



**The Abdus Salam
International Centre for Theoretical Physics**



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Conference on Superconductor-Insulator Transitions

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Compensation driven superconductor-insulator transition

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Compensation driven superconductor-insulator transition

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The Abdus Salam
International Center
of Theoretical
Physics, Trieste

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Superconductivity and Coulomb interactions

Clean, granular systems

Josephson junction arrays
(Fazio, Schön, ...)

$E_J > E_C \rightarrow$ Superconductivity

$E_J < E_C \rightarrow$ Insulator

Simply exponential **pair** transport (see K. Efetov's talk)

At lowest T:

$$\begin{array}{l} E_J > E_C \rightarrow G = G_0 \exp(T_0 / T) \\ E_J < E_C \rightarrow R = R_0 \exp(T_0 / T) \end{array}$$

What if there is strong disorder (generic)? $\delta E_C \sim E_C$

Insulator: gap is destroyed \rightarrow a priori no simple activation!

What if there are no pre-structured grains?

Do “effective grains” form due to the disorder configuration?

SIT in strong disorder:

Localization and delocalization of
Cooper pairs
in Coulomb disorder

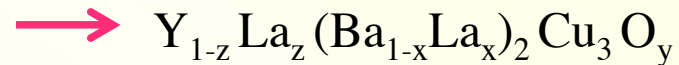
Similar analysis for neutral cold atoms in random disorder potentials, see
Falko, Nattermann and Pokrovski (08); Shklovskii (08)

Compensated high Tc materials

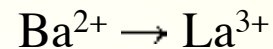
K. Segawa and Y. Ando, PRB 74, 100508 (2006)



Doping n-type carriers by La-substitution for Ba



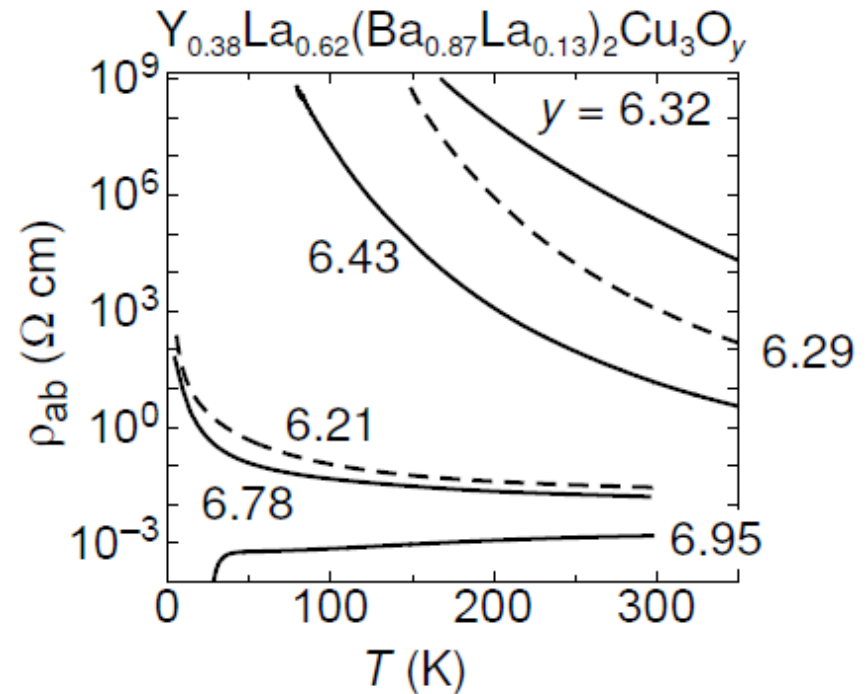
n-type doping controlled by x



Vary p-type doping by
annealing oxygen content y

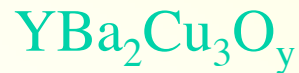
$$6.21 < y < 6.95$$

$y < 6.32$: n-type doping
 $y = 6.32$: fully compensated
 $6.32 < y$: p-type doping

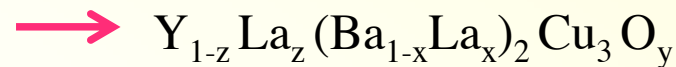


Compensated high T_c materials

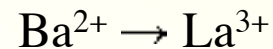
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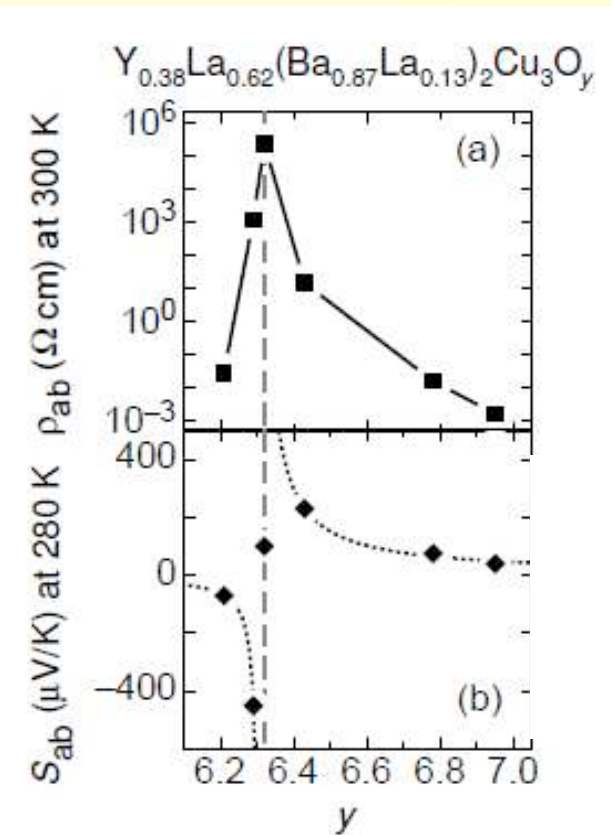
n-type doping controlled by x



