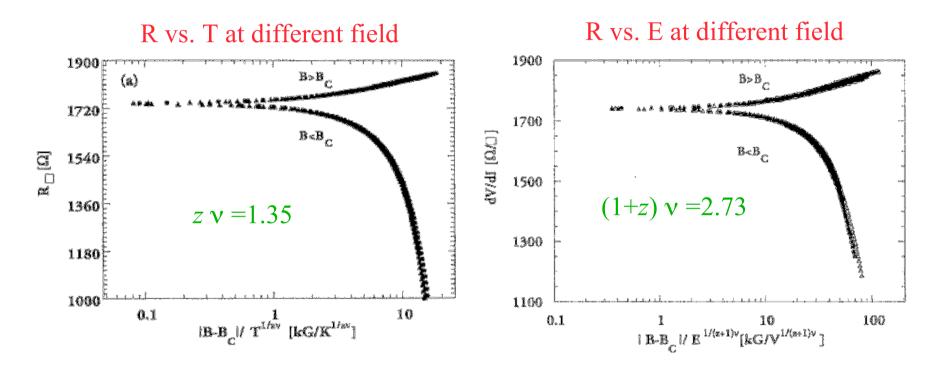


## Crossing points:





## Scaling (MoGe)



Solve for z and  $\nu$ :

$$v = 1.35$$
;  $z = 1.00$ 

Note:  $v \approx 1.35$  represents a <u>classical</u> percolation exponent indeed - <u>a metallic phase intervenes!</u>

## Properties of the Superconducting phase

- Zero-resistance Resistance will approach zero in an activated fashion
- •Vortices are localized with disorder: **VORTEX GLASS**

For Vortex Glass phase: 
$$\xi_g \sim T^{-\nu}g$$

I-V will have non-linear component that sets in at

$$I_{nl}(T) \sim T/\xi_g \sim T^{1+\nu_g}$$

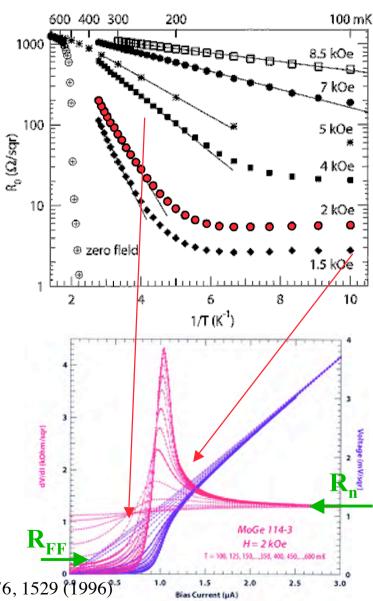
(at T=0 no linear resistance as R=0)

## A "Metallic State" emerges at low-T

#### For MoGe Films:

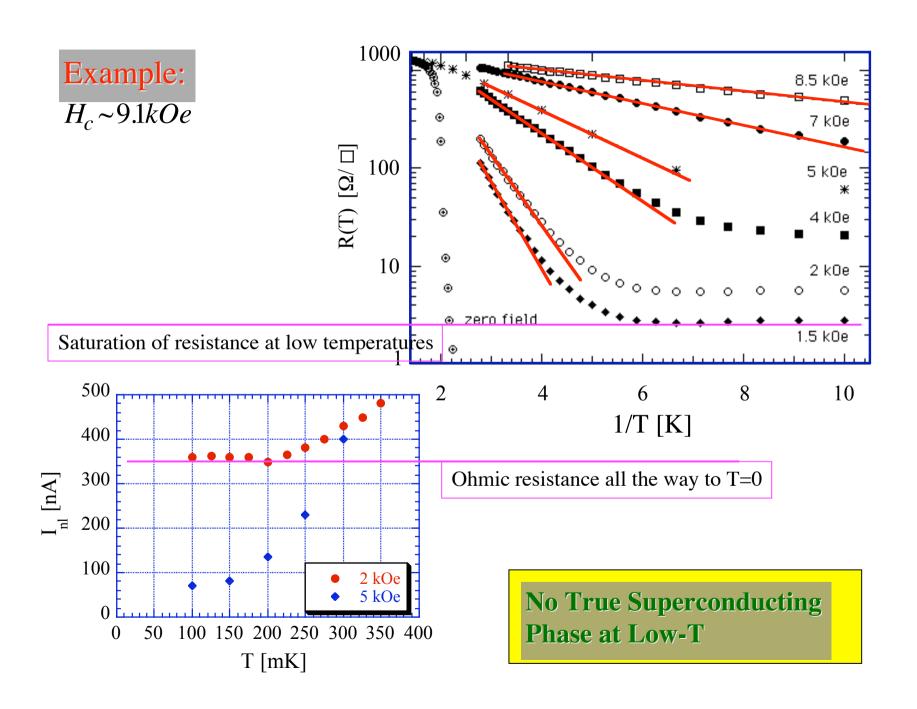
•A new metallic phase with resistance 2-3 orders of magnitude lower than the "Fermi" resistance:

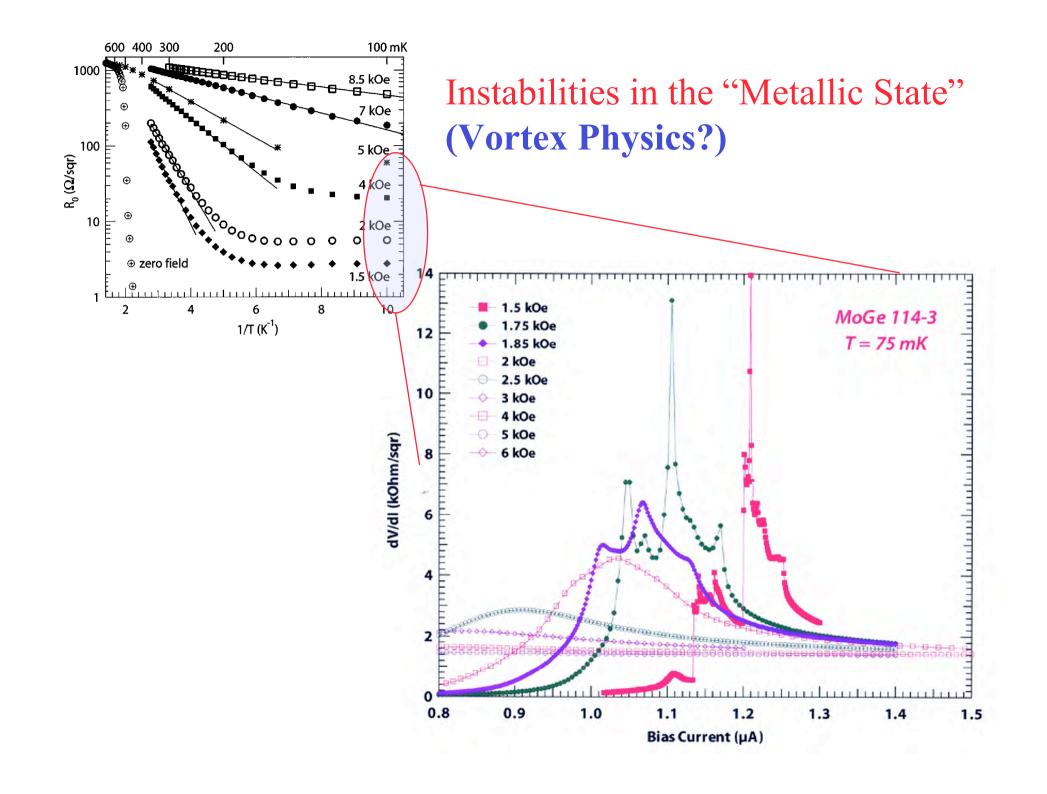
•The metallic phase is almost a superconductor. Internal Josephson couplings dominate:



D. Ephron, A. Yazdani, A. Kapitulnik, and M. R. Beasley, Phys. Rev. Lett. 76, 1529 (1996)

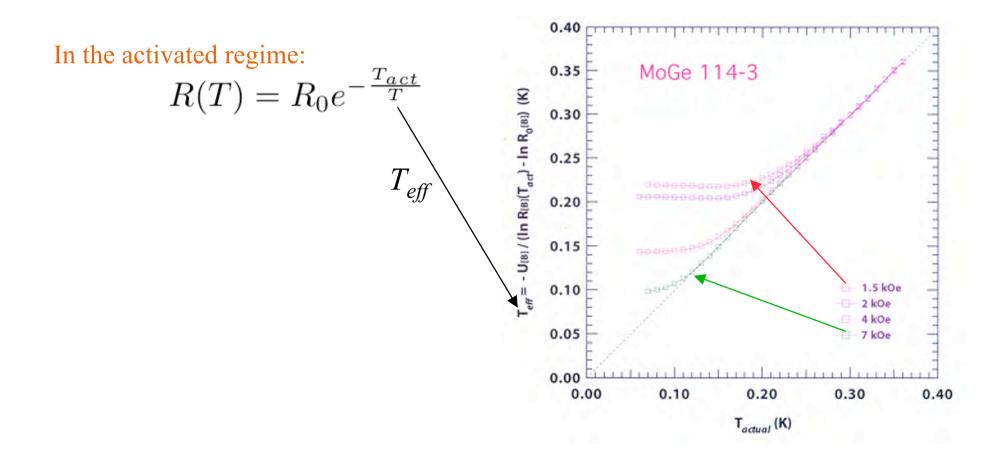
N. Mason and A. Kapitulnik, Phys. Rev. Lett. 82, 5341 (1999).

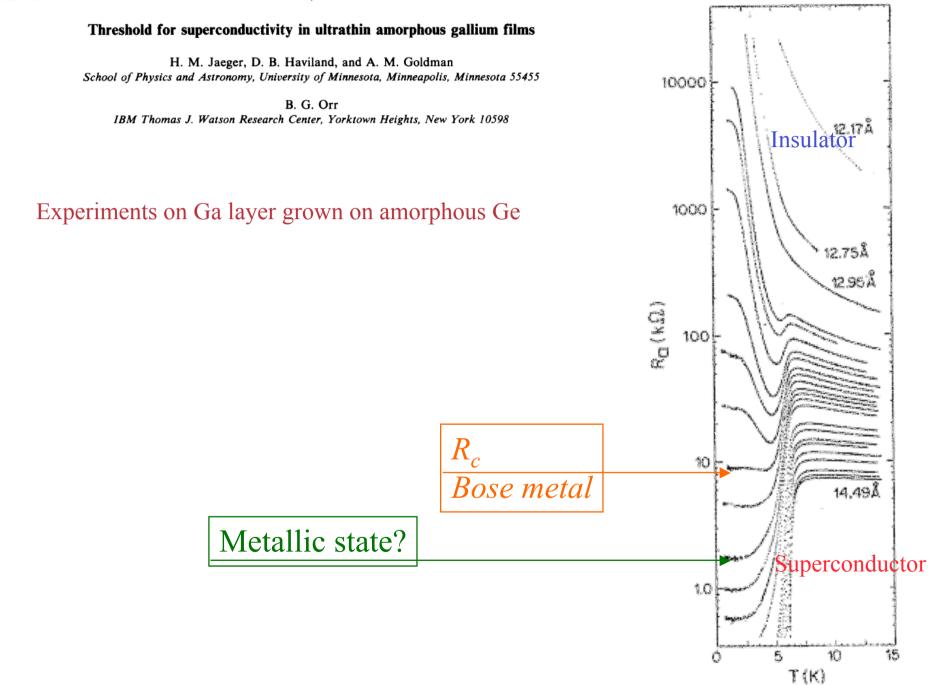




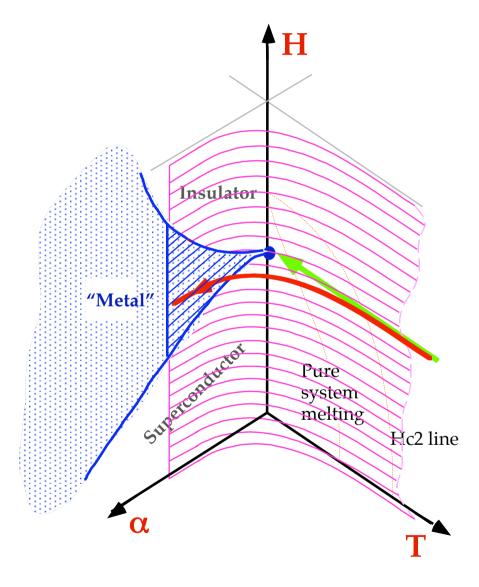
## It is not heating!

