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IAEA International Atomic Energy Agency

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Conference on Superconductor-Insulator Transitions

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Pseudogaped superconductivity of localized electrons

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Fractal Superconductivity near Localization Edge

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Short publication: Phys Rev Lett. **98**, 027001 (2007)

Superconductity v/s Localization

- Granular systems with Coulomb interaction

K.Efetov 1980 et al ***"Bosonic mechanism"***

- Coulomb-induced suppression of Tc in uniform films ***"Fermionic mechanism"***

A.Finkelstein 1987 et al

- Competition of Cooper pairing and localization (no Coulomb)

Imry-Strongin, **Ma-Lee**, Kotliar-Kapitulnik, Bulaevsky-Sadovsky(mid-80's)

Ghosal, Randeria, Trivedi 1998-2001

There will be no grains and no Coulomb in this talk !

Plan of the talk

1. Motivation from experiments
2. BCS-like theory for critical eigenstates

   - transition temperature
   - local order parameter
3. Superconductivity with pseudogap
   - transition temperature v/s pseudogap
   - tunnelling conductance
   - spectral weight
4. Conclusions and open problems

Major exp. data calling for a new theory

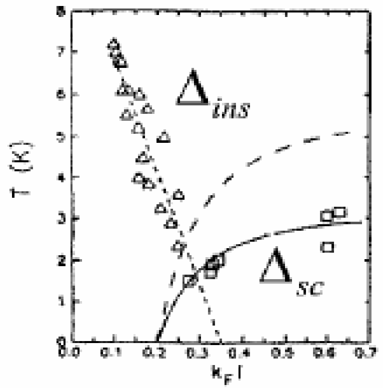
- Activated resistivity

in insulating a-InO_x

D.Shahar-Z.Ovadyahu 1992,

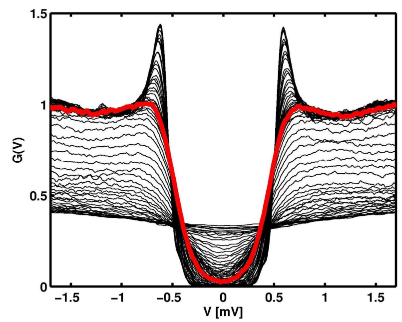
V.Gantmakher et al 1996

T_0 = 3 – 15 K



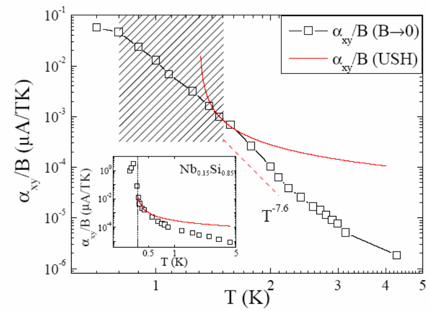
- Local tunnelling data

B.Sacepe et al 2007-8



- Nernst effect above T_c

P.Spathis, H.Aubin et al 2008



Class of relevant materials

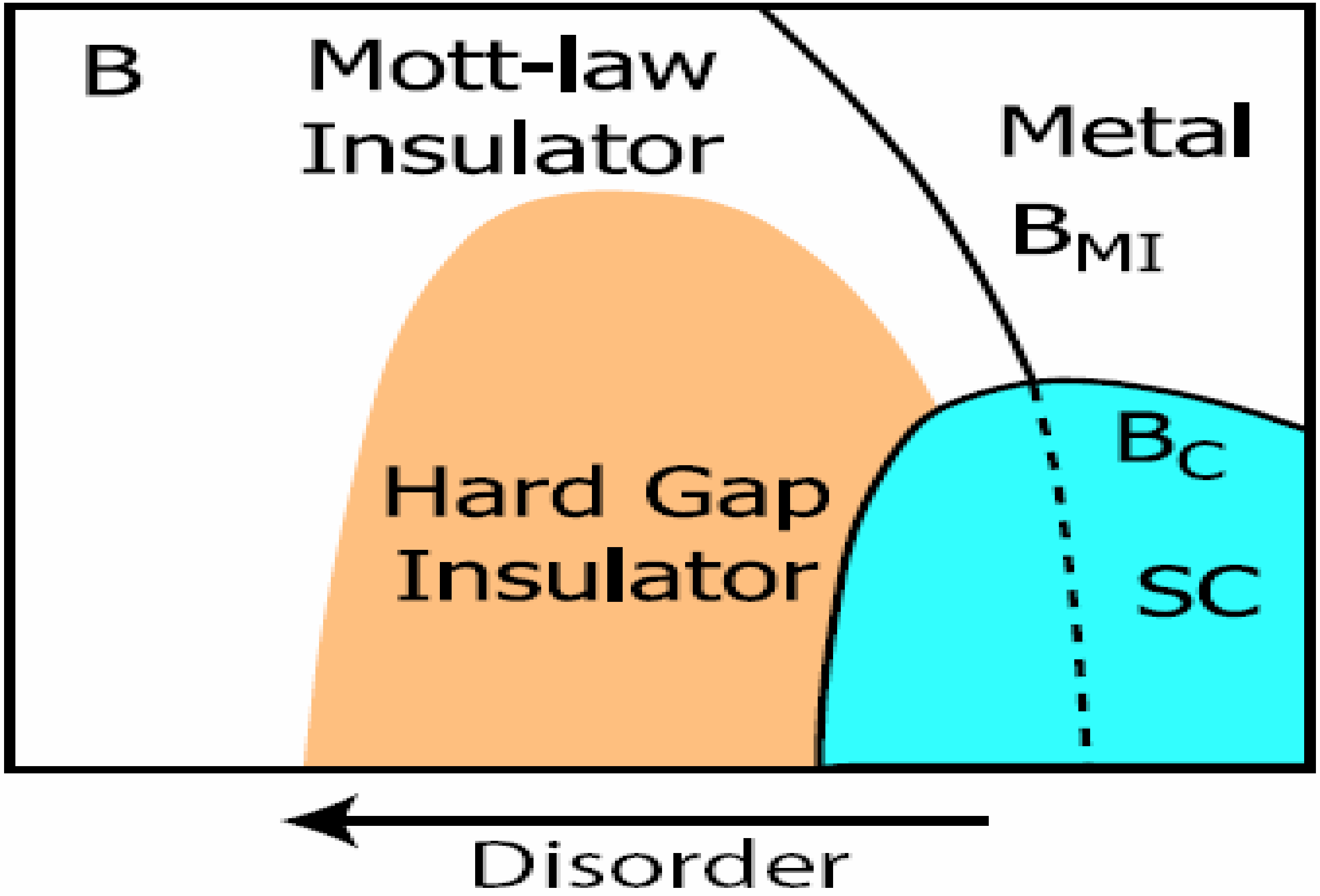
- Amorphously disordered
  (no structural grains)
- Low carrier density
  (around 10^{21} cm^{-3} at low temp.)

Examples:

InO_x NbN_x thick films or bulk (+ B-doped Diamond?)

TiN thin films Be, Bi (ultra thin films)

Phase Diagram



Theoretical model

Simplest BCS attraction model,

but for critical (or weakly)

localized electrons

$$H = H_0 \ - \ g \int d^3r \ \Psi_\uparrow^\dagger \Psi_\downarrow^\dagger \Psi_\downarrow \Psi_\uparrow$$

$\boldsymbol{\Psi} = \Sigma\, c_j\, \Psi_j\,(r)$ Basis of localized eigenfunctions

M. Ma and P. Lee (1985) : S-I transition at $\delta \approx T_c$

Superconductivity at the Localization Threshold: $T_c \gg \delta_L$

Now we will consider the case of Fermi energy very close to the mobility edge:

single-electron states are extended but **fractal** and populate small fraction of the whole volume

How BCS theory should be modified to account for eigenstate's fractality ?

Method: combination of analitic theory and numerical data for Anderson mobility edge model

Mean-Field Eq. for T_c

$$\Delta(r) = \int K_T(r, r')\Delta(r')d^d r' \qquad (9)$$

where kernel $\hat{K}_T$ is equal to

$$K_T(r, r') = \frac{\lambda}{2\nu_0} \sum_{ij} \frac{\tanh\frac{\xi_i}{2T} + \tanh\frac{\xi_j}{2T}}{\xi_i + \xi_j} \psi_i(r)\psi_j(r)\psi_i(r')\psi_j(r') \qquad (10)$$

Standard averaging over space $\Delta(r) \to \overline{\Delta}$ leads to "Anderson theorem" result: totally incorrect in the present situation.

The reason: critical eigenstates $\psi_j(r)$ are strongly correlated in real 3D space, they fill some small **submanifold** of the whole space only.