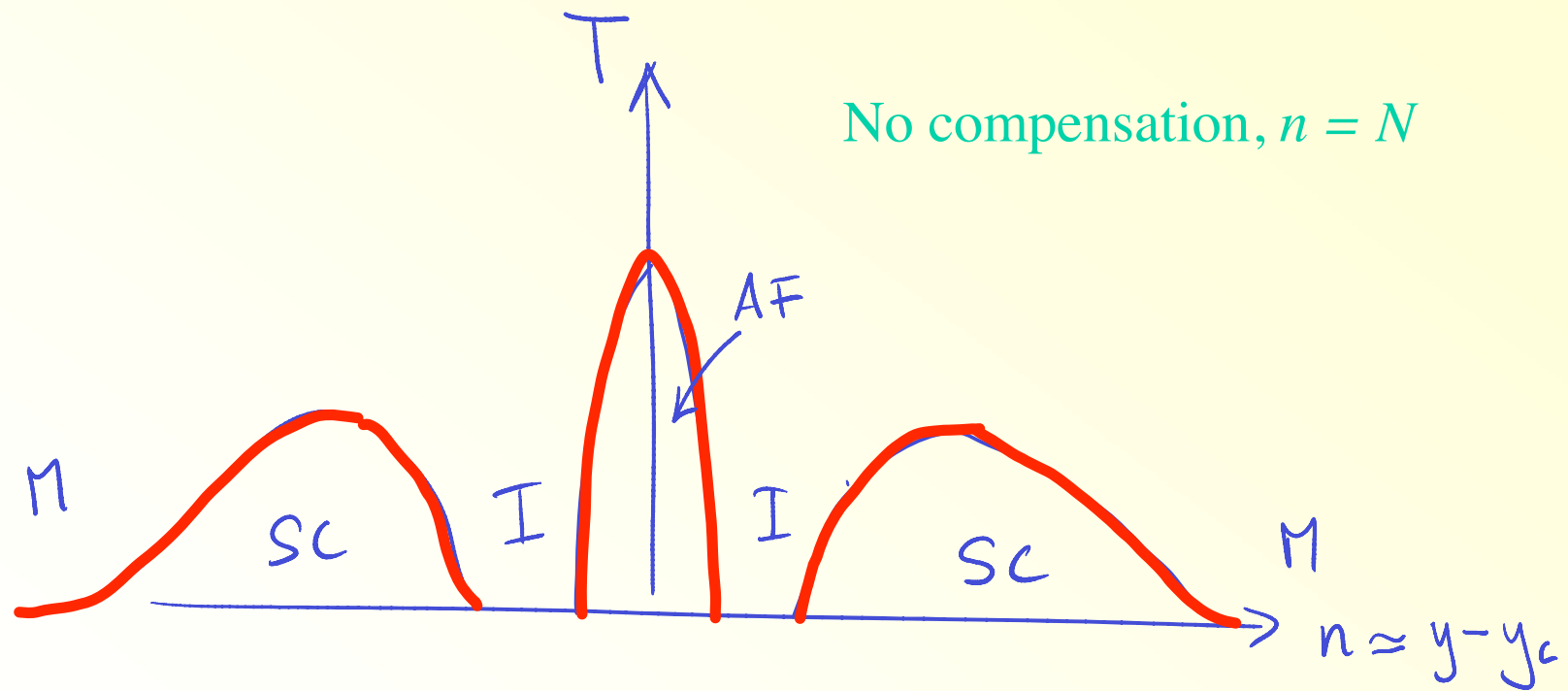
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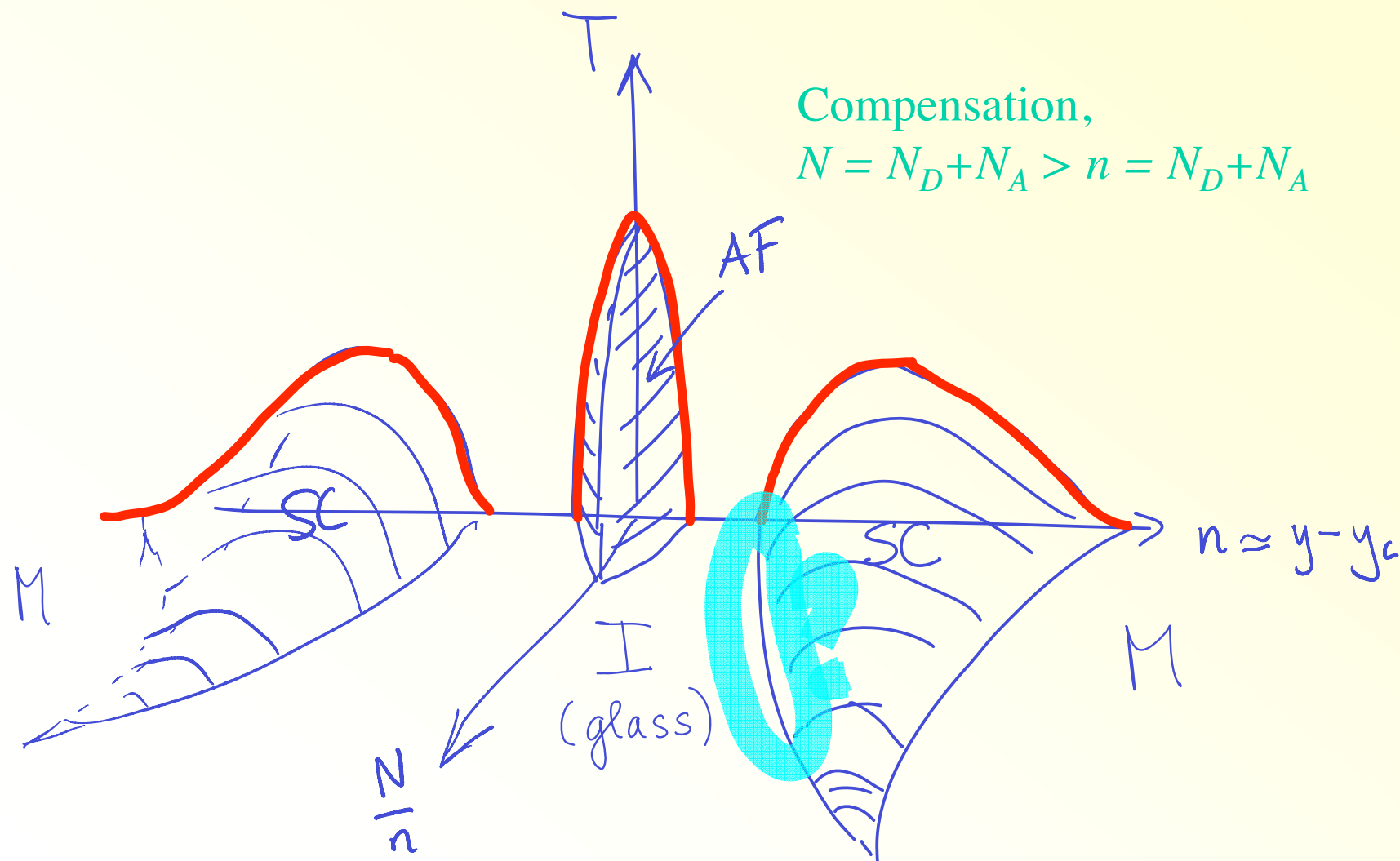
$y < 6.32$: n-type doping
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Expected phase diagram