- 1. Similar Sample (with no superconductivity, same n,  $R_{\square}$ ) shows Weak Localization log(T) down to 50 mK
- 2. Compare effective activation temperature to actual temperature: (similar treatment to that used in MQT by J. Clarke et al.)

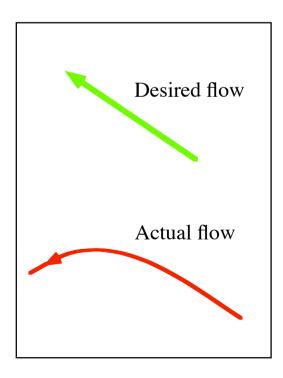




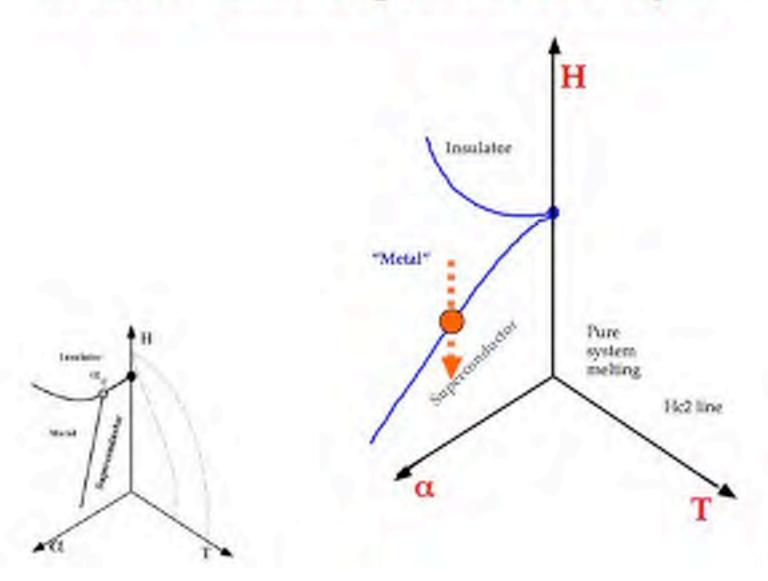
## Proposed flow in phase space during experiment:



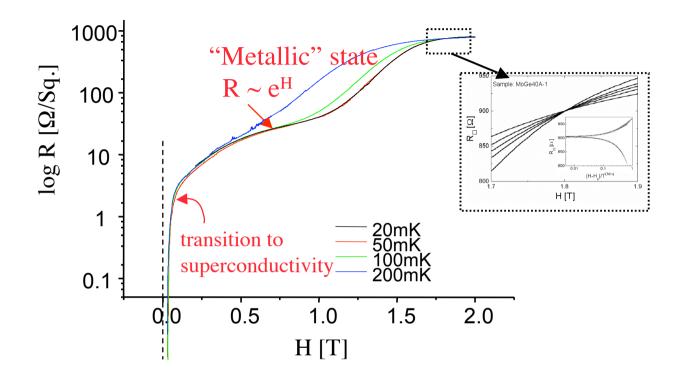
α can be e.g. dissipation



# Is there "True Superconductivity"?

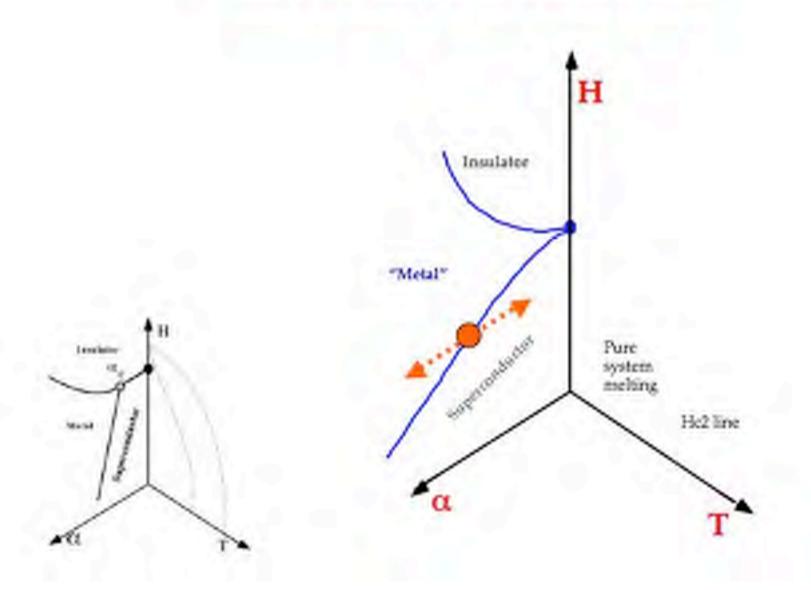


•At low fields the metallic phase undergoes a transition to a true superconductor.



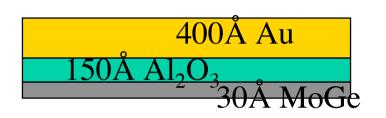
- •There is an underlying Superconductor-Insulator transition that is Bose-dominated that the metallic phase "knows" about.
- •The almost superconductor-insulator transition is percolation like With classical-percolation exponent.

# Introducing a Ground Plane



Modify properties of transition with metallic plane near sample ...