In fact one should define T_c as the divergence temperature of the Cooper ladder

$$\mathcal{C} = \left(1 - \hat{K}\right)^{-1}$$

Thus averaging procedure should be applied to $\mathcal{C}$ instead of K

We expand $\mathcal{C}$ in powers of K and average over disorder realizations. Keeping main sequence of resulting diagramms only, we come to the following equation for determination of T_c:

$$\Phi(\xi) = \frac{\lambda}{2} \int \frac{d\xi' \tanh(\xi'/2T)}{\xi'} M(\xi - \xi')\Phi(\xi') \qquad (11)$$

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)} d^d r \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

For critical eigenstates

$$L_{\text{loc}} \to \infty$$

one finds

$$M(\omega) = \left(\frac{E_0}{\omega}\right)^{\gamma}$$

where

$$\gamma = 1 - \frac{D_2}{d}$$

$D_2 \approx 1.3$ in 3D

is a measure of fractality

Usual "dirty superconductor":

$$M(\omega) = 1 \qquad \gamma = 0$$

3D Anderson model: $\gamma = 0.57$

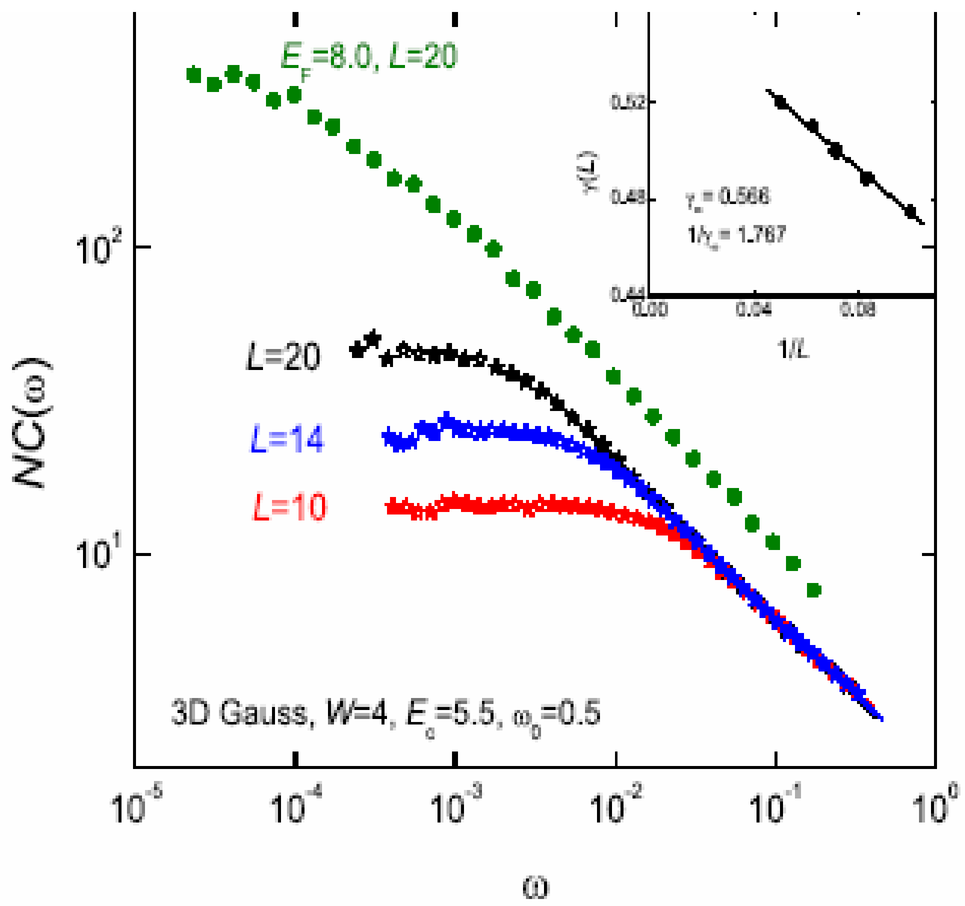


FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

Modified mean-field approximation for critical temperature T_c

$$\Delta(\xi) = \frac{\lambda}{2}\int d\zeta\eta(\zeta)M(\xi-\zeta)\Delta(\zeta)$$

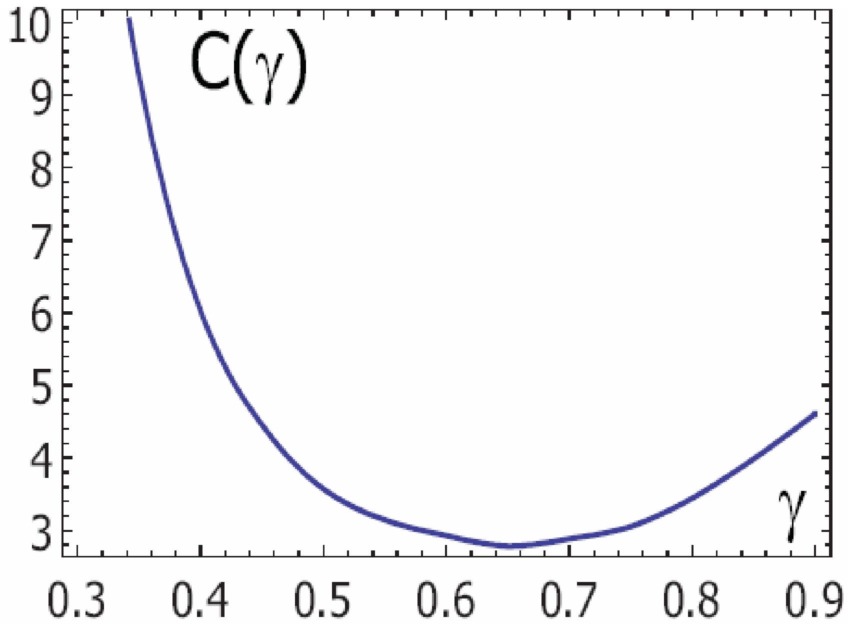
$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

$$T_c^0(\lambda,\gamma) = E_0\lambda^{1/\gamma}C(\gamma)$$

For small λ this T_c is higher than BCS value !

arxiv:0810.2915 Y.Yanase & N.Yorozu: T_c for doped diamond, Si and SiC

Numerical solution of MMFA



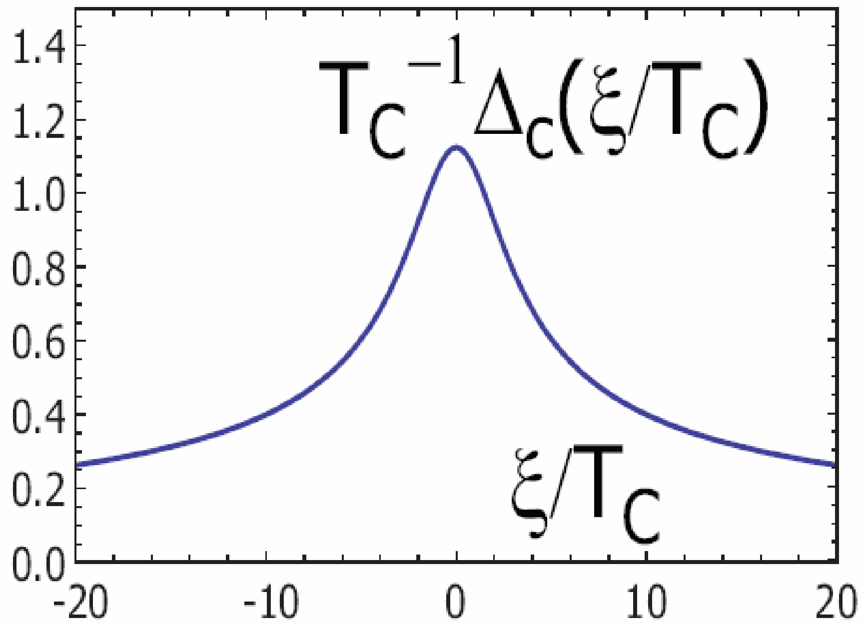


FIG. 13: Functional dependence $\Delta(\xi)$ for $T = T_c$ at $\gamma = 0.57$.

Virial expansion method

(A.Larkin & D.Khmelnitsky 1970)

$$F = \sum_{n=1}^{\infty} \mathcal{F}^{(n)} = \sum_i F_i + \sum_{i>j} (F_{ij} - F_i - F_j)$$
$$+ \sum_{i>j>k} (F_{ijk} - F_{ij} - F_{jk} - F_{ik} + F_i + F_j + F_k) + \ldots$$

$$V_\Delta = -\sum_j (\Delta S_j^+ + \Delta^* S_j^-)$$

$$\chi(T) = -\frac{\partial^2 F}{\partial \Delta \partial \Delta^*} = \sum_{M=1} \chi_M(T)$$

$$\chi_1 = \sum_i \chi_i^{(1)} \qquad (148)$$
$$\chi_2 = \sum_{n>m} (\chi_{nm}^{(2)} - \chi_n^{(1)} - \chi_m^{(1)})$$
$$\chi_3 = \sum_{n>m>l} (\chi_{nml}^{(3)} - \chi_{nl}^{(2)} - \chi_{ml}^{(2)} - \chi_{nm}^{(2)} + \chi_n^{(1)} + \chi_m^{(1)} + \chi_l^{(1)})$$

$$\lim_{n\to\infty} \frac{\chi^{(n+1)}(T_c)}{\chi^{(n)}(T_c)} = 1 \quad \Rightarrow \quad \frac{\chi^{(3)}(T_c)}{\chi^{(2)}(T_c)} = 1$$

