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Giant Nernst effect due to Fluctuating Cooper Pairs in Superconductors

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Giant Nernst effect due to fluctuating Cooper pairs in superconductors

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Outline

- Nernst effect
- Fluctuation effects in superconductors
- Microscopic theory of the fluctuation Nernst effect
- Comparison with other approaches

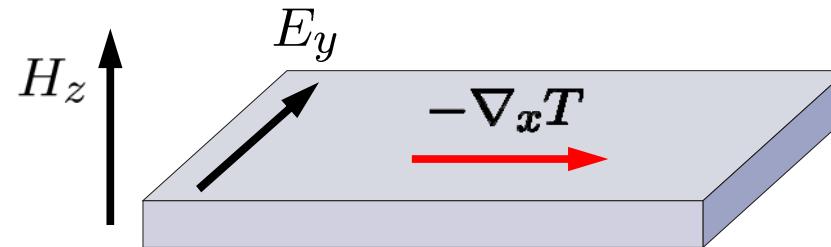
Nernst-Ettingshausen effect

A. V. Ettingshausen and W. Nernst (1886)



Walther Nernst
(1864–1941)

Nobel Prize
in Chemistry (1920)
for thermoelectricity



$$\begin{cases} j_\alpha^e = \sigma_{\alpha\beta} E_\beta - \beta_{\alpha\beta} \nabla_\beta T & \text{Onsager relation:} \\ j_\alpha^Q = \gamma_{\alpha\beta} E_\beta - \kappa_{\alpha\beta} \nabla_\beta T & \hat{\gamma}(\mathbf{H}) = T \hat{\beta}(-\mathbf{H}) \end{cases}$$

Nernst coefficient:

$$\nu_N = \frac{E_y}{(-\nabla_x T) H_z} = \frac{1}{H} \frac{\beta_{xy} \sigma_{xx} - \beta_{xx} \sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

Nernst effect in normal metals

E. H. Sondheimer (1948)

$$\nu_N = \frac{1}{H} \frac{\beta_{xy}\sigma_{xx} - \beta_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

In metals, the thermoelectric tensor $\beta_{\alpha\beta}$ is due to the particle-hole asymmetry:

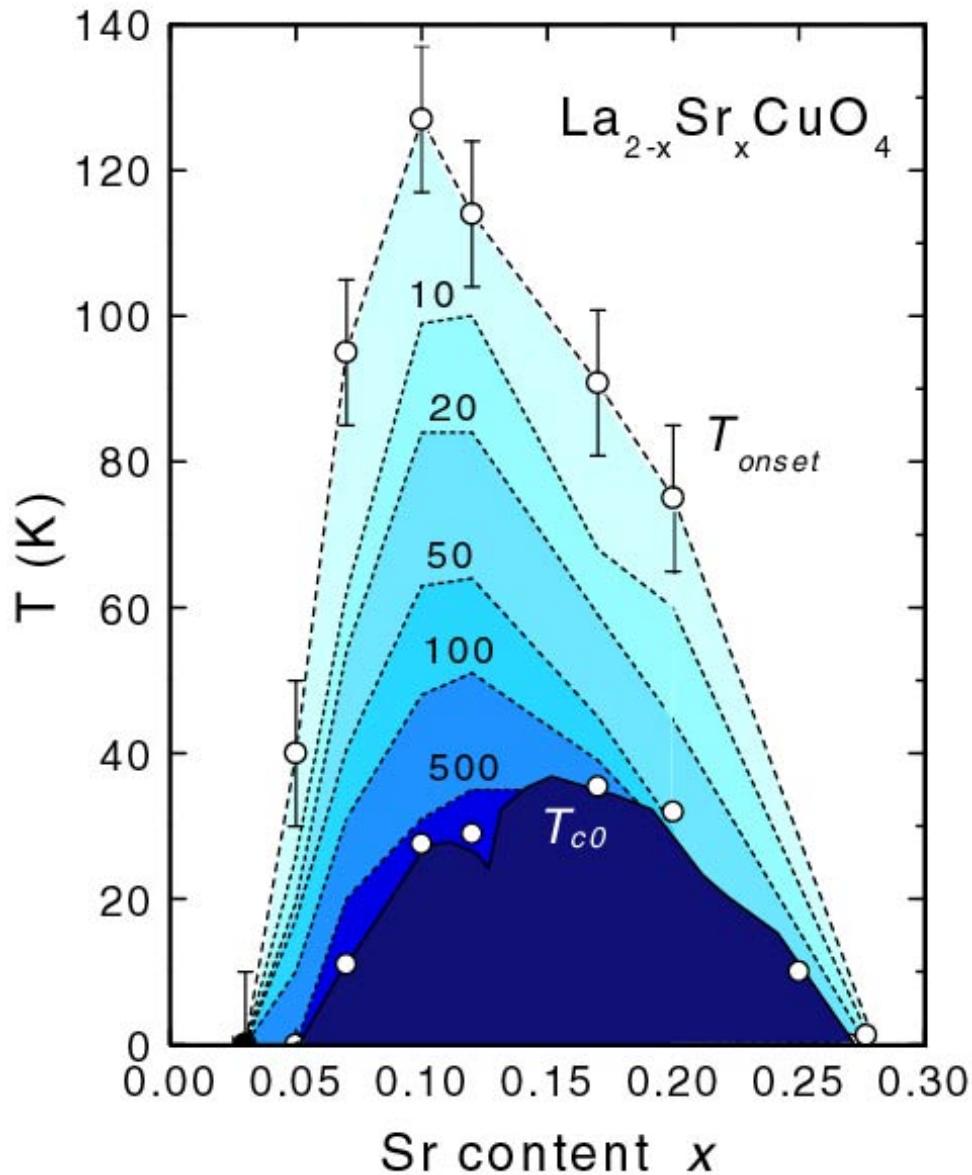
$$\beta_{\alpha\beta} = \frac{\pi^2 c T}{3 e} \frac{\partial \sigma_{\alpha\beta}}{\partial \mu}$$

$$\nu_N \approx \frac{1}{H} \frac{\beta_{xy}\sigma_{xx} - \beta_{xx}\sigma_{xy}}{\sigma_{xx}^2} = \frac{\pi^2 c T}{3 e H} \frac{\partial}{\partial \mu} \left(\frac{\sigma_{xy}}{\sigma_{xx}} \right) = \frac{\pi^2 c T}{3 e H} \frac{\partial (\omega_c \tau)}{\partial \mu}$$

Sondheimer formula:
$$\nu_N = \frac{\pi^2 T}{3 m} \frac{\partial \tau(\varepsilon)}{\partial \varepsilon} \Big|_{\varepsilon=\mu}$$

Nernst effect in superconductors

Nernst effect in cuprates



Y. Wang et al. (2001)

$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

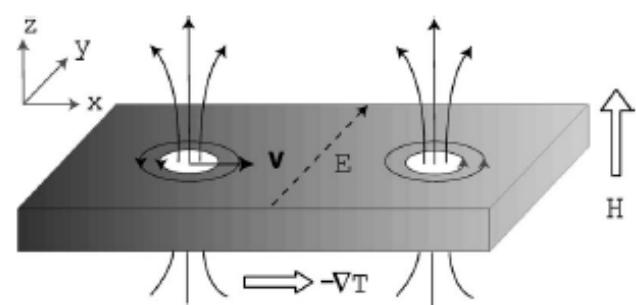
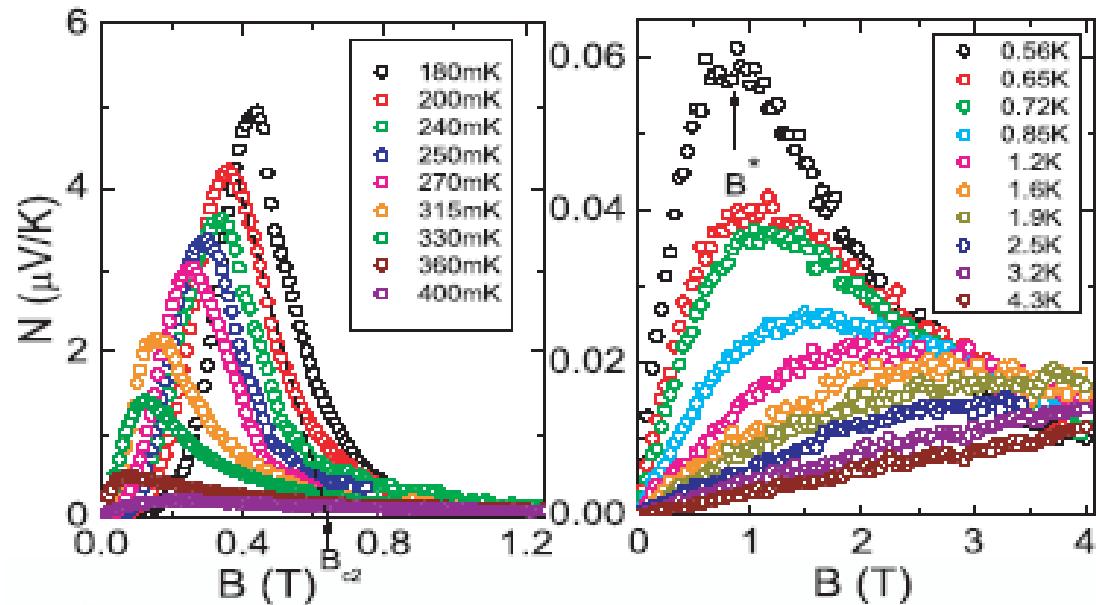