Expected phase diagram



Analysis of the SIT in terms of a
scaling analysis

-

All numerical prefactors will be
neglected

The compensation driven metal-insulator transition (fermions)

Uncompensated semiconductors (3d): Mott's criterion

Effective Bohr radius:
$$a = \frac{\hbar^2 \kappa}{me^2}$$

Metal-insulator transition (MIT):
$$n_{\text{MIT}} a^3 = N_{D,\text{MIT}} a^3 = 0.02 = \mathcal{O}(1)$$

Overlapping hydrogen-like wavefunctions \rightarrow delocalization

With BCS instability in the metal \rightarrow SIT:

$$n_{\text{SIT}} a^3 \approx n_{\text{MIT}} a^3 \sim 1$$

The compensation driven metal-insulator transition (fermions)

Metal-Insulator transition in strongly compensated semiconductors

Non-trivial regime: $Na^3 \gg 1$
Heavy doping $N = N_D + N_A$

Most carriers are captured by doping ions \rightarrow
Excess carriers in the conduction band: $n = N_A - N_D \ll N$

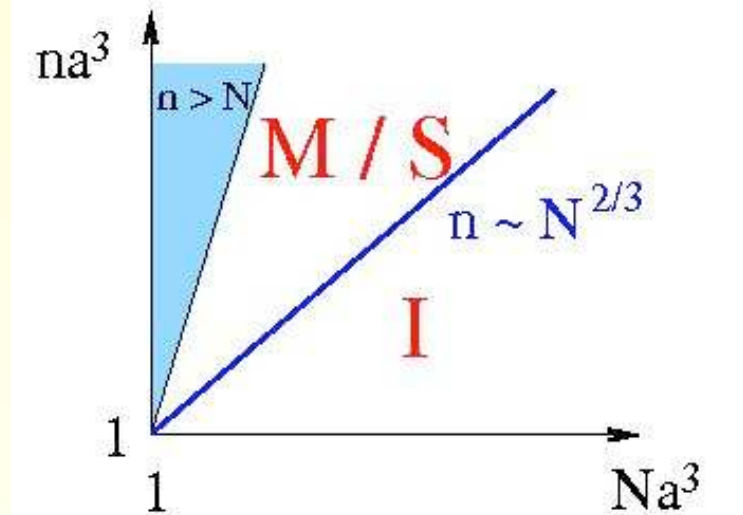
\rightarrow Strong disorder from N random charged impurities!