T_c from 3 different calculations

Modified MFA equation leads to:

$$T_c = (6.5 \pm 0.8)\lambda^{1.77}$$

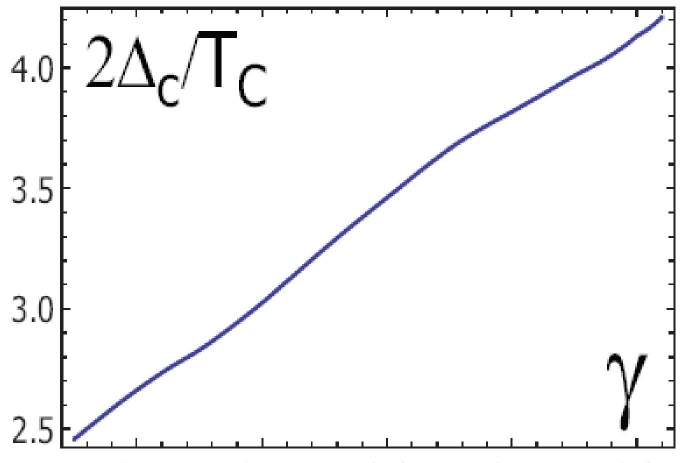
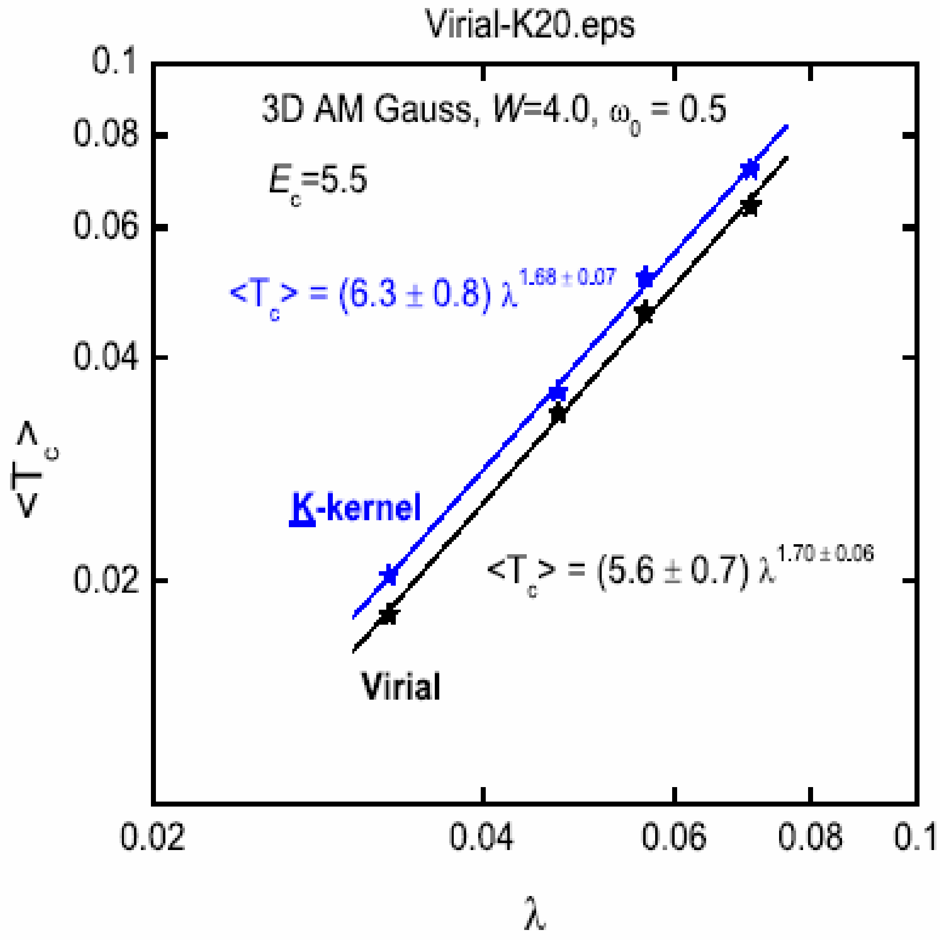


FIG. 16: Ratio $2\Delta(0)/T_c$ as function of γ.



Order parameter in real space

$$\tilde{\Delta}(\mathbf{r}) = \frac{g}{2}\sum_k \Delta_k \eta_k \psi_k^2(\mathbf{r})$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

$\Delta_k \Rightarrow \Delta(\xi)$ for $\xi = \xi_k$

$$\overline{(\tilde{\Delta}(\mathbf{r}))^2} \equiv \frac{1}{\mathcal{V}}\int d^d\mathbf{r}\tilde{\Delta}^2(\mathbf{r}) = \lambda\int_0^\infty d\xi\eta(\xi)\Delta_c^2(\xi)$$

$$\overline{\tilde{\Delta}(\mathbf{r})} \equiv \frac{1}{\mathcal{V}}\int d^d\mathbf{r}\tilde{\Delta}(\mathbf{r}) = \lambda\int_0^\infty d\xi\eta(\xi)\Delta_c(\xi)$$

Fluctuations of SC order parameter

With Prob = p << 1 Δ(r) = Δ , otherwise Δ(r) =0 ⇨

SC fraction = $$\frac{\left(\overline{\tilde{\Delta}(\mathbf{r})}\right)^2}{\overline{(\tilde{\Delta}(\mathbf{r}))^2}} = \lambda Q(\gamma) = \frac{Q(\gamma)}{C^{\gamma}(\gamma)}\left(\frac{T_c}{E_0}\right)^{\gamma} \ll 1$$

prefactor ≈ 1.7 for γ = 0.57

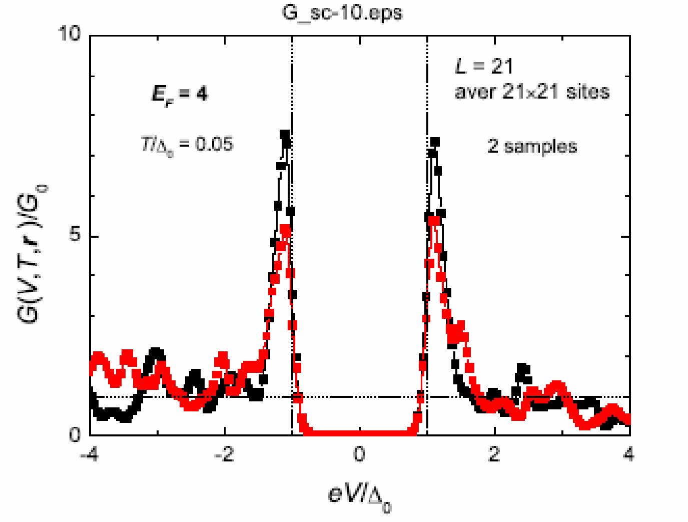
Higher moments: $$\frac{\left(\overline{\tilde{\Delta}(\mathbf{r})}\right)^n}{\overline{(\tilde{\Delta}(\mathbf{r}))^n}} \propto (T_c/E_0)^{(1-d_n/d)(n-1)}$$

$$\langle P_q \rangle \sim \ell^{-(d-d_q)(q-1)} L^{-d_q(q-1)} \propto L^{-d_q(q-1)}$$

Tunnelling DoS

$$\nu(\varepsilon, \mathbf{r}) = \frac{1}{2} \sum_j \left(1 + \frac{\xi_j}{\varepsilon}\right) \left[\delta(\varepsilon - \varepsilon_j) + \delta(\varepsilon + \varepsilon_j)\right] \psi_j^2(\mathbf{r})$$

$$\nu(\varepsilon) = \nu_0 \left| \frac{d\xi(\varepsilon)}{d\varepsilon} \right|$$



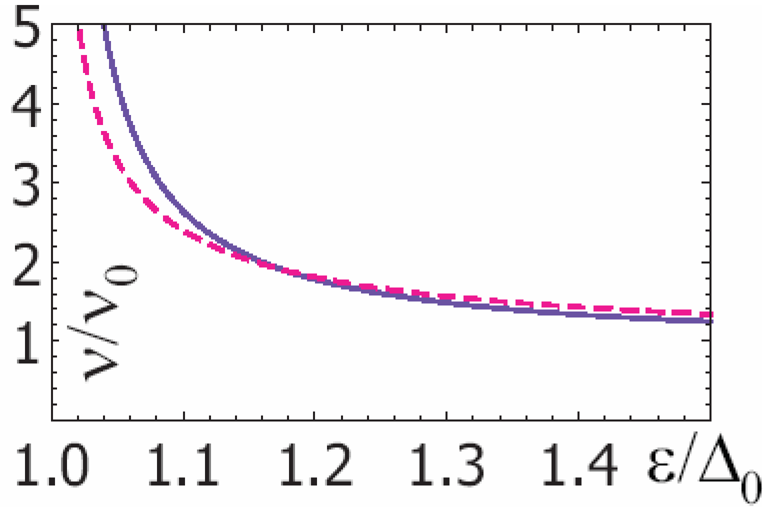


FIG. 23: Average zero-temperature density of states for t critial 3DAM solution (full line) together with BCS dens of states with the same gap value (dashed line).