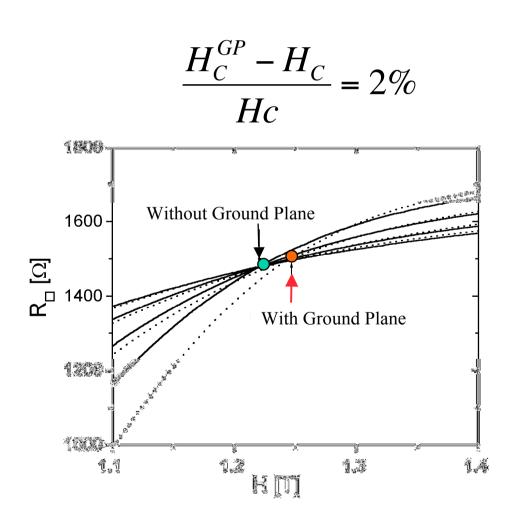
- $\bigstar$  Change dynamical scaling exponent  $\Omega \sim \xi^{-z}$ ?
  - z=1 for charged bosons w/ long-range Coulomb repulsion
  - z=2 for neutral (or screened) bosons

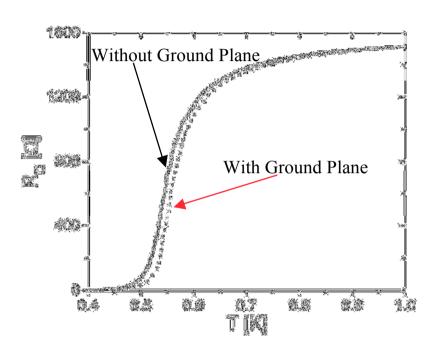


In pure boson picture, screening length should be  $<\xi \sim \xi_{GL} \, |H-H_c|/H_c$   $\xi_{min} \sim 150 \, \text{Å}$ 

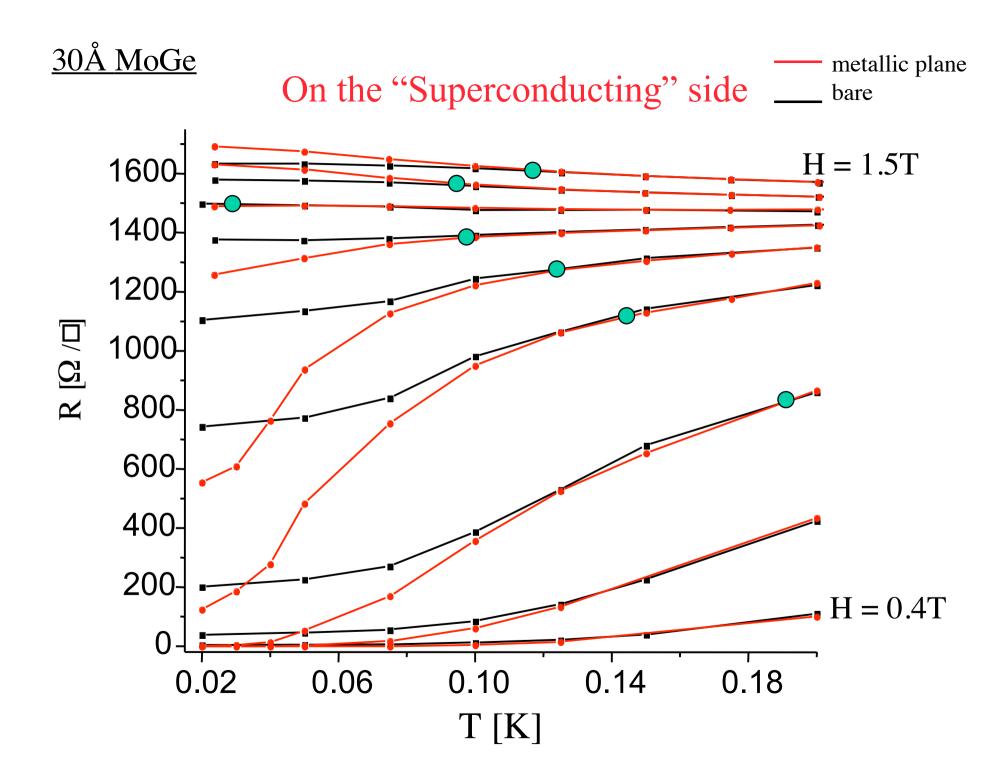
★ Metallic plane could change dissipative environment diss. ~ 1/R

