2035-10

Conference on Superconductor-Insulator Transitions

18 - 23 May 2009

Phase transitions in strongly disordered magnets and superconductors on Bethe lattice

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Strongly disordered magnets and superconductors.

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1. Very brief summary of common theoretical models (aka preconceived ideas).
2. Experimental evidence: disordered films versus Josephson arrays.
3. Summary: plausible and implausible models for the SI transition in disordered films
4. Bethe lattice model and its solution: qualitative picture of the SI transition.
5. Conclusions.

Collaborators: M. Feigelman, V. Kravtsov, M. Mezard.

Alternative scenarios of superconductor-insulator transitions

- Fermi model (suppression of fermion pairing by Coulomb interaction.

- Bose model (preformed Cooper pairs)

➢ Competition between Coulomb repulsion and Cooper pair hopping

➢ Competition between disorder and Cooper pair hopping

Fermionic model of superconductor-insulator transition

- Disorder increases Coulomb interaction and thus decreases the pairing interaction (sum of Coulomb and phonon attraction). In perturbation theory:

$$\lambda(\varepsilon) = \lambda_0 - \frac{1}{24\pi g} Log\left(\frac{1}{\varepsilon\tau}\right)$$

$$\frac{\delta T_c}{T_c} = -\frac{\delta\lambda}{\lambda^2}$$

If $\lambda(\omega_D)<0$ – attraction is gone and superconductivity disappears *(Finkelstein 1987)*

→ The state formed is likely to be a bad metal (no reasons for a large insulator gap)
No reason for 'superconducting' gap above T_c sensitive to magnetic fields.

→ This scenario might happen in films, not in Josephson arrays.

Bose model (preformed Cooper pairs)

- Competition between Coulomb repulsion and Cooper pair hopping:

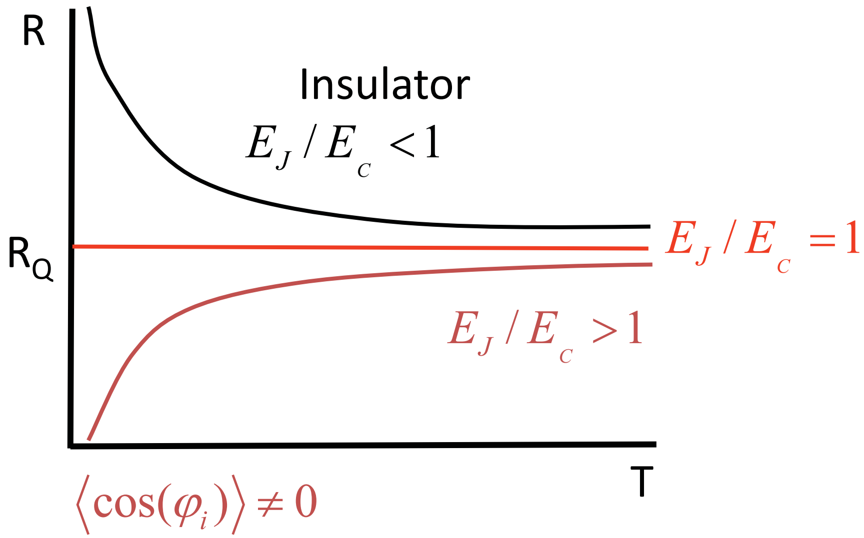
  Duality charge-vortex: both charge-charge and vortex-vortex interaction are Log(R) in 2D.

  Vortex motion generates voltage: $V=\phi_0\, j_V$

  Charge motion generates current: $I=2e\, j_c$

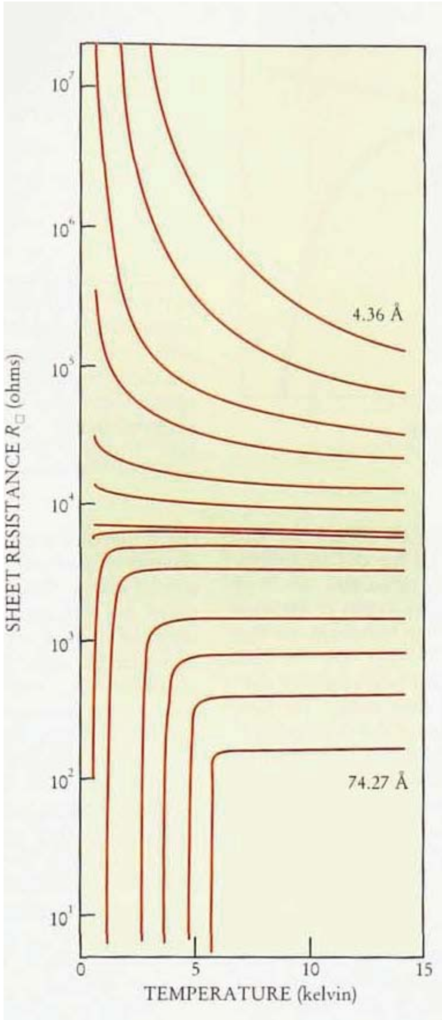
  At the self-dual point the currents are equal → $R_Q=V/I=h/(2e)^2=6.5\text{k}\Omega$.

M. Fisher 1990

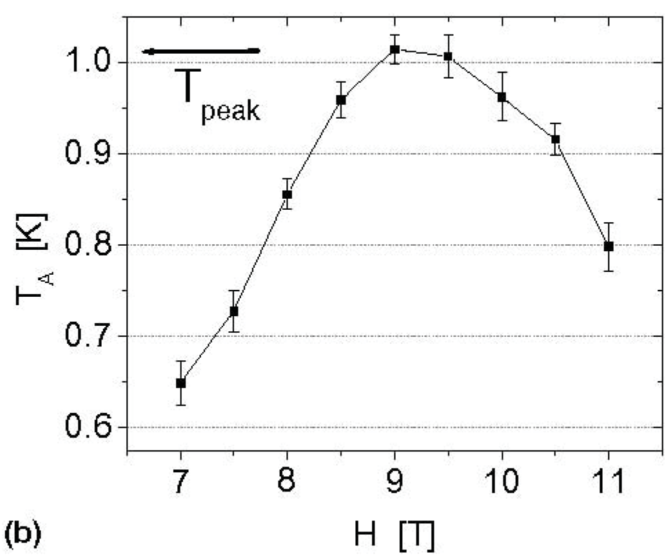


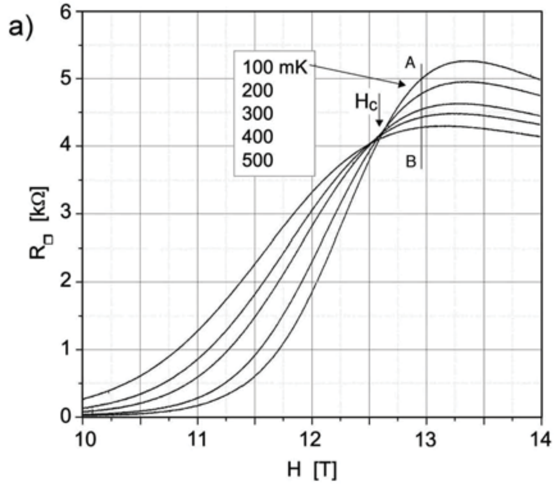
In superconducting films have cores → friction
→ vortex motion is not similar to charges
→ duality is much less likely in the films
→ intermediate normal metal is more likely in Josephson arrays (no vortex cores)

