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Towards Realization of Topological Order in Josephson Arrays

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Superconducting Nanocircuits for Topologically Protected Qubits

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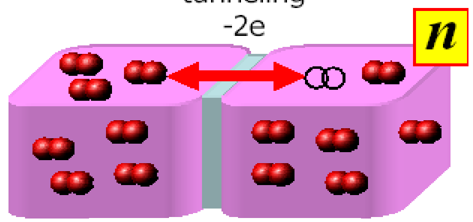
Trieste, May 19, 2009

Outline

- Topological order in large Josephson arrays
- Need for protection of superconducting qubits
- Protection in small Josephson arrays
- Proof-of-concept experiments with small Josephson arrays

Josephson Junctions: non-linear and non-dissipative at ultra-low *T*



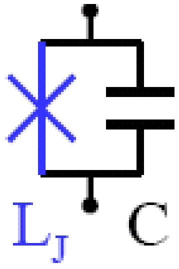
$$\varphi = \theta_1 - \theta_2$$

- superconducting phase difference

Josephson relations:

$$I_S = I_C \sin\varphi$$

$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$



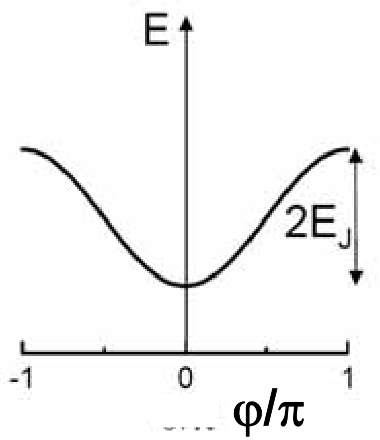
- nonlinear *LC* resonator

Josephson energy

$$\boxed{E_J = -E_{J0}\cos\varphi = -\frac{I_C\Phi_0}{2\pi}\cos\varphi}$$

I_C- the critical current

Φ_0 – the magnetic flux quantum



Charging energy

$$\boxed{E_C = \frac{e^2}{2C_J}}$$

C_J - the capacitance of the tunnel junction

E_J/E_C ~ (JJ area)^2 , typically, E_J/E_C ~1 for an area ~ 0.01 μm^2.

Charge and phase – conjugate variables

$$\hat{H} = E_C(\hat{n} - n_0)^2 - E_{J0}\cos\varphi - I\varphi$$

current

$$\hat{n} = i\frac{\partial}{\partial\varphi}$$

- number of Cooper pairs

$$[\hat{n}, \hat{\varphi}] = -i$$

The uncertainty relation for a superconductor: $\Delta n \cdot \Delta\varphi \geq 1$

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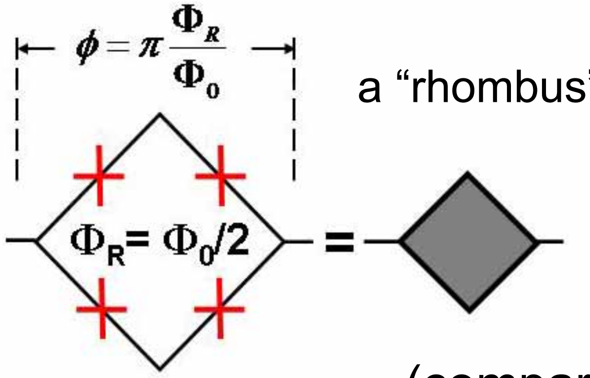
"cos2φ" Josephson element

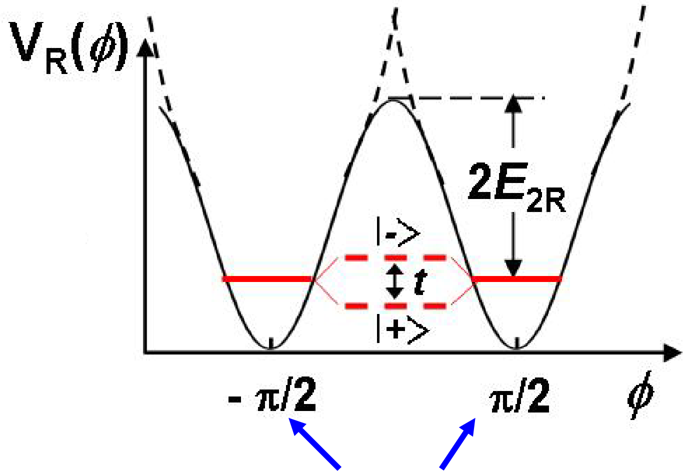
B. Doucot and J. Vidal, *PRL*. **88**, 227005 (2002)
L. Ioffe and M. Feigelman, *PRB* **66**, 224503 (2002)



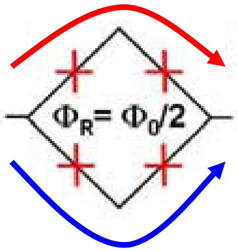
a "rhombus"

$$V_R = E_{2R} \cos 2\phi$$
$$E_{2R} \sim E_{J0}$$

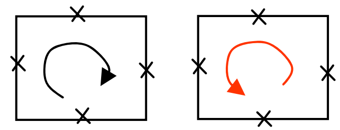
(compare with $E_J = -E_{J0} \cos\phi$)



Classical states: Δφ = ± π/2
(Δφ = ± π/4 across each JJ)

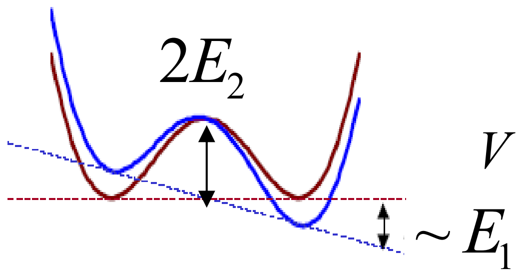


destructive interference between two tunneling processes



$$|+\rangle = \frac{1}{\sqrt{2}}\left(|\circlearrowright\rangle + |\circlearrowleft\rangle\right)$$
$$|-\rangle = \frac{1}{\sqrt{2}}\left(|\circlearrowright\rangle - |\circlearrowleft\rangle\right)$$

This rhombus is a faulty qubit (***unprotected*** against local noises): deformations of the potential $V_R(\phi)$ lead to ***dephasing***.