Delocalization transition upon tuning n :

$$n_{\text{MIT}}(N) = \frac{N}{(Na^3)^{1/3}} = \frac{N^{2/3}}{a}$$

$$\rightarrow n_{\text{MIT}} a^3 = (Na^3)^{2/3} \gg 1$$

$$\rightarrow n_{\text{MIT}} \ll N$$



Experimentally confirmed in compensated Ge

MIT: Derivation

Efros and Shklovskii (1971)

1. Non-linear screening of the disorder

Random charge density in volume R^d : $n_{\text{net imp}}(R) \sim \frac{(NR^3)^{1/2}}{R^3}$

Non-linear screening scale R_s : $n_{\text{net imp}}(R) \sim n \rightarrow R_s(n) \sim \frac{N^{1/3}}{n^{2/3}}$

Roughness of the disorder potential: $eV_{\text{dis,Cb}}(R_s) \sim \frac{e^2}{\kappa R_s} (NR_s^3)^{1/2} \sim \frac{e^2 N^{2/3}}{\kappa n^{1/3}}$

2. Delocalization condition

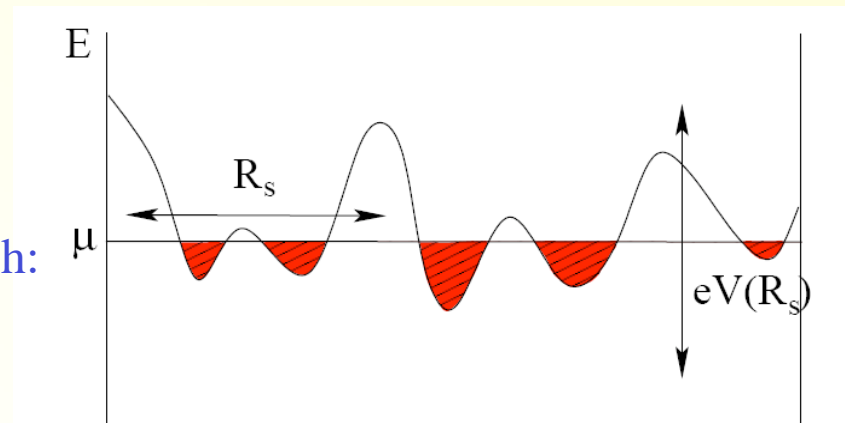
Fermi energy of excess carriers:

$$E_F \sim \hbar^2 n^{2/3} / m$$

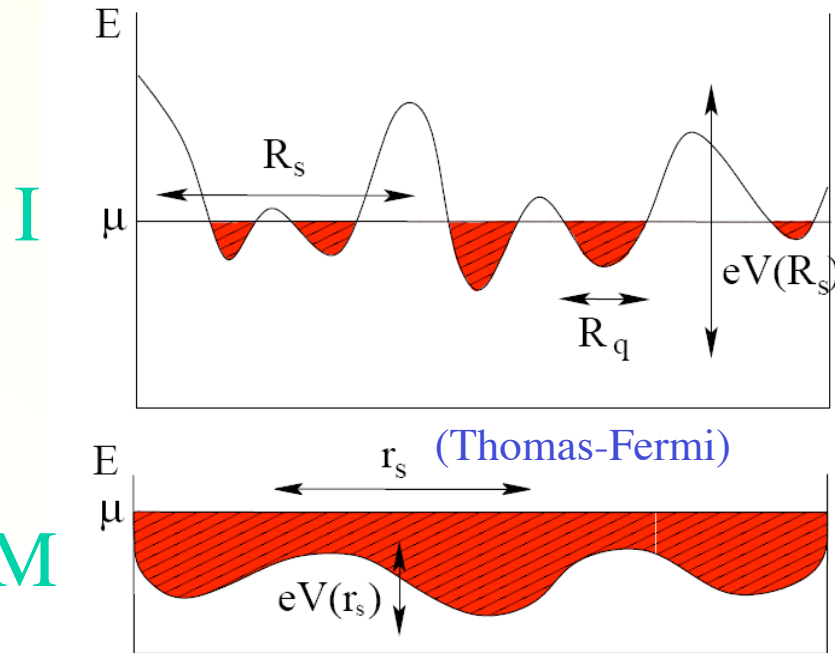
Delocalization if disorder is weak enough:

$$E_F \geq V(R_s) \Leftrightarrow \boxed{n_{\text{MIT}} \sim \frac{N}{(Na^3)^{1/3}}}$$

Consistency condition for scaling analysis: $NR_s^3 \gg 1 \Leftrightarrow Na^3 \gg 1$



Droplets in the insulator



In the insulator: $n < n_{\text{MIT}}$

Small droplets (Fermi lakes in the deepest wells) of size $R_q < R_s$!