Superconductor-Insulator: experimental evidence



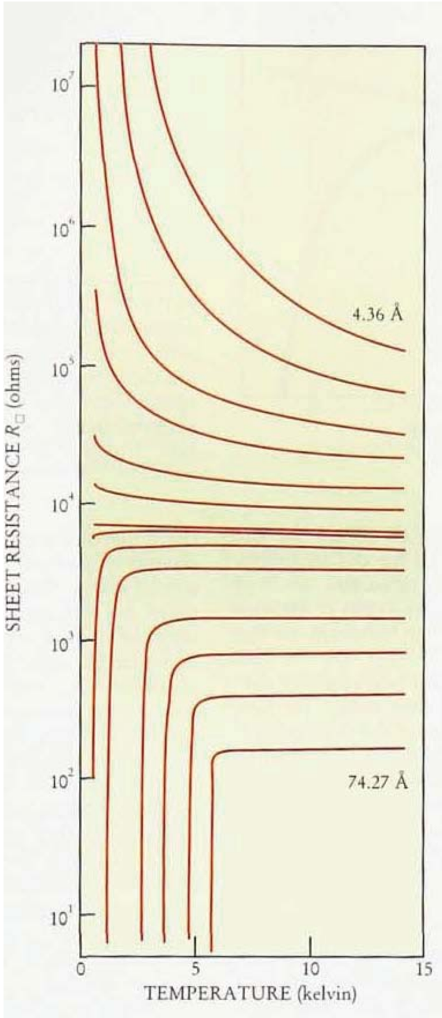
First look: critical behavior as predicted by boson duality *(Haviland, Liu, Goldman 1989, 1991)*



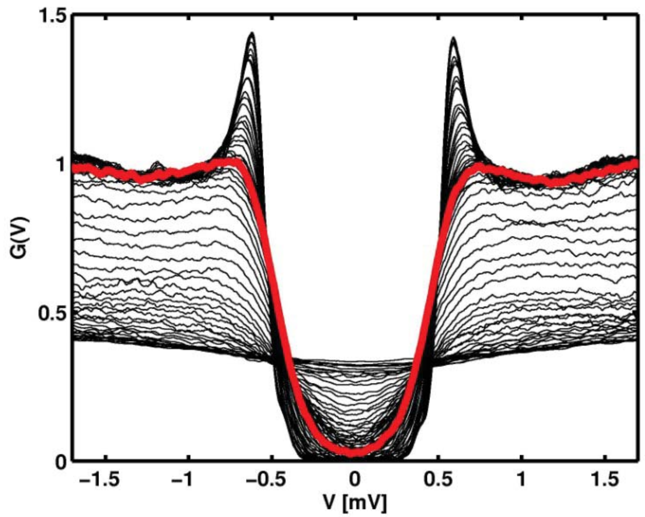


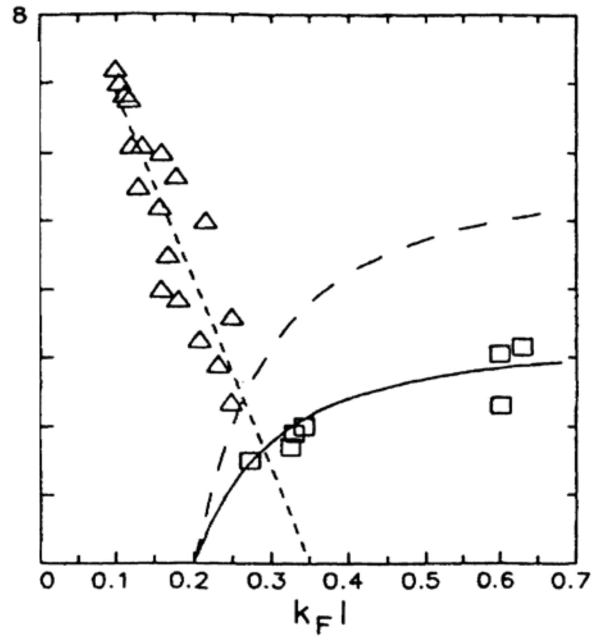
Quantum critical behavior as a function of magnetic field (*Kapitulnik 2008*). Activation dependence of the resistance above transition are difficult to reconcile with fermionic scenario.

Superconductor-Insulator: experimental evidence



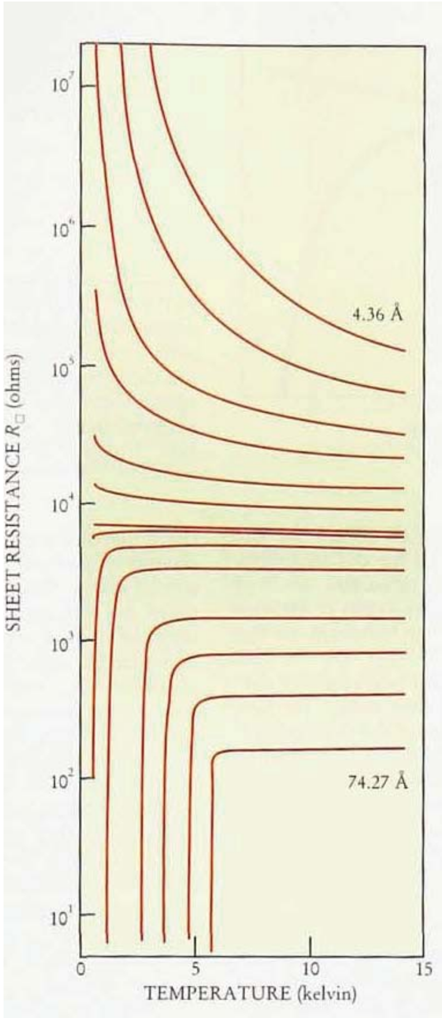
If duality arguments are correct, the transport close to the transition is carried by Cooper pairs/vortices with very small gap





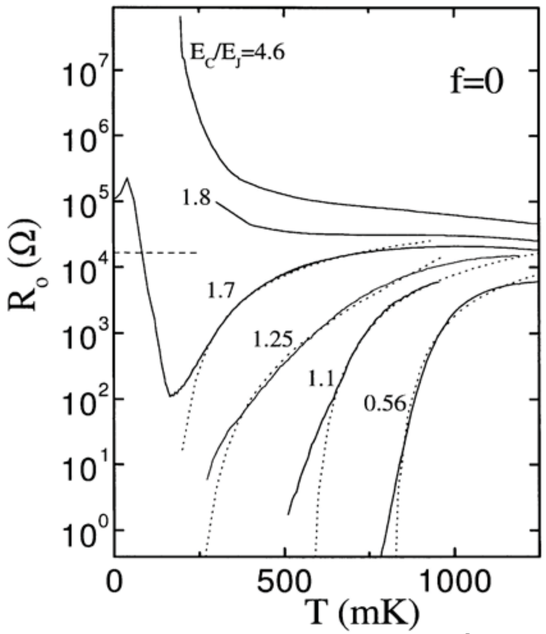
Direct evidence for the gap above the transition (Chapelier, Sacepe). Activation behavior does not show gap suppression at the critical point as a function of the disorder (Sahar, Ovaduyahu, 1992)!