$$V(\phi) \approx E_{2R} \cos(2\phi) + E_{1R} \cos(\phi)$$

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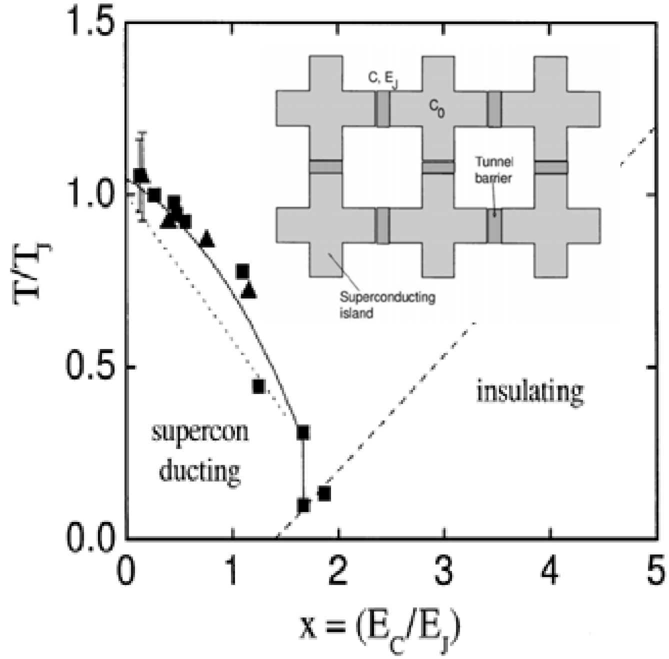
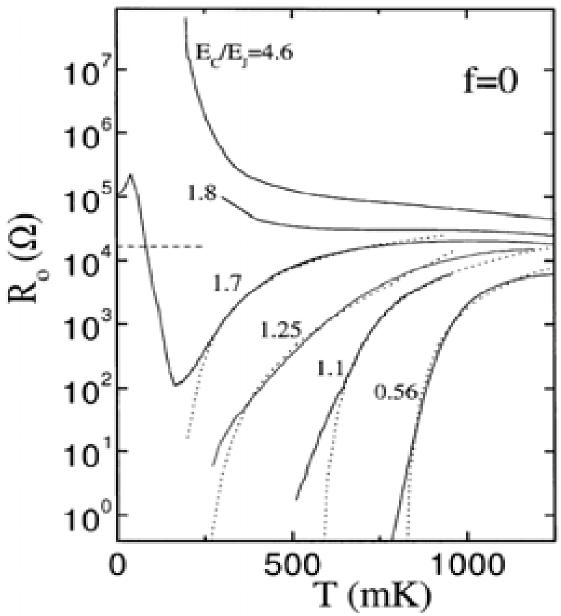
Conventional and Unconventional Josephson Arrays

Conventional Arrays

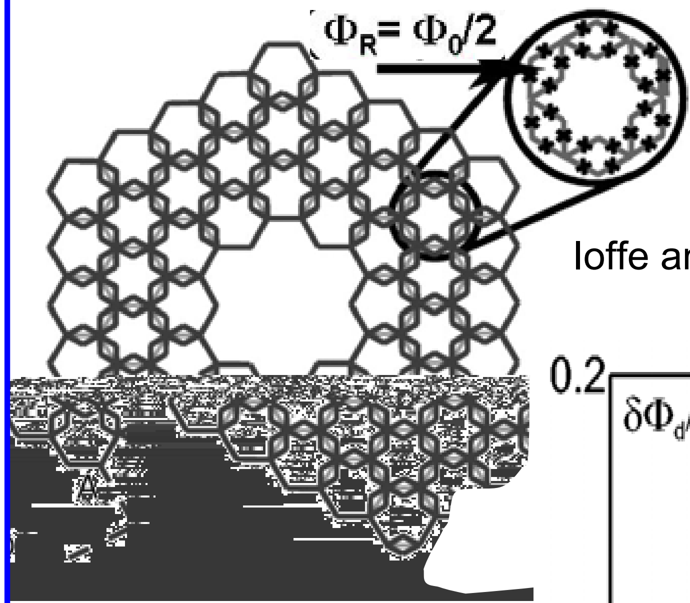




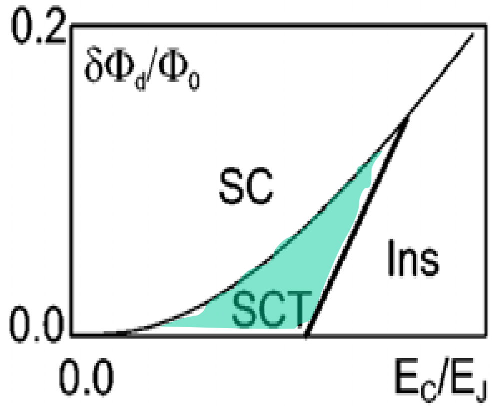
van der Zant *et al*, '96

Localization of Cooper pairs at E_J/E_C~0.5 (the normal-state JJ resistance ~ $10^4\ \Omega$).

Unconventional Arrays



Ioffe and Feigel'man, 2002



Quantum fluctuations are enhanced due to "bi-stable" elements and lower coordination number.

Novel phase: the condensate of ***pairs*** of Cooper pairs in the absence of coherence in the single-Cooper–pair condensate (charge transport: coherent co-tunneling of pairs of Copper pairs, ***objects with charge 4e***).

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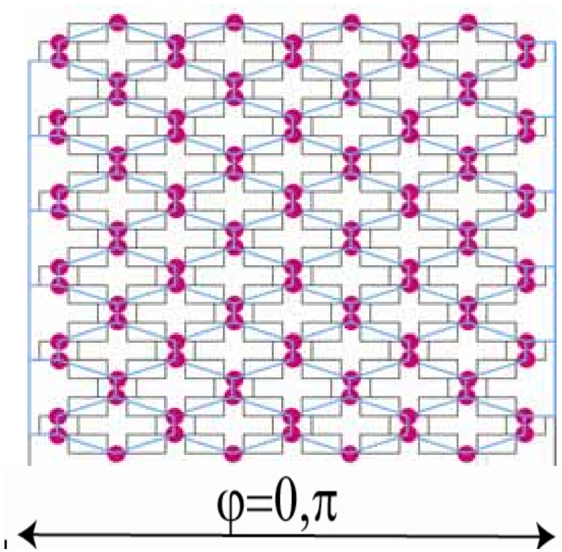
Topological Protection in Unconventional Josephson Arrays



The (degenerate) ground states of the array correspond to the phase difference across the array $\Delta\varphi = 0, \pi$.

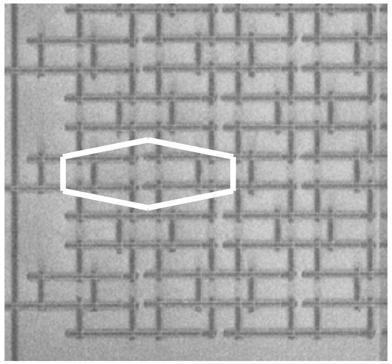
These logical states are protected from local noises by the topological order parameter.