Typical size of the droplets:

$$\mu(n = (N / R_q^3)^{1/2}) = eV_{\text{Cb}}(R_q)$$

$$\rightarrow R_q = \frac{a}{(Na^3)^{1/9}} = R_s(n) \cdot (n / n_{\text{MIT}})^{2/3}$$

Transport: Variable range hopping between the droplets!

Compensated superconductors with preformed pairs

Assume strong coupling mechanism (“glue”)

→ preformed Cooper pairs of size ξ
(finite pairing energy E_{pair} - no nodal quasiparticles)

Possible systems with preformed pairs:

- Underdoped high T_c materials
- Bipolarons
- [Anderson pseudospins (doubly occupied localized wavefunctions)]

- How do bosons modify non-linear screening and delocalization?
- How does the BEC-BCS crossover manifest itself?

$$n\xi^3 = 1$$

- What is the transport on the insulating side?

BEC – BCS crossover

BEC – BCS crossover : $n\xi^3 \sim 1$

- For $n_{\text{SIT}}\xi^3 > 1$ the transition remains the same as with fermions
- Distinctly “bosonic” behavior occurs at the SIT when

$$n_{\text{SIT}}\xi^3 < 1 \quad \Leftrightarrow \quad \xi < a$$

→ Needed: small pairs, large Bohr radii

→ $n_{\text{SIT}}(N) = ?$

BEC regime – 3d SC

1. Nonlinear screening with $(2e)$'s instead of e 's, but otherwise no difference

$$eV_{\text{Cb}}(n) = eV_{\text{Cb}}(R_s) \sim e^2 N^{2/3} / n^{1/3}$$

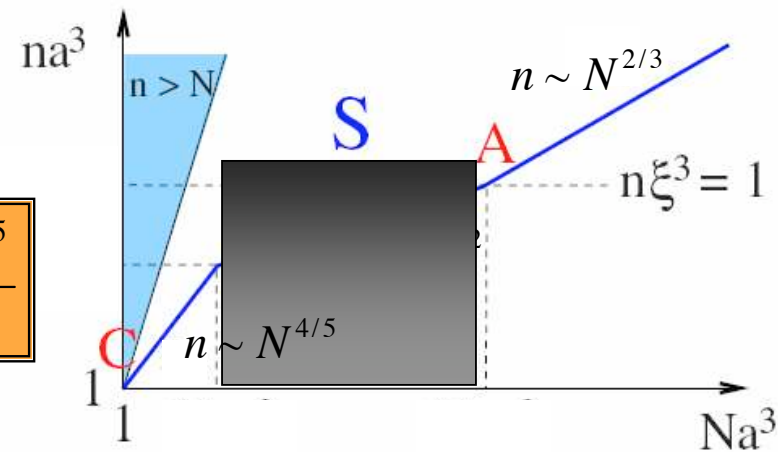
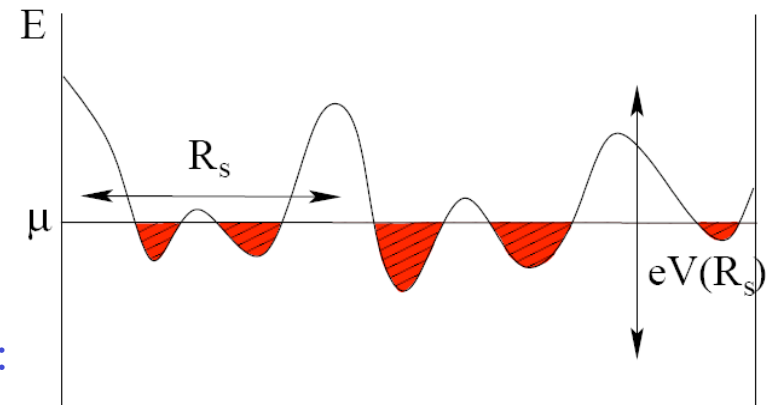
2. Very low density

Energy of confinement to screening volume:

$$\mu(n, R) = \frac{\hbar^2}{mR^2}$$

Delocalization of the BEC condensate:

$$\mu(n, R_s) \sim eV_{\text{Cb}}(n) \rightarrow n_{\text{SIT}}(N) = \frac{N^{4/5}}{a^{3/5}}$$