Asymmetry:

$$\nu_a(\varepsilon, \mathbf{r}) = \tfrac{1}{2}(\nu(\varepsilon, \mathbf{r}) - \nu(-\varepsilon, \mathbf{r}).$$

$$\overline{\nu_a^2(\varepsilon, \mathbf{r})} = \frac{1}{2} \left(\nu(\varepsilon) \frac{\xi(\varepsilon)}{\varepsilon}\right)^2 \left[M(0) - M(2\xi(\varepsilon))\right]$$

Neglected : off-diagonal terms

$$M_{ijkl} = \int d\mathbf{r} \psi_i^*(\mathbf{r}) \psi_j^*(\mathbf{r}) \psi_k(\mathbf{r}) \psi_l(\mathbf{r}) \,,$$

Non-pair-wise terms with 3 or 4 different eigenstates were omitted

To estimate the accuracy we derived effective Ginzburg-Landau functional taking these terms into account

$F[\Psi(\mathbf{r})]$ defined in terms of an envelope function

$$\Psi(\mathbf{r}) = \Delta(\mathbf{r})/\tilde{\Delta}(\mathbf{r}) \qquad \tilde{\Delta}(\mathbf{r}) = \frac{g}{2}\sum_k \Delta_k \eta_k \psi_k^2(\mathbf{r})$$

$$F_{GL}[\Psi(\mathbf{r})] = \nu_0 T_c^2 \int d\mathbf{r} \left(a(\mathbf{r})\Psi^2(\mathbf{r}) + \frac{b}{2}\Psi^4(\mathbf{r}) + C|\nabla\Psi(\mathbf{r})|^2 \right)$$

$$\mathrm{Gi} \sim \frac{b^2}{C^3(\nu_0 T_c)^2} \sim 1 \qquad \mathrm{Gi}_d \sim \frac{W^2}{C^3} \sim 1$$

Superconductivity at the Mobility Edge: major features

- Critical temperature T_c is well-defined through the whole system in spite of strong $\Delta(r)$ fluctuations
- Local DoS strongly fluctuates in real space; it results in asymmetric tunnel conductance

  $G(V,r) \neq G(-V,r)$
- Both thermal (Gi) and mesoscopic (Gi_d) fluctuational parameters of the GL functional are of order unity

Superconductivity with Pseudogap

Now we move Fermi-level into the range of localized eigenstates

Local pairing in addition to collective pairing

Local pairing energy

1. Parity gap in ultrasmall grains

K. Matveev and A. Larkin 1997

$\Delta \ll \delta$: **no many-body correlations**

$\Delta_P = \frac{1}{2}\lambda\delta$ $\lambda_R = \lambda/(1 - \lambda \log(\epsilon_0/\delta)).$ $\Delta_P = \dfrac{\delta}{2\ln\frac{\delta}{\Delta}}$

2. Parity gap for Anderson-localized eigenstates

The increase of thermodynamic potential Ω due to addition of *odd* electron to the ground-state is

$$\delta\Omega_{\mathrm{oe}} = \xi_{m+1} = \xi_{m+1} - \tilde{\xi}_{m+1} + \tilde{\xi}_{m+1} = \frac{g}{2} M_{m+1} + O(\mathcal{V}^{-1})$$

$$\tilde{\xi}_j = \xi_j - \frac{g}{2} M_j.$$

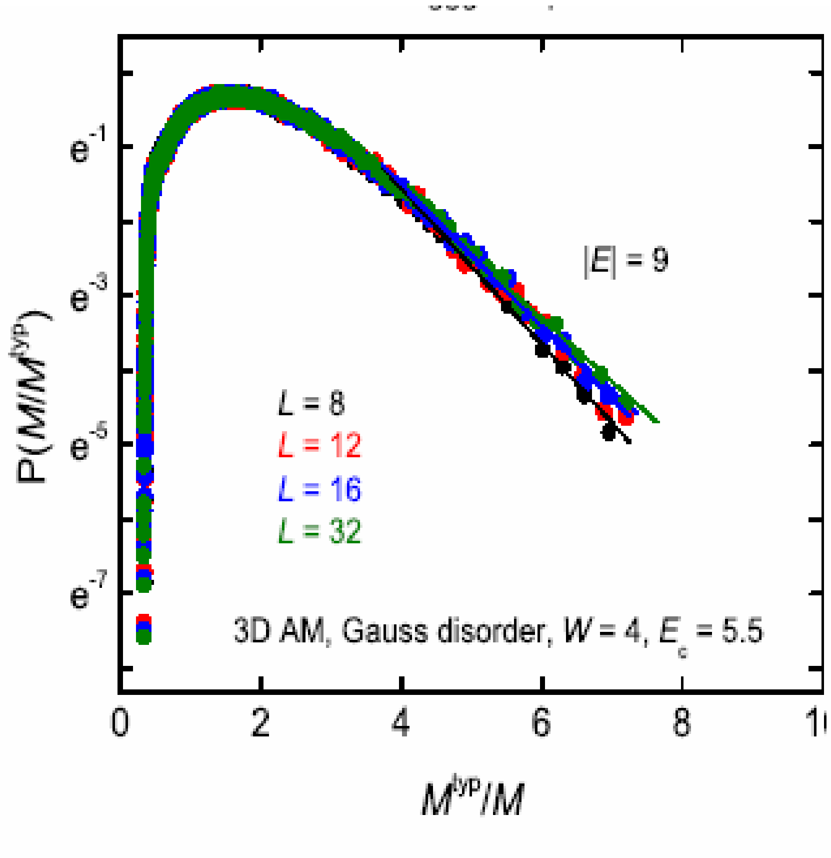
Energy of two single-particle excitations due to depairing:

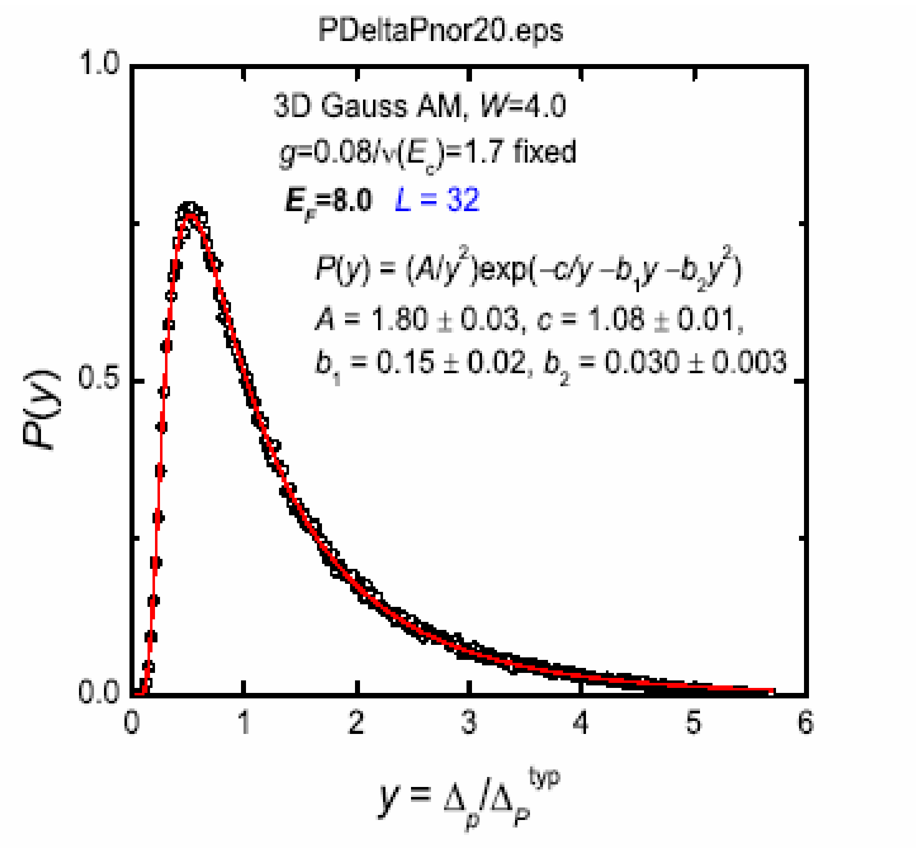
$$2\Delta_P = \xi_{m+1} - \xi_m + gM_m = \frac{g}{2}(M_m + M_{m+1}) + O(\mathcal{V}^{-1})$$