Topological excitations:

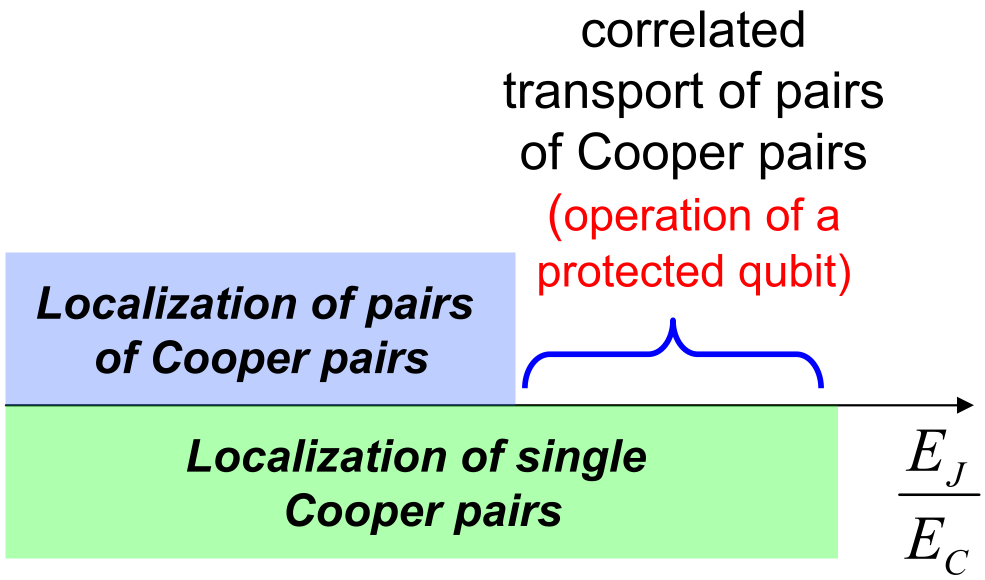
low-energy (gapped) excitations - charges 2e

high-energy excitations - "half-vortices" ($\Phi = \Phi_0 / 2$)

L. Ioffe and M. Feigelman, *PRB* **66**, 224503 (2002)
B. Doucot and J. Vidal, *PRL*. **88**, 227005 (2002)
B. Doucot *et al*., *PRB* **71**, 024505 (2005)
D. Bacon et al., (2006)

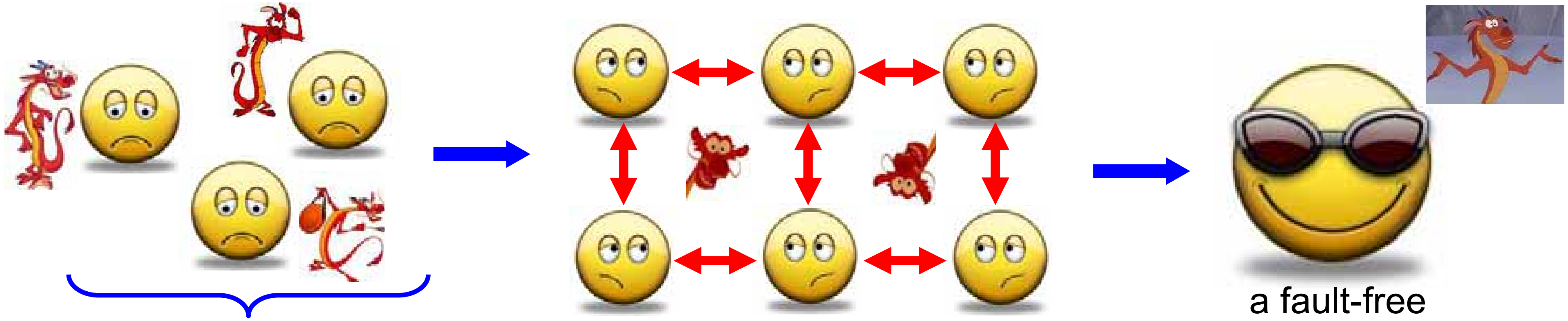


Decoupling from local noises is expected to grow ***exponentially*** with the array size.



Topological Protection from Noises

A. Kitaev: protected subspace created by a topological degeneracy of the ground state.



“Faulty” physical qubits: Josephson elements sensitive to noises

highly-entangled array of interacting Josephson elements

a fault-free logical qubit

Idea: to prevent errors by building a fault-free (topologically protected) logical qubit from “faulty” physical qubits with properly engineered interactions between them. The non-local logical states of such an array would be less sensitive to local noises, and the decoupling is expected to grow with the array size.

Hardware implementation of the software error correction!

From Protected Arrays to Qubits

protected Josephson array

Large (though incomplete) set of operations can be performed in the *protected* state (Kitaev *et al*.).

Even qubits protected only in their "idle" state are very useful devices.

In order to perform a generic gate operation, the qubit protection has to be removed for a short time:

$$t \approx \tau_0 \sim 10^{-2} \tau_d$$

the decoherence time of the *unprotected* qubit

- much reduced decoherence

Also, only a small fraction of qubits are addressed at a given moment of time, whereas all other qubits involved in computation remain in the protected state. Thus, the possibility to “turn off” decoherence in the idle state dramatically reduces the redundancy.

Advantages of topological protection:

- dramatically reduced redundancy
- no need to manipulate and control many individual physical qubits

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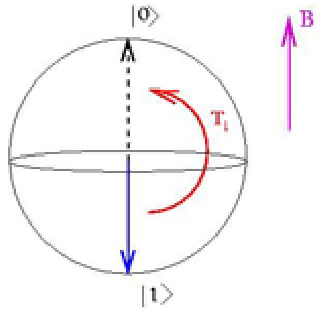
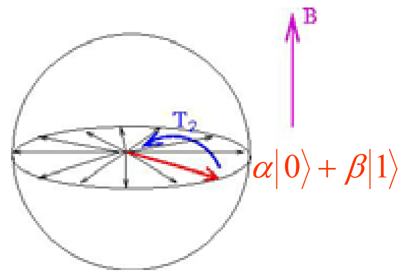
Relaxation and Dephasing

Decoherence – interaction with uncontrollable degrees of freedom in the qubit's environment not involved in computation ("**noise**").



T_1 = Relaxation time



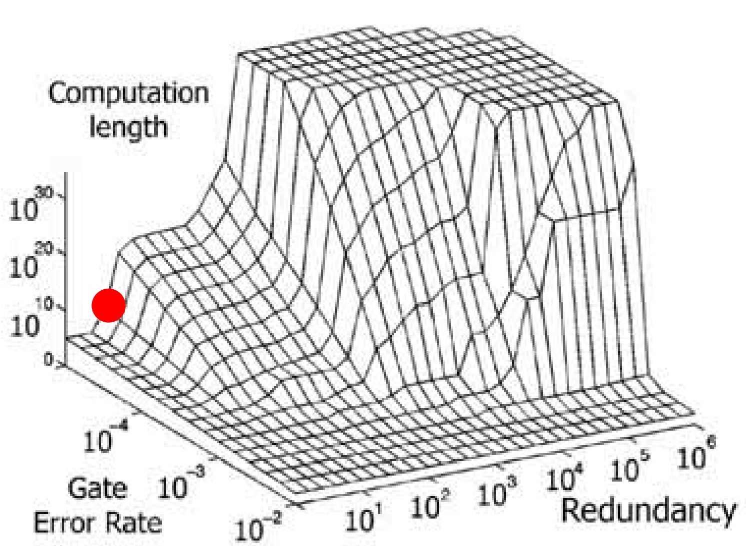
T_2 = Dephasing time

The error rate : $\boxed{\varepsilon \equiv \frac{\tau_0}{\tau_d}}$

τ_0 - the operation time

$\tau_d \approx \min(T_1, T_2)$ - the decoherence time

A. Steane



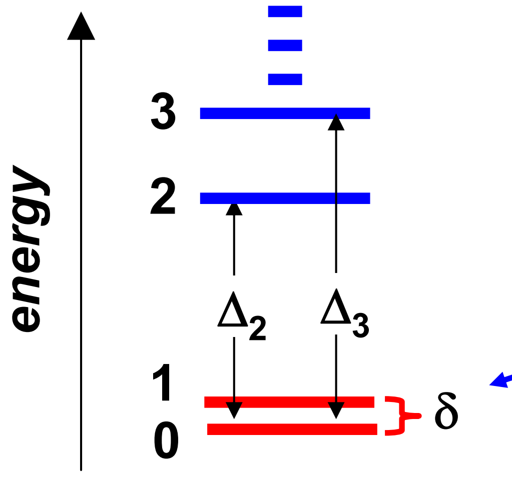
The "fault tolerance" theorem: once ***a sufficient accuracy and isolation*** is attained, there is a strategy for correcting errors so that one can carry out indefinitely long computations.