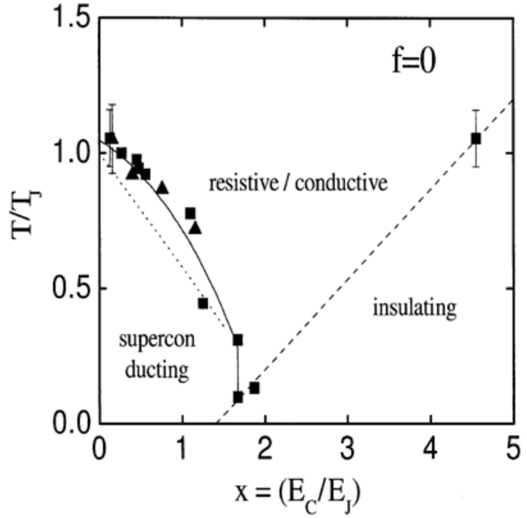
Superconductor-Insulator: experimental evidence



First look: critical behavior as predicted by boson duality *(Haviland, Liu, Goldman 1989, 1991)*

If duality arguments are correct, the same behavior should be observed in Josephson arrays…

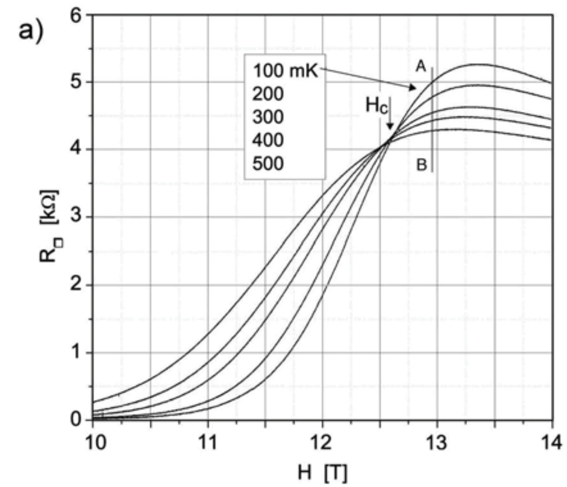




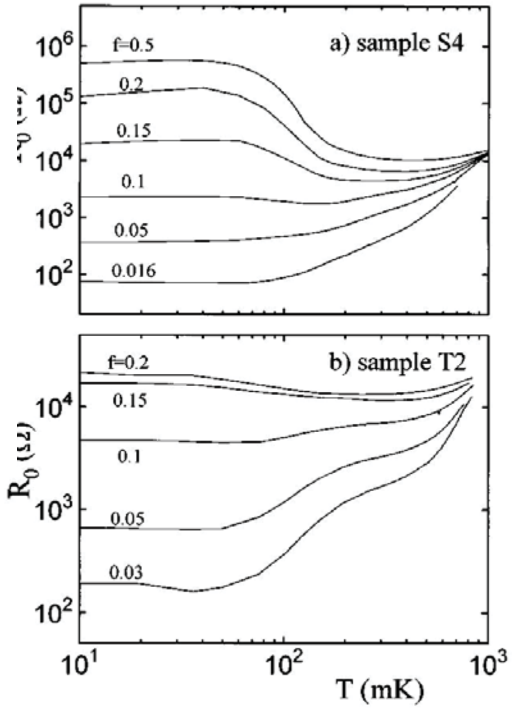
At zero field simple Josephson arrays show roughly the critical behavior. However, the critical R is not universal. (*Zant and Mooji, 1996*) and critical value of E_J/E_c differs.

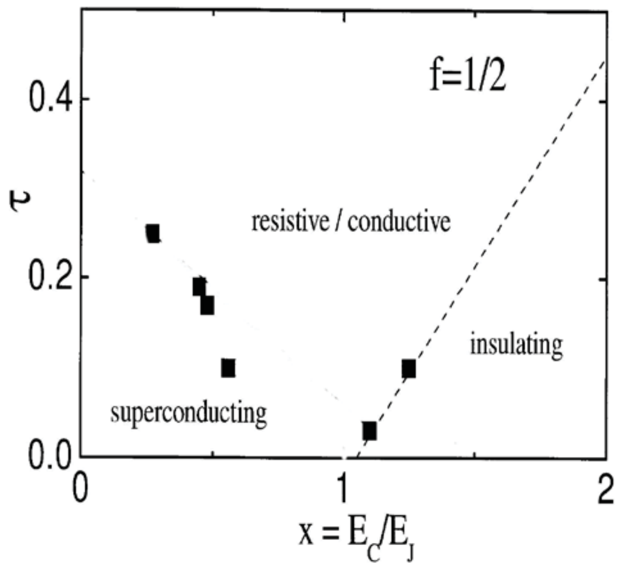
Superconductor-Insulator: experimental evidence

If Josephson/Coulomb model is correct, the same behavior should be observed in Josephson arrays…



Disordered films (*Kapitulnik*)

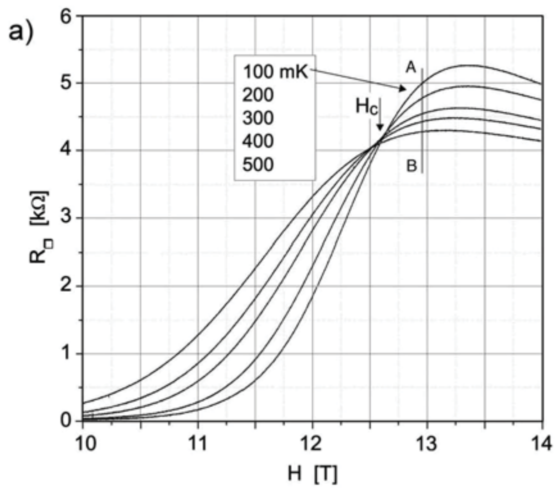




At non-zero field simple Josephson arrays show temperature independent resistance with values that change by orders of magnitude. (*Zant and Mooji, 1996*)

Superconductor-Insulator: experimental evidence

If Josephson/Coulomb model is correct, the same behavior should be observed in Josephson arrays…



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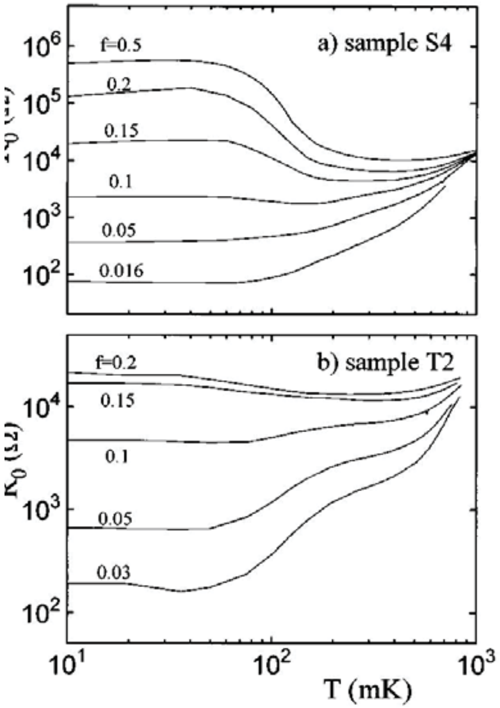