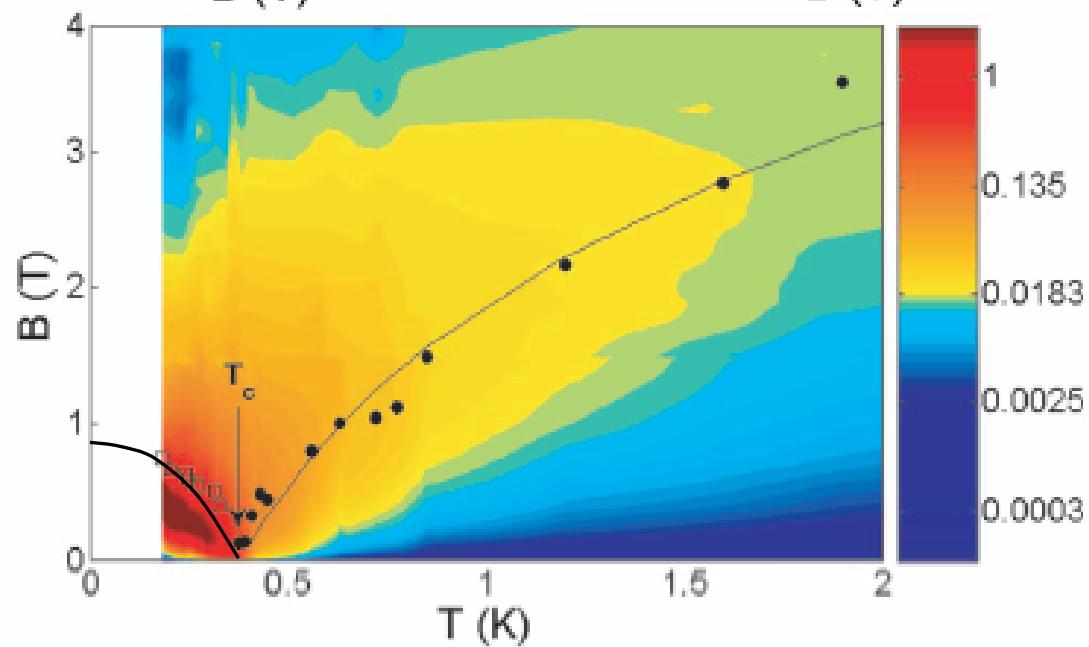
FIG. 1. The vortex-Nernst effect in a type-II superconductor. Concentric circles represent vortices.

Nernst effect in conventional superconductors



A. Pourret et al.,
Nature Phys. (2006);
Phys. Rev. B (2007)

$\text{Nb}_{0.15}\text{Si}_{0.85}$ film

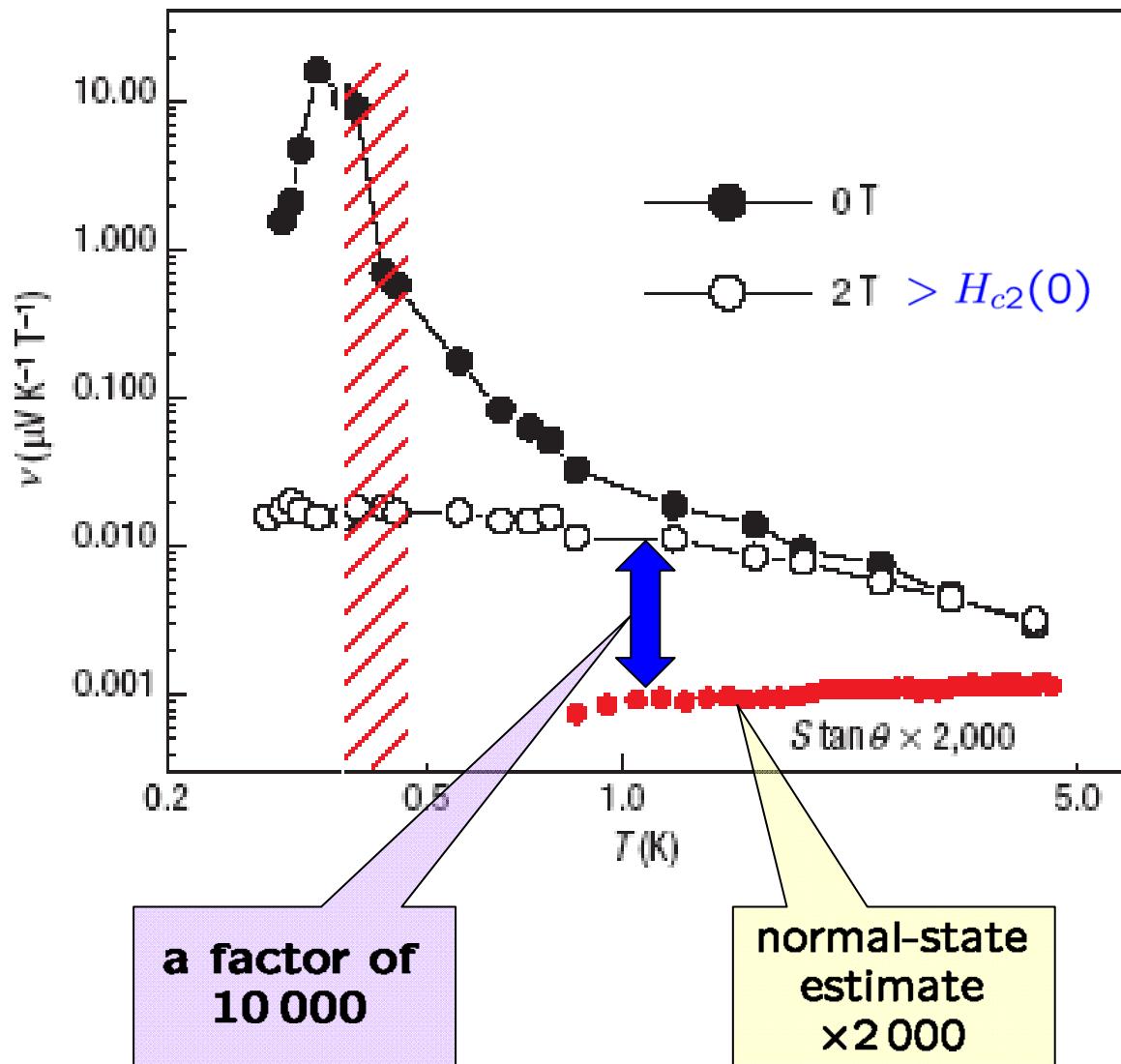


thickness
 $d = 35$ nm

$T_c = 0.38$ K

$R_{\square} = 350$ Ohm

Nernst effect in conventional superconductors



A. Pourret et al.,
Nature Phys. (2006);
Phys. Rev. B (2007)

$\text{Nb}_{0.15}\text{Si}_{0.85}$ film

thickness
 $d = 35\text{ nm}$

$T_c = 0.38\text{ K}$

$R_\square = 350\text{ Ohm}$

Origin of the Nernst effect in NbSi?