How good should be "accuracy and isolation" ?

Local noises,
realistic redundancy: $\varepsilon < 10^{-5}$

For correlated (non-local) noises, the threshold for QC increases dramatically.

What It Takes to Build a Good Qubit



the qubit spectrum

$$\boxed{\varepsilon \equiv \frac{\tau_0}{\tau_d}}$$

$\tau_d \approx \min(T_1, T_2)$ - the decoherence time

τ_0 - the operation time

two "isolated" energy levels used as the ***logical states***

- small δ (states "0" and "1" are almost degenerate) $\Rightarrow$ long ***energy relaxation time*** (***T_1***)

- external noise should not affect the energy splitting between states "0" and "1" $\Rightarrow$ long ***dephasing time*** (***T_2***)

$$e^{i(E_1 - E_0)t} = e^{i\delta t} \quad \Rightarrow \quad phase \propto \int \delta dt$$

- a large gap Δ_2 which separates logical states of a qubit from the rest of the spectrum

Δ_2 determines ***the shortest operation time*** of a qubit: $\tau_0 > \dfrac{h}{\Delta_2}$

$$\boxed{\varepsilon > \frac{h}{\tau_d \Delta_2}}$$

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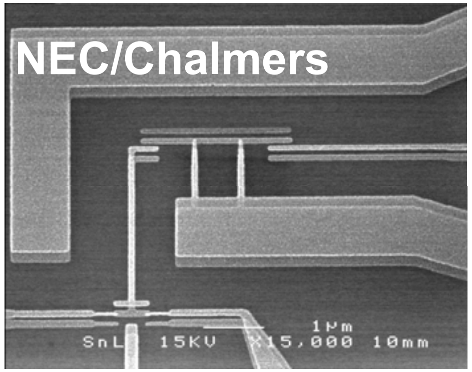
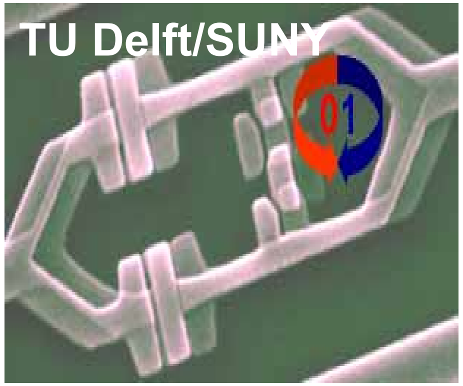
State-of-the-Art Superconducting Qubits

Charge

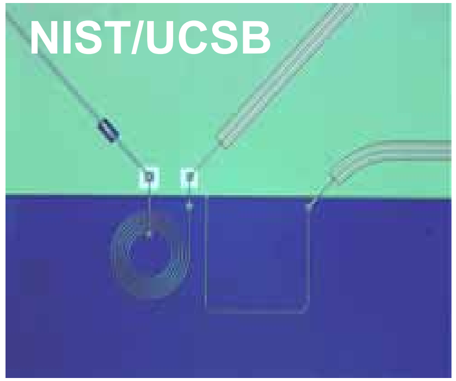


Charge/Phase

Flux



Phase



small (0.3) ⟵ E_J / E_C ⟶ **large (10^4)**

All these qubits behave as (different) ***macroscopic quantum two-level systems.***

typical qubit energies $E \sim 0.5K \sim 10GHz$

typical experimental temperature $T \sim 0.05K$

Main Issue: decoherence rate is high for all *scalable* qubits

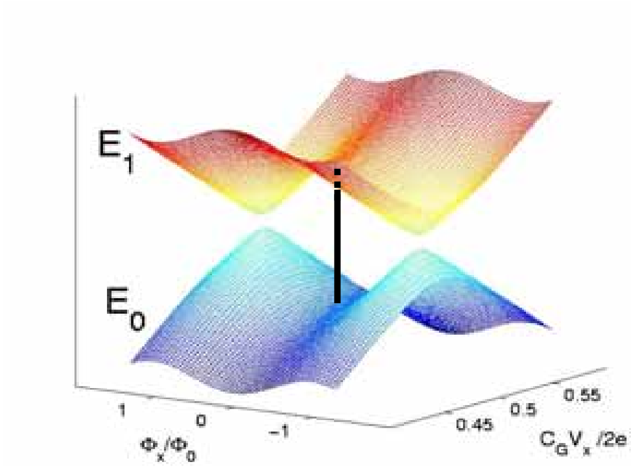
the decoherence time

the operation time

Design	Group	τ_d, μs	$\tau_{0,}$ μs	$\varepsilon = \tau_0/\tau_d$
Phase qubit	**UCSB**	**0.5**	**~0.01**	**~10^{-2} -10^{-1}**
Flux qubit	**NEC**	**1-2**	**~ 0.2**	**~10^{-2} -10^{-1}**
Transmon	**Yale**	**10**	**~0.05**	**~10^{-2}**
Quantronium	**Saclay**	**0.3**	**~ 0.01**	**~10^{-2} -10^{-1}**

Strong decoherence: s.c. qubits are sensitive to ***local noises***

- charge noise
- flux noise
- critical current fluctuations
- quasiparticle poisoning



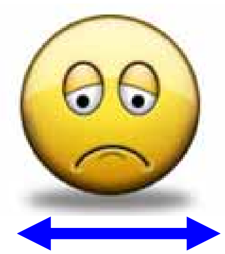
Vion *et al.*, CEA Saclay

Conventional Approaches to Increasing τ_d:

1. By noise suppression via materials research.

2. By operating a qubit within the "sweet spot" in the space of operational parameters (***first***-order decoupling).

The first-order protection $\varepsilon > 10^{-2}$



$\varepsilon < 10^{-5}$ The threshold for large-scale QC with local noises

Alternative approaches to qubit's protection are needed!