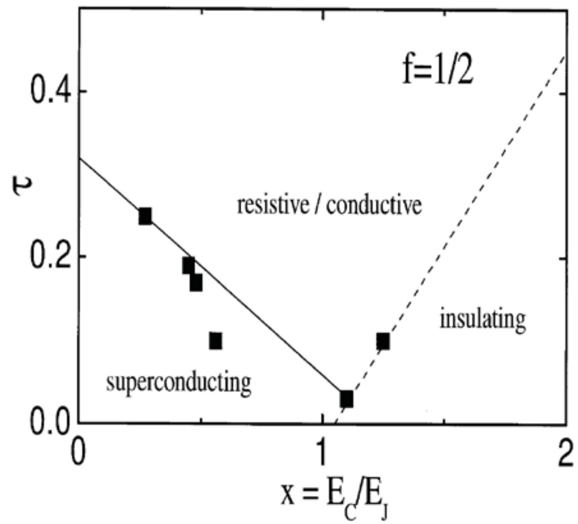

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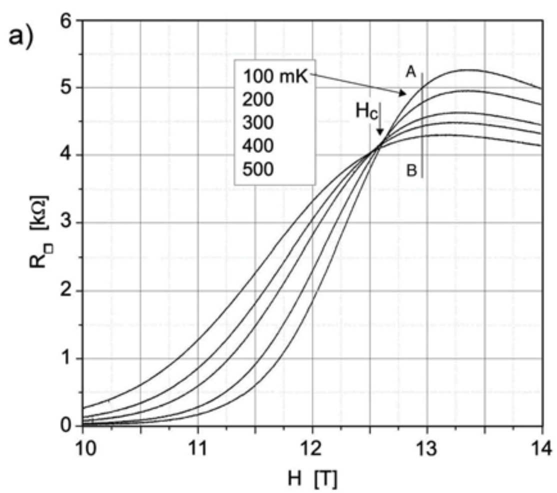
Superconductor-Insulator: experimental evidence

If Josephson/Coulomb model is correct, the same behavior should be observed in Josephson arrays…
BUT IT IS NOT



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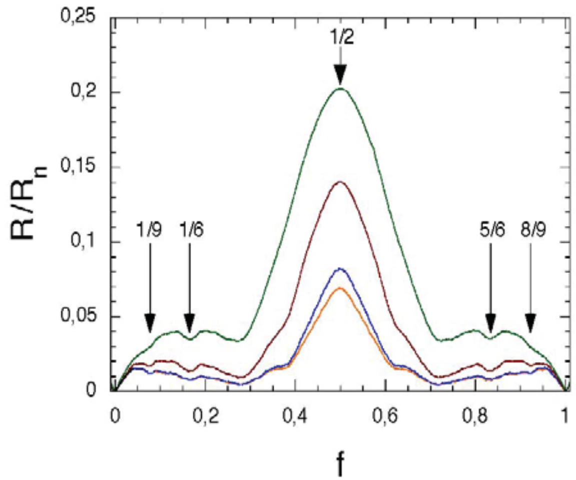
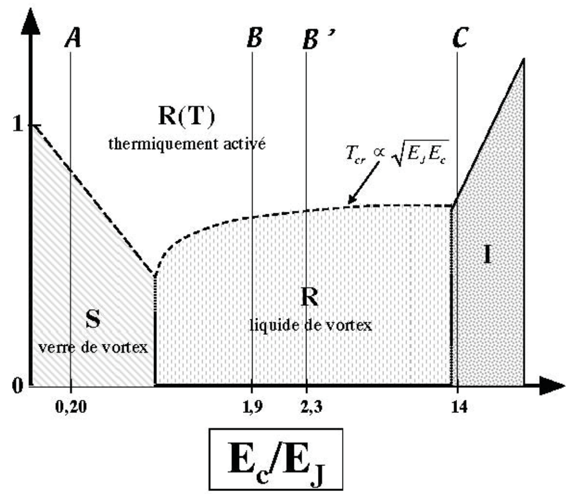
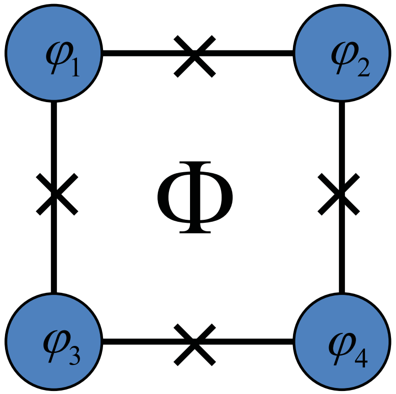




At non-zero field Josephson arrays of more complex (dice) geometry show temperature independent resistance in a wide range of E_J/E_c. (*Pannetier and Serret 2002*)

Josephson Arrays

Elementary building block

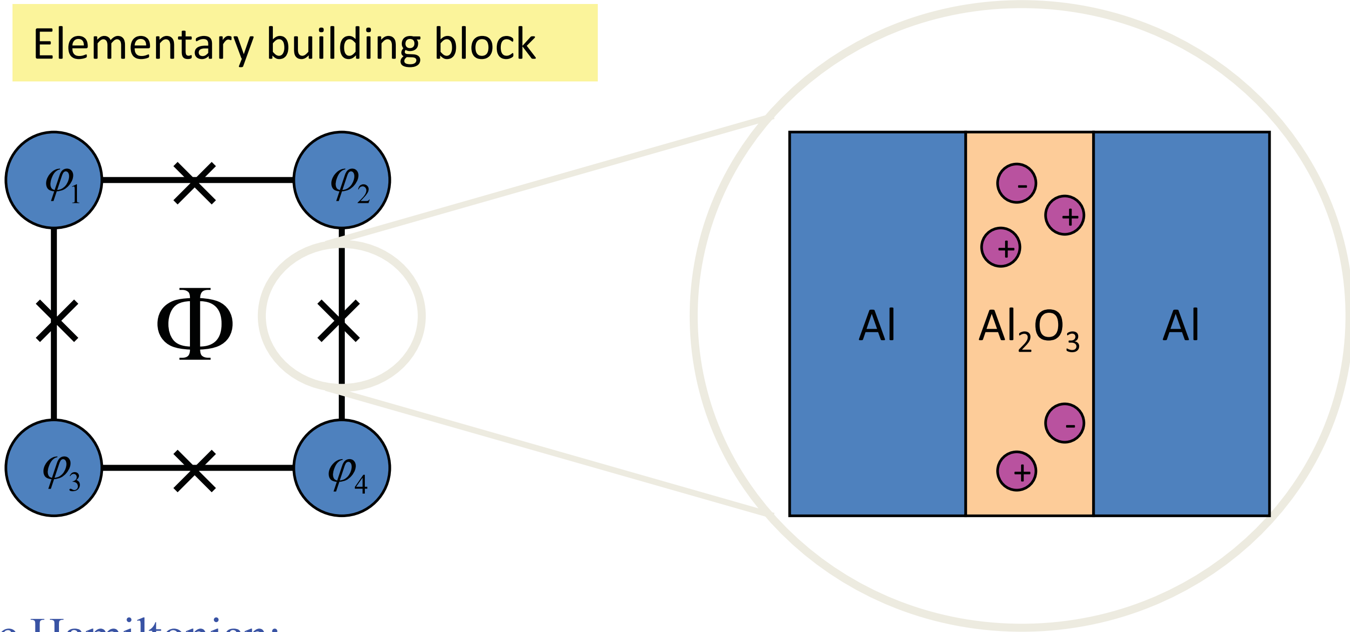


Ideal Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1} q_i q_j + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

Josephson Arrays



More realistic Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1}(q_i + Q_i)(q_j + Q_j) + (E_J + \delta E_J)\cos(\varphi_i - \varphi_j - 2\pi\frac{\Phi_{ij} + \delta\Phi}{\Phi_0}) \quad q_i = 2e\, i\frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

$Q_i = Q_i^0 + Q_i(t)$ - induced charge (static and fluctuating)

$\delta\Phi = \delta\Phi^0 + \delta\Phi(t)$ - static flux due to area scatter and flux noise

$\delta E_J = \delta E_J^0 + \delta E_J(t)$ - static scatter of Josephson energies and their time dependent fluctuations.

Films vs Arrays: summary

- Fermionic mechanism of superconductor-insulator transition is not a likely scenario for most disordered films displaying SI (InO, TiN)

- Quantum critical point behavior is not observed in magnetic field behavior in Josephson arrays (where it was actually expected). Instead there is a wide regime of T-independent resistance that varies in value by many orders.