A. Pourret et al., Nature Phys. (2006);

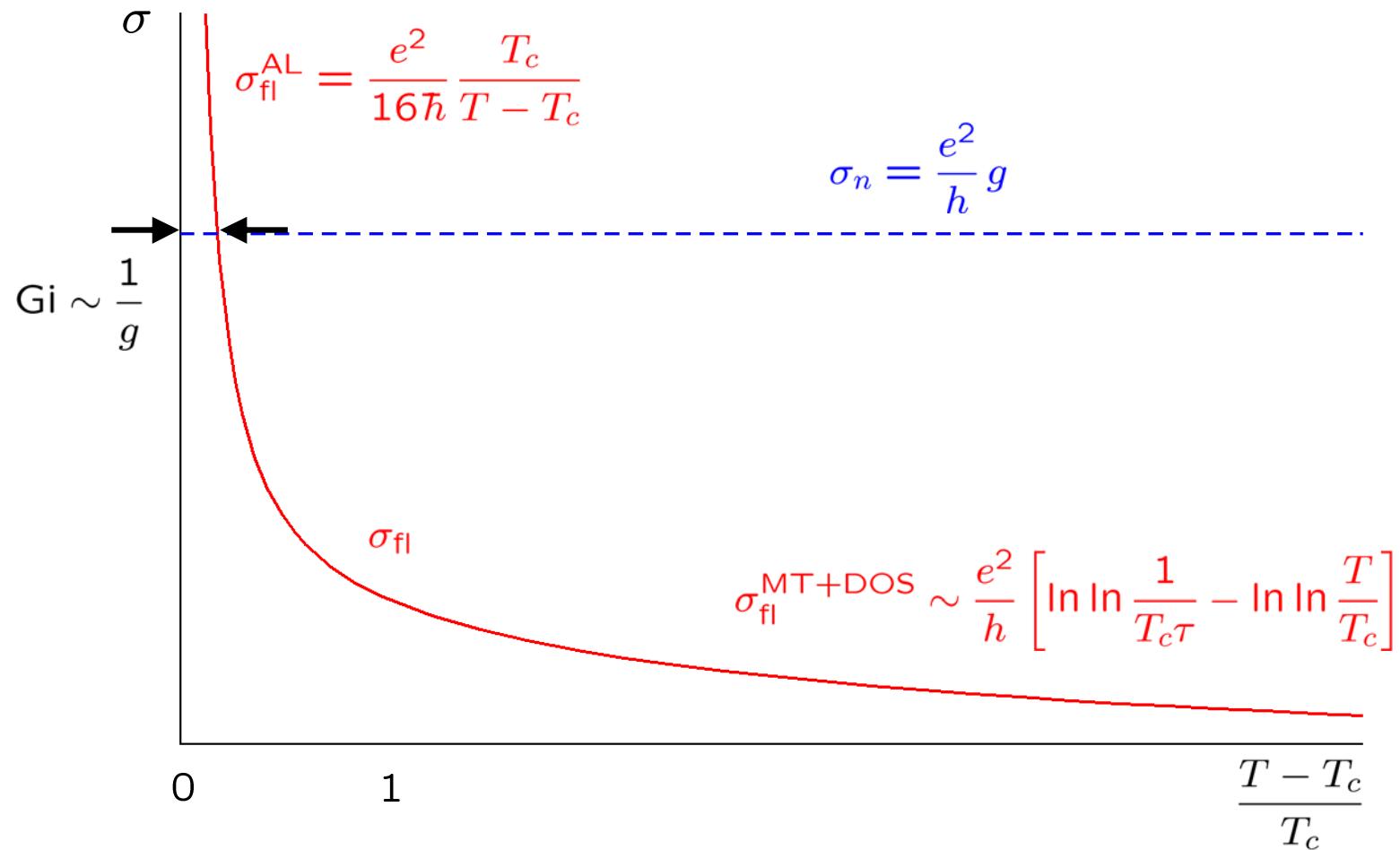
Could this signal be caused by phase fluctuations of the superconducting order parameter? This is also unlikely. In contrast to the underdoped cuprates, the carrier density in $\text{Nb}_{0.15}\text{Si}_{0.85}$ is comparable to any conventional metal. As the ‘phase stiffness’ of a superconductor is determined by its superfluid density¹⁹, there is no reason to speculate on the presence of preformed Cooper pairs without phase coherence in a wide temperature window above T_c as has been the case in the pseudogap state of the cuprates. In contrast to granular superconductors²⁰, decreasing the thickness leads to a shift of the sharp superconducting transition and does not reveal a temperature scale other than the mean-field BCS (Bardeen–Cooper–Schrieffer) critical temperature. The variation of T_c with thickness has been attributed to the enhancement of the Coulomb interactions with the increase in the sheet resistance, R_{square} (ref. 21).

On the other hand, there is no reason to doubt the presence of amplitude fluctuations of the superconducting order parameter

Fluctuation effects in conventional superconductors

Fluctuation corrections to conductivity (2D)

Aslamazov–Larkin + Maki–Thompson + Density-of-states



Fluctuation corrections to thermoelectricity

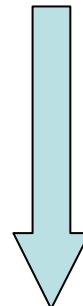
Close to T_c :

S. Ullah and A. T. Dorsey (1990)

I. Ussishkin, S. L. Sondhi, D. A. Huse (2002)

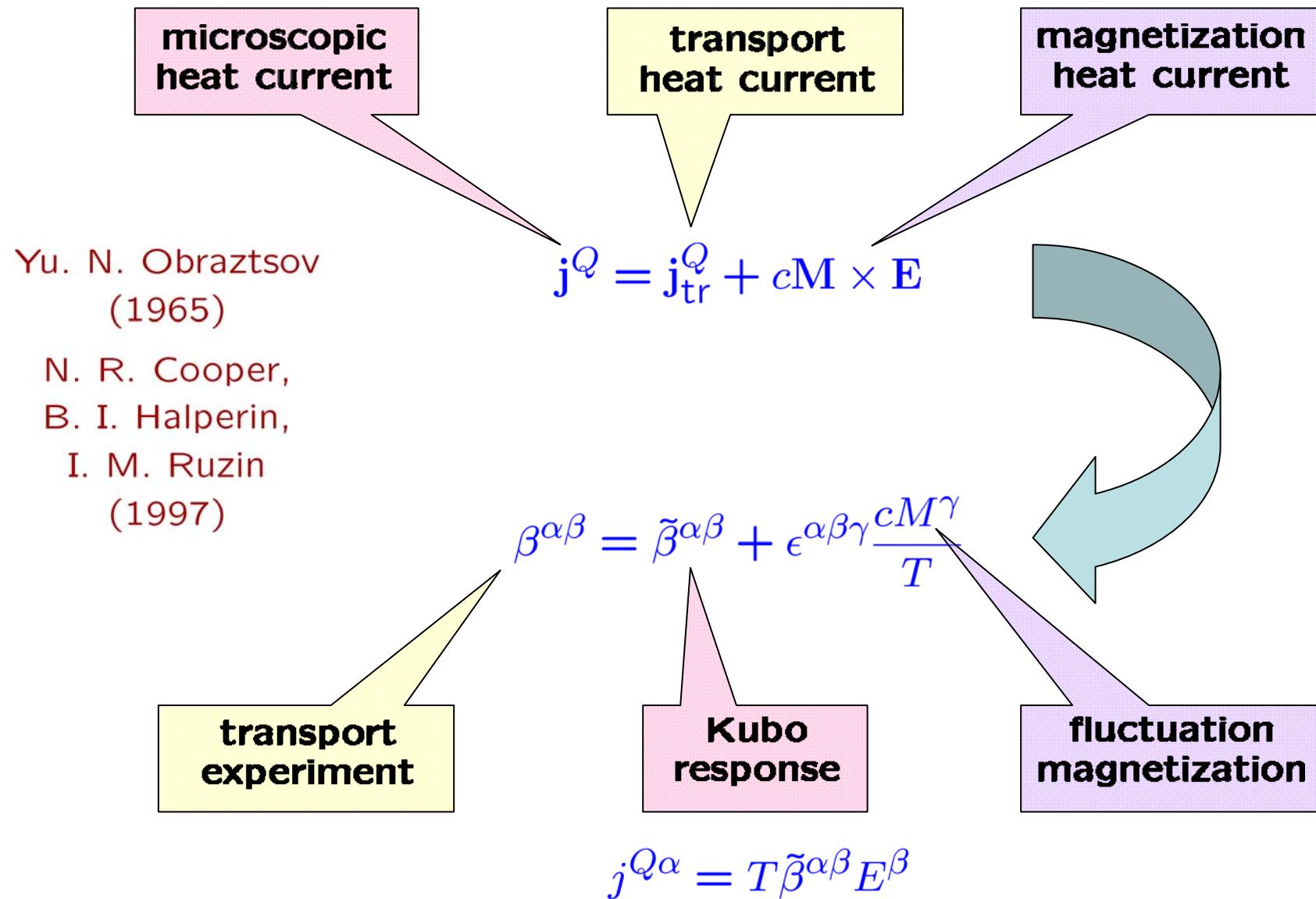
$$\beta_n \propto \begin{pmatrix} \frac{T}{E_F} & \frac{T}{E_F} \omega_c \tau \\ -\frac{T}{E_F} \omega_c \tau & \frac{T}{E_F} \end{pmatrix}$$

$$\beta_{\text{fluct}} \propto \begin{pmatrix} \frac{T}{E_F} \ln \frac{T_c}{T - T_c} & \frac{H}{\frac{T_c}{T - T_c} H_{c2}(0)} \\ -\frac{T}{E_F} \ln \frac{T_c}{T - T_c} & \frac{H}{\frac{T}{E_F} \ln \frac{T_c}{T - T_c}} \end{pmatrix}$$



$$\nu_N = \frac{1}{H} \frac{\beta_{xy}\sigma_{xx} - \beta_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \xrightarrow{\text{red}} \frac{\beta_{xy}}{H\sigma_{xx}}$$