BEC regime – 3d SC

1. Nonlinear screening with $(2e)$'s instead of $1e$'s, but otherwise no difference

$$eV_{\text{Cb}}(n) = eV_{\text{Cb}}(R_s) \sim e^2 N^{2/3} / n^{1/3}$$

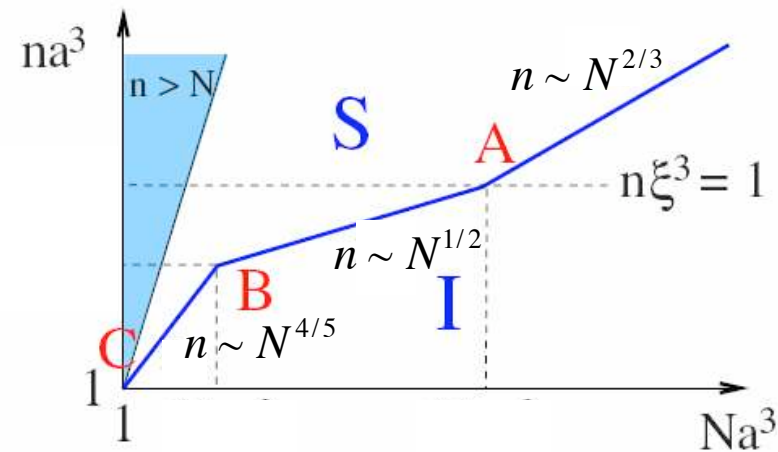
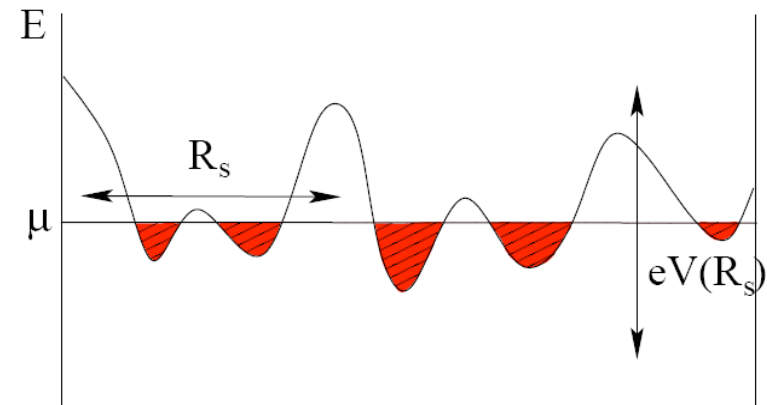
3. Chemical potential for bosons:
Bose gas with scattering length ξ

$$\mu(n) = \frac{\hbar^2}{2m} n \xi < E_F(n)!$$

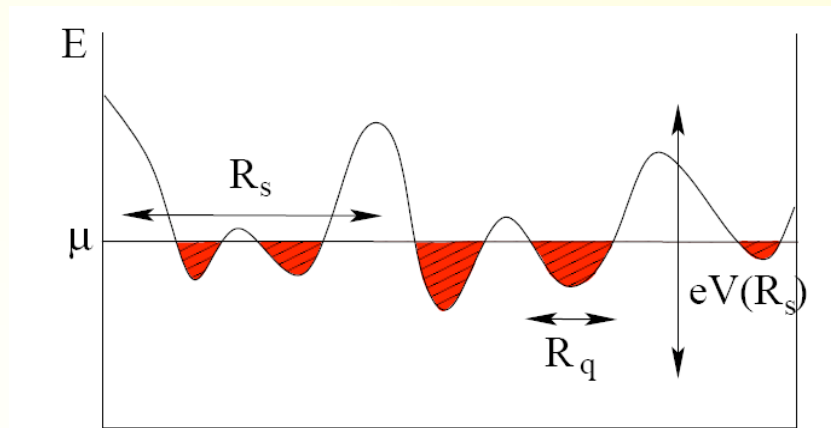
4. Bose delocalization criterion:

$$\mu(n) \sim eV_{\text{Cb}}(n)$$

$$\rightarrow n_{\text{SIT}}(N) = \frac{N^{1/2}}{(a\xi)^{3/4}}$$



The Bose insulator



In the insulator: $n < n_{\text{SIT}}$

Small droplets (boson lakes in the deepest wells) of size $R_q < R_s$!

Typical size of the droplets:

$$\mu(n = (N / R_q^3)^{1/2}) = eV_{\text{Cb}}(R_q)$$

BEC regime: $\rightarrow R_q = (a\xi)^{1/2} < R_s(n)$

Nature of the ground state

Level spacing in a droplet ($\pm 1e$): $\delta \sim (d\mu / dn R_q^3)^{-1} = e^2 / R_q$ BEC: $\xi < a$

Pair-breaking energy:

$$E_{\text{pair}} \sim \hbar^2 / m\xi^2$$

$$\rightarrow E_{\text{pair}} > \delta$$

\rightarrow Breaking pairs is unfavorable, all electrons are paired!
Single electron excitations are gapped!

Properties of the insulator

Tunneling

- Single particle gap
- SC spectrum of small droplets (and corresponding coherence peaks)