- Quantum critical behavior is observed in films.

Conclusion:

Superconductor-insulator transition is not driven by Coulomb vs. Josephson competition.

Bose model (preformed Cooper pairs)

- Competition between Cooper pairing and disorder, i.e. no Coulomb interaction. *(Ma and Lee, 1985, Kapitulnik and Kotliar 1985)*

  Potential disorder does not affect the superconductivity provided that

  $T_c \gg \delta_L = 1/\nu_o \xi^D$ – level spacing in the volume of localization.

  For $T_c \ll \delta_L \ll \omega_D$ *local* pairing is still possible leading to parity gap:

  all low lying excitations are Cooper pairs localized in fractal eigenstates of localization problem (Feigelman, Kravtsov and others).

  Superconductor-insulator transition happens when boson hopping M_{ij} between these states is comparable to the spread of the individual energies. Model Hamiltonian:

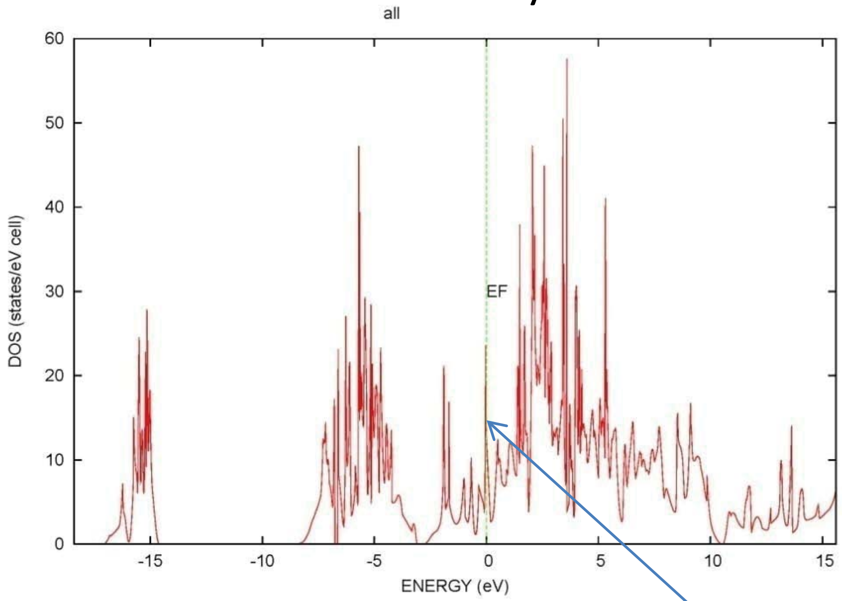
$$H = \sum_j \xi_j c_j^{\dagger} c_j - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} c_j^{\dagger} c_j^{\dagger} c_i c_i \quad \text{with } M_{ii} \gg Z M_{ij}$$

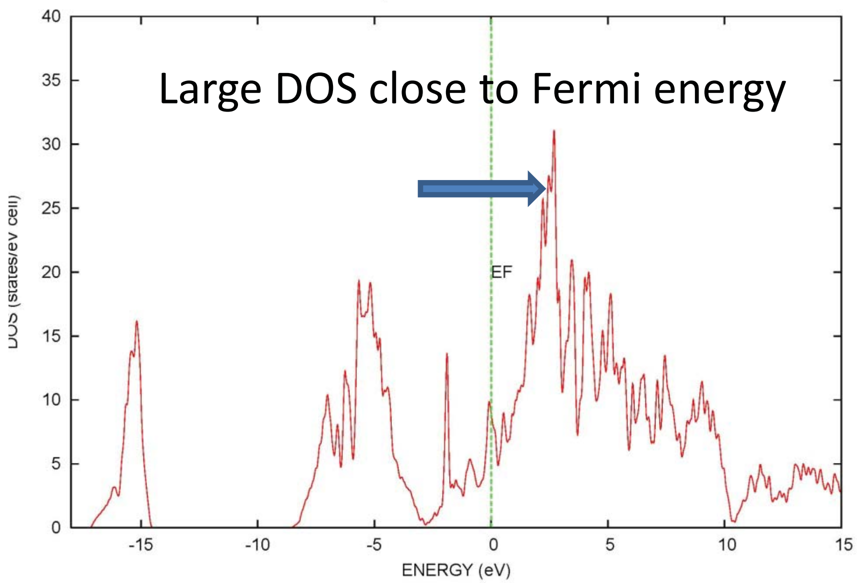
In the insulating phase the transport is via Cooper pair hopping. What is the gap?

Why no Coulomb?

- Speculation: large screening by electrons away but close to the Fermi surface (i.e. large dielectric constant of the host insulator)

Density of states of TiN clusters (V. Anisimov)





Unstable Fermi level position (similar to Efros-Shklovski)

Toy model of superconductor-insulator transition driven by disorder with purely attractive interaction and strong preformed pairing.

- In the basis of exact single particle states . Close to insulator-metal transition localized single particle states are large and have many overlaps.

$$H = \sum_j \xi_j c_j{}^\dagger c_j - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} c_j{}^\dagger c_j{}^\dagger c_i c_i \quad \text{with } M_{ii} \gg Z M_{ij},\, Z \gg 1$$

- Leave out single particle states (spin representation):

$$H = -\sum_j 2\xi_j \sigma_j{}^z - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} (\sigma_i{}^+ \sigma_j{}^- + \sigma_i{}^- \sigma_j{}^+) \quad Z \gg 1$$

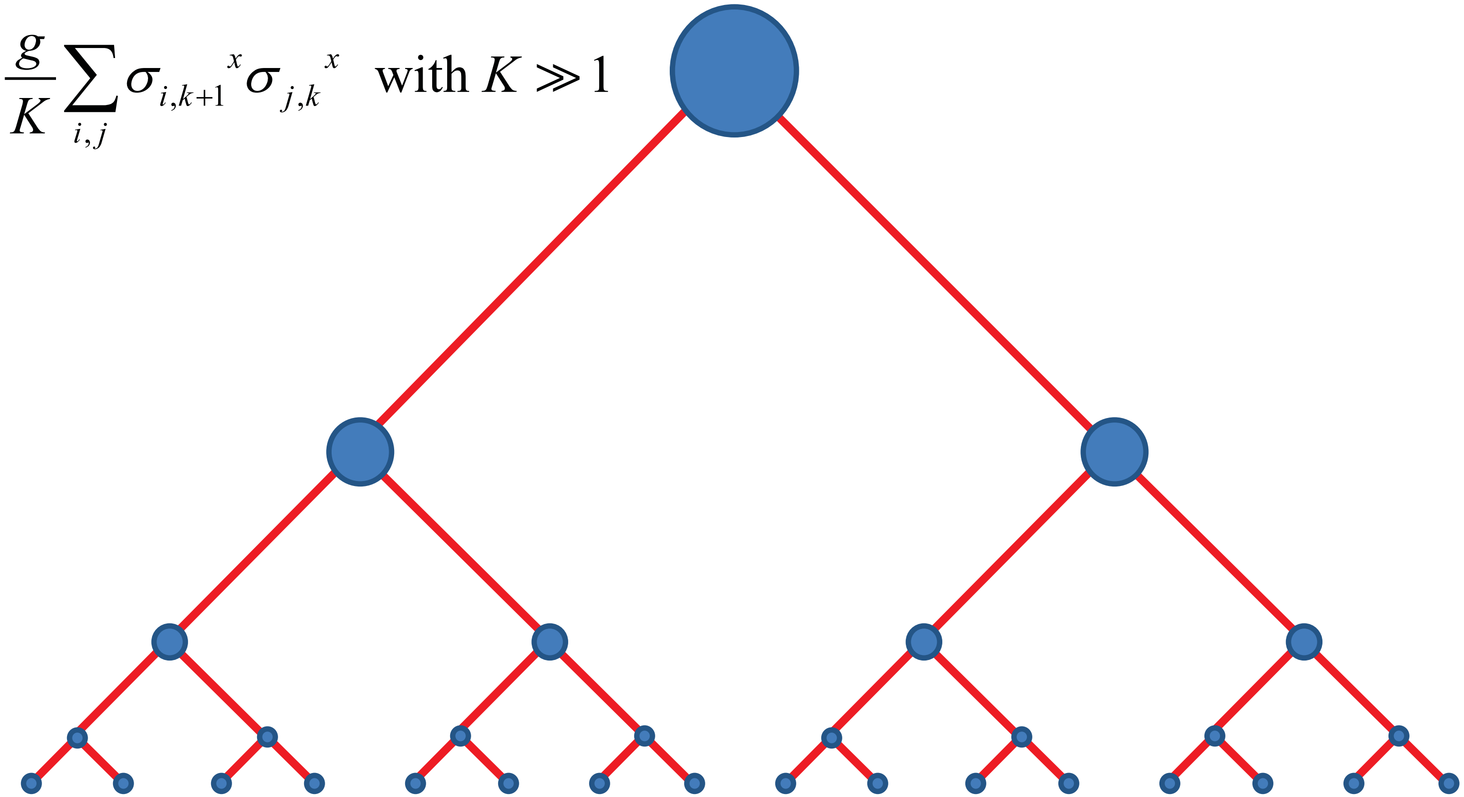
- What general properties of the quantum transition in the models in random field? Applies also to strongly disordered magnets (paramagnet-ferromagnet transition).