Role of magnetization



Vicinity of T_c : TDGL approach

S. Ullah and A. T. Dorsey (1990)

I. Ussishkin, S. L. Sondhi, D. A. Huse (2002)

$$\beta^{xy} = \beta_0 \frac{\pi e D H}{48c(T - T_c)} \sim \beta_0 \frac{\xi^2(T)}{l_H^2}$$

thermoelectric
conductance quantum

$$\beta_0 = \frac{k_B e}{\pi \hbar} = 6.68 \frac{nA}{K}$$

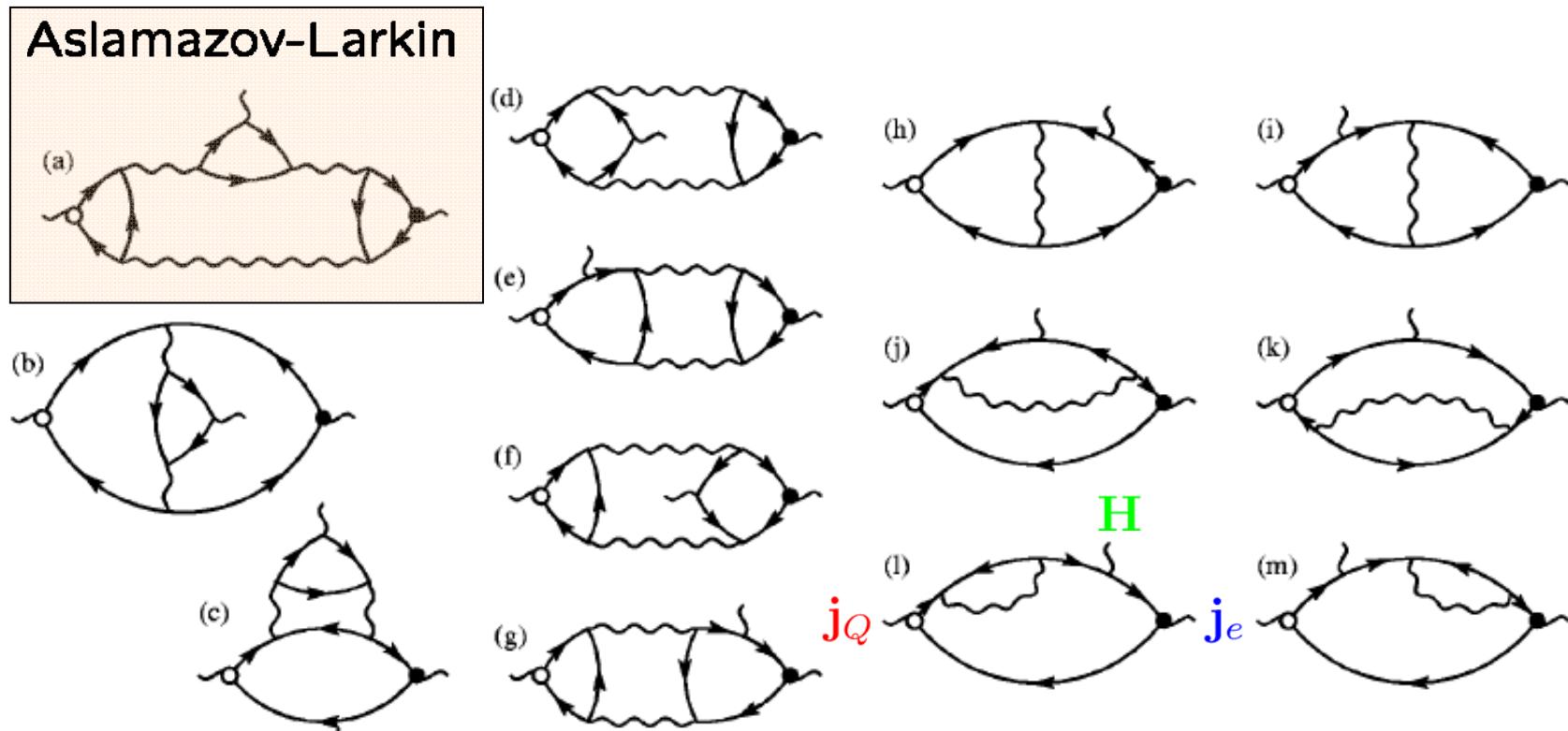
$$\beta^{xy} \propto 3 - 2$$

Kubo

magnetization

Vicinity of T_c : microscopic approach

I. Ussishkin, Phys. Rev. B (2003)



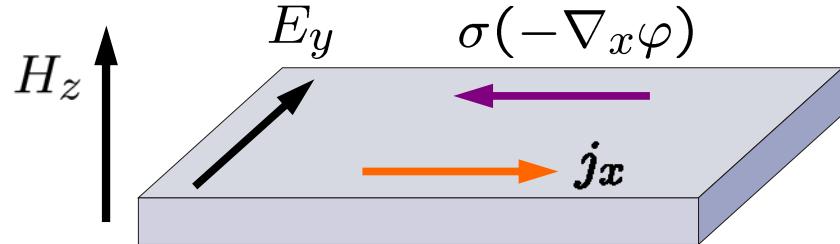
(cooperons are not shown)

Handwaving

Qualitative explanation of the Nernst effect

Nernst effect & chemical potential

- Drift velocity: $\bar{v}_x = cE_y/H_z$
- Drift current: $j_x = ne\bar{v}_x$
- Compensated by $\nabla_x\varphi = j_x/\sigma$
- Electroneutrality: $\nabla_x\mu = -e\nabla_x\varphi$
- Temperature gradient: $\nabla_x\mu = (d\mu/dT)\nabla_xT$



$$\nu_N \equiv \frac{E_y}{(\nabla_x T)H_z} = -\frac{\sigma}{ne^2c} \frac{d\mu(T)}{dT}$$

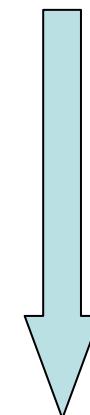
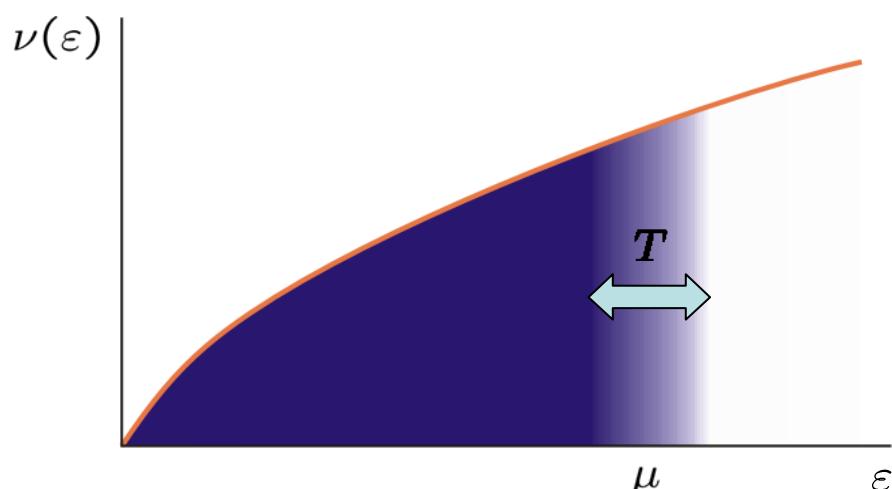
works for any types of carriers

Nernst effect: degenerate Fermi gas

$$\nu_N = -\frac{\sigma}{ne^2c} \frac{d\mu(T)}{dT}$$

Chemical potential of a degenerate Fermi gas:

$$\mu(T) = \mu_0 - \frac{\pi^2}{6} T^2 \frac{d \ln \nu(\mu)}{d\mu}$$



$$\nu_N = -\frac{\pi^2}{3} \frac{T}{m} \frac{\partial \tau(\mu)}{\partial \mu} \sim \frac{\tau}{m} \frac{T}{\mu}$$

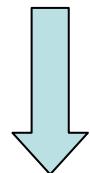
origin of
smallness

Nernst effect: fluctuating Cooper pairs near T_c

$$\nu_N = -\frac{\sigma_{\text{c.p.}}}{n_{\text{c.p.}} e^2 c} \frac{d\mu_{\text{c.p.}}(T)}{dT}$$

In the GL region close to T_c :

- density of Cooper pairs: $n = \langle |\Psi(\mathbf{r})|^2 \rangle = \frac{m T_c}{\pi} \ln \frac{T_c}{T - T_c}$
- chemical potential of Cooper pairs: $\mu(T) = T_c - T$
- paraconductivity: $\sigma^{\text{AL}} = (e^2/16) T_c/(T - T_c)$



$$\nu_N \sim \frac{1}{mc(T - T_c)}$$

(TDGL result with an unknown coefficient;
magnetization not taken into account)

Microscopic theory of the fluctuation Nernst effect

Heat current operator

J. S. Langer (1962); J. M. Luttinger (1964)

$$H = \int h(\mathbf{r}) d\mathbf{r}, \quad h(\mathbf{r}) = \frac{1}{2m} \nabla \psi_\sigma^\dagger \nabla \psi_\sigma + V_{\text{imp}}(\mathbf{r}) \psi_\sigma^\dagger \psi_\sigma + \lambda \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