## Effects on T<sub>c</sub> and H<sub>c</sub>

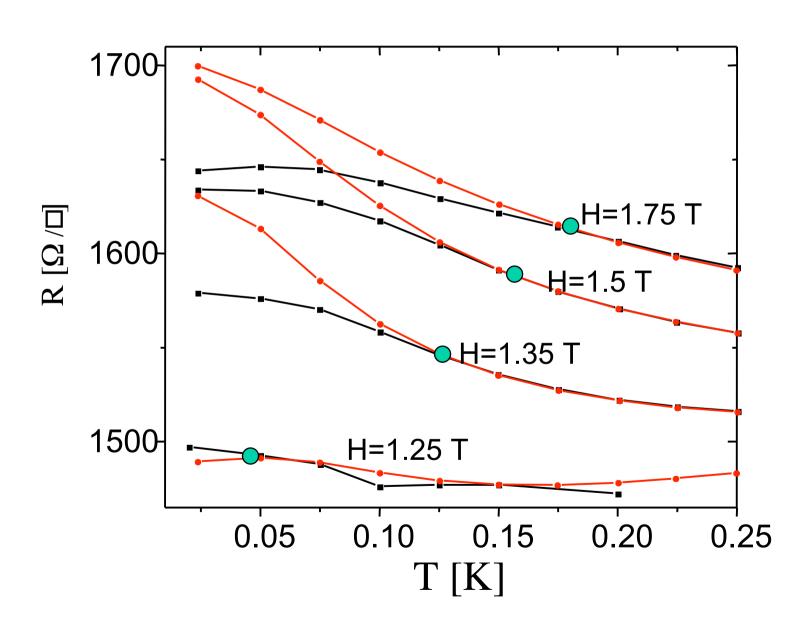




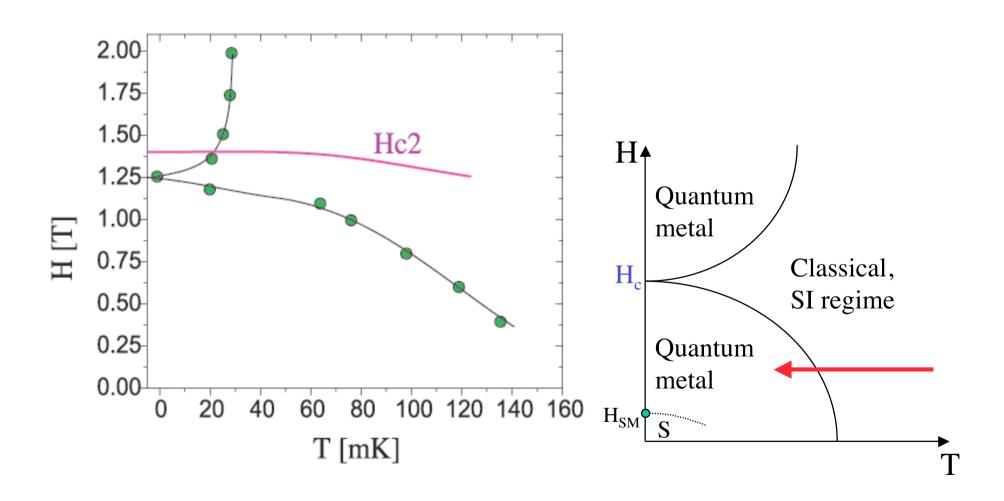
$$\frac{T_C^{GP} - T_C}{Tc} = 2\%$$



### On the "Insulating" side

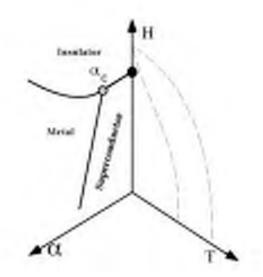


Ground plane starts to have an effect when system crosses over into quantum metal phase:

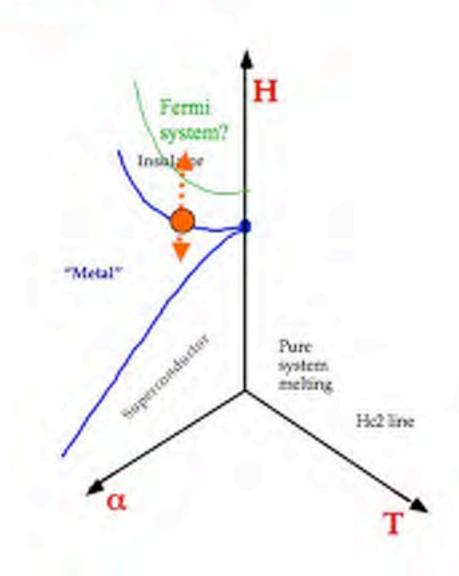


#### Conclusions for weak insulators:

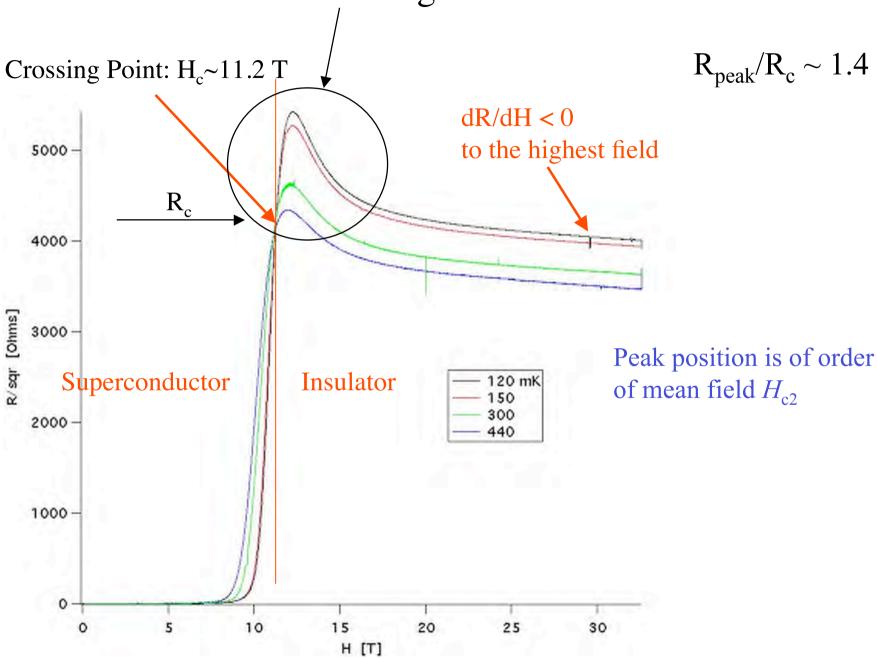
- •A new metallic phase with resistance 2-3 orders of magnitude lower than the "Fermi" resistance
- •The metallic phase is almost a superconductor. Internal Josephson couplings.
- •At low fields the metallic phase undergoes a transition to a true superconductor
- •There is an underlying Superconductor-Insulator transition that is Bose-dominated that the metallic phase "knows" about.
- •The almost superconductor-insulator transition is percolation like With classical-percolation exponent.



## How good is the insulator?

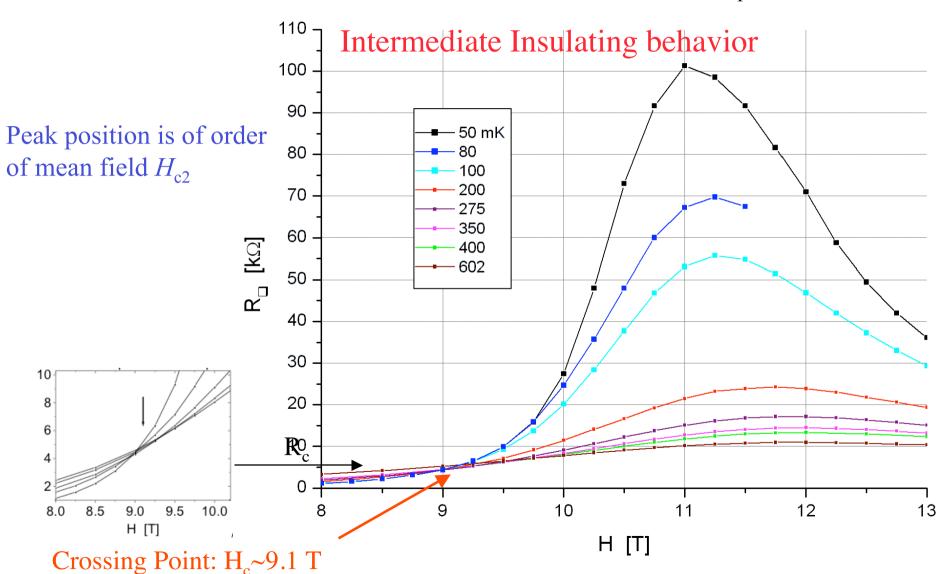


For weak disorder Insulating behavior is "weak"