$$\langle M_i \rangle = 3\ell^{-(d-d_2)} L_{\mathrm{loc}}^{-d_2},$$

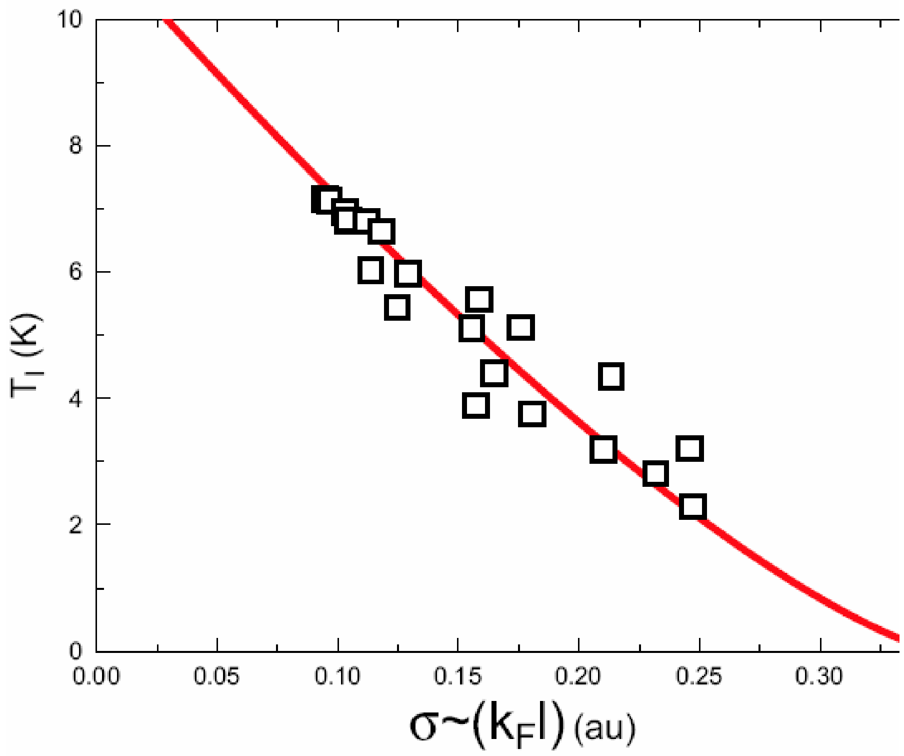
$$\Delta_P = \frac{3}{2} g \ell^{-3} (L_{loc}/\ell)^{-d_2} = \frac{3\lambda}{2} E_0 \left(\frac{E_c - E_F}{E_0} \right)^{\nu d_2}$$

P(M) distribution





Activation energy T_I from Shahar-Ovadyahu exp. and fit to theory



The fit was obtained with single fitting parameter

$$A \approx 0.5\lambda E_0$$

Example of consistent choice:

λ **= 0.05** E_0 **= 400 K**

Tunnelling conductance

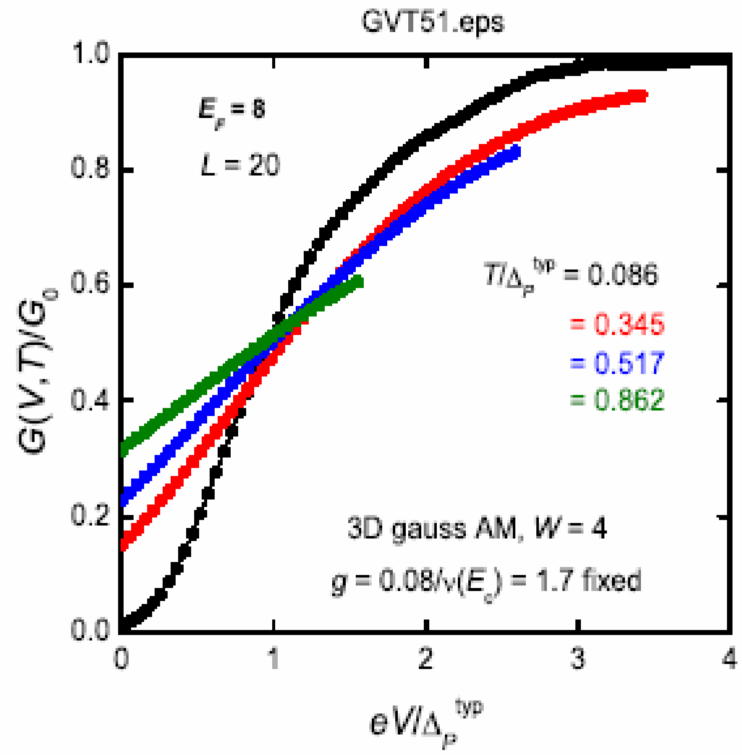


FIG. 30: (Color online) Ensemble-averaged tunneling conductance, Eq.(176), for several values of temperature below the typical local gap Δ_P^{typ}.

$$\frac{G(V,T,\mathrm{r})}{G_0} = \nu_0^{-1} \int d\varepsilon \nu(\varepsilon,\mathrm{r}) \left(-\frac{\partial f(\varepsilon - eV)}{\partial \varepsilon} \right)$$

$$\nu_n(\varepsilon) = \nu_0(\varepsilon) \int_0^{\infty} \frac{e^{-yz} + \cosh\frac{\varepsilon}{T}}{\cosh yz + \cosh\frac{\varepsilon}{T}} \cdot P(y)dy$$

Critical temperature in the pseudogap regime

MFA:

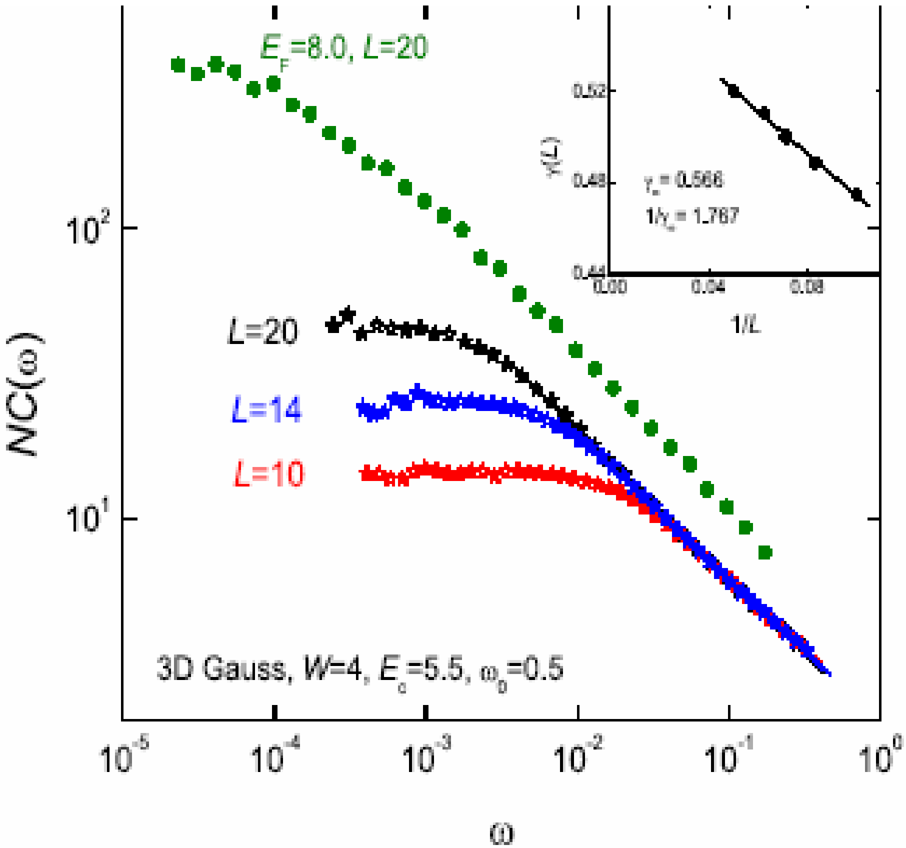
$$\Delta(\xi) = \frac{\lambda}{2} \int d\zeta \eta(\zeta) M(\xi - \zeta) \Delta(\zeta)$$

Take the same $\eta_i \equiv \eta_{ii} = \xi_i^{-1} \tanh(\xi_i / 2T).$

but use M(ω) specific for localized states

MFA is good as far as $Z \sim \nu_0 T_c L_{loc}^d$ **is large**

Correlation function M(ω)



No saturation at **ω < δ$_L$** :
M(ω) ~ ln² (δ$_L$ / ω)
(Cuevas & Kravtsov 1997)

Superconductivity with
Tc < δ$_L$ is possible

This region was not found previously

Here "local gap" exceeds **SC gap :**

$$\Delta_P = \frac{1}{2D^\gamma(\gamma)}\delta_L \left(\frac{\Delta(0)}{\delta_L}\right)^\gamma$$

FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

Critical temperature in the pseudogap regime

MFA:

$$\Delta(\xi) = \frac{\lambda}{2}\int d\zeta \eta(\zeta) M(\xi-\zeta)\Delta(\zeta)$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

We need to estimate $Z \sim \nu_0 T_c L_{loc}^d$

$$R_\omega \approx 2L_{\text{loc}} \ln\frac{\delta_L}{\omega} \gg L_{\text{loc}}$$

$$R_0^2 = \frac{\sum_{ij} r_{ij}^2 M_{ij}}{\sum_{ij} M_{ij}}$$

$$Z_{\text{eff}} \equiv \nu_0 T_c R_0^3 = 8\frac{T_c}{\delta_L}\ln^3\frac{\delta_L}{T_c}$$

It is nearly constant in a very broad range of $\frac{\delta_L}{T_c}$

Virial expansion results:

T_c versus Pseudogap

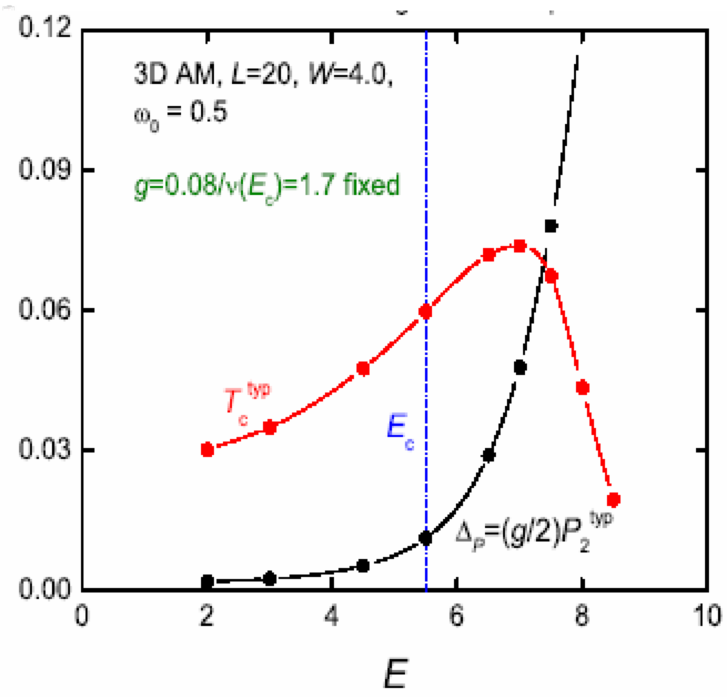


FIG. 25: (Color online) Virial expansion results for T_c (red points) and typical pseudogap Δ_P (black) as functions of E_F. The model with fixed value of the attraction coupling constant $g = 1.7$ was used; pairing susceptibilities were calculated using equations derived in Appendix B.

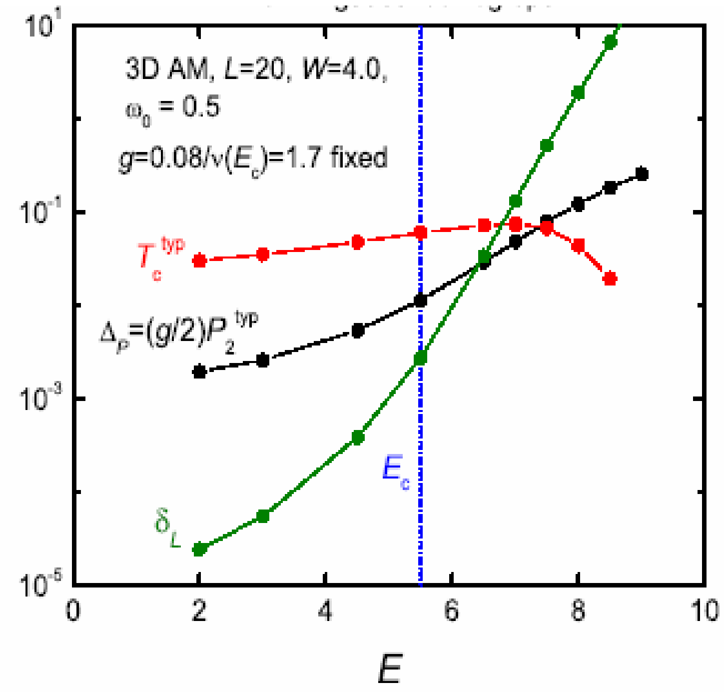


FIG. 26: (Color online) Virial results for T_c (red points), typical pseudogap Δ_P (black) and the corresponding level spacing δ_L (green), as functions of E_F on semi-logarithmic scale.

Transition exists even at $\delta_L \gg T_c$

Single-electron states suppressed by pseudogap

„Pseudo spin" representation:

$$S^+_\mu = a^+_{\mu\uparrow} a^+_{\mu\downarrow} \qquad S^-_\mu = a_{\mu\uparrow} a_{\mu\downarrow}$$

$$2S^z_\mu = a^+_{\mu\uparrow} a_{\mu\uparrow} + a^+_{\mu\downarrow} a_{\mu\downarrow}$$

H_{BCS} acts on Even sector: all states which are 2-filled or empty

$$\hat{H} = \sum_\mu 2\xi_\mu S^z_\mu - g \sum_{\mu,\nu} M_{\mu\nu} S^+_\mu S^-_\nu + \sum_{B_\mu} \left(\xi_\mu + \frac{G_\mu}{2}\right)$$

B: „blocked" states

$$\bar{M}_{\mu\nu} = \frac{1}{\nu V} M(\xi_\mu - \xi_\nu)$$

D.o.S (ν), total volume (V)

"Pseudospin" approximation

$Z \sim \nu_0 T_c L^d_{loc}$ **Effective number of interacting neighbours**

Third Scenario

- Bosonic mechanism: preformed Cooper pairs + competition Josephson v/s Coulomb – S I T in arrays
- Fermionic mechanism: suppressed Cooper attraction, no paring – S M T

- Pseudospin mechanism: individually localized pairs

- S I T in amorphous media, fractal superconductivity

SIT occurs at small Z and lead to paired insulator

Average tunnelling conductance

$$\nu_{sc}(\varepsilon) = \int_0^\infty P(y) \left[\frac{\nu_{sc}^{(0)}(\varepsilon - yzT)}{e^{yz-\varepsilon/T}+1} + \frac{\nu_{sc}^{(0)}(\varepsilon + yzT)}{e^{yz+\varepsilon/T}+1} \right] dy$$

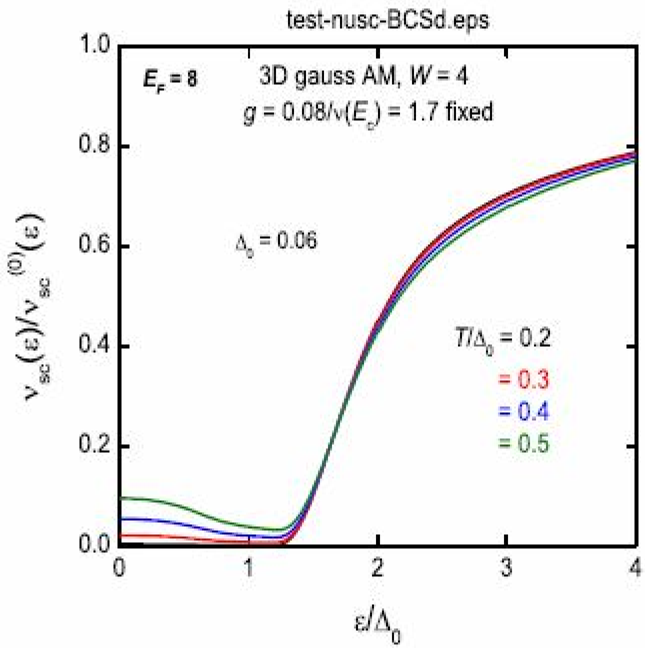


FIG. 31: (Color online) Ensemble-averaged tunneling DoS, Eq.(181), for several values of temperature much below the value of superconductive gap Δ_0. The latter is related to the typical local gap $\Delta_P^{\rm typ}$ as $\Delta_0 = 0.5\Delta_P^{\rm typ}$.