$$H = -\sum_j 2\xi_j \sigma_j{}^z - \frac{\lambda}{\nu} \sum_{(ij)} M_{ij} \sigma_i{}^x \sigma_j{}^x \quad \text{with } Z \gg 1$$

Toy model of superconductor-insulator transition driven by disorder with purely attractive interaction and strong preformed pairing.

- Because number of neighbors is large the loops can be neglected. The model on Bethe lattice is believed to reproduce the main features of the transition and phases on both sides (formally we ignore small 1/Z effects but keep 1/Log(Z):

$$H = -\sum_j \xi_j \sigma_j^{\ z} - \frac{g}{K}\sum_{i,j} \sigma_{i,k+1}^{\ x} \sigma_{j,k}^{\ x} \quad \text{with } K \gg 1$$



Toy model of superconductor-insulator transition driven by disorder with purely attractive interaction and strong preformed pairing.

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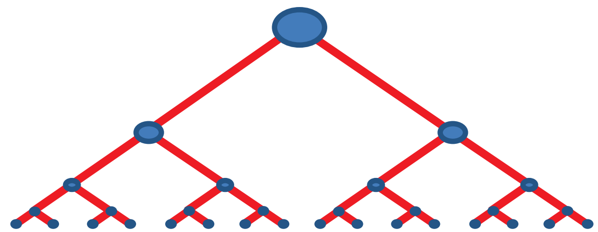


$$H = \sum_j \xi_j \sigma_j^{\ z} - \frac{g}{K} \sum_{i,j} \sigma_{i,k+1}^{\ x} \sigma_{j,k}^{\ x} \quad \text{with } K \gg 1$$

Questions to ask (“ask me no questions, i'll give 'ee no lies”):

- Nature of the phases on both sides of the transition at T=0: what are the main properties of the disordered phase?
  a. Homogeneity of the ordered phase.
  b. Local level broadening in the disordered phase.
  c. Critical value of *g.*

- Temperature dependence of the transition in the vicinity of T=0 (phase diagram).

Model solution 1: cavity equations.

Main idea: cavity equations.
Introduce effective field that simulates the effect of spins at higher levels:

$$H = -\xi_0 \sigma_0^z - h_0 \sigma_0^x \qquad \left\langle \sigma_0^x \right\rangle_0 = \frac{h_0}{\sqrt{h_0^{\,2} + \xi_0^{\,2}}} \mathrm{Tanh}\left[\frac{\sqrt{h_0^{\,2} + \xi_0^{\,2}}}{T}\right]$$

$$H = -\xi_0 \sigma_0^z - \sum_j (\xi_j \sigma_j^z + \sigma_0^x \sigma_j^x + h_j \sigma_j^x) \quad \text{Choose } h_0 \text{ so that } \left\langle \sigma_0^x \right\rangle_H = \left\langle \sigma_0^x \right\rangle_0$$

Roughly - this approximation is sufficient to get the transition temperature to O(1/K):

$$h_{k+1} = \frac{g}{K} \sum_j \frac{h_{k,j}}{\sqrt{\xi_{k,j}^{\,2} + h_{k,j}^{\,2}}} \mathrm{Tanh} \frac{\sqrt{\xi_{k,j}^{\,2} + h_{k,j}^{\,2}}}{T}$$

If averaged over uniform distribution of ξ we get usual BCS-like equation:

$$h = g \int_0^\infty \frac{d\xi h}{\sqrt{\xi^2 + h^2}} \mathrm{Tanh}\left[\frac{\sqrt{\xi^2 + h^2}}{T}\right]$$

that tells us that $T_c > 0$ for any $g > 0$.

Model solution 2: equation for T_c.

To find T_c we need to find when infinitely small field applied at the boundary leads to large field in the center:

$$h_0 = Z h_N \qquad Z = \sum_{\{i[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i[k]}/T]}{\xi_{k,i[k]}}$$

That is whether *Z=exp(fN)* with *f>0* ("magnet" or "superconductor") or *f<0* (paramagnet)?

Non-trivial physics is due to the fact that *Z* is not necessarily self-averaging quantity! Consider higher moments:

$$K\left\langle \left[\frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i}/T]}{\xi_{k,i}} \right]^n \right\rangle = \sqrt{\frac{3\pi}{4K}} K^{1-n} g^n / T^{n-1}$$

The moments diverge at *T=g/K* which becomes higher than 'average' T_c*=exp(-1/g).*

Model solution 3: equation for T_c.

$$h_0 = Z h_N \qquad Z = \sum_{\{i[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i[k]}/T]}{\xi_{k,i[k]}}$$

Z=exp(fN) with *f>0* ("magnet" or "superconductor") or *f<0* (paramagnet, non-supercond)?

Non-trivial physics is due to the fact that *Z* is not necessarily self-averaging quantity!
For T<g/K Z is not self-averaging and typical Z_{typ}=expN<f> might be different from <Z>.
Typical lattice shows the transition when <f> > 0.
To find average <f> use replica trick:

$$Z^n = \sum_{\{i_a[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i_a[k]}/T]}{\xi_{k,i_a[k]}} \qquad a = 1..n$$

Solve the problem for n replicas and continue to n=0. Similar problems were solved in the context of directed polymer physics (*Derrida and Spohn*).
Replica symmetric solution (i.e. all replicas are independent) gives the BCS-like result.
However at low T(g) replica symmetry breaks down.