Transport

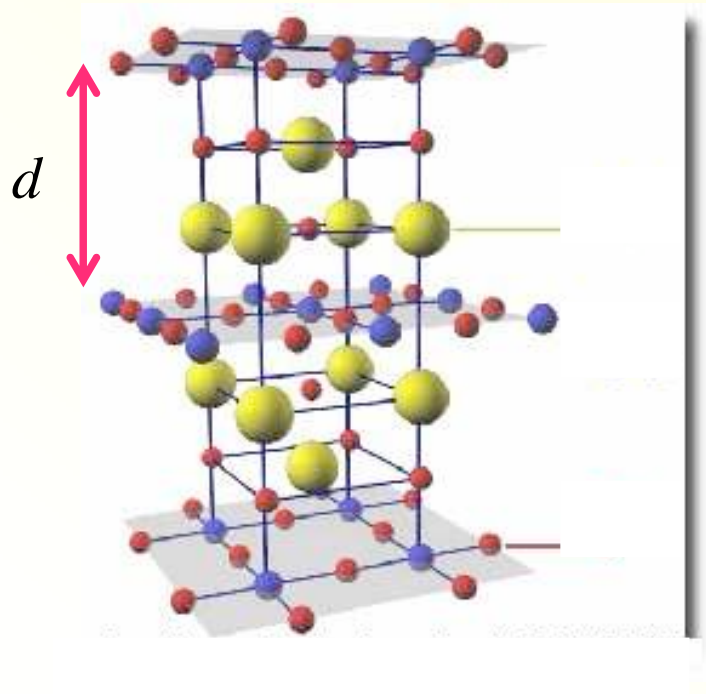
- At low T: Variable range hopping of pairs between droplets!
- In the BEC regime (strong coupling, small pairs) : always 2e-transport
- In the BCS regime (weak coupling): 1e-transport when $\Delta < \delta$.

$$\sigma = \sigma_0 \exp[-(T_{ES} / T)^{1/2}]$$

$$T_{ES} = 2.7 \cdot (2e)^2 / \kappa \xi_{2e}$$

Layered superconductors

Examples: Cuprates (CuO compounds), pnictides (FeAs compounds)



d : Distance between layers hosting the carriers

Differences 2d vs. 3d:

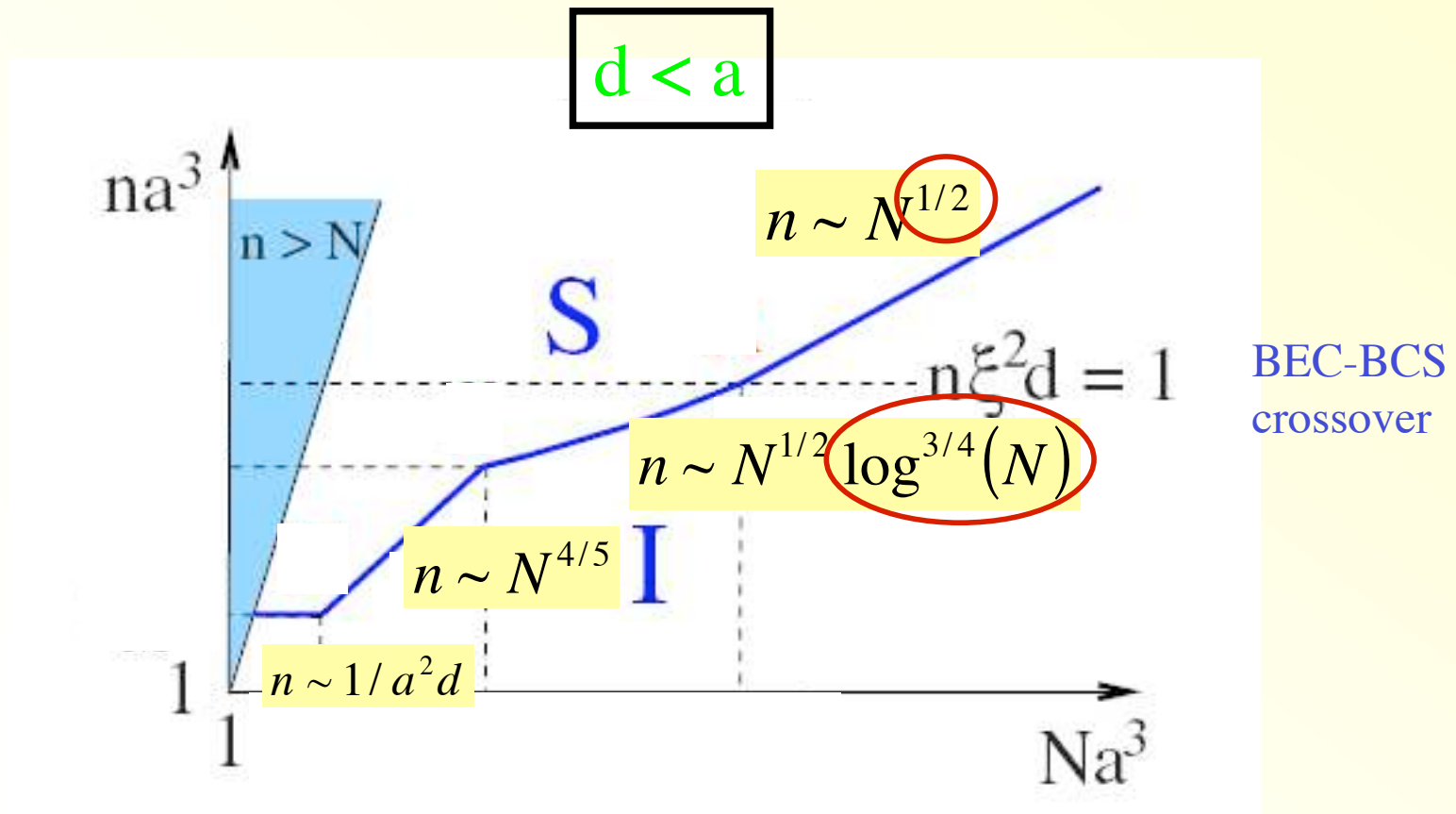
1. Nonlinear screening is modified when $d > R_s$; need to account for anisotropic dielectric constant