- electric current $\dot{\rho} + \nabla \mathbf{j}^e = 0, \quad \rho(\mathbf{r}) = e \psi_\sigma^\dagger \psi_\sigma, \quad \dot{\rho} = i[H, \rho]$

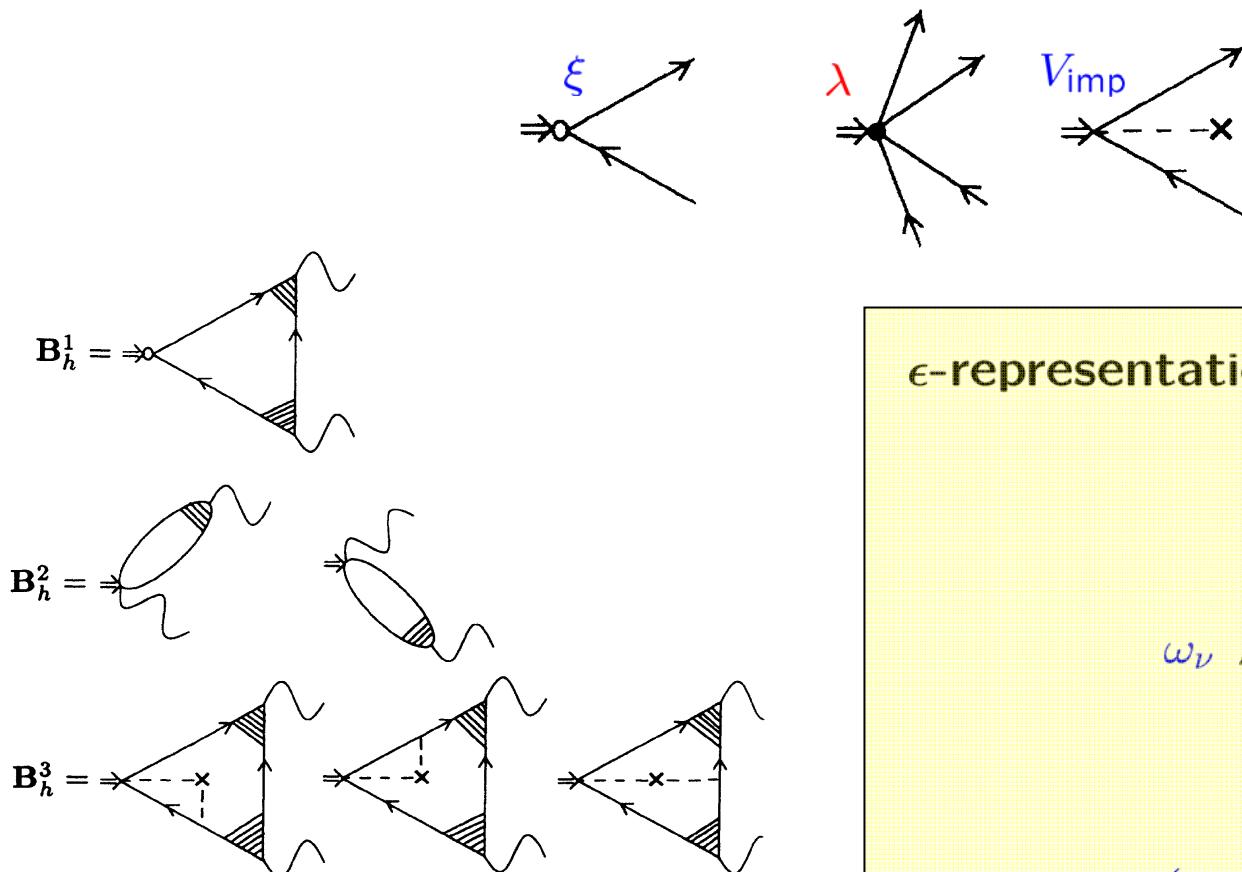
$$\mathbf{j}_\alpha^e(\mathbf{r}) = \frac{ie}{2m} \frac{\partial \psi_\sigma^\dagger}{\partial x_\alpha} \psi_\sigma + \text{h.c.}$$

- heat current $\dot{h} + \nabla \mathbf{j}^E = 0, \quad \mathbf{j}^Q = \mathbf{j}^E - (\mu/e) \mathbf{j}^e$

$$\mathbf{j}_\alpha^Q(\mathbf{r}) = \frac{i}{2m} \frac{\partial \psi_\sigma^\dagger}{\partial x_\alpha} \left(-\frac{1}{2m} \nabla^2 - \mu + V_{\text{imp}}(\mathbf{r}) \right) \psi_\sigma + \frac{i}{2m} \frac{\partial \psi_\uparrow^\dagger}{\partial x_\alpha} \lambda \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow + \text{h.c.}$$

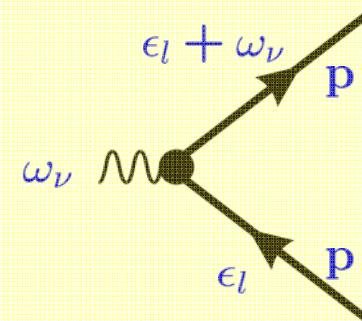
Heat current operator: diagrammatics

$$\mathbf{j}_\alpha^Q(\mathbf{r}) = \frac{i}{2m} \frac{\partial \psi_\sigma^\dagger}{\partial x_\alpha} \left(-\frac{1}{2m} \nabla^2 - \mu + V_{\text{imp}}(\mathbf{r}) \right) \psi_\sigma + \frac{i}{2m} \frac{\partial \psi_\uparrow^\dagger}{\partial x_\alpha} \lambda \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow + \text{h.c.}$$



M. Yu. Reizer, A. V. Sergeev (1994)

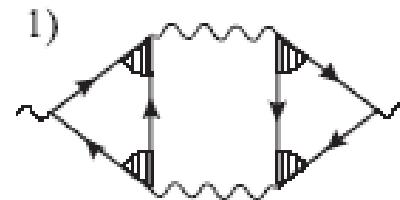
ϵ -representation of the heat vertex



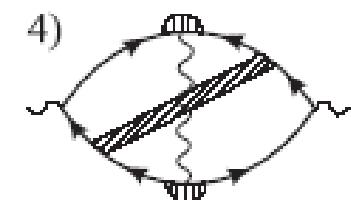
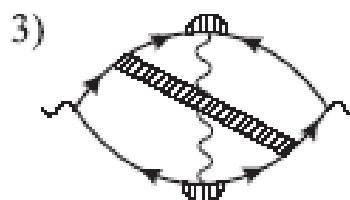
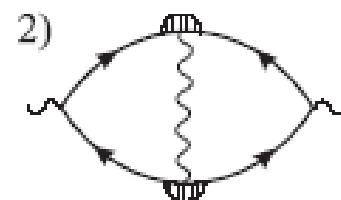
$$\mathbf{j}^Q = \sum_{\mathbf{p}, \epsilon_l} \mathbf{v} \left(i\epsilon_l + \frac{i\omega_\nu}{2} \right) a_p^\dagger (\epsilon_l + \omega_\nu) a_p (\epsilon_l)$$

Hierarchy of one-loop diagrams

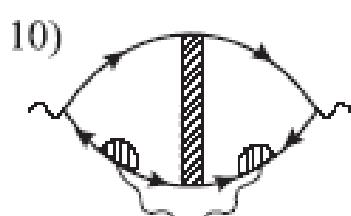
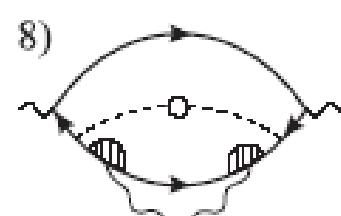
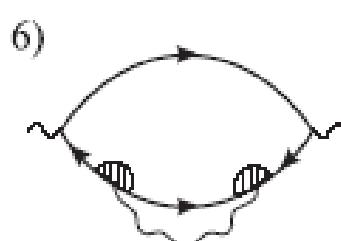
Aslamazov-Larkin



Maki-Thompson



Density-of-states



Aslamazov-Larkin diagram

The diagram shows a central system represented by two vertical rectangles connected by a wavy line. The left rectangle is labeled $L_n(\Omega_k)$ and the right one $L_m(\Omega_k + \omega_\nu)$. A green shaded region labeled "electric block" contains the expression $\hat{q}_{mn}^x B_{nm}^{(e)}$. A pink shaded region labeled "heat block" contains the expression $\hat{q}_{nm}^y B_{nm}^{(Q)}$. Arrows point from the labels to their respective regions. A curved arrow labeled $i\omega_\nu \rightarrow \omega + i0$ points from the heat block towards the central system.