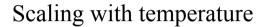


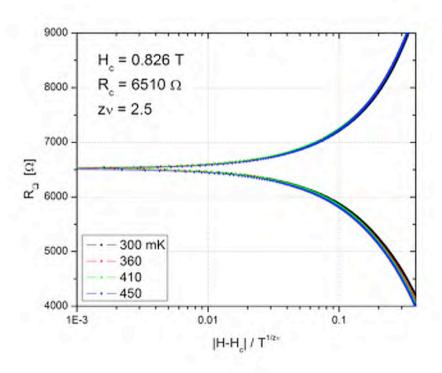
#### Intermediate disorder



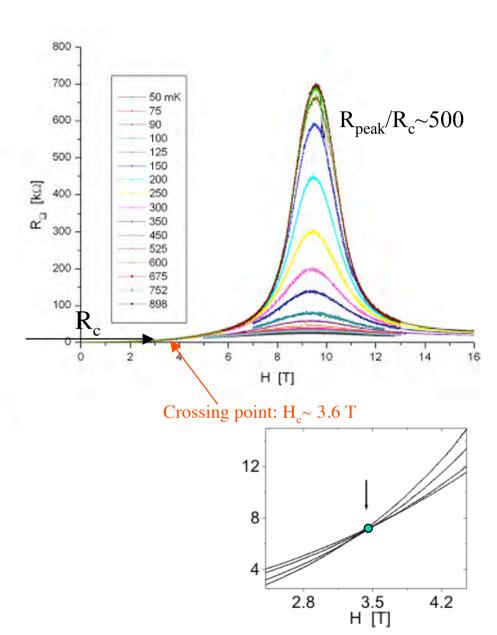


## Strong disorder behavior (InO<sub>x</sub>)

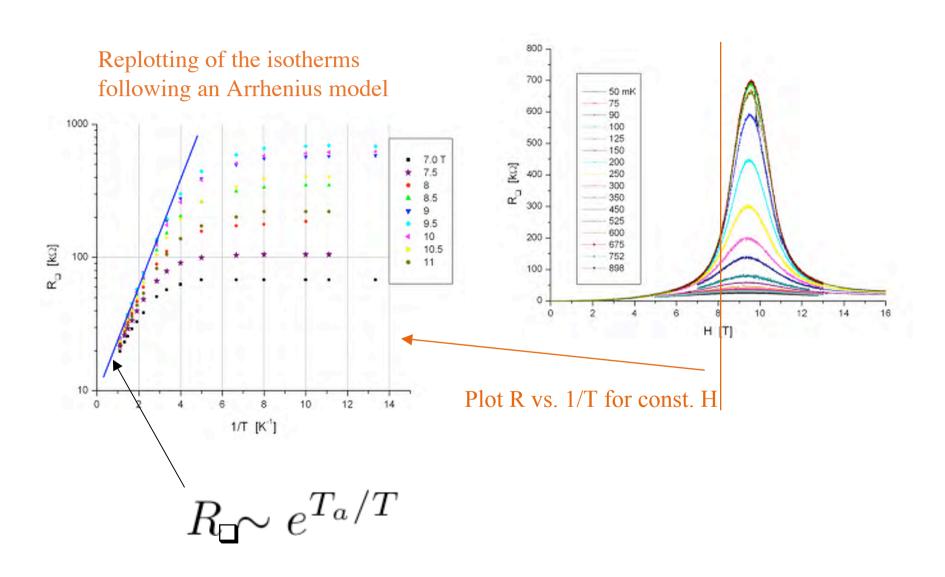




Observed v is consistent with quantum percolation: zv = 2.5

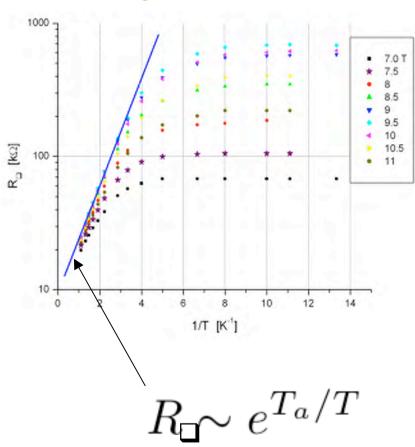


### Activated behavior of the strong insulator

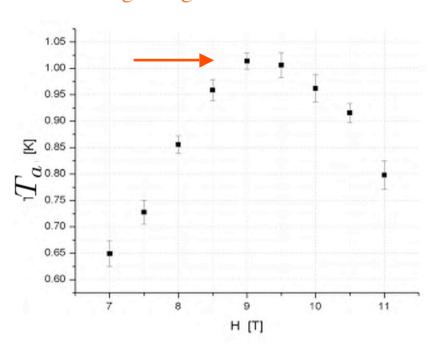


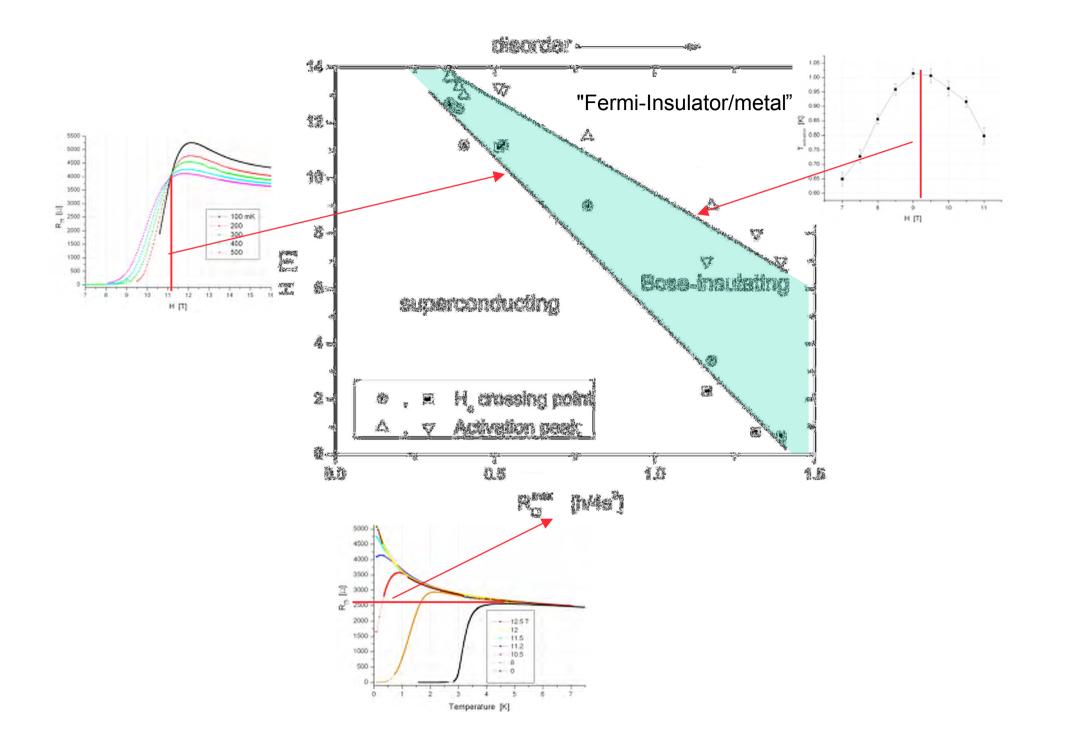
#### Repeat for each field:

Replotting of the isotherms following an Arrhenius model



# Activation temperature at high magnetic fields





### Construction of Summary phase diagram:

Scale the conductance and field to produce a global phase diagram:

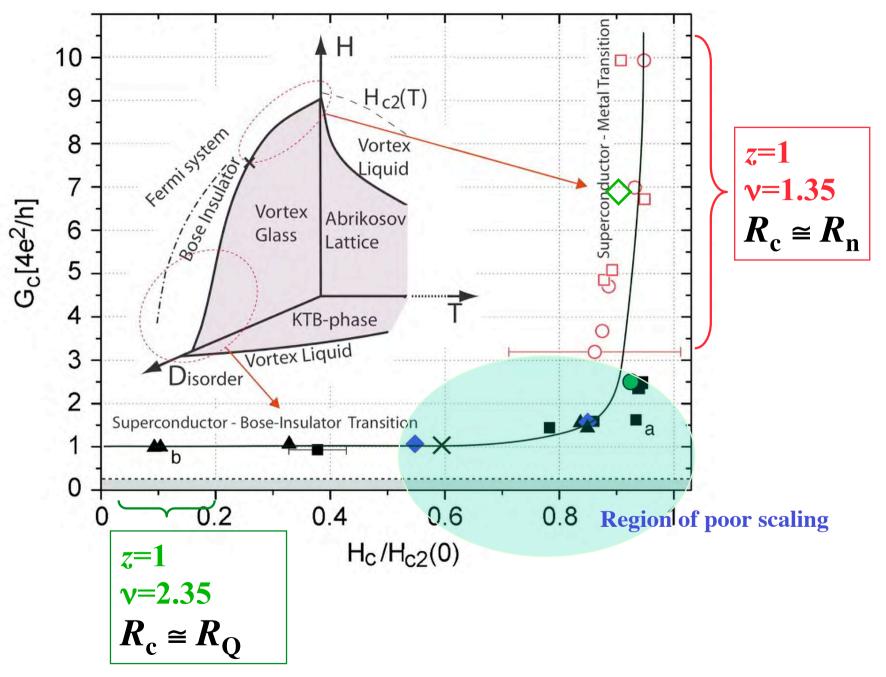
→ Scale the critical conductance by the quantum conductance of pairs:

$$G_Q = \frac{4e^2}{h} \approx [6.5 \text{ k}\Omega]^{-1}$$

 $\rightarrow$  Scale the critical field by the (mean field) upper critical field -  $H_{c2}(0)$ .