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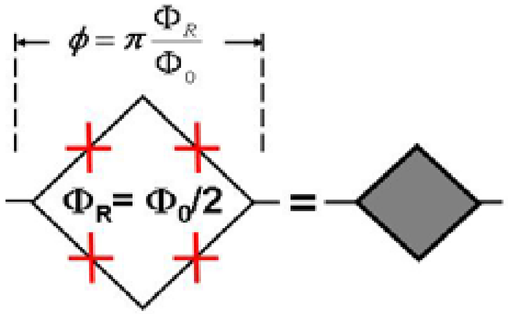
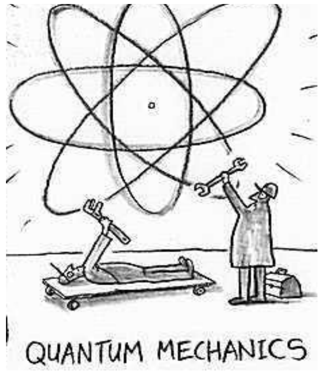
The Art of Designing Small Protected Arrays

Three design levels:

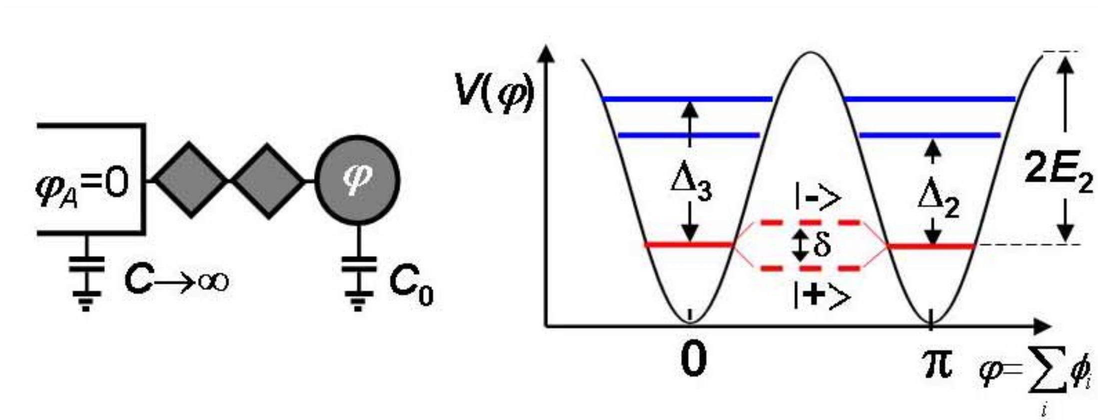
- "cos2φ" Josephson element
  – a "***faulty***" physical qubit





- a chain of *N* such elements
  – the simplest ***protected*** qubit

Logical variable: the phase of the rightmost "island" (either 0 or π)

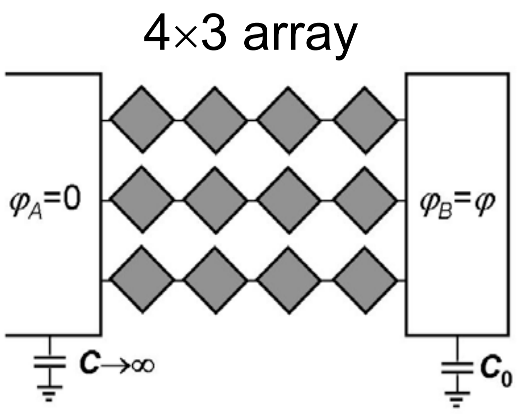


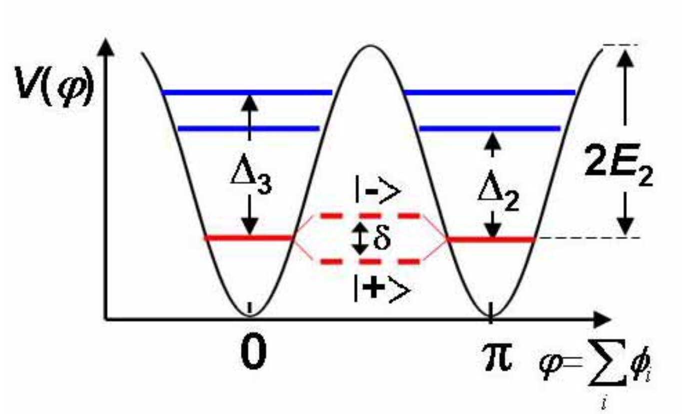
Energy relaxation is suppressed :
Single rhombus cannot flip its phase by π - it costs a large energy $2E_2$. Simultaneous flips of pairs of rhombi are permitted, but these flips do not affect the logical variable.

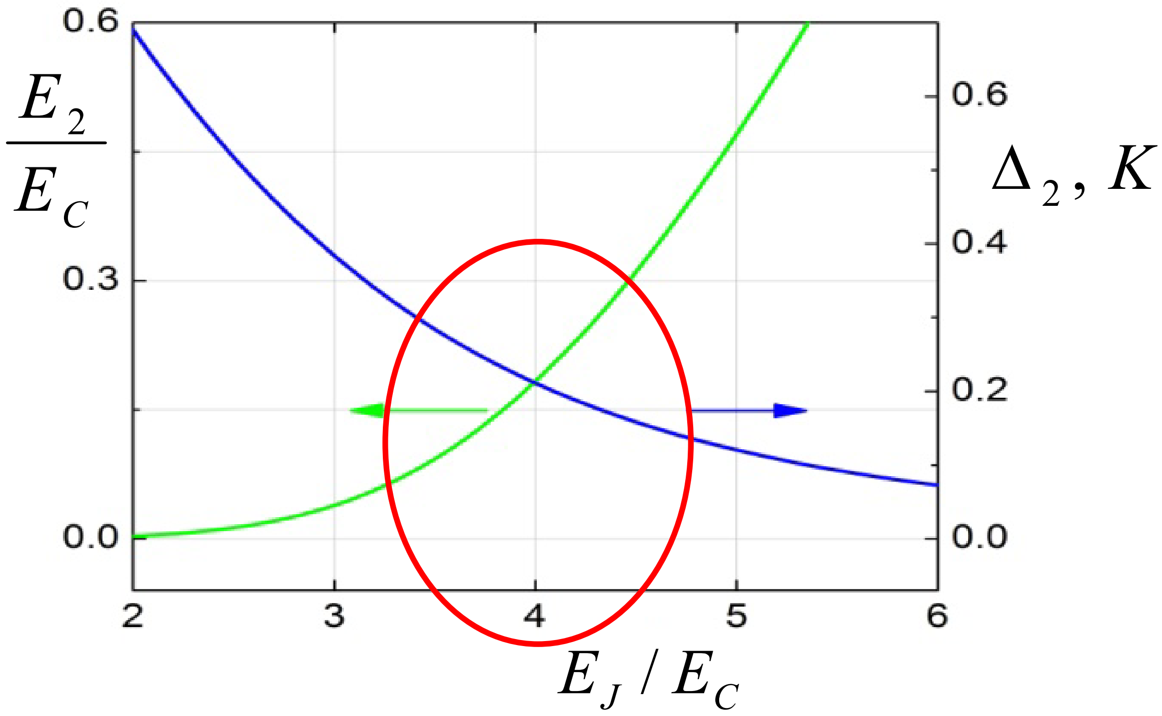
Dephasing is also suppressed :
the energies of individual rhombi (affected by noise) enter the energies of logical states ***symmetrically***, the change in energy induced by noise is ***the same*** for both logical states.