- Fluctuation propagator:

$$L_n(\Omega) = -\nu^{-1} \left[\ln \frac{T}{T_c} + \psi \left(\frac{1}{2} + \frac{|\Omega| + 4eDH(n+1/2)}{4\pi T} \right) - \psi \left(\frac{1}{2} \right) \right]$$

- Momentum operator in the Landau basis:

$$\hat{q}_{mn}^{x,y} = \sqrt{eH/c} \binom{i}{1} (\sqrt{m} \delta_{m,n+1} \mp \sqrt{n} \delta_{n,m+1})$$

- Electric & heat blocks:

3

Electric and heat vertices

- Electric vertex:

$$B_{nm}^{(e)}(\Omega_k, \omega_\nu) = e\nu D \left[\frac{\psi_m(\omega_\nu + |\Omega_k|) - \psi_n(|\Omega_k|)}{\omega_\nu + \alpha_m - \alpha_n} + \frac{\psi_n(\omega_\nu + |\Omega_{k+\nu}|) - \psi_m(|\Omega_{k+\nu}|)}{\omega_\nu - \alpha_m + \alpha_n} \right]$$

- Heat vertex:

$$\begin{aligned} B_{nm}^{(Q)}(\Omega_k, \omega_\nu) = & \frac{-i\nu D}{2} \left[\frac{(\Omega_k - \alpha_m)\psi_m(|\Omega_k| + \omega_\nu) - (\Omega_{k+\nu} - \alpha_n)\psi_n(|\Omega_k|)}{\omega_\nu + \alpha_m - \alpha_n} \right. \\ & \left. + \frac{(\Omega_{k+\nu} + \alpha_n)\psi_n(|\Omega_{k+\nu}| + \omega_\nu) - (\Omega_k + \alpha_m)\psi_m(|\Omega_{k+\nu}|)}{\omega_\nu + \alpha_n - \alpha_m} \right] \end{aligned}$$

$$\alpha_n \equiv 4eDH(n + 1/2), \quad \psi_n(\Omega) \equiv \psi \left(\frac{1}{2} + \frac{|\Omega| + \alpha_n}{4\pi T} \right)$$

General formula

- Analytic continuation, AL + DOS + Magnetization:

$$\beta^{xy} = \frac{e}{2\pi^2 iT} \sum_m (m+1) \left[\int_0^\Lambda dz \coth \frac{z}{2T} P_m^{c(\text{tot})}(z) + \int_0^\Lambda dz \frac{1}{2T \sinh^2 \frac{z}{2T}} P_m^{s(\text{tot})}(z) \right]$$

$$P_m^{c(\text{tot})}(z) = 2 \operatorname{Im} \left[L_m^R T_{mm+1} - L_{m+1}^R T_{m+1m} \right]$$

$$P_m^{s(\text{tot})}(z) = 2 \operatorname{Re} \left[L_m^R S_{mm+1} - L_{m+1}^R S_{m+1m} - z \log \frac{L_m^A}{L_m^R} + b_m L_{m+1}^R L_m^A \right]$$

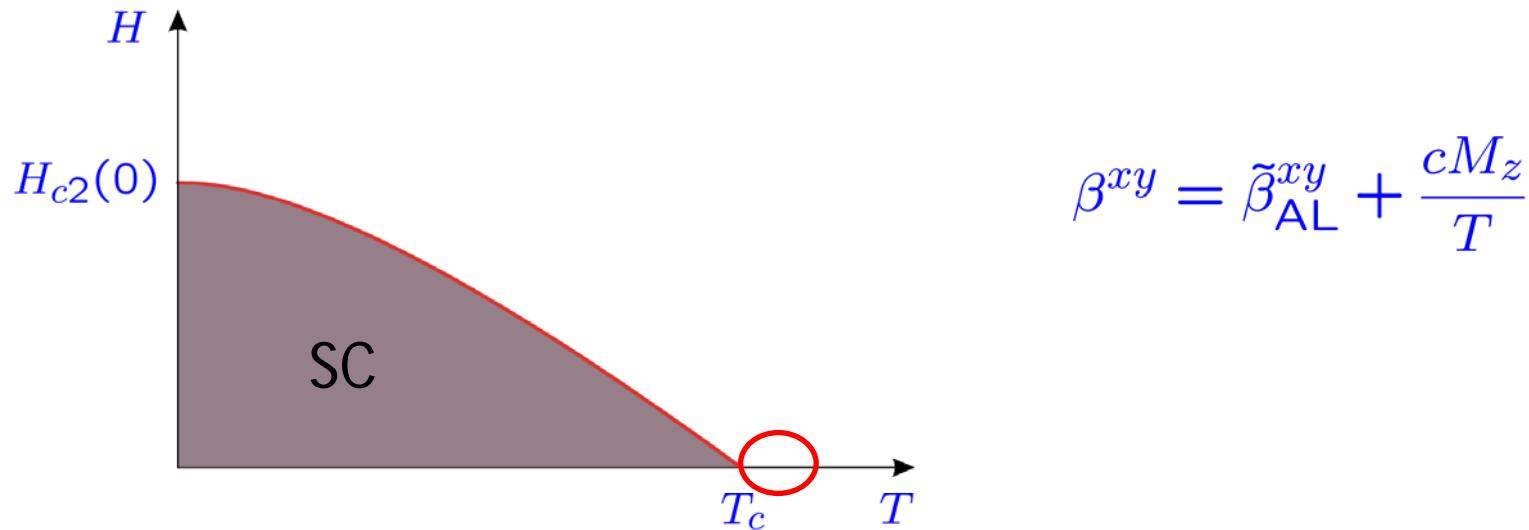
$$T_{nm} = \frac{1}{16\pi T} \left[(\alpha_m - iz)(\psi'_n(-iz) - \psi'_m(-iz)) + \frac{\alpha^2}{4\pi T(\alpha_m - \alpha_n)} (\alpha_n - iz)\psi''_n(-iz) \right]$$

$$S_{nm} = -\frac{i}{8} \left[(2iz + \alpha_n - \alpha_m)\psi_n(-iz) + (\alpha_m - \alpha_n)\psi_n(iz) - 2iz\psi_m(-iz) - \frac{\alpha^2(iz + \alpha_n)}{4\pi T(\alpha_m - \alpha_n)} (\psi'_n(-iz) - \psi'_n(iz)) \right]$$

$$b_m = -\frac{i}{4} (\psi_{m+1}(iz) - \psi_m(-iz)) \operatorname{Im} \left[(\alpha_m + iz)\psi_m(iz) - (\alpha_{m+1} + iz)\psi_{m+1}(iz) \right]$$

Results

Vicinity of T_c

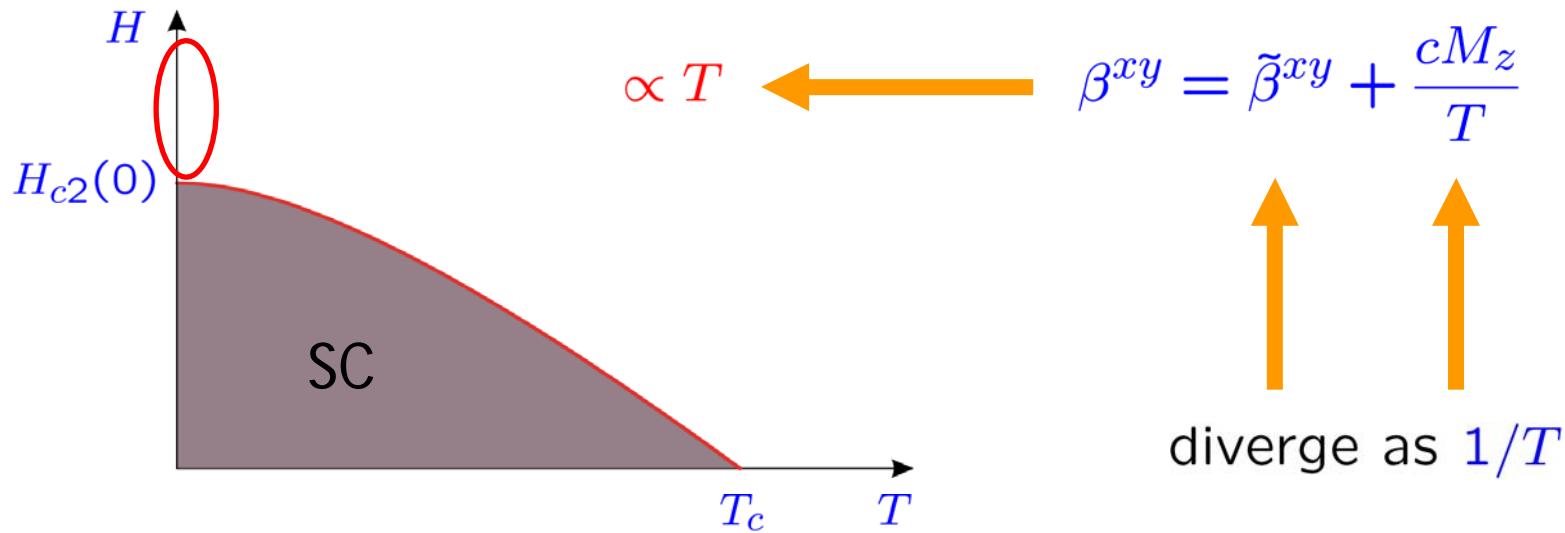


$$\beta^{xy} = \beta_0 \frac{\pi e D H}{12c(T - T_c)} = 4\beta_{\text{USH}}^{xy}$$