\* The upper critical field is found from the low field slope and from the **peak of the** magnetoresistance (which is found to be close to  $H_{c2}(0)$ ).

Myles A. Steiner, Nicholas P. Breznay and Aharon Kapitulnik, Phys. Rev. B 77, 212501 (2008).



(Samples used: amph-InO<sub>x</sub>, poly-InOx,TaN, Ta, amph-MoGe)

#### Conclusions:

- 2-D superconductors in **magnetic field** exhibit a tendency towards a superconductor-insulator transition.
- In the regime of weak disorder critical behavior (as observed from finite temperature) is consistent with classical percolation, but is disrupted by the appearance of **metallic phase**.
  - True superconductivity is achieved at lower fields.
- In the regime of strong disorder critical behavior (as observed from finite temperature) is consistent with quantum percolation.

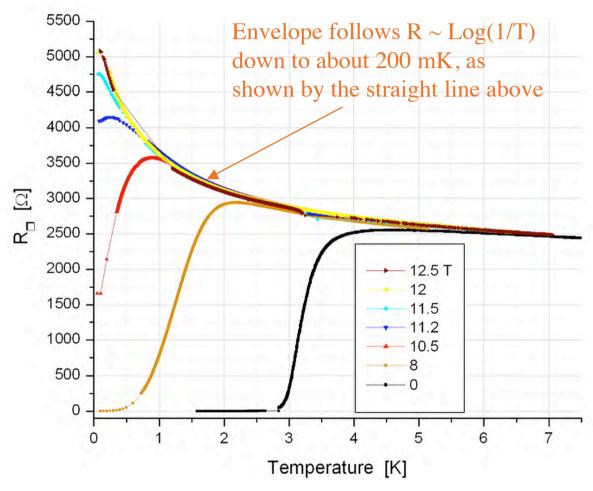
0.2 0.4 0.6

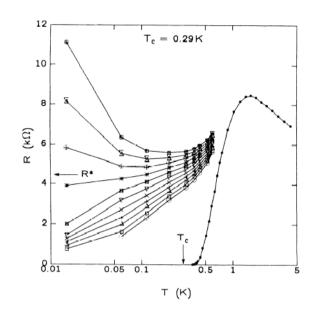
→ In these films a strong insulating behavior is expected at low T.