The Art of Designing Small Protected Arrays (cont'd)

- The goal of hierarchical construction: ***simultaneous realization of large E_2 and Δ_2***







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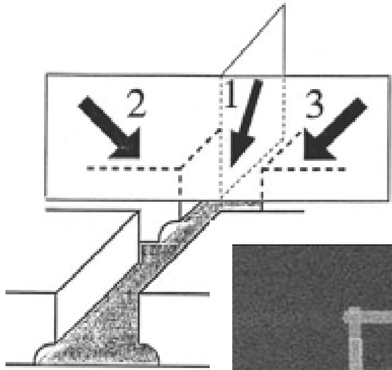
Device Fabrication

Requirements:

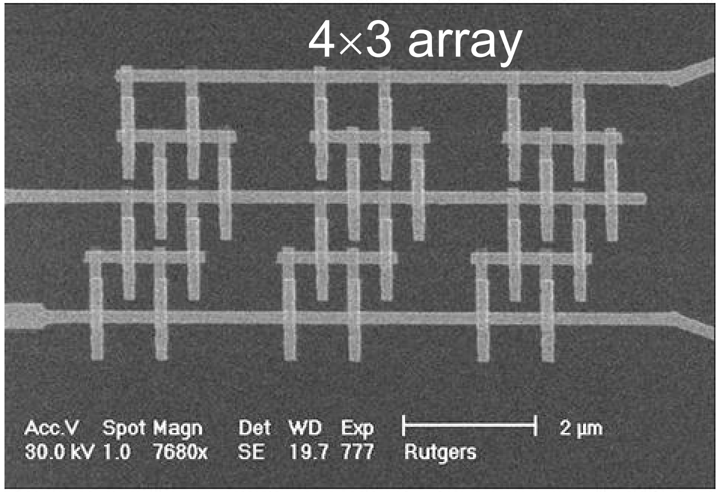
- typical in-plane JJ dimensions: ~0.2 μm (E_J/E_C~3-6)
- narrow margins of JJ parameters [$E_J/E_C \propto$ (area)^2]

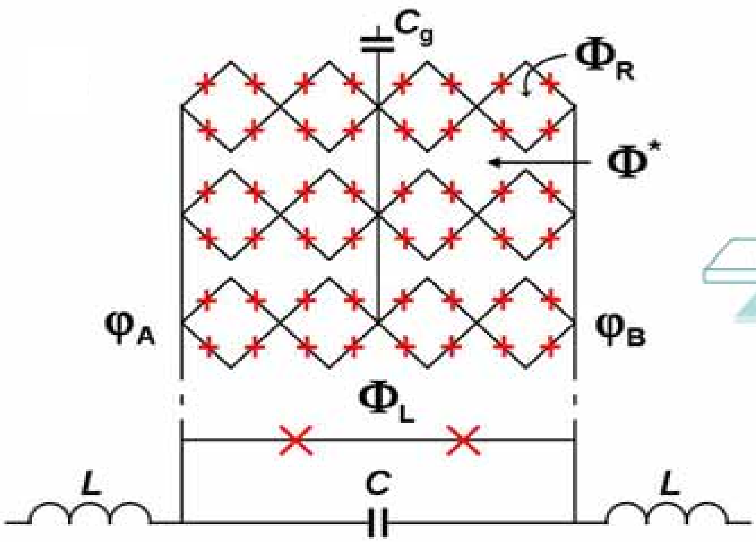
Experimental realization:

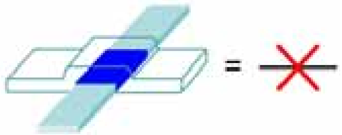
- "Manhattan pattern" nanolithography
  (better reproducibility than a "shadow" mask)
- multi-angle deposition of Al
- well-controlled oxidation



multi-angle deposition







Total – 50 Josephson junctions

To probe $E(\varphi)$, the array is included in a SQUID loop that fixes φ_A and φ_B (φ_A-φ_B is controlled by flux Φ_L).

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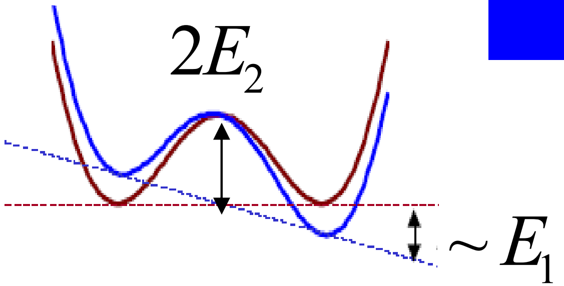
The Idea of Proof-of-Concept Experiments



The current of SQUID switching in the normal state, I_{SW}:

controlled by flux in the SQUID loop

$$V(\varphi) \propto E_2 \cos(2\varphi) + E_1 \cos(\varphi)$$

$$I_{SW}(\varphi) = I_2 \cos(2\varphi) + I_1 \cos(\varphi)$$

$$\begin{cases} I_2 = \dfrac{4\pi}{\Phi_0} E_2 \\ I_1 = \dfrac{2\pi}{\Phi_0} E_1 \end{cases}$$

By measuring the switching current, I_{SW}, and its dependence on φ, one can find $\boldsymbol{E_2}$ and the degree of suppression of $\boldsymbol{E_1}$.

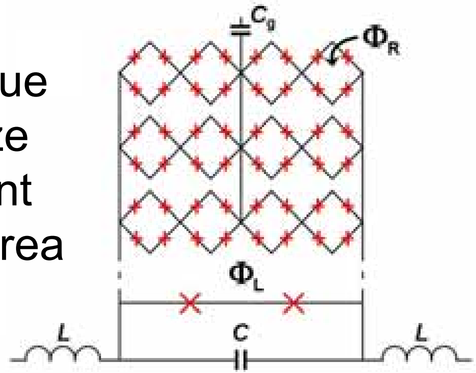
If E_1 is negligible, the array is decoupled from local noises.

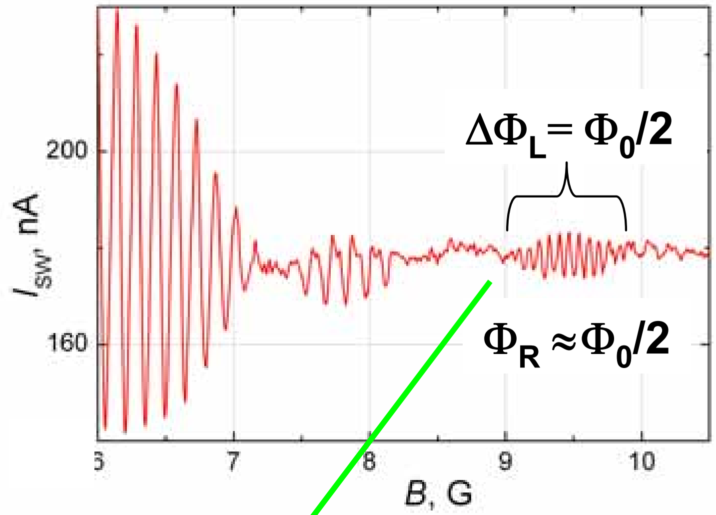
Noise Imitation: the response of the array to ***static*** deviations of the flux in each rhombus from its optimal value $\Phi_0/2$ mimics the qubit response to the flux noise