$$\beta^{xy} \propto 2 \times 3_{\text{AL}} - 2_{\text{M}} = 4$$

$$\beta_{\text{USH}}^{xy} \propto 3_{\text{AL}} - 2_{\text{M}} = 1$$

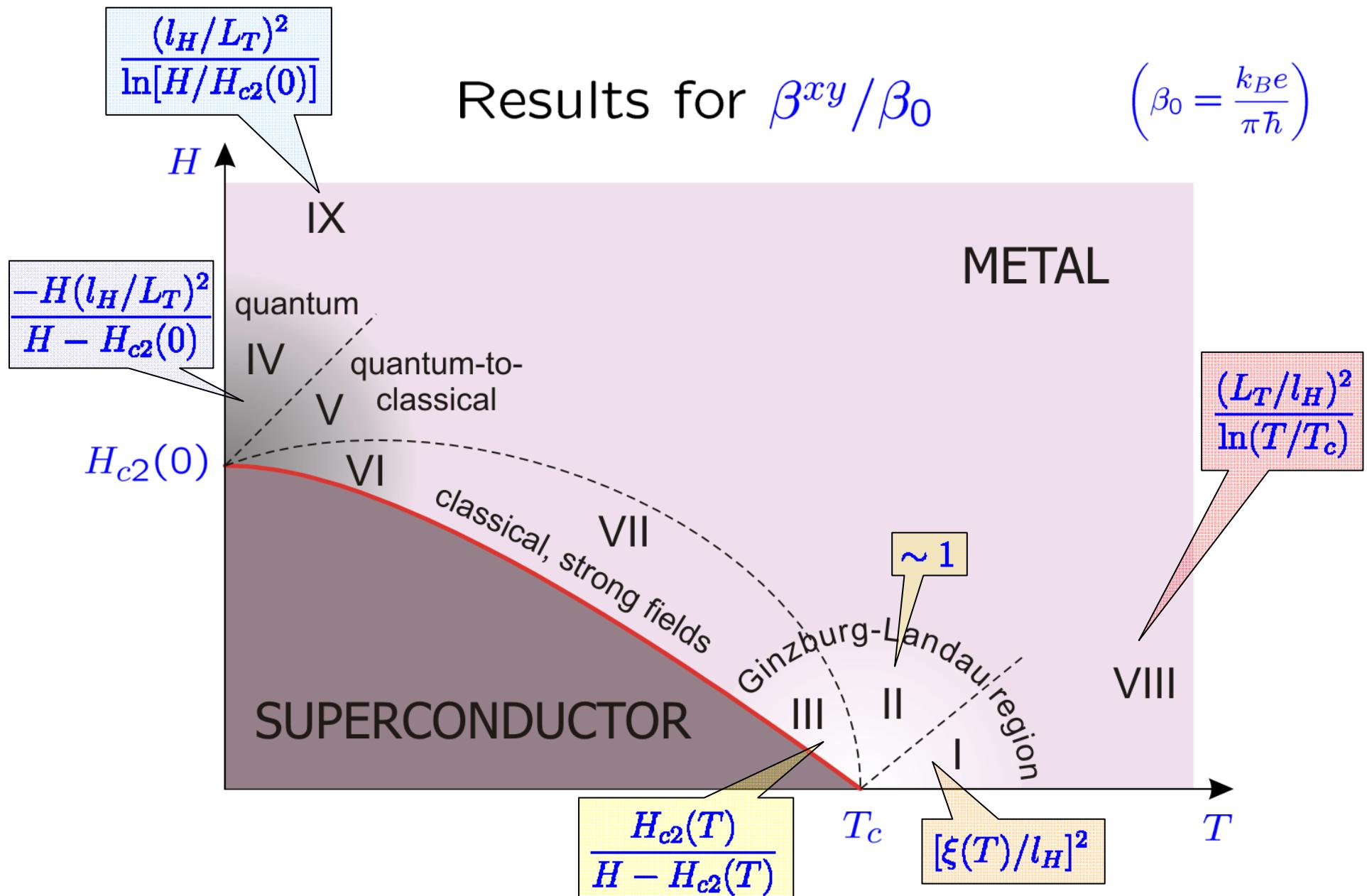
$T \rightarrow 0$ & the Third Law of Thermodynamics



$$\beta^{xy} = -\beta_0 \frac{\pi c T}{9eD[H - H_{c2}(0)]}$$

$$\frac{H - H_{c2}(0)}{H_{c2}(0)} \ll 1$$

Asymptotic regimes in the phase diagram

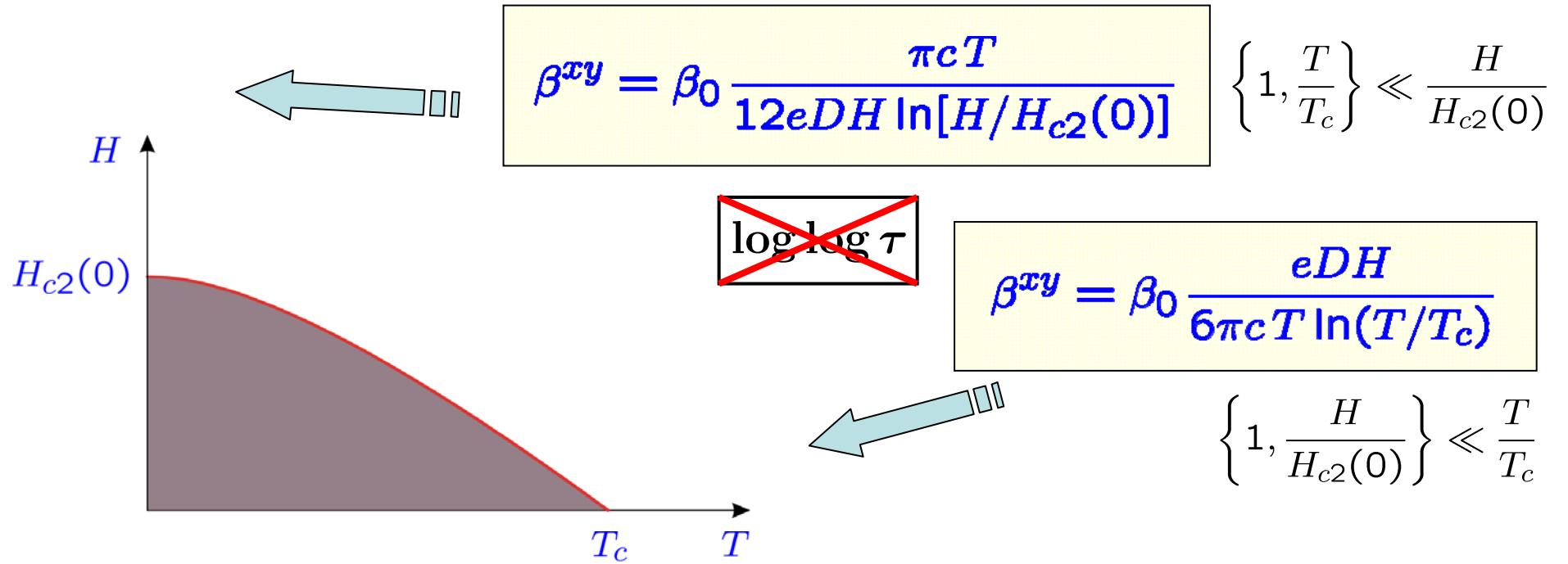


Asymptotic regimes: analytic results

	Regimes	β^{xy}/β_0
I	$\tilde{h} \ll \epsilon \ll 1$	$\tilde{h}/3\epsilon$
II	$\epsilon \ll \tilde{h} \ll 1$	$1 - (\ln 2)/2$
III	$\epsilon + \tilde{h} \ll h \ll 1$	$\tilde{h}/(\epsilon + \tilde{h})$
IV	$t \ll \eta \ll 1$	$-2\gamma t/9\eta$
V	$t^2/\ln(1/t) \ll \eta \ll t \ll 1$	$\ln(t/\eta)$
VI	$\eta \ll t^2/\ln(1/t) \ll 1$	$8\gamma^2 t^2/3\eta$
VII	$\eta \rightarrow 0$	$\frac{1}{\eta} \left[1 + \frac{h}{4\gamma t} \frac{\psi''(1/2+h/4\gamma t)}{\psi'(1/2+h/4\gamma t)} \right]$
VIII	$(1, h) \ll t$	$\frac{eDH}{6\pi cT \ln(T/T_c)}$
IX	$(1, t) \ll h$	$\frac{\pi cT}{12eDH \ln[H/H_{c2}(0)]}$

$$\epsilon = \ln \frac{T}{T_c}, \quad t = \frac{T}{T_c}, \quad h = \frac{H}{H_{c2}(0)}, \quad \tilde{h} = \frac{\pi^2 \gamma H}{8H_{c2}(0)}, \quad \eta = \frac{H - H_{c2}(T)}{H_{c2}(T)}, \quad \gamma = 1.78\dots$$