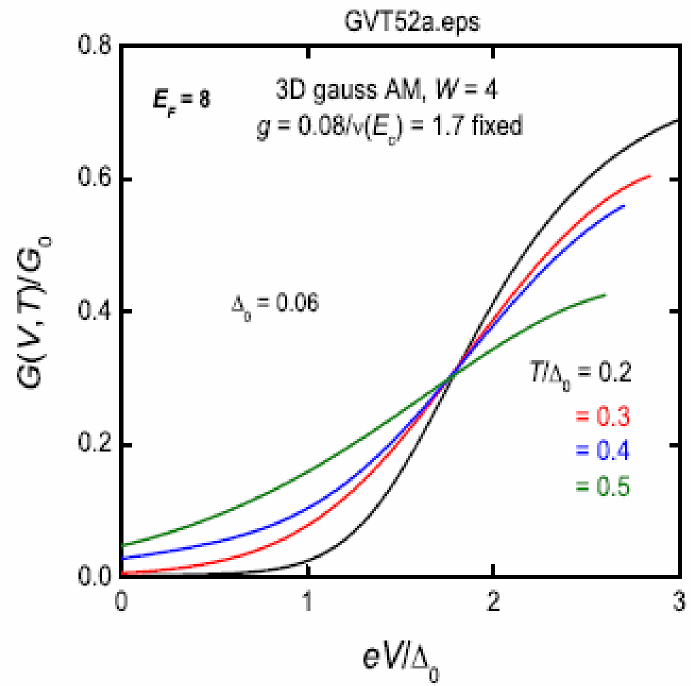
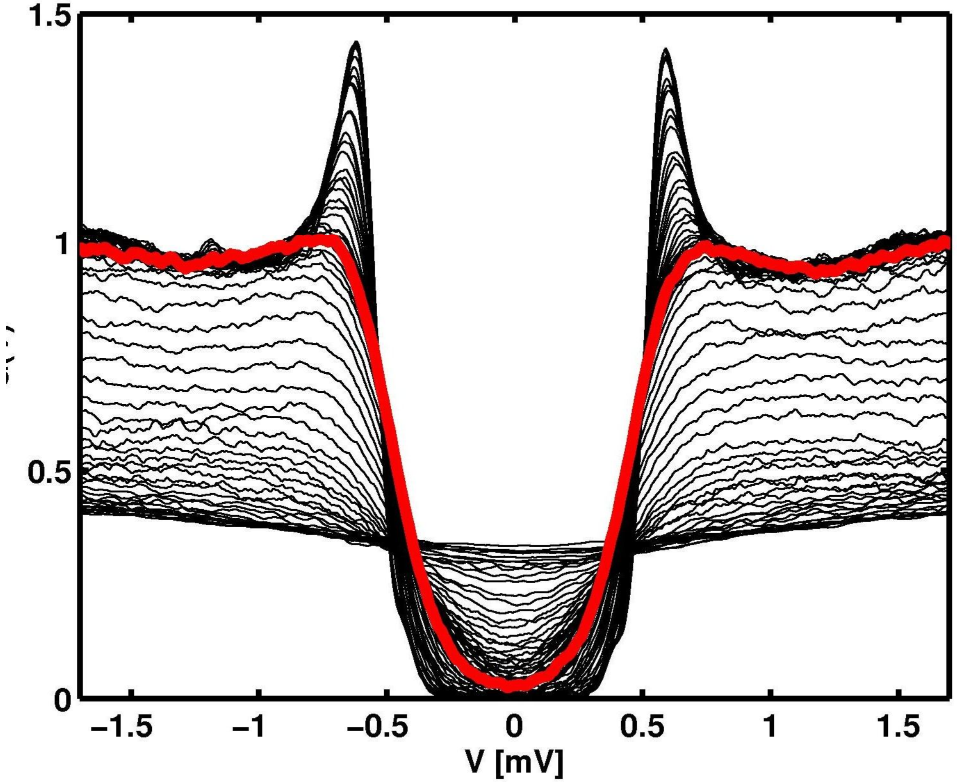
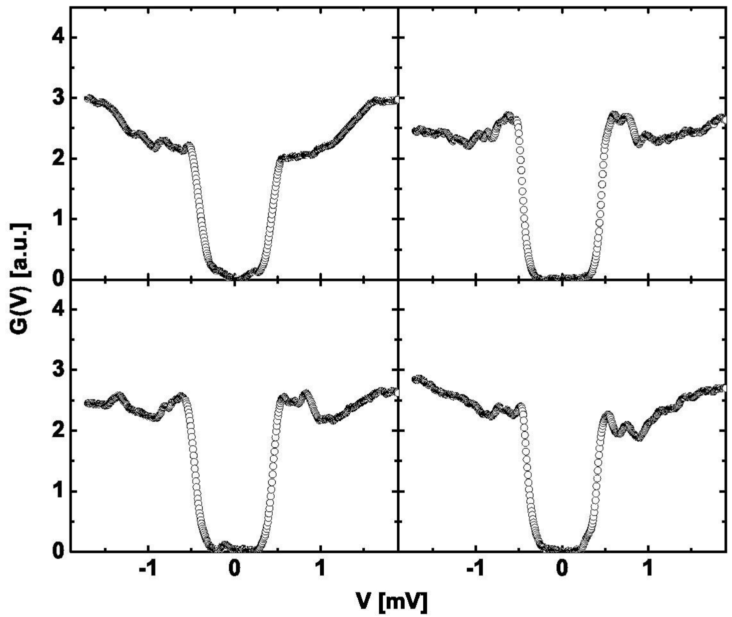


FIG. 32: (Color online) Average tunnelling conductance in superconductive state, for several values of temperature much below the value of superconductive gap $\Delta_0 = 0.5\Delta_P^{\rm typ}$.

Strong local pseudogap above T_c :
experiment B.Sacepe et al





At T=Tc - almost fully developed gap but no coherence peak

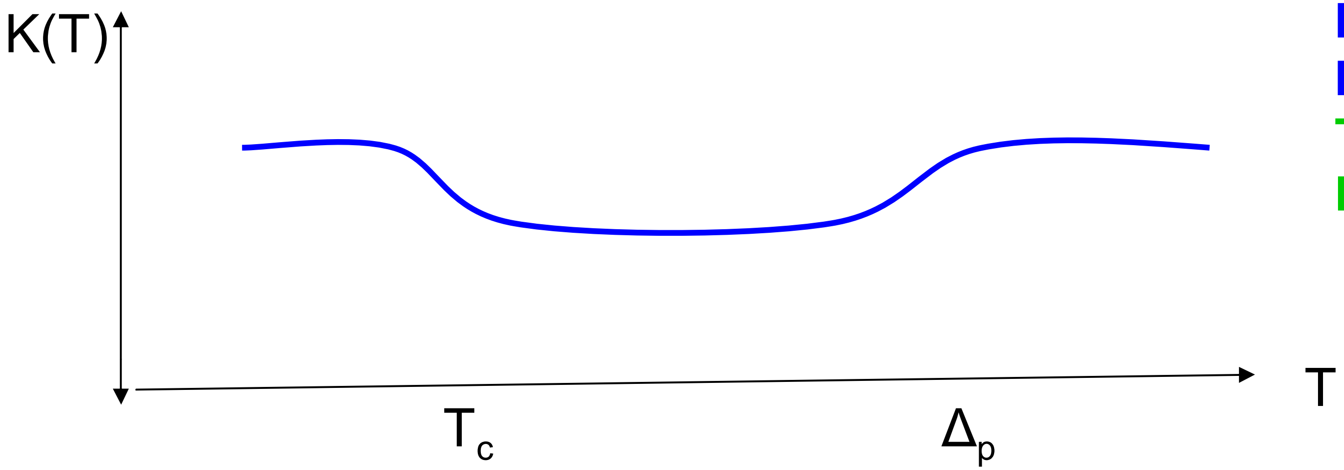
Full Spectral Weight K(T)

$$K(T) = \int^{\Omega} d\omega\, \sigma(\omega, T) \qquad \Omega < \Delta_p$$

is usually (BCS) const across T_c :

contributions from superconductive response and from DoS suppression cancel each other

Not valid for underdoped HTSC :



Experiment:
D.Basov et al 1994
Theory:
L.Ioffe & A.Millis 1999

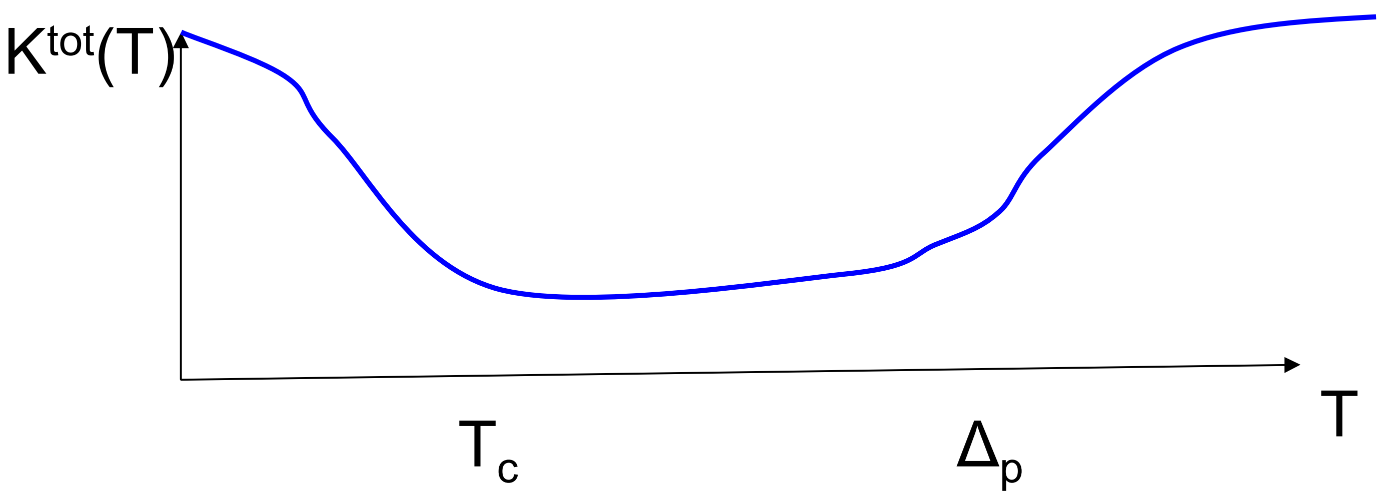
Full Spectral Weight K(T)

$$K^{tot}(T) = \frac{2}{\pi}\int_0^{\Omega_{max}} \Re\sigma(\omega,T)d\omega + \rho_s(T) \equiv K(T) + \rho_s(T)$$