2. Delocalization criterion:

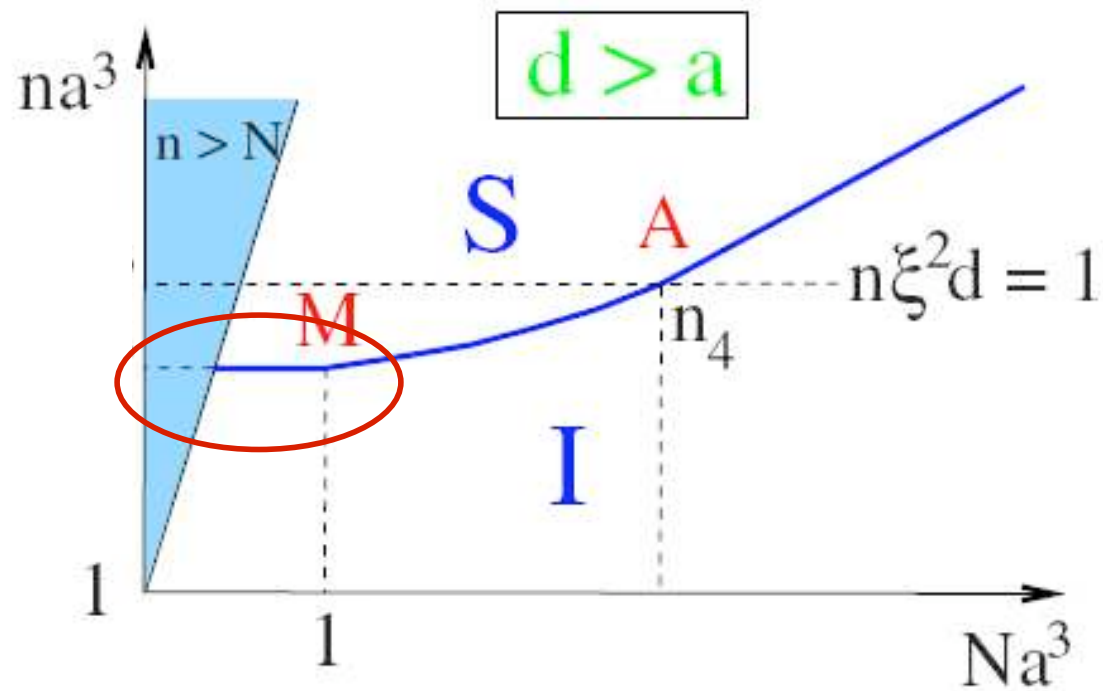
$$\mu_{2D}(n) = eV_{Cb}(n, N)$$

Only extra log at the BCS-BEC crossover!
$$\mu_{2D, BEC}(n) = \frac{\hbar^2 n d}{m \log(1/\xi^2 n d)} = \frac{E_F(n)}{\log(1/\xi^2 n d)}$$

SIT Phase diagram – 2D

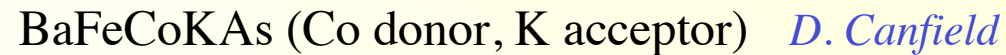
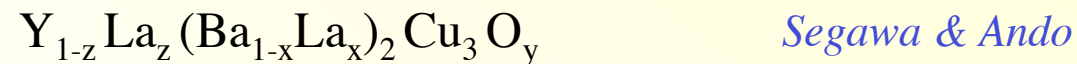


SIT Phase diagram – 2D



Applicability to high T_c ?

Candidate systems:



Condition for the SI transition to occur in the BEC regime:

$$n_{\text{SIT}} \xi^3 < 1 \quad \Leftrightarrow \quad \xi < a$$

Parameters for typical underdoped high T_c 's:

$$a = \frac{\hbar^2 \kappa}{m_{\text{pair}} (2e)^2}$$

$$\text{BSCCO: } a \sim 4 - 5 \text{ \AA}$$

$$\xi_{\text{typ}} \approx 1 - 2 \text{ nm}$$



At the border of
BEC-BCS
crossover

BEC-regime in bipolarons

Condition for the SI transition to occur in the BEC regime:

$$n_{\text{SIT}} \xi^3 < 1 \quad \Leftrightarrow \quad \xi < a$$

Two independent parameters:

- Electron-phonon coupling: $\alpha > \alpha_c = 2.9$ (in 2d)
- Ratio between electronic and static dielectric constant:
 $\eta = \kappa_{\text{el}}/\kappa$ needs to be $\eta \gg 1$:

$$\frac{\xi}{a} \sim \alpha^2 \eta < 1$$

Summary

- SIT in the presence of strong Coulomb disorder:
Delocalization of preformed pairs in a self-consistently screened disorder potential
- Non-linear screening is less efficient with bosons
(exclusion principle less effective in BEC regime)

$$n_{SIT}^{(bosons)} > n_{SIT}^{(fermions)}$$

- Low T transport in the insulator in the BEC regime is always dominated by pairs