Away from T_c

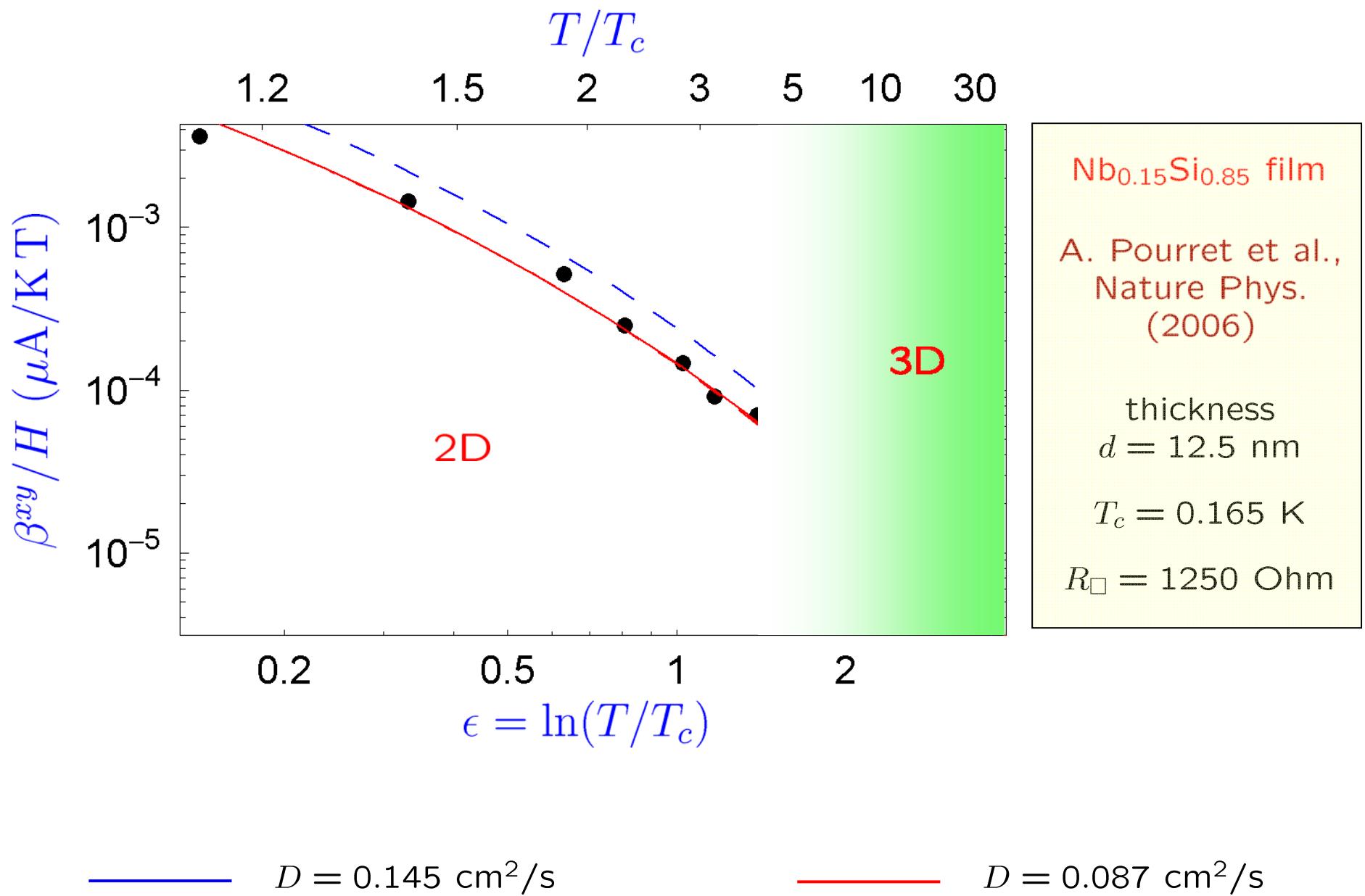


Relative value of the effect:

$$\frac{\beta^{xy}}{\beta_n^{xy}} = \frac{l}{d} \left(\frac{1}{T\tau} \right)^2 = \frac{l}{d} \left(\frac{L_T}{l} \right)^4$$

$\sim 10^3 \dots 10^5$ for NbSi

Comparison with experiment



Comparison with other approaches

Heat current in TDGL (1)

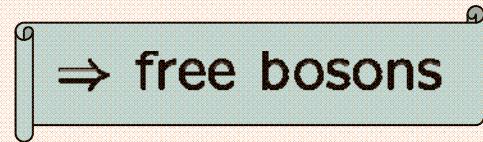
$$\Gamma \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = -\frac{\delta F}{\delta \Psi^*(\mathbf{r}, t)} + \zeta(\mathbf{r}, t), \quad \langle \zeta(\mathbf{r}, t) \zeta^*(\mathbf{r}', t') \rangle = 2\Gamma T \delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

Phenomenology

A. Schmid (1966)

$$\mathbf{j}^e = -i \frac{(2e)}{2(2m)} \Psi^* \nabla \Psi + \text{c.c.}$$

$$\mathbf{j}^Q = -\frac{1}{2(2m)} \frac{\partial \Psi^*}{\partial t} \nabla \Psi + \text{c.c.}$$



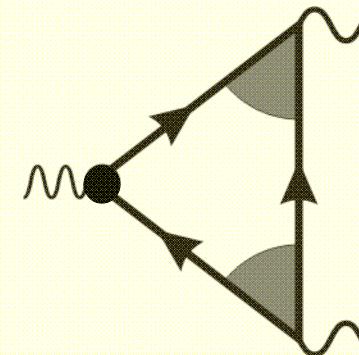
Microscopic derivation

C. Caroli, K. Maki (1967)

M. Yu. Reizer, A. V. Sergeev (1994)

$$\mathbf{j}^e = -i \frac{\pi e \nu D}{4T} \Delta^* \nabla \Delta + \text{c.c.}$$

$$\mathbf{j}^Q = -\frac{\pi \nu D}{4T} \frac{\partial \Delta^*}{\partial t} \nabla \Delta + \text{c.c.}$$

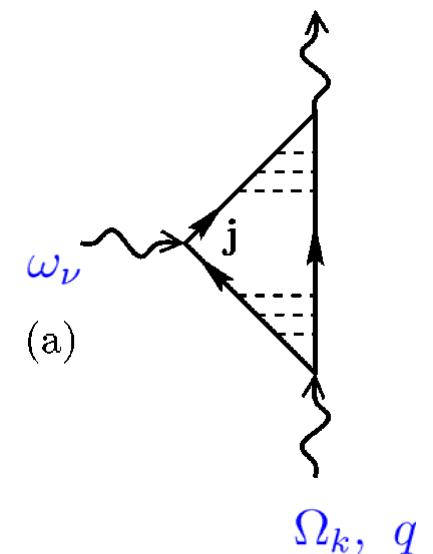


Heat current in TDGL (2)

The result for the electric current vertex \mathbf{J}^e is well known (and is needed, e.g., for the conductivity¹³). Our main concern here is with the heat current vertex \mathbf{J}^Q , for which we establish the following result: At $Q = \Omega_m = 0$ [for conventions regarding incoming and outgoing energies and momenta, see Fig. 3(a)], the electric and heat current vertices are related by

I. Ussishkin,
Phys. Rev. B 68,
024517 (2003)

$$\mathbf{J}^Q = -\frac{i\omega_m}{2e}\mathbf{J}^e. \quad (21)$$



$$B^{(Q)} = -\frac{i}{2e} \left(\Omega_k + \frac{\omega_\nu}{2} \right) B^{(e)}$$

$$\tilde{\beta}_{\text{Uss}}^{xy} \propto 1$$

$$\begin{aligned} B^{(Q)RR} &= -\frac{i}{2e}(\Omega_k + \omega_\nu)B^{(e)} \\ B^{(Q)RA} &= -\frac{i}{2e}(2\Omega_k + \omega_\nu)B^{(e)} \\ B^{(Q)AA} &= -\frac{i}{2e}(\Omega_k)B^{(e)} \end{aligned}$$

$$\tilde{\beta}^{xy} \propto 2$$

Kinetic-equation approach

K. Michaeli and A. M. Finkelstein, arXiv:0812.4268

- Quantum kinetic equation instead of Kubo-Matsubara
- Derivation of the magnetization term
- Similar cancellations at $T \rightarrow 0$ and far above the transition
- Coincides with our asymptotics up to various factors $O(1)$
- Reproduces the TDGL result close to T_c

Conclusions

- Microscopic theory of the Nernst effect at arbitrary T and H above the transition line
- Giant Nernst signal due to fluctuating Cooper pairs well above the transition line

Open problems

- Consistency between Matsubara-Kubo, TDGL and kinetic-equation approaches