 $H_{c}/H_{c2}(0)$ 

# Observations Relevant to High-Tc

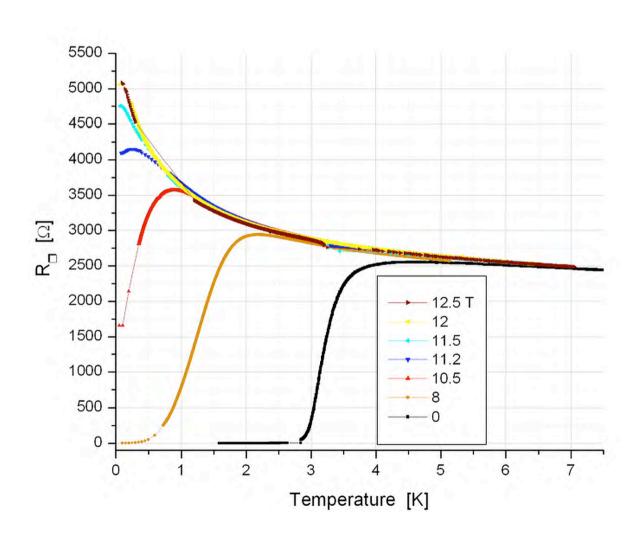
#### Resistive Transitions in a Field





M.A. Steiner, G. Boebinger and A. Kapitulnik, Phys. Rev. Lett. 94, 107008 (2005)...

#### Back to the envelope behavior: $R \sim logT$



## Logarithmic Divergence of both In-Plane and Out-of-Plane Normal-State Resistivities of Superconducting La<sub>2-x</sub>Sr<sub>x</sub>CuO<sub>4</sub> in the Zero-Temperature Limit

Yoichi Ando,\* G. S. Boebinger, and A. Passner

AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974

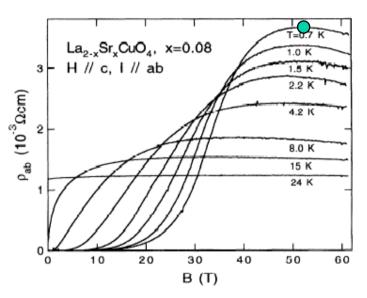


FIG. 1. In-plane resistivity  $\rho_{ab}$  versus magnetic field for the  $x = 0.08 \text{ La}_{2-x} \text{Sr}_x \text{CuO}_4$  single crystal at various temperatures.

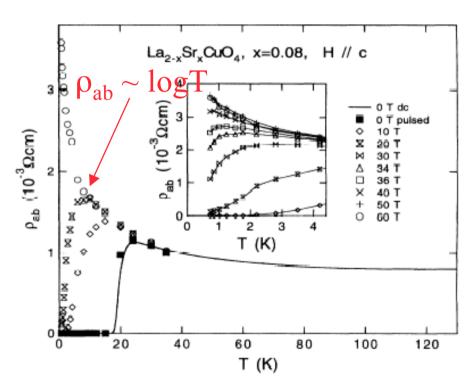
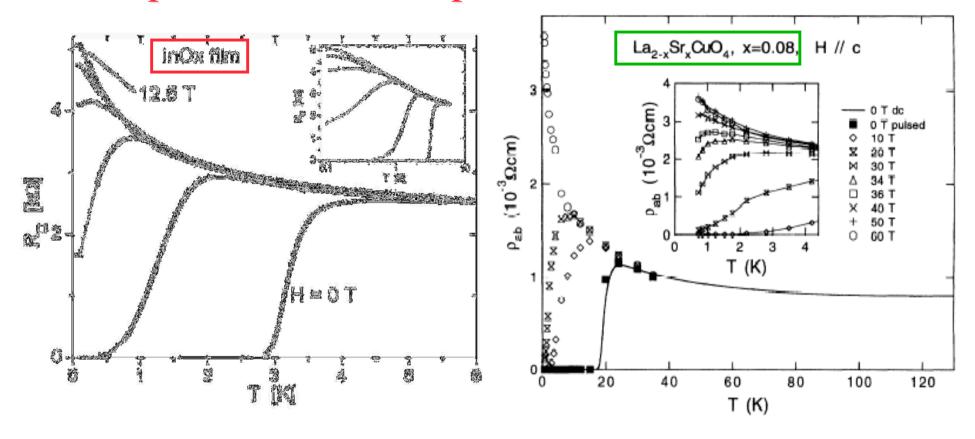


FIG. 2. Temperature dependence of  $\rho_{ab}$  in 0, 10, 20, and 60 T, obtained from the pulsed magnetic field data. The solid line shows the zero-field resistive transition. The inset contains the low-temperature data.

### Comparison of Envelopes



In-plane resistivity measured with a perpendicular magnetic field

- Amorphous, type II BCS film
- Field rolls over at  $\sim 12.5 \text{ T}$

- High-Tc, layered structure
- Field saturates at  $\sim 50 \text{ T}$

The "logT" phase in high-Tc IS NOT the "normal state"

There are still a large number of pairs in this phase.

High-Tc in a field could be close to a field-tuned SIT where pairs persist much above the mean field  $H_{c2}$ .

Results support the Nernst effect measurements\* which find "vortices" at fields much above where the "normal state" is achieved.

\* Z. A. Xu, N. P. Ong, Y. Wang, T. Kakeshita, and S. Uchida, Nature 406, 486 (2000).

Yayu Wang, N. P. Ong, 1 Z. A. Xu, T. Kakeshita, S. Uchida, D. A. Bonn, R. Liang, and W. N. Hardy, Phys. Rev. Lett. 88, 257003 (2002).

C. Capan, K. Behnia, J. Hinderer, A. G. M. Jansen, W. Lang, C. Marcenat, C. Marin, and J. Flouquet, Phys. Rev. Lett. 88, 056601 (2002).