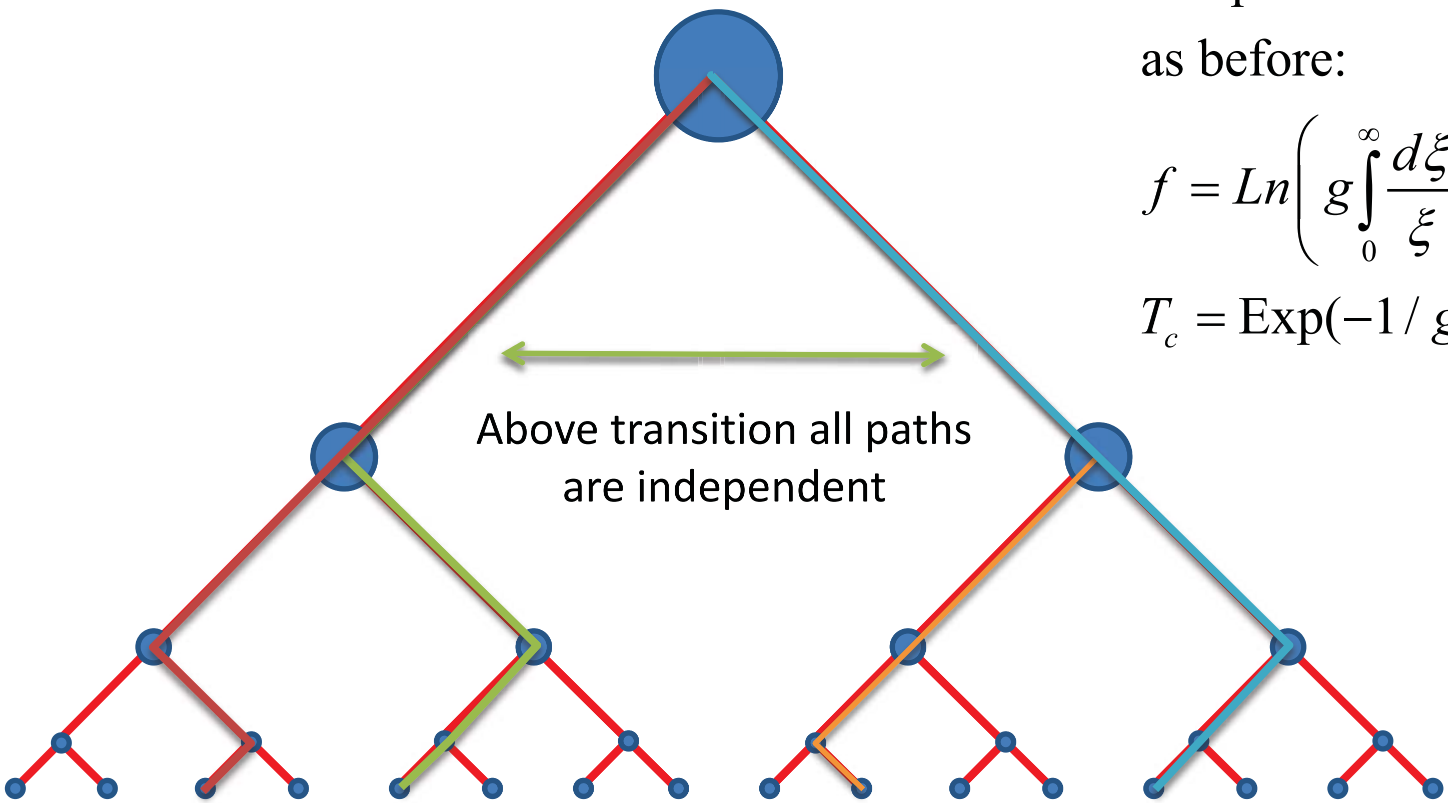
Model solution 4: equation for T_c in one step RSB.

$$Z^n = \sum_{\{i_a[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i_a[k]}/T]}{\xi_{k,i_a[k]}} \quad a = 1..n$$

Assumption that all paths are independent leads the same result as before:

$$f = Ln\left(g \int_0^\infty \frac{d\xi}{\xi} \mathrm{Tanh}\left[\frac{\xi}{T}\right]\right)$$

$$T_c = \mathrm{Exp}(-1/g)$$

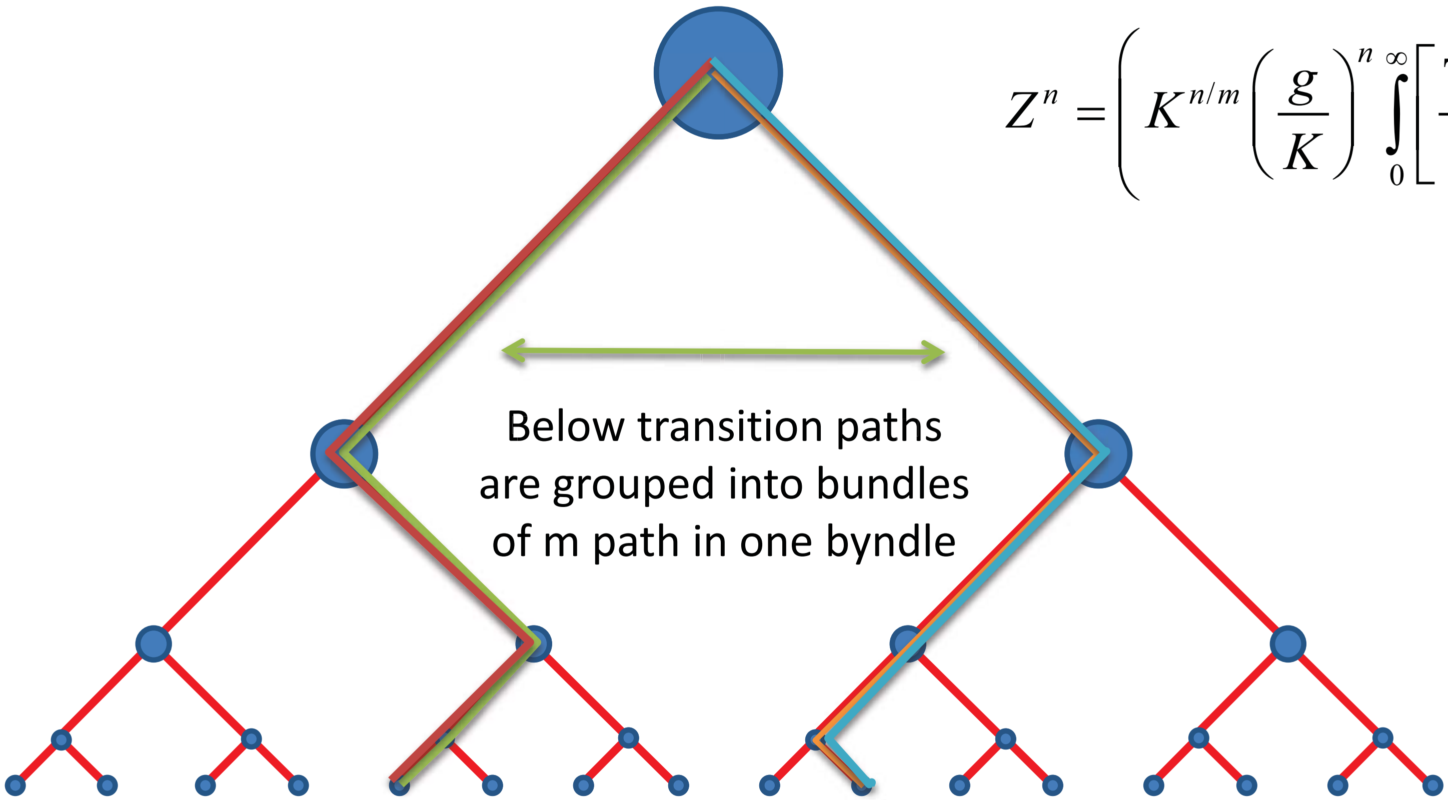


Model solution 5: equation for T_c in one step RSB.