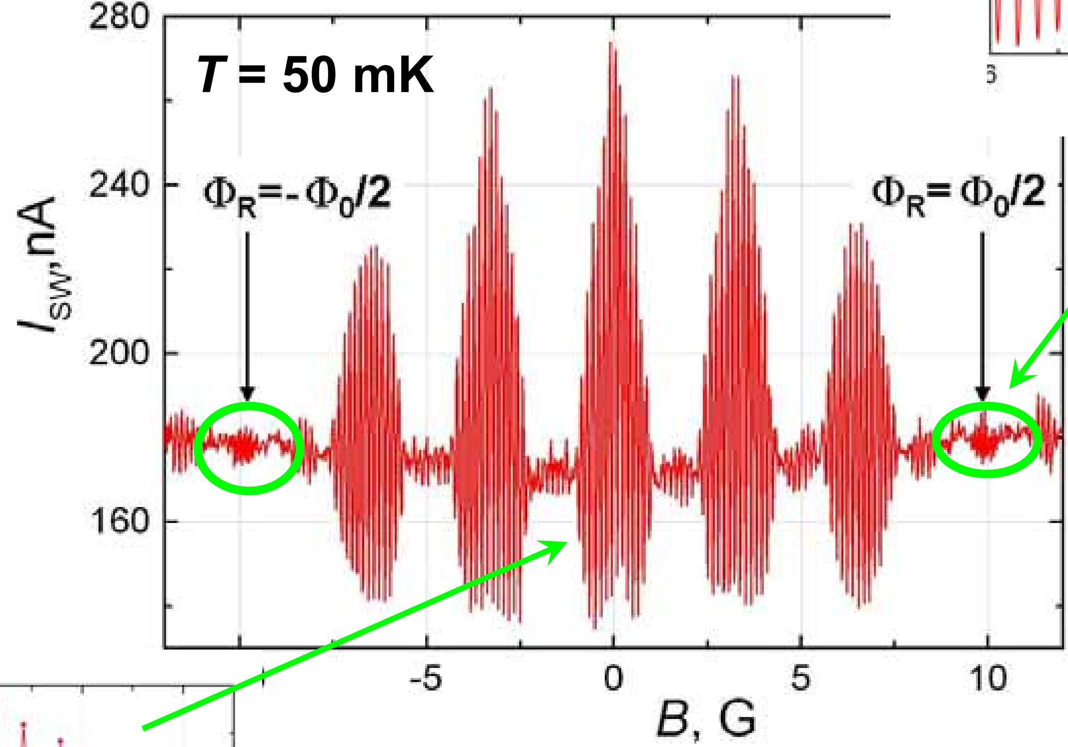
the effective flux noise $$\delta\Phi_{eff} = \underbrace{\left(\frac{\delta\Phi}{\Phi_0} \frac{E_J}{\Delta_2} \right)^{N-1}}_{N^{\text{th}}\text{ order protection}} \delta\Phi \qquad \delta\Phi = \Phi - \Phi_0 / 2$$

If $\boxed{\dfrac{\delta\Phi}{\Phi_0} \dfrac{E_J}{\Delta_2} << 1}$ deformations of the potential $V(\varphi)$ (and, thus, ***dephasing***) are greatly reduced → indirect information on Δ_2

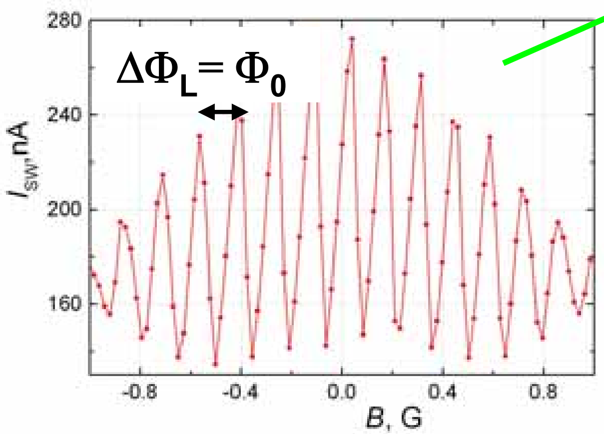
The beating pattern - due to the intermediate-size loops between adjacent rhombi chains with an area 4×(rhombus area)





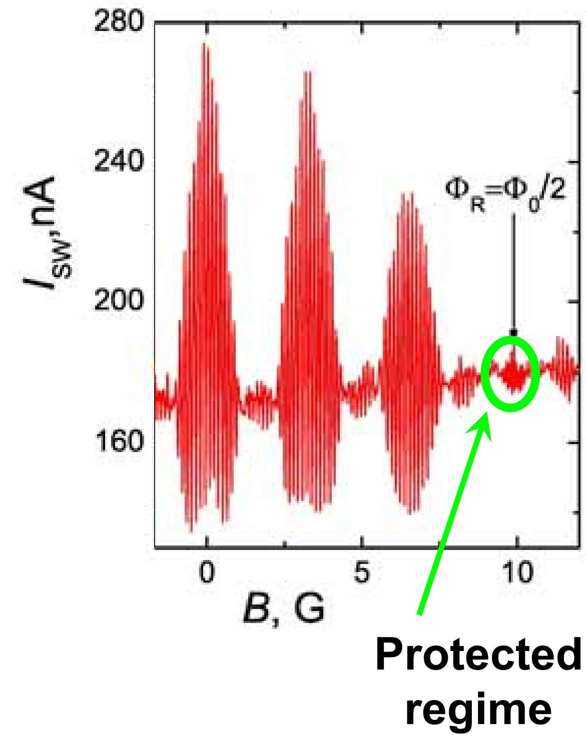


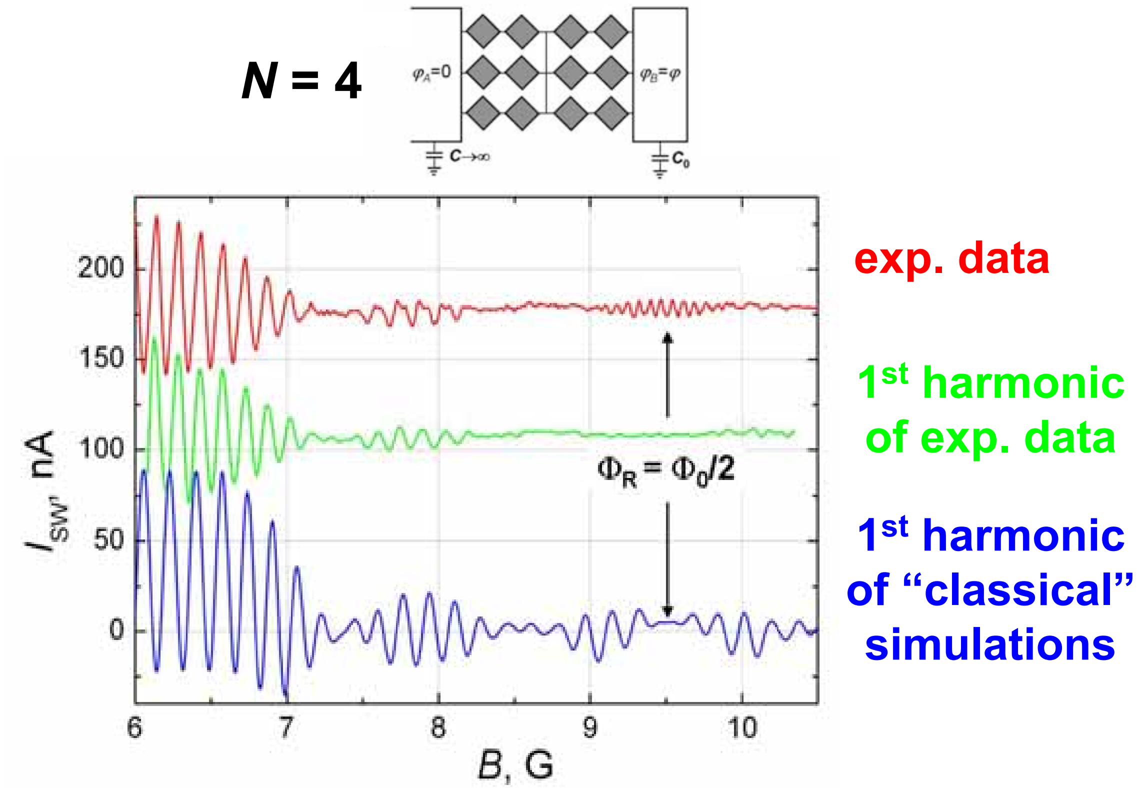
Protected regime:
$\Phi_R \cong (n+1/2)\,\Phi_0$
$E_1 \approx 0$