$$\frac{K(2T_c) - K(T_c)}{K(T_c)} \sim \frac{T_c}{\Delta_P} \ll 1$$

$$\frac{K(T_c)}{\rho_s(0)} \sim \frac{gM_{ij}}{T_c} \sim \frac{1}{Z_{\text{eff}}}$$

$$\frac{K(T_c) - K(0)}{K(T_c)} \sim \frac{1}{2}.$$

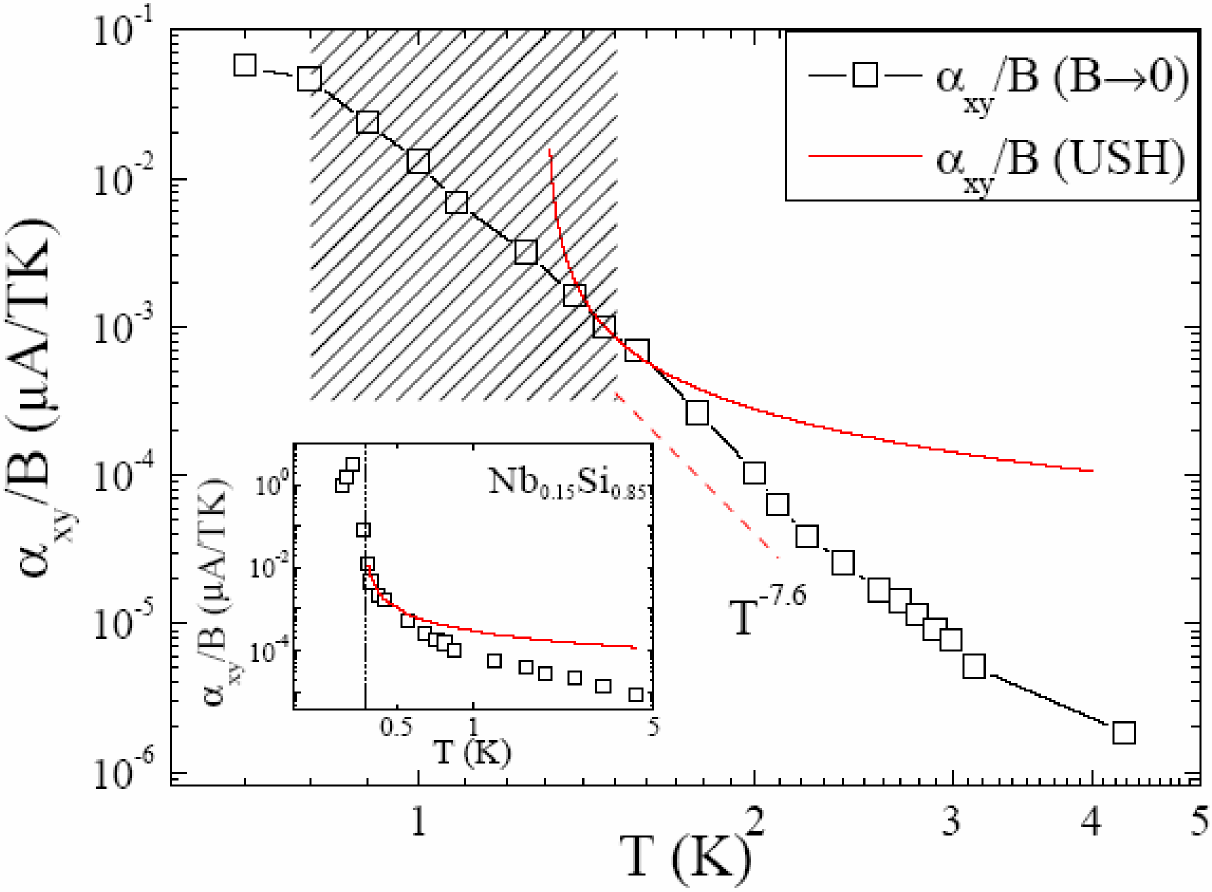


Qualitative features of "Pseudogaped Superconductivity":

- STM DoS evolution with T
- Nonconservation of full spectral weight across T_c
- Anomalous Nernst effect above T_c

Nernst coeff. in a-InOx

P.Spathis, H. Aubin *et al* 2007



Similarity to underdoped HTSC

Exponent 7.6 ??

No way to describe InO_x data by Gaussian fluctuations contrary to NbSi case: ***M.Serbyn et al, Phys.Rev.Lett. 102, 067001 (2009)***
K.Michaeli and A.Finkelstein arxiv:0902.2732

"Phase fluctuations" ? Where the amplitude comes from ?

S-I Transition

- Hamiltonian of the pseudospin array:

$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$

$$Z \sim \nu_0 T_c L_{loc}^d$$

At Z << 1   Insulating state is realized: localized pairs

How to desribe quantum phase transition ?

See talk by Lev Ioffe

"First-order" transition ?

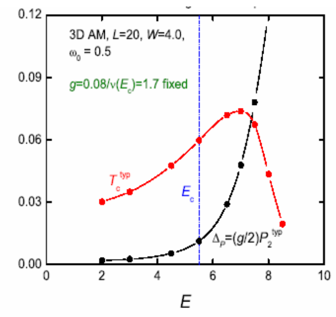


FIG. 25: (Color online) Virial expansion results for T_c (red points) and typical pseudogap Δ_P (black) as functions of E_F. The model with fixed value of the attraction coupling constant $g = 1.7$ was used; pairing susceptibilities were calculated using equations derived in Appendix B.

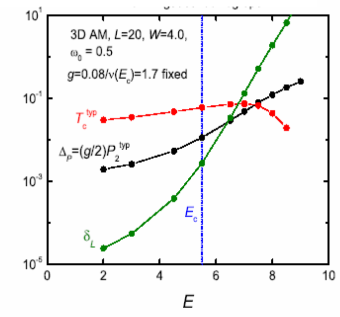


FIG. 26: (Color online) Virial results for T_c (red points), typical pseudogap Δ_P (black) and the corresponding level spacing δ_L (green), as functions of E_F on semi-logarithmic scale.

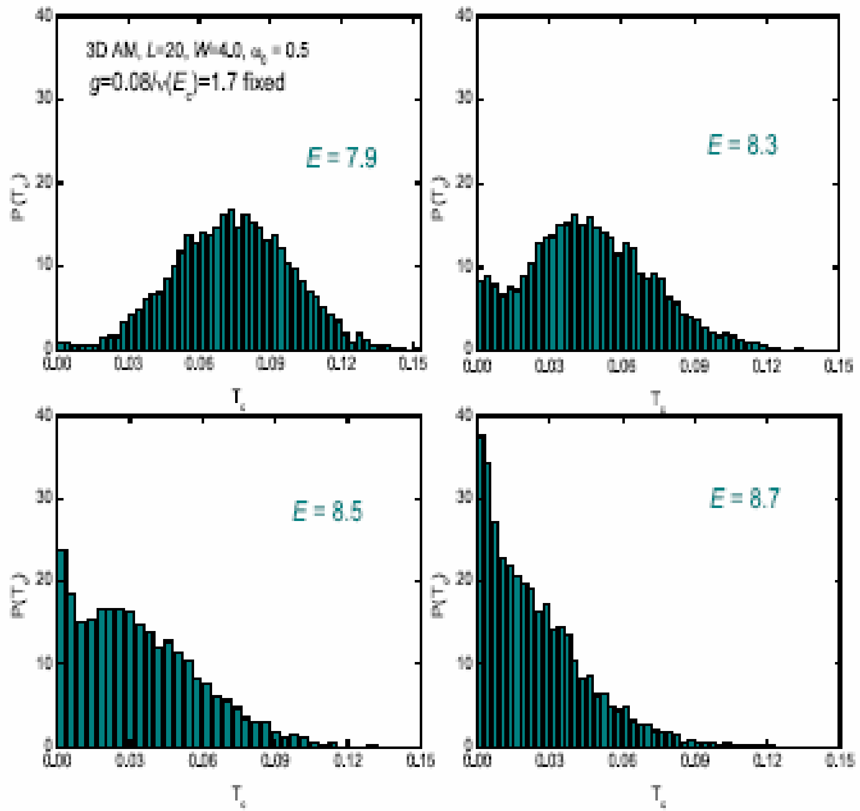


FIG. 27: Distribution functions $P(T_c)$ obtained with virial expanstion for different Fermi energies. All values of T_c was obtained here in the pseudospin approximation $\Delta_P \gg T_c$,

Conclusions

- Pairing of electrons on localized states leads to hard gap and Arrhenius resistivity for 1e transport
- Pairing on nearly-critical states produces fractal superconductivity with relatively high T_c but very small superconductive density
- Pseudogap behaviour is generic near S-I transition, with "insulating gap" exceeding T_c

Major unsolved problems (theor)

- 1. Role of Coulomb enchancement near mobility edge ? (this effect was treated by Finkelstein for metal thin-film case)
- 2. How to include magnetic field into the "fractal" scheme ?
- 3. Transition between pseudogap SC and insulator. Why Cooper pair transport is activated ?
- 4. Rectangular gap in local tunnelling ?
- 5. Size-dependence of SIT (Kowal-Ovadyahu 2007)