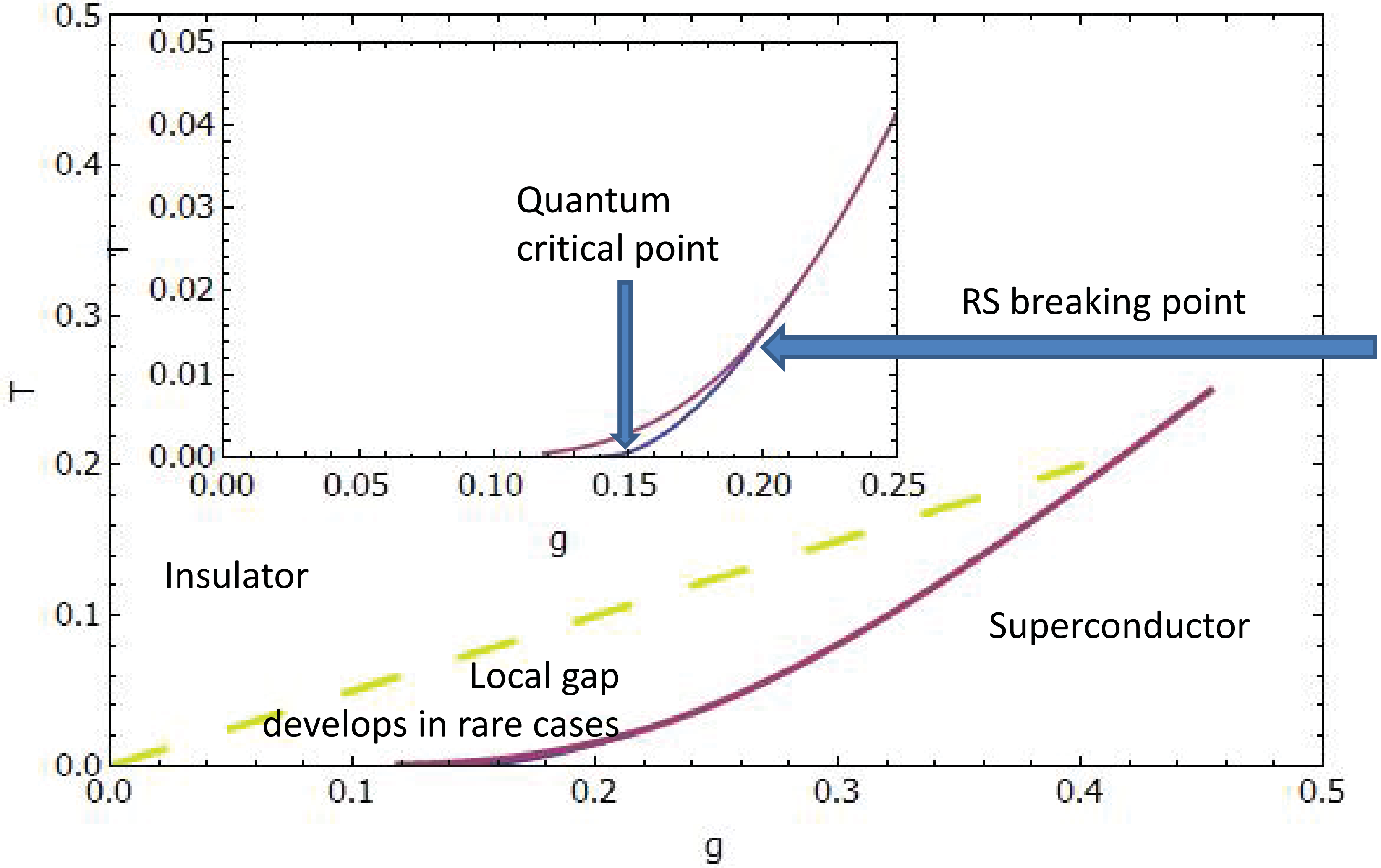
$$Z^n = \sum_{\{i_a[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i_a[k]}/T]}{\xi_{k,i_a[k]}} \quad a=1..n$$

Bundled average (n/m bundles):

$$Z^n = \left(K^{n/m} \left(\frac{g}{K}\right)^n \int_0^\infty \left[\frac{\mathrm{Tanh}(\xi/T)}{\xi}\right]^m d\xi \right)^N$$

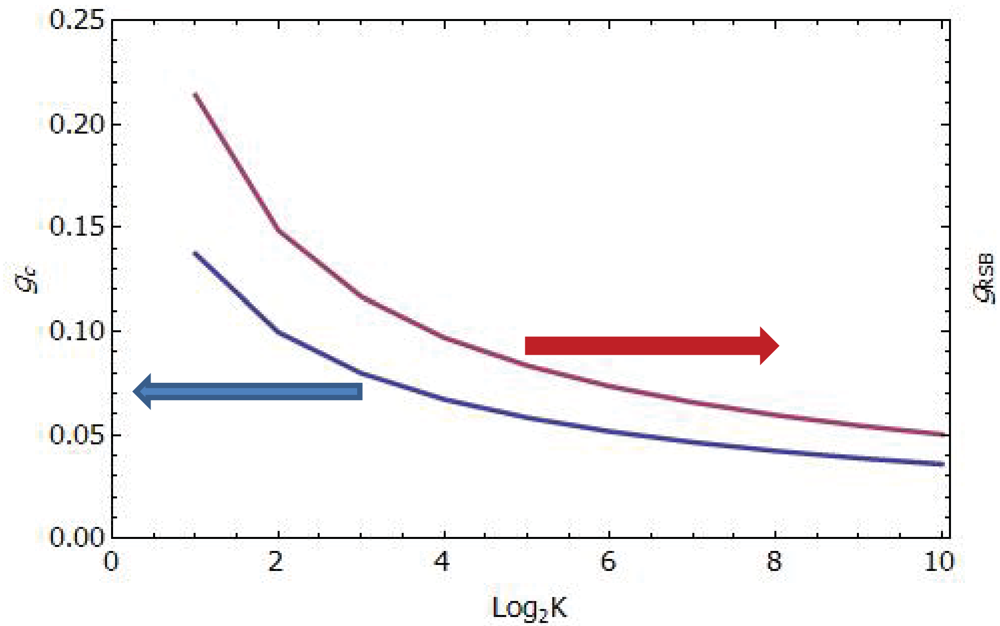


Phase diagram of Bethe lattice model



Phase diagram of Bethe lattice model

Critical value of the interaction corresponding to quantum critical point.



Properties of the insulating phase.

- **Perturbation theory estimate.**

  Relaxation rate of the central spin with low ξ is due to its coupling to the boundary spins:

$$\Gamma_0 = \sum_{i[k]} \prod_k \left(\frac{(g/K)^2}{\xi_{i[k]}{}^2 + (g/K)^2} \right) \Gamma_{i[N]}$$

  which is dominated by the spins with the energies ξ~g/K.

1. For g<1 the probability to find one such neighbor is g<1, so probability to find the chain of length N of such spins is g^N«1.
2. 1D spin chain can be mapped onto the non-interacting fermions in random potential, in this problem all states are localized.

- **Conclusion:** all levels retain zero width in the insulating phase at T=0.

Conclusions

1. Properties of the disordered superconducting films exhibiting SI transition ask for a different model than Josephson arrays.
2. Good candidate is the model with no Coulomb repulsion (equivalent to magnet in random field)
3. Solution of magnet in random field on Bethe lattice shows formation of a very inhomogeneous (non-self averaging) phase at low T close to quantum critical point.
4. Insulating phase is characterized by zero level width at T=0.