Unprotected regime:
Φ_R far from $(n+1/2)\,\Phi_0$
E_1 is large

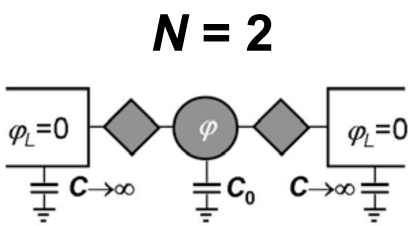
Protected Regime: Correlated Transport of Pairs of Cooper

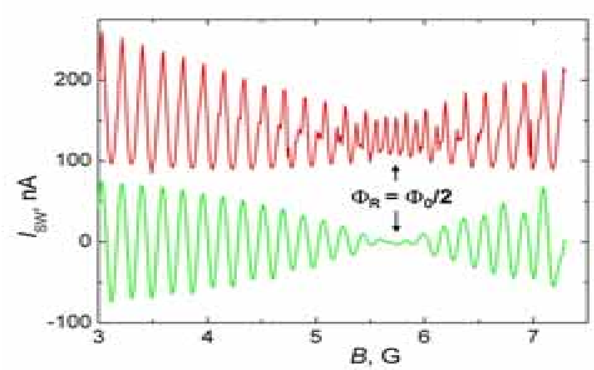




Vanishing of oscillations with $\Delta\varphi=2\pi$ is a sign of protection from static variations of Φ. Charge transport in this regime is due to the correlated transport of ***pairs of Cooper pairs with charge 4e***, tunnelling of single Cooper pairs across the array is blocked.

Protection diminishes with decreasing *N*:





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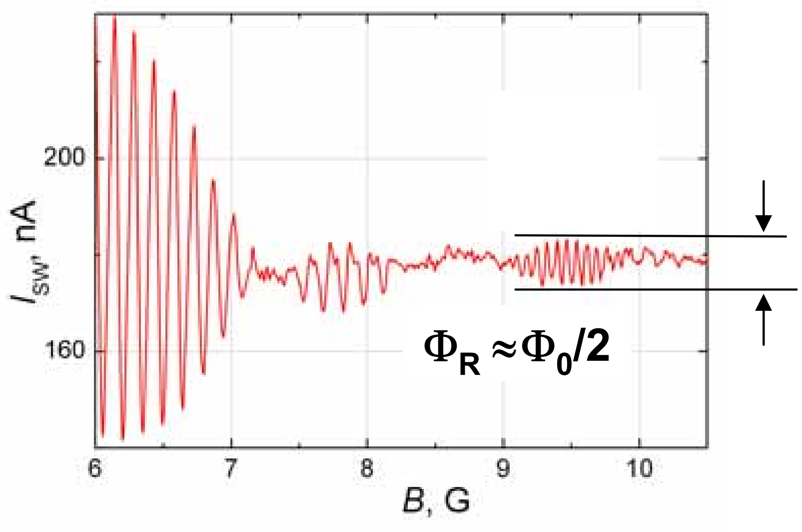
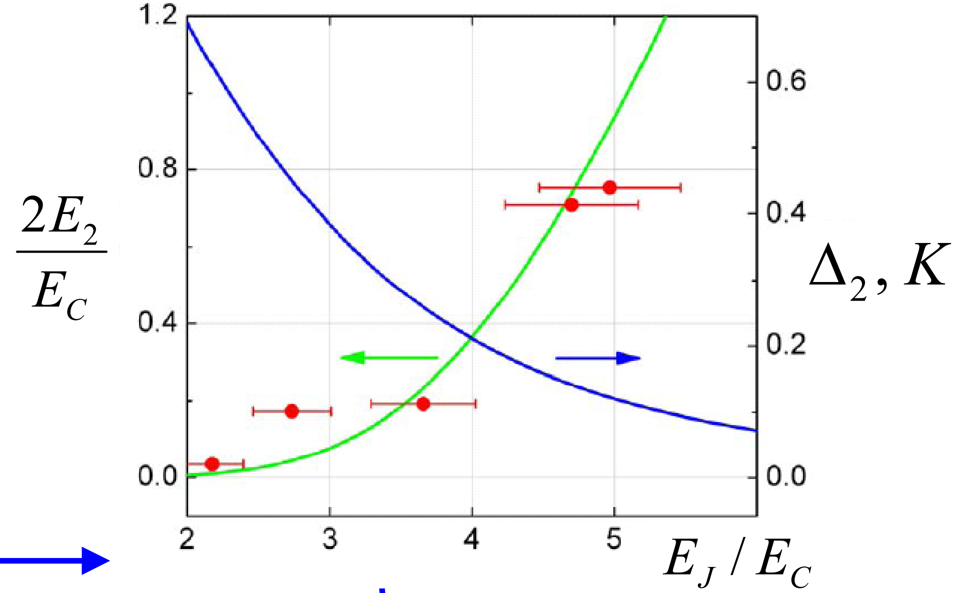
Comparison with the Theory



$$2I_2 = \frac{8eE_2}{\hbar}$$



The measured value of E_2 is in good agreement with numerical simulations

"Sweet spot":
relatively large $\boldsymbol{\Delta_2}$ and $\boldsymbol{I_2}$

The experiment shows that even a relatively small array can be protected against variations of external parameters well beyond the linear order.

S. Gladchenko, D. Olaya, E. Dupont-Ferrier, B. Douçot, L.B. Ioffe, and M.E. Gershenson, "Superconducting Nanocircuits for Topologically Protected Qubits", *Nature Physics* **5**, 48 (2009).

Summary

- Even a small Josephson array is protected against $\delta\Phi$ well beyond the linear order:

$$V(\varphi) \approx -E_2 \cos(2\varphi) - \cancel{E_1} \cos(\varphi)$$

- We observed correlated transport of pairs of Cooper pairs in the chain of $\cos 2\varphi$ Josephson elements in the regime when quantum fluctuations are strong.

- The measured values of E_2 and $\Delta E_2(V_g)$ are in good agreement with numerical simulations.

***A long way to protected qubits… Next steps*:**

- microwave spectroscopy of the arrays
- measurements of the decay and dephasing time
- development of the single-qubit gates, demonstration of the operation of topologically-protected qubits.

Post-docs and grad students are welcome !!!